Abstract

In 2013, Aschenbrenner et al. investigated and developed a notion of VC-density for NIP structures, an analog of geometric dimension in an abstract setting [1]. Their applications included a bound for p-adic numbers, an object of great interest and a very active area of research in mathematics. My research concentrates on improving and expanding techniques of that paper to improve the known bounds as well as computing VC-density for other NIP structures of interest. I am able to obtain new bounds for the additive reduct of p-adic numbers, Henselian valued fields, and certain families of graphs. Recent research by Chernikov and Starchenko in 2015 [2] suggests that having good bounds on VC-density in p-adic numbers opens a path for applications to incidence combinatorics (e.g. Szemeredi-Trotter theorem).

Introduction

The concept of VC-dimension was first introduced in 1971 by Vapnik and Chervonenkis for set systems in a probabilistic setting (see [1]). The theory grew rapidly and found wide use in geometric combinatorics, computational learning theory, and machine learning. Around the same time Shelah was developing the notion of NIP (”not having the independence property”), a natural tameness property of (complete theories of) structures in model theory. In 1992 Laskowski noticed the connection between the two: theories where all uniformly definable families of sets have finite VC-dimension are exactly NIP theories. It is a wide class of theories including algebraically closed fields, differentially closed fields, modules, free groups, o-minimal structures, and ordered abelian groups. A variety of valued fields fall into this category as well, including the p-adic numbers.

P-adic numbers were first introduced by Hensel in 1897, and over the following century a powerful theory was developed around them with numerous applications across a variety of disciplines, primarily in number theory, but also in physics and computer science. In 1965 Ax, Kochen and Ershov axiomatized the theory of p-adic numbers and proved a quantifier elimination result. A key insight was to connect properties of the value group and residue field to the properties of the valued field itself. In 1984 Denef proved a cell decomposition result for more general valued fields. This result was soon generalized to p-adic subanalytic and rigid analytic extensions, allowing for the later development of a more powerful technique of motivic integration. The conjunction of those model theoretic results allowed to solve a number of outstanding open problems in number theory (e.g., Artin's Conjecture on p-adic homogeneous forms).

In 1997, Karpinski and Macintyre computed VC-density bounds for o-minimal structures and asked about similar bounds for p-adic numbers. VC-density is a concept closely related to VC-dimension. It comes up naturally in combinatorics with relation to packings, Hamming metric, entropic dimension and discrepancy. VC-density is also the decisive parameter in the Epsilon-Approximation Theorem, which is one of the crucial tools for applying VC theory in computational geometry. In a model theoretic setting it is computed for families of uniformly definable sets. In 2013, Aschenbrenner, Dolich, Haskell, Macpherson, and Starchenko computed a bound for VC-density in p-adic numbers and a number of other NIP structures [1]. They observed connections to dp-rank and dp-minimality, notions first introduced by Shelah. In well behaved NIP structures families of uniformly definable sets tend to have VC-density bounded by a multiple of their dimension, a simple linear behavior. In a lot of cases including p-adic numbers this bound is not known to be optimal. My research concentrates on improving those bounds and adapting those techniques to compute VC-density in other common NIP structures of interest to mathematicians.

Some of the other well behaved NIP structures are Shelah-Spencer graphs and flat graphs. Shelah-Spencer graphs are limit structures for random graphs arising naturally in a combinatorial context. Their model theory was studied by Baldwin, Shi, and Shelah in 1997, and later work of Laskowski in 2006 [4] have provided a quantifier simplification result. Flat graphs were first studied by Podewski-Ziegler in 1978, showing that those are stable [8], and later results gave a criterion for super stability. Flat graphs also come up naturally in combinatorics in work of Nesetril and Ossona de Mendez [6].

Research Plan

The first chapter of my dissertation concentrates on Shelah-Spencer graphs. I have shown that they have infinite dp-rank, so they are poorly behaved as NIP structures. I have also shown that one can obtain arbitrarily high VC-density when looking at uniformly definable families in a fixed dimension. However I'm able to bound VC-density of individual formulas in terms of edge density of the graphs they define.

