Diffeomorphic image registration taking topological differences into account

Metamorphosis on brain MRI containing Glioblastomas

THÈSE DE DOCTORAT MATHÉMATIQUES

Spécialité: Mathématiques Appliquées

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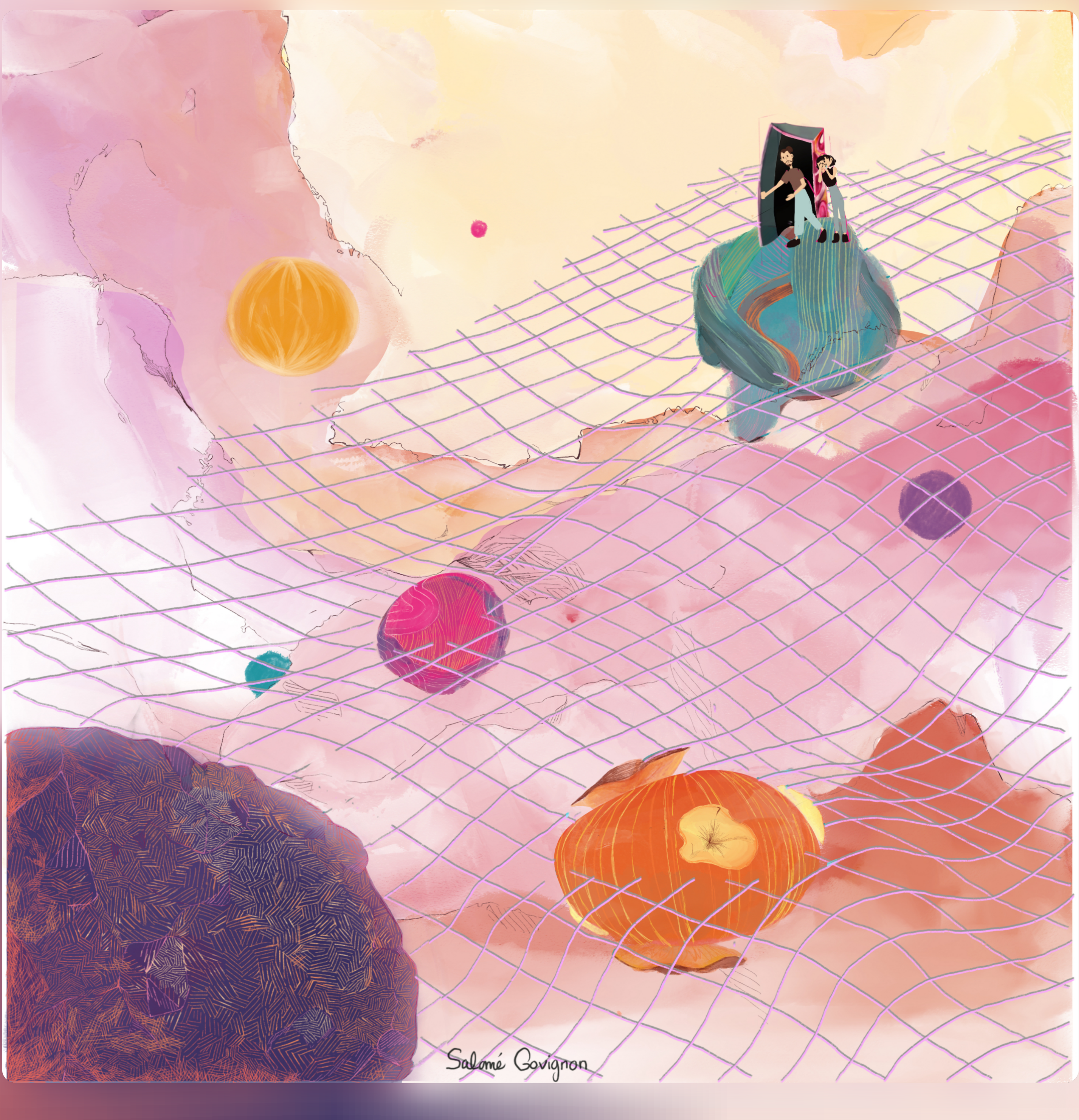
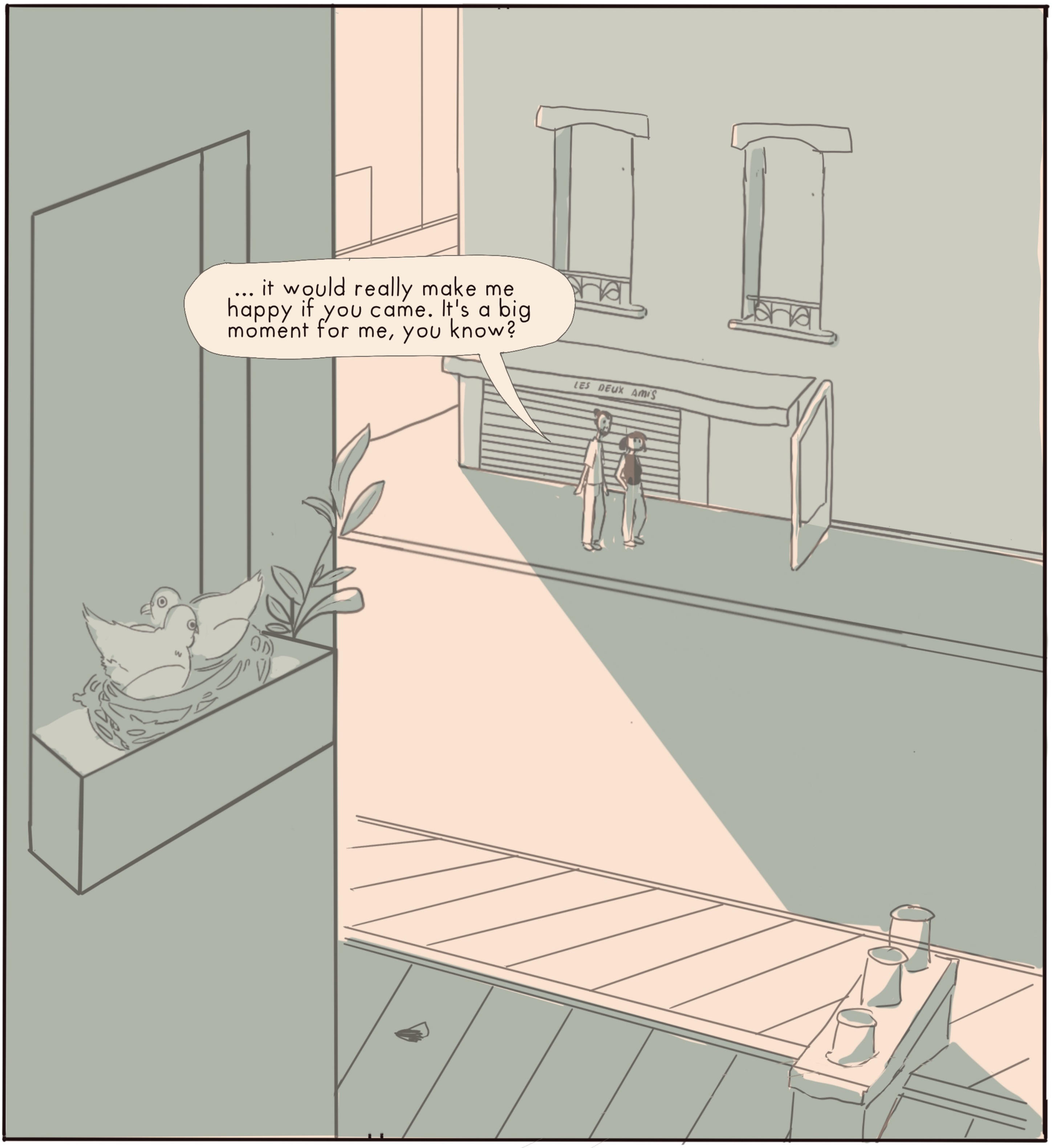
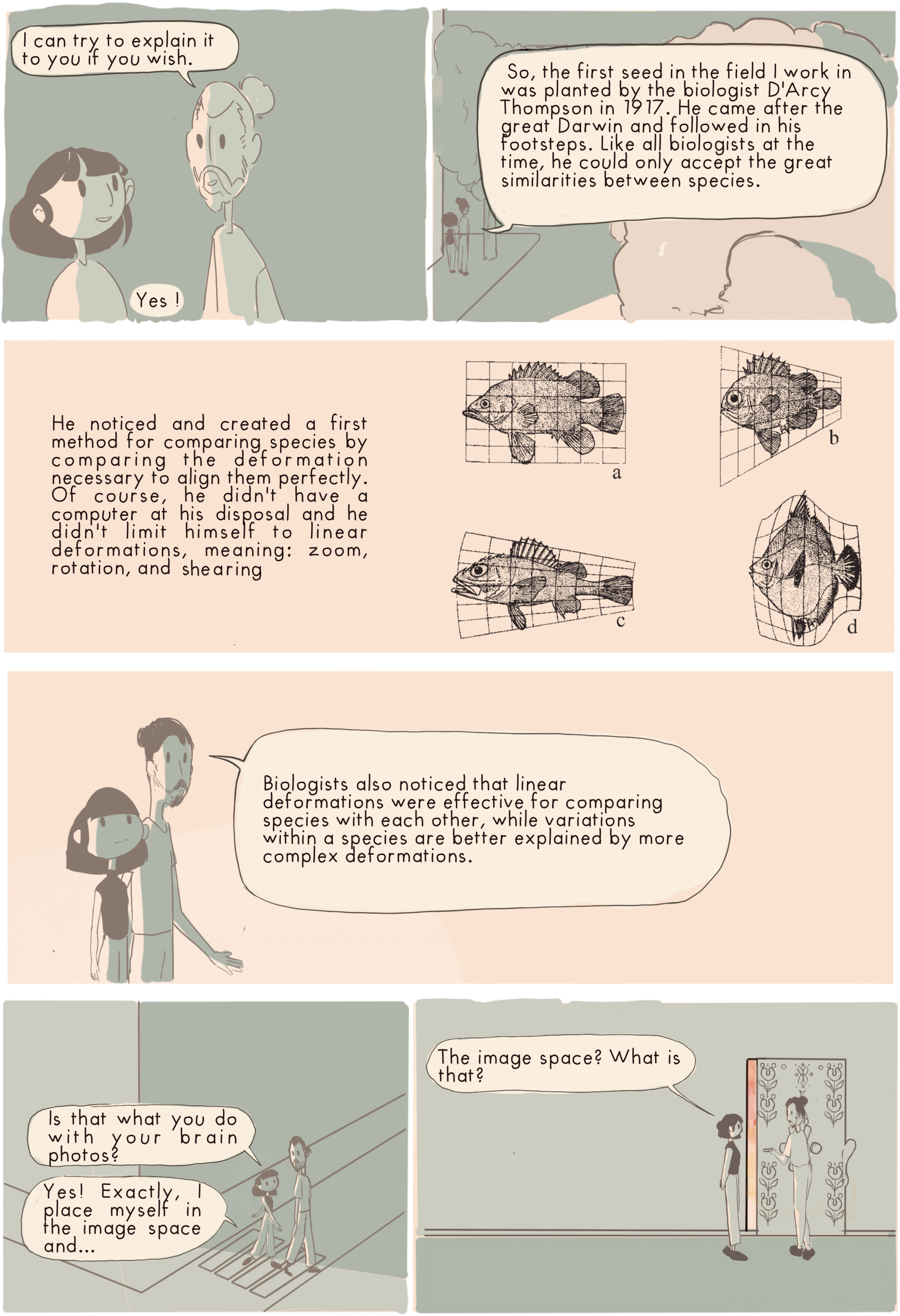



