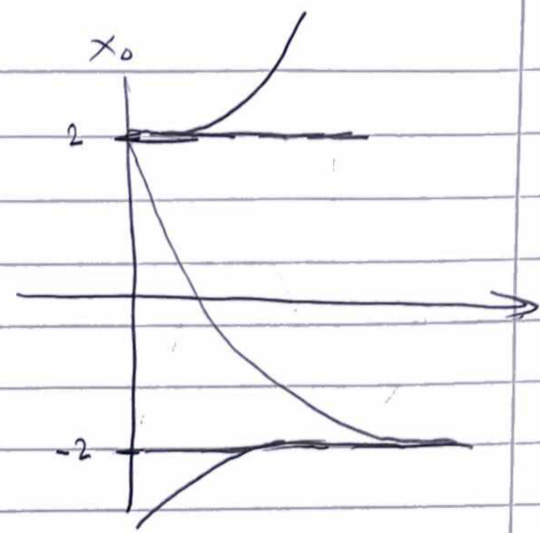
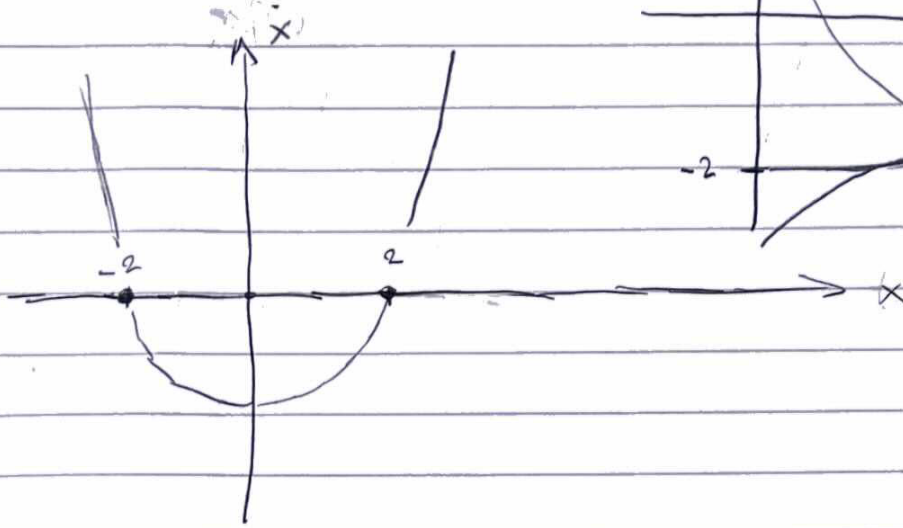
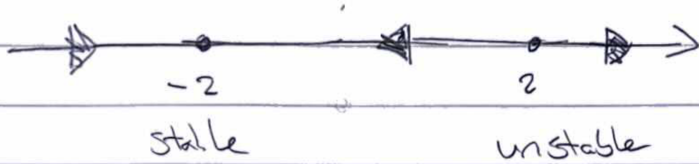
Exercise 2.2.1

