

Steady state:

$$-(vC - DC')' = 0 \quad (\dot{C} = 0)$$

No flux condition:

$$vC - DC' = 0 \quad \text{at } z \in \{0, H\}$$

( $vC - DC' = J$  is Fick's first law)

By the conservation law:

$$\dot{C} + J' = 0 \quad \overset{\dot{C}=0}{\Rightarrow} \quad J' = 0$$

We know  $J = 0$  at  $z \in \{0, H\}$  but because  $J' = 0$  it must hold for the whole interval. We can hence just solve

$$vC - DC' = 0$$

to get  $C$ .

$$DC' = vC \Rightarrow$$

$$C' = \frac{vC}{D} \Rightarrow$$

$$C = \frac{1}{Z} \exp\left(\frac{v}{D} z\right)$$

$Z$  is the normalization constant which we find by

$$Z = \int_0^H \exp\left(\frac{v}{D} z\right) dz$$

$$= D/v \left( \exp\left(\frac{vH}{D}\right) - 1 \right)$$