Scale larger than the horizon k << aH

Ecs. generales para cualquier componente.

$$\dot\delta = -(1+w)\left(\theta - 3\dot\phi\right) - \cancel{\mathcal{H}\left(\frac{\delta p}{\rho} - w\delta\right)}$$

$$\dot\theta = -\mathcal{H}\left(1-3w\right)\theta - \cancel{\frac{\dot w}{1+w}\theta} + \cancel{\frac{1}{1+w}\frac{k^2\delta p}{\rho}} + \cancel{k^2\psi}$$

Assume perfect barotropic fluid $\delta p = w\,\delta\rho$

Assume also ϕ = const

$$\ddot\delta + 3aH(1-3w)\dot\delta = 0$$

If you want to solve the eqs just replace the epoch and the fluid

δ = constant

δ = decaying

xq no hay un término de fuente proporcional a δ. (x eso δ no aumenta)

$$k^2\phi + 3aH(\cancel{\phi + aH\psi}) = -4\pi G a^2\rho\delta$$

Perturbations outside the horizon do not grow

Scale smaller than the horizon k >> aH

$$\dot\delta = -(1+w)\left(\theta - 3\dot\phi\right) - \mathcal{H}\left(\frac{\delta p}{\rho} - w\delta\right)$$

$$\dot\theta = -\mathcal{H}\left(1-3w\right)\theta - \frac{\dot w}{1+w}\theta + \frac{1}{1+w}\frac{k^2\delta p}{\rho} + k^2\psi$$

Pressureless matter, w=0 but with some c_s

$$\begin{aligned}\dot\delta &= -\theta + \cancel{3\dot\phi} - \cancel{\mathcal{H}c_s^2\delta}\,,\\ \dot\theta &= -\mathcal{H}\theta + c_s^2k^2\delta + k^2\phi\end{aligned}$$

k >> aH

$\phi \sim \delta/k^2$

ec. de Einstein

$$\delta'' + \mathcal{H}\delta' + (c_s^2k^2 + k^2\phi) = 0$$

$$k^2\phi = -4\pi G a^{-1}\delta$$

We find $\ddot\delta + \mathcal{H}\dot\delta + \boxed{\left(c_s^2k^2 - \frac{3}{2}\mathcal{H}^2\right)}\delta = 0$

$$c_s^2k^2 - \frac{3}{2}\mathcal{H}^2 \to \begin{cases}\geq 0 & \text{oscillating}\\ < 0 & \text{growing}\end{cases}$$

$2\pi/\lambda$

Jeans scale $\lambda_J = c_s\sqrt{\frac{\pi}{G\rho}}$

Jean scale es básicamente la $\vec v$ del sonido. (perturbaø más pequeña que puede crecer).

↳ A escalas más pequeñas, la perturbación no crece.

In MDE

Pure CDM w=cs=0

$$\ddot\delta + \mathcal{H}\dot\delta - \frac{3}{2}\mathcal{H}^2\delta = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau} = a^2H\frac{\mathrm{d}}{\mathrm{d}a}$$

$$\delta'' + \left(\frac{3}{a} + \frac{H'}{H}\right)\delta' - \frac{3}{2\,a^2}\delta = 0$$

→ Pure CDM: pudo desacoplarse antes que la radiación.

The presence of H might lead to different growth: we can replace any dominant component, but it's not that easy. Remind that the gravitational potential is the sum of all the components, hence it will take the dominant component at that time.

$$H = H_0\sqrt{\Omega_{m,0}a^{-3}}$$

$$\delta'' + \frac{3}{2a}\delta' - \frac{3}{2a^2}\delta = 0$$

cuando a la materia le cuesta acoplarse, δ disminuye un poco xq la expansión va muy rápido

$$\delta = c_1\,a + c_2\,a^{-3/2}$$

sol. que crece

sol. que decrece.

During the epoch? They'll grow slower. So MDE is the most interesting epoch for LSS.

Extra info: we said that in linear theory each wave evolves independently and what we did is

$$\delta = \delta(a,\,k) = \delta(a)\delta(k)$$

xq cualquier escala evoluciona independiente de las otras

In RDE

Pure CDM w=cs=0

$$\left.\begin{aligned}\dot\delta &= -\left(\theta - 3\dot\phi\right)\\ \dot\theta &= -\mathcal{H}\theta + k^2\psi\end{aligned}\right\}$$

para CDM (comparamos $\delta_r \ll \delta_m$)

$$\frac{\mathrm{d}}{\mathrm{d}\tau} = a^2H\frac{\mathrm{d}}{\mathrm{d}a}$$

· CDM se pudo desacoplar del baño térmico mucho antes que los bariones.

And the potential is:

$$k^2\phi = -4\pi G a^2\sum_s \rho_s\left[\delta_s + \frac{3aH}{k^2}(1+w_s)\theta_s\right]$$

→ Ahora tenemos que incluir todas las componentes! → componente que domina es la radiaø.

And we saw that for pure CDM in MDE the potential had only CDM. If we assume we are in RDE (and still small scales) then

$$k^2\phi = -4\pi G a^2\rho_r\delta_r \sim 0$$

Small wrt CDM

→ fluctuaø pequeña con respecto a la CDM.

$$\delta'' + \left(\frac{3}{a} + \frac{H'}{H}\right)\delta' = 0$$

And with: $H = H_0\sqrt{\Omega_{r,0}a^{-4}}$ $\Longrightarrow$ $\delta = \text{const.} + \ln(a)$ → no crece la perturbaø.

In RDE

Pure radiation w=cs=1/3

ϕ tiene δr

$$\dot{\delta}_r = -\frac{4}{3}\left(\theta_r - 3\dot{\phi}\right)$$

$$\dot{\theta}_r = \frac{k^2}{4}\delta_r + k^2\psi$$

perturbaçøes de radiaçø dado por la siguiente ecuaçø:

And the small scale solution

$$\ddot{\delta}_r + \frac{k^2}{3}\delta_r + \frac{4}{3}k^2\phi = 0 \xrightarrow[\text{c/r al factor de escala.}]{\text{pasando a la ec.}} \delta_r'' + \left(\frac{2}{a} + \frac{H'}{H}\right)\delta_r' + \boxed{\left(\frac{k^2}{3} - \frac{3}{2\,a^2}H_0^2\Omega_{r,0}\right)}\delta_r = 0$$

Jeans scale

For smaller scales, radiation perturbations do not grow, they oscillate

For small scales but larger than the Jeans scale they can grow. But how large?

- $H^2 = H_0^2\Omega_{r,0}\,a^{-4}$ $\Rightarrow$ $\delta_r'' + \frac{k^2}{3}\delta_r = 0 \longrightarrow \delta_r \propto \cos(ka/\sqrt{3})$

  $\frac{H'}{H} = -\frac{2}{a}$ $\qquad k^2 \gg H_0^2\Omega_{r,0}$

No puede crecer → oscila

→ una parte de la soluçø de los fotones de la CMB. Bariones acoplados con los fotones.

⇓

La T° de los fotones que se liberan de la materia tendrá impresa el mov. oscilatorio.

slide 10 cosmo

$\boxed{\text{Synchronous gauge}}$ → Tiene bajo control perturbaciones que entran o salen del horizonte causal.

↳ Estamos encima de la perturbaø. ⟹ podemos considerar perturbaøes fuera du horizonte.

$ds^2 = a^2\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$

* utilizando un gauge u otro cambian efectos que no son físicos. → crean artefactos de la perturbaø.

$B = \eta$

We do not want to evaluate the Einstein equations in a different gauge but we rather want to derive the transformation law relating two arbitrary gauges.

The components g_{00} and g_{0i} of the metric tensor in the synchronous gauge are by definition unperturbed.

The line element is given by

no tiene un potencial gravitacional.

$$ds^2 = a^2\left[-d\tau^2 + (\delta_{ij} + h_{ij}dx^i dx^i)\right]$$

por teo. de Helmholtz.

We can decompose, as before → descomposiø SVT

$$h_{ij} = \underbrace{\frac{1}{3}h\,\delta_{ij}}_{\text{traza}} + \underbrace{h^{\parallel}_{ij}}_{\text{escalar}} + \underbrace{h^{\perp}_{ij}}_{\text{vector}} + \underbrace{h^{T}_{ij}}_{\text{tensorial}}\,, \qquad \Longrightarrow$$

como potencial escalar

conectada a un escalar. η

$$\begin{aligned} h^{\parallel}_{ij} &= \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)B\,, \\ h^{\perp}_{ij} &= \partial_i A_j + \partial_j A_i\,, \qquad \partial_i A_i = 0\,, \end{aligned}$$

Scalars

As we want to work in Fourier space, and for the scalar part

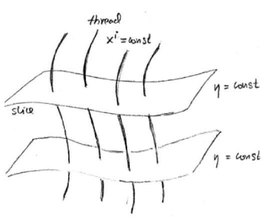
$$h_{ij}(\vec{x},\tau) = \int d^3k\, e^{i\vec{k}\cdot\vec{x}}\left\{\hat{k}_i\hat{k}_j h(\vec{k},\tau) + \left(\hat{k}_i\hat{k}_i - \frac{1}{3}\delta_{ij}\right)6\eta(\vec{k},\tau)\right\}$$

It's widely use, but there are serious disadvantages:

1) Since the choice of the initial hypersurface and its coordinate assignments are arbitrary, the synchronous gauge conditions do not fix the gauge degrees of freedom completely.

2) Since the coordinates are defined by freely falling observers, coordinate singularities arise when two observers' trajectories intersect each other: a point in spacetime will have two coordinate labels.

These difficulties led Bardeen (1980) to formulate gauge-invariant quantities.



Hypersurfaces

We need to know the equations in Synchronous gauge because CAMB (and CLASS) use this gauge.

· Elección de hypersuperficies a t constante. (slice)

↳ calcularlo a ≠ t:

ver clase 9!

Let's derive the transformation law starting from the Newtonian gauge $\quad \delta g_{\mu\nu} = a^2\begin{pmatrix} -2\psi & w_i \\ w_i & 2\phi\delta_{ij} + \chi_{ij} \end{pmatrix}$

pasar de Newtonian Gauge a synchronous $\qquad$ (h)

where we have substituted h_{ij} with χ_{ij} to make a distinction between the two gauges.

Consider a general coordinate transformation from a coordinate system x^μ to another $\hat{x}^\mu$

$$x^\mu \to \hat{x}^\mu = x^\mu + d^\mu(x^\nu)\,.$$

We write the time and the spatial parts separately as (and using bold to indicate vectors)

$$\begin{aligned} \hat{x}^0 &= x^0 + \alpha(\mathbf{x},\tau) \\ \hat{\mathbf{x}} &= \mathbf{x} + \nabla\beta(\mathbf{x},\tau) + \epsilon(\mathbf{x},\tau)\,, \qquad \nabla\cdot\epsilon(\mathbf{x},\tau) = 0 \end{aligned}$$

where the vector **d** has been decomposed into a longitudinal component

$$\nabla\beta \quad (\nabla\times\nabla\beta = 0) \qquad \text{And} \qquad \epsilon \quad (\nabla\cdot\epsilon = 0)$$

From GR we have the transformation of scalar, vector, and tensor quantities

Scalar $\quad s = s \qquad$ ⟹ The scalar is invariant and so it has to be the metric ds²

Vector $\quad w^{\tilde{\alpha}} = X^{\tilde{\alpha}}_{\beta} w^{\beta}$

Tensors $\left\{ \begin{aligned} A^{\tilde{\alpha}}_{\;\tilde{\beta}} &= X^{\tilde{\alpha}}_{\gamma} X^{\hat{\delta}}_{\tilde{\beta}} A^{\hat{\gamma}}_{\;\hat{\delta}} \\ B_{\tilde{\alpha}\tilde{\beta}} &= X^{\hat{\gamma}}_{\tilde{\alpha}} X^{\hat{\delta}}_{\tilde{\beta}} B_{\hat{\gamma}\hat{\delta}} \end{aligned} \right.$

where

jacobiano $\qquad$ d (aplicar la transformaø)

$$\begin{aligned} X^{\tilde{\alpha}}_{\beta} &\equiv \frac{\partial\tilde{x}^{\alpha}}{\partial\tilde{x}^{\beta}} = \delta^{\alpha}_{\beta} + \xi^{\alpha}_{\;,\beta} \\ X^{\hat{\delta}}_{\tilde{\beta}} &\equiv \frac{\partial\tilde{x}^{\delta}}{\partial\tilde{x}^{\beta}} = \delta^{\delta}_{\beta} - \xi^{\delta}_{\;,\beta}\,. \end{aligned}$$

gorrito es el nuevo sistema

x los símbolos de Christoffel

⇓

This implies the transformation law for the metric tensor

⇓

vieja

$$\tilde{g}_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial\tilde{x}^{\mu}}\frac{\partial x^{\beta}}{\partial\tilde{x}^{\nu}}g_{\alpha\beta}$$

nueva

With $$x^\mu \to \hat{x}^\mu = x^\mu + d^\mu(x^\nu)\,.$$
$$\Downarrow$$
$$\hat{g}_{\mu\nu}(x) = g_{\mu\nu}(x) - g_{\mu\beta}(x)\partial_\nu d^\beta - g_{\alpha\nu}(x)\partial_\mu d^\alpha - d^\alpha \partial_\alpha g_{\mu\nu}(x) + \mathcal{O}(d^2)$$

calculadas en la misma coordenada x (no necesariam. son iguales)

We note that both sides of this equation are evaluated at the same coordinate values x in the two gauges, which do not correspond to the same physical point in general.

