UNIVERSITY OF CAMBRIDGE

# Application for Employment

In Progress

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|---|---|
| Applicant Name | Jaclyn Lang |
| Position applied for | Research Associate |
| Department | Department of Pure Mathematics and Mathematical Statistics |
| Vacancy reference | LF07414: Research Associate |

## PERSONAL DETAILS

| | |
|---|---|
| Last name | Lang |
| First name(s) | Jaclyn |
| Title | Ms |
| Current address | UCLA Mathematics Department<br>Box 951555<br>Los Angeles<br>CA<br>90095<br>United States of America |
| Primary phone number | +1 303 587 4174 |
| Secondary phone number | |
| Email address | jaclynlang@math.ucla.edu |
| Immigration status | Are you a settled worker (i.e. do you have the permanent right to work in the UK – for example as a British or EEA citizen)?<br>**No**<br>If no, do you already have temporary permission to work in the UK?<br>**No**<br>If yes, please specify your visa type and end date: |
| UK NI Number | |
| Availability or Notice Period | available after June 2016 |

## REFERENCES

### 1) Individual Referee

| | |
|---|---|
| Name | Professor Haruzo Hida |
| Position | Professor of Mathematics |
| Address | UCLA Mathematics Department<br>Box 951555<br>Los Angeles<br>CA<br>90095<br>United States of America |
| Telephone number | +1 310 206 3382 |
| E-mail address | hida@math.ucla.edu |
| At what point in the recruitment process may references be gathered? | At any point in the process |

### 2) Individual Referee

| | |
|---|---|
| Name | Professor Jacques Tilouine |
| Position | Professor of Mathematics |
| Address | Université Paris 13, LAGA<br>99 Av. J.-B. Clément<br>Villetaneuse<br>93430<br>France |
| Telephone number | +33 1 49 40 40 87 |
| E-mail address | jacques.tilouine@free.fr |
| At what point in the recruitment process may references be gathered? | At any point in the process |

## APPLICANT DECLARATION & DATA CONSENT

When you are ready to submit your final application, you must agree to the declaration below and click Submit Application.

By doing so, you are confirming that:

- You have understood and accept how the University will use and store your personal data, having read the section on Storage and Use of Applicant Data on our HR web pages.
- The information you have given in your web-based application for employment and any supporting documents is correct and complete.
- I understand that the documents I have uploaded to support my application have been converted to pdf format, and I confirm that I have checked them and they are an accurate representation of the originals.
- You understand that you will not be able to make any changes after submitting your application.
- You understand that failure to disclose any relevant information or the provision of false information may lead to dismissal/withdrawal of any offer of employment made to you.
- You understand that the University of Cambridge may check all or any of the information provided as part of your application or given by your referees.
- You understand that an appointment, if offered, will be subject to the receipt of references, and the outcome of any relevant pre-employment checks, which the University regards as satisfactory.

Signature: Date:

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## Contact Information

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Website: http://www.math.ucla.edu/~jaclynlang/

## Education

University of California, mathematics, Ph.D., 2016 (expected)

Thesis adviser: Haruzo Hida

Thesis title: Images of Galois representations associated to Hida families

University of Cambridge, pure mathematics, CASM, 2010

Bryn Mawr College, mathematics, B.A./M.A., 2009

*summa cum laude*, with honors in mathematics

## Publications and Preprints

1. *On images of Galois representations in non-CM Hida families.* Submitted to *Algebra and Number Theory.* http://www.math.ucla.edu/~jaclynlang/I_0_level_existence.pdf
2. (with J. Balakrishnan, M. Çiperiani, B. Mirza, and R. Newton) *Shadow lines in the arithmetic of elliptic curves.* To appear in *Women in numbers 3 Proceedings.*
3. (with M. Daub, M. Merling, N. Pitiwan, A. Pacelli, and M. Rosen) *Function fields with class number indivisible by a prime $\ell$. Acta Arith.*, Volume 150, No. 4, (2011), 339–359.