The second chapter of my dissertation concentrates on graphs and graph-like structures. I have answered an open question from [1], computing VC-density for trees viewed as a partial order. The main idea is to adapt a technique of Parigot [7] to partition trees into weakly interacting parts, with simple bounds of VC-density on each. Similar partitions come up in the Podewski-Ziegler analysis of flat graphs [8]. I am able to use that technique to show that flat graphs are dp-minimal, an important first step before establishing bounds on VC-density. The first of my remaining research goals is to apply this partition to compute VC-densities for specific families of flat graphs.

The third chapter of my dissertation deals with p-adic numbers and valued fields. I have shown that VC-density is linear for an additive reduct of p-adic numbers (using a cell decomposition result from the work of Leenknegt in 2013 [5]). I will explore other reducts described in that paper, to see if my techniques apply to those as well. I have also shown that a Henselian valued field of equi-characteristic zero has linear one-dimensional VC-density if its value group and its residue field have that property. This is along the lines of the results of Ax-Kochen mentioned before. The second of my remaining research goals is to adapt those techniques to higher dimensions, as well as applying them to RV sorts introduced by Flenner in 2011 [3].

My dissertation will therefore consist of VC-density computations for partial order trees, Shelah-Spencer graphs, flat graphs, and various valued fields, as well as any additional applications I am able to find after discussion with my advisor and my colleagues.

Research Timeline

I propose a start date of October 2016.

- March through September 2016: I will prepare and submit papers on my results for trees and Shelah-Spencer graphs. I will research families of flat graphs to see which of my techniques apply in that setting. I will generalize my result for valued fields from one dimension to multiple dimensions. I will also use this time to attend conferences to discuss my results with other mathematicians and get advice on further applications of my research.

- October 2016: I will research p-adic number reducts and RV sorts to see if my valued field techniques apply.

- November 2016: I will prepare and submit a paper containing my results for p-adic numbers and valued fields.

- December 2016: I will write an introduction to my thesis, defining VC-density and summarizing known results and computations.
- January 2017: I will write the first chapter of my thesis on Shelah-Spencer graphs.
- February 2017: I will write the second chapter of my thesis on trees and flat graphs.
- March 2017: I will write the third and final chapter of my thesis on p-adic numbers and valued fields. At the end of the month I will submit the thesis to my advisor.
- April 2017: I will make revisions to my thesis suggested by my advisor and submit the thesis to my committee. I will start preparing for the defense.
- May 2017: I will implement revisions to my thesis given by my committee and resubmit the final version. I will complete the defense.

References

[1] M. Aschenbrenner, A. Dolich, D. Haskell, D. Macpherson, S. Starchenko, *Vapnik-Chervonenkis density in some theories without the independence property*, I, preprint (2011)

[2] Artem Chernikov, Sergei Starchenko, *Regularity lemma for distal structures*, arXiv:1507.01482

[3] Joseph Flenner. *Relative decidability and definability in Henselian valued fields.* The Journal of Symbolic Logic, 76(04):12401260, 2011.

[4] Michael C. Laskowski, *A simpler axiomatization of the Shelah-Spencer almost sure theories*, Israel J. Math. **161** (2007), 157186. MR MR2 350161

[5] E. Leenknegt. *Reducts of p-adically closed fields*, Archive for Mathematical Logic **20 pp.**

[6] J. Nesetril and P. Ossona de Mendez. *On nowhere dense graphs.* European Journal of Combinatorics, 32(4):600617, 2011

[7] Michel Parigot. Théories d'arbres. *Journal of Symbolic Logic*, 47, 1982.

[8] Klaus-Peter Podewski and Martin Ziegler. Stable graphs. *Fund. Math.*, 100:101-107, 1978.