To obtain the previous one, we need to transform the tensor, from GR:

pasamos del viejo al nuevo (pequeña perturbaciones, rotación).

$$\begin{aligned} B_{\tilde\mu\tilde\nu}(\tilde P) &= X^{\hat\rho}_{\tilde\mu} X^{\hat\sigma}_{\tilde\nu} B_{\hat\rho\hat\sigma}(\tilde P) = (\delta^\rho_\mu - \xi^\rho{}_{,\mu})(\delta^\sigma_\nu - \xi^\sigma{}_{,\nu}) \left[B_{\hat\rho\hat\sigma}(\hat P) - \frac{\partial \bar B_{\rho\sigma}}{\partial x^\alpha}(\bar P)\xi^\alpha \right] \\ &= B_{\hat\mu\hat\nu}(\hat P) - \xi^\rho{}_{,\mu} B_{\hat\rho\hat\nu}(\hat P) - \xi^\sigma{}_{,\nu} B_{\hat\mu\hat\sigma}(\hat P) - \frac{\partial \bar B_{\mu\nu}}{\partial x^\alpha}(\bar P)\xi^\alpha \\ &= B_{\hat\mu\hat\nu}(\hat P) - \xi^\rho{}_{,\mu} \bar B_{\rho\nu}(\bar P) - \xi^\sigma{}_{,\nu} \bar B_{\mu\sigma}(\bar P) - \frac{\partial \bar B_{\mu\nu}}{\partial x^\alpha}(\bar P)\xi^\alpha\,, \end{aligned}$$

evaluado en un punto $\tilde P$ a x ($\tilde P$)

$$\hat{g}_{\mu\nu}(x) = g_{\mu\nu}(x) - g_{\mu\beta}(x)\partial_\nu d^\beta - g_{\alpha\nu}(x)\partial_\mu d^\alpha - d^\alpha \partial_\alpha g_{\mu\nu}(x) + \mathcal{O}(d^2)$$

Let's do the 00 component:

$$\hat{g}_{00}(x) = g_0(x) - g_{0\beta}(x)\partial_0 d^\beta - g_{\alpha 0}(x)\partial_0 d^\alpha - d^\alpha\partial_\alpha g_{00}(x)$$

And with the dummy indices:

$$\hat{g}_{00}(x) = g_0(x) - g_{00}(x)\partial_0 d^0 - g_{0i}(x)\partial_0 d^i - g_{00}(x)\partial_0 d^0 - g_{i0}(x)\partial_0 d^i - d^0\partial_0 g_{00}(x) - d^i\partial_i g_{00}(x)$$

$$\hat{g}_{00}(x) = g_0(x) - 2g_{00}(x)\partial_0 d^0 - 2g_{0i}(x)\partial_0 d^i - d^0\partial_0 g_{00}(x) - d^i\partial_i g_{00}(x)$$

Since the transformations are between perturbed systems, they are of the same order of the perturbations

$$\begin{aligned} \hat{x}^0 &= x^0 + \alpha(\mathbf{x},\tau) \\ \hat{\mathbf{x}} &= \mathbf{x} + \nabla\beta(\mathbf{x},\tau) + \epsilon(\mathbf{x},\tau)\,, \qquad \nabla\cdot\epsilon(\mathbf{x},\tau) = 0 \end{aligned}$$

The 00 component of the metric tensor in the Newtonian gauge is

$$g_{00} = -a^2(1+2\psi)$$

This one $$\hat{g}_{00}(x) = g_0(x) - 2g_{00}(x)\partial_0 d^0 - 2\overset{0}{g_{0i}}(x)\partial_0 d^i - \boxed{d^0}\partial_0 g_{00}(x) - d^i\partial_i g_{00}(x)$$

becomes

potencial al ... vieja.

$$-a^2(\cancel{1} + 2\hat\psi) = -a^2(\cancel{1} + 2\psi) + 2a^2(1 + \cancel{2\psi})\dot\alpha + \alpha\left[2a\dot a + \cancel{2a^2\phi}\right] + d^i\partial_i\left[\cancel{a^2(1+2\psi)}\right]$$

$\propto (2a\dot a(1+2\psi) + 2\dot\psi a^2)$

Are the same

Are 2nd order perturbations

And we're left with

$$\hat\psi = \psi - \dot\alpha + \frac{\dot a}{a}\alpha$$

The metric tensor components transform:

$$\begin{aligned} \hat\psi(\mathbf{x},\tau) &= \psi(\mathbf{x},\tau) - \dot\alpha(\mathbf{x},\tau) - \frac{\dot a}{a}\alpha(\mathbf{x},\tau)\,, \\ \hat w_i(\mathbf{x},\tau) &= w_i(\mathbf{x},\tau) + \partial_i\alpha(\mathbf{x},\tau) - \partial_i\dot\beta(\mathbf{x},\tau) - \dot\epsilon_i(\mathbf{x},\tau)\,, \\ \hat\phi(\mathbf{x},\tau) &= \phi(\mathbf{x},\tau) + \frac{1}{3}\nabla^2\beta(\mathbf{x},\tau) + \frac{\dot a}{a}\alpha(\mathbf{x},\tau)\,, \\ \hat\chi_{ij}(\mathbf{x},\tau) &= \chi_{ij}(\mathbf{x},\tau) - 2\left\{\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\beta(\mathbf{x},\tau) + \frac{1}{2}(\partial_i\epsilon_j + \partial_j\epsilon_i)\right\}. \end{aligned}$$

curvatura de la componente espacial

Since wi and χ_{ij} are a vector and a tensor, respectively, we can further decompose them into longitudinal and transverse parts:

salen estos términos

$$\begin{aligned} \hat w_i^{\parallel}(\mathbf{x},\tau) &= w_i^{\parallel}(\mathbf{x},\tau) + \partial_i\alpha(\mathbf{x},\tau) - \partial_i\dot\beta(\mathbf{x},\tau)\,, \\ \hat w_i^{\perp}(\mathbf{x},\tau) &= w_i^{\perp}(\mathbf{x},\tau) - \dot\epsilon_i(\mathbf{x},\tau)\,, \end{aligned}$$

$$\begin{aligned} \hat\chi_{ij}^{\parallel}(\mathbf{x},\tau) &= \chi_{ij}^{\parallel}(\mathbf{x},\tau) - 2\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\beta(\mathbf{x},\tau) \\ \hat\chi_{ij}^{\perp}(\mathbf{x},\tau) &= \chi_{ij}^{\perp}(\mathbf{x},\tau) - (\partial_i\epsilon_j + \partial_j\epsilon_i)\,, \\ \hat\chi_{ij}^{T}(\mathbf{x},\tau) &= \chi_{ij}^{T}(\mathbf{x},\tau)\,. \end{aligned}$$

Let us now relate the Newtonian and the synchronous gauges

$$\begin{aligned} \alpha(\mathbf{x},\tau) &= \dot\beta(\mathbf{x},\tau) + \xi(\tau)\,, \\ \epsilon_i(\mathbf{x},\tau) &= \epsilon_i(\tau)\,, \\ h_{ij}^{\parallel}(\mathbf{x},\tau) &= -3\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij\nabla^2}\right)\beta(\mathbf{x},\tau)\,, \\ \partial_i\epsilon_j + \partial_j\epsilon_i &= 0\,, \end{aligned}$$

These come considering the previous ones, for instance:

$$\hat w_i^{\parallel} = w_i^{\parallel} + \partial_i\alpha - \partial_i\dot\beta$$

componente 0i

$$\Downarrow$$

$$0 = 0 + \partial_i\alpha - \partial_i\dot\beta$$

where ξ(τ) is a arbitrary function of time, reflecting the gauge freedom associated with the coordinate transformation. This transformation corresponds to a global redefinition of time with no physical significance therefore we shall set to zero.

The two potentials are related as

$$\begin{aligned} \psi(\mathbf{x},\tau) &= \ddot\beta(\mathbf{x},\tau) + \frac{\dot a}{a}\dot\beta(\mathbf{x},\tau) \\ \phi(\mathbf{x},\tau) = &= -\frac{1}{6}h(\mathbf{x},\tau) - \frac{1}{3}\nabla^2\beta(\mathbf{x},\tau) - \frac{\dot a}{a}\dot\beta(\mathbf{x},\tau) \end{aligned}$$

Where β is determined by h_{ij} parallel

We defined: $$h_{ij}(\vec x,\tau) = \int d^3k\, e^{i\vec k\cdot\vec x}\left\{\hat k_i\hat k_j h(\vec k,\tau) + \left(\hat k_i\hat k_i - \frac{1}{3}\delta_{ij}\right)6\eta(\vec k,\tau)\right\}$$

How to relate to the scalar modes?

$$h_{ij} = \frac{1}{3}h\delta_{ij} + h_{ij}^{\parallel} \qquad h_{ij}^{\parallel}(\vec x,\tau) = \int d^3k\, e^{i\vec k\cdot\vec x}\left(\hat k_i\hat k_i - \frac{1}{3}\delta_{ij}\right)\left[h(\vec k,\tau) + 6\eta(\vec k,\tau)\right]$$

el grad. de β.

With

$$\beta(\vec x,\tau) = \int d^3k\, e^{i\vec k\cdot\vec x}\frac{1}{2k^2}\left\{h(\vec k,\tau) + 6\eta(\vec k,\tau)\right\}$$

And we finally have:

$$\begin{aligned} \psi(\vec k,\tau) &= \frac{1}{2k^2}\left\{\ddot h(\vec k,\tau) + 6\ddot\eta(\vec k,\tau) + \frac{\dot a}{a}\left[\dot h(\vec k,\tau) + 6\dot\eta(\vec k,\tau)\right]\right\}, \\ \phi(\vec k,\tau) &= \eta(\vec k,\tau) - \frac{1}{2k^2}\frac{\dot a}{a}\left[\dot h(\vec k,\tau) + 6\dot\eta(\vec k,\tau)\right]. \end{aligned}$$

How the Einstein equations will look like:

synchronous.

$$\left(\begin{aligned} k^2\eta - \frac{1}{2}\frac{\dot a}{a}\dot h &= 4\pi G a^2 \delta T^0{}_0(\mathrm{Syn})\,, \\ k^2\dot\eta &= 4\pi G a^2(\bar\rho + \bar P)\theta(\mathrm{Syn})\,, \\ \ddot h + 2\frac{\dot a}{a}\dot h - 2k^2\eta &= -8\pi G a^2\delta T^i{}_i(\mathrm{Syn})\,, \\ \ddot h + 6\ddot\eta + 2\frac{\dot a}{a}\left(\dot h + 6\dot\eta\right) - 2k^2\eta &= -24\pi G a^2(\bar\rho + \bar P)\sigma(\mathrm{Syn})\,. \end{aligned}\right)$$

Newtoniano.

$$\theta = -H(1-3w)\theta - \frac{\dot w}{1+w}\theta + \frac{1}{1+w}\frac{k^2\delta P}{\bar\rho} + k^2\psi$$

hay un potencial (ya vemos encima de la perturbación).

Where we need to transform also the EM tensor

$$T^\mu{}_\nu(\mathrm{Syn}) = \frac{\partial\hat x^\mu}{\partial x^\sigma}\frac{\partial x^\rho}{\partial\hat x^\nu}T^\sigma{}_\rho(\mathrm{Con})\,, \qquad \Longrightarrow$$

El Gauge Newtoniano (longitudinal).

$$\begin{aligned} \delta(\mathrm{Syn}) &= \delta(\mathrm{Con}) - \alpha\frac{\dot{\bar\rho}}{\bar\rho}\,, \\ \theta(\mathrm{Syn}) &= \theta(\mathrm{Con}) - \alpha k^2\,, \\ \delta P(\mathrm{Syn}) &= \delta P(\mathrm{Con}) - \alpha\dot{\bar P}\,, \\ \sigma(\mathrm{Syn}) &= \sigma(\mathrm{Con})\,. \end{aligned}$$

Background.

ecs. tensoriales son invariantes.