**Research Interests** Galois representations, modular forms, elliptic curves, $p$-adic interpolation, $p$-adic $L$-functions

## Honors and Awards

- *Charles E. and Sue K. Young Graduate Student Award*, 2015, ($10,000), 4/year out of all UCLA graduate students, UCLA Graduate Division
- *Teaching Award*, 2014, 4/year out of UCLA math graduate teaching assistants, UCLA Department of Mathematics
- *Edward A. Bouchet Graduate Honor Society Inductee*, 2014, 5/year out of all graduate students at UCLA, UCLA Graduate Division
- *NSF Graduate Research Fellowship*, 2010, (∼$121,500), 2,000/year nationally, National Science Foundation
- *Eugene V. Cota Robles Fellowship*, 2010, (∼$96,000), 71/year out of all graduate students at UCLA, UCLA Graduate Division

- *Churchill Scholarship*, 2009, (∼\$50,000), 14/year nationally, The Winston Churchill Foundation of the United States

**Talks**
**Invited lectures:**

Five Colleges Number Theory Seminar, Amherst College (September 2015), *Images of Galois representations of Hida families*

Number Theory Seminar, Massachusetts Institute of Technology (September 2015), *Images of Galois representations of Hida families*

Mathematics Colloquium, Loyola Marymount University (February 2015), *p-adic interpolation*

Number Theory Seminar, University of Texas at Austin (March 2014), *Images of non-CM Galois representations associated to Hida families of modular forms*

Mathematics Colloquium, California State Polytechnic University (January 2014), *p-adic interpolation*

**Contributed talks:**

AMS Western Sectional Meeting, CSU - Fullerton (October 2015), *Images of Galois representations associated to Hida families*

BU-Keio U. Workshop, Boston University (September 2015), *Images of Galois representations associated to Hida families*

Number Theory Conference, University of Illinois at Urbana-Champaign (August 2015), *Images of Galois representations associated to Hida families*

Graduate Summer School on New Geometric Techniques in Number Theory, Mathematical Sciences Research Institute (July 2013), *On images of Galois representations associated to non-CM Hida families of modular forms*

Women in Mathematics in Southern California Symposium, Loyola Marymount University (October 2012), *Introduction to p-adic modular forms*

**Talks at home institution:**

UCLA Number Theory Seminar, March 2014: *On the image of non-CM Galois representations attached to Hida families*

Advancement to Candidacy, June 2013: *Images of Big Galois Representations*

UCLA Participating Number Theory Seminar:

Winter 2014: *Deformation Theory towards Serre's Conjecture* (4 lectures)
Fall 2014: *Iwasawa's Theorem and the Main Conjecture* (2 lectures)
Spring 2014: *Heuristics for completed cohomology* (2 lectures)
Winter 2014: *Abelian class field theory, via duality theorems*

Fall 2013: *Duality for abelian varieties over local fields and Global Duality Theorems* (2 lectures)

Spring 2013: *Serre's proof of a special case of the Mumford-Tate Conjecture* (2 lectures)

Winter 2013: *Introduction to Abelian Varieties* (2.5 lectures)

Fall 2012: *Families of $p$-adic modular forms* (2 lectures)

Spring 2012: *Classifying pro-$p$ subgroups of* $\mathrm{SL}(2, A)$ *for a $p$-adic ring $A$*

UCLA Graduate Student Seminar:

October 2014: *The Art of Giving a Math Talk*

January 2014: *What is the BSD conjecture?*

November 2012: *The Local-Global Principle*

Part III 2010 Lent seminars: *How to add points on a hyperelliptic curve of genus two*

Part III 2009 Michaelmas seminars: *The Local-Global Principle*

## Conference and Workshops Attended

*Boston University/Keio University Workshop: Number Theory*, September 2015, Boston University (funded participant)

Sage Days 69: Women in Sage 6, September 2015, La Jolla (funded participant)

*Illinois Number Theory Conference*, August 2015, University of Illinois - Urbana-Champaign (funded participant)

*$p$-adic methods in number theory: A conference inspired by the mathematics of Robert Coleman*, May 2015, University of California - Berkeley (funded participant)

