I have read the paper “Short retractions of CAT(1) spaces” by Lytchak and Petrunin submitted to the Proceedings of AMS. This is a very good paper and I strongly recommend it be accepted for publication.

It generalizes to the $CAT(1)$ case a well known result that in a $CAT(0)$ space nearest point projection to a closed convex subset is 1-Lipschitz. Despite the latter result being quite easy and well-known the generalization to the $CAT(1)$ case is not trivial. In order to prove it the authors develop a theory of gradient curves of time dependent semiconcave functions on $CAT(K)$ spaces and use it to prove the main result. The theory can be of use for other future applications. The authors also show that there main result implies that for a $CAT(1)$ space U of radius at most $\pi/2$ the product $U \times U$ admits a 1-Lipschitz retraction to the diagonal. This can be used to derive existence and uniqueness of various analytic problems in $CAT(1)$ spaces. While all those analytic applications are already known they follow very quickly and easily from the main results of this paper.

The paper is written well and I only have a few small comments.

(1) I find the notations for closed and open balls somewhat odd. Closed balls are denoted as $\bar{B}[p, r]$ and open ones as $B(p, r)$. Is there a reason why the first notation uses square brackets while the second uses round brackets?
(2) on page 2 before Theorem 1.3 the authors mention that L^p spaces are 1-complemented. This doesn't strike me as obvious. I would like an explanation or a reference added here.
(3) bottom of page 2: The authors claim that under the assumption $|p - x| < \pi/2$ one gets uniqueness in (a)-(d) using Corollary 1.2. I don't understand why one gets uniqueness in (c). It seems wrong to me. The argument they give using Corollary 1.2 does not produce a conformal map. Please clarify this point.
(4) page 5: in the third bullet point the authors claim that “the family B_t is decreasing in t”. They should say a few words about why that's true.
(5) page 5, line 4 after the third bullet point “given a point $x \in B(p, \pi)$...” it should be clarified that the ball is taken in U not W.
(6) page 7, line 9: “the restriction of f_t to Ω' is semiconcave” should be “the restriction of f_t to Ω' is λ-concave”
(7) page 8, last sentence of proof of A.2: “Therefore the first inequality implies the second one.” It's rather unclear which inequalities are

meant here. The authors should clarify that they mean the first two dispalyed inequalities in the statement of A.2. Perhaps give these inequalities labels.

(8) page 10: The authors sketch an alternate proof of Theorem 1.1. They explain how to construct a short retraction Φ but don't explain why the map they've constructed is short. They end the paper by saying that this can be done by straightforward calculations. However, it's not at all obvious how to prove that Φ is short. I feel that if the authors want to include a sketch of an alternate proof of Theorem 1.1 they should provide enough explanation so that a reader could fill in details themselves. I don't find that the current explanation meets that requirement and would like the authors to expand it.