$\Longrightarrow$ and we arrive

$$\begin{aligned} \dot\delta &= -(1+w)\left(\theta + \frac{\dot h}{2}\right) - 3\frac{\dot a}{a}\left(\frac{\delta P}{\delta\rho} - w\right)\delta\,, \\ \dot\theta &= -\frac{\dot a}{a}(1-3w)\theta - \frac{\dot w}{1+w}\theta + \frac{\delta P/\delta\rho}{1+w}k^2\delta - k^2\sigma\,, \end{aligned}$$

→ no hay un potencial.
· Ψ no está afectada por un potencial (ya no lo ve ya me muevo con él)

Slide 12 cosmo

Condiciones iniciales adiabáticas

- Todos los modos deben haber sido superhorizonte en el pasado.
- Comienza la evolución del universo perturbado en t_{ini} con los modos fuera del horizonte $\frac{k}{a} \ll H$

perturbaø no puede crecer xq no está en contacto causal pero evoluciona en el tiempo.

$\rightarrow \rho(t,\vec{x}) = \bar{\rho}(t) + \delta\rho(t,\vec{x})$ → soluø para modos superhorizon (no crecen) ⇒ para $\lambda > R_H$ $\rho(t,x) \sim \rho(t)$ ya que es una solución de background.

↳ $\delta = 0$

$= \bar{\rho}(t + \delta t(t,\vec{x}))$ → hay que esperar que la perturbación entre al horizonte. (cada perturbaø entra en ≠ tiempo).

↳ depende de la escala

- En tiempos ≠, el radio de Hubble es ≠ pero las soluøes son similares a background.

$$= \bar{\rho}(t) + \dot{\rho}\,\delta t(t,\vec{x})$$

$$\Rightarrow \frac{\delta\rho(t,\vec{x})}{\dot{\bar{\rho}}} = \delta t(t,\vec{x})$$

Now, since at $\lambda < R_H$ then all superhorizon modes behave as background solutions and all the species are in thermal equilibrium, this amounts to say that for two the species a and b (and all)

↳ no pierde ni gana calor

$$\delta t_a(t,\vec{x}) = \delta t_b(t,\vec{x}) \longrightarrow \frac{\delta\rho_a(t,\vec{x})}{\dot{\bar{\rho}}_a} = \frac{\delta\rho_b(t,\vec{x})}{\dot{\bar{\rho}}_b}$$

↳ 2 componentes de materia que entran al mismo tiempo con la misma escala.

and for the continuity equation $\dot{\bar{\rho}} + 3H(\bar{\rho} + \bar{p}) = 0 \longrightarrow \frac{\delta\rho_a(t,\vec{x})}{\bar{\rho}_a + \bar{p}_a} = \frac{\delta\rho_b(t,\vec{x})}{\bar{\rho}_b + \bar{p}_b}$

→ condiciones adiabático.

Perturbations respecting this relation are called adiabatic.

We can repeat the same calculations for other quantities like pressure and temperature:

$$\frac{\delta\rho(t,\vec{x})}{\dot{\bar{\rho}}} = \frac{\delta p(t,\vec{x})}{\dot{\bar{p}}} = \frac{\delta T(t,\vec{x})}{\dot{\bar{T}}} = \delta t(t,\vec{x}) \qquad \text{Valid for all the species}$$

↗ temp.

at very early times, t_{ini}, all perturbations must be determined by a single perturbation $\delta t(x, t)$.

→ condiciones adiabáticas:

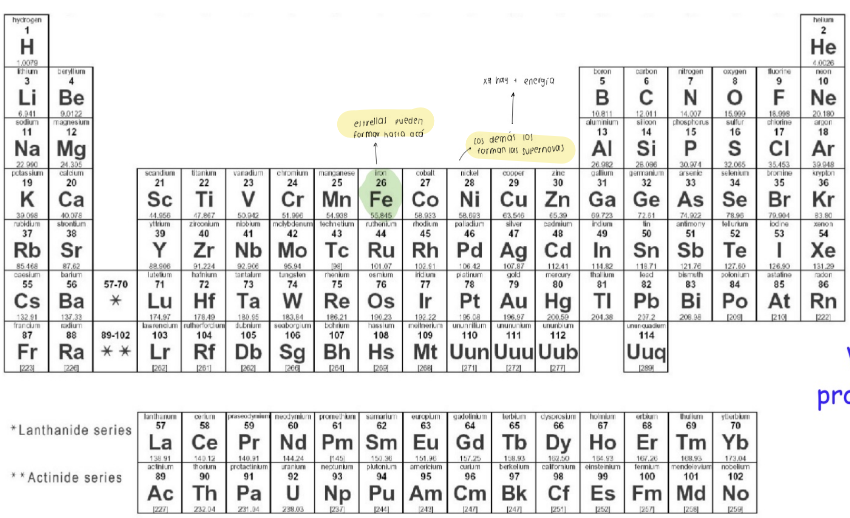
T° grandes y todo está acoplado

⇓

todo está en equilibrio termodinámico.

Big bang nucleosynthesis

Where do all elements come from?

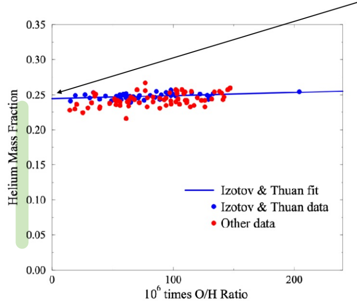


What's been produced where?

Stellar evolution worked until 1920/30 and calculations has been made.

^4He in the Universe cannot be accounted:

If Milky Way has 1010 years, it would generated only 4% vs 24% observed*



Where we considered that all the stars in the MW are made of H and they burn to create He

"Metallicity" = indicator of stellar processes

These are HII regions, where stellar evolution is high.

*https://arxiv.org/pdf/astro-ph/9903300.pdf

· sólo se observa el 0,25% del helio de las estrellas pero el 4% es lo que debe haber

↓

BBN

↓

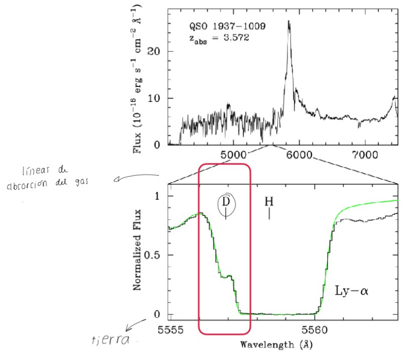
Helio que se formó en el Big Bang.

Another problems is where the Deuterium come from? It can't all come from star, since stars use D, so even when a star explodes there's no D left since it has been used. It is simply impossible to have D.

But it has been observed!

Quasar far away

Gas made of whatever we have in the universe



Quasi-stellar radio source

https://arxiv.org/pdf/astro-ph/9903300.pdf

Cosmological considerations

At z = 0 (today)

T_{CMB} = 2.73 K

$\rho_b = 5 \times 10^{-31}$ g/cm^3

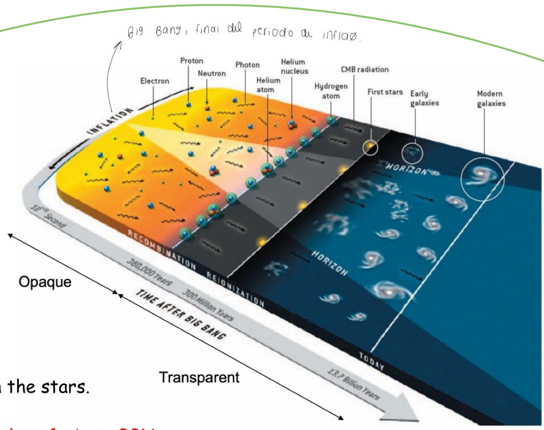
$\propto a^{-1}$

$\propto a^{-3}$

At z = 10^{10}

$T_{CMB} = 10^{10}$ K

ρ_b = 80 g/cm^3



· T° escala como a^{-1}

· ρ 〃 〃 a^{-3}

materia antes estaba más comprimida.

Very similar condition for nuclear fusion! As in the stars.

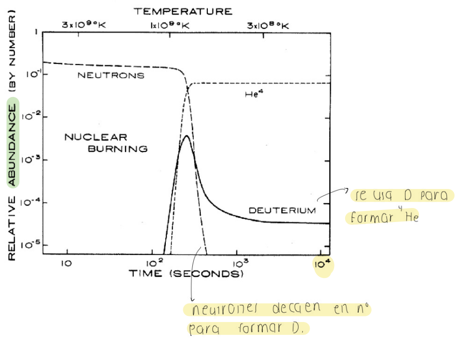
Cosmological nuclear fusion = BBN

PRIMEVAL HELIUM ABUNDANCE AND THE PRIMEVAL FIREBALL*

P. J. E. Peebles

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received 7 February 1966)



Evaluated numerical the abundance

$$\frac{dn}{dt} = -\langle \sigma v \rangle (n^2 - n_{eq}^2)$$

↓

$\vec{v}$ de las interacoes

→ cross-section

· Deuterio: isótopo del H.

→ densidad en n° de los elem. que queremos considerar.

Why he did not continue? Was he lazy?

Big Bang Nucleosynthesis

No stable elements with A = 5 or A = 8

BBN:

ρ_b = 80 g/cm^3

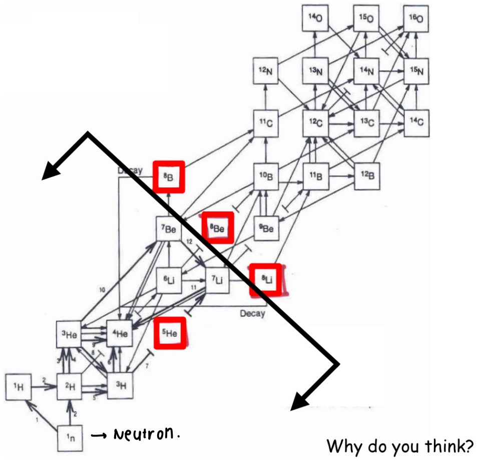
unable to process anything heavier than ^{7}Li, ^{7}Be due to very short timescale for fusion

Sun:

ρ_b = 150 g/cm^3

able to process elements heavier than ^{7}Li, due to high time timescale for fusion

↳ tienen + tiempo también para formarlos.



Why do you think?

* via unstable Be (tiple-alpha-process)

· la BBN dura poco desde 3 min a 10.000 s. ⟶ decaen los elem. que tienen A = 5 o A = 8

· Para las estrellas dura más tiempo el proceso por lo que se llegan a formar elem. más pesados y más estables.

→ BBN → forma núcleos ~~no~~ átomos.

Big Bang Nucleosynthesis

BBN summary: → se forman núcleos no átomos

Production of H, 2H, 3H, 3He, 4He, 7Be, 7Li

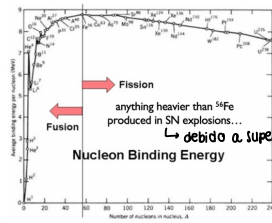
Deuterio, Tritio, Helio 3, Helio 4, Berilio 3, Litio 7

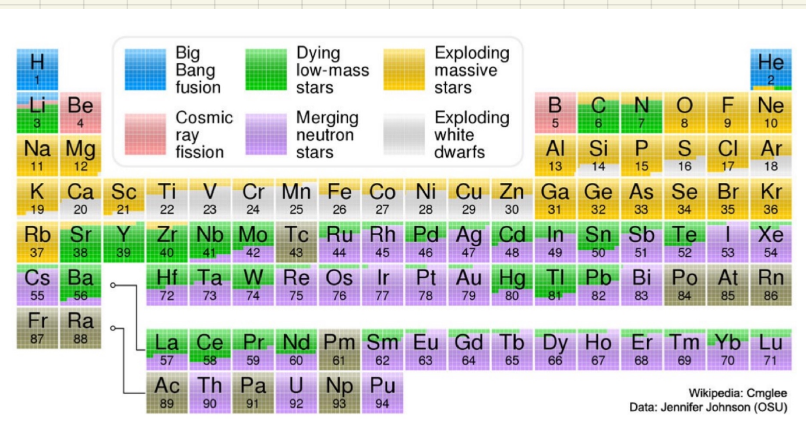
fracc de He y a la fracc total de elem. en el universo.

