Ex1 On page 70 we are given the mean and covariance matrix of a finite dimensional brownian motion. B = (Bt, Bt2, Bt3, ..., Btn) E[B]=(0,0,0,...,0) $\mathbb{E}[\overline{B}B] = \operatorname{Cov}(\overline{B}) = \begin{bmatrix} t_1 & t_2 & \cdots & t_2 \\ \vdots & \vdots & \ddots & \vdots \end{bmatrix}$ to to --- to this is also confirmed by the simulation.

We define $Y_{\varepsilon} = B_{\varepsilon}^{T} - t$ and now we want to show $E[Y_{\varepsilon} \mid \mathcal{T}_{s}] = Y_{s} \text{for} S \notin \mathcal{T}_{\varepsilon}$ $C.f. S. 83 \text{ [in the text. not as thm, pefor luming].}$ We note that $E[Y_{\varepsilon} \mid \mathcal{T}_{s}] = E[B_{\varepsilon}^{T} \mid \mathcal{T}_{s}] - E[t \mid \mathcal{T}_{s}]$ $= E[B_{\varepsilon}^{T} \mid \mathcal{T}_{s}] - t \text{(because t is a const.)}$ We now work with $E[B_{\varepsilon}^{T} \mid \mathcal{T}_{s}]$ $= E[(B_{\varepsilon} - B_{s})^{T} \mid \mathcal{T}_{s}] + E[B_{s}^{T} \mid \mathcal{T}_{s}] + E[2((B_{\varepsilon} - B_{s})B_{s}) \mid \mathcal{T}_{s}]$ $= E[(B_{\varepsilon} - B_{s})^{T} \mid \mathcal{T}_{s}] + E[B_{s}^{T} \mid \mathcal{T}_{s}] + E[2((B_{\varepsilon} - B_{s})B_{s}) \mid \mathcal{T}_{s}]$ $= E[(B_{\varepsilon} - B_{s})^{T} \mid \mathcal{T}_{s}] + E[B_{\varepsilon}^{T} \mid \mathcal{T}_{s}] + E[B_{\varepsilon}^$	Ex 4: Show that {Bt-t} is a martingale.
$E[Y_t \mid \mathcal{T}_s] = Y_s \qquad \text{for} \qquad S \text{ L} $ $C.f. S. 83 \text{ [in the text. not as thm, pefor lumma].}$ $We \text{note that}$ $E[Y_t \mid \mathcal{T}_s] = E[B_t^2 \mid \mathcal{T}_s] - E[t \mid \mathcal{T}_s]$ $= E[B_t^2 \mid \mathcal{T}_s] - t \qquad \text{(because t is a const.)}$ $We \text{now work with } E[B_t^2 \mid \mathcal{T}_s]$ $= E[(B_t - B_s)^2 \mid \mathcal{T}_s] + E[B_s^2 \mid \mathcal{T}_s] + E[2((B_t - B_s)B_s)]\mathcal{T}_s]$ $= E[(B_t - B_s)^2 \mid \mathcal{T}_s] + E[B_s^2 \mid \mathcal{T}_s] + E[2((B_t - B_s)B_s)]\mathcal{T}_s]$ $= E[(B_t - B_s)^2 \mid \mathcal{T}_s] + E[B_t^2 \mid \mathcal{T}_s] + E[2((B_t - B_s)B_s)]\mathcal{T}_s]$ $= E[Y_t \mid \mathcal{T}_s] = (t - S) + B_s^2 + O$ $= E[Y_t \mid \mathcal{T}_s] = (t - S) + B_s^2 - t - B_s^2 - S = Y_s$ $= 2B_s(0) \text{(onstant under \mathcal{T}_s in each 1 and 2 is a constant under \mathcal{T}_s in each 2 in each 2 is a constant under 2 in each 2 is a constant under 2 in each 2 is a constant under 2 in each 2 in$	v
We now work with $E[B_{\epsilon}^{2} \mathcal{F}_{5}]$ $E[B_{t}^{2} \mathcal{F}_{5}] = E[(B_{\epsilon}-B_{s})+B_{s})^{2} \mathcal{F}_{5}]$ $= E[(B_{\epsilon}-B_{s})^{2} \mathcal{F}_{5}] + E[B_{s}^{2} \mathcal{F}_{5}] + E[2((B_{\epsilon}-B_{s})B_{s}) \mathcal{F}_{5}]$ Variance of the $= E[(B_{\epsilon}-B_{s})^{2}] + B_{s}^{2} + 0$ increment is by deft $= (t-s) + B_{s}^{2}$ Thus B_{s} is known under B_{s} is $E[Y_{\epsilon} \mathcal{F}_{5}] = (t-s) + B_{s}^{2} - t - B_{s}^{2} - s = Y_{s}$ $E[Y_{\epsilon} \mathcal{F}_{5}] = (t-s) + B_{s}^{2} - t - B_{s}^{2} - s = Y_{s}$ $= 2B_{s}(0) \leftarrow \text{we know it under } \mathcal{F}_{5}, \dots$ we know it under \mathcal{F}_{5}, \dots	
We now work with $E[B_{\epsilon}^{2} \mathcal{F}_{5}]$ $E[B_{t}^{2} \mathcal{F}_{5}] = E[(B_{\epsilon}-B_{s})+B_{s})^{2} \mathcal{F}_{5}]$ $= E[(B_{\epsilon}-B_{s})^{2} \mathcal{F}_{5}] + E[B_{s}^{2} \mathcal{F}_{5}] + E[2((B_{\epsilon}-B_{s})B_{s}) \mathcal{F}_{5}]$ Variance of the $= E[(B_{\epsilon}-B_{s})^{2}] + B_{s}^{2} + 0$ increment is by deft $= (t-s) + B_{s}^{2}$ Thus B_{s} is known under B_{s} is $E[Y_{\epsilon} \mathcal{F}_{5}] = (t-s) + B_{s}^{2} - t - B_{s}^{2} - s = Y_{s}$ $E[Y_{\epsilon} \mathcal{F}_{5}] = (t-s) + B_{s}^{2} - t - B_{s}^{2} - s = Y_{s}$ $= 2B_{s}(0) \leftarrow \text{we know it under } \mathcal{F}_{5}, \dots$ we know it under \mathcal{F}_{5}, \dots	We note that $ \frac{\mathbb{E}[Y_t \mid \mathcal{F}_s] = \mathbb{E}[B_t^2 \mid \mathcal{F}_s] - \mathbb{E}[t \mid \mathcal{F}_s]}{= \mathbb{E}[B_t^2 \mid \mathcal{F}_s] - t} $ (because t is
$= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[2((B_t - B_s)B_s) \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[2((B_t - B_s)B_s) \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[2((B_t - B_s)B_s) \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)^2 \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right]$ $= \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_s \right] + \mathbb{E} \left[(B_t - B_s)B_s \mathcal{F}_$	
$= 2B_s(0) \leftarrow \text{ we know it under } \mathcal{F}_s \text{ i}$	$= \mathbb{E}\left[\left(B_{t} - B_{s}\right)^{2} \left \mathcal{F}_{s}\right] + \mathbb{E}\left[B_{s}^{2} \left \mathcal{F}_{s}\right] + \mathbb{E}\left[2\left(\left(B_{t} - B_{s}\right)B_{s}\right)\right] \mathcal{F}_{s}\right]$ $= \mathbb{E}\left[\left(B_{t} - B_{s}\right)^{2}\right] + B_{s}^{2} + O\left(B_{t} - B_{s}\right)B_{s}\right] \mathcal{F}_{s}$ increment is by define the convergence of the con
Because we know	
IE[Be-Bs Fs] is an incremeant from Bs to Bution in Bepection	E[B_B_B_S F_s] is an
is O.	

