A Tiny Example
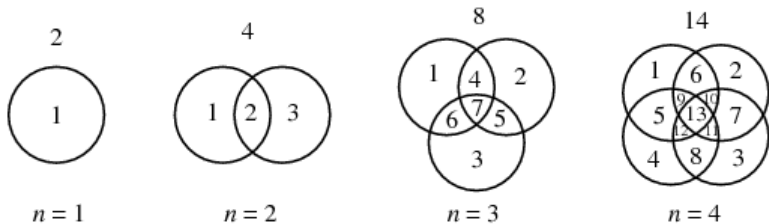
Andrew Mertz and William Slough

June 15, 2005

Suppose we have an (infinite) collection of sets $\mathcal{F}$.
We define a shatter function $\pi_{\mathcal{F}}(n)$

$$\pi_{\mathcal{F}}(n) = \max\{\# \text{ of atoms in boolean algebra generated by } S$$
$$\mid S \subset \mathcal{F} \text{ with } |S| = n\}$$

Example: Let $\mathcal{F}$ consist of all discs on a plane.

$$\pi_{\mathcal{F}}(1) = 2 \quad \pi_{\mathcal{F}}(2) = 4 \quad \pi_{\mathcal{F}}(3) = 8 \quad \pi_{\mathcal{F}}(4) = 14$$

$$\pi_{\mathcal{F}}(n) = n^2 - n + 2$$

More examples:

1. Lines on a plane $\pi_{\mathcal{F}}(n) = n^2/2 + n/2 + 1$
2. Disks on a plane $\pi_{\mathcal{F}}(n) = n^2 - n + 2$
3. Balls in $\mathbb{R}^3$ $\pi_{\mathcal{F}}(n) = n^3/3 - n^2 + 8n/3$
4. Intervals on a line $\pi_{\mathcal{F}}(n) = 2n$
5. Half-planes on a plane $\pi_{\mathcal{F}}(n) = n(n+1)/2 + 1$
6. Finite subsets of $\mathbb{N}$ $\pi_{\mathcal{F}}(n) = 2^n$
7. Polygons in a plane $\pi_{\mathcal{F}}(n) = 2^n$