

# Stuve diagram in L<sup>A</sup>T<sub>E</sub>X

## Dry adiabats calculation

Given the following dry adiabats formula<sup>1</sup> <sup>2</sup>:

$$\phi = T \cdot \left( \frac{p_0}{p} \right)^k$$

We know that  $\phi$  refers to the  $x$  axis, and  $p$  to the  $y$  axis, so we can rearrange to solve for  $p$ :

$$\begin{aligned} \frac{\phi}{T} &= \left( \frac{p_0}{p} \right)^k \\ \left( \frac{\phi}{T} \right)^{\frac{1}{k}} &= \frac{p_0}{p} \\ \left( \frac{T}{\phi} \right)^{\frac{1}{k}} &= \frac{p}{p_0} \\ p_0 \cdot \left( \frac{T}{\phi} \right)^{\frac{1}{k}} &= p \\ p &= p_0 \cdot \left( \frac{T}{\phi} \right)^{\frac{1}{k}} \end{aligned}$$

We know that  $\phi$  refers to the  $x$  axis, and  $p$  to the  $y$  axis, while  $p_0$  is the initial pressure with a standart value of 1000 hPa, and  $k$  the Poisson constant, the ratio of the gas constant to the specific heat capacity at constant pressure for an ideal diatomic gas<sup>3</sup>.

$$\phi = x; \quad p = y; \quad p_0 = 1000; \quad k = 0.286$$

We can replace variables:

$$y = 1000 \cdot \left( \frac{T}{x} \right)^{\frac{1}{0.286}}$$

And knowing that temperature should be in Kelvin:

$$y = 1000 \cdot \left( \frac{T + 273.15}{x + 273.15} \right)^{\frac{1}{0.286}}$$

Which gives us a formula to use in the L<sup>A</sup>T<sub>E</sub>X diagram.

$$1000 * ((x + 273.15) / (\text{\textbackslash}T + 273.15))^{(1/0.286)}$$

Also, we shoult take into account that in Stuve diagrams we want to draw the dry adiabats as straight lines. Thus  $y$  axis is not logarithmic, but progressing in a ratio of  $y^k$ , or  $y^{0.286}$ .

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<sup>1</sup>Stull, Roland: *Practical Meteorology: An Algebra-based Survey of Atmospheric Science*, chapter 3, p. 61, equation 3.12, 2018

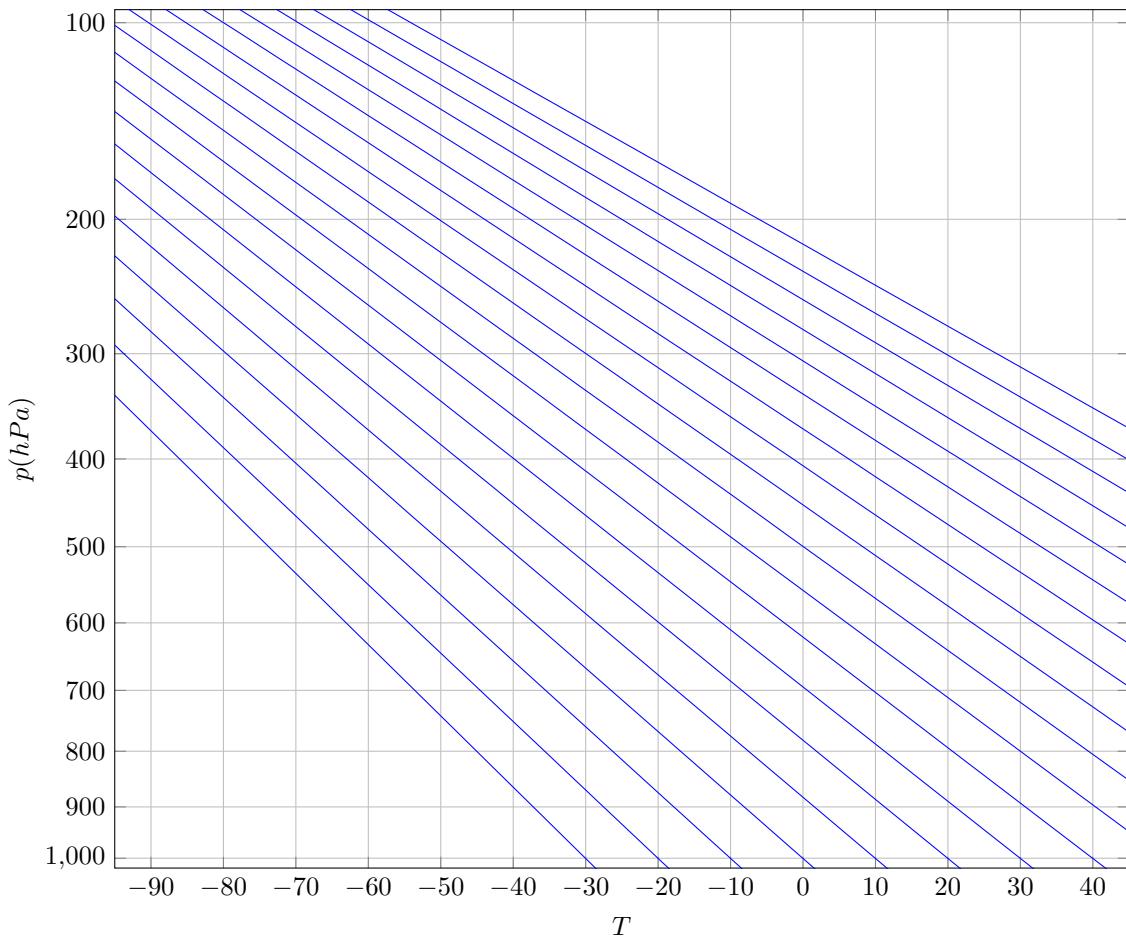
<sup>2</sup>[www.igf.fuw.edu.pl/m/courses\\_materials.../thermodynamic\\_diagrams.pdf](http://www.igf.fuw.edu.pl/m/courses_materials.../thermodynamic_diagrams.pdf)