*Southern California Number Theory Day*, May 2015, University of California - San Diego

*Southern California Number Theory Day*, April 2015, California Institute of Technology

*$p$-adic methods in the theory of classical automorphic forms*, March 2015, Centre de recherches mathématiques, Montreal (funded participant)

*Automorphic forms, Shimura varieties, Galois representations and $L$-functions*, December 2014, Mathematical Sciences Research Institute, Berkeley (funded participant)

*Southern California Number Theory Day*, October 2014, University of California - Irvine

*Introductory Workshop: New Geometric Techniques in Number Theory*, August 2014, Mathematical Sciences Research Institute, Berkeley (funded participant)

*Connections for Women: New Geometric Techniques in Number Theory*, August 2014, Mathematical Sciences Research Institute, Berkeley (funded participant)

*Graduate Summer School: Counting Arithmetic Objects*, June 2014, Centre de recherches mathématiques, Montreal (funded participant) *p-adic variation in number theory*, June 2014, Boston University (funded participant)

*Women in Numbers 3*, April 2014, Banff International Research Station, Banff, Shadow Lines project group (funded participant)

*11th Annual Yale Bouchet Conference on Diversity and Graduate Education*, March 2014, Yale University (funded participant)

*Arizona Winter School: Arithmetic Statistics*, March 2014, University of Arizona, Bjorn Poonen's Project Group (funded participant)

*Hot Topics Workshop: Perfectoid Spaces and their Applications*, March 2014, Mathematical Sciences Research Institute, Berkeley (funded participant)

*Conference on Stark's Conjectures and related topics*, September 2013, University of California - San Diego (funded participant)

*Graduate Summer School: New Geometric Techniques in Number Theory*, July 2013, Mathematical Sciences Research Institute, Berkeley (funded participant)

*p-adic modular forms, L-functions, and Galois representations*, May 2013, University of California - Los Angeles

*Cohomology of Arithmetic Groups Graduate Workshop*, May 2013, Chicago (funded participant)

*Arizona Winter School: Modular Forms and Modular Curves*, March 2013, University of Arizona, Frank Calegari's Project Group (funded participant)

*p-adic modular forms and arithmetic*, June 2012, University of California - Los Angeles

*Joint Mathematics Meetings* January 2013, 2012 (funded participant)

## Teaching Experience

University of California - Los Angeles

Teaching Assistant Consultant, Fall 2013 (trained new teaching assistants)

Teaching Fellow, Discrete Mathematics, Spring 2014

Teaching Fellow, Group Theory, Winter 2014

Teaching Fellow, Integration and Infinite Series, Fall 2013

Teaching Assistant, Integration and Infinite Series, Summer 2013

Teaching Assistant, Integration and Infinite Series, Spring 2011

Teaching Assistant for Summer Program for Women in Mathematics, George Washington University, Summer 2012

Counselor for Program in Mathematics for Young Scientists (PROMYS), Boston University, Summer 2010

Bryn Mawr College

Problem Session Holder and Grader, Abstract Algebra II, Spring 2009

Problem Session Holder and Grader, Abstract Algebra I, Fall 2008

Peer Instructor, Linear Algebra, Spring 2008

Peer Instructor, Multivariable Calculus, Fall 2007

Problem Session Holder and Grader, Transitions to Higher Mathematics, Spring 2007

Problem Session Holder, Calculus 101, Fall 2006

Grader, Multivariable Calculus (enriched), Fall 2006

## Service and Outreach Activities

Served as a referee for *Math Research Letters* and *Coates' 70th Birthday Conference Proceedings*

Contributed functionality to the open source software package Sage

Organized and ran advising workshops for UCLA math graduate students applying for NSF Graduate Research Fellowship, 2014, 2015

Co-created and ran a booth on the Monty Hall Problem at UCLA EmpowHer STEM Day, 2014, 2015

President of Graduate Student Organization in the UCLA mathematics department, 2012-2014

Co-founded and co-organized UCLA women in math group (2010-present)

