Lesson 4: Stars and Bars

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April 27, 2019

Problem 1 (Binomial formula)**.**
Show that

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}ab^{n-1} + b^n$$

Problem 2.
Given an exam with three problems, how many ways are there to assign point values to each problem so that the whole exam adds up to 100 points?

Problem 3.
Find the number of ways to write a positive integer n as an ordered sum of k positive integers. Here "ordered" means that $3 = 1 + 2$ and $3 = 2 + 1$ would be different representations of 3 as a sum of 2 numbers.
a) Simply show us the correct formula, no proof needed.

b) Now prove your formula.

Hint: First, pick some small values of n and k, and compute the answer in those specific examples. Use these examples to formulate a conjecture about how the answer depends on n and k (it might be useful to look for the answers you get in small cases in the Pascal's triangle). Once you have a formula which seems to work in specific cases, try to find a general proof for it.

Problem 4.
a) Find a number which occurs in the Pascal's triangle at least 4 times.

b) Find a number which occurs in the Pascal's triangle at least 5 times.

Problem 5.
Let AA_1 and BB_1 be altitudes in a triangle $\triangle ABC$. Show that $CA_1 \cdot CB = CB_1 \cdot CA$.

Problem 6.
Let K and N be points on the sides AB and AC of the triangle $\triangle ABC$ such that $AK = KB$ and $AN = 2NC$. Let P be the intersection of NK with the median AM. Find the ratio AP/PM.