UNIVERSITY OF CAMBRIDGE

Application for Employment

In Progress

Applicant Name	Jaclyn Lang
Position applied for	University Lecturer
Department	Department of Pure Mathematics and Mathematical Statistics
Vacancy reference	LF07381: University Lecturer

PERSONAL DETAILS

Last name	Lang
First name(s)	Jaclyn
Title	Ms
Current address	UCLA Mathematics Department Box 951555 Los Angeles CA 90095 United States of America
Primary phone number	+1 303 587 4174
Secondary phone number	
Email address	jaclynlang@math.ucla.edu
Immigration status	Are you a settled worker (i.e. do you have the permanent right to work in the UK – for example as a British or EEA citizen)? **No** If no, do you already have temporary permission to work in the UK? **No** If yes, please specify your visa type and end date:
UK NI Number	
Availability or Notice Period	available after June 2016

REFERENCES

1) Individual Referee

Name	Professor Haruzo Hida
Position	Professor of Mathematics
Address	UCLA mathematics department Box 951555 Los Angeles California 90095 United States of America
Telephone number	+1 310 206 3382
E-mail address	hida@math.ucla.edu
At what point in the recruitment process may references be gathered?	At any point in the process

2) Individual Referee

Name	Professor Jacques Tilouine
Position	Professor of Mathematics
Address	Université Paris 13 99 Av. J.-B. Clément Villetaneuse 93430 France
Telephone number	+33 1 49 40 40 87
E-mail address	jacques.tilouine@free.fr
At what point in the recruitment process may references be gathered?	At any point in the process

3) Individual Referee

Name	Professor Chandrashekhar Khare
Position	Professor of Mathematics
Address	UCLA mathematics department Box 951555 Los Angeles California 90095 United States of America
Telephone number	+1 310 825 2082
E-mail address	shekhar@math.ucla.edu
At what point in the recruitment process may references be gathered?	At any point in the process

APPLICANT DECLARATION & DATA CONSENT

When you are ready to submit your final application, you must agree to the declaration below and click Submit Application.

By doing so, you are confirming that:

- You have understood and accept how the University will use and store your personal data, having read the section on Storage and Use of Applicant Data on our HR web pages.
- The information you have given in your web-based application for employment and any supporting documents is correct and complete.
- I understand that the documents I have uploaded to support my application have been converted to pdf format, and I confirm that I have checked them and they are an accurate representation of the originals.
- You understand that you will not be able to make any changes after submitting your application.
- You understand that failure to disclose any relevant information or the provision of false information may lead to dismissal/withdrawal of any offer of employment made to you.
- You understand that the University of Cambridge may check all or any of the information provided as part of your application or given by your referees.
- You understand that an appointment, if offered, will be subject to the receipt of references, and the outcome of any relevant pre-employment checks, which the University regards as satisfactory.

Signature: Date:

This page is intentionally blank

Contact Information

Address:	UCLA Mathematics Department Box 951555 Los Angeles, CA 90095-1555
Telephone:	(303) 587-4174
E-mail:	jaclynlang@math.ucla.edu
Website:	`http://www.math.ucla.edu/~jaclynlang/`

Education

University of California, mathematics, Ph.D., 2016 (expected)

Thesis adviser: Haruzo Hida

Thesis title: Images of Galois representations associated to Hida families

University of Cambridge, pure mathematics, CASM, 2010

Bryn Mawr College, mathematics, B.A./M.A., 2009

summa cum laude, with honors in mathematics

Publications and Preprints

1. *On images of Galois representations in non-CM Hida families.* Submitted to *Algebra and Number Theory.* `http://www.math.ucla.edu/~jaclynlang/I_0_level_existence.pdf`
2. (with J. Balakrishnan, M. Çiperiani, B. Mirza, and R. Newton) *Shadow lines in the arithmetic of elliptic curves.* To appear in *Women in numbers 3 Proceedings.*
3. (with M. Daub, M. Merling, N. Pitiwan, A. Pacelli, and M. Rosen) *Function fields with class number indivisible by a prime ℓ. Acta Arith.*, Volume 150, No. 4, (2011), 339–359.

Research Interests Galois representations, modular forms, elliptic curves, p-adic interpolation, p-adic L-functions

Honors and Awards

- *Charles E. and Sue K. Young Graduate Student Award*, 2015, ($10,000), 4/year out of all UCLA graduate students, UCLA Graduate Division
- *Teaching Award*, 2014, 4/year out of UCLA math graduate teaching assistants, UCLA Department of Mathematics
- *Edward A. Bouchet Graduate Honor Society Inductee*, 2014, 5/year out of all graduate students at UCLA, UCLA Graduate Division
- *NSF Graduate Research Fellowship*, 2010, (∼$121,500), 2,000/year nationally, National Science Foundation
- *Eugene V. Cota Robles Fellowship*, 2010, (∼$96,000), 71/year out of all graduate students at UCLA, UCLA Graduate Division