<sup>3</sup>See <https://resources.eumetrain.org/data/2/28/Content/theta.htm>

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\begin{axis}[
[...]
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]

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## Isohumes calculation (Work in Progress)

### First try

Given the following formula<sup>4</sup>:

$$r_s = \varepsilon \cdot \frac{e_s(T)}{p - e_s(T)}$$

And assuming that  $e_s(T)$  is the Tetens equation<sup>5</sup>:

$$e_s(T) = 0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)$$

We know that  $r_s$  refers to the  $x$  axis, and  $p$  to the  $y$  axis.

$$r_s = x; \quad p = y$$

$$x = \varepsilon \cdot \frac{e_s(T)}{y - e_s(T)}$$

Then we can solve for  $y$ :

$$x = \frac{\varepsilon \cdot e_s(T)}{y - e_s(T)}$$

$$y - e_s(T) = \frac{\varepsilon \cdot e_s(T)}{x}$$

$$y = \frac{\varepsilon \cdot e_s(T)}{x} + e_s(T)$$

$$y = \frac{(\varepsilon \cdot e_s(T)) + (x \cdot e_s(T))}{x}$$

$$y = \frac{(\varepsilon \cdot 0.61078 \cdot \exp(\frac{12.27 \cdot T}{237.3 + T})) + (x \cdot 0.61078 \cdot \exp(\frac{12.27 \cdot T}{237.3 + T}))}{x}$$

We also know that  $\varepsilon$  is a constant: the ratio between the gas constant for dry air and the gas constant for water vapor<sup>6</sup>:

$$\varepsilon = 0.622$$

We replace  $\varepsilon$ :

$$y = \frac{(0.622 \cdot 0.61078 \cdot \exp(\frac{12.27 \cdot T}{237.3 + T})) + (x \cdot 0.61078 \cdot \exp(\frac{12.27 \cdot T}{237.3 + T}))}{x}$$

$$y = \frac{(0.23620100952 \cdot \exp(\frac{12.27 \cdot T}{237.3 + T})) + (x \cdot 0.61078 \cdot \exp(\frac{12.27 \cdot T}{237.3 + T}))}{x}$$

