SUPERFLAT GRAPHS ARE DP-MINIMAL

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Abstract. We show that the theory of superflat graphs is dp-minimal.

We work with an infinite graph G and a subset of vertices $V \subset V(G)$. Say that V is n-connected if there aren't a set of $n-1$ vertices removing which disconnects every pair of vertices in V. Connectivity of V is the smallest n such that V are n-connected.

Lemma 0.1. *Suppose $\{a, b\}$ in G have finite connectivity $n+1$. Then there are finitely many n-point sets that disconnect a from b.*

Corollary 0.2. *Suppose a finite $V \subset V(G)$ has finite connectivity $n+1$. Then there are finitely many n-point sets that disconnect V.*

Definition 0.3. Suppose $V \subset V(G)$ has finite connectivity $n+1$. Let connectivity hull of V to be union of all n-point sets that disconnect it.

From now on work in a flat graph. It is stable so all the indiscernible sequences are totally indiscernible.

Lemma 0.4. *Suppose we have an indiscernible sequence over parameter set A in a flat graph $(a_i)_{i\in I}$ with I countable. Then (a_i)*

Suppose we have an indiscernible sequence over parameter set A in a flat graph $(a_i)_{i\in I}$ with I countable. Fix n. By a flatness result we can find an infinite $J \subset I$ and a finite set B' such that each pair from $(a_j)_{j\in J}$ have infinite distance over B'. Using total indispensability we have an automorphism sending $(a_j)_{j\in J}$ to $(a_i)_{i\in I}$. Denote image of B' under this automorphism as B.

Theorem 0.5. *Suppose we have an indiscernible sequence over parameter set A in a flat graph $(a_i)_{i\in I}$ with I countable. Fix $n \in \mathbb{N}$. Then there exists a finite set B such that (a_i) is indiscernible over $A \cup B$ and $\forall i \neq j\ d_B(a_i, a_j) \geq n$.*

Corollary 0.6. *Let $(a_i)_{i\in I}$ be a countable indiscernible sequence over A. Then there is a countable B such that (a_i) is indiscernible over $A \cup B$ and*

$$\forall i \neq j\ d_B(a_i, a_j) = \infty$$

That is every indiscernible sequence can be upgraded to have infinite distance over its parameter set.

Lemma 0.7. *Suppose $a \equiv_A b$ and $d_A(a, c) = d_A(b, c) = \infty$. Then $a \equiv_{Ac} b$*

Theorem 0.8. *Let G be a flat graph with $(a_i)_{i\in\mathbb{Q}}$ indiscernible over A and $b \in G$. There exists $c \in \mathbb{Q}$ such that $(a_i)_{i\in\{\mathbb{Q}-c\}}$ is indiscernible over Ab.*

Corollary 0.9. *Flat graphs are dp-minimal.*

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