- *Churchill Scholarship*, 2009, (~$50,000), 14/year nationally, The Winston Churchill Foundation of the United States

Talks
Invited lectures:

Five Colleges Number Theory Seminar, Amherst College (September 2015), *Images of Galois representations of Hida families*

Number Theory Seminar, Massachusetts Institute of Technology (September 2015), *Images of Galois representations of Hida families*

Mathematics Colloquium, Loyola Marymount University (February 2015), *p-adic interpolation*

Number Theory Seminar, University of Texas at Austin (March 2014), *Images of non-CM Galois representations associated to Hida families of modular forms*

Mathematics Colloquium, California State Polytechnic University (January 2014), *p-adic interpolation*

Contributed talks:

AMS Western Sectional Meeting, CSU - Fullerton (October 2015), *Images of Galois representations associated to Hida families*

BU-Keio U. Workshop, Boston University (September 2015), *Images of Galois representations associated to Hida families*

Number Theory Conference, University of Illinois at Urbana-Champaign (August 2015), *Images of Galois representations associated to Hida families*

Graduate Summer School on New Geometric Techniques in Number Theory, Mathematical Sciences Research Institute (July 2013), *On images of Galois representations associated to non-CM Hida families of modular forms*

Women in Mathematics in Southern California Symposium, Loyola Marymount University (October 2012), *Introduction to p-adic modular forms*

Talks at home institution:

UCLA Number Theory Seminar, March 2014: *On the image of non-CM Galois representations attached to Hida families*

Advancement to Candidacy, June 2013: *Images of Big Galois Representations*

UCLA Participating Number Theory Seminar:

Winter 2014: *Deformation Theory towards Serre's Conjecture* (4 lectures)
Fall 2014: *Iwasawa's Theorem and the Main Conjecture* (2 lectures)
Spring 2014: *Heuristics for completed cohomology* (2 lectures)
Winter 2014: *Abelian class field theory, via duality theorems*

Fall 2013: *Duality for abelian varieties over local fields and Global Duality Theorems* (2 lectures)

Spring 2013: *Serre's proof of a special case of the Mumford-Tate Conjecture* (2 lectures)

Winter 2013: *Introduction to Abelian Varieties* (2.5 lectures)

Fall 2012: *Families of p-adic modular forms* (2 lectures)

Spring 2012: *Classifying pro-p subgroups of* $\mathrm{SL}(2, A)$ *for a p-adic ring A*

UCLA Graduate Student Seminar:

October 2014: *The Art of Giving a Math Talk*

January 2014: *What is the BSD conjecture?*

November 2012: *The Local-Global Principle*

Part III 2010 Lent seminars: *How to add points on a hyperelliptic curve of genus two*

Part III 2009 Michaelmas seminars: *The Local-Global Principle*

Conference and Workshops Attended

Boston University/Keio University Workshop: Number Theory, September 2015, Boston University (funded participant)

Sage Days 69: Women in Sage 6, September 2015, La Jolla (funded participant)

Illinois Number Theory Conference, August 2015, University of Illinois - Urbana-Champaign (funded participant)

p-adic methods in number theory: A conference inspired by the mathematics of Robert Coleman, May 2015, University of California - Berkeley (funded participant)

Southern California Number Theory Day, May 2015, University of California - San Diego

Southern California Number Theory Day, April 2015, California Institute of Technology

p-adic methods in the theory of classical automorphic forms, March 2015, Centre de recherches mathématiques, Montreal (funded participant)

Automorphic forms, Shimura varieties, Galois representations and L-functions, December 2014, Mathematical Sciences Research Institute, Berkeley (funded participant)

Southern California Number Theory Day, October 2014, University of California - Irvine

Introductory Workshop: New Geometric Techniques in Number Theory, August 2014, Mathematical Sciences Research Institute, Berkeley (funded participant)

Connections for Women: New Geometric Techniques in Number Theory, August 2014, Mathematical Sciences Research Institute, Berkeley (funded participant)

Graduate Summer School: Counting Arithmetic Objects, June 2014, Centre de recherches mathématiques, Montreal (funded participant) *p-adic variation in number theory*, June 2014, Boston University (funded participant)

Women in Numbers 3, April 2014, Banff International Research Station, Banff, Shadow Lines project group (funded participant)

11th Annual Yale Bouchet Conference on Diversity and Graduate Education, March 2014, Yale University (funded participant)

Arizona Winter School: Arithmetic Statistics, March 2014, University of Arizona, Bjorn Poonen's Project Group (funded participant)

Hot Topics Workshop: Perfectoid Spaces and their Applications, March 2014, Mathematical Sciences Research Institute, Berkeley (funded participant)

Conference on Stark's Conjectures and related topics, September 2013, University of California - San Diego (funded participant)

