02 INFORMATION ABOUT PRINCIPAL INVESTIGATORS/PROJECT DIRECTORS(PI/PD) and co-PRINCIPAL INVESTIGATORS/co-PROJECT DIRECTORS

Submit only ONE copy of this form **for each PI/PD and co-PI/PD** identified on the proposal. The form(s) should be attached to the original proposal as specified in GPG Section II.C.a. Submission of this information is voluntary and is not a precondition of award. This information will not be disclosed to external peer reviewers. ***DO NOT INCLUDE THIS FORM WITH ANY OF THE OTHER COPIES OF YOUR PROPOSAL AS THIS MAY COMPROMISE THE CONFIDENTIALITY OF THE INFORMATION.***

PI/PD Name: Jaclyn A Lang

Gender: ☐ Male ☒ Female

Ethnicity: (Choose one response) ☐ Hispanic or Latino ☒ Not Hispanic or Latino

Race: (Select one or more)
☐ American Indian or Alaska Native
☐ Asian
☐ Black or African American
☐ Native Hawaiian or Other Pacific Islander
☒ White

Disability Status: (Select one or more)
☐ Hearing Impairment
☐ Visual Impairment
☐ Mobility/Orthopedic Impairment
☐ Other
☒ None

Citizenship: (Choose one) ☒ U.S. Citizen ☐ Permanent Resident ☐ Other non-U.S. Citizen

Check here if you do not wish to provide any or all of the above information (excluding PI/PD name): ☐

REQUIRED: Check here if you are currently serving (or have previously served) as a PI, co-PI or PD on any federally funded project ☐

Ethnicity Definition:
Hispanic or Latino. A person of Mexican, Puerto Rican, Cuban, South or Central American, or other Spanish culture or origin, regardless of race.
Race Definitions:
American Indian or Alaska Native. A person having origins in any of the original peoples of North and South America (including Central America), and who maintains tribal affiliation or community attachment.
Asian. A person having origins in any of the original peoples of the Far East, Southeast Asia, or the Indian subcontinent including, for example, Cambodia, China, India, Japan, Korea, Malaysia, Pakistan, the Philippine Islands, Thailand, and Vietnam.
Black or African American. A person having origins in any of the black racial groups of Africa.
Native Hawaiian or Other Pacific Islander. A person having origins in any of the original peoples of Hawaii, Guam, Samoa, or other Pacific Islands.
White. A person having origins in any of the original peoples of Europe, the Middle East, or North Africa.

WHY THIS INFORMATION IS BEING REQUESTED:

The Federal Government has a continuing commitment to monitor the operation of its review and award processes to identify and address any inequities based on gender, race, ethnicity, or disability of its proposed PIs/PDs. To gather information needed for this important task, the proposer should submit a single copy of this form for each identified PI/PD with each proposal. Submission of the requested information is voluntary and will not affect the organization's eligibility for an award. However, information not submitted will seriously undermine the statistical validity, and therefore the usefulness, of information recieved from others. Any individual not wishing to submit some or all the information should check the box provided for this purpose. (The exceptions are the PI/PD name and the information about prior Federal support, the last question above.)

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List of Suggested Reviewers or Reviewers Not To Include (optional)

SUGGESTED REVIEWERS:
Not Listed

REVIEWERS NOT TO INCLUDE:
Not Listed

COVER SHEET FOR PROPOSAL TO THE NATIONAL SCIENCE FOUNDATION

PROGRAM ANNOUNCEMENT/SOLICITATION NO./CLOSING DATE/if not in response to a program announcement/solicitation enter NSF 15-1

NSF 14-582

FOR NSF USE ONLY

NSF PROPOSAL NUMBER

FOR CONSIDERATION BY NSF ORGANIZATION UNIT(S) (Indicate the most specific unit known, i.e. program, division, etc.)

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DATE RECEIVED	NUMBER OF COPIES	DIVISION ASSIGNED	FUND CODE	DUNS# (Data Universal Numbering System)	FILE LOCATION
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☐ AN ACCOMPLISHMENT-BASED RENEWAL

IS THIS PROPOSAL BEING SUBMITTED TO ANOTHER FEDERAL AGENCY? YES ☐ NO ☒ IF YES, LIST ACRONYM(S)

NAME OF ORGANIZATION TO WHICH AWARD SHOULD BE MADE

Lang Jaclyn A

AWARDEE ORGANIZATION CODE (IF KNOWN)

P269962704

ADDRESS OF AWARDEE ORGANIZATION, INCLUDING 9 DIGIT ZIP CODE

11849 Goshen Ave. #6
Los Angeles, CA 90049

NAME OF PRIMARY PLACE OF PERF

Universite Paris 13

ADDRESS OF PRIMARY PLACE OF PERF, INCLUDING 9 DIGIT ZIP CODE

Universite Paris 13
Avenue J-B Clement, 93430
Villetaneuse ,FR.

IS AWARDEE ORGANIZATION (Check All That Apply) (See GPG II.C For Definitions)
☐ SMALL BUSINESS ☐ MINORITY BUSINESS ☐ FOR-PROFIT ORGANIZATION ☐ WOMAN-OWNED BUSINESS

☐ IF THIS IS A PRELIMINARY PROPOSAL THEN CHECK HERE

TITLE OF PROPOSED PROJECT **PostDoctoral Research Fellowship**

REQUESTED AMOUNT	PROPOSED DURATION (1-60 MONTHS)	REQUESTED STARTING DATE	SHOW RELATED PRELIMINARY PROPOSAL NO. IF APPLICABLE
$ **150,000**	**48** months	**10/01/16**	

THIS PROPOSAL INCLUDES ANY OF THE ITEMS LISTED BELOW

☐ BEGINNING INVESTIGATOR (GPG I.G.2)
☐ DISCLOSURE OF LOBBYING ACTIVITIES (GPG II.C.1.e)
☐ PROPRIETARY & PRIVILEGED INFORMATION (GPG I.D, II.C.1.d)
☐ HISTORIC PLACES (GPG II.C.2.j)
☐ VERTEBRATE ANIMALS (GPG II.D.6) IACUC App. Date _____
PHS Animal Welfare Assurance Number _____
☒ FUNDING MECHANISM **Fellowship**

☐ HUMAN SUBJECTS (GPG II.D.7) Human Subjects Assurance Number _____
Exemption Subsection _____ or IRB App. Date _____
☒ INTERNATIONAL ACTIVITIES: COUNTRY/COUNTRIES INVOLVED (GPG II.C.2.j)
FR
☒ COLLABORATIVE STATUS
Not a collaborative proposal

PI/PD DEPARTMENT
mathematics

PI/PD FAX NUMBER

PI/PD POSTAL ADDRESS
UCLA Mathematics Department
Box 951555
Los Angeles, CA 90095
United States

NAMES (TYPED)	High Degree	Yr of Degree	Telephone Number	Email Address
PI/PD NAME **Jaclyn A Lang**	**PhD**	**2016**	**303-587-4174**	**jaclynlang@math.ucla.edu**
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				
CO-PI/PD				

CERTIFICATION PAGE

Certification for Authorized Organizational Representative (or Equivalent) or Individual Applicant

By electronically signing and submitting this proposal, the Authorized Organizational Representative (AOR) or Individual Applicant is: (1) certifying that statements made herein are true and complete to the best of his/her knowledge; and (2) agreeing to accept the obligation to comply with NSF award terms and conditions if an award is made as a result of this application. Further, the applicant is hereby providing certifications regarding conflict of interest (when applicable), drug-free workplace, debarment and suspension, lobbying activities (see below), nondiscrimination, flood hazard insurance (when applicable), responsible conduct of research, organizational support, Federal tax obligations, unpaid Federal tax liability, and criminal convictions as set forth in the NSF Proposal & Award Policies & Procedures Guide,Part I: the Grant Proposal Guide (GPG). Willful provision of false information in this application and its supporting documents or in reports required under an ensuing award is a criminal offense (U.S. Code, Title 18, Section 1001).

