Lesson 9: Game

Anton Lykov & Konstantin Miagkov

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Problem 1.
Magnus played in a chess tournament with 20 games, and earned 12.5 points. How many more wins than losses did Magnus have? In a chess tournament, you earn 1 point for a win, 0.5 for a draw, and 0 for a loss.

Problem 2.
Amy makes two 4-digit numbers using each of the digits 1,2,3,4,5,6,7, and 8 exactly once. If Amy makes the numbers so that adding them gives the smallest possible total, what is the total?

Problem 3.
Mike has 130 details. He can build a toy windmill using 5 details, a ship using 7 details, and a plane using 14 details. A plane costs 19 coins, ship - 8 coins, and windmill - 6 coins. What is the largest amount of coins Mike can earn?

Problem 4.
Find a 10-digit number where the first digit is equal to how many 0's are in the number, the second digit is equal to how many 1's are in the number, the third digit is equal to how many 2's are in the number, all the way up to the last digit, which is equal to how many 9's are in the number.

Problem 5.
Mark 6 points on the plane such that for each point there would be exactly three other points at a distance 1 from it.

Problem 6.
In the $\triangle ABC$ the median BM is perpendicular to the angle bisector AD. If $AC = 12$, find AB.

Problem 7.
Find a point (x, y) that satisfies both $y = x^3 + 3x^2 + 1$ and $y = x^3 + 2x^2 + 2x$.

Problem 8.
Find the smallest positive integer n such that any positive integer a has the same last digit as a^n.

Problem 9.
Let x and y denote the real roots of the equation $x^2 - 3^{2011}x + 3^{4020} = 0$. Find

$$\log_3\left(\frac{x^3 + y^3}{2}\right)$$

Problem 10.
The IQs of Jon (21), John (22), and Johhn (23) follow a quadratic function $f(x) = -45x^2 + bx + c$, where x represents the age in years after 20 years old. So for example, John's IQ at 22 years old is $f(2)$. Given that John's IQ is 130 and Johhn's IQ is 100, what is Jon's IQ?

Problem 11.
Find all real values of m for which the equations $mx - 1000 = 1001$ and $1001x = m - 1000x$ have a common root.

Problem 12.
Put the following numbers in the increasing order: $2^{222}, 2^{22^2}, 222^2, 2^{2^{2^2}}, 2^{2^{22}}, 22^{22}, 22^{2^2}$.

Problem 13.
What's the biggest number of non-overlapping 1×4 tiles that can be placed in a 6×6 square?

Problem 14.
Jon chose with three distinct nonzero digits. John added up all two-digit numbers using only those three digits, and his result was 231. What digits did Jon choose?

Problem 15.
Some squares of the 100×100 board have stones on them. For every stone on the board, either its row or its column contain exactly one stone. What is the maximal possible number of stones on the board?

Problem 16.
You know that $a + b + c = 7$ and

$$\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{a+c} = 0.7$$

Find

$$\frac{c}{a+b} + \frac{a}{b+c} + \frac{b}{a+c}$$