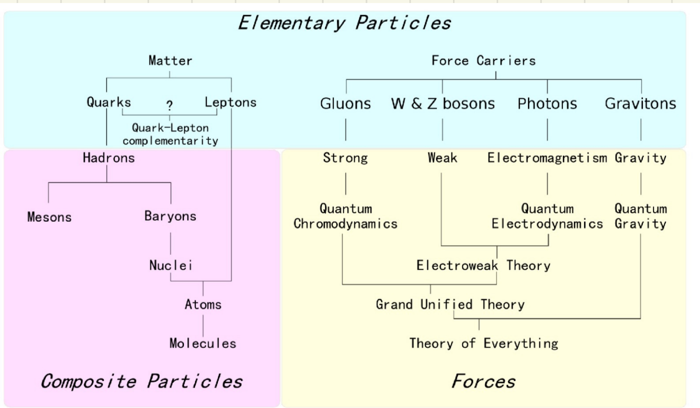
$Y_{\mathrm{He}} \sim 0.24$

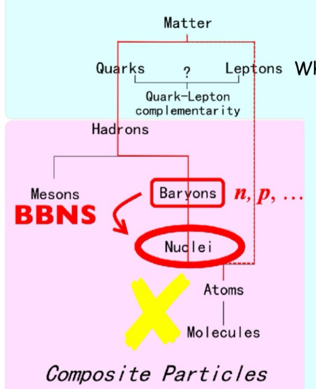
$Y_{\mathrm{H}} \sim 0.73$

All the other elements are produced in stars









When do we start having free protons and neutrons?

$$\nu_e + n \Longleftrightarrow p + e^-$$

$$e^+ + n \Longleftrightarrow p + \bar{\nu}_e$$ antineutrino

$$n \Longleftrightarrow p + e^- + \bar{\nu}_e$$

p and n held in equilibrium via weak interaction

BBN can only starts after neutrinos decoupling!

· hay que esperar a que ν se desacoplen para que no interactúen con n y p y se puedan formar núcleos.

Event	time t	redshift z	temperature T
Inflation	10^{-34} s (?)	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μs	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	$\boxed{60 \text{ kyr}}$	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

→ se forman los bariones.

→ desacople. (depende de la tipología de partículas)

se empiezan a formar los átomos ← Recombination

CMB ← Photon decoupling

We are talking about nuclei and not atoms

Synthesis of elements

Hydrogen: $p + e^- \to H + \gamma$

Deuterium: $p + n \to D + \gamma$

this requires free electrons, but they are still coupled to the photons:

→ Deuterio permite formar hasta el Litio 7.

All A>2 require D and n for synthesis (as catalyst)

(Remember: H, 2H, 3H, 3He, 4He, 7Be, 7Li)

→ Energía de enlace de D ~ 2MeV

energía de enlace de D. ← Binding energy of D

Neutrinos decoupling

$$E_b \sim 2\,\text{MeV}; \quad kT_\nu \sim 0.8\,\text{MeV}$$

??? → ν se desacoplan ~0,8 MeV

But D easily photo-dissociated by photons until $kT_D \sim 0.086\,\text{MeV}$ (ca. $t \sim 100\,\text{s}$)

→ Hay que esperar a kT ~ 0,086 MeV

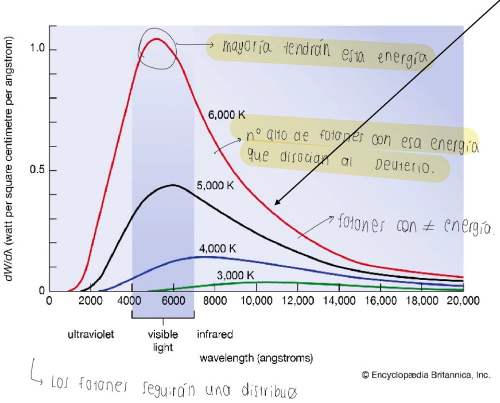
Why we need to wait until the thermal bath is 0.086 << 2 MeV?

→ Hay que esperar que pare

But D easily photo-dissociated by photons until $kT_\nu \sim 0.086\,\text{MeV}$ (ca. $t \sim 100\,\text{s}$)

→ Hay muchos fotones que pueden foto-disociar al deuterio aún.

Photones de la CMB siguen la distrib. de radiaø de cuerpo negro.



Los fotones seguirán una distribuø

$$u(T) = \frac{2\,h\,\nu^3}{c^2}\,\frac{1}{e^{h\nu/kT} - 1}$$

for temperatures $kT_D < E_b$ there will still be lots of higher energy photons in the Planck distribution

-> deuterium production (and all successive nuclei) sensitively depends on baryon-to-photon ratio!

All A>2 require D and n for synthesis (as catalyst) (Remember: H, 2H, 3H, 3He, 4He, 7Be, 7Li)

Deuterium bottleneck:

para 4He necesito 2D.

- Too few D -> important fusion agent is missing
- Too much D -> locks up neutrons for further synthesis

Todos los neutrones en el Deuterio y no quedan neutrones para formar otros elementos.

-> deuterium production (and all successive nuclei) sensitively depends on baryon-to-photon ratio!

cant. de bariones

$$\eta = \frac{n_b}{n_\gamma} = 10^{-10} \cdot 274\,\Omega_b h^2$$

cant. de fotones.

* muchos + fotones que bariones.

Big Bang Nucleosynthesis

Full network of nuclei reactions

$(p + e^- \rightarrow H + \gamma)$

$p + n \rightarrow D + \gamma$

$D + n \rightarrow {}^3H + \gamma$

$D + D \rightarrow {}^3H + p$

$D + p \rightarrow {}^3He + \gamma$

$D + D \rightarrow {}^3He + n$

${}^3He + n \rightarrow {}^3H + p$

$D + D \rightarrow {}^4He + \gamma$

$D + {}^3He \rightarrow {}^4He + p$

$D + {}^3H \rightarrow {}^4He + n$

${}^3He + {}^3He \rightarrow {}^4He + 2p$

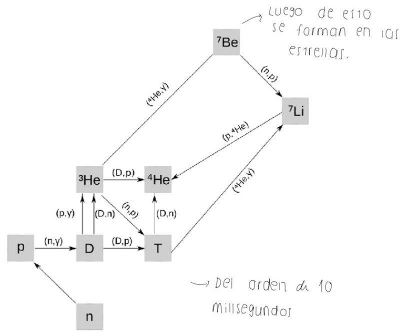
${}^3He + n \rightarrow {}^4He + \gamma$

${}^3H + p \rightarrow {}^4He + \gamma$

${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

${}^7Be \rightarrow {}^7Li + e^+ + \nu_e$

${}^3H + {}^4He \rightarrow {}^7Li + e^+ + \nu_e$



We don't require electrons, just protons and neutrons to form nuclei, with the limitations of forming up to A=7

* Necesitamos un ligero exceso de materia c/r a la antimateria.

Thermal equilibrium of radiation & matter

$$\gamma + \gamma \Longleftrightarrow P + \bar{P}$$

Interaction rate of particles: $\Gamma_c \propto n\,\sigma\,v$

nº de part. $\bar{v}$

cross section: prob. de que puedan chocar

Expansion rate of the Universe: $\Gamma_e \propto H$

$$\frac{\Gamma_c}{H} < 1 \quad \rightarrow \quad \text{"Freeze-out"}$$

↓ Desacople

The freeze-out depends on the temperature window which depends on H:

calcular expansión según la T°.

Radiation domination:

$$T \propto a^{-1}$$
$$H \propto a^{-2} \quad \rightarrow H \propto T^2$$

⇒ BBN is sensitive to cosmology

And Thermal History of the Universe.

Thermal equilibrium of radiation & matter

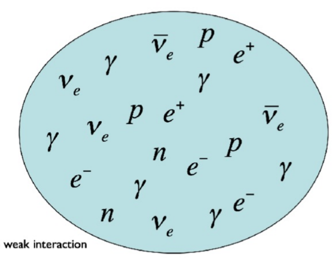
$$\nu_e + n \Longleftrightarrow p + e^-$$
$$e^+ + n \Longleftrightarrow p + \bar{\nu}_e$$
$$n \Longleftrightarrow p + e^- + \bar{\nu}_e$$

Weak interaction

Inventory	
Relativistic particles	e^-, e^+
Decoupled relativistic particles	ν
Decoupled non-relativistic particles	p, n

- Ecuación de Boltzman para resolverlo
  - ↳ para la especie no relativista

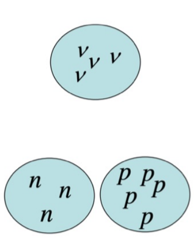
Thermal bath



Weak interaction freezes out at

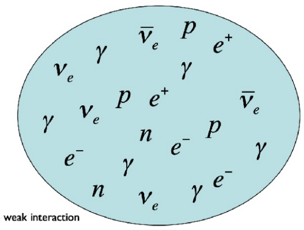
T ~ 0.8 MeV

↓ Neutrinos desacoplados



*only ν_e contributes to weak interaction

Neutron-to-proton ratio, i.e. for non-relativistic particles:



*only ν_e contributes to weak interaction

$$\nu_e + n \Longleftrightarrow p + e^-$$
$$e^+ + n \Longleftrightarrow p + \bar{\nu}_e$$
$$n \Longleftrightarrow p + e^- + \bar{\nu}_e$$
$$e^- + \gamma \Longleftrightarrow e^- + \gamma$$

razón entre neutrones y protones.

$$n_A = g_A \left(\frac{m_A k_B T}{2\pi \hbar}\right)^{3/2} e^{-(m_A - \mu_A)c^2/k_B T} \longrightarrow \frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-Q/k_B T} e^{(\mu_n - \mu_p)c^2/k_B T}$$

diferencias de las masas

$m_n \neq m_p \quad \rightarrow \quad Q = m_n - m_p = 1.293\,\mathrm{MeV}$

Which is the binding energy of D

What about the chemical potentials?

Big Bang Nucleosynthesis

We can't ignore the chemical potentials

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-Q/k_B T} e^{(\mu_n - \mu_p)c^2/k_B T}$$

· potencial químico se conserva!

Some "out-of-the-blue" calculations… chemistry!

The chemical potential is the energy absorbed or released during the chemical reaction

And it obeys to the the law (chemical equilibriums)*

$$A + B \Longleftrightarrow C + D \xrightarrow{\text{Chemical equilibrium}} \mu_A + \mu_B = \mu_C + \mu_D$$

But what's this reaction?

β-decay $\quad \nu_e + n \Longleftrightarrow p + e^- \quad \rightarrow \mu_{\nu_e} + \mu_n = \mu_p + \mu_e$

neutrinos se desacoplan

* chemical reactions are much faster than cosmic expansion…

Big Bang Nucleosynthesis

These chemical potentials can be calculated

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-Q/k_B T} e^{(\mu_e - \mu_{\nu_e})c^2/k_B T}$$

But how? Remember that the Universe is charge neutral.
Electrons and positrons have to be equal to protons.

nº de positrones

$$n_{e^-} - n_{e^+} = n_p \longrightarrow$$

nº de e^- totales en el universo
=
nº de protones (universo neutro)

We use baryon-to-photon ratio

$$n_{e^-} - n_{e^+} = \frac{2T^3}{6\pi^2}\left[\pi^2\left(\frac{\mu_e}{T}\right) + \left(\frac{\mu_e}{T}\right)^3\right] \longrightarrow$$

func de distrib. relativista

$$n_p \sim \eta\, n_\gamma = \eta \frac{2\zeta(3)}{\pi^2} T^3$$

$$\frac{2T^3}{6\pi^2}\left[\pi^2\left(\frac{\mu_e}{T}\right) + \left(\frac{\mu_e}{T}\right)^3\right] = \eta \frac{2\zeta(3)}{\pi^2} T^3$$

$$\left(\frac{\mu_e}{T}\right) + \left(\frac{\mu_e}{T}\right)^3 = \eta \frac{6\zeta(3)}{\pi^2} \quad \Longrightarrow \quad \left(\frac{\mu_e}{T}\right) \sim \eta \frac{6\zeta(3)}{\pi^2}$$

Which is given by observations: $\eta \sim 10^{-10}$

$$\mu_e \ll T \Longrightarrow \mu_e \sim 0$$

T° del baño

potencial químico de los e^-.

With neutrinos

$\frac{\mu_{\nu_e}}{T} < 0{,}2$ medido hoy. $\xrightarrow{BBN}$ $a = 10^{-10} \Rightarrow T = 10^{10}$ $\frac{\mu_{\nu e}}{T} \sim 0$

↓ despreciable en esa época

$$\frac{n_n}{n_p} = \left(\frac{m_n}{m_p}\right)^{3/2} e^{-Q/k_B T} \qquad Q = 1{,}293 \text{ MeV}.$$

$k_B T > 1.3 \text{ MeV} \Leftrightarrow n_n \approx n_p$ → T° del baño en el que está

$k_B T < 1{,}3 \text{ MeV} \Rightarrow n_n < n_p$

Freeze out: $k_B T < 0{,}8 \text{ MeV} < Q$

↓

$\frac{n_n}{n_p} \approx \frac{1}{6}$ → 6 protones x cada neutrón.

↓

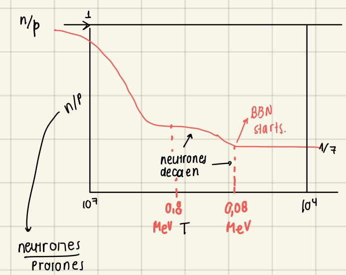
No es lo observado!

