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To the Hiring Committee:

My name is Jaclyn Lang. I am a graduate student at the University of California, Los Angeles (UCLA), working in the area of algebraic number theory, specifically p-adic families of Galois representations and modular forms. I expect to receive a Ph.D. from UCLA in June 2016, under the direction of Haruzo Hida. I am applying for the Zeeman Lectureship in your department. I am particularly interested in working with David Loeffler and Samir Siksek on questions related to Galois representations and their images. I am also interested in working with John Cremona on a computational project, more details of which can be found in my research statement.

With my application, I include a curriculum vitae, a list of publications, a research statement, and a teaching statement. Three letters of reference attesting to my research have been sent to Georgina Copeland (Georgina.Copeland@warwick.ac.uk). They are from Haruzo Hida (hida@math.ucla.edu), Chandrashekhar Khare (shekhar@math.ucla.edu), and Jacques Tilouine (jacques.tilouine@free.fr). Please let me know if there is anything else I can provide. Thank you in advance for your consideration.

Sincerely,

Jaclyn Lang

Jaclyn Lang
University of California, Los Angeles

List of Publications

[1] Lang, Jaclyn, 'On the image of the Galois representation associated to a non-CM Hida family', submitted to *Algebra and Number Theory*, 2015, `http://www.math.ucla.edu/~jaclynlang/I_0_level_existence.pdf`.

[2] Balakrishnan, Jennifer, and Mirela Çiperiani, Jaclyn Lang, Bahare Mirza, Rachel Newton, 'Shadow lines in the arithmetic of elliptic curves', to appear in *Women in Numbers 3 Proceedings*, 2015, `https://www.ma.utexas.edu/users/mirela/ShadowLines.pdf`.

[3] Daub, Michael, and Jaclyn Lang, Mona Merling, Allison Pacelli, Natee Pitiwan, Michael Rosen, 'Function fields with class number indivisible by a prime ℓ', *Acta Arith.* **150** (2011) 339-359.

Overview

My research is in the area of algebraic number theory. Primarily I study modular forms, the Galois representations associated to them, and p-adic families of such objects. I am also interested in the arithmetic of elliptic curves. The main result of my thesis is that, in a qualitative sense, the Galois representation associated to an ordinary p-adic family of modular forms has "large" image. My future research plans include improving the result to a quantitative form and obtaining a complete description of the images of such Galois representations.

Galois representations are fundamental objects of study in modern number theory. They are the only known tools for systematically studying the absolute Galois group of the rational numbers, $G_{\mathbb{Q}}$, and geometric Galois representations are one side of the celebrated Langlands correspondence. A Galois representation is a continuous homomorphism $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(A)$ for a topological ring A. Most known examples of Galois representations arise from an action of $G_{\mathbb{Q}}$ on the cohomology of varieties defined over $\mathbb{Q}$, or by putting such representations into p-adic families.

A fundamental problem is to determine the image of a given Galois representation. This was first done by Serre [15] for the p-adic Galois representations $\rho_{E,p}$ associated to an elliptic curve $E_{/\mathbb{Q}}$. He showed that if E does not have complex multiplication (CM) then $\rho_{E,p}$ is surjective for all but finitely many primes p. Furthermore the image of $\rho_{E,p}$ is open for all p. Serre's result is an example of a general pattern governing the expected behavior of images of Galois representations.

Heuristic. *The image of a Galois representation should be as large as possible, subject to the symmetries of the geometric object from which it arose.*

The notion of "symmetry" is vague and depends on the situation. In the case of elliptic curves, the relevant symmetry is complex multiplication, a condition that means that the elliptic curve has a larger endomorphism ring than usual. In the 1980s, Ribet and Momose determined, up to finite error, the image of a Galois representation coming from a classical modular form without CM and thus showed that such images are "large" [12, 14]. Their proof introduced new symmetries of modular forms known as "conjugate self-twists". Indeed, if $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathcal{O})$ arises from a modular form, then one can talk about the subring $\mathcal{O}_0$ of $\mathcal{O}$ fixed by all conjugate self-twists. Ribet and Momose proved that the intersection of $\mathrm{Im}\,\rho$ with $\mathrm{SL}_2(\mathcal{O}_0)$ is open in $\mathrm{SL}_2(\mathcal{O}_0)$.