University of California, Los Angeles

GRADUATE Student Copy Transcript Report

Student Information

Name: BOBKOV, ANTON
UCLA ID: 603557936
Date of Birth: 07/12/XXXX
Version: 08/2014 | SAITONE
Generation Date: February 19, 2016 | 01:19:13 AM
This output is generated only once per hour. Any data changes from this time will be reflected in 1 hour.

Program of Study

Admit Date: 09/19/2011
GRADUATE DIVISION

Major:
MATHEMATICS

Degrees | Certificates Awarded

CANDIDATE IN PHILOSOPHY Awarded June 12, 2015
in MATHEMATICS

Graduate Degree Progress

MATHEMATICS for DOCTOR OF PHILOSOPHY Degree
06/03/2015 Qualifying Oral Exam Passed
06/05/2015 Advanced To Candidacy

Previous Degrees

BACHELOR OF ARTS AND SCIENCE Awarded June 10, 2011 from UCLA
in PHYSICS-BA
in MATHEMATICS
With Departmental Highest Honors Awarded
Magna Cum Laude
With College Honors

Language Exams

11/24/2014 RUSSIAN DEPARTMENTAL EXAM PASSED.

California Residence Status

Resident

Fall Quarter 2011

Major:
MATHEMATICS (PHD)

MATHEMATICAL LOGIC	MATH 220A	4.0	16.0	A+
DESCRIPTVE SET THRY	MATH 223D	4.0	16.0	A
DIFFERENTL TOPOLOGY	MATH 225A	4.0	16.0	A
REAL ANALYSIS	MATH 245A	4.0	9.2	C+

	Atm	Psd	Pts	GPA
Term Total	16.0	16.0	57.2	3.575

Winter Quarter 2012

MATHEMATICAL LOGIC	MATH 220B	4.0	16.0	A
TOPICS-MODEL THEORY	MATH 223M	4.0	16.0	A
DIFFERENTL GEOMETRY	MATH 225B	4.0	16.0	A

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	48.0	4.000

Spring Quarter 2012

ALGEBRAIC TOPOLOGY	MATH 225C	4.0	16.0	A
ALGEBRA	MATH 290C	4.0	0.0	S
DIRECTED INDIV STDY	MATH 596	4.0	16.0	A

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	32.0	4.000

Fall Quarter 2012

COMMUTATIVE ALGEBRA	MATH 215A	4.0	14.8	A-
TCHNG APRNTC PRCTCM	MATH 375	2.0	0.0	S
TCHNG APRNTC PRCTCM	MATH 375	2.0	0.0	S
TEACHG COLLEGE MATH	MATH 495	2.0	0.0	S
DIRECTED INDIV STDY	MATH 596	4.0	16.0	A

	Atm	Psd	Pts	GPA
Term Total	14.0	14.0	30.8	3.850

Winter Quarter 2013

INTR-ALGEBRAIC GEOM	MATH 214A	4.0	16.0	A
TCHNG APRNTC PRCTCM	MATH 375	2.0	0.0	S
TCHNG APRNTC PRCTCM	MATH 375	2.0	0.0	S
DIRECTED INDIV STDY	MATH 596	4.0	16.0	A

