Homework 3 Problem 2 Solution

Konstantin Miagkov

October 28, 2018

Problem 2.
Let I be the intersection of diagonals of $ABCD$, which is incidentally also
the center of the inscribed circle. Let E be the point at which the inscribed
circle is tangent to AB, and F – the point of tangency to AD. Then note
that $AE = AF$ and $IE = IF$. Then $\triangle AEI = \triangle AFI$, which in turn implies
$\angle BAI = \angle DAI$. Similarly, we can show that $\angle BCI = \angle DCI$. The two angle
equalities imply that $\triangle ABC = \triangle ADC$, which means that $AB = CD$ and
$AD = BC$. Now we can remember that $AB + CD = AD + BC$, and we get
$AB = CD = AD = BC$ which means that $ABCD$ is a rhombus.