In the 1980s Hida developed his theory of p-adic families of (ordinary) modular forms. To such a family F, Hida associated a Galois representation $\rho_F : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{I})$ for a certain ring $\mathbb{I}$. From ρ_F one obtains a mod p representation $\bar{\rho}_F$. One can consider conjugate self-twists of F and form the ring $\mathbb{I}_0$ fixed by all such twists. The following theorem is the main result of my thesis [8].

Theorem 1 (Lang [8])**.** *Let F be a non-CM Hida family. Assume that $\bar{\rho}_F$ is absolutely irreducible and satisfies a technical but mild regularity condition. Then there is a nonzero $\mathbb{I}_0$-ideal $\mathfrak{a}_0$ such that the image of ρ_F contains all matrices in $\mathrm{SL}_2(\mathbb{I}_0)$ that are congruent to the identity modulo $\mathfrak{a}_0$.*

I have a secondary interest in the arithmetic of elliptic curves. My group at the Women in Numbers 3 workshop studied shadow lines of elliptic curves, an invariant first defined by Mazur and Rubin [10]. Using explicit class field theory, we developed an algorithm to compute the shadow line of a triple (E, K, p), where $E_{/\mathbb{Q}}$ is an elliptic curve, K is an imaginary quadratic field, and p is a rational prime of good reduction for E that splits in K. We implemented our algorithm in Sage [16] and computed the first examples of shadow lines. The resulting paper will appear in the proceedings volume [1].

Thesis

Background. We now give some definitions and notation to make the statement of Theorem 1 more precise. Fix a prime p, and assume for simplicity that $p \geq 5$. Let $\Lambda = \mathbb{Z}_p[[T]]$ and $\mathbb{I}$ be an integral domain that is finite flat over Λ. Fix embeddings of an algebraic closure $\overline{\mathbb{Q}}$ of $\mathbb{Q}$ into $\mathbb{C}$ and $\overline{\mathbb{Q}}_p$.

Definition 1 (Hida [5], Wiles [17])**.** A formal power series $F = \sum_{n=1}^{\infty} A_n q^n \in \mathbb{I}[[q]]$ is a *Hida family* if $A_p \in \mathbb{I}^\times$ and for every integer $k \geq 2$ and every prime ideal $\mathfrak{P}$ of $\mathbb{I}$ lying over $(1 + T - (1+p)^k)\Lambda$, we have:

- $A_n \bmod \mathfrak{P}$ is in $\overline{\mathbb{Q}}$ (rather than just $\overline{\mathbb{Q}}_p$), and
- $f_{\mathfrak{P}} := \sum_{n=1}^{\infty} (A_n \bmod \mathfrak{P}) q^n$ gives the q-expansion of a classical modular form of weight k.

Hida showed that every p-ordinary classical modular form of weight at least 2 arises from a unique such family [5]. Furthermore, there is a Galois representation $\rho_F : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{I})$ that is unramified almost everywhere. For all primes ℓ at which ρ_F is unramified, $\operatorname{tr} \rho_F(\mathrm{Frob}_\ell) = A_\ell$, where Frob_ℓ is the conjugacy class of a Frobenius element at ℓ [4].

Definition 2. An automorphism σ of $\mathbb{I}$ is a *conjugate self-twist* of a Hida family $F = \sum_{n=1}^{\infty} A_n q^n \in \mathbb{I}[[q]]$ if there exists a non-trivial Dirichlet character η_σ such that

$$\sigma(A_\ell) = \eta_\sigma(\ell) A_\ell$$

for almost all primes ℓ. We say F has *complex multiplication (CM)* if the identity automorphism is a conjugate self-twist of F.