1) Neutron inestable (880 s) → tiempo limitado.
2) Fácil de disociar es el Deuterio (0,086 MeV)

⟶ How many neutrons are left?

$$\frac{n_n}{n_p} \approx \frac{1}{6} e^{-t/\tau_n} \approx \frac{1}{7}$$

↓ t asociado a la T° del ambiente así que cambia.



Helium abundance:

$$X_{He} = Y = \frac{m_{He}}{m_{He} + m_H} = \frac{4 m_p n_{He}}{4 m_p n_{He} + m_p n_H} = \frac{2 n_n}{2n_n + \underbrace{(n_p - n_n)}_{\text{protones not in 4He}}} = 0{,}25.$$

→ Abundancia de Helio en nuestro universo

↓

Tiene relación con lo que vemos ahora.

p que no están en en Helio.

- $\frac{n_n}{n_p} = \frac{1}{7}$
- $n_{He} = \frac{n_n}{2}$
- $m_p \approx m_n$

Eq nuclear: materia no relativista: $A = N_n + Z = \# n + \# p.$

$$n_A = g_A \left(\frac{m_A k_B T}{2\pi\hbar}\right)^{3/2} e^{-(m_A - \mu_A)c^2 \ldots}$$

$X_A = \frac{A n_A}{n_b}$, $n_b = \sum A_i n_i$ → la suma de todos los elem. que ya hemos creado antes.

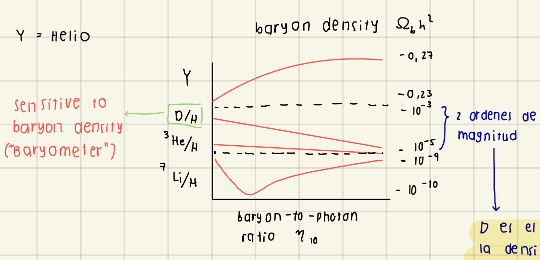
fracción de elementos de la especie A.

Big Bang Nucleosynthesis. 24 · sept.

- ^3H → 3 protones y 6 neutrones
- Neutrones son part. inestables.
  - ↳ más neutrones tienen que juntarse con los protones
  - ↳ Decaen en el tiempo.

  } hace que disminuya su mass fraction.

* Y = Helio



D es el más sensible a la variación de la densidad de Bariones.

· Hay una tensión cuando medimos distinta

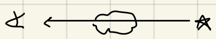
· D y ^3He son utilizados para formar los otros y por eso baja la curva.
↓
es el carburante de todo lo demás

Big bang Nucleosynthesis. → es sensible a todas las cantidades de acá:

g_* : $g_* \nearrow \;\to\; T_f \nearrow \;\to\; n/p \nearrow \;\to\; {}^4He \nearrow$

Ly-α clouds → estado de excitaø del átomo de H desde el nivel 1 al 2.

· Quasars: muchas ondas de radio, no es estrella xq no podrían formar lo observado.
(quasi stellar)



↓ radiaø (luz golpea la materia)
↓ se absorbe en el átomo

Líneas de absorción

$\eta \simeq 2.7 \cdot 10^{-5}$ — vemos + bariones que lo esperado

$\Omega_b h^2 \approx 0,0214$ — lo esperado

$\eta \simeq 1.6 \cdot 10^{-5}$

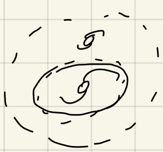
Nearby dwarf galaxies:

↳ galaxias enanas que tuvieron tiempo para crecer por lo que se piensa que tienen elem. pesados.

* metal en cosmología es todo lo + pesado que el Li

Galactic halos

↳ Materia oscura: Tiene efectos gravitacionales que no se ven.



HII regions

↳ Nube con gas colapsando. → formaø estelar. → Necesitamos una formaø estelar que ilumine el gas para verlo.

↓

Observaøes dan un nº ≠ al esperado (mayor).

hay una concentraø mayor de bariones en estas regiones (materia oscura hace que colapsen).

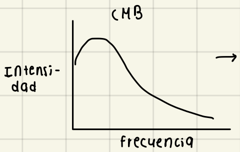
CMB

Fluctuaciones
- primarias → creadas en inflación
- secundarias → creadas after photon decoupling

- barotropic fluids $p = w\rho c^2$
  - radiación $w = 1/3$ → $T \propto R^{-1}$ — decae con el factor de escala
  - matter $w = 0$ → $T \propto R^{-2}$

- adiabatically

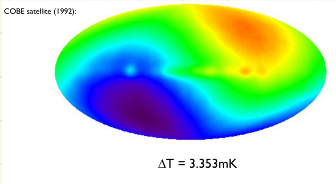
* radiación de cuerpo negro se enfría pero se mantiene en eq. térmico.



→ radiación de cuerpo negro

$T = 2{,}725$ K

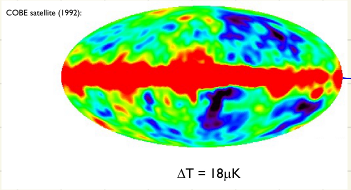
- dipolo
  - debido al movimiento local → $\Delta T = 3{,}35$ mK
  - vel. peculiar (redshift + fuerte)



* monopolo nos da la T° media del universo

- galaxias se mueven hacia un gran atractor (hacia una zona). (no se considera)

- A órdenes superiores → otras fluctuaciones → características del CMB.
  $\Delta T = 18\,\mu K$

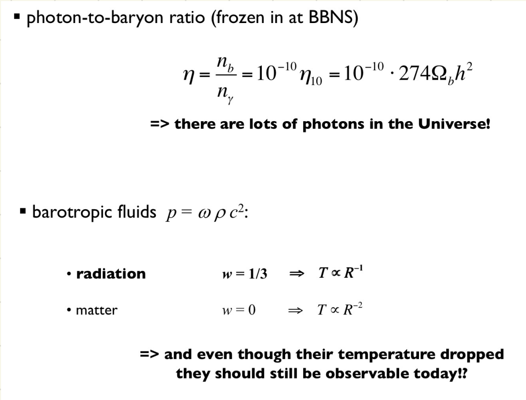


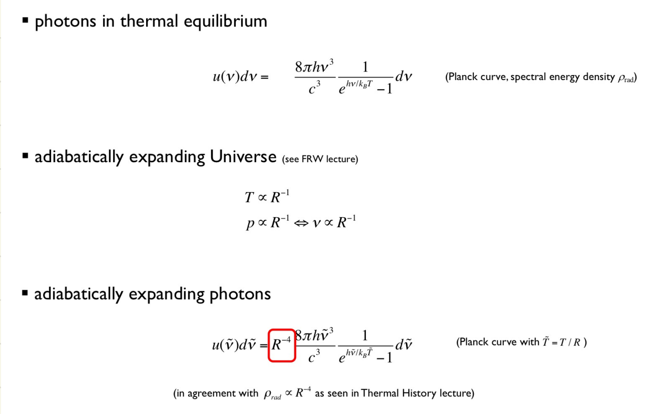
→ plano galáctico (nuestra galaxia) que hay que sustraerlo.

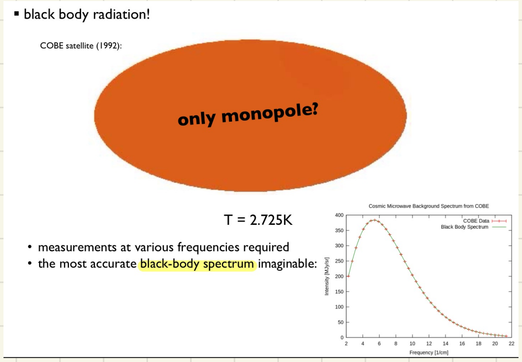
(cuadripolo hacia arriba).

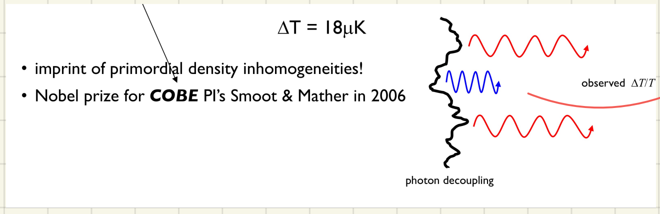
- No es tan directo que zonas en que está más caliente es xq hay más densidad.

Slide 14



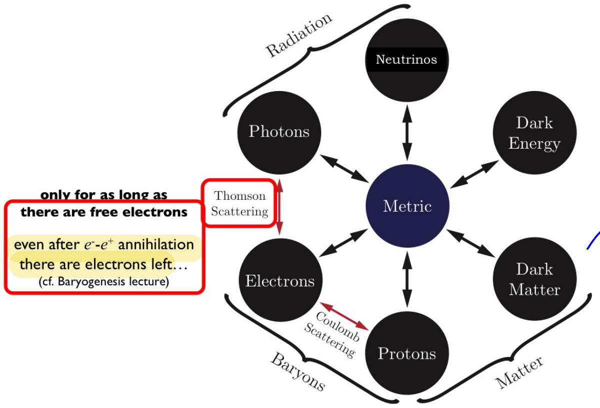






fotones salen de zonas con + densidad (más fríos)

- interactions between different forms of matter



· Hay electrones libres hasta que se desacoplen de los fotones.
(Después de la BBN)
↳ cuando e^- forman átomos no hay scattering.

no tiene interacción con las otras excepto por la métrica (efectos gravitacionales)

- CMBR origin

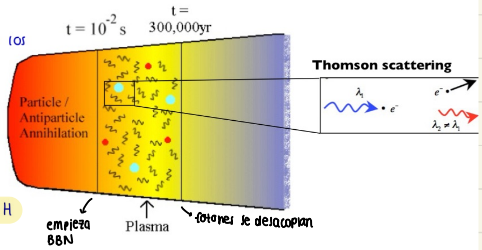
prior to recombination — recombinación de p con e^- para formar los átomos.

- electrons and photons couple via Thomson scattering
- Universe is opaque for radiation

↓

Hay fotones energéticos que pueden ionizar los átomos de H y sacan los e^-

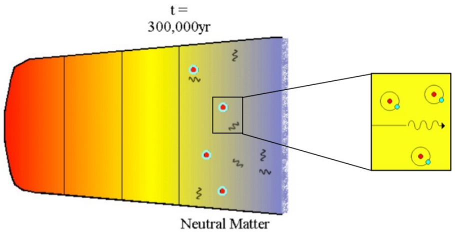
↓



after decoupling — de los γ con los p.

- electrons are bound to protons
- photons are free to travel

↳ Aquí fotones pueden viajar libremente.
↳ completamente desacoplados los e^- con los fotones.



- CMBR origin calculation – hydrogen recombination

$$e^- + p \leftrightarrow H + \gamma$$

we are interested in the fraction of free electrons:

those are the ones participating in the scattering with photons!

$$e^- + \gamma \leftrightarrow e^- + \gamma \quad \to \text{Thomson}$$

$$n_e = g_e \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_e-\mu_e)c^2/kT}$$

$$n_p = g_p \left(\frac{m_p kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_p-\mu_p)c^2/kT}$$

$$n_H = g_H \left(\frac{m_H kT}{2\pi\hbar^2}\right)^{3/2} e^{-(m_H-\mu_H)c^2/kT}$$

$$\left(\frac{n_H}{n_e n_p}\right) = \frac{g_H}{g_e g_p}\left(\frac{m_H}{m_e m_p}\frac{2\pi\hbar}{kT}\right)^{3/2} e^{\underbrace{(m_e+m_p-m_H)c^2}/kT}$$

B_H: binding energy of hydrogen
↓
Energía para tener el átomo de H.

$$n_\gamma = \frac{2\zeta(3)}{\pi^2}\left(\frac{k}{\hbar c}\right)^3 T^3$$

fraction of free electrons:

$$X_e = \frac{n_e}{n_b}$$

$$n_b = \eta n_\gamma = \eta \frac{2\zeta(3)}{\pi^2}\left(\frac{k}{\hbar c}\right)^3 T^3$$

$$n_b \approx n_p + n_H = n_e + \boxed{n_H}$$

(ignoring all nuclei A>1 and assuming charge neutrality)

$$1 = X_e\left(1 + n_e\left(\frac{2\pi\hbar}{m_e kT}\right)^{3/2} e^{B_H/kT}\right)$$

$$\frac{1}{X_e} = 1 + n_e\left(\frac{2\pi\hbar}{m_e kT}\right)^{3/2} e^{B_H/kT}$$

$$\frac{1}{X_e} - 1 = n_e\left(\frac{2\pi\hbar}{m_e kT}\right)^{3/2} e^{B_H/kT}$$

$$\frac{1-X_e}{X_e} = n_e\left(\frac{2\pi\hbar}{m_e kT}\right)^{3/2} e^{B_H/kT} = X_e n_b \left(\frac{2\pi\hbar}{m_e kT}\right)^{3/2} e^{B_H/kT}$$

$$\frac{1-X_e}{X_e^2} = \eta \frac{2\zeta(3)}{\pi^2}\left(\frac{k}{\hbar c}\right)^3 T^3 \left(\frac{2\pi\hbar}{m_e kT}\right)^{3/2} e^{B_H/kT}$$

fraction of free electrons:

$$X_e = \frac{n_e}{n_b}$$

$$n_b = \eta n_\gamma = \eta \frac{2\zeta(3)}{\pi^2}\left(\frac{k}{\hbar c}\right)^3 T^3 = n_e\left(1 + n_e\left(\frac{2\pi\hbar}{m_e kT}\right)^{3/2} e^{B_H/kT}\right)$$

