My research is in the area of algebraic number theory. I study modular forms, Galois representations, p-adic families of such objects, and elliptic curves. The main result of my thesis is that the Galois representation associated to an ordinary p-adic family of modular forms has "large" image.

The philosophy behind the study of Galois representations comes from a universal theme in mathematics: linearization. Namely, there is a complicated object that we want to understand; in our case it is the group $G_{\mathbb{Q}}$, the absolute Galois group of $\mathbb{Q}$ (the rational numbers). Just as one studies the tangent space of a complicated manifold or uses linear approximations to study a non-linear partial differential equation, representations allow us to study $G_{\mathbb{Q}}$ through its action on certain vector spaces. Thus we end up studying a collection of matrices known as the *image of the representation*, a kind of "shadow" of $G_{\mathbb{Q}}$ inside matrices, which are more concrete objects.

More formally, if A is a ring, write $\mathrm{GL}_n(A)$ for the collection of all invertible $(n \times n)$-matrices with entries in A and $\mathrm{SL}_n(A)$ for the collection of matrices in $\mathrm{GL}_n(A)$ that have determinant 1. A *Galois representation* is a (continuous) homomorphism $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_n(A)$. The *image* of ρ is all matrices in $\mathrm{GL}_n(A)$ of the form $\rho(\sigma)$ for $\sigma \in G_{\mathbb{Q}}$. Elliptic curves defined over $\mathbb{Q}$ give rise to interesting Galois representations. If E is such an elliptic curve, then for each prime p there is a Galois representation $\rho_{E,p} : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{Z}_p)$, where $\mathbb{Z}_p$ is the *p-adic integers*.

The larger the image of a Galois representation, the more information we can recover about $G_{\mathbb{Q}}$. The following definition quantifies what it means for a representation to have large image. Given a representation $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(A)$ and a subring A_0 of A, we say ρ is *A_0-full* if there is a non-zero A_0-ideal $\mathfrak{a}$ such that the image of ρ contains all elements of $\mathrm{SL}_2(A_0)$ that are congruent to the identity modulo $\mathfrak{a}$. In the 1960s, Serre studied the images of Galois representations associated to elliptic curves. His work implies that, in the generic case (when E does not have something called *complex multiplication (CM)*), $\rho_{E,p}$ is $\mathbb{Z}_p$-full. Serre's result is an example of a general pattern governing the expected behavior of images of Galois representations that arise from geometry.

Heuristic. *The image of a Galois representation should be as large as possible, subject to the symmetries of the geometric object from which it arose.*

The notion of "symmetry" depends on the geometric object. The relevant symmetry for an elliptic curve E is CM, a condition that means that E has extra symmetries. In the 1980s, Ribet and Momose determined, up to finite error, the image of a Galois representation $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathcal{O})$ associated to a classical modular form without CM. They showed that ρ is $\mathcal{O}_0$-full for a certain subring $\mathcal{O}_0$ of $\mathcal{O}$ cut out by new symmetries of modular forms known as "conjugate self-twists".

In the 1980s Hida developed his theory of p-adic families of (ordinary) modular forms. To such a family F, Hida associated a Galois representation $\rho_F : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{I})$ for a certain ring $\mathbb{I}$. From ρ_F one obtains a mod p representation $\overline{\rho}_F$. One can consider conjugate self-twists of F and form the ring $\mathbb{I}_0$ cut out by such twists. The following theorem is the main result of my thesis.

Theorem (Lang)**.** *Let F be a non-CM Hida family. Under mild conditions on $\overline{\rho}_F$, ρ_F is $\mathbb{I}_0$-full.*