Panelist at Aftermath Conference for undergraduate math majors interested in graduate school, Harvey Mudd College, February 2013

Served on panel for undergraduate math majors interested in graduate school, University of California - Los Angeles, October 2011

## Languages

English (native)

French (intermediate reading, writing, speaking)

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## Overview

My research is in the area of algebraic number theory. Primarily I study modular forms, the Galois representations associated to them, and $p$-adic families of such objects. I am also interested in the arithmetic of elliptic curves. The main result of my thesis is that, in a qualitative sense, the Galois representation associated to an ordinary $p$-adic family of modular forms has "large" image. My future research plans include improving the result to a quantitative form and obtaining a complete description of the images of such Galois representations.

A fundamental problem is to determine the image of a given Galois representation. This was first done by Serre [15] for the $p$-adic Galois representations $\rho_{E,p}$ associated to an elliptic curve $E_{/\mathbb{Q}}$. He showed that if $E$ does not have complex multiplication (CM) then $\rho_{E,p}$ is surjective for all but finitely many primes $p$. Furthermore the image of $\rho_{E,p}$ is open for all $p$. Serre's result is an example of a general pattern governing the expected behavior of images of Galois representations.

**Heuristic.** *The image of a Galois representation should be as large as possible, subject to the symmetries of the geometric object from which it arose.*

The notion of "symmetry" is vague and depends on the situation. In the case of elliptic curves, the relevant symmetry is complex multiplication. In the 1980s, Ribet and Momose determined, up to finite error, the image of a Galois representation coming from a classical modular form without CM and thus showed that such images are "large" [12, 14]. Their proof introduced new symmetries of modular forms known as "conjugate self-twists". Indeed, if $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathcal{O})$ arises from a modular form, then one can talk about the subring $\mathcal{O}_0$ of $\mathcal{O}$ fixed by all conjugate self-twists. Ribet and Momose proved that the intersection of $\operatorname{Im}\rho$ with $\mathrm{SL}_2(\mathcal{O}_0)$ is open in $\mathrm{SL}_2(\mathcal{O}_0)$.

In the 1980s, Hida developed his theory of $p$-adic families of (ordinary) modular forms. To such a family $F$, Hida associated a Galois representation $\rho_F : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{I})$ for a certain ring $\mathbb{I}$. From $\rho_F$ one obtains a mod $p$ representation $\bar{\rho}_F$. One can consider conjugate self-twists of $F$ and form the ring $\mathbb{I}_0$ fixed by all such twists. The following theorem is the main result of my thesis [8].

**Theorem 1** (Lang [8])**.** *Let $F$ be a non-CM Hida family. Assume that $\bar{\rho}_F$ is absolutely irreducible and satisfies a technical but mild regularity condition. Then there is a nonzero $\mathbb{I}_0$-ideal $\mathfrak{a}_0$ such that the image of $\rho_F$ contains all matrices in $\mathrm{SL}_2(\mathbb{I}_0)$ that are congruent to the identity modulo $\mathfrak{a}_0$.*

I have a secondary interest in the arithmetic of elliptic curves. My group at the Women in Numbers 3 workshop studied shadow lines of elliptic curves, an invariant first defined by Mazur and Rubin [10]. Using explicit class field theory, we developed an algorithm to compute the shadow line of a triple $(E, K, p)$, where $E_{/\mathbb{Q}}$ is an elliptic curve, $K$ is an imaginary quadratic field, and $p$ is a rational prime of good reduction for $E$ that splits in $K$. We implemented our algorithm in Sage [16] and computed the first examples of shadow lines. The resulting paper will appear in the proceedings volume [1].

## Thesis

***Background.*** I now give some definitions and notation to make the statement of Theorem 1 more precise. Fix a prime $p$, and assume for simplicity that $p \geq 5$. Let $\Lambda = \mathbb{Z}_p[[T]]$ and $\mathbb{I}$ be an integral domain that is finite flat over $\Lambda$. Fix embeddings of an algebraic closure $\overline{\mathbb{Q}}$ of $\mathbb{Q}$ into $\mathbb{C}$ and $\overline{\mathbb{Q}}_p$.

