Assignment 2

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By: Anton Campbell, 100975168

Question 1 - Simple Cases

Two simple cases of 2-D electrostatic potential were created. The y dimension length was set to be $W = 60$. The x dimension length was set to be $L = 50$. The conductivity was set at $\sigma = 1$ everywhere.

For case 1, the top and bottom we set as free by setting $\frac{dV}{dy} = 0$. The left boundary ($x = 0$) was set to $V = V_0 = 1V$. The right boundary ($x = L$) was set to $V = 0$.

For case 2, the top and bottom were set to $V = 0$. The left and right were set to $V = V_0 = 1V$. The analytical solution was computed using:

$$V(x, y) = \frac{4V_0}{\pi} \sum_{n=1,3,5,...}^{\infty} \frac{1}{n} \frac{cosh\left(\frac{n\pi x}{a}\right)}{cosh\left(\frac{n\pi b}{a}\right)} sin\left(\frac{n\pi y}{a}\right)$$

where $a = W$ and $b = L/2$.

(a)

```
box_potential_1a()
```

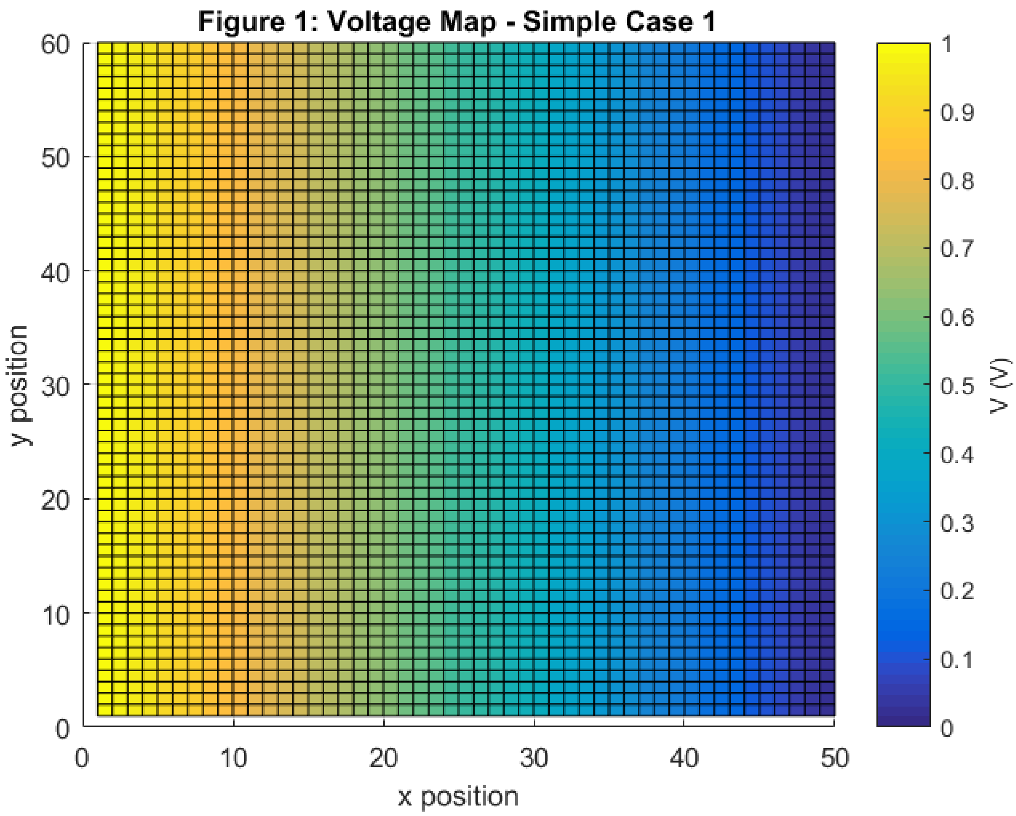


Figure 1 shows the electrostatic potential for case 1 that was solved using the finite difference method. The voltage changes linearly between the left boundary and the right boundary. The voltage is uniform in the y direction.

