

- (1) Let $A \in M_n(F)$. Define

$$V_A = \text{span}\{I, A, A^2, A^3, \dots\} \subset M_n(F)$$

Show that $\dim(V_A) = \deg(q_A)$

- (2) Now suppose that $A, B \in M_n(F)$ commute, that is $AB = BA$. Define

$$V_{A,B} = \text{span}\{A^i B^j \mid i, j \in \mathbb{N}\} \subset M_n(F)$$

Show that

$$\dim(V_{A,B}) \leq \dim(V_A) \dim(V_B)$$

- (3) Using previous results show that

$$\deg(q_{A+B}) \leq \deg(q_A) \deg(q_B)$$

$$\deg(q_{AB}) \leq \deg(q_A) \deg(q_B)$$