**Definition 1** (Hida [5], Wiles [17]). A formal power series $F = \sum_{n=1}^{\infty} A_n q^n \in \mathbb{I}[[q]]$ is a *Hida family* if $A_p \in \mathbb{I}^\times$ and for every integer $k \geq 2$ and every prime ideal $\mathfrak{P}$ of $\mathbb{I}$ lying over $(1 + T - (1+p)^k)\Lambda$, we have:

- $A_n \bmod \mathfrak{P}$ is in $\overline{\mathbb{Q}}$ (rather than just $\overline{\mathbb{Q}}_p$), and
- $f_{\mathfrak{P}} := \sum_{n=1}^{\infty} (A_n \bmod \mathfrak{P}) q^n$ gives the $q$-expansion of a classical modular form of weight $k$.

Hida showed that every $p$-ordinary classical modular form of weight at least 2 arises from a unique such family [5]. Furthermore, there is a Galois representation $\rho_F : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{I})$ that is unramified almost everywhere. For all primes $\ell$ at which $\rho_F$ is unramified, $\operatorname{tr}\rho_F(\mathrm{Frob}_\ell) = A_\ell$, where $\mathrm{Frob}_\ell$ is the conjugacy class of a Frobenius element at $\ell$ [4].

**Definition 2.** An automorphism $\sigma$ of $\mathbb{I}$ is a *conjugate self-twist* of a Hida family $F = \sum_{n=1}^{\infty} A_n q^n \in \mathbb{I}[[q]]$ if there exists a non-trivial Dirichlet character $\eta_\sigma$ such that

$$\sigma(A_\ell) = \eta_\sigma(\ell) A_\ell$$

for almost all primes $\ell$. We say $F$ has *complex multiplication (CM)* if the identity automorphism is a conjugate self-twist of $F$.

***Ideas in the Proof of Theorem 1.*** The key result is a lifting theorem showing when a conjugate self-twist of an arithmetic specialization $f_{\mathfrak{P}}$ of $F$ can be lifted to a conjugate self-twist of the entire family $F$. The ideas that go into the lifting theorem will be briefly explained in the next paragraph. Using the lifting theorem, there is a series of reduction steps to conclude that the theorem of Ribet and Momose is sufficient to prove Theorem 1. An important tool in these reduction steps is a $\mathbb{Z}_p$-Lie algebra of Pink [13] that is associated to $\operatorname{Im}\rho_F$, which allows us to reduce our problem to linear algebra. Hida observed that the ordinarity of $\rho_F$ can be used to give Pink's Lie algebra a $\Lambda$-algebra structure [6], and this structure is critical to the proof.

I now give a brief outline of the proof of the lifting theorem. A conjugate self-twist $\sigma$ of an arithmetic specialization $f_{\mathfrak{P}}$ of $F$ induces an automorphism $\bar{\sigma}$ of the residue field $\mathbb{F}$. Using deformation theory, I lift $\bar{\sigma}$ to an automorphism $\Sigma$ of the entire universal deformation ring $R_{\bar{\rho}_F}$ of $\bar{\rho}_F$ and show that $\Sigma$ satisfies certain properties. Using automorphic methods I show that $\Sigma$ preserves a certain Hecke algebra, $\mathbb{T}$. Since $\operatorname{Spec}\mathbb{I}$ is an irreducible component of $\operatorname{Spec}\mathbb{T}$, we see that $\Sigma$ must send $\operatorname{Spec}\mathbb{I}$ to another irreducible component $\Sigma^*\operatorname{Spec}\mathbb{I}$ of $\operatorname{Spec}\mathbb{T}$. The properties of $\Sigma$ force the two components $\operatorname{Spec}\mathbb{I}$ and $\Sigma^*\operatorname{Spec}\mathbb{I}$ to intersect at an arithmetic point. Since the Hecke algebra is étale over $\Lambda$ at arithmetic points, $\Sigma$ must descend to an automorphism of $\mathbb{I}$, as desired.

