ex 1.

a)

$\text{conv}(X) = \{(0,6), (0,0), (2,0), (6,3)\}$

b)

$$\max \quad x_1 + 3x_2$$

s.t.

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda_1 \begin{pmatrix} 0 \\ 6 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 2 \\ 0 \end{pmatrix} + \lambda_4 \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$

$$\sum_{i=1}^{4} \lambda_i = 1$$

$$\lambda_i \geq 0 \quad \forall i \in \{1, \dots, 4\}$$

c)

$$\max \quad x_1 + 3x_2$$

s.t.

$$x_2 \geq 0$$

$$x_1 \geq 0$$

$$-3/2 - 3/4\, x_1 + x_2 \geq 0$$

$$6 + 1/2\, x_1 - x_2 \geq 0$$

Ex2

a) See drawing. blue is the new line added to A1.

b) New extreme points of A1: $X_{A_1} = \{(1,1),(2,2),(1,2)\}$
We use Minkowsky-Weyl

$$\max \; x + 2y$$
$$\text{s.t.}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 2 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$
$$\sum_{i=1}^{3} \lambda_i = 1$$
$$\lambda_i \geq 0 \quad \forall i \in \{1,2,3)$$
$$3x + y \leq 13$$
$$-x - 3y \leq -7$$
$$x, y \in \mathbb{Z}$$

$$\Downarrow$$

$$\max \; (\lambda_1 + 2\lambda_2 + \lambda_3) + 2(\lambda_1 + 2\lambda_2 + 2\lambda_3)$$
$$\text{s.t.} \; 3(\lambda_1 + 2\lambda_2 + \lambda_3) + (\lambda_1 + 2\lambda_2 + 2\lambda_3) \leq 13$$
$$-(\lambda_1 + 2\lambda_2 + \lambda_3) - 3(\lambda_1 + 2\lambda_2 + 2\lambda_3) \leq -7$$
$$(\lambda_1 + 2\lambda_2 + \lambda_3) \in \mathbb{Z} \qquad (*)$$
$$(\lambda_1 + 2\lambda_2 + 2\lambda_3) \in \mathbb{Z}$$
$$\sum \lambda_i = 1$$
$$\lambda_i \geq 0$$

c) If we relax $(*)$ and solve it we obtain an upper bound of 6. This is also the optimal solution.

d) The more the better, but there is probably a catch. Probably adding more constraints in the convex hull is very expensive.

Ex 3

a)

	1	2	3	4	5	6	7	8	9	10	11	12
x_1	1				1		1			1	1	
x_2		1			1			1	1		1	1
x_3			1			1		1		1	1	1
x_4				1		1	1		1			1

	λ_1	λ_2	λ_3	λ_4
x_1	1	1	1	1
x_2	0	0	1	0
x_3	1	1	1	0
x_4	0	1	0	0

← feasible if constraint 1 is convexified.

So

$$\max \; (\lambda_1+\lambda_2+\lambda_3+\lambda_4) + (\lambda_3) + (\lambda_1+\lambda_2+\lambda_3) + (\lambda_2)$$

s.t.

$$(\lambda_1+\lambda_2+\lambda_3+\lambda_4) + (\lambda_3) + 4(\lambda_1+\lambda_2+\lambda_3) + (\lambda_2) \leq 3 \qquad (\square)$$

$$\sum_{i\in\{1:4\}} \lambda_i = 1$$

$$\lambda_i \geq 0$$

d) If we convexify constraint 2 all binary combinat are valid except those containing x_3

i.e.

x_1	1	0	0	1	0	1
x_2	0	1	0	1	1	0
x_3	0	0	0	0	0	0
x_4	0	0	1	0	1	1

$(\ldots)$ $\qquad (*)$

If we solve $(\square)$ we obtain a upper bound of 1.8 but the LP corresponding to $(*)$ gives 1 wich is also the optimal solution.