(b)

```
box_potential_1b()
```

Figure 2: Voltage Map for Finite Difference Method - Simple Case 2

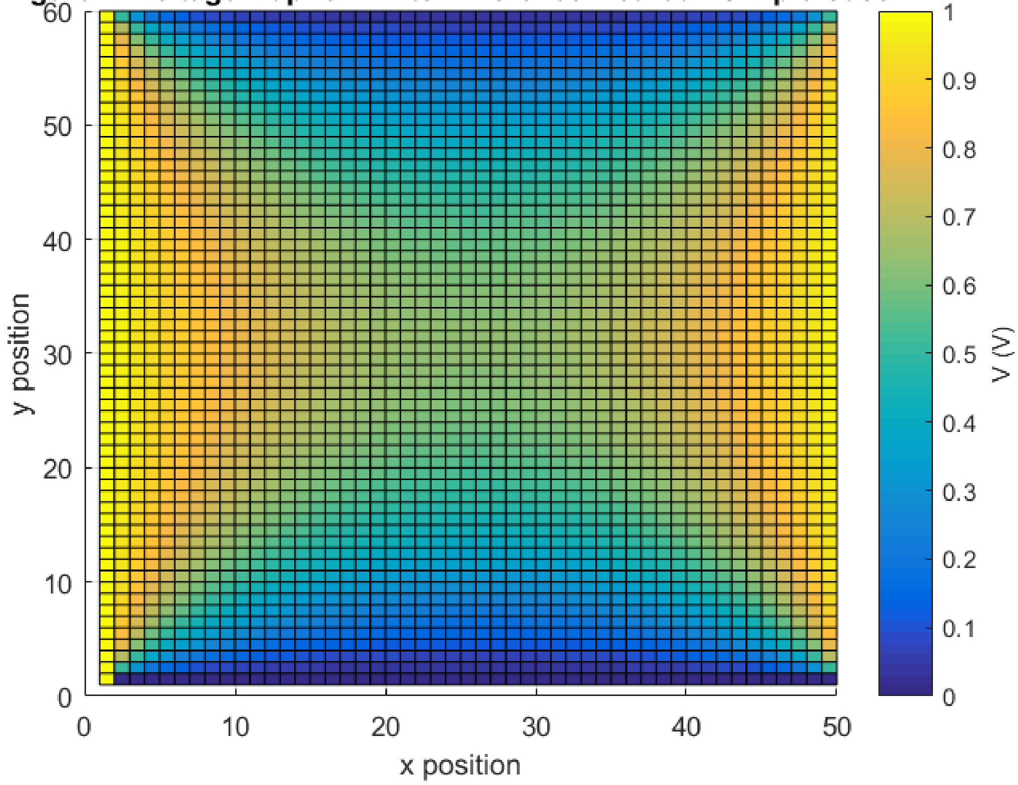


Figure 3: Voltage Map for Analytical Solution - Simple Case 2

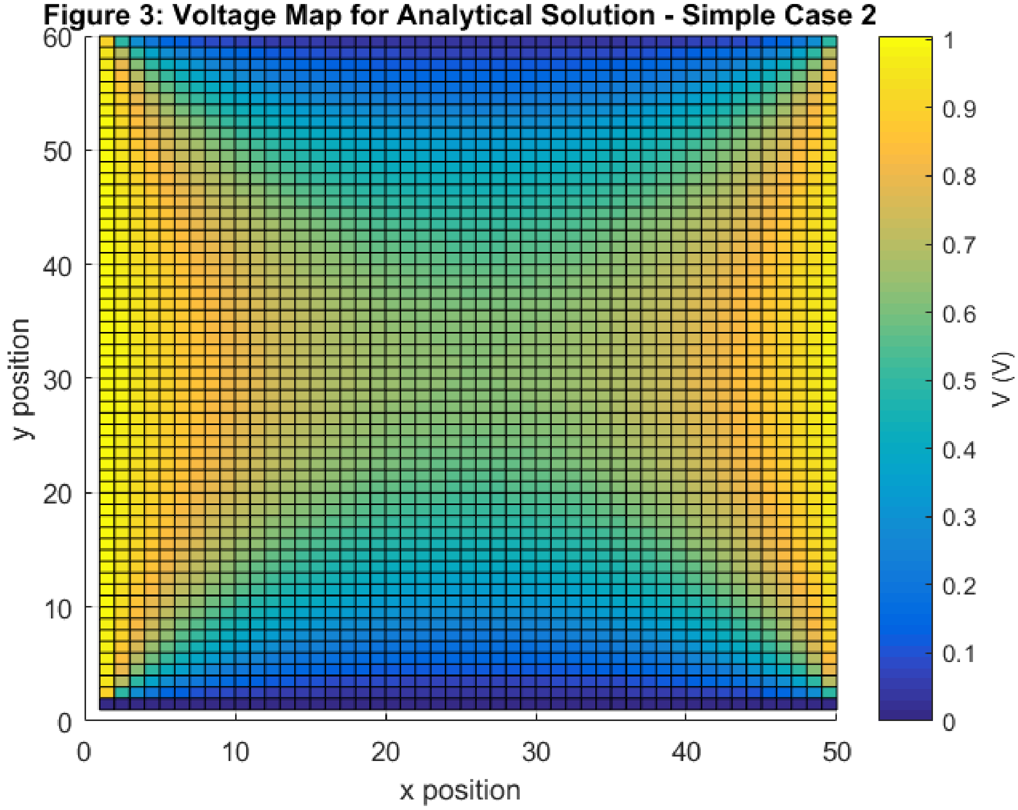


Figure 2 shows the electrostatic potential for case 2 that was solved using the finite difference method. Figure 3 shows the electrostatic potential for case 2 that was solved using the analytical series solution. The voltage map for both solutions has a saddle shape. As seen, both solutions are nearly identical. The corners of the analytical solution are slightly low.

