Exam 2020

- Optimazation And Data Analytics
- Student name: Anton Sakarias Rørbæk Sihm
- Student number: 201504954
- AU id : au535993

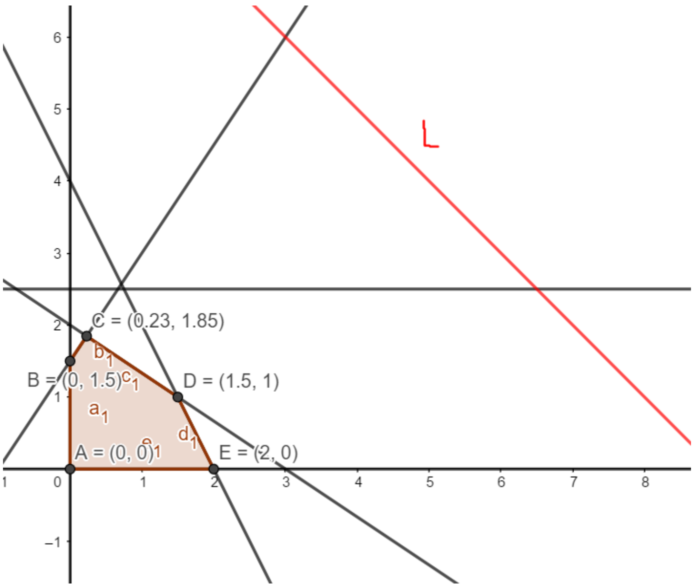
Exercise 1

Exercise 1

Consider the following optimization problem:

Maximize:	$f(x_1, x_2) = 4x_1 + 3x_2$	
Subject to:	$2x_1 + 3x_2 \leq 6;$	(A)
	$-3x_1 + 2x_2 \leq 3;$	(B)
	$2x_2 \leq 5;$	(C)
	$2x_1 + x_2 \leq 4;$	(D)
and	$x_1 \geq 0; x_2 \geq 0;$	

a) Sketch the feasible set in 2 dimensions, including the level set: $L_9 = \{(x_1, x_2) | f(x_1, x_2) = 9\}$.



From this the get the following cadinate solutions:

```
pointA = [0,0];
pointB = [0,1.5];
pointC = [0.23,1.85];
pointD = [1.5,1];
pointE = [2,0];

syms x1 x2
f = 4*x1 + 3*x2;
%max pointA
x1 = pointA(1);
x2 = pointA(2);
max_A = subs(f)
```

max_A = 0

```
%max pointB
x1 = pointB(1);
x2 = pointB(2);
max_B= double(subs(f))
```

```
max_B =
       9/2
```

```
%max pointC
x1 = pointC(1);
x2 = pointC(2);
max_C= double(subs(f))
```

```
max_C =
     647/100
```

```
%max pointD
x1 = pointD(1);
x2 = pointD(2);
max_D = double(subs(f))
```

```
max_D =
       9
```

```
%max pointE
x1 = pointE(1);
x2 = pointE(2);
max_E= double(subs(f))
```

```
max_E =
       8
```

```
fprintf("Our maxpoint is [x1, x2]  = [ %.2f, %.2f ] with max value : %.2f ", pointD, max_D );
```

```
Our maxpoint is [x1, x2]  = [ 1.50, 1.00 ] with max value : 9.00
```

b) Write out the simplex tableaux for the problem and show the first step needed to bring in a new variable into the solution (e.g. argue what column and row to choose, and what elementary operations are needed for the first reductions). Use the simplex algorithm to find the maximum for f on the feasible set (you may use MatLab).