Ideas in the Proof of Theorem 1. The key result is a lifting theorem showing when a conjugate self-twist of an arithmetic specialization $f_{\mathfrak{P}}$ of F can be lifted to a conjugate self-twist of the entire family F. The ideas that go into the lifting theorem will be briefly explained in the next paragraph. Using the lifting theorem, there is a series of reduction steps to conclude that the theorem of Ribet and Momose is sufficient to prove Theorem 1. An important tool in these reduction steps is a $\mathbb{Z}_p$-Lie algebra of Pink [13] that is associated to $\operatorname{Im} \rho_F$, which allows us to reduce our problem to linear algebra. Hida observed that the ordinarity of ρ_F can be used to give Pink's Lie algebra a Λ-algebra structure [6], and this structure is critical to the proof.

We now give a brief outline of the proof of the lifting theorem. A conjugate self-twist σ of an arithmetic specialization $f_{\mathfrak{P}}$ of F induces an automorphism $\bar{\sigma}$ of the residue field $\mathbb{F}$. Using deformation theory, I lift $\bar{\sigma}$ to an automorphism Σ of the entire universal deformation ring $R_{\bar{\rho}_F}$ of $\bar{\rho}_F$ and show that Σ satisfies certain properties. Using automorphic methods I show that Σ preserves a certain Hecke algebra, $\mathbb{T}$. Since $\operatorname{Spec} \mathbb{I}$ is an irreducible component of $\operatorname{Spec} \mathbb{T}$, we see that Σ must send $\operatorname{Spec} \mathbb{I}$ to another irreducible component $\Sigma^* \operatorname{Spec} \mathbb{I}$ of $\operatorname{Spec} \mathbb{T}$. The properties of Σ force the two components $\operatorname{Spec} \mathbb{I}$ and $\Sigma^* \operatorname{Spec} \mathbb{I}$ to intersect at an arithmetic point. Since the Hecke algebra is étale over Λ at arithmetic points, Σ must descend to an automorphism of $\mathbb{I}$, as desired.

Future work

Determining the $\mathbb{I}_0$-level and relation to p-adic L-functions. Theorem 1 guarantees that there is a non-zero $\mathbb{I}_0$-ideal $\mathfrak{a}_0$ such that the image of ρ_F contains all determinant 1 matrices that are congruent to the identity modulo $\mathfrak{a}_0$. The largest such ideal is called the $\mathbb{I}_0$-*level* of ρ_F and is denoted $\mathfrak{c}_{0,F}$. A natural question is to determine the $\mathbb{I}_0$-level. I plan to prove the following conjecture, which is a generalization of Hida's Theorem II in [6].

Conjecture 1. *Let F and p be as in Theorem 1.*

1. *If $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\mathbb{F}_p)$, then $\mathfrak{c}_{0,F} = \mathbb{I}_0$. That is, $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\mathbb{I}_0)$.*
2. *Suppose that $\bar{\rho}_F$ is absolutely irreducible and $\bar{\rho}_F \cong \operatorname{Ind}_M^{\mathbb{Q}} \bar{\psi}$ for an imaginary quadratic field M in which p splits and a character $\bar{\psi} : \mathrm{Gal}(\overline{\mathbb{Q}}/M) \to \overline{\mathbb{F}}_p^{\times}$. Assume M is the only such quadratic field. Under minor conditions on the tame level of F, there is a product $\mathcal{L}_0$ of anticyclotomic Katz p-adic L-functions such that $\mathfrak{c}_{0,F}$ is a factor of $\mathcal{L}_0$. Furthermore, every prime factor of $\mathcal{L}_0$ is a factor of $\mathfrak{c}_{0,F}$ for some F.*

Note that even when $\mathbb{I} = \Lambda$, Conjecture 1 is stronger than Hida's Theorem II [6]. Furthermore, Conjecture 1.1 is a natural extension of the work of Mazur-Wiles [11] and Fischman [3].

In proving case (1) of the conjecture, I will make use of Manoharmayum's recent work that shows $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(W)$ for a finite unramified extension W of $\mathbb{Z}_p$ [9]. This will be combined with the Λ-module structure on the Pink Lie algebra associated to $\operatorname{Im}\rho_F$ that was used in the proof of Theorem 1 to get the desired result.