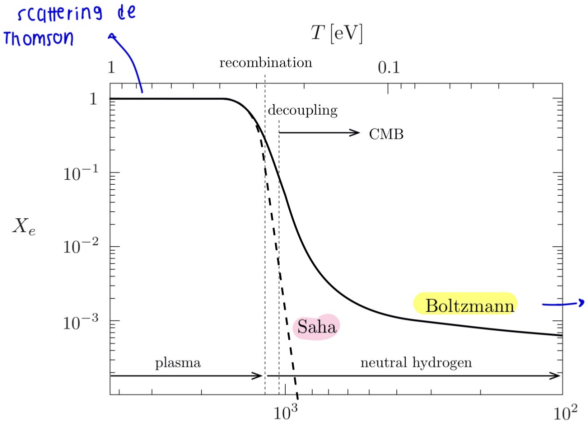
$$n_b \approx n_p + n_H = n_e + n_H$$

$$n_H = n_e^2 \left(\frac{2\pi\hbar}{m_e kT}\right)^{3/2} e^{B_H/kT}$$

$$\boxed{\frac{1-X_e}{X_e^2} = \frac{2\zeta(3)}{\pi^2}\eta\left(\frac{2\pi kT}{\hbar^2 m_e}\right)^{3/2} e^{B_H/kT}}$$

(Saha equation)

se conservan e^- libres. ← scattering de Thomson



→ Nos entrega fracción de e^- libres c/r a los bariones.

Boltzmann → la realidad

Binding energy de H:

$B_H \to T = 13{,}6$ eV

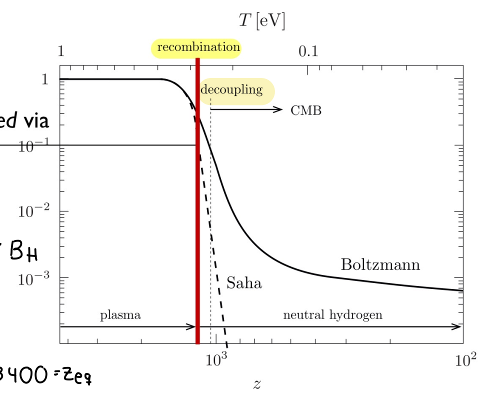
recombination defined via $X_e = 0.1$

↓

$T_{rec} = 0.31\,\text{eV} \ll B_H$

↓ lo podemos ver con el redshift.

$z_{rec} = 1330 \ll 3400 = z_{eq}$



▪ CMBR origin calculation – photon decoupling

- hydrogen recombination: $T_{rec} = 0.31\text{eV}$, $z_{rec} = 1330$
- photon decoupling: $e^- + \gamma \leftrightarrow e^- + \gamma$

decoupling condition: $\boxed{\Gamma}/H < 1$

Decoupling: expansión del universo domina sobre scattering

fracción de e^- : 0,1.

$$\Gamma_\gamma \approx n_e\,\sigma_T c = n_b X_e\,\sigma_T c = \eta \frac{2\zeta(3)}{\pi^2}\left(\frac{k}{\hbar c}\right)^3 T^3\, X_e \sigma_T c$$

rate de interacciones

$$H = \left(H_0^2 \Omega_{m,0} R^{-3}\right)^{1/2}$$ matter domination (as $z_{rec} << z_{eq}$):

→ Recombinación ocurre en MDE.

$\boxed{T \propto R^{-1}}$ for photons

↳ decae como el factor de escala.

$$H = H_0\sqrt{\Omega_{m,0}}\left(\frac{T}{T_0}\right)^{3/2}$$ (T is the temperature of the photons!)

$$\eta \frac{2\zeta(3)}{\pi^2}\left(\frac{k}{\hbar c}\right)^3 T_{dec}^3 X_e \sigma_T c \approx H_0\sqrt{\Omega_{m,0}}\left(\frac{T_{dec}}{T_0}\right)^{3/2}$$

- use Saha equation for $X_e(T_{dec})$
- solve for T_{dec}

→ se obtiene T_{dec}

↓

$z_{dec} = 1100$

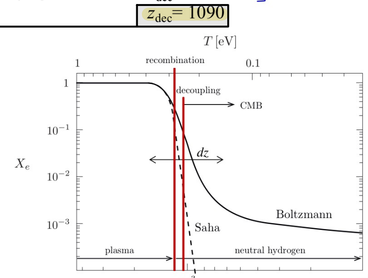
▪ CMBR origin calculation – photon decoupling

- hydrogen recombination: $T_{rec} = 0.31\text{eV}$, $z_{rec} = 1330$
- photon decoupling: $T_{dec} = 0.27\text{eV}$, $\boxed{z_{dec} = 1090}$

→ en este periodo, γ no viajan libres.

last scattering surface! ←

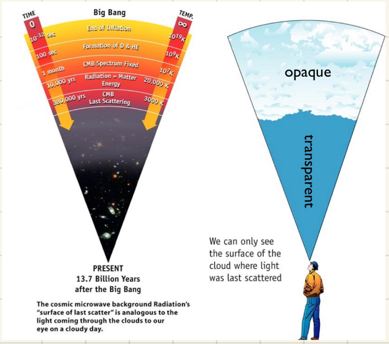
↳ último momento en que hicieron scattering de Thomson.



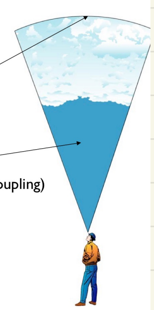
$\tau \sim 0{,}09$.

↑ qué tan difícil es para los fotones viajar.

redshift interval: scattering probability in $[z, z+dz]$: $p(z)dz = e^{-\tau}\frac{d\tau}{dz}dz$ (τ: optical depth)

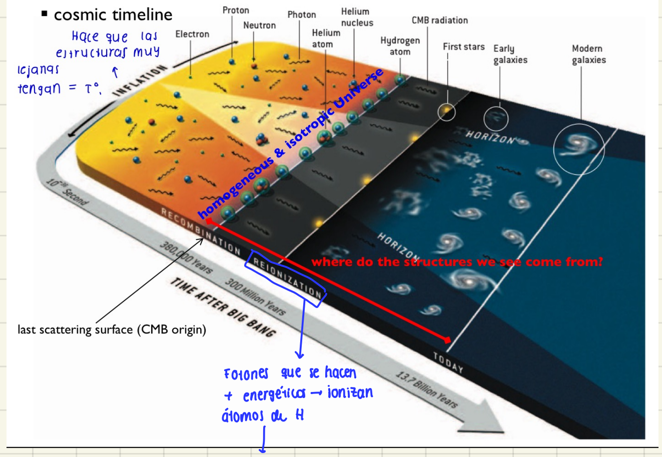


- discovery
- origin
- **CMB fluctuations**
  - primary (created during inflation)
  - secondary (created after photon decoupling)



- cosmic timeline

Hace que las estructuras muy lejanas tengan = T°.



last scattering surface (CMB origin)

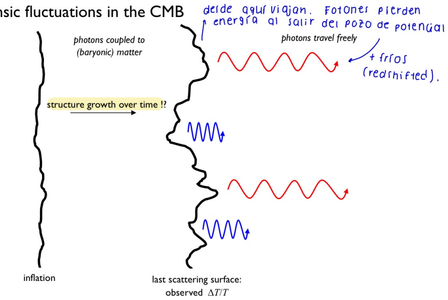
Fotones que se hacen + energéticos → ionizan átomos de H

x la formaø de las 1° estrellas

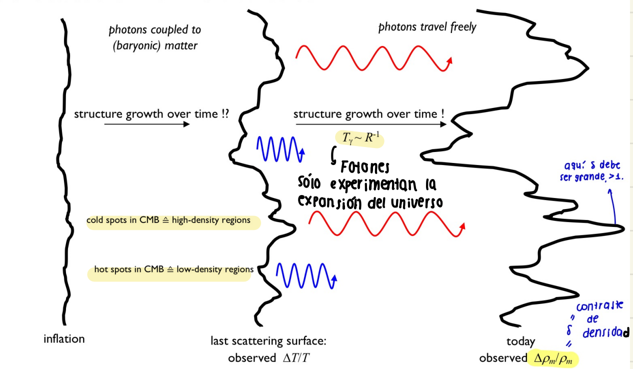
- seed inhomogeneities and their relation to temperature fluctuations:
  - primordial matter perturbations lead to…
  - …intrinsic fluctuations in the CMB

desde aquí viajan. Fotones pierden energía al salir del pozo de potencial

+ fríos (redshifted). ya que hay un pozo de potencial mayor.



…sic fluctuations in CMB are conserved



→ tiene que ser muy gr…

- (seed) inhomogeneities and their relation to temperature fluctuations:

observed $(\Delta\rho_m/\rho_m)_0$ | predicted $(\Delta T/T)_0 = (\Delta T/T)_{\rm dec}$

xq todos los fotones vienen influenciados de la misma manera

(la $\Delta T/T_c$ es la misma)

theoretical structure formation

radiation remains thermal

se quedan en eq. térmico!

$(\Delta\rho_m/\rho_m)_{\rm dec}$ → predicted $(\Delta T/T)_{\rm dec}$

some relation

$\Delta T/T = k\ \Delta\rho_m/\rho_m$

$T \sim 3000$

$\left(\frac{\Delta\rho_m}{\rho_m}\right)_{dec}$

relaø : $\Delta T/T = k\,\Delta\rho_m/\rho_m$

$\left(\frac{\Delta\rho_m}{\rho_m}\right)_0$

- theoretical structure formation (see "LSS" lecture)

época de materia

$$\boxed{\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G\rho_m\delta_m} \quad \text{with } \delta_m = \frac{\Delta\rho_m}{\rho_m}$$

- solution: $\delta_{m,0} = \delta_{m,\rm dec}\, a$
- today (lower limit!): $\delta_{m,0} \geq 1$
- decoupling: $z_{dec} \approx 1100$

$\delta_{m,dec} \geq 10^{-3}$ (lower limit!)

relation $\Delta T/T = k\ \Delta\rho_m/\rho_m$:

condiø adiabática ← **a) relation of $\Delta\rho_m/\rho_m$ to $\Delta\rho_r/\rho_r$**

b) relation of $\Delta\rho_r/\rho_r$ to $\Delta T/T$

deriv. c/r a a (R)

$$\rho_m \propto R^{-3} \Rightarrow \Delta\rho_m \propto -3R^{-4}\Delta R$$

$$\rho_r \propto R^{-4} \Rightarrow \Delta\rho_r \propto -4R^{-5}\Delta R$$

Δ is with respects to time, but we want spatial perturbations…

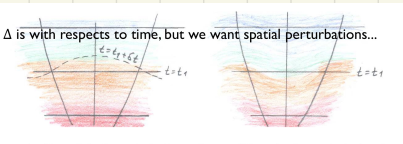


Figure 1: For adiabatic perturbations, the conditions in the perturbed universe (right) at $(t_1, \boldsymbol{x})$ equal conditions in the (homogeneous) background universe (left) at some time $t_1 + \delta t(\boldsymbol{x})$.

$$\left.\begin{aligned}\frac{\Delta\rho_m}{\rho_m} &= -3\frac{\Delta R}{R}\\ \frac{\Delta\rho_r}{\rho_r} &= -4\frac{\Delta R}{R}\end{aligned}\right\} \quad \boxed{\frac{\Delta\rho_m}{\rho_m} = \frac{3}{4}\frac{\Delta\rho_r}{\rho_r}}$$

condición adiabática.