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Consider the following optimization problem:

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Subject to: $2x_1 + 3x_2 \le 6;$ (A)

$-3x_1 + 2x_2 \le 3;$ (B)

$2x_2 \le 5;$ (C)

$2x_1 + x_2 \le 4;$ (D)

and $x_1 \ge 0; x_2 \ge 0;$

```
% Simplex tuable for the problem
% Step 1 : change inquality to equality by adding slacks var and setting c to
% minus with slack var M.
%     x1   x2,    s1   s2    s3    s4    M     B
A = [ 2,    3,    1,   0,    0,    0,    0,    6;
     -3,    2,    0,   1,    0,    0,    0,    3;
      0,    2,    0,   0,    1,    0,    0,    5;
      2,    1,    0,   0,    0,    1,    0,    4;
      -4,   -3,   0,   0,    0,    0,    1     0]
```

```
A =
      2       3       1       0       0       0       0       6
     -3       2       0       1       0       0       0       3
      0       2       0       0       1       0       0       5
      2       1       0       0       0       1       0       4
     -4      -3       0       0       0       0       1       0
```

```
% Step two: find most negative at bottom row-->  we take x1 = -4
% Step three: we will find the most positive ratio than is smallets
% B/entry  for all entries
% 6/2 = 3  |   3/-3 = -1   | 4/2 = 2  -->  we take R4
% we will use r4 to create the pivot colum, since it makes b none negative.

% R4 *(1/2)
A(4,:) = A(4,:)*(1/2);

% R1 - 2R4
A(1,:) = A(1,:) - 2*A(4,:);

% R2 + 3R4
A(2,:) = A(2,:) + 3*A(4,:);

% R5 + 4R4
A(5,:) = A(5,:) + 4*A(4,:);

% after 1 iteration of the simplex algortihm:
A
```

```
A =
       0          2          1          0          0         -1          0          2
       0        7/2          0          1          0        3/2          0          9
       0          2          0          0          1          0          0          5
       1        1/2          0          0          0        1/2          0          2
       0         -1          0          0          0          2          1          8
```

```
clear x1 x2
% This is the first iteration
% lets finish the rest computationally
x1 = optimvar('x1');
x2 = optimvar('x2');

prob = optimproblem('Objective', 4*x1 +3*x2,'ObjectiveSense','max');
prob.Constraints.c1 = 2*x1 + 3*x2  <= 6;
prob.Constraints.c2 = -3*x1 + 2*x2  <= 3;
prob.Constraints.c3 = 2*x2 <= 5;
prob.Constraints.c4 = 2*x1 + x2 <= 4;
prob.Constraints.c5 = x2 >= 0;
prob.Constraints.c5 = x1 >= 0;

problem = prob2struct(prob);
[sol,fval,exitflag,output] = linprog(problem);
```

```
Optimal solution found.
```

```
max_value = 4*sol(1) + 3*sol(2);
fprintf("Our maxpoint is [x1, x2]  = [ %.2f,  %.2f] with max value : %.2f ", sol, max_value );
```

```
Our maxpoint is [x1, x2]  = [ 1.50,  1.00] with max value : 9.00
```

```
% we can indeed conclude that this is the same as found via the geografic
% method in 2d.
```

c) Find the marginal values corresponding to the inequality (A), (B), (C) and (D).

```
% lets compute then final simplex tablue:
matrix=[1 0 0 1 0 0;
        4 1 0 0 1 0;
        8 4 1 0 0 1];

b=[1;4;16];
c=-[4;2;1;0;0;0];
v=[4;5;6];

matrix=[ 2  3  1  0  0  0;
        -3  2  0  1  0  0;
         0  2  0  0  1  0;
         2  1  0  0  0  1];

b=[6;3;5;4];
c=-[4;3;0;0;0;0];
v=[3;4;5;6];

simplex(c,matrix,b,v,1)
```

```
      -3          2          0          1          0          0          3
       0          2          0          0          1          0          5
       2          1          0          0          0          1          4
      -4         -3          0          0          0          0          0

Pivot point:
       4          1

New tableau:
       0          2          1          0          0         -1          2
       0        7/2          0          1          0        3/2          9
       0          2          0          0          1          0          5
       1        1/2          0          0          0        1/2          2
       0         -1          0          0          0          2          8

Pivot point:
       1          2

New tableau:
       0          1        1/2          0          0       -1/2          1
       0          0       -7/4          1          0       13/4       11/2
       0          0         -1          0          1          1          3
       1          0       -1/4          0          0        3/4        3/2
       0          0        1/2          0          0        3/2          9
ans =
     3/2
       1
       0
```

```
    11/2
     3
     0
```

```
% At the final tablue we can see the solution the the dual problem as well. Which
% is the marginal values : A = 1/2    B = 0    C = 0    D = 3/2
% This is the input values for the dual problem, that gives the minimize
% solution.
```

d) How much can you decrease the right hand side of inequality (C), without changing the maximum for f (obtained in b), argue for answer.

```
% Since the inequality C has no contribution to the maximum point we can
% acutally set it to infinite with changing the maximum point.
```