The analytical solution was performed for a sum going up to $n = 115$. At this point the analytical solution is approaching the numerical solution. The numerical solution using using the finite difference method requires more memory. The numerical solution gives a distinct solution for the given mesh spacing. The analytical solution can require iteration.

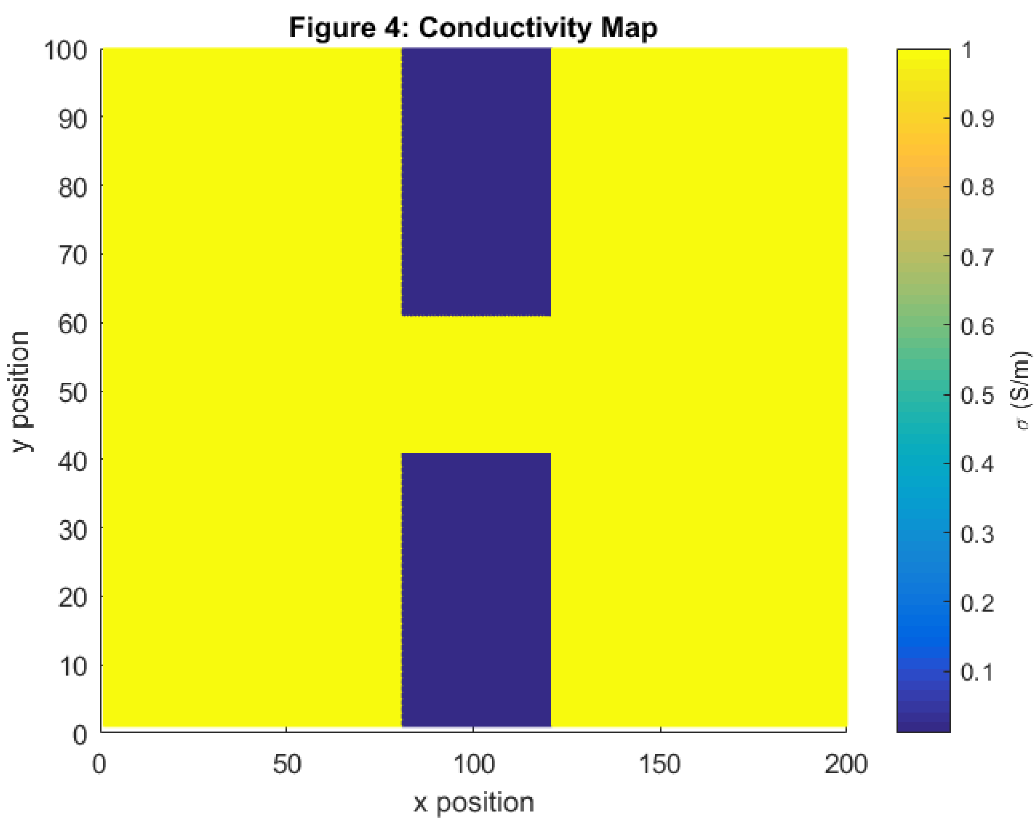
Question 2 - Bottle Neck

Case 1 was from Question 1 was modified by adding a "bottle-neck". The y dimension length was set to be $W = 100$. The x dimension length was set to be $L = 200$. Two rectangular "boxes" with low conductivity of $\sigma = 10^{-2}$ were created with heights of $W_b = 40$ and widths of $L_b = 40$. The conductivity was set at $\sigma = 1$ outside the boxes.