Certification Regarding Conflict of Interest

The AOR is required to complete certifications stating that the organization has implemented and is enforcing a written policy on conflicts of interest (COI), consistent with the provisions of AAG Chapter IV.A.; that, to the best of his/her knowledge, all financial disclosures required by the conflict of interest policy were made; and that conflicts of interest, if any, were, or prior to the organization's expenditure of any funds under the award, will be, satisfactorily managed, reduced or eliminated in accordance with the organization's conflict of interest policy. Conflicts that cannot be satisfactorily managed, reduced or eliminated and research that proceeds without the imposition of conditions or restrictions when a conflict of interest exists, must be disclosed to NSF via use of the Notifications and Requests Module in FastLane.

Drug Free Work Place Certification

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent), is providing the Drug Free Work Place Certification contained in Exhibit II-3 of the Grant Proposal Guide.

Debarment and Suspension Certification

(If answer "yes", please provide explanation.)

Is the organization or its principals presently debarred, suspended, proposed for debarment, declared ineligible, or voluntarily excluded from covered transactions by any Federal department or agency? Yes ☐ No ☒

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant is providing the Debarment and Suspension Certification contained in Exhibit II-4 of the Grant Proposal Guide.

Certification Regarding Lobbying

This certification is required for an award of a Federal contract, grant, or cooperative agreement exceeding $100,000 and for an award of a Federal loan or a commitment providing for the United States to insure or guarantee a loan exceeding $150,000.

Certification for Contracts, Grants, Loans and Cooperative Agreements

The undersigned certifies, to the best of his or her knowledge and belief, that:
(1) No Federal appropriated funds have been paid or will be paid, by or on behalf of the undersigned, to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with the awarding of any Federal contract, the making of any Federal grant, the making of any Federal loan, the entering into of any cooperative agreement, and the extension, continuation, renewal, amendment, or modification of any Federal contract, grant, loan, or cooperative agreement.
(2) If any funds other than Federal appropriated funds have been paid or will be paid to any person for influencing or attempting to influence an officer or employee of any agency, a Member of Congress, an officer or employee of Congress, or an employee of a Member of Congress in connection with this Federal contract, grant, loan, or cooperative agreement, the undersigned shall complete and submit Standard Form-LLL, ''Disclosure of Lobbying Activities,'' in accordance with its instructions.
(3) The undersigned shall require that the language of this certification be included in the award documents for all subawards at all tiers including subcontracts, subgrants, and contracts under grants, loans, and cooperative agreements and that all subrecipients shall certify and disclose accordingly.

This certification is a material representation of fact upon which reliance was placed when this transaction was made or entered into. Submission of this certification is a prerequisite for making or entering into this transaction imposed by section 1352, Title 31, U.S. Code. Any person who fails to file the required certification shall be subject to a civil penalty of not less than $10,000 and not more than $100,000 for each such failure.

Certification Regarding Nondiscrimination

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Certification Regarding Flood Hazard Insurance

Two sections of the National Flood Insurance Act of 1968 (42 USC §4012a and §4106) bar Federal agencies from giving financial assistance for acquisition or construction purposes in any area identified by the Federal Emergency Management Agency (FEMA) as having special flood hazards unless the:
(1) community in which that area is located participates in the national flood insurance program; and
(2) building (and any related equipment) is covered by adequate flood insurance.

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) or Individual Applicant located in FEMA-designated special flood hazard areas is certifying that adequate flood insurance has been or will be obtained in the following situations:
(1) for NSF grants for the construction of a building or facility, regardless of the dollar amount of the grant; and
(2) for other NSF grants when more than $25,000 has been budgeted in the proposal for repair, alteration or improvement (construction) of a building or facility.

Certification Regarding Responsible Conduct of Research (RCR)
(This certification is not applicable to proposals for conferences, symposia, and workshops.)

By electronically signing the Certification Pages, the Authorized Organizational Representative is certifying that, in accordance with the NSF Proposal & Award Policies & Procedures Guide, Part II, Award & Administration Guide (AAG) Chapter IV.B., the institution has a plan in place to provide appropriate training and oversight in the responsible and ethical conduct of research to undergraduates, graduate students and postdoctoral researchers who will be supported by NSF to conduct research. The AOR shall require that the language of this certification be included in any award documents for all subawards at all tiers.

CERTIFICATION PAGE - CONTINUED

Certification Regarding Organizational Support

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that there is organizational support for the proposal as required by Section 526 of the America COMPETES Reauthorization Act of 2010. This support extends to the portion of the proposal developed to satisfy the Broader Impacts Review Criterion as well as the Intellectual Merit Review Criterion, and any additional review criteria specified in the solicitation. Organizational support will be made available, as described in the proposal, in order to address the broader impacts and intellectual merit activities to be undertaken.

Certification Regarding Federal Tax Obligations

When the proposal exceeds $5,000,000, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Federal tax obligations. By electronically signing the Certification pages, the Authorized Organizational Representative is certifying that, to the best of their knowledge and belief, the proposing organization:
(1) has filed all Federal tax returns required during the three years preceding this certification;
(2) has not been convicted of a criminal offense under the Internal Revenue Code of 1986; and
(3) has not, more than 90 days prior to this certification, been notified of any unpaid Federal tax assessment for which the liability remains unsatisfied, unless the assessment is the subject of an installment agreement or offer in compromise that has been approved by the Internal Revenue Service and is not in default, or the assessment is the subject of a non-frivolous administrative or judicial proceeding.

Certification Regarding Unpaid Federal Tax Liability

When the proposing organization is a corporation, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Federal Tax Liability:

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that the corporation has no unpaid Federal tax liability that has been assessed, for which all judicial and administrative remedies have been exhausted or lapsed, and that is not being paid in a timely manner pursuant to an agreement with the authority responsible for collecting the tax liability.

Certification Regarding Criminal Convictions

When the proposing organization is a corporation, the Authorized Organizational Representative (or equivalent) is required to complete the following certification regarding Criminal Convictions:

By electronically signing the Certification Pages, the Authorized Organizational Representative (or equivalent) is certifying that the corporation has not been convicted of a felony criminal violation under any Federal law within the 24 months preceding the date on which the certification is signed.

