Lesson 2: Set theory and Probability

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Problem 1.
Let $A = \{57, 91, 179, 239\}, B = \{91, 239, 2014\}, C = \{2, 57, 239, 2014\}, D = \{2, 91, 2014, 2017\}$ and E is the set of all flying crocodiles. Draw the Venn diagram and find the following sets:

1. $A \cap B$
2. $(A \cap B) \cup D$
3. $C \cap (D \cap B)$
4. $(A \cup B) \cap (C \cup D)$
5. $(A \cap B) \cup (C \cap D)$
6. $(A \cap (B \cap C)) \cap D$
7. $(A \cup (B \cap C)) \cap D$
8. $(C \cap A) \cup ((A \cup (C \cap D)) \cap B)$
9. Is $A \subset B$ true?
10. Is $E \subset A$ true?

Problem 2.
Calculate the probability of flipping a fair coin three times and getting 2H and 1T

1. In this order
2. In any order

Problem 3.
Calculate the probability of randomly rearranging tiles with the letters of FORMULA and getting a word starting with a vowel.

Problem 4.
Calculate the probability of rolling a six-sided die three times and getting a sum of three outcomes equal to 10.

Problem 5.
Let $\Omega = \{\omega_1, \omega_2, ..., \omega_n\}$ be a sample space. Each ω_i is called an **elementary (atomic) event**. Recall from the lecture that probability function P satisfies 2 axioms:

1. $P(\omega) \geq 0$ for any $\omega \in \Omega$
2. $P(\omega_1) + P(\omega_2) + ... + P(\omega_n) = 1$

Probability of any **event** $A = \{\omega_{k_1}, \omega_{k_2}, ..., \omega_{k_m}\} \subset \Omega$ is defined as the sum of probabilities of its elementary events:

$$P(A) = P(\omega_{k_1}) + P(\omega_{k_2}) + ... + P(\omega_{k_m})$$

For any subsets $A, B \subset \Omega$, prove the following:

1. $P(\varnothing) = 0$
2. $0 \leq P(A) \leq 1$
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
4. $P(A^c) = 1 - P(A)$

Problem 6.
In a bucket, there are m white and $(n - m)$ black balls. Each turn, Michael and Peter take one ball from the bucket, Michael starts. The one who takes the white ball first wins. Find the probability of Peter winning this game if:

1. $n = 5, m = 1$
2. $n = 7, m = 2$
3. n, m - any natural numbers (find a formula of n and m).

Problem 7.
Suppose that birthdays are equally distributed between all months of the year.

1. Find the probability that in a group of n people, two share the same birthday month.
2. Find the smallest n so that this probability is $\geq 50\%$.

Problem 8.
You have two congruent cardboard triangles, and one of their angles is equal to α. Place them on the plane in such a way, that three vertices form an angle equal to $\alpha/2$. Using any instruments, including pencils, is not allowed!