Graduate Summer School: New Geometric Techniques in Number Theory, July 2013, Mathematical Sciences Research Institute, Berkeley (funded participant)

p-adic modular forms, L-functions, and Galois representations, May 2013, University of California - Los Angeles

Cohomology of Arithmetic Groups Graduate Workshop, May 2013, Chicago (funded participant)

Arizona Winter School: Modular Forms and Modular Curves, March 2013, University of Arizona, Frank Calegari's Project Group (funded participant)

p-adic modular forms and arithmetic, June 2012, University of California - Los Angeles

Joint Mathematics Meetings January 2013, 2012 (funded participant)

Teaching Experience

University of California - Los Angeles

Teaching Assistant Consultant, Fall 2013 (trained new teaching assistants)

Teaching Fellow, Discrete Mathematics, Spring 2014

Teaching Fellow, Group Theory, Winter 2014

Teaching Fellow, Integration and Infinite Series, Fall 2013

Teaching Assistant, Integration and Infinite Series, Summer 2013

Teaching Assistant, Integration and Infinite Series, Spring 2011

Teaching Assistant for Summer Program for Women in Mathematics, George Washington University, Summer 2012

Counselor for Program in Mathematics for Young Scientists (PROMYS), Boston University, Summer 2010

Bryn Mawr College

Problem Session Holder and Grader, Abstract Algebra II, Spring 2009

Problem Session Holder and Grader, Abstract Algebra I, Fall 2008

Peer Instructor, Linear Algebra, Spring 2008

Peer Instructor, Multivariable Calculus, Fall 2007

Problem Session Holder and Grader, Transitions to Higher Mathematics, Spring 2007

Problem Session Holder, Calculus 101, Fall 2006

Grader, Multivariable Calculus (enriched), Fall 2006

Service and Outreach Activities

Served as a referee for *Math Research Letters* and *Coates' 70th Birthday Conference Proceedings*

Contributed functionality to the open source software package Sage

Organized and ran advising workshops for UCLA math graduate students applying for NSF Graduate Research Fellowship, 2014, 2015

Co-created and ran a booth on the Monty Hall Problem at UCLA EmpowHer STEM Day, 2014, 2015

President of Graduate Student Organization in the UCLA mathematics department, 2012-2014

Co-founded and co-organized UCLA women in math group (2010-present)

Panelist at Aftermath Conference for undergraduate math majors interested in graduate school, Harvey Mudd College, February 2013

Served on panel for undergraduate math majors interested in graduate school, University of California - Los Angeles, October 2011

Languages

English (native)

French (intermediate reading, writing, speaking)

This page is intentionally blank

Overview

My research is in the area of algebraic number theory. Primarily I study modular forms, the Galois representations associated to them, and p-adic families of such objects. I am also interested in the arithmetic of elliptic curves. The main result of my thesis is that, in a qualitative sense, the Galois representation associated to an ordinary p-adic family of modular forms has "large" image. My future research plans include improving the result to a quantitative form and obtaining a complete description of the images of such Galois representations.

Galois representations are fundamental objects of study in modern number theory. They are the only known tools for systematically studying the absolute Galois group of the rational numbers, $G_{\mathbb{Q}}$, and geometric Galois representations are one side of the celebrated Langlands correspondence. A Galois representation is a continuous homomorphism $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(A)$ for a topological ring A. Most known examples of Galois representations arise from an action of $G_{\mathbb{Q}}$ on the cohomology of varieties defined over $\mathbb{Q}$, or by putting such representations into p-adic families.

A fundamental problem is to determine the image of a given Galois representation. This was first done by Serre [15] for the p-adic Galois representations $\rho_{E,p}$ associated to an elliptic curve $E_{/\mathbb{Q}}$. He showed that if E does not have complex multiplication (CM) then $\rho_{E,p}$ is surjective for all but finitely many primes p. Furthermore the image of $\rho_{E,p}$ is open for all p. Serre's result is an example of a general pattern governing the expected behavior of images of Galois representations.

Heuristic. *The image of a Galois representation should be as large as possible, subject to the symmetries of the geometric object from which it arose.*

The notion of "symmetry" is vague and depends on the situation. In the case of elliptic curves, the relevant symmetry is complex multiplication, a condition that means that the elliptic curve has a larger endomorphism ring than usual. In the 1980s, Ribet and Momose determined, up to finite error, the image of a Galois representation coming from a classical modular form without CM and thus showed that such images are "large" [12, 14]. Their proof introduced new symmetries of modular forms known as "conjugate self-twists". Indeed, if $\rho : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathcal{O})$ arises from a modular form, then one can talk about the subring $\mathcal{O}_0$ of $\mathcal{O}$ fixed by all conjugate self-twists. Ribet and Momose proved that the intersection of $\mathrm{Im}\,\rho$ with $\mathrm{SL}_2(\mathcal{O}_0)$ is open in $\mathrm{SL}_2(\mathcal{O}_0)$.