AUTHORIZED ORGANIZATIONAL REPRESENTATIVE	SIGNATURE	DATE
NAME		

TELEPHONE NUMBER	EMAIL ADDRESS	FAX NUMBER

Application Form

National Science Foundation
Mathematical Sciences Postdoctoral
Research Fellowships

1

NAME

Lang	Jaclyn	A.		
Last	First	MI	TITLE	Other Names Used

MAILING ADDRESS for correspondence. It is imperative that you keep NSF informed of a current mailing address.
UCLA Mathematics Department
Los Angeles CA 90095

CURRENT ADDRESS (if different from Permanent Mailing Address):

CURRENT E-MAIL ADDRESS: jaclynlang@math.ucla.edu

TELEPHONE NUMBERS:
At Home: (303)587-4174
At your institution: (303)587-4174
FAX number:

CURRENT POSITION
Department Mathematics
Institution University of California, Los Angeles
Type of Institution university
Title of Position graduate student
Date this position started 08/09/2010
Source of support NSF GRFP, Cota-Robles, UCLA

CITIZENSHIP (check one)
✖ US Citizen or National ___ Permanent Resident ___ Foreign National(give country: ___)
If you are a Permanent Resident you must provide your Permanent Resident Registration Number: ___

HIGHEST DEGREE EARNED: PhD YEAR CONFERRED: 2016

DOCTORAL DEGREE
Department and Institution Mathematics University of California, Los Angeles
Date awarded or anticipated date of receipt 06/09/2016

PROPOSED FELLOWSHIP INSTITUTION including department and address
Universite Paris 13 Mathematics
Avenue J-B Clement, 93430
Paris, France

PROPOSED SPONSORING SCIENTIFIC ADVISOR OR MENTOR including title, e-mail address, telephone and FAX numbers:
Tilouine Jacques
Professeur jacques.tilouine@free.fr
(331)494-0408 (331)494-0356
(Telephone) (FAX)

BRIEF TITLE of your proposed research or field of study:
Images of Galois representations in p-adic families

Application Form

National Science Foundation
Mathematical Sciences Postdoctoral
Research Fellowships

1

REFERENCES(do not use your mentor).

	Name	Department	Institution	Phone
1.	Poonen Bjorn	Mathematics	MIT	(617)258-8164
2.	Khare Chandrashekhar	mathematics	UCLA	(310)825-2082
3.	Hida Haruzo	mathematics	UCLA	(310)206-3382
4.				

EXPECTED STARTING DATE OF FELLOWSHIP: 10/2016 (Month, year) DURATION OF TENURE: 48 (# of months)

OTHER SUPPORT

Have you applied for any other fellowships or similar appointments for all or part of the tenure herein requested? YES

If so, name of agency or program U.S. Fulbright student program

Have you ever received any NSF postdoctoral fellowship? NO

If yes, when and from what program? _____

Please provide the following information:

OTHER INSTITUTION AND SPONSOR if short-term international experience is proposed

_____

Application Form (Part B)

National Science Foundation
Mathematical Sciences Postdoctoral
Research Fellowships

2

NAME : Lang (last) Jaclyn (first) A (middle)

TITLE OF RESEARCH PLAN : Images of Galois representations in p-adic families

1. Provide two-digit classifications of your proposed research according to the classification system list on page 7:
   Primary code 11 Secondary areas 14 13 0

2. It is normally desirable to select an institution different from where you received your doctoral degree or where you are currently employed; please provide justification (on separate sheet) if your proposed fellowship institution is the same as either of the above. It is not necessary to provide justification if you have been currently employed at your proposed fellowship institution for less than one year and it is not where you received your doctoral degree; if that is the case, please check here: ____

   No Justification provided.

3. Do you plan to use the Fellowship for 18 academic months full-time? X

   or the Instructorship for 9 months full-time and 18 months half-time? ____

   If half-time, have you secured a suitable commitment for a teaching position at your host institution?

   ____ Yes ____ No

4. Specify the area of mathematical science(pure mathematics, applied mathematics, statistics) for your proposed research: Pure Mathematics

Application Form (Part B)

National Science Foundation
Mathematical Sciences Postdoctoral
Research Fellowships

2

NAME : Lang (last) Jaclyn (first) A (middle) S.S.N : *** ** ****

JUSTIFICATION:

Project Summary–Jaclyn Lang

Overview. The Principal Investigator (PI) proposes to study the images of Galois representations attached to modular forms and p-adic families of such objects. Of particular interest are representations associated to Hida families. The PI has already proved a qualitative result showing that, in the non-CM case, such representations have large images. In particular, her previous work identifies a ring $\mathbb{I}_0$, cut out by certain symmetries of the p-adic family, such that the image of the Galois representations contains the kernel of reduction modulo some $\mathbb{I}_0$-ideal $\mathfrak{a}_0$. The **first research objective** of the proposed project is to quantitatively study $\mathfrak{a}_0$ and obtain a precise description of $\mathfrak{a}_0$ based on the shape of the residual representation. This is expected to yield a relationship between images and Katz p-adic L-functions.

The **second research objective** is to quantitatively study the images of Galois representations associated to classical modular forms by computing the extent to which they fail to contain SL_2 of an appropriate ring. This will be done by relating the image of such representations to the congruence number of the corresponding modular form and then using Sage [23] to compute congruence numbers. The congruence number is expected to be related to certain values of the Katz p-adic L-function.

The **third research objective** is to prove a p-adic analogue of the Mumford-Tate Conjecture for Hida families. Let ρ be the Galois representation of a non-CM Hida family. Hida has conjectured that there is a simple algebraic group G, defined over $\mathbb{Q}_p$, such that the image of each p-adic specialization of ρ can be obtained from G through an appropriate base change, at least up to abelian error. The group G plays a role analogous to the Mumford-Tate group for compatible systems of Galois representations. The goal of this project is to show that G exists and give a precise description of G. It is expected that the finer information about the images of Galois representations attached to Hida families in the first objective will be useful in achieving this goal.

The **fourth research objective** is to study the above questions in the non-ordinary setting and for representations valued in groups bigger than GL_2. This line of inquiry has already been started in two of the Sponsoring Scientist Tilouine's collaborations [2, 9].

Intellectual Merit. Completion of the proposed objectives will give number theorists a precise understanding of the images of Galois representations associated to Hida families while also relating those images important objects such as p-adic L-functions and congruence numbers. The second objective will provide the first significant computations of images of classical modular Galois representations that do not come from elliptic curves. The third research objective will prove a heretofore unknown uniformity among images of classical Galois representations in a Hida family. In particular, it will provide a p-adic uniformity of images of Galois representations in a Hida family analogous to the uniformity of images of Galois representations in a compatible system given by the classical Mumford-Tate Conjecture. Finally, the fourth objective will begin to investigate the generality of the phenomena discovered through the first three objectives.

Broader Impacts. Knowledge of the images of Galois representations has been useful for various methods in number theory including the construction of Euler systems [11, 13] and solutions to the inverse Galois problem [24, 25]. The PI hopes to engage undergraduate and masters students in the coding aspects of the second research objective. Furthermore, the programs developed in the second research objective, along with the data they generate, will be made publicly available.