In proving case (2), I will relate the $\mathbb{I}_0$-level to the congruence ideal of F as in Hida's proof of Theorem II [6]. The connection to Katz p-adic L-functions is then obtained by relating the congruence ideal to the p-adic L-function through known cases of the Main Conjecture of Iwasawa Theory. The idea is that replacing the Λ-level in Hida's work with the more precise $\mathbb{I}_0$-level will allow me to remove the ambiguity of the square factors that show up in Theorem II [6].

Proving Conjecture 1 would yield refined information about the images of Galois representations attached to Hida families. It is the first step in completely determining the images of such representations.

Computing $\mathcal{O}_0$-levels of classical Galois representations. The goal of this project is to compute the level of Galois representations coming from classical modular forms and thus completely determine the image of such a representation. Let f be a non-CM classical Hecke eigenform, $\mathfrak{p}$ a prime of the ring of integers of the field generated by the Fourier coefficients of f, and $\rho_{f,\mathfrak{p}} : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathcal{O})$ the associated p-adic representation. Let π be a uniformizer of the subring $\mathcal{O}_0$ of $\mathcal{O}$ fixed by all conjugate self-twists. By the work of Ribet [14] and Momose [12], there is a minimal non-negative integer $c(f,\mathfrak{p})$ such that $\operatorname{Im}\rho_f$ contains all matrices of determinant 1 that are congruent to the identity modulo $\pi^{c(f,\mathfrak{p})}$. Their work shows that $c(f,\mathfrak{p}) = 0$ for all but finitely many primes $\mathfrak{p}$. However, relatively little is known about the case when $c(f,\mathfrak{p})$ is positive and the weight of f is greater than 2. I plan to study how $c(f,\mathfrak{p})$ changes as f varies over the classical specializations of a non-CM Hida family that is congruent to a CM family.

This project will have both theoretical and computational components. First, I will establish a relationship between $c(f,\mathfrak{p})$ and the congruence number of f, which should also be related to values of the Katz p-adic L-function, as suggested by the proof of Theorem II in [6]. Once this is established, I will create a method in the open source software Sage [16] to compute $c(f, \mathfrak{p})$ by computing the congruence number of f. This should be relatively straightforward since Sage can already compute congruence numbers. Using the new functionality, I will create a large data set of levels of classical Galois representations in Hida families, which will likely lead to new conjectures to be studied theoretically. I have experience working with Sage from my Women in Numbers 3 project [1] and from leading a project at Sage Days 69. Indeed, my Sage Days project consisted of writing a method to test whether a modular form is CM, a first step in the eventual program to compute $c(f, \mathfrak{p})$. I hope to involve undergradutes in the computational aspects of the project. The programming should be straightforward and would be a good way for them to learn about modular forms.

Analogue of the Mumford-Tate Conjecture in p-adic families. Another way to describe the work of Ribet and Momose is that they proved the Mumford-Tate Conjecture for compatible systems of Galois representations associated to classical modular forms. Hida has proposed an analogue of the Mumford-Tate Conjecture for p-adic families of Galois representations. For an arithmetic prime $\mathfrak{P}$ of $\mathbb{I}$, write $\mathrm{MT}_{\mathfrak{P}}$ for the Mumford-Tate group of the compatible system containing $\rho_{f_{\mathfrak{P}}}$, so $\mathrm{MT}_{\mathfrak{P}}$ is an algebraic group over $\mathbb{Q}$. Let $\kappa(\mathfrak{P}) = \mathbb{I}_{\mathfrak{P}}/\mathfrak{P}_{\mathfrak{P}}$, and write $G_{\mathfrak{P}}$ for the Zariski closure of $\operatorname{Im} \rho_{f_{\mathfrak{P}}}$ in $\mathrm{GL}_2(\kappa(\mathfrak{P}))$. Let $G_{\mathfrak{P}}^{\circ}$ be the connected component of the identity of $G_{\mathfrak{P}}$ and $G'_{\mathfrak{P}}$ the (closed) derived subgroup of $G_{\mathfrak{P}}$. Finally, let Γ_F denote the group generated by the conjugate self-twists of a non-CM Hida family F.