Ex5: (Ex20 in the book) Let X be a random variable such
that E[X] < \infty, and let \(\int_E: E \ge 0 \) be a filtration. Show
that the process {Mt: t=0} given by
$M_t = IE[X \mid \mathcal{F}_t]$
is a martingale, with {Ft; E205.
By clif 4.5.1. a martingale Me is defined by the following 3 properties.
1) The process Mt is adapted to the filtration For all times t20, F[Mt] < 02 3) F[Mt] Fs] = Ms, whenever t2520.
2) for all times £20, [] MEJ < X
3) #[Milts]=11s, Whenever C-5-0.
We start with projects 3). We have
We start with property 3). We have $E[M_t \mathcal{F}_s] = E[E[X_s \mathcal{F}_s]] \mathcal{F}_s] \qquad t \ge s$
from the def. of filtrations we know Fs = Fe and hence the
land of iterated expectations hold (EX)=E[X173)
$\mathbb{E}[\mathbb{E}[\times \mathcal{F}_s] = \mathbb{E}[\times \mathcal{F}_s] = M_s$
Now 2)
$\mathbb{E}[M_t] = \mathbb{E}[\mathbb{E}[X] \times [\mathcal{F}_t]] = \mathbb{E}[X] \times \mathbb{E}[$
Lastly property 1) follows from how we defined Mt.
Hence Mt is a martingale.

Ex6 (Ex4.11) Show that if {Mt: t=0} is a martingale such
that IE[IM_12] < 00 for all t, then the increments
Mt-Ms and Mr-Mn
are uncorrelated whenever 0555±5M±V.
· We first see that given the filtration FE
E[Mv-Mulfi]=E[Mvlfi]-E[Mulfi]
= Mt - Mt (Martingale prop)
= O
Next we can use tower property and get
Next we can use tower property and get E[(Mt-Ms)(Mv-Mn)] = E[E[(Mt-Ms)(Mv-Mn) Ft]]
= E[(Me-Ms) E[Mv-Mul Fe]]
= E[(M&-Ms).O]
= 0
Next show that the variance of increments is additive:
Next show that the variance of increments is additive: V[Mn-Ms] = V[Mn-Mt] + V[Mt+Ms]
· Variance of the sum of two random variables is given by
V[X+Y] = V[X] + V[Y] + Cov[X,Y]
So .
$V[M_n - M_s] = V[(M_n - M_t) + (M_t - M_s)]$
= V[Mn-Me] + V[Me-Ms] + Car[(Mn-Me), (Mt-Ms]
= V[Ma-Me]+V[Me-Ms] (because 90, c.f. first part)
of this exercise
Show that the variance is increasing
· We let 0535t, Hen
V[M=]=V[M=]+V[M=-M=]=V[M=7

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