## Future work

***Determining the $\mathbb{I}_0$-level and relation to $p$-adic $L$-functions.*** Theorem 1 guarantees that there is a non-zero $\mathbb{I}_0$-ideal $\mathfrak{a}_0$ such that the image of $\rho_F$ contains all determinant 1 matrices that are congruent to the identity modulo $\mathfrak{a}_0$. The largest such ideal is called the $\mathbb{I}_0$-*level* of $\rho_F$ and is denoted $\mathfrak{c}_{0,F}$. A natural question is to determine the $\mathbb{I}_0$-level. I plan to prove the following conjecture, which is a generalization of Hida's Theorem II in [6].

**Conjecture 1.** *Let $F$ and $p$ be as in Theorem 1.*

1. *If* $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\mathbb{F}_p)$*, then* $\mathfrak{c}_{0,F} = \mathbb{I}_0$*. That is,* $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\mathbb{I}_0)$.
2. *Suppose that* $\bar{\rho}_F$ *is absolutely irreducible and* $\bar{\rho}_F \cong \operatorname{Ind}_M^{\mathbb{Q}} \bar{\psi}$ *for an imaginary quadratic field* $M$ *in which* $p$ *splits and a character* $\bar{\psi} : \mathrm{Gal}(\overline{\mathbb{Q}}/M) \to \overline{\mathbb{F}}_p^{\times}$*. Assume* $M$ *is the only such quadratic field. Under minor conditions on the tame level of* $F$*, there is a product* $\mathcal{L}_0$ *of anticyclotomic Katz* $p$*-adic* $L$*-functions such that* $\mathfrak{c}_{0,F}$ *is a factor of* $\mathcal{L}_0$*. Furthermore, every prime factor of* $\mathcal{L}_0$ *is a factor of* $\mathfrak{c}_{0,F}$ *for some* $F$.

Note that even when $\mathbb{I} = \Lambda$, Conjecture 1 is stronger than Hida's Theorem II [6]. Furthermore, Conjecture 1.1 is a natural extension of the work of Mazur-Wiles [11] and Fischman [3].

In proving case (1) of the conjecture, I will make use of Manoharmayum's recent work that shows $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(W)$ for a finite unramified extension $W$ of $\mathbb{Z}_p$ [9]. This will be combined with the $\Lambda$-module structure on the Pink Lie algebra associated to $\operatorname{Im}\rho_F$ that was used in the proof of Theorem 1 to get the desired result.

In proving case (2), I will relate the $\mathbb{I}_0$-level to the congruence ideal of $F$ as in Hida's proof of Theorem II [6]. The connection to Katz $p$-adic $L$-functions is then obtained by relating the congruence ideal to the $p$-adic $L$-function through known cases of the Main Conjecture of Iwasawa Theory. The idea is that replacing the $\Lambda$-level in Hida's work with the more precise $\mathbb{I}_0$-level will allow me to remove the ambiguity of the square factors that show up in Theorem II [6].

Proving Conjecture 1 would yield refined information about the images of Galois representations attached to Hida families. It is the first step in completely determining the images of such representations.

***Analogue of the Mumford-Tate Conjecture in*** $p$***-adic families.*** Another way to describe the work of Ribet and Momose is that they proved the Mumford-Tate Conjecture for compatible systems of Galois representations associated to classical modular forms. Hida has proposed an analogue of the Mumford-Tate Conjecture for $p$-adic families of Galois representations. For an arithmetic prime $\mathfrak{P}$ of $\mathbb{I}$, write $\mathrm{MT}_{\mathfrak{P}}$ for the Mumford-Tate group of the compatible system containing $\rho_{f_{\mathfrak{P}}}$, so $\mathrm{MT}_{\mathfrak{P}}$ is an algebraic group over $\mathbb{Q}$. Let $\kappa(\mathfrak{P}) = \mathbb{I}_{\mathfrak{P}}/\mathfrak{P}_{\mathfrak{P}}$, and write $G_{\mathfrak{P}}$ for the Zariski closure of $\operatorname{Im}\rho_{f_{\mathfrak{P}}}$ in $\mathrm{GL}_2(\kappa(\mathfrak{P}))$. Let $G_{\mathfrak{P}}^{\circ}$ be the connected component of the identity of $G_{\mathfrak{P}}$ and $G'_{\mathfrak{P}}$ the (closed) derived subgroup of $G_{\mathfrak{P}}$. Finally, let $\Gamma_F$ denote the group generated by the conjugate self-twists of a non-CM Hida family $F$.