... it would really make me happy if you came. It's a big moment for me, you know?
LES DEUX AMIS


But I won't understand anything! I don't do math, I just know that you look at brains from time to time.

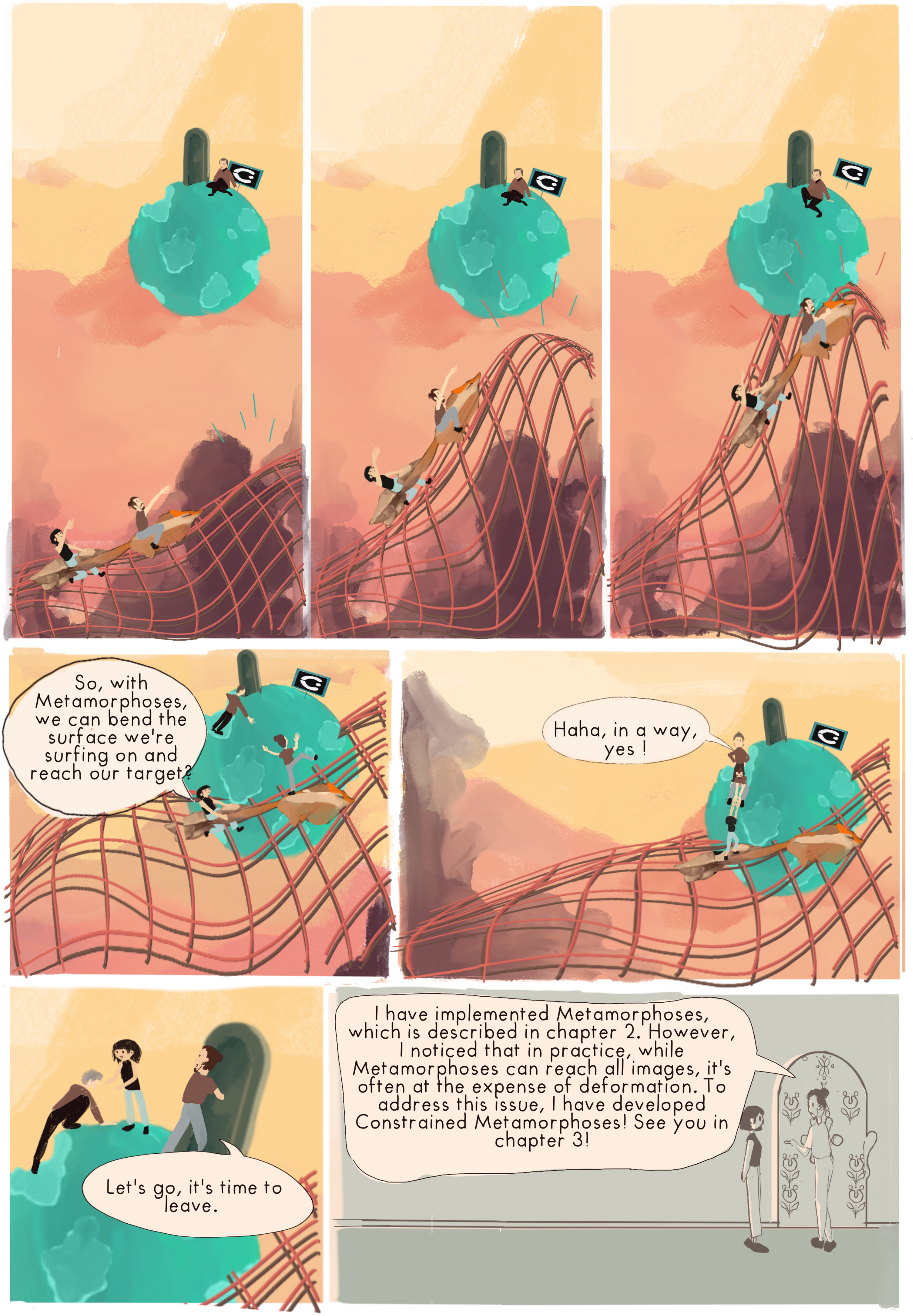

I can try to explain it to you if you wish.
Yes !
So, the first seed in the field I work in was planted by the biologist D'Arcy Thompson in 1917. He came after the great Darwin and followed in his footsteps. Like all biologists at the time, he could only accept the great similarities between species.
He noticed and created a first method for comparing species by comparing the deformation necessary to align them perfectly. Of course, he didn't have a computer at his disposal and he didn't limit himself to linear deformations, meaning: zoom, rotation, and shearing
a
b
c
d
Biologists also noticed that linear deformations were effective for comparing species with each other, while variations within a species are better explained by more complex deformations.
Is that what you do with your brain photos?
Yes! Exactly, I place myself in the image space and...
The image space? What is that?


It's a high-dimensional space where every point is a possible image variation. For example, for a 100x100 pixel image, there are 10,000 dimensions.
Yes, it's mind-boggling. And if you randomly pick an image, it's very likely to look like this:
I come on, let's try to find a brain.


We'll never get there, it's too tiring...
AAAA
HAAAA
HAAAAAA


Outch !
No, my name is Ulf, Grenander
Wow, how do you manage to move around so easily?
Thanks to our geodesic surfboards!
What's that? And what are those lines that appeared?
MICHEL MILLER
A group of diffeomorphic deformations!
As you saw, it's difficult to move around in the space of images, which is of far too high dimension. Since we are interested only in images of organs that have the same overall structure, we know that we can reach one image by deforming another.