The PI has an extensive record of mentoring underrepresented minorities and women in STEM disciplines as well as sharing her research with a wide variety of audiences. She plans to continue her mentoring as a postdoc and hopes to work with the European Women in Mathematics to organize conferences for women in mathematics in Paris. She is acquiring the French language skills needed for her to share her research with both young students and current researchers.

TABLE OF CONTENTS

For font size and page formatting specifications, see GPG section II.B.2.

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Table of Contents	1	
Project Description (Including Results from Prior NSF Support) (not to exceed 15 pages) **(Exceed only if allowed by a specific program announcement/solicitation or if approved in advance by the appropriate NSF Assistant Director or designee)**	5	
References Cited	2	
Biographical Sketches (Not to exceed 2 pages each)	2	
Budget (Plus up to 3 pages of budget justification)	0	
Current and Pending Support	0	
Facilities, Equipment and Other Resources	0	
Special Information/Supplementary Documents (Data Management Plan, Mentoring Plan and Other Supplementary Documents)	3	
Appendix (List below.) **(Include only if allowed by a specific program announcement/ solicitation or if approved in advance by the appropriate NSF Assistant Director or designee)**		
Appendix Items:		

*Proposers may select any numbering mechanism for the proposal. The entire proposal however, must be paginated. Complete both columns only if the proposal is numbered consecutively.

NSF MSPRF – PROJECT DESCRIPTION: IMAGES OF p-ADIC FAMILIES OF GALOIS REPRESENTATIONS

JACLYN LANG

INTRODUCTION

Galois representations are fundamental objects of study in modern number theory. They are the only known tools for systematically studying the absolute Galois group of the rational numbers, $G_{\mathbb{Q}}$, and geometric Galois representations are one side of the celebrated Langlands correspondence. A Galois representation is a continuous homomorphism $\rho : G_{\mathbb{Q}} \to G(A)$ for some linear algebraic group G and topological ring A. Most of the known examples of Galois representations arise from an action of $G_{\mathbb{Q}}$ on the cohomology of varieties defined over a number field, or by putting such representations into p-adic families.

A fundamental problem is to determine the image of a given Galois representation. This was first done by Serre [21] for the p-adic Galois representation $\rho_{E,p}$ associated to an elliptic curve E over $\mathbb{Q}$. He showed that, so long as E does not have complex multiplication (CM), $\rho_{E,p}$ is surjective for all but finitely many primes p. Furthermore the image of $\rho_{E,p}$ is open for all primes p. Serre's result is an example of a general pattern governing the expected behavior of images of Galois representations.

Heuristic. *The image of a Galois representation should be as large as possible, subject to the symmetries of the geometric object from which it arose.*

The notion of "symmetry" is vague and depends on the situation. In the case of elliptic curves, the relevant symmetry is complex multiplication, a condition that means that the elliptic curve has a larger endomorphism ring than usual.

Galois representations also arise from modular forms. Given a normalized cuspidal Hecke eigenform $f = \sum_{n=1}^{\infty} a_n q^n$ of level N and an embedding $\iota_p : \overline{\mathbb{Q}} \hookrightarrow \overline{\mathbb{Q}}_p$, Deligne [3] showed that there is a finite extension $\mathcal{O}$ of $\mathbb{Z}_p$ and a Galois representation $\rho_{f,\iota_p} : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathcal{O})$ that is unramified at primes ℓ not dividing Np and satisfies $\operatorname{tr} \rho_{f,\iota_p}(\mathrm{Frob}_\ell) = a_\ell$ for all such ℓ. The image of such a Galois representation was determined up to finite error by Ribet and Momose [16, 19]. Ribet introduced a new symmetry of a modular form that can be viewed as a weakening of the CM condition.

Definition 1. [Ribet [18]] Let f be as above and let K be the number field generated by $\{a_n : n \in \mathbb{Z}_{\geq 1}\}$. An automoprhism σ of K is a *conjugate self-twist* of f if there is a non-trivial Dirichlet character η_σ such that $a_\ell^\sigma = \eta_\sigma(\ell) a_\ell$ for almost all primes ℓ. (If the identity automorphism is a conjugate self-twist of f then f has *CM*.)

Let $\mathcal{O}_0$ be the ring of integers in the field fixed by all conjugate self-twists of f. Write $\mathcal{O}_{0,\mathfrak{p}}$ for the closure of $\iota_p(\mathcal{O}_0)$ in $\overline{\mathbb{Q}}_p$. It is a discrete valuation ring whose maximal ideal will be denote by $\mathfrak{p}$. For any commutative ring A and ideal $\mathfrak{a}$ of A, write

$$\Gamma_A(\mathfrak{a}) = \ker(\mathrm{SL}_2(A) \to \mathrm{SL}_2(A/\mathfrak{a})).$$

Ribet and Momose proved the following theorem.

Theorem 2 (Ribet [19], Momose [16])**.** *Let f be a non-CM modular form and fix an embedding $\iota_p : \overline{\mathbb{Q}} \hookrightarrow \overline{\mathbb{Q}}_p$. Then the image of ρ_{f,ι_p} contains an open subgroup of $\mathrm{SL}_2(\mathcal{O}_{0,\mathfrak{p}})$, i.e. $\operatorname{Im} \rho_{f,\iota_p} \supseteq \Gamma_{\mathcal{O}_{0,\mathfrak{p}}}(\mathfrak{p}^n)$*

for some non-negative integer n. Furthermore, $\operatorname{Im}\rho_{f,\iota_p} \supseteq \mathrm{SL}_2(\mathcal{O}_{0,\mathfrak{p}})$ *for almost all* ι_p*, as* p *ranges over all primes.*

Let $c(f,\mathfrak{p})$ be the smallest non-negative integer such that $\operatorname{Im}\rho_{f,\mathfrak{p}} \supseteq \Gamma_{\mathcal{O}_{0,\mathfrak{p}}}(\mathfrak{p}^{c(f,\mathfrak{p})})$, which exists by Theorem 2. It is called the $\mathcal{O}_{0,\mathfrak{p}}$-*level* of $\rho_{f,\mathfrak{p}}$.

In the 1980s, Hida showed that if a_p is a p-adic unit, then f can be put into a p-adic family of modular forms. Let $\Lambda = \mathbb{Z}_p[[T]]$ and $\mathbb{I}$ an integral domain that is finite flat over Λ. An *arithmetic* prime of Λ is a prime of the form

$$P_{k,\zeta} = (1 + T - \zeta(1+p)^k)\Lambda, \tag{1}$$

for some integer $k \geq 2$ and p-power root of unity ζ. Primes of $\mathbb{I}$ lying over such primes are also called *arithmetic*. A formal power series $F = \sum_{n=1}^{\infty} A_n q^n$ is a *Hida family* if $A_p \in \mathbb{I}^\times$ and for every arithmetic prime $\mathfrak{P}$ of $\mathbb{I}$: $A_n \bmod \mathfrak{P} \in \iota_p(\overline{\mathbb{Q}})$ (rather than just in $\overline{\mathbb{Q}}_p$) and $f_{\mathfrak{P}} := \sum_{n=1}^{\infty}(A_n \bmod \mathfrak{P})q^n$ is the q-expansion of a classical modular form of weight k with appropriate level and nebentypus. Hida showed that whenever $a_p(f)$ is a p-adic unit, there is a unique Hida family F and arithmetic prime $\mathfrak{P}$ such that $f = f_{\mathfrak{P}}$ [6].

