Lesson 8: Miscellaneous

Konstantin Miagkov

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Problem 1.
Find all values of a for which the quadratic equation $x^2 - ax + a - 1$ has the smallest possible sum of squares of roots.

Problem 2.
A placement of 8 queens on a standard 8×8 chess board is called *correct* if no two queens are in the same row, column or on the same diagonal. Prove that the number of correct placements of 8 queens is even.

Problem 3.
Find all real numbers a such that $an(n + 2)(n + 4)$ is an integer for any integer n.

Problem 4.
2019 integers are written on the board. The sum of any three integers on the board is also on the board. What is the least possible number of zeroes out of the 2019 integers?

Problem 5.
Find all pairs $q < p$ of prime numbers such that $2p - 1$ is divisible by q and $2q - 1$ is divisible by p.

Problem 6.
I have a $3 \times n$ board, with green, red, and blue chips on it. There are n chips of each color. Prove that is it possible to rearrange the chips on each row of the board so that in each column there is exactly one chip of each color.

Problem 7.
Bran chose $n + 1$ numbers out of the integers from 1 to $2n$. Show that one of the chosen numbers is divisible by another chosen number.