$\dot{x} = 4x^2 - 16$



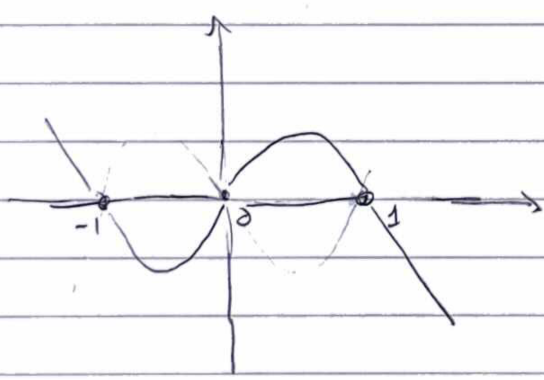


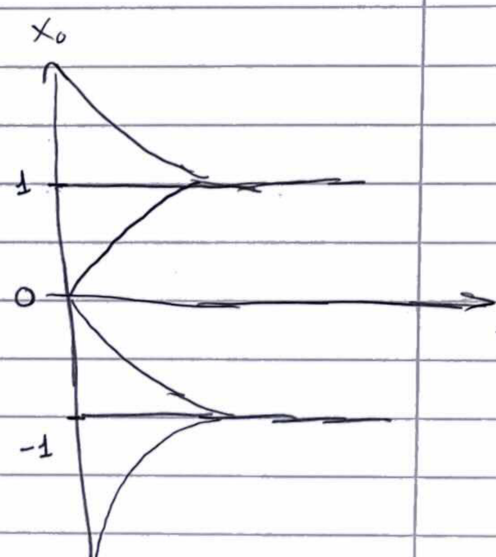


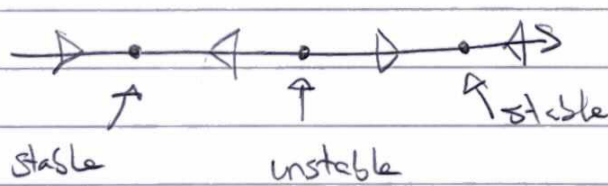
Phase-diagram

Exercise 2.2.3

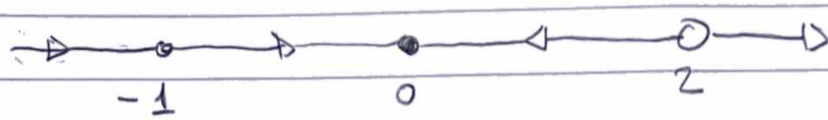
$\dot{x} = x - x^3$

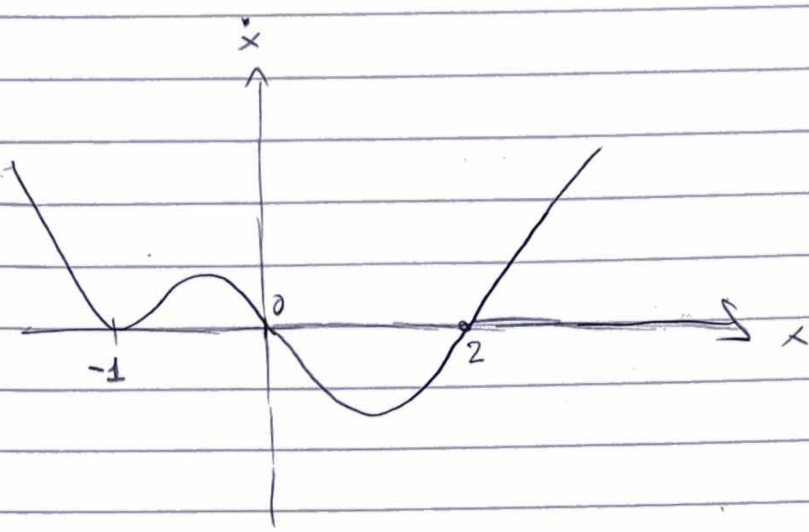






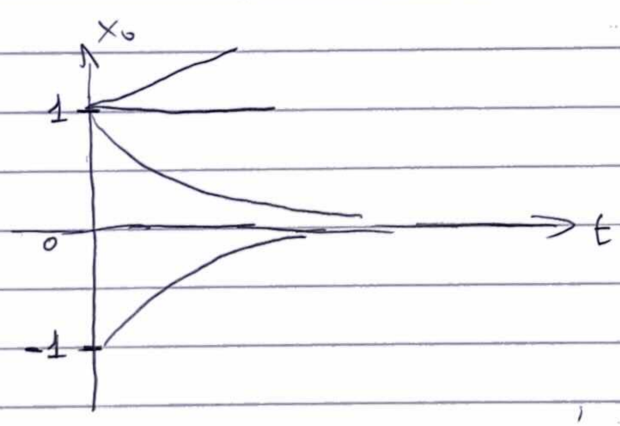
Exercise 2.2.8 (Backwards)

