# Lesson 2: Vieta's formula

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**Definition 1.**
We say that $x_0$ is a root of a function $f(x)$ if $f(x_0) = 0$.

**Problem 1.**
**a)** Let $ax^2 + bx + c = 0$ be a quadratic equation. Show that if it has two distinct real roots $x_0, x_1$, then $ax^2+bx+c = a(x-x_0)(x-x_1)$. Hint: consider the difference between $ax^2 + bx + c$ and $a(x - x_0)(x - x_1)$. What degree is it? How many roots does it have?

**b)** Show that a quadratic equation cannot have more than two distinct real roots.

**c)** Now suppose that $ax^2 + bx + c$ has exactly one real root $x_0$. Show that $ax^2 + bx + c = a(x - x_0)^2$.

**Problem 2.**
**a)** [Vieta's formulas] Consider a quadratic equation $ax^2 + bx + c = 0$ with real roots $x_0, x_1$. Show that

$$x_0 + x_1 = -b/a$$

$$x_0 x_1 = c/a$$

These are called *Vieta's formulas*. You may use the result of problem 1 even if you did not solve it.

**b)** Given any two real numbers $x_0, x_1$ with $x_0 + x_1 = u$ and $x_0 x_1 = v$, show that both $x_0$ and $x_1$ are roots of the quadratic equation $x^2 - ux + v = 0$.

**Problem 3.**
**a)** Let $x_0, x_1$ be roots of a quadratic equation $ax^2 + bx + c = 0$. Find the formula for $x_0^2 + x_1^2$ in terms of $a, b, c$.

**b)** Let $x_0, x_1$ be roots of $x^2 + bx + c = 0$. Find the formula for $x_0^3 + x_1^3$ in terms of $b, c$.

**Problem 4.**
Consider a circle whose diameter is the side $AB$ of the triangle $ABC$. Show that if that circle contains the midpoint of $AC$, then $\triangle ABC$ is isosceles.

**Problem 5.**
Let $AC$ be a diameter of a circle, and $B$ be a point on the circle distinct from $A$ and $C$. Let $P$ be the foot of the perpendicular from $A$ to the tangent to the circle at $B$. Show that $AB$ is the angle bisector of $\angle PAC$.