Lesson 8: Quadratic Equations Mix

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December 2, 2018

Read all the problems before starting to solve any!

Problem 1.
Prove that if b is any real number and $a \neq 0$, then the quadratic equation $ax^2+bx-a=0$ has two distinct real roots.

Problem 2.
$f(x) = x^2 + px + q$ is an even function. Find (with proof) the value of p.

Problem 3.
Points A, B, C lie on a circle of radius r, and $AC = r$. Find $\angle ABC$.
Hint: Is there only one solution?

Problem 4.
a) Find p and q, if 10 and -15 are the roots of the quadratic equation $x^2 + px + q$.

b) Sum of squares of roots of a quadratic equation $x^2 + px - 3$ is equal to 10. Find p.

Problem 5.
a) What is the minimum value of the function $f(x) = x^2 + 10x - 13$?

b) What is its maximum value?

Problem 6.
Let O be the center of a circle and A be a point outside of it. Let AM and AN be tangents to the circle, and let B be the intersection of line AO with the circle furthest from A. Given that $AN \parallel BM$, find $\angle OAM$ and prove that $AMBN$ is a rhombus.