The primary objects that the PI proposes to study are Galois representations associated to Hida families. Hida showed [5] that (so long as F is an eigenform and a certain irreducibility criteria is satisfied) there is a big Galois representation

$$\rho_F : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{I})$$

that is unramified outside a finite set of primes including p, and such that for all unramified primes ℓ, one has $\operatorname{tr}\rho_F(\mathrm{Frob}_\ell) = A_\ell$. Since $\mathbb{I}$ is a local ring, reduction modulo the unique maximal ideal yields the mod p residual representation $\bar{\rho}_F$ of ρ_F. Note that the definitions of CM and conjugate self-twist carry over to Hida families by replacing K with the field of fractions $Q(\mathbb{I})$ of $\mathbb{I}$ and using the q-expansion of F in Definition 1. Let Γ_F denote the group generated by all conjugate self-twists of F and $\mathbb{I}_0$ the integral closure of Λ in $Q(\mathbb{I})^{\Gamma_F}$.

Until recently, the known results describing the image of ρ_F for non-CM F required that $\operatorname{Im}\bar{\rho}_F \supseteq \mathrm{SL}_2(\mathbb{F}_p)$. Under this assumption, Mazur and Wiles proved that $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\Lambda)$ if $\mathbb{I} = \Lambda$ [15]. Later Fischman improved their result by showing that $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\mathbb{I}_0)$ when $\mathbb{I}$ is a power series ring [4]. Recently Hida proved that ρ_F has big image assuming only a minor regularity condition on the residual representation.

Theorem 3 (Hida [8])**.** *Let $p > 3$ and let F be a non-CM Hida family that satisfies a minor regularity condition. Then there is a non-zero ideal $\mathfrak{a}$ of Λ such that, in an appropriate basis,* $\Gamma_\Lambda(\mathfrak{a}) \subseteq \operatorname{Im}\rho_F$.

The largest Λ-ideal $\mathfrak{a}$ such that $\Gamma_\Lambda(\mathfrak{a}) \subseteq \operatorname{Im}\rho_F$ is called the Λ-*level* $\mathfrak{c}_F$ of ρ_F. Hida determined the Λ-level of F in many cases, including those described in the following theorem.

Theorem 4 (Hida [8])**.** *Let F and p be as in Theorem 3.*

- *(a) If* $\operatorname{Im}\bar{\rho}_F \supseteq \mathrm{SL}_2(\mathbb{F}_p)$*, then* $\mathfrak{c}_F \supseteq \mathfrak{m}_\Lambda^n$ *for some non-negative integer n, where $\mathfrak{m}_\Lambda$ denotes the unique maximal ideal of Λ.*
- *(b) Suppose that $\bar{\rho}_F$ is absolutely irreducible and* $\bar{\rho}_F \cong \mathrm{Ind}_M^{\mathbb{Q}}\bar{\psi}$ *for an imaginary quadratic field M in which p splits and a character* $\bar{\psi} : \mathrm{Gal}(\overline{\mathbb{Q}}/M) \to \overline{\mathbb{F}}_p^\times$*. Assume M is the only such quadratic field. Under minor conditions on the tame level of F, there is a product $\mathcal{L}$ of anticyclotomic Katz p-adic L-functions such that* $\mathfrak{c}_F | \mathcal{L}^2$*. Furthermore, for every prime divisor P of $\mathcal{L}$ there exists a Hida family F such that* $P | \mathfrak{c}_F$.

The goal of the PI is to understand the images of Galois representations ρ_F as precisely as possible by using conjugate self-twists of Hida families.

PAST WORK OF THE PI

The past work of the PI, which was supported by an NSF Graduate Research Fellowship (Grant No. DGE-1144087), is a generalization of Hida's Theorem 3 and an analogue of Theorem 2. The main result of the PI's paper [10] is the following theorem.

Theorem 5 (Lang [10])**.** *Let F be a non-CM Hida family such that $\bar{\rho}_F$ is absolutely irreducible and satisfies a minor $\mathbb{Z}_p$-regularity condition. Then there is a non-zero ideal $\mathfrak{a}_0$ of $\mathbb{I}_0$ and a basis for ρ_F such that $\operatorname{Im}\rho_F \supseteq \Gamma_{\mathbb{I}_0}(\mathfrak{a}_0)$.*

As in the Λ-adic case, the $\mathbb{I}_0$-*level* $\mathfrak{c}_{0,F}$ is the largest $\mathbb{I}_0$-ideal for which $\Gamma_{\mathbb{I}_0}(\mathfrak{c}_{0,F}) \subseteq \operatorname{Im}\rho_F$. Part of the proposed project (see below) is to determine the $\mathbb{I}_0$-level.

The proof of Theorem 5 goes as follows. First, the PI proves the following lifting theorem, which shows when conjugate self-twists of specializations of F can be lifted to conjugate self-twists of all of F.

Theorem 6 (Lang [10])**.** *Let $\mathfrak{P}$ be an arithmetic prime of $\mathbb{I}$ and σ be a conjugate self-twist of $f_{\mathfrak{P}}$ that is also an automorphism of the local field $\mathbb{Q}_p(\{a(n, f_{\mathfrak{P}}) : n \in \mathbb{Z}^+\})$. Then σ can be lifted to a conjugate self-twist of F.*

Using the lifting theorem, there is a series of algebraic reduction steps to conclude that Theorem 2 implies Theorem 5. One tool that is particularly important in the reduction steps is a $\mathbb{Z}_p$-Lie algebra of Pink [17] that can be associated to $\operatorname{Im}\rho_F$ as in [8]. It is critical to the proof that this Lie algebra can be endowed with the structure of a Λ-module. Doing so depends crucially on the fact that ρ_F is ordinary. In particular, it is known [7, Theorem 4.3.2] that for the inertia group I_p, $\operatorname{Im}\rho_F|_{I_p}$ contains an element of the form $\left(\begin{smallmatrix} 1+T & * \\ 0 & 1 \end{smallmatrix}\right)$.

RESEARCH OBJECTIVES

Determining the $\mathbb{I}_0$-level and relation to p-adic L-functions. The first project is to strengthen Hida's Theorem 4 by replacing the Λ-level by the $\mathbb{I}_0$-level, enabling the PI to prove the following.