Exercise 2

Exercise 2

Consider the function defined $g(x_1, x_2) = e^{-2x_1-3x_2}$

a) Find the gradient for the function g.

```
clearvars;
warning('off');
syms x1 x2
g = exp(1)^(-2*x1-3*x2);

% parciel diff for x1 for first index of the gradient.
% parciel diff for x2 for second index of the gradient.

gradiant_g = gradient(g, [x1, x2])
```

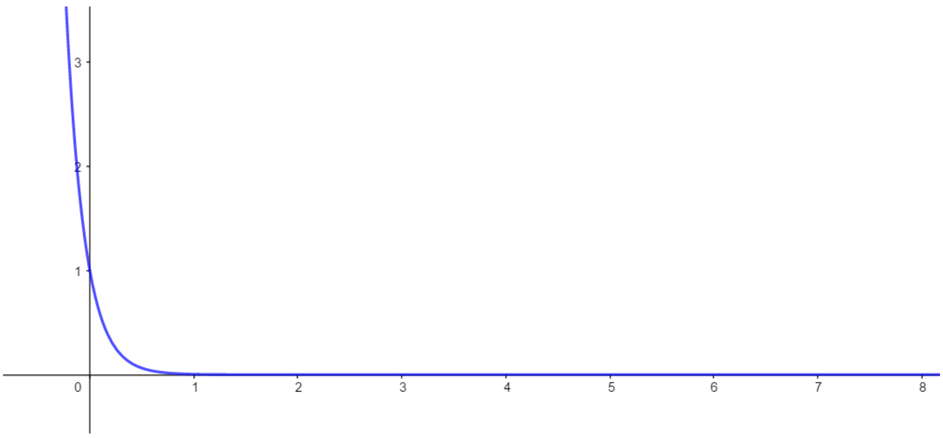
gradiant_g =

$$\begin{pmatrix} -2\,\dfrac{1}{\left(\dfrac{3060513257434037}{1125899906842624}\right)^{2x_1+3x_2}}\log\left(\dfrac{3060513257434037}{1125899906842624}\right) \\ -3\,\dfrac{1}{\left(\dfrac{3060513257434037}{1125899906842624}\right)^{2x_1+3x_2}}\log\left(\dfrac{3060513257434037}{1125899906842624}\right) \end{pmatrix}$$

Let $c > 0$.

b) Argue that all level sets: $L_c = \{(x_1, x_2) | g(x_1, x_2) = c\}$ are parallel straight lines, and find the slope of straight lines.

```
% function g plotted.
```




```
% When c > 0 we will have straight line in g, and the lvl set will be
% paralel.

% We can differentiate the function to get a function for the slope.
% for c> 0 this will be 0 for the lvl set and the function g.
```

Now let $h(x_1, x_2) = {x_1}^2 + {x_2}^2 - 13.$

c) Find the minimum and maximum of f subject to the constraint $h(x_1, x_2) = 0$ (argue for your calculations).