Starting from an image, we will only reach a portion of the space of images, but it is precisely those that interest us! Remarkably, this subset of the space is a manifold, that is, a surface on which we know how to move! Following a deformation is like following a path in the space of images!
Our surfboards are precisely for that, they follow a straight line on this space that we call geodesic. And the best part about all of this is that they measure the distance they have traveled based on the "amount" of deformation, which allows us to compare the images with each other.
Great, do you know where we can find these geodesic surfs ?
Haha, we're in the world of math, everything is in your head, just think about it really hard!


That's right! By the way, if you want to get out of here, you'll find an exit in this image.
Thank you for the information!
But won't we get lost?
Good question! But no, don't worry, this method allows us to build the space of shapes, also known as an "Atlas".
From a series of images, we construct an image, the "Template", which is at the center of the generated surface. That is, at the point where the sum of distances to reach each image is the shortest.
An average image is not enough to make an Atlas, is it?
Yes, you are right. We find the Template, but we also store all the deformation paths from this point to all the other images.
Well, it's time to get out of here. The image seems to be in that direction.


So, with Metamorphoses, we can bend the surface we're surfing on and reach our target?
Haha, in a way, yes !
Let's go, it's time to leave.
I have implemented Metamorphoses, which is described in chapter 2. However, I noticed that in practice, while Metamorphoses can reach all images, it's often at the expense of deformation. To address this issue, I have developed Constrained Metamorphoses! See you in chapter 3!