Conjecture 2 (Hida)**.** *Assume F is non-CM. There is a simple algebraic group G', defined over $\mathbb{Q}_p$, such that for all arithmetic primes $\mathfrak{P}$ of $\mathbb{I}$ one has $G'_{\mathfrak{P}} \cong G' \times_{\mathbb{Q}_p} \kappa(\mathfrak{P})$ and $\mathrm{Res}_{\mathbb{Q}_p}^{\kappa(\mathfrak{P})} G_{\mathfrak{P}}$ is (the ordinary factor of) $\mathrm{MT}_{\mathfrak{P}} \times_{\mathbb{Q}} \mathbb{Q}_p$. Furthermore, the component group $G_{\mathfrak{P}}/G_{\mathfrak{P}}^0$ is canonically isomorphic to the Pontryagin dual of the decomposition group of $\mathfrak{P}$ in Γ_F.*

By obtaining a sufficiently precise understanding of images of Galois representations attached to Hida families through the first project, I plan to prove results along the lines of Conjecture 2. I have some preliminary results relating the Pontryagin dual of Γ_F to the quotient $(\operatorname{Im} \rho_F)/(\operatorname{Im} \rho_F|_H)$ for a certain finite index normal subgroup H of $G_{\mathbb{Q}}$.

Completing this research objective, or even any preliminary results in this direction, would reveal that the images of classical specializations of the Galois representation attached to a Hida family are even more related to one another than previously thought. Not only would they arise as specializations of some group in $\mathrm{GL}_2(\mathbb{I})$, they could all be found simply by base change from a single group, at least up to abelian error.

Other settings. The above projects can be studied in more general settings than Hida families for GL_2. Hida and Tilouine proved an analogue of Hida's Theorem II [6] for GSp_4-representations associated to Hida families of Seigel modular forms [7]. There are two main difficulties they overcome in their work: the types of symmetries are much more complicated than CM versus non-CM, and Pink's theory of Lie algebras is only valid for SL_2. The tools they developed to overcome these problems could be applied to study analogues of the above questions for bigger groups.

Tilouine and his collaborators proved an analogue of Theorem 1 in the non-ordinary GL_2-setting [2] by building on the ideas in my thesis. They introduce the relative Sen operator to create a Λ-algebra structure on the Lie algebra of $\operatorname{Im} \rho_F$ as their representation is not ordinary. This idea will be useful in studying the above questions in the non-ordinary setting.

References

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Teaching at UCLA

I have been a Teaching Assistant (TA) for lower and upper division courses as well as a Teaching Assistant Consultant (TAC). At UCLA, TAs hold weekly discussion sections to review concepts and examples from the lecture, hold office hours, proctor and grade exams, and sometimes grade homework. The TAC position is awarded to experienced TAs who have a strong record of quality instruction. The TAC teaches and develops the curriculum for MATH 495 — the course instructing first year TAs in effective teaching techniques — and participates in a quarter-long campus-wide seminar on pedagogy through the Office of Instructional Development. In 2014 I was awarded a Distinguished Teaching Award from the UCLA mathematics department based on evaluations from both students and faculty. In my classroom I focus on developing problem solving and technical communication skills, while exposing my students to practical skills like programming.

Problem Solving. I expect my students to improve their problem solving skills through my classes. I emphasize Polya's techniques from his classic book *How to Solve It* such as trying to visualize the problem. Visualization can be especially powerful in Group Theory where the material can seem abstract but in fact is capturing something visual, namely symmetries of objects. When I was a TA for Group Theory I had the students make equilateral triangles and squares and use them to understand multiplication in dihedral groups. For a homework problem about a dodecahedron, I brought in toothpicks and gumdrops so that students could make their own dodecahedra and understand their symmetries. Another technique that Polya stresses is the importance of reflecting on your work. In Group Theory I required students to self-evaluate their homework by numerically ranking how well they answered each question. This did not affect their grades, but it forced them to reflect on their work and learning process while helping me give individualized feedback.