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	32.0	4.000

Spring Quarter 2013

INTR-ALGEBRAIC GEOM	MATH 214B	4.0	16.0	A
TCHNG APRNTC PRCTCM	MATH 375	4.0	0.0	S
DIRECTED INDIV STDY	MATH 596	4.0	16.0	A

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	32.0	4.000

Fall Quarter 2013

COMBINATORL THEORY	MATH 206A	4.0	0.0	S
DIRECTED INDIV STDY	MATH 596	8.0	32.0	A

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	32.0	4.000

Winter Quarter 2014

COMBINATORL THEORY	MATH 206B	4.0	0.0	S
DIRECTED INDIV STDY	MATH 596	8.0	32.0	A

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	32.0	4.000

Spring Quarter 2014

LOGIC	MATH 290D	4.0	0.0	S
DIRECTED INDIV STDY	MATH 596	8.0	32.0	A

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	32.0	4.000

Fall Quarter 2014

SEMINAR IN LOGIC	MATH 285D	4.0	16.0	A
DIRECTED INDIV STDY	MATH 596	8.0	0.0	S

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	16.0	4.000

Winter Quarter 2015

TCHNG APRNTC PRCTCM	COMPTNG 375	4.0	0.0	S
DIRECTED INDIV STDY	MATH 596	8.0	0.0	S

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	0.0	0.000

Spring Quarter 2015

Majors:
MATHEMATICS (PHD)
(New) MATHEMATICS (CPH)

TCHNG APRNTC PRCTCM	COMPTNG 375	4.0	0.0	S
DIRECTED INDIV STDY	MATH 596	8.0	0.0	S

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	0.0	0.000

Fall Quarter 2015

TCHNG APRNTC PRCTCM	COMPTNG 375	2.0	0.0	S
SEMINAR IN LOGIC	MATH 285D	4.0	0.0	S
RSRCH IN MATH	MATH 599	6.0	0.0	S

	Atm	Psd	Pts	GPA
Term Total	12.0	12.0	0.0	0.000

Winter Quarter 2016

*** Courses In Progress ***

SEM-COMBINATORICS	MATH 285N	4.0
TCHNG APRNTC PRCTCM	MATH 375	2.0
RSRCH IN MATH	MATH 599	8.0

	Atm	Psd	Pts	GPA
Term Total	0.0	0.0	0.0	0.000

GRADUATE Totals

	Atm	Psd	Pts	GPA
Satisfactory/Unsatisfactory Total	74.0	74.0	N/a	N/a
Graded Total	88.0	88.0	N/a	N/a
Cumulative Total	162.0	162.0	344.0	3.909

Total Completed Units 162.0

END OF RECORD
NO ENTRIES BELOW THIS LINE

Personal statement

P-adic numbers are a simple, yet a very deep construction. They were only discovered a hundred years ago, but could have been studied in classical mathematics when number theory was just forming. Their construction is simple enough to explain at the undergraduate level, yet has a very rich number theoretic structure. Normally the real numbers are constructed by first taking rational numbers in decimal form and allowing infinite decimal sequences after the decimal point. Letting decimals be infinite before the decimal point yields a well behaved mathematical object as well, but with a drastically different behavior from real numbers, now depending on the base in which the decimals were written. When the base is a prime number p, this constructs p-adic numbers. These were first studied exclusively within number theory, but later found applications in other areas of math, physics, and computer science. My research will allow for a finer understanding of the finite structure of polynomially definable sets in p-adic numbers. In my career as an educator I hope to increase exposure to this elegant and rich construction for students both inside and outside of mathematics.

My research lies in the area of model theory, a branch of formal logic. Model theory began with Gödel and Malcev in the 1930s, but first matured as a subject in the work of Abraham Robinson, Tarski, Vaught, and others in the 1950s. Model theory studies sets definable by first order formulas in a variety of mathematical objects. Restricting to subsets definable by simple formulas gives access to an array of powerful techniques such as indiscernible sequences and nonstandard extensions. These allow insights not otherwise accessible by classical methods. Nonstandard real numbers, for example, formalize the notion of infinitesimals. Model theory is an extremely flexible field with applications in many areas of mathematics including algebra, analysis, geometry, number theory, and combinatorics as well as some applications to computer science and quantum mechanics. In my career as a mathematician I hope to expose researchers in other fields to model theoretic methods allowing them to explore alternative approaches to classical mathematical objects.