We will use the Lagrange multiplier for the problem

$\nabla f(x) + \lambda * \nabla h(x) = 0$

We can solve the follwing problem by solving 3 linequeations with 3 uknowns

```
% I Assume that the f  function is meant to be g?????? I will ref to
% function g as f
clearvars
syms x1 x2
f = exp(1)^(-2*x1-3*x2);
gradiant_f = gradient(f, [x1,x2])
```

gradiant_f =

$$\begin{pmatrix} -2 \dfrac{1}{\left(\frac{3060513257434037}{1125899906842624}\right)^{2x_1+3x_2}} \log\left(\frac{3060513257434037}{1125899906842624}\right) \\ -3 \dfrac{1}{\left(\frac{3060513257434037}{1125899906842624}\right)^{2x_1+3x_2}} \log\left(\frac{3060513257434037}{1125899906842624}\right) \end{pmatrix}$$

```
clear x1 x2
syms x1 x2 lambda

h = x1^2 + x2^2 -13;
gradiant_h = gradient(h, [x1,x2])
```

gradiant_h =

$$\begin{pmatrix} 2x_1 \\ 2x_2 \end{pmatrix}$$

```
% Lagrange multiplier
Lagrange_multiplier = gradiant_f + lambda*gradiant_h == 0
```

Lagrange_multiplier =

$$\begin{pmatrix} 2\lambda x_1 - 2 \dfrac{1}{\left(\frac{3060513257434037}{1125899906842624}\right)^{2x_1+3x_2}} \log\left(\frac{3060513257434037}{1125899906842624}\right) = 0 \\ 2\lambda x_2 - 3 \dfrac{1}{\left(\frac{3060513257434037}{1125899906842624}\right)^{2x_1+3x_2}} \log\left(\frac{3060513257434037}{1125899906842624}\right) = 0 \end{pmatrix}$$

```
% we now have 2 equation and 3 unknowns
% we can use the fuction h = 0 as the last one:
equation1 = Lagrange_multiplier(1)
```

equation1 =

$$2\lambda x_1 - 2 \frac{1}{\left(\frac{3060513257434037}{1125899906842624}\right)^{2x_1+3x_2}} \log\left(\frac{3060513257434037}{1125899906842624}\right) = 0$$

```
equation2 = Lagrange_multiplier(2)
```

equation2 =

$$2\lambda x_2 - 3 \frac{1}{\left(\frac{3060513257434037}{1125899906842624}\right)^{2x_1+3x_2}} \log\left(\frac{3060513257434037}{1125899906842624}\right) = 0$$

```
equation3 =  x1^2 + x2^2 -13 == 0
```

equation3 = ${x_1}^2 + {x_2}^2 - 13 = 0$

```
[x1,x2,lambda] = solve([equation1, equation2, equation3], [x1,x2, lambda]);
x1 = double(x1)
```

```
x1 =
       2
```

```
x2 = double(x2)
```

```
x2 =
       3
```

```
lambda = double(lambda)
```

```
lambda =
       1/884827
```

```
% candidate coordinates
s1 = [x1(1), x2(1)];
% Solutions
%(x1-2)^2 + (x2-1)^2;
f1 = exp(1)^(-2*s1(1)-3*s1(2));
fprintf("maximum: %.2f at point : (%.2f, %.2f)", f1, s1);
```

```
maximum: 0.00 at point : (2.00, 3.00)
```

Exercise 3

Exercise 3

Answer the following with **ONE sentence** per question.

1) Are Genetic Algorithm methods guaranteed to find the global optimum if the search is run for enough iterations (Yes/No)?

```
% Even tho we run for long enough, we will have higher probalitity. But we
% cannot corfirm that we will find the global optimum.
% NO.
```

2) Name one example of a stochastic optimization method, what does it mean is it *stochastic*?

```
% Stocastic means that the algortihm have some randomness included.
% The generic method is stochastic. When candidate solutions have at high
% fitness, the highere probalilyty there are for some to spawn childrin, by
% doing crossovers.
```

3) What are the main operators for Genetic Algorithms?

```
% 1. Candidate solutions are represented by chromosomes
% 2. uses Exploration or explortation to explore new solutions
% 3. Fitness funtion decides how good a  chromosome is
% 4. 2 Chrosomes have to possibility to spawn childrin if thier fitness is
% high.
```

4) List two terminating conditions for iterative optimization algorithms.

```
% 1. When the error is acceptable. Neuton method: one could be that with
% the results does not change decimals after 5 decimals we converge.
% 2. Number of interations. When we have limited computational power, we
% might be satisfied at some interation.
```