Technical Communication. A central goal of my classes is for students to learn technical communication skills, both written and oral, formal and informal. For example, I developed a worksheet for the first discussion section of Integration and Infinite Series that immediately gets the students talking to one another. The worksheet asks students to sketch graphs of standard functions, evaluate limits, recall derivative rules, recall trigonometry, and remember how exponentiation and logarithms behave. (The worksheet is on my website: `www.math.ucla.edu/~jaclynlang/`) The key to this assignment is that students not only give an answer for each question but the name of a person with whom they solved it. Furthermore, they cannot answer more than four questions with the same person. Thus the students practice discussing mathematics while reviewing the prerequisite material for the course. This activity allows me to meet the students and gauge the background of the class. The informal communication skills that students develop through activities such as this will be useful in any career that involves technical teamwork.

Programming. As computers become more powerful and ubiquitous, programming skills are becoming critical in many disciplines and careers. Math courses are a natural place to introduce students to the basics of programming, and computational problems can lead to student research. As a Calculus TA, I used the open source software Sage to supplement the numerical analysis material in the course. Students wrote simple programs in Sage Math Cloud that implemented numerical integration techniques and error bounds. Sage Math Cloud is well suited for assignments and classroom activities. Students add me as a collaborator, and I can give feedback directly on their code. Furthermore, some of my research projects involve computations in Sage. By bringing Sage into the classroom, I can offer interested students opportunities to deepen their knowledge by conducting research with me or working on Sage development projects.

Teaching beyond UCLA

My teaching style has been influenced by my experiences teaching in less traditional settings. As a student at Bryn Mawr College, I ran problem sessions for a number of math courses from Calculus to Abstract Algebra. The focus of these sessions was on deepening conceptual understanding rather than just doing homework problems. This one-on-one time with students naturally hones the problem solving and communication skills mentioned above, and many of the interactive activities I have developed for discussion sections at UCLA were inspired by my experience at Bryn Mawr.

In summer 2010 I was a counselor for the Program in Mathematics for Young Scientists (PROMYS) at Boston University, a six-week program for talented high school students. Three students were assigned to me, and I was responsible for guiding their learning and grading their work. My students had strikingly different needs: one came from a strong background in competition math, one was severely unprepared for the program, and the last was talented but struggled with mathematical confidence. It was challenging to find the right balance of support while maintaining high expectations when the students were so different. When I teach I remember this experience to remind myself of the vastly different challenges and backgrounds of my students.

During summer 2012 I served as a TA at the Summer Program for Women in Mathematics (SPWM) at George Washington University. The program is for women math majors entering their final year of college. It exposes them to math beyond the usual undergraduate curriculum as well as careers for mathematicians with advanced degrees. Besides helping the students learn about normed division algebras and semigroups, I served as a mentor, counseling them about life in graduate school and other career options. Evidently this was appreciated as one of the student evaluations called me "one of the most helpful parts of the program". I have incorporated mentoring into my classroom at UCLA by encouraging students to attend departmental activities and apply for some of the math opportunities from a comprehensive list posted on my website.

Supporting Diversity through Teaching

I am interested in bolstering participation of underrepresented minorities in mathematics. Twice I have partnered with other graduate students at UCLA to bring underprivileged middle school girls to UCLA for EmpowHer STEM Day. The girls spend a day participating in science experiments and demonstrations. My group explained basic probability using the Monty Hall Problem. My advocacy for women in math and diversity in STEM led to an interest in stereotype threat research and its implications for teaching mathematics. I have read books on the subject including *Whistling Vivaldi: How Stereotypes Affect Us and What We Can Do*, by Claude Steele, and I attended the related lecture series on Women in STEM curated by UCLA social psychologist Professor Jenessa Shapiro in 2011.

As a TAC, part of my contribution to the curriculum for MATH 495 was to introduce a day when Professor Shapiro gave a lecture on stereotype threat along with tips for combating its effects in the classroom. For example, research has shown that students who believe that math skills are malleable and grow with practice perform better than those who believe math ability is a fixed genetic trait. The difference is especially dramatic for students who are part of an underrepresented group. By teaching Polya's problem solving techniques, emphasizing communication and group work, and explicitly stressing the malleability of mathematical ability in my classes, my students internalize both the mathematics and the confidence to succeed in technical work in the future.