**Conjecture 2** (Hida)**.** *Assume* $F$ *is non-CM. There is a simple algebraic group* $G'$*, defined over* $\mathbb{Q}_p$*, such that for all arithmetic primes* $\mathfrak{P}$ *of* $\mathbb{I}$ *one has* $G'_{\mathfrak{P}} \cong G' \times_{\mathbb{Q}_p} \kappa(\mathfrak{P})$ *and* $\mathrm{Res}_{\mathbb{Q}_p}^{\kappa(\mathfrak{P})} G_{\mathfrak{P}}$ *is (the ordinary factor of)* $\mathrm{MT}_{\mathfrak{P}} \times_{\mathbb{Q}} \mathbb{Q}_p$*. Furthermore, the component group* $G_{\mathfrak{P}}/G_{\mathfrak{P}}^0$ *is canonically isomorphic to the Pontryagin dual of the decomposition group of* $\mathfrak{P}$ *in* $\Gamma_F$.

By obtaining a sufficiently precise understanding of images of Galois representations attached to Hida families through the first project, I plan to prove results along the lines of Conjecture 2. I have some preliminary results relating the Pontryagin dual of $\Gamma_F$ to the quotient $(\operatorname{Im}\rho_F)/(\operatorname{Im}\rho_F|_H)$ for a certain finite index normal subgroup $H$ of $G_{\mathbb{Q}}$.

Completing this research objective, or even any preliminary results in this direction, would reveal that the images of classical specializations of the Galois representation attached to a Hida family are even more related to one another than previously thought. Not only would they arise as specializations of some group in $\mathrm{GL}_2(\mathbb{I})$, they could all be found simply by base change from a single group, at least up to abelian error.

## References


[1] BALAKRISHNAN, J., ÇIPERIANI, M., LANG, J., MIRZA, B., AND NEWTON, R. Shadow lines in the arithmetic of elliptic curves. to appear, 2015.

[2] CONTI, A., IOVITA, A., AND TILOUINE, J. Big image of Galois representations attached to finite-slope $p$-adic families of modular forms. July 2015.

[3] FISCHMAN, A. On the image of $\Lambda$-adic Galois representations. *Ann. Inst. Fourier (Grenoble) 52*, 2 (2002), 351–378.

[4] HIDA, H. Galois representations into $\mathrm{GL}_2(\mathbf{Z}_p[[X]])$ attached to ordinary cusp forms. *Invent. Math. 85*, 3 (1986), 545–613.

[5] HIDA, H. Iwasawa modules attached to congruences of cusp forms. *Ann. Sci. École Norm. Sup. (4) 19*, 2 (1986), 231–273.

[6] HIDA, H. Big Galois representations and $p$-adic $L$-functions. *Compos. Math. 151*, 4 (2015), 603–664.

[7] HIDA, H., AND TILOUINE, J. Big image of Galois representations and the congruence ideal. In *Arithmetic and Geometry* (2014), L. Dieulefait, G. Faltings, D. R. Heath-Brown, Y. V. Manin, B. Z. Moroz, and J.-P. Wintenberger, Eds., vol. 420, Cambridge University Press, pp. 217–254.

[8] LANG, J. On the image of the Galois representation associated to a non-CM Hida family. `http://www.math.ucla.edu/~jaclynlang/I_0_level_existence.pdf`, submitted, July 2015.

[9] MANOHARMAYUM, J. A structure theorem for subgroups of $\mathrm{GL}_n$ over complete local Noetherian rings with large residual image. *Proc. Amer. Math. Soc. 143*, 7 (2015), 2743–2758.

