week2 - ex1

Multiple choice knapsack (slide 39, uge 2)

$$\max_{x} \sum_{j \in N} p_j x_j$$

s.t. (1)

$$\sum_{j \in N} w_j x_j \leq Q$$

$$\sum_{j \in N_i} x_j = 1 \quad \forall i \in \{1, \dots, \beta\}$$

$$x_j \in \{0, 1\}$$

Item (x_i)	1	2	3	4	5	6	7	8	9
Weight (w)	1	6	3	1	8	4	1	7	3
Profit (p)	2	12	6	1	13	7	1	10	4

$\beta = 3$, $N_1 = \{1,2,3\}$, $N_2 = \{4,5,6\}$, $N_3 = \{7,8,9\}$, $Q = 10$

1) Write up a Julia program to solve the above directly.

2) LP relax the above

3) Danzig-Wolfe relax using $\sum wx \leq Q$ as $\bar{X}^1$

We find all index sets

$$x_i w \leq Q$$

where $x_i \in \{0,1\}^9$, i.e.

$$\bar{X}^1 = \{x_i\} \quad \forall x_i \in x_i w \leq Q$$

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3) con't

we rewrite (1)

$$\max_{\lambda} \; p^T (\bar{X}^1 \lambda)$$

$$\text{s.t.} \quad \sum_{N_i} \bar{X}^1 [N_i,:] \; \lambda = 1 \quad : \pi_i \quad \forall i \in \{1, \dots, \beta\} \qquad (2a)$$

$$\mathbf{1} \lambda = 1 \qquad : \kappa$$

$$\lambda \geq 0$$

If we want to use (2a) as the restricted master problem we also need the subproblem

$$\max_{x} \; \bar{p}^* = p^T x - \pi N x - \kappa \qquad (2b)$$

$$\text{s.t.} \quad \omega^T x \leq Q$$

$$x \in \mathbb{Z}$$

We implement it in Julia.

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We now again perform Danzig-Wolf reformulation to (1) but convexify

$$\sum_{j \in N_i} x_j = 1 \qquad \forall i \in \{1, \dots, \beta\}$$

We write up the master program

$$\begin{aligned}
&\max_{\lambda} \quad && p^T(\bar{X}^1 \lambda) \\
(3a) \quad &\text{s.t.} && \omega^T(\bar{X}^1 \lambda) \leq Q && : \pi \\
& && \mathbf{1}\lambda = 1 && : K \\
& && \lambda \geq 0
\end{aligned}$$

and the subprogram

$$\begin{aligned}
&\max_{x} \quad && \bar{p}^* = p^T x - \pi \omega^T x - K \\
&\text{s.t.} && Nx = \mathbf{1} \\
& && x \in \mathbb{Z}
\end{aligned}$$