In the 1980s Hida developed his theory of p-adic families of (ordinary) modular forms. To such a family F, Hida associated a Galois representation $\rho_F : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{I})$ for a certain ring $\mathbb{I}$. From ρ_F one obtains a mod p representation $\bar{\rho}_F$. One can consider conjugate self-twists of F and form the ring $\mathbb{I}_0$ fixed by all such twists. The following theorem is the main result of my thesis [8].

Theorem 1 (Lang [8]). *Let F be a non-CM Hida family. Assume that $\bar{\rho}_F$ is absolutely irreducible and satisfies a technical but mild regularity condition. Then there is a nonzero $\mathbb{I}_0$-ideal $\mathfrak{a}_0$ such that the image of ρ_F contains all matrices in $\mathrm{SL}_2(\mathbb{I}_0)$ that are congruent to the identity modulo $\mathfrak{a}_0$.*

I have a secondary interest in the arithmetic of elliptic curves. My group at the Women in Numbers 3 workshop studied shadow lines of elliptic curves, an invariant first defined by Mazur and Rubin [10]. Using explicit class field theory, we developed an algorithm to compute the shadow line of a triple (E, K, p), where $E_{/\mathbb{Q}}$ is an elliptic curve, K is an imaginary quadratic field, and p is a rational prime of good reduction for E that splits in K. We implemented our algorithm in Sage [16] and computed the first examples of shadow lines. The resulting paper will appear in the proceedings volume [1].

Thesis

Background. I now give some definitions and notation to make the statement of Theorem 1 more precise. Fix a prime p, and assume for simplicity that $p \geq 5$. Let $\Lambda = \mathbb{Z}_p[[T]]$ and $\mathbb{I}$ be an integral domain that is finite flat over Λ. Fix embeddings of an algebraic closure $\overline{\mathbb{Q}}$ of $\mathbb{Q}$ into $\mathbb{C}$ and $\overline{\mathbb{Q}}_p$.

Definition 1 (Hida [5], Wiles [17])**.** A formal power series $F = \sum_{n=1}^{\infty} A_n q^n \in \mathbb{I}[[q]]$ is a *Hida family* if $A_p \in \mathbb{I}^{\times}$ and for every integer $k \geq 2$ and every prime ideal $\mathfrak{P}$ of $\mathbb{I}$ lying over $(1+T-(1+p)^k)\Lambda$, we have:

- $A_n \bmod \mathfrak{P}$ is in $\overline{\mathbb{Q}}$ (rather than just $\overline{\mathbb{Q}}_p$), and
- $f_{\mathfrak{P}} := \sum_{n=1}^{\infty}(A_n \bmod \mathfrak{P})q^n$ gives the q-expansion of a classical modular form of weight k.

Hida showed that every p-ordinary classical modular form of weight at least 2 arises from a unique such family [5]. Furthermore, there is a Galois representation $\rho_F : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{I})$ that is unramified almost everywhere. For all primes ℓ at which ρ_F is unramified, $\operatorname{tr}\rho_F(\mathrm{Frob}_\ell) = A_\ell$, where Frob_ℓ is the conjugacy class of a Frobenius element at ℓ [4].

Definition 2. An automorphism σ of $\mathbb{I}$ is a *conjugate self-twist* of a Hida family $F = \sum_{n=1}^{\infty} A_n q^n \in \mathbb{I}[[q]]$ if there exists a non-trivial Dirichlet character η_σ such that

$$\sigma(A_\ell) = \eta_\sigma(\ell)A_\ell$$

for almost all primes ℓ. We say F has *complex multiplication (CM)* if the identity automorphism is a conjugate self-twist of F.

Ideas in the Proof of Theorem 1. The key result is a lifting theorem showing when a conjugate self-twist of an arithmetic specialization $f_{\mathfrak{P}}$ of F can be lifted to a conjugate self-twist of the entire family F. The ideas that go into the lifting theorem will be briefly explained in the next paragraph. Using the lifting theorem, there is a series of reduction steps to conclude that the theorem of Ribet and Momose is sufficient to prove Theorem 1. An important tool in these reduction steps is a $\mathbb{Z}_p$-Lie algebra of Pink [13] that is associated to $\operatorname{Im}\rho_F$, which allows us to reduce our problem to linear algebra. Hida observed that the ordinarity of ρ_F can be used to give Pink's Lie algebra a Λ-algebra structure [6], and this structure is critical to the proof.