• radiation density: $\rho_r \propto T^4 \Rightarrow \Delta\rho_r \propto 4T^3\Delta T = 4\frac{\rho_r}{T}\Delta T \Rightarrow \boxed{\frac{\Delta\rho_r}{\rho_r} = 4\frac{\Delta T}{T}}$

▪ theoretical structure formation (see "LSS" lecture)

$$\boxed{\ddot{\delta}_m + 2H\dot{\delta}_m = 4\pi G\rho_m\delta_m} \quad \text{with } \delta_m = \frac{\Delta\rho_m}{\rho_m}$$

• solution: $\delta_{m,0} = \delta_{m,\mathrm{dec}}\, a$

• today (lower limit!): $\delta_{m,0} \geq 1$

• decoupling: $z_{dec} \approx 1100$

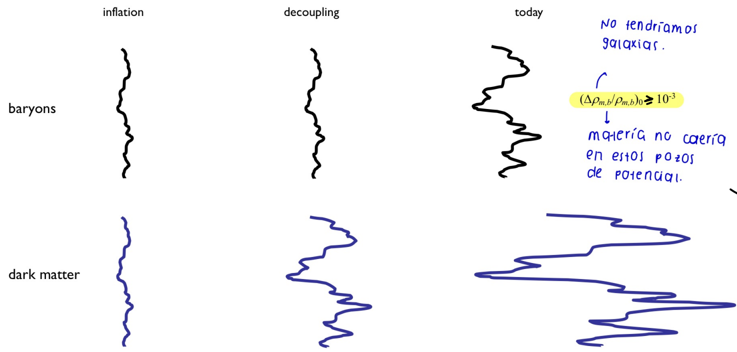
$\delta_{m,dec} \geq 10^{-3}$ (lower limit!)

• adiabatic perturbations: $\Delta\rho_m/\rho_m = (3/4)\, \Delta\rho_r/\rho_r$

• radiation density: $\Delta\rho_r/\rho_r = 4\,\Delta T/T$

$$\boxed{\frac{\Delta T}{T} = \frac{1}{3}\frac{\Delta\rho_m}{\rho_m}}$$

pero lo observado es 10^{-5}



No tendríamos galaxias.

$(\Delta\rho_{m,b}/\rho_{m,b})_0 \gtrsim 10^{-3}$

materia no caería en estos pozos de potencial.

→ lo real: $(\Delta\rho_{m,b}/\rho_{m,b})_0 \sim 1$

we require some matter that already formed structures before z_{dec}!

→ Necesitamos materia oscura

↓

que no interactúe con los fotones.

▪ quantifying fluctuations on a sphere

$$\boxed{\frac{\Delta T}{T}(\vartheta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{+l} a_{lm}Y_{lm}(\vartheta,\varphi)}$$

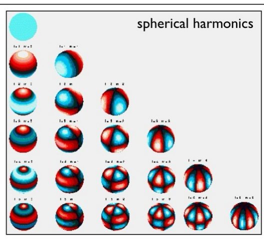
$Y_{lm}(\vartheta,\varphi)$: spherical harmonics

(complete orthonormal set of functions on the surface of a sphere)

$l=0$ → monopole (not measurable)

dipolo no lo consideramos. ← $l=1$ → dipole (Local Group motion)

$l>1$ → anisotropies we're after!



disminuimos las distancias entre 2 puntos que observamos.

→ + simétrico el sistema → + multipolos.

· Sistema muy simétrico a escalas grandes ⇒ debemos ir a escalas pequeñas para ver las ≠ de T°.

$$\frac{\Delta T}{T}(\vartheta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=-l}^{+l} a_{lm}Y_{lm}(\vartheta,\varphi)$$

ortonormales

isotropy ≙ rotational invariance

la media es 0.

$$\boxed{C_l = \frac{1}{2l+1}\sum_{m=-l}^{+l}|a_{lm}|^2}$$

→ Medimos la varianza.

C_l: power spectrum of temperature fluctuations

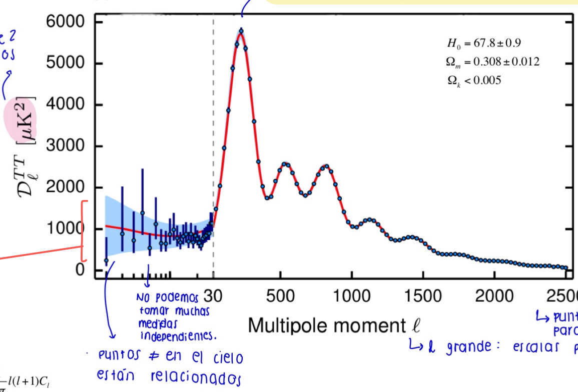
cosmic variance = multiplicidad ; cuantos modos indep. podemos observar → por cada ℓ tengo un m.

· Densidad es igual al background

▪ quantifying fluctuations – Planck 2015 (Ade et al., astro-ph/1502.02114)

peaks de la CMB: puntos con mayor ≠ de T°.

T° de 2 puntos



COSMIC VARIANCE

No podemos tomar muchas medidas independientes.

puntos ≠ en el cielo están relacionados

↳ puntos + cercanos para ℓ + grande.

↳ ℓ grande: escalas pequeñas

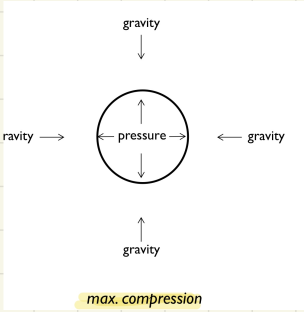
$D_l = \frac{1}{2\pi}l(l+1)C_l$

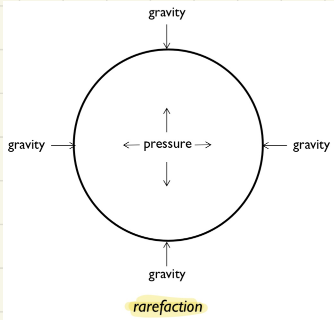
cosmic variance (mucho error xq para bajo ℓ tengo bajas m: menos medidas indep.).

Fotones llegan de la superficie del scattering.

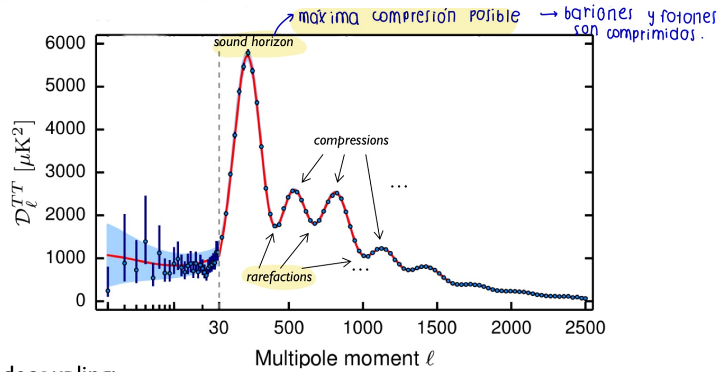
↳ Bao: oscilación acústica de bariones por la compresión y rarefacción de nuestro fluido con bariones y fotones.

· Radiaø explota ⇒ oscilaø acústica





- gravity vs. radiation pressure ➔ oscillations



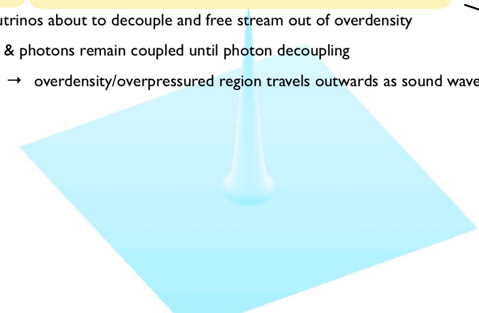
decoupling:

- oscillations are frozen
- photons are caught at extremes → translation into temperature fluctuations

▪ baryonic acoustic oscillations

- gravity vs. radiation pressure ➔ oscillations ➔ sound waves $c_s = \sqrt{\frac{\partial p}{\partial \rho}} \approx \frac{c}{\sqrt{3}}$
- overdensity in DM, neutrinos, gas & photons:
  - DM is decoupled and hence able to gravitationally collapse right away
  - neutrinos about to decouple and free stream out of overdensity
  - gas & photons remain coupled until photon decoupling
    - → overdensity/overpressured region travels outwards as sound wave

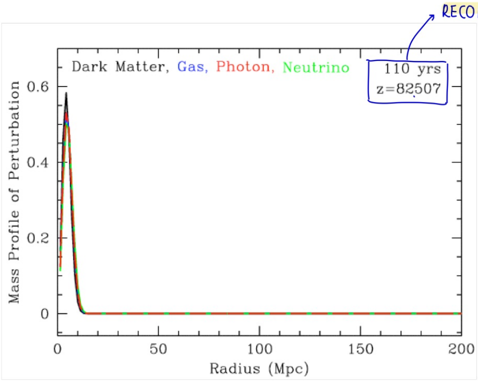
→ Materia oscura afecta los pozos de potencial.



▪ baryonic acoustic oscillations

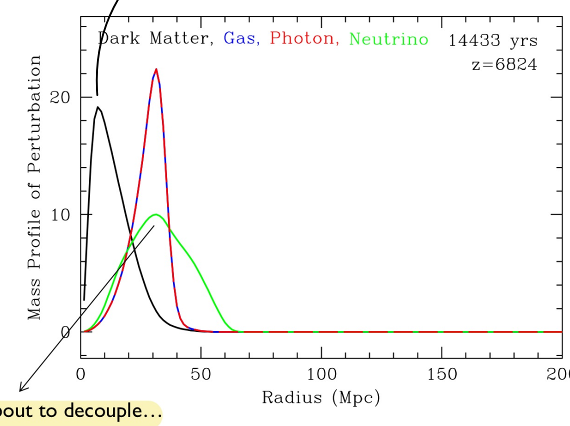
- gravity vs. radiation pressure ➔ oscillations ➔ sound waves $c_s = \sqrt{\frac{\partial p}{\partial \rho}} \approx \frac{c}{\sqrt{3}}$

RECORDAR!
desacoplo: $z = 1.100$
300.000 años

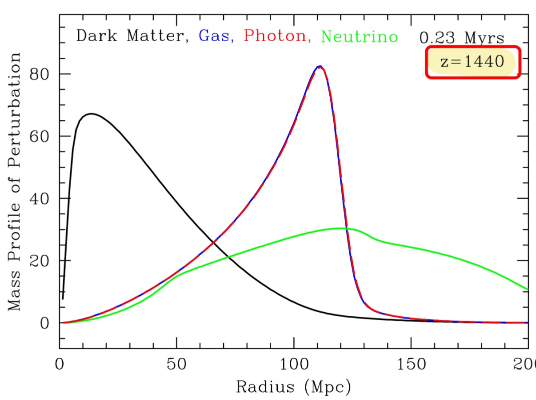


Eisenstein et al. (2007)

DM desacoplada



neutrinos are about to decouple…

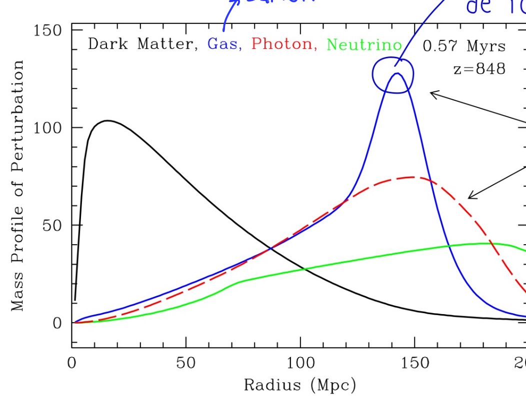


~ **recombination**

Eisenstein et al. (20…

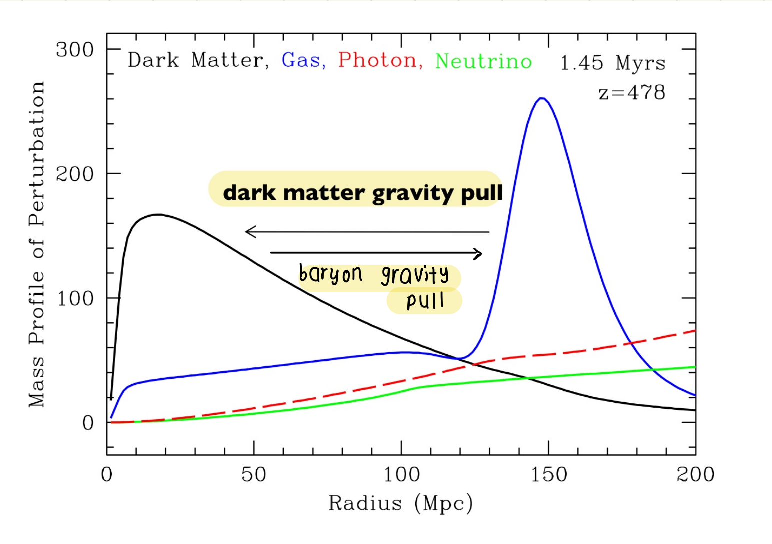
Bariones

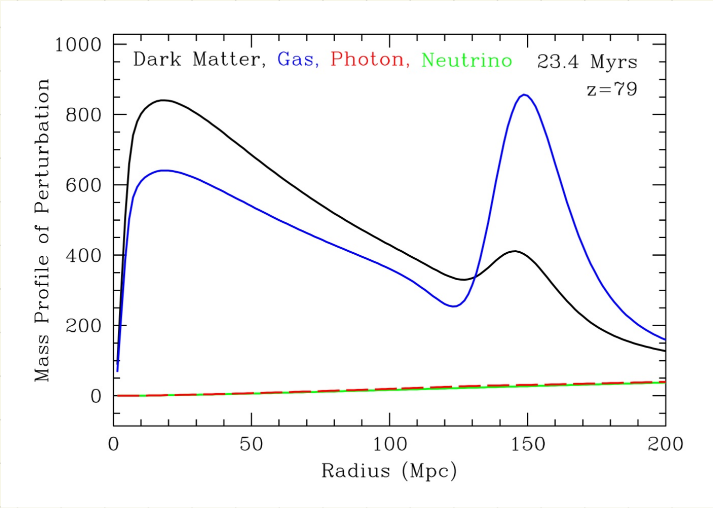
peak por la gravedad de la DM.



baryons and photons decoupled!

