Assume that we have three points:

x	$x^{(1)}$	$x^{(2)}$	$x^{(3)}$
y	$y^{(1)}$	$y^{(2)}$	$y^{(3)}$

and we want to draw a line

(1) $$y = w_0 + w_1 x$$

minimizing the sum of squared errors

$$L = \left(y^{(1)} - w_0 - w_1 x^{(1)}\right)^2 + \left(y^{(2)} - w_0 - w_1 x^{(2)}\right)^2 + \left(y^{(3)} - w_0 - w_1 x^{(3)}\right)^2.$$

This is a quadratic function; therefore, it has a unique minimum.

Problem 1. Calculate the derivatives:

$$\frac{\partial L}{\partial w_0} =$$

$$\frac{\partial L}{\partial w_1} =$$

Factor out w_0 and w_1 and equate the derivatives to zero:

$$\frac{\partial L}{\partial w_0} =$$

$$\frac{\partial L}{\partial w_1} =$$

Divide the equations by 6 and rewrite them using the following notation: $\overline{x} = \dfrac{x^{(1)} + x^{(2)} + x^{(3)}}{3}$, $\overline{y} = \dfrac{y^{(1)} + y^{(2)} + y^{(3)}}{3}$,
$\overline{x^2} = \dfrac{x^{(1)2} + x^{(2)2} + x^{(3)2}}{3}$, $\overline{xy} = \dfrac{x^{(1)}y^{(1)} + x^{(2)}y^{(2)} + x^{(3)}y^{(3)}}{3}$,

$Eq.1:$

$Eq.2:$

Solve the equations for w_0 and w_1

$w_0 =$

$w_1 =$

Problem 2. Given the following data

x	1	2	3
y	1	$\frac{3}{2}$	$\frac{5}{2}$

Calculate weights and plot the line (1)

$w_0 =$

$w_1 =$