Lesson 2: Weighings, Logic and Geometric Constructions

Konstantin Miagkov

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Problem 1.
There are six coins on the table, one of them is fake and weighs differently than the real ones. Show how to determine the fake coin using a scale at most 3 times. In this problem, the scale simply shows the weight of all coins on it in ounces.

Problem 2.
Out of five coins three coins three are real and two are fake. Both fake coins weigh the same, but their weight is different from the weight of the real coins. What is the least number of times you have to use the balance scale to guarantee that you can find at least one real coin?

Problem 3.
A square is split into 100 rectangles using 9 vertical and 9 horizontal lines. Exactly 9 of those rectangles are squares – show that two of those squares have the same side length.

Problem 4.
Suppose you have five positive integers, and you computed all ten of their pairwise sums. Is it possible that the ten pairwise sums all have different last digits?

In the following two problems, "construct" means "construct using straightedge and compass".

Problem 5.
a) Given a line ℓ and a point P on this line, show how to construct a line through P perpendicular to ℓ.

b) Do the same if the point P is not on ℓ.

Problem 6.
Given a line ℓ and a point P not on ℓ, show how to construct a line through P parallel to ℓ.