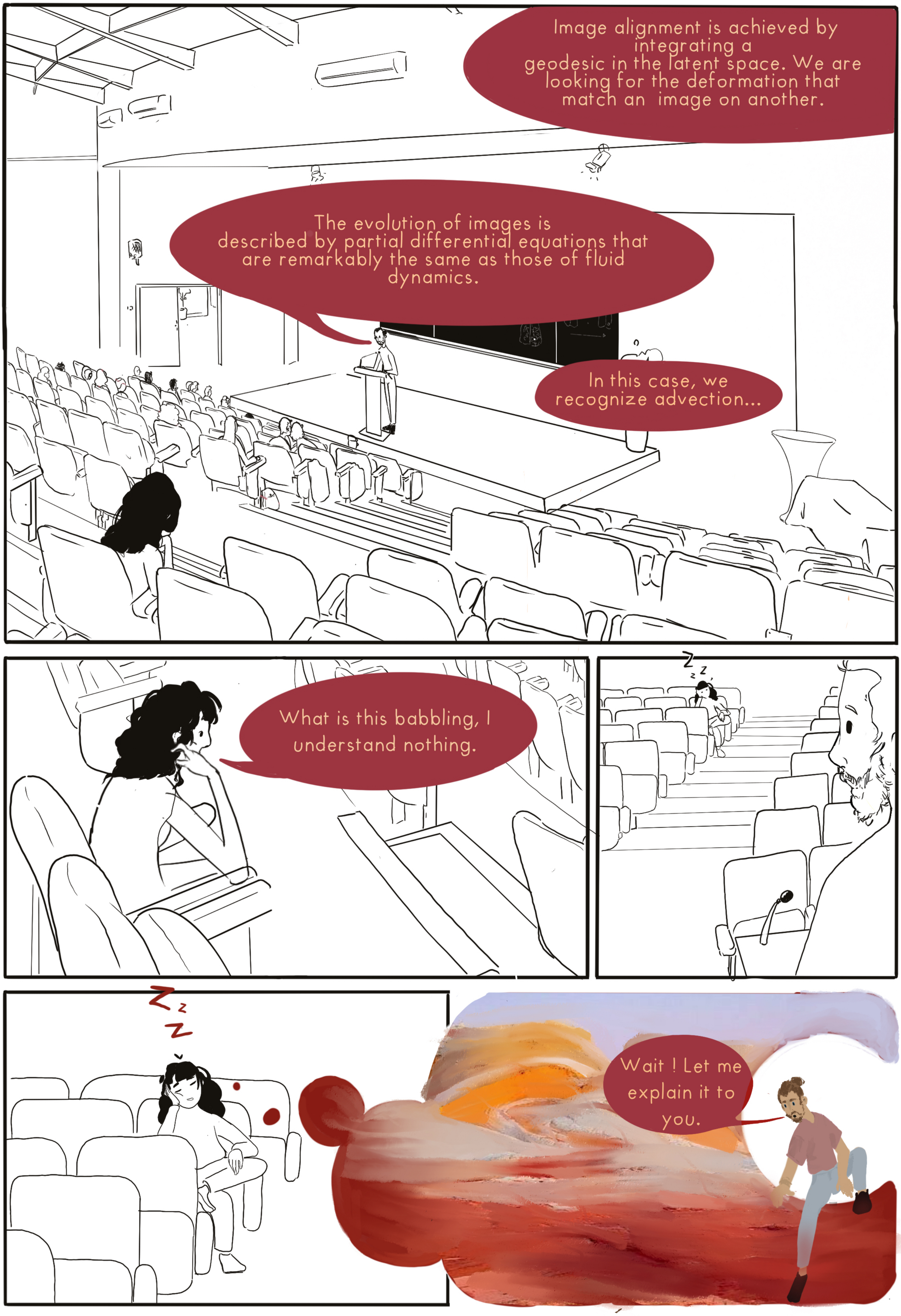
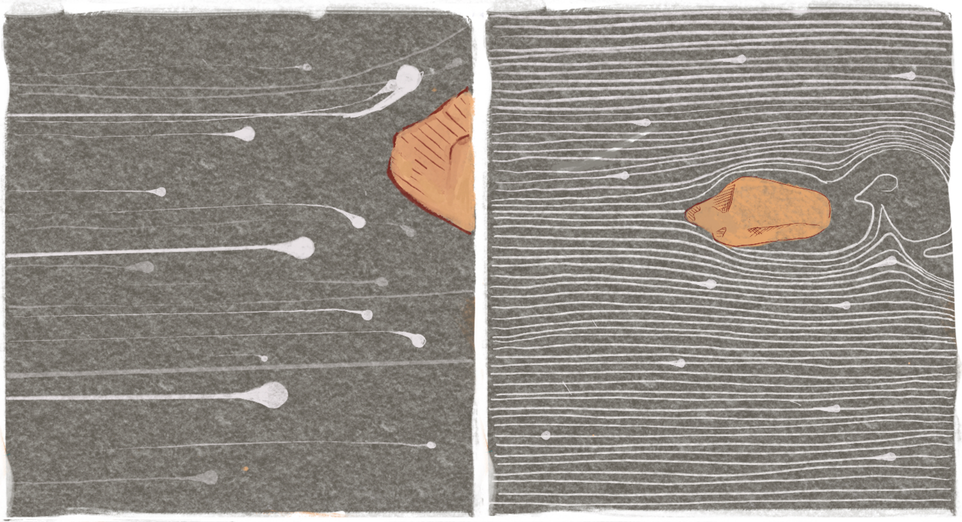


