

Lesson 4: Probability III

Anton Lykov

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Definition 1 (Random Variable - informal).

A random variable is a quantifiable experiment. That is, an experiment with numerical outcomes. Random variables are usually denoted by capital letters (X, Y, Z, etc.) For example, we could define random variables X = the number of heads after tossing 5 fair coins.

Definition 2 (Random Variable - formal).

Let Ω be a sample space, and \mathbf{P} be a Probability Function on Ω . A **random variable** X is a function $X : \Omega \rightarrow \mathbb{R}$.

Definition 3 (Probability Mass Function).

The **probability mass function** of X , denoted $p_X : \mathbb{R} \rightarrow [0, 1]$ is defined by:

$$p_X(x) = \mathbf{P}(X = x) = \mathbf{P}(\{X = x\}) = \mathbf{P}(\{\omega \in \Omega : X(\omega) = x\})$$

Definition 4 (Expected Value).

The **expected value** of a random variable X is defined as:

$$\sum_{x \in \mathbb{R}} xp_X(x)$$

1 Introductory problems

Problem 1.

Let X be a discrete random variable on a sample space Ω . Prove that $\sum p_X(x) = 1$.

2 Advanced problems

Problem 2.

Suppose you flip a biased coin that lands heads with probability p . Let X be the number of heads in 3 throws. Find the probability mass function of X and $\mathbb{E}(X)$.