(a)

```
box_potential_2a()
```

The relative current througth the left contact is: 0.23853.
The relative current througth the right contact is: 0.23853.



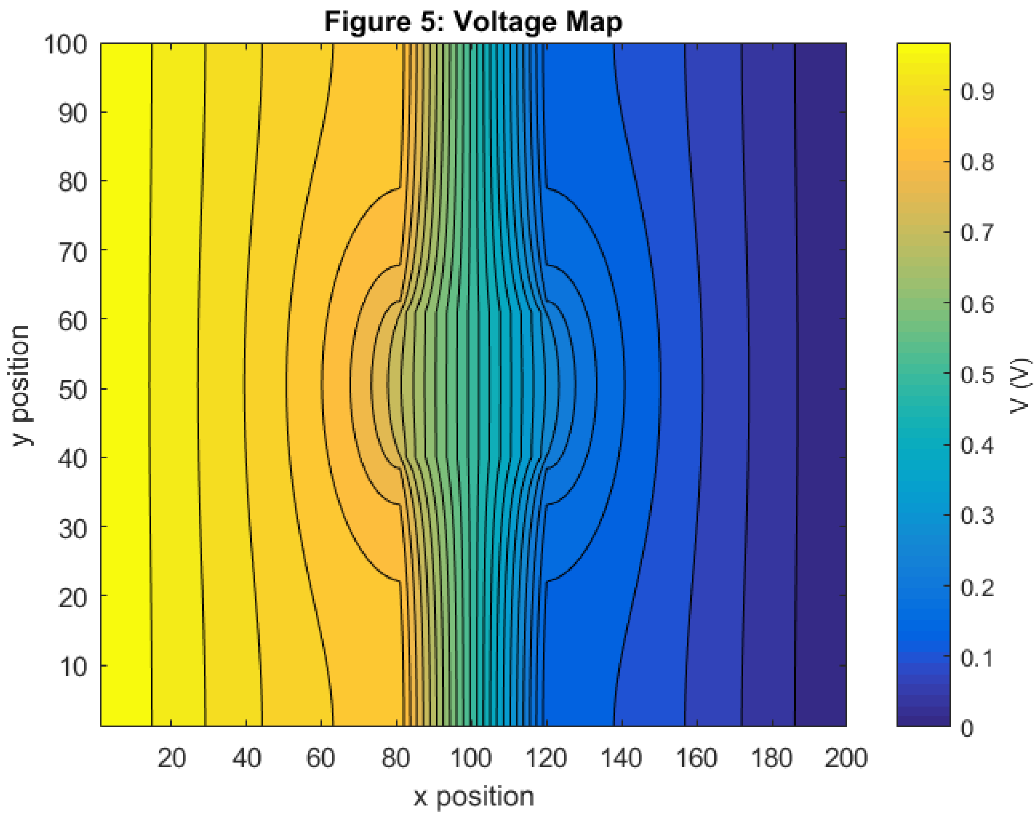


Figure 5: Voltage Map
y position
x position
V (V)
100
90
80
70
60
50
40
30
20
10
20
40
60
80
100
120
140
160
180
200
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0