I now give a brief outline of the proof of the lifting theorem. A conjugate self-twist σ of an arithmetic specialization $f_{\mathfrak{P}}$ of F induces an automorphism $\bar{\sigma}$ of the residue field $\mathbb{F}$. Using deformation theory, I lift $\bar{\sigma}$ to an automorphism Σ of the entire universal deformation ring $R_{\bar{\rho}_F}$ of $\bar{\rho}_F$ and show that Σ satisfies certain properties. Using automorphic methods I show that Σ preserves a certain Hecke algebra, $\mathbb{T}$. Since $\operatorname{Spec}\mathbb{I}$ is an irreducible component of $\operatorname{Spec}\mathbb{T}$, we see that Σ must send $\operatorname{Spec}\mathbb{I}$ to another irreducible component $\Sigma^*\operatorname{Spec}\mathbb{I}$ of $\operatorname{Spec}\mathbb{T}$. The properties of Σ force the two components $\operatorname{Spec}\mathbb{I}$ and $\Sigma^*\operatorname{Spec}\mathbb{I}$ to intersect at an arithmetic point. Since the Hecke algebra is étale over Λ at arithmetic points, Σ must descend to an automorphism of $\mathbb{I}$, as desired.

Future work

Determining the $\mathbb{I}_0$-level and relation to p-adic L-functions. Theorem 1 guarantees that there is a non-zero $\mathbb{I}_0$-ideal $\mathfrak{a}_0$ such that the image of ρ_F contains all determinant 1 matrices that are congruent to the identity modulo $\mathfrak{a}_0$. The largest such ideal is called the $\mathbb{I}_0$-*level* of ρ_F and is denoted $\mathfrak{c}_{0,F}$.

A natural question is to determine the $\mathbb{I}_0$-level. I plan to prove the following conjecture, which is a generalization of Hida's Theorem II in [6].

Conjecture 1. *Let F and p be as in Theorem 1.*

1. *If $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\mathbb{F}_p)$, then $\mathfrak{c}_{0,F} = \mathbb{I}_0$. That is, $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(\mathbb{I}_0)$.*
2. *Suppose that $\bar{\rho}_F$ is absolutely irreducible and $\bar{\rho}_F \cong \operatorname{Ind}_M^{\mathbb{Q}} \bar{\psi}$ for an imaginary quadratic field M in which p splits and a character $\bar{\psi} : \operatorname{Gal}(\overline{\mathbb{Q}}/M) \to \overline{\mathbb{F}}_p^{\times}$. Assume M is the only such quadratic field. Under minor conditions on the tame level of F, there is a product $\mathcal{L}_0$ of anticyclotomic Katz p-adic L-functions such that $\mathfrak{c}_{0,F}$ is a factor of $\mathcal{L}_0$. Furthermore, every prime factor of $\mathcal{L}_0$ is a factor of $\mathfrak{c}_{0,F}$ for some F.*

Note that even when $\mathbb{I} = \Lambda$, Conjecture 1 is stronger than Hida's Theorem II [6]. Furthermore, Conjecture 1.1 is a natural extension of the work of Mazur-Wiles [11] and Fischman [3].

In proving case (1) of the conjecture, I will make use of Manoharmayum's recent work that shows $\operatorname{Im}\rho_F \supseteq \mathrm{SL}_2(W)$ for a finite unramified extension W of $\mathbb{Z}_p$ [9]. This will be combined with the Λ-module structure on the Pink Lie algebra associated to $\operatorname{Im}\rho_F$ that was used in the proof of Theorem 1 to get the desired result.

In proving case (2), I will relate the $\mathbb{I}_0$-level to the congruence ideal of F as in Hida's proof of Theorem II [6]. The connection to Katz p-adic L-functions is then obtained by relating the congruence ideal to the p-adic L-function through known cases of the Main Conjecture of Iwasawa Theory. The idea is that replacing the Λ-level in Hida's work with the more precise $\mathbb{I}_0$-level will allow me to remove the ambiguity of the square factors that show up in Theorem II [6].

Proving Conjecture 1 would yield refined information about the images of Galois representations attached to Hida families. It is the first step in completely determining the images of such representations.

Computing $\mathcal{O}_0$-levels of classical Galois representations. The goal of this project is to compute the level of Galois representations coming from classical modular forms and thus completely determine the image of such a representation. Let f be a non-CM classical Hecke eigenform, $\mathfrak{p}$ a prime of the ring of integers of the field generated by the Fourier coefficients of f, and $\rho_{f,\mathfrak{p}} : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathcal{O})$ the associated p-adic representation. Let π be a uniformizer of the subring $\mathcal{O}_0$ of $\mathcal{O}$ fixed by all conjugate self-twists. By the work of Ribet [14] and Momose [12], there is a minimal non-negative integer $c(f,\mathfrak{p})$ such that $\operatorname{Im}\rho_f$ contains all matrices of determinant 1 that are congruent to the identity modulo $\pi^{c(f,\mathfrak{p})}$. Their work shows that $c(f,\mathfrak{p}) = 0$ for all but finitely many primes $\mathfrak{p}$. However, relatively little is known about the case when $c(f,\mathfrak{p})$ is positive and the weight of f is greater than 2. I plan to study how $c(f,\mathfrak{p})$ changes as f varies over the classical specializations of a non-CM Hida family that is congruent to a CM family.

