



## Homework assignment No. 05

Due May 6, 2015

### Task 5.1: Euler vs. Runge Kutta

2+3 P

Create a new 2D experiment. Integrate a tangent curve in forward direction starting from the seed point  $\mathbf{x} = (1, 0)^T$  in the vector field

$$\mathbf{v}(\mathbf{x}) = \begin{pmatrix} -y \\ \frac{x}{2} \end{pmatrix}.$$

Implement the integration using:

- (a) the Euler method,
- (b) the Runge-Kutta method of 4th order.

Allow the user to specify the *number of integration steps* and the *step size* for each integration method independently. Draw the resulting tangent curves with different colors.

*Hint: Our goal is to compare the two integration schemes. See Figure 1 for how it can look like.*

### Task 5.2: Full-blown Stream Line Integrator

10 P

Implement a versatile set of functions for integrating stream lines in 2D vector fields. You will need these functions in later assignments.

To test these functions, create a new 2D experiment where the user can load a 2D vector field and integrate a stream line from any seed point. Display the seed and the stream line in the graphics window. Include the following functionality:

- (a) Allow integration in forward and backward direction.
- (b) Allow different step sizes.
- (c) Allow integration in the direction field (normalized vector field).
- (d) Stop the integration after a certain number of steps.
- (e) Stop the integration after a certain arc length of the stream line.
- (f) Stop the integration at the boundary of the domain.
- (g) Stop the integration at zeros of the vector field.
- (h) Stop the integration when the velocity becomes too slow.

The assignment comes with several data sets for testing. Make sure to test the last point (slow velocity) with *Sink.am*.

### Task 5.3: Seeding of Stream Lines

2+3 P

Load a 2D vector field and display its stream lines.

- (a) Seed  $n$  stream lines randomly in the domain of the vector field. The user defines  $n$ .
- (b) Seed the stream lines on an uniform grid. The number of grid points in each direction can be defined by the user. The bounding box of the grid corresponds to the bounding box of the vector field.
- (c) (Bonus Points: +5) Seed  $n$  stream lines such that the distribution of the seeds corresponds to the distribution of the magnitude of the vector field, i.e., there are more seeds in areas with high magnitude and fewer seeds in areas with low magnitude.

Good data sets for testing are *ANoise2CT4.am*, *Cylinderclose2CT10.am*, and *boussinesq2CT522.am*.

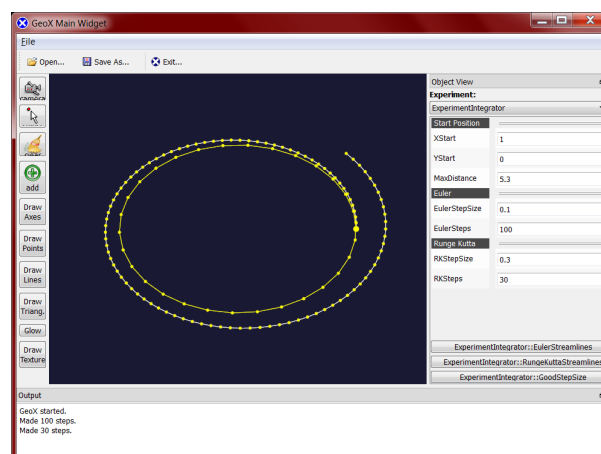


Figure 1: The Runge-Kutta method approximates the ellipse well with just 30 samples, whereas the Euler method cannot approximate it well with 100 samples.