Conjecture 1. *Under the assumptions of Theorem 4:*

- *(a) If $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\mathbb{F}_p)$, then $\mathfrak{c}_{0,F} = \mathbb{I}_0$. That is, $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\mathbb{I}_0)$.*
- *(b) Under the hypotheses of Theorem 4(b) there is an $\mathbb{I}_0$-analogue $\mathcal{L}_0$ of $\mathcal{L}$ such that $\mathfrak{c}_{0,F}$ is a factor of $\mathcal{L}_0$. Furthermore, every prime factor of $\mathcal{L}_0$ divides $\mathfrak{c}_{0,F}$ for some F.*

Note that even when $\mathbb{I} = \Lambda$, Conjecture 1 is stronger than Theorem 4. Furthermore, Conjecture 1(a) is a natural extension of the work of Mazur-Wiles [15] and Fischman [4].

In proving case (a) of the conjecture, the PI will make use of Manoharmayum's recent work that shows $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(W)$ for a finite unramified extension W of $\mathbb{Z}_p$ [14]. This will be combined with the Λ-module structure on the Pink Lie algebra associated to $\operatorname{Im}\rho_F$ that was used in the proof of Theorem 5 to get the desired result.

In proving case (b), the PI will relate the $\mathbb{I}_0$-level to the congruence ideal of F as in the proof of Theorem 4(b). The connection to Katz p-adic L-functions is then obtained by relating the congruence ideal to the p-adic L-function through known cases of the Main Conjecture of Iwasawa Theory. The idea is that replacing the Λ-level with the more precise $\mathbb{I}_0$-level will allow the PI to remove the ambiguity of the square factors that show up in Theorem 4(b).

Proving Conjecture 1 would yield refined information about the images of Galois representations attached to Hida families. It is the first step in completely determining the images of such representations. Once the $\mathbb{I}_0$-level of a representation ρ_F is understood, one can study the image of the representation ρ_F mod $\mathfrak{c}_{0,F}$ to obtain a complete understanding of $\operatorname{Im}\rho_F$.

Computing $\mathcal{O}_{0,\mathfrak{p}}$-levels of classical Galois representations. Let f be a non-CM classical Hecke eigenform. By Theorem 2, $c(f,\mathfrak{p}) = 0$ for all but finitely many primes $\mathfrak{p}$ of the ring of integers $\mathcal{O}$ generated by the Fourier coefficients of f. However, relatively little is known about the case when $c(f,\mathfrak{p})$ is positive. The PI will study how $c(f,\mathfrak{p})$ changes as f varies over the classical specializations of a non-CM Hida family that is congruent to a CM family.

This project will have both theoretical and computational components. First, the PI will establish a relationship between $c(f,\mathfrak{p})$ and the congruence number of f, which should also be related to values of the Katz p-adic L-function, as suggested by the proof of Theorem 4(b). Once this is established, the PI will create a method in the open-source software Sage [23] to compute $c(f,\mathfrak{p})$ by computing the congruence number of f. This should be relatively straightforward since Sage can already compute congruence numbers, and the PI hopes to involve undergraduates in the coding process. Using the new functionality, the PI will create a large data set of levels of classical Galois representations in Hida families, which will likely lead to new conjectures to be studied theoretically. The PI has experience working with Sage from her Women in Numbers 3 project [1] and from leading a project at Sage Days 69. Indeed, her project consisted of writing a method to test whether a modular form is CM, a first step in the eventual program to compute $c(f,\mathfrak{p})$.

Most of the computation that has been done surrounding levels of classical Galois representations has been for Galois representations arising from elliptic curves over $\mathbb{Q}$ [20, 22, 26]. Relatively little is known for higher weight forms. Completing the computational aspect of this project will begin to fill that gap in the literature. Furthermore, computational data often reveals interesting patterns and questions that can be studied theoretically. This project will bring number theorists close to being able to completely and computationally determine the image of a Galois representation associated to a classical modular form.

Analogue of the Mumford-Tate Conjecture in p-adic families. One can rephrase the work of Ribet and Momose by saying that they proved the Mumford-Tate Conjecture for compatible systems of Galois representations associated to classical modular forms. Hida has proposed an analogue of the Mumford-Tate Conjecture for p-adic families of Galois representations. For an arithmetic prime $\mathfrak{P}$ of $\mathbb{I}$, write $\mathrm{MT}_{\mathfrak{P}}$ for the Mumford-Tate group of the compatible system containing $\rho_{f_{\mathfrak{P}}}$, so $\mathrm{MT}_{\mathfrak{P}}$ is an algebraic group over $\mathbb{Q}$. Let $\kappa(\mathfrak{P}) = \mathbb{I}_{\mathfrak{P}}/\mathfrak{P}_{\mathfrak{P}}$, and write $G_{\mathfrak{P}}$ for the Zariski closure of $\operatorname{Im}\rho_{f_{\mathfrak{P}}}$ in $\mathrm{GL}_2(\kappa(\mathfrak{P}))$. Let $G^{\circ}_{\mathfrak{P}}$ be the connected component of the identity of $G_{\mathfrak{P}}$ and $G'_{\mathfrak{P}}$ the (closed) derived subgroup of $G_{\mathfrak{P}}$.

Conjecture 2 (Hida)**.** *Assume F is non-CM. There is a simple algebraic group G', defined over $\mathbb{Q}_p$, such that for all arithmetic primes $\mathfrak{P}$ of $\mathbb{I}$ one has $G'_{\mathfrak{P}} \cong G' \times_{\mathbb{Q}_p} \kappa(\mathfrak{P})$ and $\mathrm{Res}^{\kappa(\mathfrak{P})}_{\mathbb{Q}_p} G_{\mathfrak{P}}$ is (the ordinary factor of) $\mathrm{MT}_{\mathfrak{P}} \times_{\mathbb{Q}} \mathbb{Q}_p$. Furthermore, the component group $G_{\mathfrak{P}}/G^0_{\mathfrak{P}}$ is canonically isomorphic to the Pontryagin dual of the decomposition group of $\mathfrak{P}$ in Γ_F.*

By obtaining a sufficiently precise understanding of images of Galois representations attached to Hida families through the first project, the PI hopes to prove results along the lines of Conjecture 2. The PI has some preliminary results relating the Pontryagin dual of Γ_F to the quotient $(\operatorname{Im}\rho_F)/(\operatorname{Im}\rho_F|_H)$ for a certain finite index normal subgroup H of $G_{\mathbb{Q}}$.

Completing this research objective, or even any preliminary results in this direction, would reveal that the images of classical specializations of the Galois representation attached to a Hida family are even more related to one another than previously thought. Not only would they come as specializations of some group in $\mathrm{GL}_2(\mathbb{I})$, they could all be found simply by base change from a single group, at least up to abelian error.

Other settings. The above projects can be studied in more general settings than Hida families for GL_2. Hida and Tilouine proved an analogue of Theorem 3 for GSp_4-representations associated

to Hida families of Seigel modular forms [9]. There are two main difficulties they overcome in their work: the types of symmetries are much more complicated than CM versus non-CM, and Pink's theory of Lie algebras is only valid for SL_2. The tools they developed to overcome these problems could be applied to study analogues of the above questions for groups other than GL_2.