Image alignment is achieved by integrating a geodesic in the latent space. We are looking for the deformation that match an image on another.
The evolution of images is described by partial differential equations that are remarkably the same as those of fluid dynamics.
In this case, we recognize advection...
What is this babbling, I understand nothing.
Wait ! Let me explain it to you.

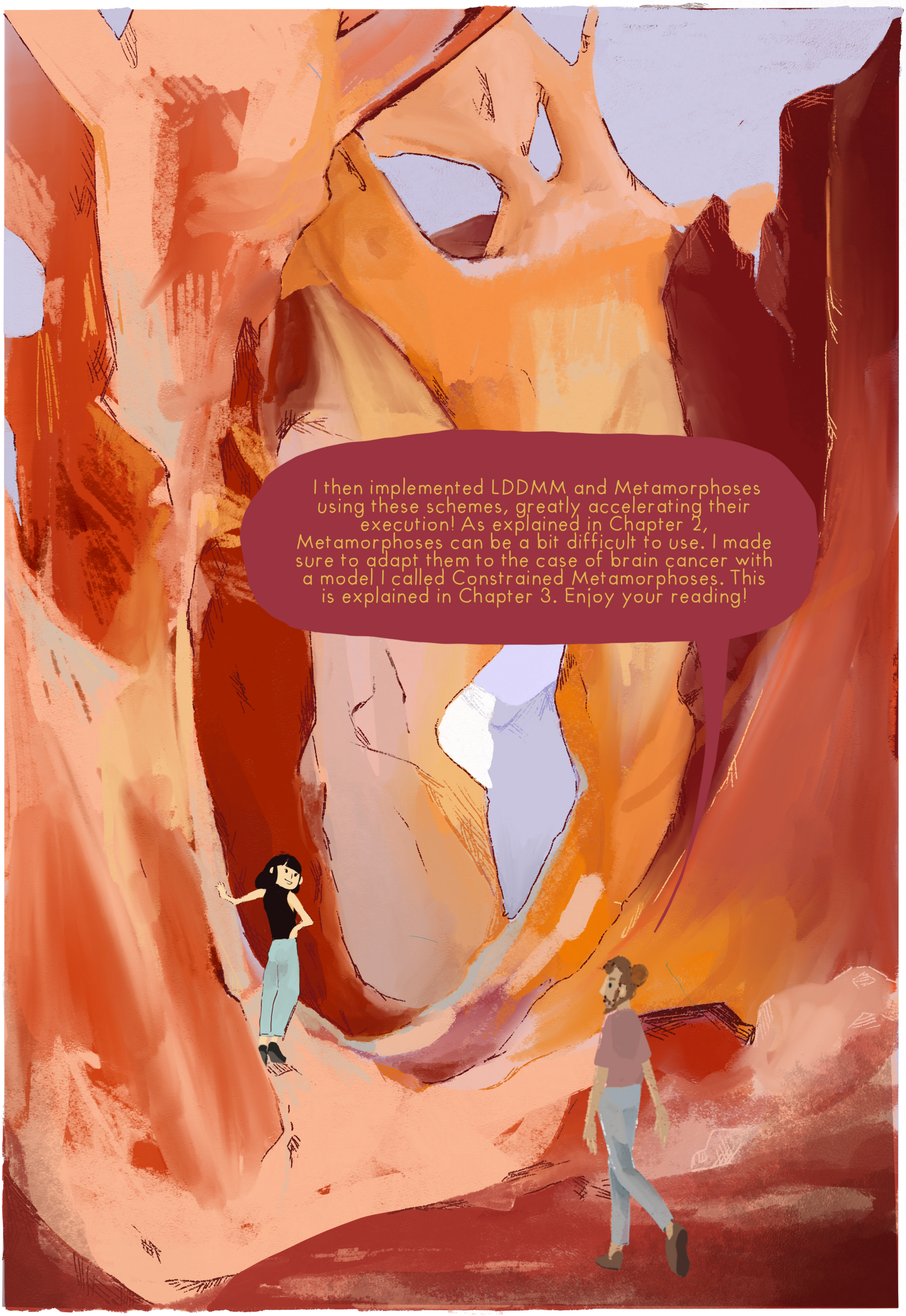

You may try, but I do not make any promises to understand.
You will see, It is not that hard.
I was aiming at implementing Metamorphoses, I had at disposition the formula of a straight line between an image pair.
I have noticed, as others before me, that these equations bear a strong resemblance to the equations of fluid dynamics.
If these equations describe the evolution of the fluid, it must be possible to perform this operation using a computer in order to represent it.
It seams difficult
Indeed
Thankfully, a whole branch of physics tries to answer that. But I know nothing about it...
Let's see Lagrange, maybe he will give us an idea.



A fluid can be represented by a collection of particles, in water it would be water molecules.
In the liquid state, the flow of the fluid is continuous and regular, a mathematician could say differentiable. This means that locally, the particles are going in the same general direction. To reformulate it: if we take a particle at random at a time t, its neighboring molecule will have a similar trajectory.
This property means that to describe the flow as a whole, it's sufficient to follow a few well-chosen particles!


This seems like a tiring technique and in practice
it wouldn't be useful for me. The particles are likely to move too far away from each other, or worse, leave the image...
Oh, look, there's Euler, he'll surely have another idea for studying flows.
134,135 ...
Excuse me sir, but why are you counting?
Oh ?! ... Hello youngsters, I didn't hear you coming.