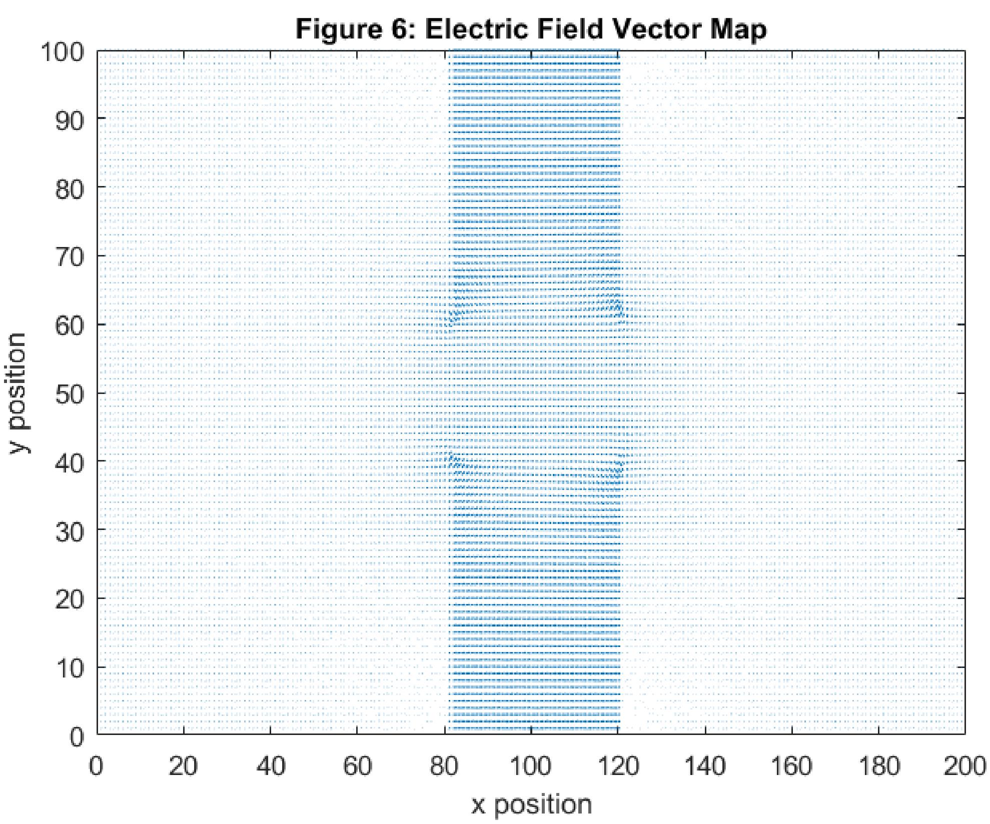
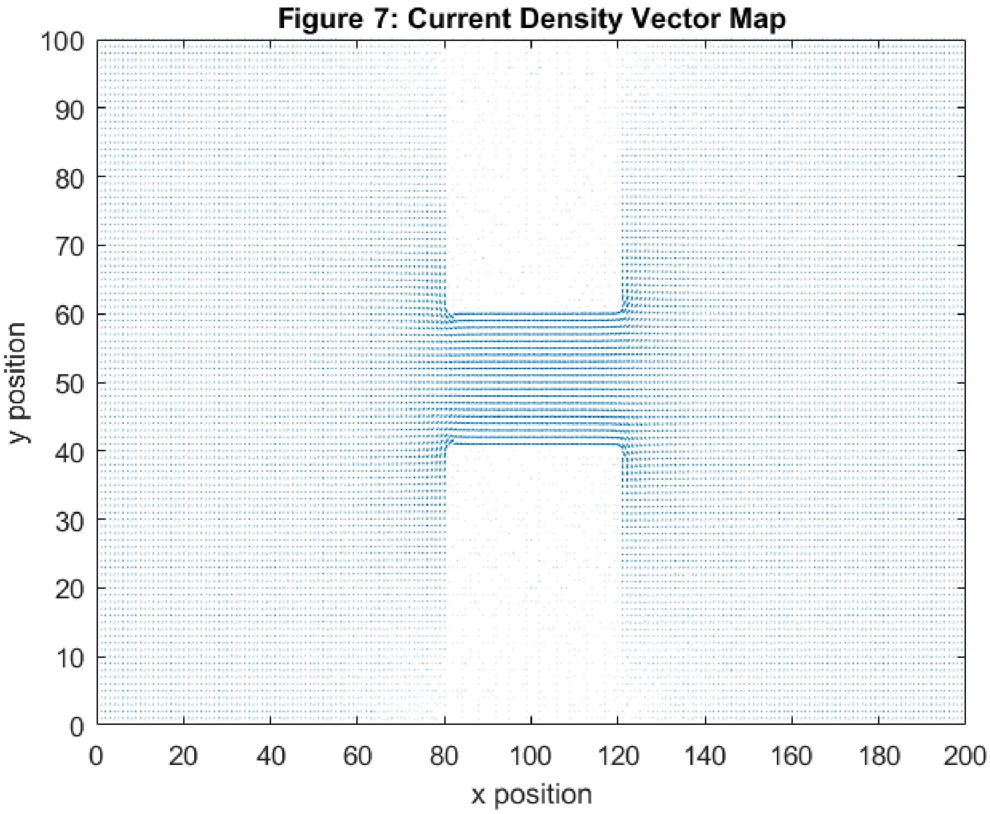


Figure 6: Electric Field Vector Map
y position
x position
100
90
80
70
60
50
40
30
20
10
0
0
20
40
60
80
100
120
140
160
180
200



The relative current through the left contact is 0.23853 which is the same as the current through the right contact.

Figure 4 shows the conductivity map. The conductivity is $\sigma = 10^{-2}$ inside the "boxes" and $\sigma = 1$ outside the boxes.