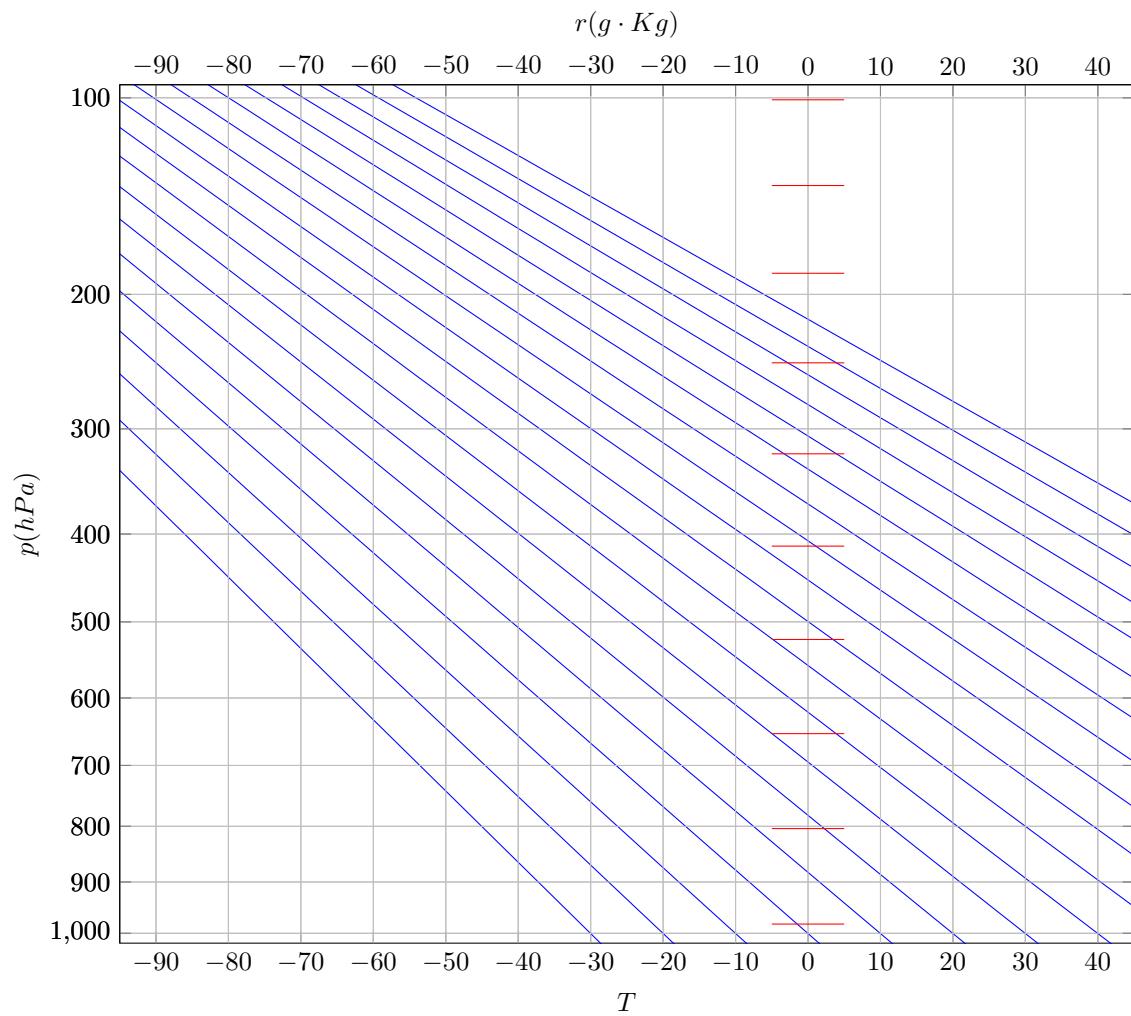
Which we should be able to use in LaTeX:

<sup>4</sup>See [www.igf.fuw.edu.pl/m/courses\\_materials.../thermodynamic\\_diagrams.pdf](http://www.igf.fuw.edu.pl/m/courses_materials.../thermodynamic_diagrams.pdf)

<sup>5</sup>See [en.wikipedia.org/wiki/Tetens\\_equation](https://en.wikipedia.org/wiki/Tetens_equation)

<sup>6</sup>See [pressbooks-dev.oer.hawaii.edu/atmo/chapter/chapter-4-water-vapor/](https://pressbooks-dev.oer.hawaii.edu/atmo/chapter/chapter-4-water-vapor/)

$$((0.23620100952 * \exp((12.27 * \text{\textbackslash}T)/(237.3 + \text{\textbackslash}T))) + \\ (x * 0.61078 * \exp((12.27 * \text{\textbackslash}T)/(237.3 + \text{\textbackslash}T))) / x) * 10$$



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## Second try

Given the following formula<sup>7</sup>:

$$r_s = \frac{\varepsilon \cdot e_s(T)}{p - e_s(T)}$$

And assuming that  $e_s(T)$  is the Tetens equation<sup>8</sup>:

$$e_s(T) = 0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)$$

We know that  $T$  refers to the  $x$  axis, and  $p$  to the  $y$  axis, so the strategy would be to solve for  $p$ .