To describe the flow, unlike Lagrange, I prefer to describe the flow by the quantity of particles that cross a given place.
And I don't like to run
And once I know the quantity of particles in my basket
, I only need to count the quantity of particles that enter and leave.
This variation is the derivative.
Wow, it's perfect for describing the evolution of an image over time. Each basket here is a pixel, and since I know the amount of variation in the number of particles in a basket based on its neighbors, I can describe the evolution of my entire image.
Very well, but I do not see the link with your thesis.
Our goal is to do an image registration. That is to find the vectors that describe the flow, which will in turn carry our image. Hopefully it will end up looking as much as possible like the target image.


If the Euler method works well, it requires making variations with small time steps and therefore doing a lot of calculations! And it is much less stable than the Lagrange method.
Fortunately, we can make sure to take the best of both methods.
Semi-Lagrangian Schemes: The idea is to choose the particles wisely. A regular grid is defined and only the particles that will reach this grid at the end of the time step are taken. To retrieve our image, simply replace the color of the pixel from the previous position of the particles. Let me explain...
One chooses the particles that arrive at the center of our pixels...
And then we retrieve the color of the cell where it came from...
And we replace the color of the cell with that one!
Finally, we can see the evolution of such a numerical scheme


I then implemented LDDMM and Metamorphoses using these schemes, greatly accelerating their execution! As explained in Chapter 2, Metamorphoses can be a bit difficult to use. I made sure to adapt them to the case of brain cancer with a model I called Constrained Metamorphoses. This is explained in Chapter 3. Enjoy your reading!