Let $S = (s_1, s_2, \dots, s_n)$ be a collection of vectors in a vector space V. Suppose we also have $W \subseteq V$ a subspace. Denote $\bar{S} = (\bar{s}_1, \bar{s}_2, \dots, \bar{s}_n)$ where $\bar{s}_i = s_i + W$. So $\bar{S}$ is a collection of vectors in V/W.

(1) Show that if S spans V then $\bar{S}$ spans V/W.
(2) Give an example of S, V, W where $\bar{S}$ spans V/W but S doesn't span V.
(3) Is it true that if S is linearly independent in V then $\bar{S}$ is linearly independent in V/W? Prove it or give a counterexample.
(4) Is it true that if $\bar{S}$ is linearly independent in V/W then S is linearly independent in V? Prove it or give a counterexample.