This project will have both theoretical and computational components. First, I will establish a relationship between $c(f,\mathfrak{p})$ and the congruence number of f, which should also be related to values of the Katz p-adic L-function, as suggested by the proof of Theorem II in [6]. Once this is established, I will create a method in the open source software Sage [16] to compute $c(f,\mathfrak{p})$ by computing the congruence number of f. This should be relatively straightforward since Sage can already compute congruence numbers. Using the new functionality, I will create a large data set of levels of classical Galois representations in Hida families, which will likely lead to new conjectures to be studied theoretically. I have experience working with Sage from my Women in Numbers 3 project [1] and from leading a project at Sage Days 69. Indeed, my Sage Days project consisted of writing a method to test whether a modular form is CM, a first step in the eventual program to compute $c(f, \mathfrak{p})$. I hope to involve undergradutes in the computational aspects of the project. The programming should be straightforward and would be a good way for them to learn about modular forms.

Analogue of the Mumford-Tate Conjecture in p-adic families. Another way to describe the work of Ribet and Momose is that they proved the Mumford-Tate Conjecture for compatible systems of Galois representations associated to classical modular forms. Hida has proposed an analogue of the Mumford-Tate Conjecture for p-adic families of Galois representations. For an arithmetic prime $\mathfrak{P}$ of $\mathbb{I}$, write $\mathrm{MT}_{\mathfrak{P}}$ for the Mumford-Tate group of the compatible system containing $\rho_{f_{\mathfrak{P}}}$, so $\mathrm{MT}_{\mathfrak{P}}$ is an algebraic group over $\mathbb{Q}$. Let $\kappa(\mathfrak{P}) = \mathbb{I}_{\mathfrak{P}}/\mathfrak{P}_{\mathfrak{P}}$, and write $G_{\mathfrak{P}}$ for the Zariski closure of $\operatorname{Im} \rho_{f_{\mathfrak{P}}}$ in $\mathrm{GL}_2(\kappa(\mathfrak{P}))$. Let $G_{\mathfrak{P}}^{\circ}$ be the connected component of the identity of $G_{\mathfrak{P}}$ and $G'_{\mathfrak{P}}$ the (closed) derived subgroup of $G_{\mathfrak{P}}$. Finally, let Γ_F denote the group generated by the conjugate self-twists of a non-CM Hida family F.

Conjecture 2 (Hida)**.** *Assume F is non-CM. There is a simple algebraic group G', defined over $\mathbb{Q}_p$, such that for all arithmetic primes $\mathfrak{P}$ of $\mathbb{I}$ one has $G'_{\mathfrak{P}} \cong G' \times_{\mathbb{Q}_p} \kappa(\mathfrak{P})$ and $\mathrm{Res}^{\kappa(\mathfrak{P})}_{\mathbb{Q}_p} G_{\mathfrak{P}}$ is (the ordinary factor of) $\mathrm{MT}_{\mathfrak{P}} \times_{\mathbb{Q}} \mathbb{Q}_p$. Furthermore, the component group $G_{\mathfrak{P}}/G^0_{\mathfrak{P}}$ is canonically isomorphic to the Pontryagin dual of the decomposition group of $\mathfrak{P}$ in Γ_F.*

By obtaining a sufficiently precise understanding of images of Galois representations attached to Hida families through the first project, I plan to prove results along the lines of Conjecture 2. I have some preliminary results relating the Pontryagin dual of Γ_F to the quotient $(\operatorname{Im} \rho_F)/(\operatorname{Im} \rho_F|_H)$ for a certain finite index normal subgroup H of $G_{\mathbb{Q}}$.

Completing this research objective, or even any preliminary results in this direction, would reveal that the images of classical specializations of the Galois representation attached to a Hida family are even more related to one another than previously thought. Not only would they arise as specializations of some group in $\mathrm{GL}_2(\mathbb{I})$, they could all be found simply by base change from a single group, at least up to abelian error.

Other settings. The above projects can be studied in more general settings than Hida families for GL_2. Hida and Tilouine proved an analogue of Hida's Theorem II [6] for GSp_4-representations associated to Hida families of Seigel modular forms [7]. There are two main difficulties they overcome in their work: the types of symmetries are much more complicated than CM versus non-CM, and Pink's theory of Lie algebras is only valid for SL_2. The tools they developed to overcome these problems could be applied to study analogues of the above questions for bigger groups.

Tilouine and his collaborators proved an analogue of Theorem 1 in the non-ordinary GL_2-setting [2] by building on the ideas in my thesis. They introduce the relative Sen operator to create a Λ-algebra structure on the Lie algebra of $\operatorname{Im} \rho_F$ as their representation is not ordinary. This idea will be useful in studying the above questions in the non-ordinary setting.

