# A Tiny Example

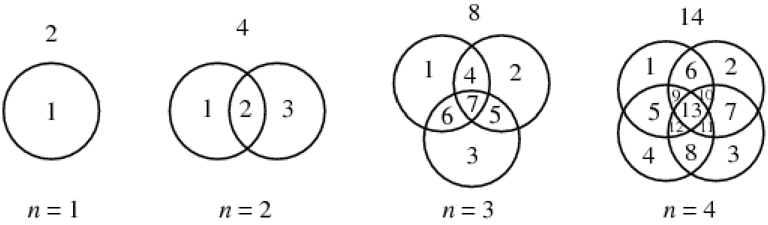
Andrew Mertz and William Slough

June 15, 2005

Suppose we have an (infinite) collection of sets $\mathcal{F}$.
We define a shatter function $\pi_{\mathcal{F}}(n)$

$$\pi_{\mathcal{F}}(n) = \max\{\#\text{ of atoms in boolean algebra generated by } S \mid S \subset \mathcal{F} \text{ with } |S| = n\}$$

Example: Let $\mathcal{F}$ consist of all discs on a plane.




$$\pi_{\mathcal{F}}(1) = 2 \quad \pi_{\mathcal{F}}(2) = 4 \quad \pi_{\mathcal{F}}(3) = 8 \quad \pi_{\mathcal{F}}(4) = 14$$

$$\pi_{\mathcal{F}}(n) = \frac{1}{2}n^2 + \frac{1}{2}n + 1$$

More examples:

1. Let $\mathcal{F}$ be a set of lines on a plane. Then
$$\pi_{\mathcal{F}}(n) = n(n+1)/2 + 1$$
2. Let $\mathcal{F}$ be a set of disks on a plane. Then
$$\pi_{\mathcal{F}}(n) = n^2 - n + 2$$
3. Let $\mathcal{F}$ be a set of balls in $\mathbb{R}^3$. Then
$$\pi_{\mathcal{F}}(n) = n(n^2 - 3n + 8)/3$$
4. Let $\mathcal{F}$ be a set of intervals on a line. Then
$$\pi_{\mathcal{F}}(n) = 2n$$
5. Let $\mathcal{F}$ be a set of half-planes. Then
$$\pi_{\mathcal{F}}(n) = n(n+1)/2 + 1$$
6. Let $\mathcal{F}$ be a collection of finite subsets of $\mathbb{N}$. Then
$$\pi_{\mathcal{F}}(n) = 2^n$$
7. Let $\mathcal{F}$ be a collection of polygons in a plane. Then
$$\pi_{\mathcal{F}}(n) = 2^n$$