

QUANTIFIER ELIMINATION IN SHELAH-SPENCER GRAPHS

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ABSTRACT. We simplify [?] proof of quantifier elimination in Shelah-Spencer graphs.

1. INTRODUCTION

Laskowski's paper [?] provides a combinatorial proof of quantifier elimination in Shelah-Spencer graphs. Here we provide a simplification of the proof using only maximal chains and avoiding technical lemmas of sections 3 and 4.

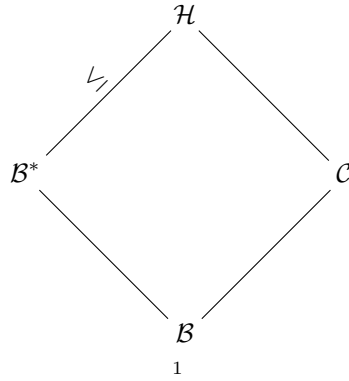
2. PRELIMINARIES

We will use notation of [?], in particular things like K_α , $\delta(\mathcal{A}/\mathcal{B})$, $X_m(\mathcal{A})$ etc.

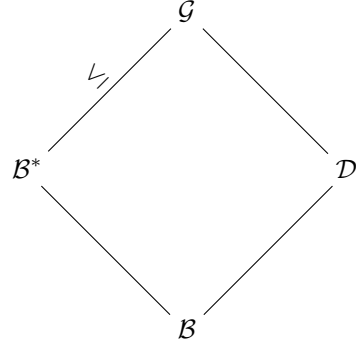
3. PROOF

Definition 3.1. Fix $\mathcal{B} \in K_\alpha$, Φ, Γ finite subsets of K_α , and $m \in \omega$ such that for each $\mathcal{C} \in \Phi$ or $\mathcal{C} \in \Gamma$ we have $\mathcal{B} \subseteq \mathcal{C}$ and $|\mathcal{C} \setminus \mathcal{B}| < m$. Define $Z(\mathcal{B}, \Phi, \Gamma, m)$ to be all $\mathcal{B}^* \in X_m(\mathcal{B})$ such that

- (1) For every $\mathcal{C} \in \Phi$ there are no \mathcal{H} with $|\mathcal{H} \setminus \mathcal{B}^*| < m$ satisfying



- (2) For every $\mathcal{D} \in \Gamma$ there is some \mathcal{G} with $|G \setminus B^*| < m$ satisfying



Definition 3.2. Let $\mathcal{M} \models S_\alpha$, $\mathcal{B} \in \mathbf{K}_\alpha$, embedding $f: \mathcal{B} \rightarrow \mathcal{M}$, Φ finite subset of \mathbf{K}_α

- (1) Say that f *omits* Φ if there are no $\mathcal{C} \in \Phi$ and $g: \mathcal{C} \rightarrow \mathcal{M}$ extending f .
- (2) Say that f *admits* Φ if for every $\mathcal{C} \in \Phi$ there is $g: \mathcal{C} \rightarrow \mathcal{M}$ extending f .

Lemma 3.3.

REFERENCES

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- [3] P. Simon, *On dp-minimal ordered structures*, J. Symbolic Logic 76 (2011), no. 2, 448460.
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