5) Name at least five considerations for effective utilization/production of scientific and technical figures.

```
% 1. Golden seach method, for find local minimum between two points (No differential needed)
% 2. Steepest decent method for finden local minimum differential needed
% 3. Swarm optimazation for finding local/global optimums
% 4. Simulated anealing for finding local/global optimums
% 5. Neuton method for finding local/global optimums
```

6) The particle swarm update formula is given by the equation below, name the parts of the equation.

$$v_i(t+1) = \overbrace{wv_i(t)}^{a} + \overbrace{c_1r_1[g(t) - x_i(t)]}^{b} + \overbrace{c_2r_2[\hat{x}_i(t) - x_i(t)]}^{c}$$

- What does $\hat{x}_i(t)$ refer to in this formula?
- What does $g(t)$ refer to in this formula?
- What does a refer to in this formula?
- What does b refer to in this formula?
- What does c refer to in this formula?

```
% 1.
% - Best position for current particle v_i up to iteration t

% 2.
% - Best position so far from any particale (GLOBAL BEST)

% 3 a.
% -inertia component: keeps particle moving in similar direction
```

```
% - Velocity of current particale v_i at iteration t is included

% 4 b.
% - social component: encourages particle to move to swarms best found
% position so far: g(t)

% 5 c.
% -cognitive component: current particle memory, encourages particle to go back to best position
```

Exercise 4

Exercise 4

A classification problem is formed by two classes. We are given a set of 2-dimensional data $\mathbf{X} = [x_1, x_2 \dots x_7]$:

$$\mathbf{X} = \begin{bmatrix} 1 & 0 & 1 & 3 & 3 & 4 & 4 \\ 1 & 1 & 0 & 1 & 2 & 1 & 2 \end{bmatrix}$$

each belonging to one of the two classes, as indicated in the class label vector $\mathbf{l} = [l_1, \dots l_7]$:

$$\mathbf{l} = \begin{bmatrix} 1 & 1 & 1 & 2 & 2 & 2 & 2 \end{bmatrix}$$

Using the above (training) vectors and the corresponding class labels, classify the following vectors:

$$x_8 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T \qquad x_9 = \begin{bmatrix} 3 & 1 \end{bmatrix}^T \qquad x_{10} = \begin{bmatrix} 2 & 2 \end{bmatrix}^T$$

a) The Nearest Class Centroid (NCC) classifier

```
%For this I will use python.
```

I did the following in python. Th steps are as follows:

```
% Calculate the centroid for each class --> mean each x_i of each class
% Calculate the dist from test sample to each centroid
% Classify by taking the smallest dist to centroid.
% The python code can be seen as follows:
```

Nearest_Centroid.py

```python
from sklearn.neighbors import NearestCentroid

def train_NCC_model(training_set, training_labels):
    ncc_model = NearestCentroid()
    ncc_model.fit(training_set, training_labels)
    return ncc_model

def classify_NCC(testing_set, trained_model):
    return trained_model.predict(testing_set)

if __name__ == '__main__':
    training_set =  [[1,1], [0,1], [1,0] ,[3,1], [3,2],[4,1], [4,2]]
    labels = [1,1,1,2,2,2,2]
    testing_set =  [[0,0], [3,1], [2,2]]


    trained_model = train_NCC_model(training_set, labels)
    predicted_classes = classify_NCC(testing_set, trained_model)
    print(predicted_classes)
```

```
%  .\Nearest_Centroid.py  yields the prediction for x_i for i = 8, 9, 10.
%  prediction =  [1 2 2]
```

b) The Bayes-based classification scheme, where:

$$p(x_i|c_k) = \frac{\|x_i - m_k\|_2^{-2}}{\sum_{m=1}^{K} \|x_i - m_m\|_2^{-2}}$$

where m_k is the class mean of class c_k, $k = 1, 2$ and $\|v\|_2^{-2} = 1/(v^T v)$

```
% For this i will use python, with the following code:
```

```
from sklearn.naive_bayes import GaussianNB

if __name__ == '__main__':
    training_set =  [[1,1], [0,1], [1,0] ,[3,1], [3,2],[4,1], [4,2]]
    labels = [1,1,1,2,2,2,2]
    testing_set =  [[0,0], [3,1], [2,2]]

    gnb = GaussianNB()
    y_pred = gnb.fit(training_set, labels).predict(testing_set)

    print(y_pred)
```