Figure 5 shows the voltage map with equipotential lines. The equipotential lines have closer spacing at the "bottle-neck". Thus, the greatest voltage drop is across the "bottle-neck". The low conductivity near the "bottle-neck" equates to a higher resistance. Since current should be conserved, voltage drop is higher for higher resistance according to the equation $V = IR$.

Figure 6 shows a map of electric field vectors. The vectors are all pointing to the right. The magnitude of the electric field is much higher inside the boxes because the voltage change is much quicker as seen in Figure 5.

Figure 7 shows a map of the current density vectors. The vectors are all pointing to the right. The current density was computed as $\vec{J} = \sigma \vec{E}$. The current density is much lower inside the "boxes" which makes sense given their low conductivity.

(b)

```
box_potential_2b()
```

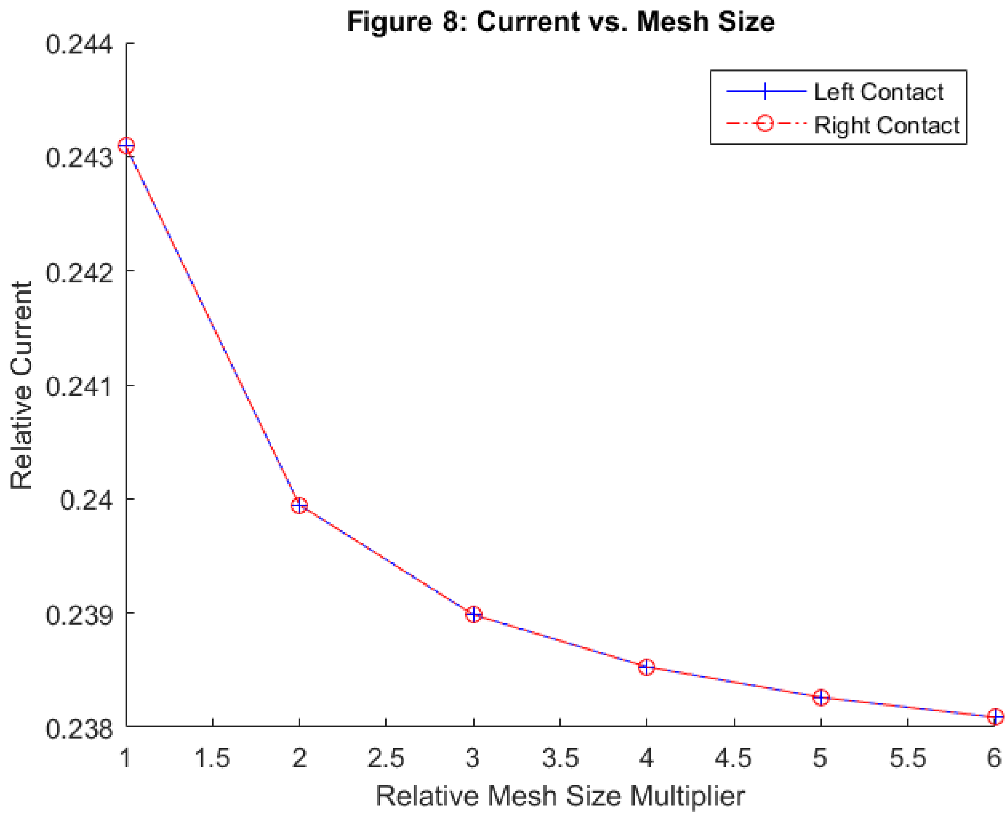


Figure 8 shows the relative current on both contacts as a function of the mesh density. The number of mesh divisions in both the x and y directions was changed using multiplying factor. The relative sizes of the "bottle-neck" was kept constant.

As the mesh density was increased, the current exponetially approached a value, which is the ideal current. As mesh density increases, the simulation more accurately represents a real situation.

