Lesson 7: Quadratic Equations VI

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1 From Last Week

Problem 1.
Let $f(x) = ax^2 + bx + c$ be a quadratic function with $a > 0$, and set $h = -b/(2a)$. Show that the f is decreasing from $-\infty$ to h and increasing from h to $+\infty$. In other words, show that for all x, y such that $x < y < h$ we have $f(x) > f(y)$, and for all x, y with $h < x < y$ we get $f(x) < f(y)$. What happens when $a < 0$? *Hint: complete the square!*

Problem 2.
Show that a quadratic equation f with two distinct real roots x_0, x_1 is an even function if and only if $x_0 = -x_1$.

Problem 3.
In a triangle ABC we have $\angle ACB = 135°$. Let $ABMN$ be a square lying to the opposite side of C with respect to AB, and let O be the intersection of its diagonals. If $AM = 12$, find OC.

2 New Problems

Problem 4.
Let $ax^2 + bx + c = 0$ be a quadratic equation with $a > 0$ and $c < 0$. Show that this equation has two distinct real roots.

Problem 5.
a) Let $f(x) = ax^2 + bx + c$ be a quadratic function. Show that if $f(y) = f(z)$ for some pair of real numbers $y \neq z$, then $y + z = -b/a$.

b) Show the converse: if $y + z = -b/a$, then $f(y) = f(z)$. If you completed both parts of the problem, congratulations – you have successfully proven that a parabola (graph of a quadratic function) is symmetric with respect to the line $x = -b/2a$.

Problem 6.
Let $ABCD$ be an inscribed quadrilateral. The angle between the lines AB and CD is $15°$, and the angle between the lines AD and BC is $45°$. Find $\angle BAD$.

Problem 7.
Suppose the angle bisectors of $\triangle ABC$ intersect its circumcircle at points A_1, B_1, C_1. Show that $AA_1 \perp B_1C_1$.