Lesson 2: Set theory and Probability

Anton Lykov

October 8, 2019

Problem 1.
Let $A = \{57, 91, 179, 239\}, B = \{91, 239, 2014\}, C = \{2, 57, 239, 2014\}, D = \{2, 91, 2014, 2017\}$ and E is the set of all flying crocodiles. Find the following:

1. $A \cap B$
2. $(A \cap B) \cup D$
3. $C \cap (D \cap B)$
4. $(A \cup B) \cap (C \cup D)$
5. $(A \cap B) \cup (C \cap D)$
6. $(D \cup A) \cap (C \cup B)$
7. $(A \cap (B \cap C)) \cap D$
8. $(A \cup (B \cap C)) \cap D$
9. $(C \cap A) \cup ((A \cup (C \cap D)) \cap B)$
10. Is $A \subset B$ true?
11. Is $E \subset A$ true?

Problem 2.
Calculate the probability of flipping a fair coin three times and getting 2H and 1T (in any order).

Problem 3.
Calculate the probability of randomly rearranging tiles with the letters of FORMULA and getting a word starting with a vowel.

Problem 4.
Calculate the probability of rolling a six-sided three times and getting a sum of three outcomes equal to 10.

Problem 5.
For any sets A, B, prove all of the following:

1. $P(\varnothing) = 0$

2. $0 \leq P(A) \leq 1$

3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4. $P(A^c) = 1 - P(A)$

Problem 6.
In a bucket, there are m white and $(n-m)$ balls. Each turn, Michael and Peter take one ball from the bucket, Michael starts. The one who takes the white ball wins. Find the probability of Peter winning this game if:

1. $n = 5, m = 1$

2. $n = 7, m = 2$

3. n, m - any natural numbers (find a formula of n and m).

Problem 7.
Suppose that birthdays are equally distributed between all months of the year.

1. Find the probability that in a group of n people, two share the same birthday month.

2. Find n so that this probability is $\geq 50\%$.

Problem 8.
You have two congruent cardboard triangles, and one of their angles is equal to α. Place them on the plane in such a way, that three vertices form an angle equal to $\alpha/2$. Using any instruments, including a pencil, is not allowed!