Lesson 9: Game

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Problem 1.
Magnus played in a chess tournament with 20 games, and earned 12.5 points. How many
more games did he win, than lose? In a chess tournament, you earn 1 point for a win,
0.5 for a draw, and 0 for a lose.

Problem 2.
Amy makes two 4-digit numbers using each of the digits 1,2,3,4,5,6,7, and 8 exactly once.
If Amy makes the numbers so that adding them gives the smallest possible total, what
is the total?

Problem 3.
Mike has 130 details. He can build a toy windmill using 5 details, a ship using 7 details,
and a plane using 14 details. A plane costs 19 coins, ship - 8 coins, and windmill - 6
coins. What is the largest amount of coins Mike can earn?

Problem 4.
Find a 10-digit number where the first digit is equal to how many 0's are in the number,
the second digit is equal to how many 1's are in the number, the third digit is equal to
how many 2's are in the number, all the way up to the last digit, which is equal to how
many 9's are in the number.

Problem 5.
Mark 6 points on the plane such that for each point there would be exactly three other
points at a distance 1 from it.

Problem 6.
Find a point (x, y) that satisfies both $y = x^3 + 3x^2 + 1$ and $y = x^3 + 2x^2 + 2x$.

Problem 7.
Find the smallest positive integer n such that any positive integer a has the same last
digit as a^n.

Problem 8.
Let x and y denote the real roots of the equation $x^2 - 3^{2011}x + 3^{4020} = 0$. Find

$$\log_3 \left(\frac{x^3 + y^3}{2} \right)$$

Problem 9.
The IQs of Jon (21), John (22), and Johhn (23) follow a quadratic function
$f(x) = -45x^2 + bx + c$ where x represents the age in years after 20 years old. So for
example, John's IQ at 22 years old is $f(2)$. Given that John's IQ is 130 and Johhn's IQ
is 100, what is Jon's IQ?