$$r_s = \frac{\varepsilon \cdot 0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)}{p - \left(0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)\right)}$$

We can solve for  $p$ :

$$r_s \cdot \left(p - \left(0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)\right)\right) = \varepsilon \cdot 0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)$$

$$p - \left(0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)\right) = \frac{\varepsilon \cdot 0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)}{r_s}$$

$$p = \frac{\varepsilon \cdot 0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)}{r_s} + \left(0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)\right)$$

$$p = \frac{\left(\varepsilon \cdot 0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)\right) + \left(r_s \cdot 0.61078 \cdot \exp\left(\frac{12.27 \cdot T}{237.3 + T}\right)\right)}{r_s}$$

$$T = x \quad p = y \quad \varepsilon = 0.622$$

$$y = \frac{\left(0.622 \cdot 0.61078 \cdot \exp\left(\frac{12.27 \cdot x}{237.3 + x}\right)\right) + \left(r_s \cdot 0.61078 \cdot \exp\left(\frac{12.27 \cdot x}{237.3 + x}\right)\right)}{r_s}$$

Which, we should be able to use in LaTeX when passing fixed values of  $r_s$ :

$$r_s = 2$$

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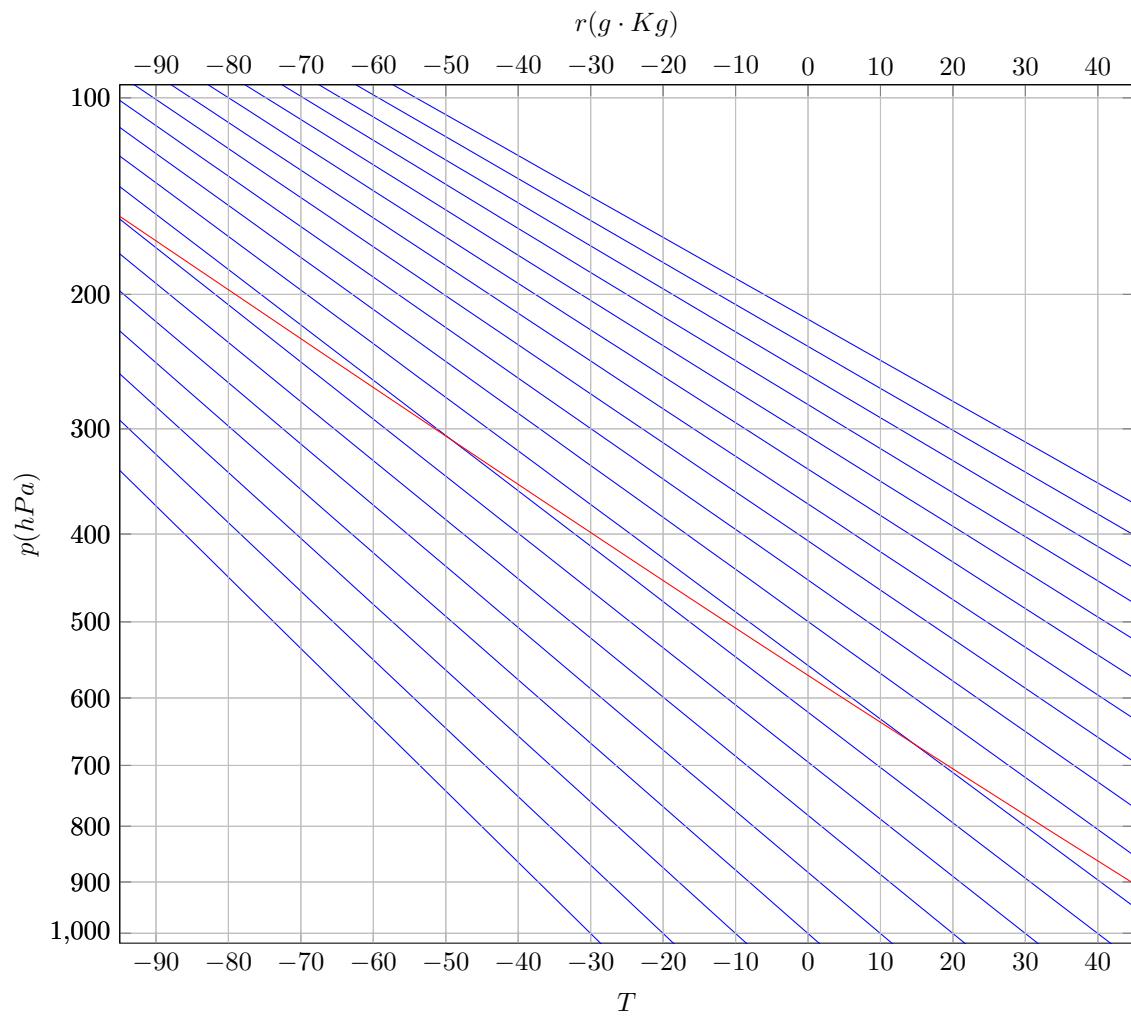
<sup>7</sup>See [www.igf.fuw.edu.pl/m/courses\\_materials.../thermodynamic\\_diagrams.pdf](http://www.igf.fuw.edu.pl/m/courses_materials.../thermodynamic_diagrams.pdf)