Tilouine and his collaborators proved an analogue of Theorem 5 in the non-ordinary GL_2-setting [2] by building on the PI's ideas in [10]. As their representation is not ordinary, they introduce the relative Sen operator to create a Λ-algebra structure on the Lie algebra of the image of ρ_F. This idea will be useful in studying the above questions in the non-ordinary setting.

Sponsoring scientist, host institution, and career development

Jacques Tilouine is the foremost senior number theorist currently working on questions about images of p-adic families of Galois representations. He has developed new tools for working in the non-ordinary setting [2] and with groups bigger than GL_2 [9]. The PI will benefit from learning those tools during her fellowship tenure. The PI will further benefit from the vibrant number theory community in Paris. There are frequent seminars and workshops with experts in her field at Université Paris-Sud, Université Diderot, and Université Paris 13. Benoît Stroh (Université Paris 13) and Gaëtan Chenevier (Université Paris-Sud) are experts on Galois representations and will be invaluable resources, exposing the PI to additional new techniques and questions in the field.

The PI will pursue a career as an academic research mathematician in the United States. The MSPRF will allow the PI to establish her research program before starting a permanent position while being mentored by some of the foremost experts in Galois representations. The mathematical relationships that the PI establishes in Paris are likely to lead to international collaborations that will bear fruit well beyond the tenure of the fellowship. The travel funds from the fellowship will facilitate her sharing her research and collaborating with mathematicians throughout Europe.

broader impacts

Detailed information about images of Galois representations is important for other methods in number theory. For example, the Euler systems recently constructed by Lei, Loeffler, and Zerbes require big image results [11, 13]. Furthermore, precise descriptions of images of Galois representations often lead to new solutions to the inverse Galois problem [24, 25].

The third proposed project will engage young researchers and generate data and Sage programs that the PI will make publicly available. The data of the $\mathcal{O}_{0,\mathfrak{p}}$-levels of classical Galois representations will be added to the L-Functions and Modular Forms Database [12]. Both the programs and the data they generate will be made available on the PI's webpage. After the PI develops a theoretical algorithm for computing $\mathcal{O}_{0,\mathfrak{p}}$-levels, she will teach young researchers the necessary background and coding skills for them to implement the algorithm and generate the desired data.

The PI has an extensive record of mentoring women in mathematics. She cofounded and organized the UCLA Women in Math group, which included mentoring young women graduate students and inviting distinguished women to campus. She served as a teaching assistant at the George Washington University's Summer Program for Women in Mathematics in 2012, where she taught and mentored undergraduate women majoring in math from across the country. She plans to continue this mentoring as a postdoc and hopes to work with the organization European Women in Mathematics to organize conferences for women in mathematics in Paris.

The PI has engaged a wide variety of people in mathematics, including underprivileged middle school girls, high school students, undergraduate math majors, mathematicians outside of number theory, and specialists in her field. She is taking French classes to improve her ability to engage with such audiences in France as a mathematical and cultural ambassador. She will share mathematics with young students while simultaneously participating in the Paris number theory seminars.

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BIOGRAPHICAL SKETCH - JACLYN LANG

UCLA Mathematics Department
Los Angeles, CA 90095-1555

jaclynlang@math.ucla.edu
www.math.ucla.edu/~jaclynlang/

PROFESSIONAL PREPARATION

Bryn Mawr College	Bryn Mawr, PA	mathematics	B.A./M.A., 2009
University of Cambridge	Cambridge, UK	pure mathematics	CASM, 2010
University of California, Los Angeles	Los Angeles, CA	mathematics	Ph.D., 2016 (expected)

APPOINTMENTS

- Cota-Robles Fellow (2010-2011, 2015-2016)
- NSF Graduate Research Fellow (2011-2013, 2014-2015)
- UCLA Math Dept. Teaching Assistant Consultant (Fall 2013)
- Summer Program for Women in Mathematics teaching assistant (Summer 2012)
- UCLA Math Dept. Teaching Assistant (Spring 2011); Teaching Fellow (2013-2014)
- Program in Mathematics for Young Scientists Counselor (Summer 2010)

PUBLICATIONS

Related to proposed project

(1) LANG, J. On images of Galois representation associated to a non-CM Hida family. submitted to *Algebra and Number Theory*, http://www.math.ucla.edu/~jaclynlang/I_0_level_existence.pdf, 2015

Other publications

(1) BALAKRISHNAN, J., ÇIPERIANI, M., LANG, J., MIRZA, B., AND NEWTON, R. Shadow lines in the arithmetic of elliptic curves. to appear in *Women in Numbers 3 Proceedings*, https://www.ma.utexas.edu/users/mirela/ShadowLines.pdf, 2015.

(2) DAUB, M., LANG, J., MERLING, M., PITIWAN, N., PACELLI, A., AND ROSEN, M. Function fields with class number indivisible by a prime ℓ. *Acta Arith.150*, 4 (2011), 339–359.

SYNERGISTIC ACTIVITIES

Increasing participation of underrepresented minorities and women in mathematics

- Co-organized UCLA Women in Math group (2010-present); activities included mentoring young women graduate students, inviting distinguished women mathematicians to campus
- Presenter at EmpowHer STEM Day (2014-2015); presented probability to underprivileged middle school girls through an interactive demonstration of the Monty Hall Problem
- Yale Bouchet Conference on Diversity and Graduate Education (2014); inducted into Bouchet Graduate Honor Society and participated in conference on techniques for increasing diversity in academia

- Curriculum development for MATH 495 (2013); co-created the curriculum for MATH 495 which teaches new math graduate students how to be effective teaching assistants; introduced a lesson on stereotype threat and diversity in the classroom

Talks

Invited talks

- Five Colleges Number Theory Seminar, Amherst College (September 2015)
- Number Theory Seminar, Massachusetts Institute of Technology (September 2015)
- Mathematics Colloquium, Loyola Marymount University (February 2015)
- Number Theory Seminar, University of Texas, Austin (March 2014)
- Mathematics Colloquium, California State Polytechnic University, Pomona (January 2014)

Contributed talks (outside home institution)

- AMS Western Sectional Meeting, CSU-Fullerton (October 2015)
- BU-Keio U. Workshop, Boston University (September 2015)
- Number Theory Conference, University of Illinois at Urbana-Champaign (August 2015)
- Graduate Summer School on "New Geometric Techniques in Number Theory", Mathematical Sciences Research Institute (July 2013)
- Women in Mathematics in Southern California Symposium, Loyola Marymount University (October 2012)

Peer review service

- Referee for *Mathematical Research Letters*
- Referee for *John Coates' 70th Birthday Conference Proceedings*

Contributions to SAGE (open source software)

- Helped create method to compute shadow lines of elliptic curves as part of Women in Numbers: 3 project.
- Created a method `has_cm()` at SAGE Days 69: Women in SAGE 6 to test whether a given modular form has CM.