References

[1] Balakrishnan, J., Çiperiani, M., Lang, J., Mirza, B., and Newton, R. Shadow lines in the arithmetic of elliptic curves. to appear, 2015.

[2] CONTI, A., IOVITA, A., AND TILOUINE, J. Big image of Galois representations attached to finite-slope p-adic families of modular forms. July 2015.

[3] FISCHMAN, A. On the image of Λ-adic Galois representations. *Ann. Inst. Fourier (Grenoble) 52*, 2 (2002), 351–378.

[4] HIDA, H. Galois representations into $\mathrm{GL}_2(\mathbf{Z}_p[[X]])$ attached to ordinary cusp forms. *Invent. Math. 85*, 3 (1986), 545–613.

[5] HIDA, H. Iwasawa modules attached to congruences of cusp forms. *Ann. Sci. École Norm. Sup. (4) 19*, 2 (1986), 231–273.

[6] HIDA, H. Big Galois representations and p-adic L-functions. *Compos. Math. 151*, 4 (2015), 603–664.

[7] HIDA, H., AND TILOUINE, J. Big image of Galois representations and the congruence ideal. In *Arithmetic and Geometry* (2014), L. Dieulefait, G. Faltings, D. R. Heath-Brown, Y. V. Manin, B. Z. Moroz, and J.-P. Wintenberger, Eds., vol. 420, Cambridge University Press, pp. 217–254.

[8] LANG, J. On the image of the Galois representation associated to a non-CM Hida family. http://www.math.ucla.edu/~jaclynlang/I_0_level_existence.pdf, submitted, July 2015.

[9] MANOHARMAYUM, J. A structure theorem for subgroups of GL_n over complete local Noetherian rings with large residual image. *Proc. Amer. Math. Soc. 143*, 7 (2015), 2743–2758.

[10] MAZUR, B., AND RUBIN, K. Studying the growth of Mordell-Weil. *Doc. Math.*, Extra Vol. (2003), 585–607.

[11] MAZUR, B., AND WILES, A. On p-adic analytic families of Galois representations. *Compos. Math. 59*, 2 (1986), 231–264.

[12] MOMOSE, F. On the l-adic representations attached to modular forms. *J. Fac. Sci. Univ. Tokyo Sect. IA Math. 28*, 1 (1981), 89–109.

[13] PINK, R. Classification of pro-p subgroups of SL_2 over a p-adic ring, where p is an odd prime. *Compos. Math. 88*, 3 (1993), 251–264.

[14] RIBET, K. A. On l-adic representations attached to modular forms. II. *Glasgow Math. J. 27* (1985), 185–194.

[15] SERRE, J.-P. *Abelian ℓ-adic representations and elliptic curves*. W. A. Benjamin, Inc., 1968.

[16] A. STEIN, W., ET AL. *Sage Mathematics Software*. The Sage Development Team, http://www.sagemath.org.

[17] WILES, A. On ordinary λ-adic representations associated to modular forms. *Invent. Math. 94*, 3 (1988), 529–573.

This page is intentionally blank

Teaching at UCLA

I have been a Teaching Assistant (TA) for lower and upper division courses as well as a Teaching Assistant Consultant (TAC). The latter position is awarded to experienced TAs with a strong record of past instruction. The TAC teaches and develops the curriculum for MATH 495 – the course instructing first year TAs in effective teaching techniques – and participates in a quarter-long campus-wide seminar on pedagogy through the Office of Instructional Development. In 2014 I was awarded a Distinguished Teaching Award from the UCLA Mathematics Department based on evaluations from students and faculty. In my classroom I focus on developing problem solving, technical communication, and programming skills, while mentoring my students, particularly those underrepresented in STEM fields.

Problem Solving. In teaching problem solving skills, I employ Polya's techniques from his classic book *How to Solve It*. For example, visualization can be a powerful tool in Group Theory, where the material can seem abstract but is capturing something visual (symmetries of objects). As a TA for Group Theory, I had the students make equilateral polygons and use them to understand multiplication in dihedral groups. I also brought in toothpicks and gumdrops so that students could make their own dodecahedra and understand their symmetries – something students remembered months later during evaluations. Another technique that Polya stresses is the importance of reflecting on past work. In Group Theory, I required students to self-evaluate their homework by ranking how well they believed they answered each question. This did not affect their grades, but it forced them to reflect on their work and learning process while helping me write "helpful comments on every homework assignment" (student evaluation).