[10] MAZUR, B., AND RUBIN, K. Studying the growth of Mordell-Weil. *Doc. Math.*, Extra Vol. (2003), 585–607.

[11] MAZUR, B., AND WILES, A. On $p$-adic analytic families of Galois representations. *Compos. Math. 59*, 2 (1986), 231–264.

[12] MOMOSE, F. On the $l$-adic representations attached to modular forms. *J. Fac. Sci. Univ. Tokyo Sect. IA Math. 28*, 1 (1981), 89–109.

[13] PINK, R. Classification of pro-$p$ subgroups of $\mathrm{SL}_2$ over a $p$-adic ring, where $p$ is an odd prime. *Compos. Math. 88*, 3 (1993), 251–264.

[14] RIBET, K. A. On $l$-adic representations attached to modular forms. II. *Glasgow Math. J. 27* (1985), 185–194.

[15] SERRE, J.-P. *Abelian $\ell$-adic representations and elliptic curves*. W. A. Benjamin, Inc., 1968.

[16] A. STEIN, W., ET AL. *Sage Mathematics Software*. The Sage Development Team, http://www.sagemath.org.

[17] WILES, A. On ordinary $\lambda$-adic representations associated to modular forms. *Invent. Math. 94*, 3 (1988), 529–573.

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Jaclyn Lang
UCLA Mathematics Department
Box 951555
Los Angeles, CA 90095
USA

November 11, 2015

DPMMS, Statistical Laboratory
Centre for Mathematical Sciences
Wilberforce Road
Cambridge CB3 0WB
United Kingdom

To the Postdoctoral Search Committee:

I am applying to the Postdoctoral Research Associate in Number Theory position, because I am excited about the recent advances in automorphy theorems by Jack Thorne and his collaborators and their Diophantine applications. While my dissertation was not on the topic of automorphy, many of the tools I used are related, including Hida families, Galois representations and their images, and p-adic interpolation. In fact, a key result I prove in my thesis is a certain “lifting theorem.” My proof uses an interplay between deformation theory and automorphic techniques, inspired by the philosophy of automorphy lifting theorems. I am interested in working with Jack Thorne to develop new applications of these tools.

I expect to receive my Ph.D. in June 2016. My dissertation, supervised by Haruzo Hida, is in the area of algebraic number theory, specifically on the images of Galois representations associated to ordinary p-adic families of modular Galois representations. The main results of my thesis are written up in a paper that is currently under review at the journal *Algebra and Number Theory*. The tools that I developed to prove my result will likely be useful in proving additional "big image theorems." Please see my research statement for more detailed information.

I have some research experience outside the area of my dissertation. I was a member of the Shadow Lines project group at Women in Numbers 3, led by Mirela Çiperiani and Jennifer Balakrishnan. We used explicit class field theory to develop and implement an algorithm to compute the "shadow line" associated to certain elliptic curve data, an invariant first introduced by Mazur and Rubin at the 2002 ICM. Through this project I gained experience working with the arithmetic of elliptic curves, explicit class field theory, and Iwasawa theory. In 2014, I was a member of Bjorn Poonen's project group at the Arizona Winter School. We proved a generalization of Poonen's analogue of Bertini's Theorem over finite fields, so I have some experience working with arithmetic geometry, particularly over finite fields.

In addition to the above research projects, I have participated in a number of classes and seminars at that are relevant to the post. I have taken classes from Chandrashekhar Khare on the proof of Serre’s Conjecture and Modularity Lifting Theorems, and I participated in number theory learning seminars on topics including completed cohomology, the Taylor-Wiles method, and arithmetic duality theorems. Furthermore, I have attended numerous conferences and graduate student workshops dedicated to automorphy lifting theorems including the MSRI summer school on New Geometric Techniques in Number Theory (2013) and the Arizona Winter School (2013).

Thank you for your time and consideration.

Sincerely,

Jaclyn Lang

Jaclyn Lang

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