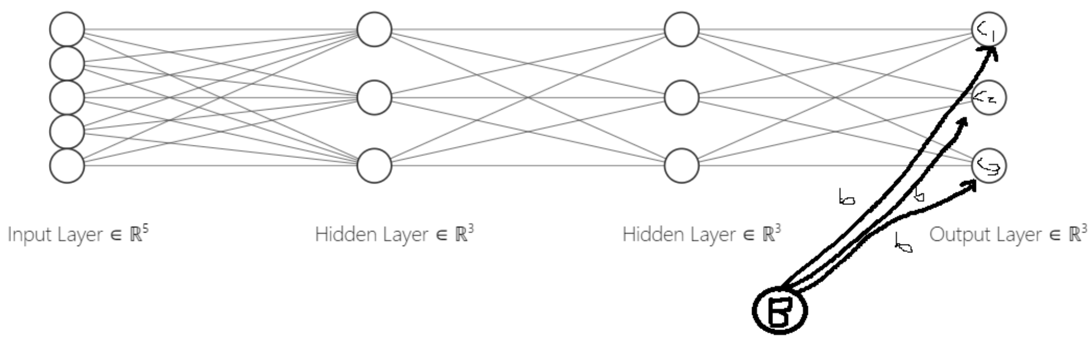
```
%.\Bayes_classification_scheme.py yields the following prediction : [1 2 2]
```

Exercise 5

Exercise 5

- Draw an artificial neural network solving a 3-class classification problem using training data $x_i \in \Re^5, i = 1, \ldots, N$. The neural network is formed by 2 hidden layers. Use b to represent the bias input.

```
% - Since the inputs are in R5 we will input to have an input of 5 neurons
% - I have chosen the 2 hidden layer size 3 randomly. But normaly this
% heavyly depends on the input problem.
% - Then we have the output layer, which consist of 3 different out clases
% - We have the bias, the shift the activation funtion from left to right.
```



- Describe the operations (equation) involved in the feedforward phase for output unit o_m and describe the variables you use.

```
% BLANK
```

- Assuming the training set has been normalized, and the activation functions are sigmoids, how should the weights for each layer be initialized?

```
% We should initilize random in the normilized space
```

Exercise 6

Exercise 6

For all questions in this exercise, we refer to applications and considerations for multilayer perceptron networks:

1) Name some advantages of Stochastic learning

```
% Since we are using the stochastic approch, the network will learn much
% faster.
```

2) Networks can learn faster from the most unexpected samples. What can you do to exploit such behaviour?

```
% Input generated input samples can exploit this, giving feeding the
% network multiple random samples.
```

3) Why it is a good practice to normalize the inputs to a neural network?

```
% The actvation functions typically works with values around -1 to 1. This
% also makes the neural network much fast.
```

4) Why and when would you consider to use Radial Basis Functions (RBF) instead of perceptron units?

```
% When we have big network, and want to train it fast.
```

5) What is underfitting?

```
% This means that the trained model is not fittet enough to do testing.
% This is caused by that the model cant find any relationship between the input samples.
```

6) Name at least two factors that affect the training process of neural networks?

```
% 1. train with data that the algorithm should be avalble to classify
% 2. train with subsets of classes of data that the algorithm should be availble to classify
% 3. train with data that the algorithm should not be able to classify, so
% that it can learn from mistakes.
% - Supervises learned can affect it, be manuely correct the network when
% doing mistakes or doing the correct.
```

7) Neural networks can be very challenging to train. Imagine you have to spend a significant amount of time optimizing the hyperparameters of your network, but the performance still suboptimal. You also tried optimizing the stopping criteria and tested the performance using regularization terms. What could be another common problem, and how could it be addressed.

```
%Blank
```