<sup>8</sup>See [en.wikipedia.org/wiki/Tetens\\_equation](https://en.wikipedia.org/wiki/Tetens_equation)

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y = ( 0.622 * 0.61078 * exp( ( 12.27 * x ) / ( 237.3 + x )) + (2 * 0.61078 *
exp( ( 12.27 * x ) / ( 237.3 + x )) ) ) / ( 2 )

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### Third try

Following the equation found on *Practical Meteorology: An Algebra-based Survey of Atmospheric Science*:<sup>9</sup>

$$T = \left[ \frac{1}{T_0} - \frac{\mathfrak{R}_v}{L_v} \cdot \ln\left(\frac{r_s \cdot p}{e_0 \cdot (r_s + \varepsilon)}\right) \right]^{-1}$$

We should be able to solve for  $p$ :

$$\frac{1}{T} = \frac{1}{T_0} - \frac{\mathfrak{R}_v}{L_v} \cdot \ln\left(\frac{r_s \cdot p}{e_0 \cdot (r_s + \varepsilon)}\right)$$

$$\frac{1}{T} + \frac{\mathfrak{R}_v}{L_v} \cdot \ln\left(\frac{r_s \cdot p}{e_0 \cdot (r_s + \varepsilon)}\right) = \frac{1}{T_0}$$

$$\frac{\mathfrak{R}_v}{L_v} \cdot \ln\left(\frac{r_s \cdot p}{e_0 \cdot (r_s + \varepsilon)}\right) = \frac{1}{T_0} - \frac{1}{T}$$

$$\frac{\mathfrak{R}_v}{L_v} \cdot \ln\left(\frac{r_s \cdot p}{e_0 \cdot (r_s + \varepsilon)}\right) = \frac{T - T_0}{T \cdot T_0}$$

$$\ln\left(\frac{r_s \cdot p}{e_0 \cdot (r_s + \varepsilon)}\right) = \frac{T - T_0}{T \cdot T_0 \cdot \frac{\mathfrak{R}_v}{L_v}}$$

$$\frac{r_s \cdot p}{e_0 \cdot (r_s + \varepsilon)} = \exp\left(\frac{T - T_0}{T \cdot T_0 \cdot \frac{\mathfrak{R}_v}{L_v}}\right)$$

$$r_s \cdot p = \exp\left(\frac{T - T_0}{T \cdot T_0 \cdot \frac{\mathfrak{R}_v}{L_v}}\right) \cdot e_0 \cdot (r_s + \varepsilon)$$

$$p = \frac{\exp\left(\frac{T - T_0}{T \cdot T_0 \cdot \frac{\mathfrak{R}_v}{L_v}}\right) \cdot e_0 \cdot (r_s + \varepsilon)}{r_s}$$

$$T_0 = 273.15 \quad e_0 = 0.6113 \quad \frac{\mathfrak{R}_v}{L_v} = 0.0001844 \quad \varepsilon = 0.622$$

$$p = \frac{\exp\left(\frac{T - 273.15}{T \cdot 273.15 \cdot 0.0001844}\right) \cdot 0.6113 \cdot (r_s + 0.622)}{r_s}$$

$$T = x \quad p = y$$

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<sup>9</sup>Stull, Roland: *Practical Meteorology: An Algebra-based Survey of Atmospheric Science*, chapter 4, p. 101, equation 4.36, 2018