My research concentrates on the concept of VC-density, a recent notion of rank in NIP theories. The study of a structure in model theory usually starts with quantifier elimination, followed by a finer analysis of definable functions and interpretability. The study of VC-density goes one step further, looking at a structure of the asymptotic growth of finite definable families. In the most geometric examples, VC-density coincides with the natural notion of dimension. However, no geometric structure is required for the definition of VC-density, thus we can get some notion of geometric dimension for families of sets given without any geometric context! In my career as a researcher I hope to further explore this notion and introduce other model theorists to its applications.

To summarize, I intend to follow a career path in academia, balancing my teaching with my research. An important part of being a mathematician is communicating and disseminating mathematical knowledge. I have enjoyed my work as a teaching assistant, and look forward to working with students at all stages of their mathematical education. Another equally important part is developing and progressing mathematical knowledge. My work in model theory has been a great motivation for me, and I plan to stay an active researcher for the rest of my mathematical career.

Anton Bobkov

CONTACT INFORMATION

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EDUCATION

University of California, Los Angeles *(graduate)* **Fall 2011 to present** *PhD*, Mathematics (in progress)

- GPA: 3.91
- Advanced to candidacy on June 5, 2015

Advisor: Matthias Aschenbrenner
Research interests: Mathematical logic, model theory, NIP theories, VC-density

University of California, Los Angeles *(undergraduate)* **Graduated Spring 2011**

- *B.S.* in Mathematics, *B.A.* in Physics
- Sherwood Prize
- Departmental Highest Honors in Mathematics, College Honors
- GPA: 3.82 (Magna Cum Laude)
- William Lowell Putnam Mathematics Competition
  - 2008 - score 30
  - 2009 - score 19

UNDERGRADUATE RESEARCH

Cryptography REU at Northern Kentucky University **Summer 2009**
Implemented a variant of MXL algorithm in computational algebra system MAGMA

Research assistant for Vladimir Vassiliev **2008 - 2011**
Numerical simulations for AGIS gamma-ray telescope. This included forward and inverse kinematics for Stewart platform, ray casting, and high precision calibration.

TEACHING

Math 31B: Integration and Infinite Series	**2012 - 2013**
Math 33A: Linear Algebra and Applications	**2012 - 2013**
PIC 10B: Intermediate Programming	**Winter 2015, Spring 2015**
PIC 20A: Principles of Java Language with Applications	**Spring 2015**
PIC 40A: Introduction to Programming for Internet	**Fall 2015**
Math 115B: Linear Algebra	**Winter 2016**
Independent Programming Projects	**Winter 2016**

AWARDS AND SCHOLARSHIPS

2011-2012:

Fees: $13,247.13 paid from unrestricted Graduate Division allocation
Stipend: $21,000 paid from departmental funds (RTG Logic)

Stipend: $4,000 paid from unrestricted Graduate Division allocation
Summer'11 Stipend: $ 3000 from departmental funds (RTG Logic)
Summer'11 Stipend:$3,000 from College funds

2012-2013:

Fees: $14,372.68 paid through TA appointments
Fee remission balance: $374.25 paid from unrestricted Graduate Division allocation
Income: $17,655.18 TA salary
Stipend: $3,344.82 paid from unrestricted Graduate Division allocation

2013-2014:

Fees: $2,070.75 partial fees paid from unrestricted Graduate Division allocation
Fees: $10,500 remainder of fees paid from departmental funds (RTG Logic)
Stipend: $ 21,000 paid from departmental funds (RTG Logic)
Stipend: $4,000 paid from unrestricted Graduate Division allocation
Summer'13: $4,000 paid from departmental funds (RTG Algebra)

2014-2015:

Fees: $15,203.10 paid through TA & GSR appointment
Fee remission balance: $378.99 paid from unrestricted Graduate Division allocation
Income: $20,621.16 TA & GSR salary
Stipend: $378.84 paid from unrestricted Graduate Division allocation

2015-2016:

Fees: $15,440.48 paid through TA appointments
TA Income: $11,121.43 (Spring TA salary yet to be paid during Spring'16)
Summer'15: $4,000 paid from unrestricted Graduate Division allocation