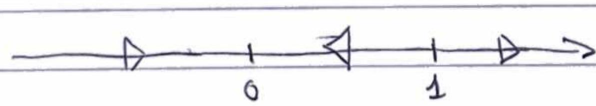




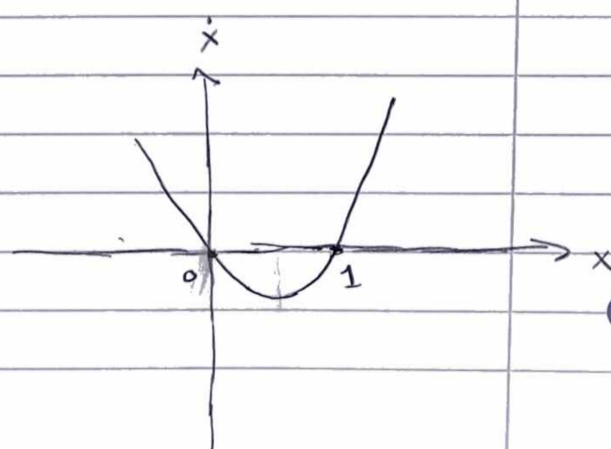
$$\dot{x} = (1+x)^2(-2+x)\,x^2$$

Exercise 2.2.9 (Backwards)





$$\underline{\underline{\dot{x} = (x^2-1)x}}$$



exersice 2

we are given

$$\dot{x}(t) = p\, x(t) \quad , \quad x(0) = x_0$$

(a) find ϕ_t

the solution is $x(t) = x_0 e^{pt}$, so

$$\phi_t(x) = x e^{pt}$$

, i.e. $\phi_0(x) = xe^0$, $\phi_1(x) = xe^p$ e.t.c.

(b) What is $\phi_1(2)$? What is $\phi_2(1)$? What is $\phi_1(x)$? What is $\phi_t(1)$?

$\phi_1(2) = 2 \cdot e^p$, $\phi_2(1) = e^{2p}$, $\phi_1(x) = xe^p$

$\phi_t(1) = e^{tp}$

(c) Draw the 1D phase-potrait for different values of p. Let $J = (a,b)$. Simplify/eval the following set:

$$\phi_t(J) = \{ \phi_t(x) \mid x \in J \}$$

ex2

$a=-2$, $b=2$, $p=1$

$$\phi_t([-2,2]) = [-2,2]e^t$$

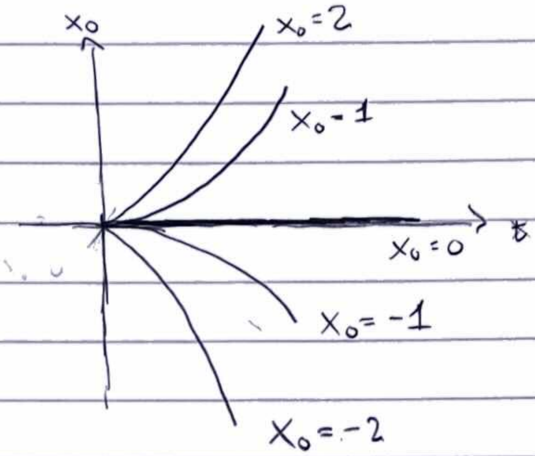
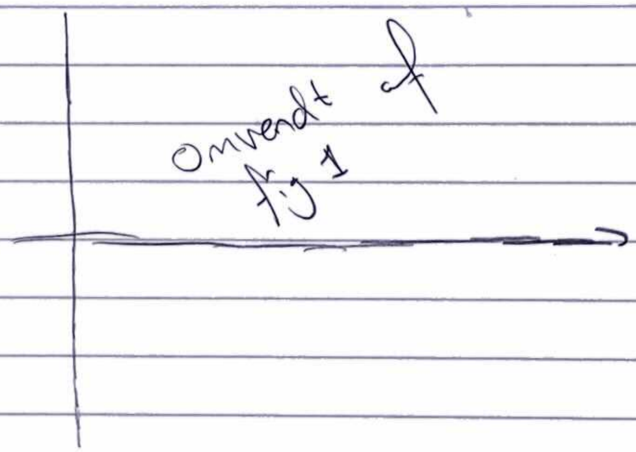


fig 1

$a=-2$, $b=2$ $p=-1$ $\phi_t([a,b]) = [-2,2]e^{-t}$



ex 2

(d) Show that the linear initial value problem defines a dynamical system by verifying each of the ingrediens above with $\phi_t(x)$ as in (a)