Leadership positions

- President of UCLA Mathematics Department Graduate Student Organization (2012-2014)
- Organizer of UCLA Mathematics Department professional development workshops: applying for the NSF Graduate Research Fellowship; how to give a math talk; panel on academic jobs; peer editing for academic job documents
- Panelist for undergraduate math majors interested in graduate school: The Aftermath Conference, Harvey Mudd College (February 2013); UCLA (October 2011)

COLLABORATORS AND OTHER AFFILIATIONS

Collaborators (Total: 9) Jennifer Balakrishnan (University of Oxford), Mirela Çiperiani (University of Texas, Austin), Michael Daub (Amazon), Bahare Mirza (McGill University), Mona Merling (Johns Hopkins University), Rachel Newton (Institut des Hautes Études Scientifiques), Allison Pacelli (Williams College), Natee Pitiwan (Chulalongkorn University), Michael Rosen (Brown University)

Graduate Advisors (Total: 3) Haruzo Hida (University of California, Los Angeles), Tom Fisher (Trinity College, University of Cambridge), Helen Grundman (Bryn Mawr College)

DATA MANAGEMENT PLAN

JACLYN LANG

1. Types of data

With the exception of the second research objective, the proposed project is theoretical in nature. As such, the most likely type of data to be produced is papers that will be published in peer-reviewed mathematical journals.

The second research objective will produce both programs in the open-source mathematical software Sage as well as a large set of data consisting of the Galois level of many classical modular forms.

2. Access to data

Preprints of the papers that are produced will be immediately made available on the website of the PI as well as posted to the arXiv e-print server. This will give other mathematical researchers and the public the ability to access the new results while they go through an often lengthy peer-review process. The papers will be promptly submitted to a peer-reviewed mathematical journal for publication.

Regarding the second research objective, the PI plans to make the Sage programs and the data they produce on the Galois level of classical modular forms publicly available on her webpage. Furthermore, she plans to add the data to the L-functions and Modular Forms Database (`http://www.lmfdb.org/`), a website with an extensive and interactive database of mathematical objects arising in number theory.

LETTER IN SUPPORT OF J. LANG'S VISIT AT U. PARIS 13

J. TILOUINE, PROFESSOR OF MATHEMATICS, UNIVERSITÉ PARIS 13
99 AV. J.-B. CLÉMENT, 93430 VILLETANEUSE, FRANCE
EMAIL : jacques.tilouine@free.fr, PHONE: +33-1-49 40 40 87

I met Dr. Lang several times, either during visits at UCLA, or on the occasion of various conferences, at MSRI (in Fall 2014), and most recently in Montreal in March 2015. Her energy and her seriousness in research struck me from the beginning. So, when she talked to me about a possibility of spending a couple of years in Paris in our group, I was really delighted.

Her subject of research is the study of the image of Galois representations associated to p-adic families of automorphic forms, inside their Mumford-Tate group. This study, initiated by Hida, consists in two parts : a qualitative one, whose goal is to establish the existence of a maximal principal congruence subgroup of the Mumford-Tate group inside the Galois image (up to suitable conjugation), and a quantitative one, which consists in determining the level of this congruence sugroup -called the Galois level, in terms of a congruence ideal (or a p-adic L function, when these two objects are related) between the given family and other, more degenerate, families. This intuition of Hida proved remarkably accurate. He found, in a beautiful paper (Compos. Math. 2015), that these two ideals are indeed closely related in the $GL(2)/\mathbf{Q}$ case . The hope is that after suitable reformulation, they might be equal. The contribution of Dr. Lang is a key step towards this reformulation. Indeed, in her thesis, she established a Λ-adic version of theorems by Ribet and Momose. They showed that the image of Galois for a given modular form is open (modulo center) in an inner form of $GL(2)$ over an explicit subring of the Hecke ring, called the conjugate self-twist Hecke subring (note that in the ordinary case, this inner form is $GL(2)$ itself). She treated the case of Hida families for $GL(2)$. Her main result concerns the definition and properties of a Galois representation defined over the Λ-adic analogue of the self-twist Hecke subring, based on deformation theory. In a work in progress, Dr. Lang found another definition of this ring as the trace ring of the adjoint representation, using an old paper by R. Pink (Compact subgroups of linear algebraic groups), as suggested by Hida. This reformulation allows generalizations to Hida families over larger groups.

In fact, in a joint paper (to appear in the Proc. of a Conf. on Serre's Conjecture, Hausdorff Institute, Bonn 2013) Hida and myself found generalizations of Hida's results to several variable Hida families over larger groups, but we needed to assume that the Λ-adic Hecke ring coincides with the (several variable) Iwasawa algebra, because we didn't tackle the problem of conjugate self twist Hecke rings.

Moreover, in a recent preprint ArXiv with my student A. Conti and A. Iovita, we proved, in the case of $GL(2)$, but in the non ordinary, finite slope case, qualitative and quantitative results similar to Hida's. In this paper, instead of using directly Hida's aforementioned paper,

Date: September 23, 2015.

we chose to use Dr. Lang's thesis, because of its excellent lisibility and the great degree of generality of the statements which make them easily applicable to our situation, despite our different setting.

It should also be mentioned that I have another PhD student, Huan Chen, at Paris 13 who is studying the image of Galois representations associated to (ordinary) Hida families for higher rank unitary groups (constructed in Geraghty's thesis). These families must be congruent to, but not equal to, families coming from smaller unitary groups by automorphic induction (as studied by Arthur-Clozel, Harris-Labesse and Labesse). He studies also Hida families on symplectic groups containing subfamilies of automorphic symmetric power lifts, as constructed by Kim-Shahidi and Clozel-Thorne. I recommended him too to read J. Lang's thesis, because of the clarity of her formulations and proofs.

These studies do obviously fit in the new technology provided by Dr. Lang's thesis and project. I do hope that concrete interaction (and even joint works) with my students will result from Dr. Lang's stay at Paris 13. I will be present at Paris 13 in 2016-2018, most of the time.

In our group, we usually conduct two activities weekly : an open seminar and semester-long workshops about important recent results (more or less democratically chosen). On these days, people have lunch together and we then have informal discussions. But besides these two meetings, we feel that a visitor does not owe daily presence at our university (although we'll provide her of course with office, computer and access to electronic and physical library). So during her stay, she will be encouraged, besides, to attend whatever seminar she likes, in Paris, Orsay or IHES, and to interact with whomever she wishes. It is to be noted that in our group, besides my students and myself, interesting interlocutors for her include

-P. Boyer, an expert in the cohomology of Shimura varieties,

-A. Mokrane, an expert in p-adic Hodge theory,

-B. Stroh, an expert in arithmetic and rigid geometry of Shimura varieties especially for applications to p-adic automorphic forms,

-F. Brumley, an expert in automorphic forms and harmonic analysis on locally symmetric domains

Our new project with Hida deals with applications of $R = T$ theorems to the relation between Galois level and congruence ideals in the $Sym^3 : GL_2 \to GSp(4)$ case for instance. While it won't interfere directly with Dr. Lang's current projects, it may also help to give them some impetus.

In conclusion, I consider that Dr. Lang's results and research program fit very well with the research themes of our group at Paris 13. I therefore expect a very fruitful stay for her, with great benefit for both parties. I totally support her application to visit us in 2016-2018.

Jacques Tilouine
Professor of Mathematics
Université de Paris 13