(c)

```
box_potential_2c()
```

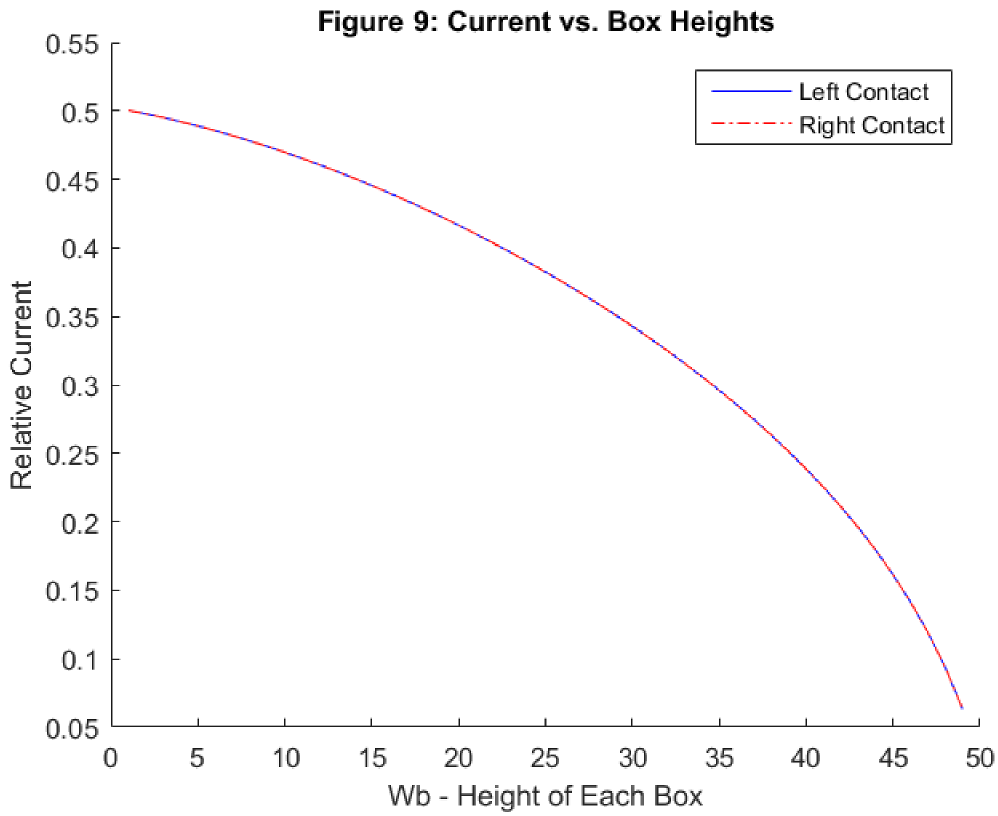


Figure 9: Current vs. Box Heights
Left Contact
Right Contact
Relative Current
Wb - Height of Each Box
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
5
10
15
20
25
30
35
40
45
50