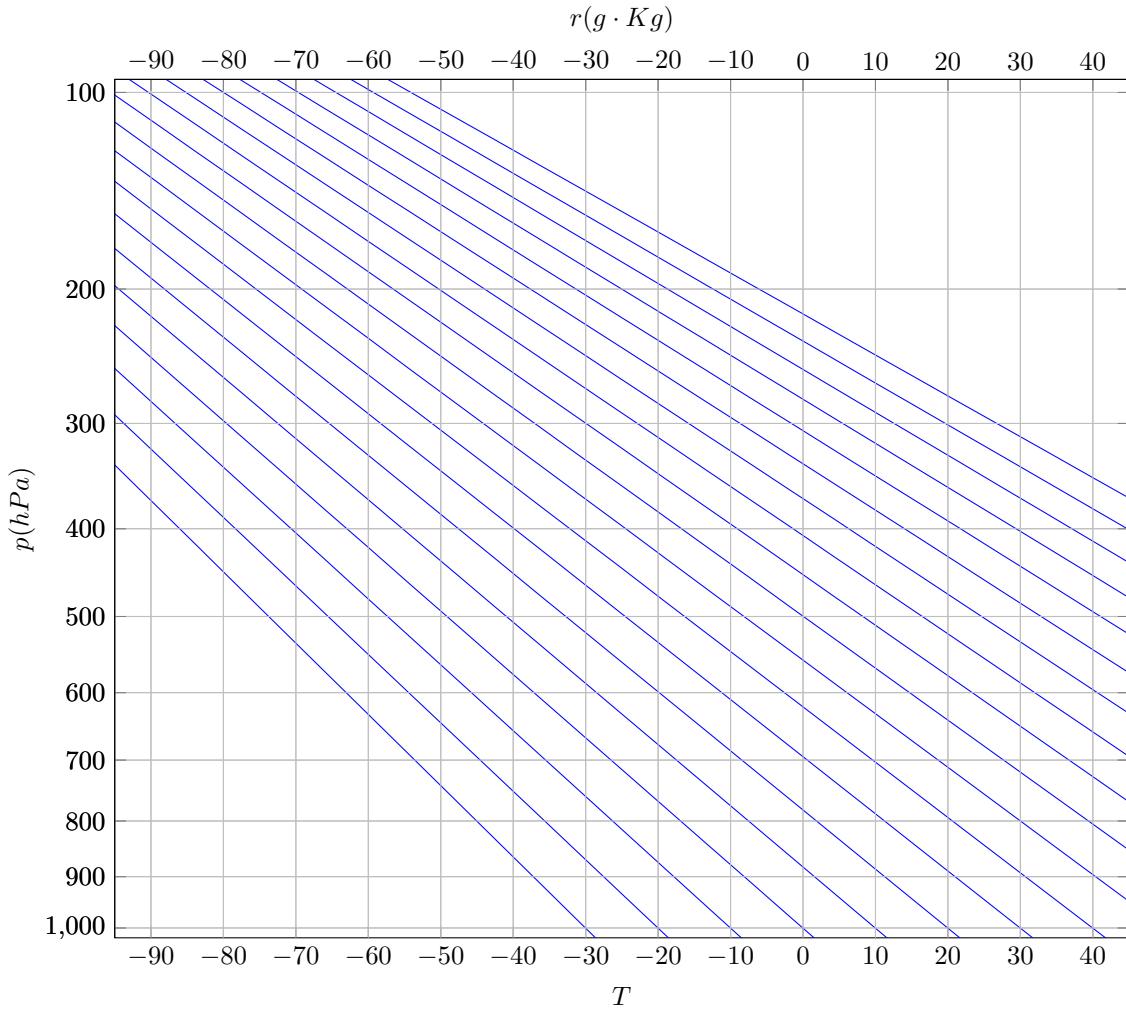
$$y = \frac{\exp\left(\frac{x-273.15}{x \cdot 273.15 - 0.0001844}\right) \cdot 0.6113 \cdot (r_s + 0.622)}{r_s}$$

Which, we should be able to use in LaTeX when passing fixed values of  $r_s$  and converting T to Kelvin:

$$r_s = 2$$

$$y = \frac{\exp\left(\frac{(x+273.15)-273.15}{(x+273.15) \cdot 273.15 - 0.0001844}\right) \cdot 0.6113 \cdot (2 + 0.622)}{2}$$

$$(\exp((x + 273.15) - 273.15) / ((x + 273.15) * 273.15 * 0.0001844)) \\ * 0.6113 * (2 + 0.622) ) / (2)$$



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