

# Lesson 7: Quadratic Equations VI

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## 1 From Last Week

### Problem 1.

Let  $f(x) = ax^2 + bx + c$  be a quadratic function with  $a > 0$ , and set  $h = -b/(2a)$ . Show that the  $f$  is decreasing from  $-\infty$  to  $h$  and increasing from  $h$  to  $+\infty$ . In other words, show that for all  $x, y$  such that  $x < y < h$  we have  $f(x) > f(y)$ , and for all  $x, y$  with  $h < x < y$  we get  $f(x) < f(y)$ . What happens when  $a < 0$ ? *Hint: complete the square!*

### Problem 2.

Show that a quadratic equation  $f$  with two distinct real roots  $x_0, x_1$  is an even function if and only if  $x_0 = -x_1$ .

### Problem 3.

In a triangle  $ABC$  we have  $\angle ACB = 135^\circ$ . Let  $ABMN$  be a square lying to the opposite side of  $C$  with respect to  $AB$ , and let  $O$  be the intersection of its diagonals. If  $AM = 12$ , find  $OC$ .

## 2 New Problems

### Problem 4.

Let  $ax^2 + bx + c = 0$  be a quadratic equation with  $a > 0$  and  $c < 0$ . Show that this equation has two distinct real roots.

### Problem 5.

a) Let  $f(x) = ax^2 + bx + c$  be a quadratic function. Show that if  $f(x_0) = f(x_1)$  for some pair of real numbers  $x_0 \neq x_1$ , then  $x_0 + x_1 = -b/a$ .

b) Show the converse: if  $x_0 + x_1 = -b/a$ , then  $f(x_0) = f(x_1)$ . If you completed both parts of the problem, congratulations – you have successfully proven that a parabola (graph of a quadratic function) is symmetric with respect to the line  $x = -b/2a$ .

### Problem 6.

Let  $ABCD$  be an inscribed quadrilateral. The angle between the lines  $AB$  and  $CD$  is  $15^\circ$ , and the angle between the lines  $AD$  and  $BC$  is  $45^\circ$ . Find  $\angle BAD$ .

### Problem 7.

Suppose the angle bisectors of  $\triangle ABC$  intersect its circumcircle at points  $A_1, B_1, C_1$ . Show that  $AA_1 \perp B_1C_1$ .