

# Homework 2: Combinations and Pascal's Triangle

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## 1 Homework

### Problem 1.

How many ways are there to arrange 2 green, 2 red and 2 blue balls in a row so that not two balls of the same color are adjacent to each other?

### Problem 2.

Let  $T$  be a point inside a circle, and  $A, B, C, D$  be points on the circle such that lines  $AB$  and  $CD$  intersect at  $T$ . Show that  $AT \cdot TB = CT \cdot TD$ .

## 2 Reading

### Solution 1 (L2.3).

*If the ratios of three binomial coefficients are*

$$\binom{n+1}{m+1} : \binom{n+1}{m} : \binom{n+1}{m-1} = 5 : 5 : 3$$

*find  $n, m$ . Since*

$$\binom{n+1}{m+1} : \binom{n+1}{m} : \binom{n+1}{m-1} = 5 : 5 : 3$$

*we know that*

$$\binom{n+1}{m+1} = \binom{n+1}{m}$$

Writing it out we get

$$\frac{(n+1)!}{(m+1)!(n-m)!} = \frac{(n+1)!}{(m)!(n+1-m)!}$$
$$(m)!(n+1-m)! = (m+1)!(n-m)!$$

Cancelling out  $m!(n-m)!$  we get  $n+1-m = m+1$  or  $n = 2m$ . Now we use

$$\binom{n+1}{m} : \binom{n+1}{m-1} = 5 : 3$$

to do the same factorial calculations:

$$\frac{3(n+1)!}{m!(n+1-m)!} = \frac{5(n+1)!}{(m-1)!(n+2-m)!}$$

$$3(m-1)!(n+2-m)! = 5m!(n+1-m)!$$

Cancelling out  $(m-1)!(n+1-m)!$  we get  $3(n+2-m) = 5m$ . Plugging in  $n = 2m$  we have

$$3m + 6 = 5m$$

$$m = 3, n = 6$$

and we are done.

**Solution 2** (H2.1).

If we want to descend to the  $n$ -th row of Pascal's triangle, we have to make  $n$  total steps, since every step takes us one row down. At each step we can choose to go left or right, so the number of ways is  $2^n$ .