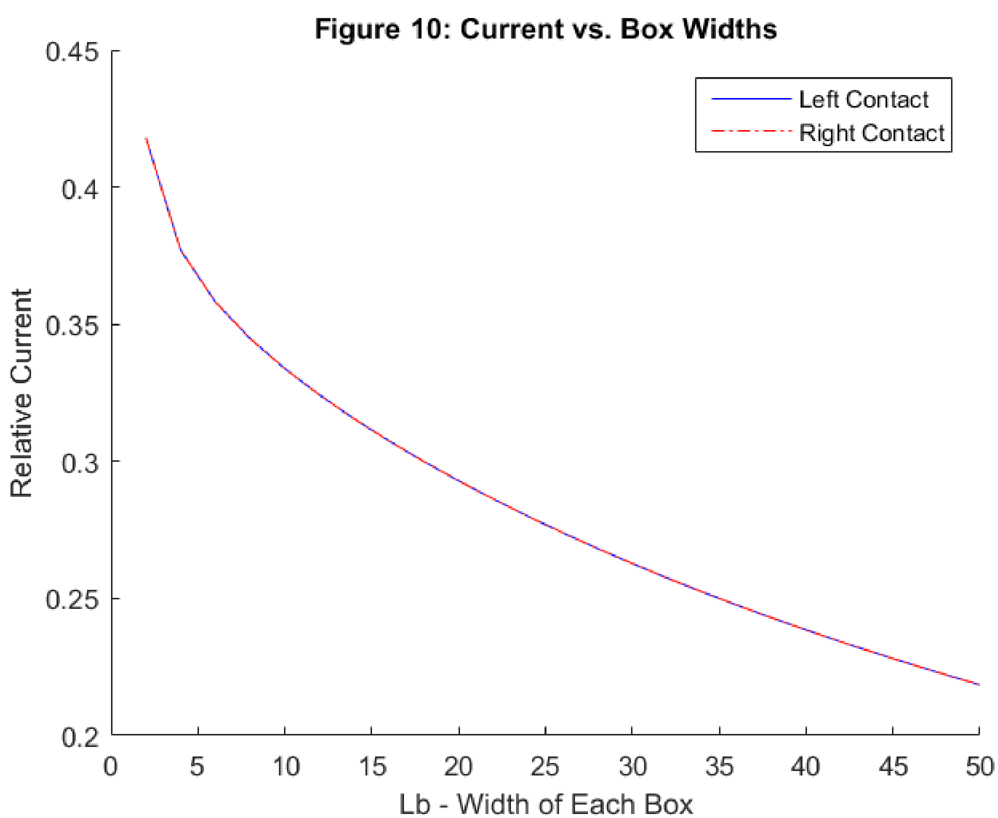


Figure 10: Current vs. Box Widths
Left Contact
Right Contact
Relative Current
Lb - Width of Each Box
0.45
0.4
0.35
0.3
0.25
0.2
0
5
10
15
20
25
30
35
40
45
50

Figure 9 shows the current as a function of the box heights, W_b. As the box heights increase, the width of the higher conductive path through the bottle neck decreases. This causes the resistance of the "bottle-neck" to increase. Since the voltages on the left and right contacts are fixed, an increased resistance decreases the current according to $V = IR$. Thus, increasing the box heights causes the relative current to decrease.

Figure 10 shows the current as a function of the box widths, L_b. As the box widths increase, the length of the "bottle-neck" path increases. This causes the resistance of the "bottle-neck" to increase. Since the voltages on the left and right contacts are fixed, an increased resistance decreases the current according to $V = IR$. Thus, increasing the box widths causes the relative current to decrease.

(d)

```
box_potential_2d()
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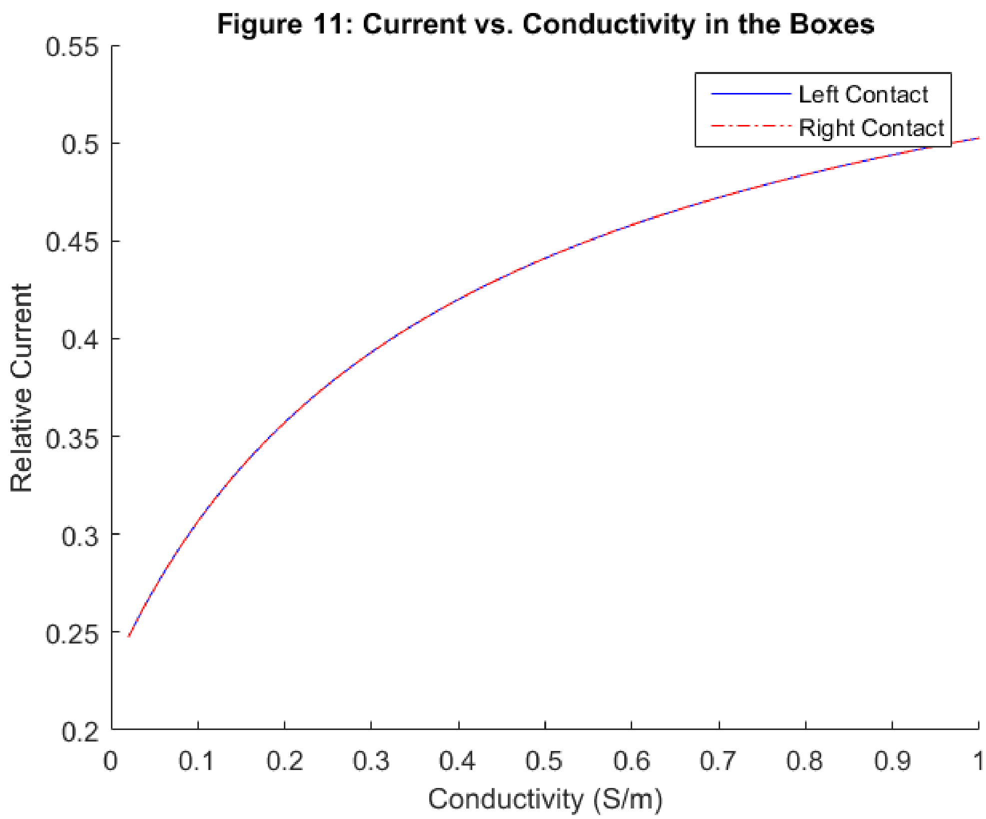

Figure 11 shows the current as a function of the conductivity in the "boxes". As the conductivity in the boxes increases, the overall resistance decreases. Since the voltages on the left and right contacts are fixed, an decreased resistance increases the current according to $V = IR$. Thus, increasing the box conductivity causes the relative current to increase.

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