Eisenstein et al. (2007)





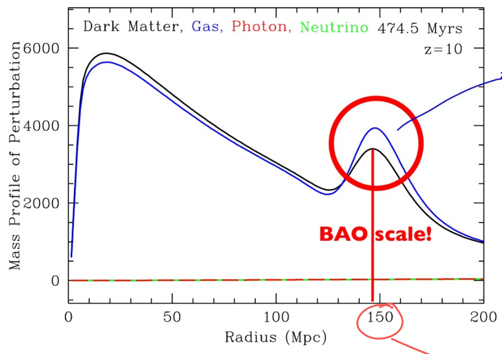
· Nos da la funØ de correlaØ de las galaxias:

▪ baryonic acoustic oscillations

• gravity vs. radiation pressure ➔ oscillations ➔ sound waves

· cuando fotones explotan hay una onda de choque
↓
empujan la materia → ahí se forman las galaxias.
↓
exceso de materia

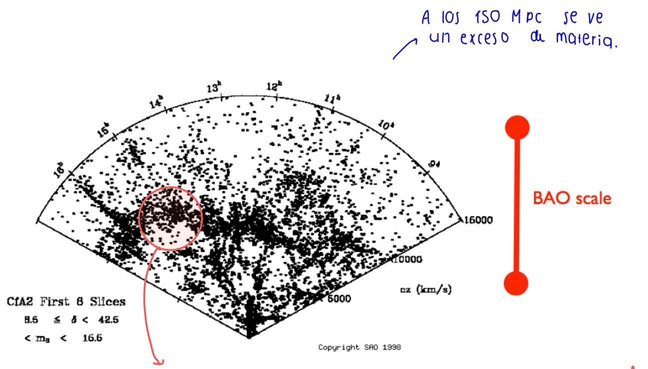
decae como r^{-3} la densidad.



→ Dist. comovente.

Eisenstein et al. (2007)

A los 150 Mpc se ve un exceso de materia.



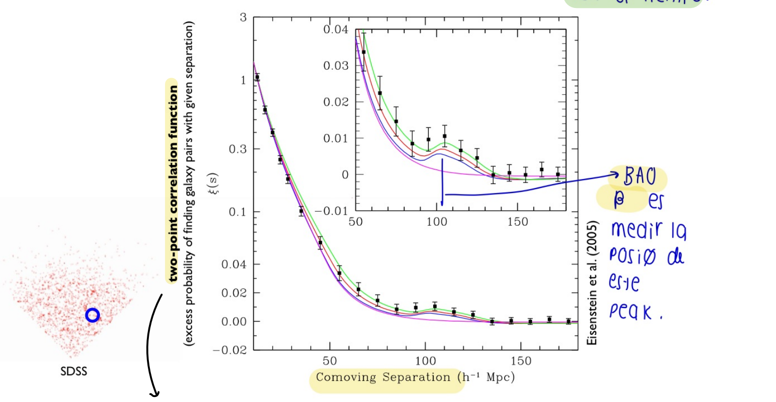
si hacemos un radio de 150 Mpc hay un exceso de materia.

lo que conocemos.
↑

▪ baryonic acoustic oscillations as a standard ruler

• discovered in SDSS survey in 2005

*lo medimos con la dist. angular pero lo cambiamos a comóvil para que no cambie con el tiempo.



→ BAO
Ø es medir la posiØ de este peak.

densidad

se utiliza BAO para medir angular diameter distance y H(z).

▪ intrinsic fluctuations – where do they come from?

→ + fluctuaø de densidad → + materia comprimida
↓
calentamos los fotones
↓
+ δT

$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta \rho_m}{\rho_m}$$ (adiabatic fluctuations)

▪ more detailed calculations:

- Sachs-Wolfe effect
- Doppler effect
- Silk damping

▪ **Sachs-Wolfe effect**

- variations in gravitational potential lead to temperature fluctuations

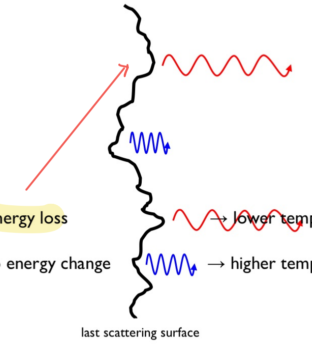
$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta \Phi}{c^2}$$

$$\Delta\theta \approx 10^{\circ}$$

dark matter over-density = potential well → energy loss → lower temperature

dark matter under-density = potential hill → no energy change → higher temperature

(+ time dilation!)

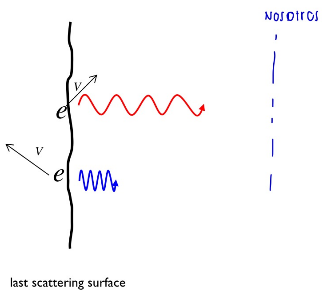


▪ **Doppler effect** → por la velocidad de los e^- en el fluido.

- last-scattering electrons have finite velocity

$$\frac{\delta T}{T} = -\frac{\vec{V}\cdot\vec{n}}{c}$$

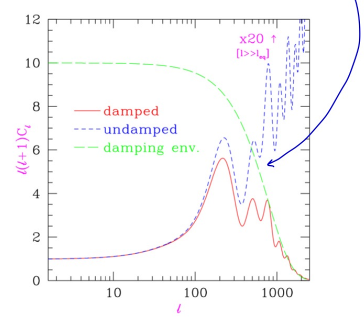
$$\Delta\theta \approx 1^{\circ}$$



▪ **Silk damping**

- photon diffusion from high to low-density regions → variaøes que pueden haber entre los fotones son pequeñas en escala menor
- electrons are dragged along via Compton interactions
- protons also follow due to Coulomb coupling to electrons

→ baryonic density fluctuations are damped! $(\Delta\theta << 1^{\circ})$



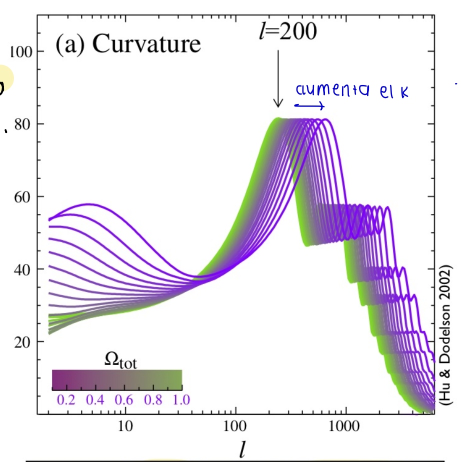
→ Fotones chocan con el gas y pierde la fluctuaø original.

▪ **sensitivity to cosmic parameters**

- curvature

· Posiø del peak del BAO tambien nos da la curvatura.

· Si aumenta más rápida# se ve lo mismo a escalas más pequeñas.

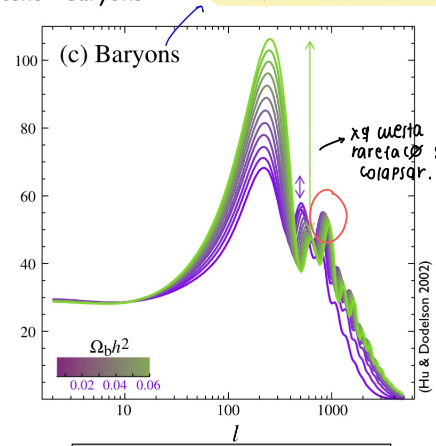


· fluctuaøes a ≠ escalas.

first peak at $l \sim 200$ => Universe is spatially flat!

- matter content – baryons

+ bariones, colapsa mucho

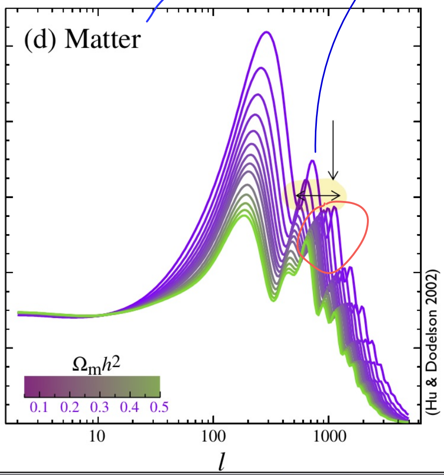


→ xq cuesta más la rarefacø y rápida# quiere colapsar.

ratio of 1[st] to 2[nd] peak => $\Omega_b h^2 \sim 0.02$

• matter content – all matter

oscura



third peak separates dark matter from baryons => $\Omega_m h^2 \sim 0.3$

· Dark matter va a modificar el espectro de potencia → indicador del 2° y 3° peak.

↓

xq va a modificar el potencial gravitacional

→ Anisotropías dependen del baryon-photon ratio (η)

→ DM se desacopló bastante antes de las anisotropías.

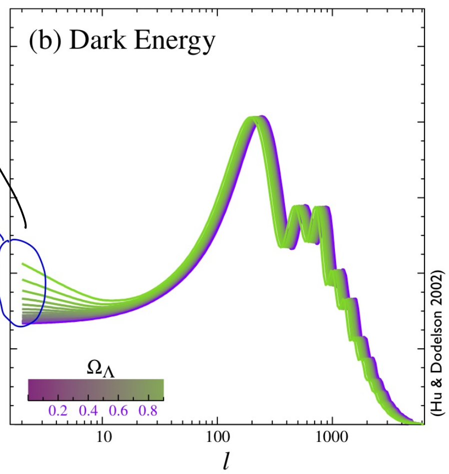
• matter content – dark energy

→ Lo contrario a la curvatura.

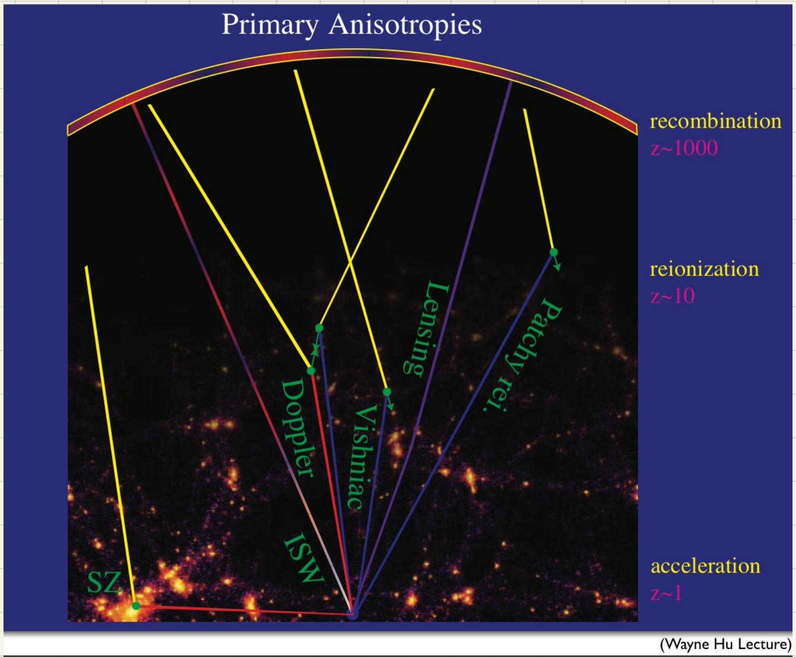
ISW effect

multipolos pequeños

COSMIC VARIANCE (se pueden hacer pocas mediciones)



$\boxed{\text{CMB fluctuations}}$ → secundarias.



(Wayne Hu Lecture)

Importantes a ≠ escalas.

▪ secondary fluctuations – where do they come from?

- integrated Sachs-Wolfe effect
- Rees-Sciama effect
- Sunyaev-Zeldovich effect (thermal & kinematic)
- Ostriker-Vishniac effect
- patchy reionisation of the Universe
- gravitational lensing