Homework 7: Invariants and Geometric Constructions

Konstantin Miagkov

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1 Homework

Problem 1.
n positive integers are written around a circle. Each minute we write the gcd of each pair of consecutive numbers between them, and then erase all the original numbers. Show that eventually all numbers will become equal.

Problem 2.
Given a line ℓ and points A_0, B_0 not on ℓ, reconstruct a triangle $\triangle ABC$ such that points A, B lie on ℓ and AA_0 and BB_0 are altitudes of $\triangle ABC$. It is guaranteed that such a triangle exists.
Hint: Show that points A, B, A_0, B_0 must lie on a circle with the center on ℓ, and find the center of this circle.