We must show (C) in the week note

$$\phi_0(x) = x \quad \wedge \quad \phi_{t+s}(x) = \phi_t \circ \phi_s(x)$$

✓ $\phi_0(x) = x \cdot e^{P \cdot 0} = x$

✓ $\phi_{t+s}(x) = x \cdot e^{P(t+s)}$ $\quad \wedge \quad \phi_t(\phi_s(x)) = \left(x \cdot e^{sP}\right)e^{tP}$

$$= x e^{P(s+t)}$$

(e) What is $(\phi_t)^{-1}$? Compare with ϕ_{-t} and (2) for $s=-t$. Discuss

(first the inverse) $\quad x = \phi_t(\phi_t^{-1}(x)) = (\phi_t^{-1}(x))e^{Pt} \quad \Rightarrow \quad$ [def of inverse $x = f(f^{-1}(x))$

$$x \cdot e^{P(-t)} = \phi_t^{-1}(x)e^{Pt} \cdot e^{P(-t)}$$
$$= \phi_t^{-1}(x)e^{0}$$
$$= \phi_t^{-1}(x)$$

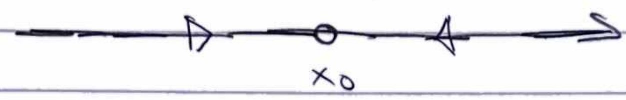
we compare with ϕ_{-t}

$$\phi_{-t}(x) = xe^{P(-t)}$$

i.e. ϕ_t^{-1} is reversing time.

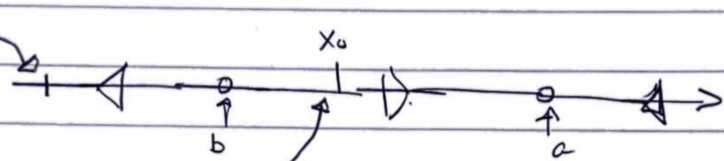
ex3

- first we assume x_0 is a solution to $f(x)$ i.e. a fix point



therefore we can move and $x(t)$ will be constant.

- Next we assume x_0 is not a fix poin



if x_0 is as then we would move towa a but no further. Therefore $x(t)$ would be strictly increasing $\forall t$.

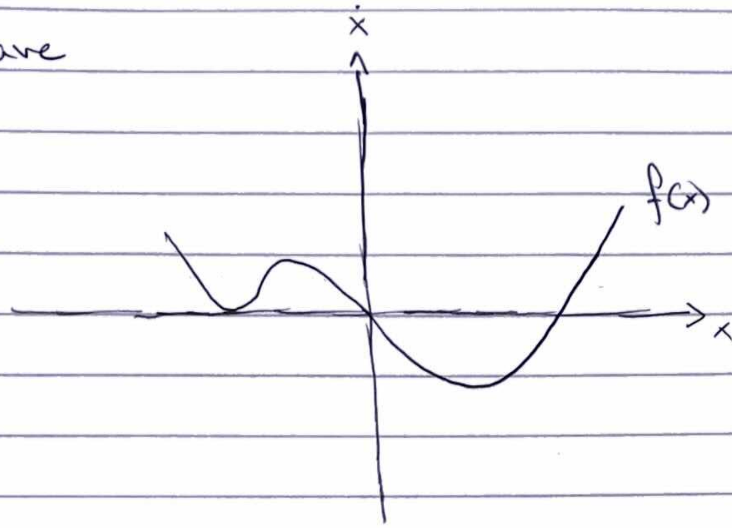
If x_0 was then ~~x(t)~~ $x(t)$ would be ~~th~~ strictly decreasing to all time.

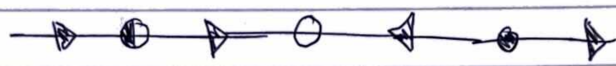
the solution $x(t)$ can not be both increasing and decreasing because it would result in violating the existence and uniqueness theorem. ~~B~~

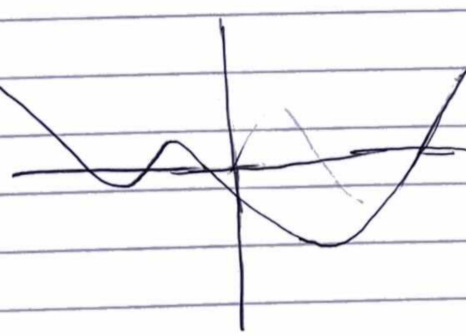
$\square$

ex 4

We have







ex 5

from ex3 we know that we can only go towords $-\infty$, ∞ or a fix point.

Doing several would mean one will violate the uniquness and existess theorem.