Technical Communication. A central goal of my classes is for students to learn technical communication skills, both written and oral, formal and informal. As one of my evaluations noted, "she encourages student participation and students interacting actively with each other." For example, I developed a worksheet for the first discussion section of Integration and Infinite Series that immediately catalyzes mathematical student discussions. The worksheet (available at `www.math.ucla.edu/~jaclynlang/`) asks students to sketch graphs of standard functions, evaluate limits, recall trigonometry, and remember rules for derivatives, exponents, and logarithms. Students not only answer each question, but name a person with whom they solved it. (The same name cannot appear more than four times.) Thus, students discuss mathematics while reviewing the prerequisite material for the course. This activity allows me to meet the students and gauge the background of the class. The informal communication skills that students develop through such activities is useful in any career involving technical teamwork. I know my students learn to value communication skills because their evaluations call me a "great communicator."

Programming. As computers become more powerful and ubiquitous, programming skills are becoming critical in many disciplines and careers. Math courses are a natural place to introduce students to the basics of programming, and computational problems can lead to student research. As a Calculus TA, I used the open source software Sage to supplement the numerical analysis material in the course. Students wrote simple programs in Sage Math Cloud that implemented numerical integration techniques and error bounds. Sage Math Cloud is well suited for assignments and classroom activities. Students add me as a collaborator, and I can give feedback directly on their code. Furthermore, some of my research projects involve computations in Sage. By bringing Sage into the classroom, I can offer interested students opportunities to deepen their knowledge by conducting research with me or working on Sage development projects.

Teaching beyond UCLA

My teaching is influenced by my experiences outside of UCLA. As a student at Bryn Mawr College, I ran problem sessions for math courses from Calculus to Abstract Algebra. I met one-on-one with students, deepening conceptual understanding rather than just doing homework problems, which naturally hones the problem solving and communication skills mentioned above. Many interactive activities I developed at UCLA were inspired by my experience at Bryn Mawr.

In summer 2010 I was a counselor for the Program in Mathematics for Young Scientists at Boston University, a six-week program for talented high school students. I was responsible for guiding the learning of three students. One had a strong background in competition math, one was severely unprepared for the program, and the last was talented but struggled with mathematical confidence. I addressed this challenge by spending hours with each individual student, away from the pressures of a group setting, to create goals that were achievable and maximized her learning. While such time is not always possible for large classes, I keep this example in mind in my classes and work with my students to articulate and achieve appropriate individual goals.

During summer 2012 I was a TA at the Summer Program for Women in Mathematics at George Washington University. The program was for women majoring in math from around the country entering their final year of college. Participants took math courses beyond the usual undergraduate curriculum and went on weekly field trips to companies and agencies that employ mathematicians with advanced degrees. I lived with the students in the dorm to help them with their courses and provide guidance on career trajectories and opportunities. Many students were curious about my experience in graduate school. These informal conversations led one student to write in her evaluations that I was "one of the most helpful parts of the program." I have incorporated mentoring into my classroom at UCLA by encouraging students to attend departmental activities and apply for math opportunities from a comprehensive list posted on my website. My students have noticed these efforts, noting on evaluations that "her concern for our education really showed."

Supporting Diversity through Teaching

I am committed to increasing participation and achievement of underrepresented minorities in mathematics. Twice I have partnered with other graduate students at UCLA to bring underprivileged middle school girls to UCLA for EmpowHer STEM Day, where they participate in science and math demonstrations. My group explained basic probability using the Monty Hall Problem.

My advocacy for diversity in STEM led to an interest in stereotype threat research and its implications for teaching mathematics. I have read books on the subject including *Whistling Vivaldi: How Stereotypes Affect Us and What We Can Do*, by Claude Steele, and I attended a related lecture series on Women in STEM curated by UCLA social psychologist Professor Jenessa Shapiro in 2011. As a TAC, part of my contribution to the curriculum for MATH 495 was to introduce a day when Professor Shapiro gave a lecture on stereotype threat and tips for combating its effects in the classroom. For example, research has shown that students who believe that math skills are malleable and grow with practice outperform those who believe math ability is a fixed genetic trait. The difference is especially dramatic for students who are part of an underrepresented group.

By teaching Polya's problem solving techniques, emphasizing communication and group work, and explicitly stressing the malleability of mathematical ability in my classes, my students internalize both the mathematics and the confidence to succeed in future technical work.

List of Publications

[1] Lang, Jaclyn, 'On the image of the Galois representation associated to a non-CM Hida family', submitted to *Algebra and Number Theory*, 2015, `http://www.math.ucla.edu/~jaclynlang/I_0_level_existence.pdf`.

[2] Balakrishnan, Jennifer, and Mirela Çiperiani, Jaclyn Lang, Bahare Mirza, Rachel Newton, 'Shadow lines in the arithmetic of elliptic curves', to appear in *Women in Numbers 3 Proceedings*, 2015, `https://www.ma.utexas.edu/users/mirela/ShadowLines.pdf`.

[3] Daub, Michael, and Jaclyn Lang, Mona Merling, Allison Pacelli, Natee Pitiwan, Michael Rosen, 'Function fields with class number indivisible by a prime ℓ', *Acta Arith.* **150** (2011) 339-359.