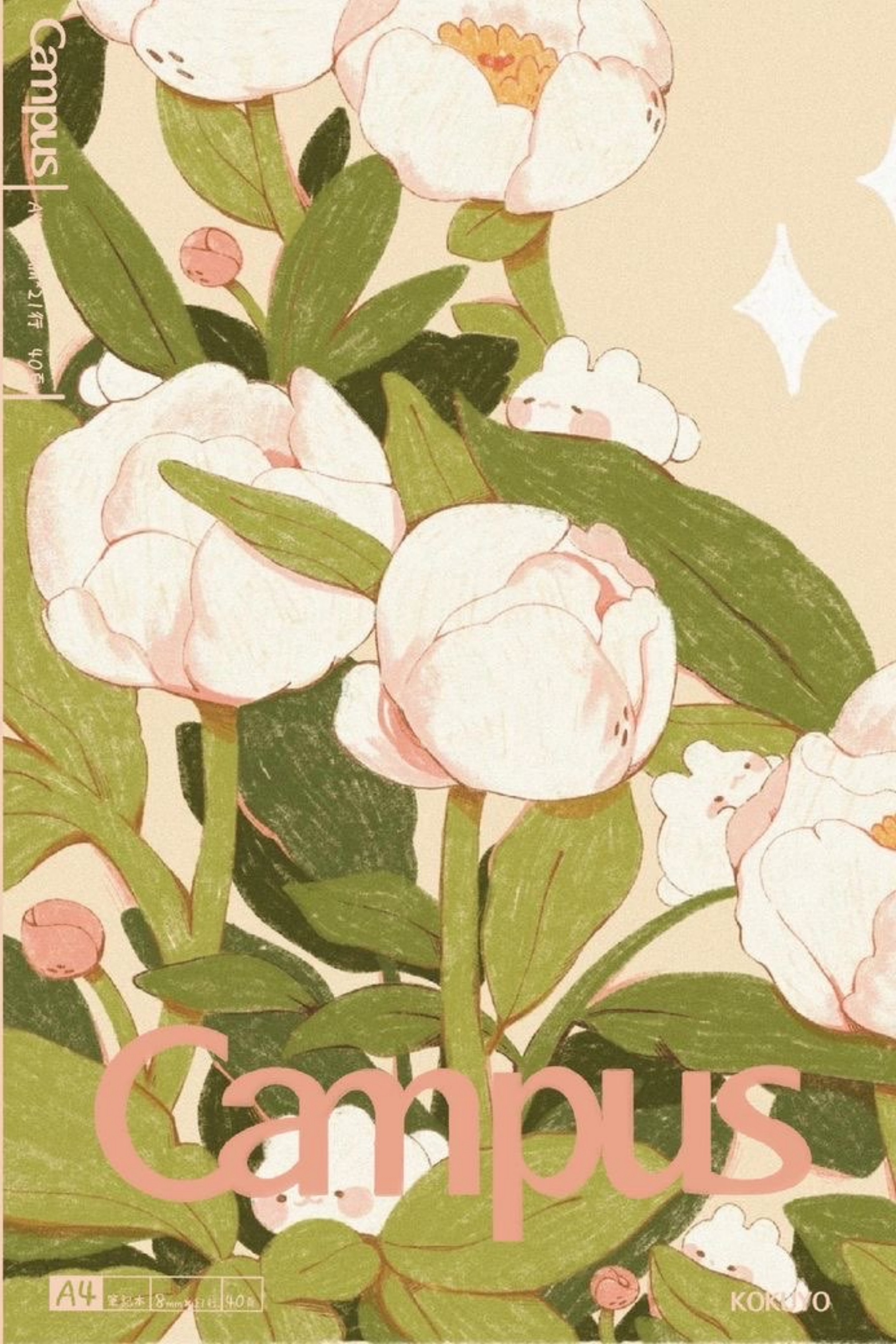

Campus
Campus
A4

Cosmology : Theory and observations

Info :

Part 1 :

- General discussion about cosmology
- Dynamics in cosmology.

Part 2 :

- CMB
- large scale structure.

Part 3 :

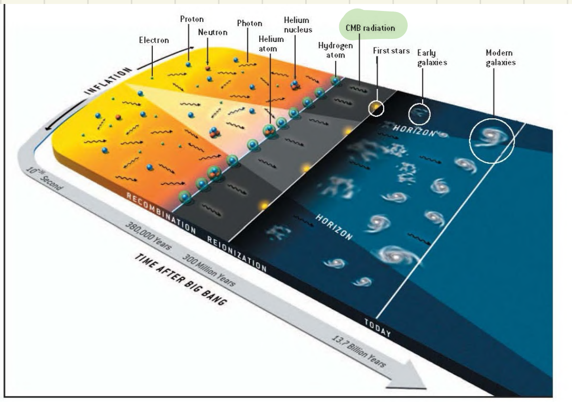
- observation cosmology

- CAMB → camb.info

* Pocas tareas : cálculos largos.

* Nombrar la nomenclatura en las tareas ya que son distintas según el libro.

cosmology is ...

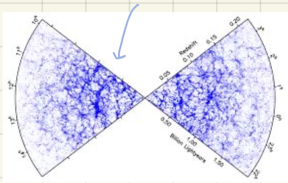


- Galaxias como puntos, se estudia el universo a gran escala.

- cosmos → orden.
- cosmogonía : orden ↔ caos . Dar una descripción del caos.
- caos : espacio que se abre.
  - └ lienzo influenciado por la cantidad de materia. Interactúa con la materia.
- vacío tiene una energía

- para universo tardío, galaxias como puntos.
- survey : distribución de galaxias
- galaxias quieren estar juntas. Toman patrones particulares.
- Gravedad responsable de las estructuras.

hay más al centro por la capacidad de recibir fotones a galaxias más lejanas.



- spectroscópico : espectro de emisión completa.
- photométrica : espectro de emisión de la galaxia medido con filtros particulares.
  - → galaxias azules (jóvenes)
  - → galaxias rojas (viejas)

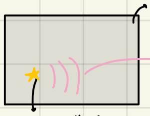
Hidrógeno ionizado.

→ Más lejos . gas ionizado (mucha energía de la CMB) . Fotones golpean el gas.

Neutral H ←→ Ionized H

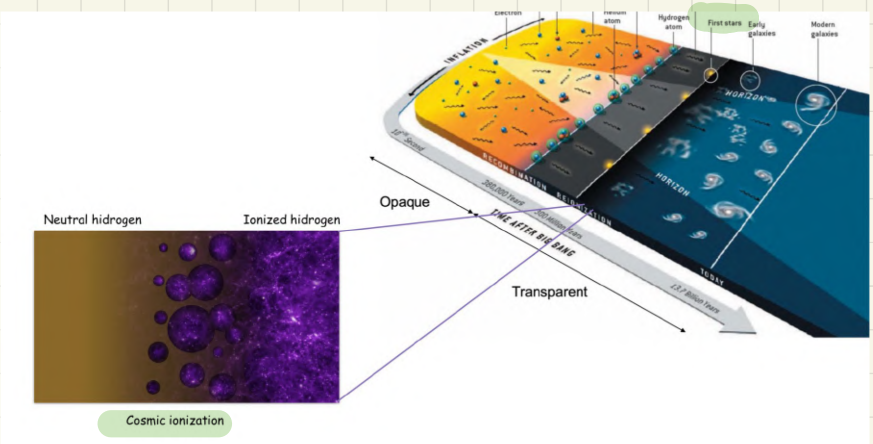
Cosmic ionization.

gas de H

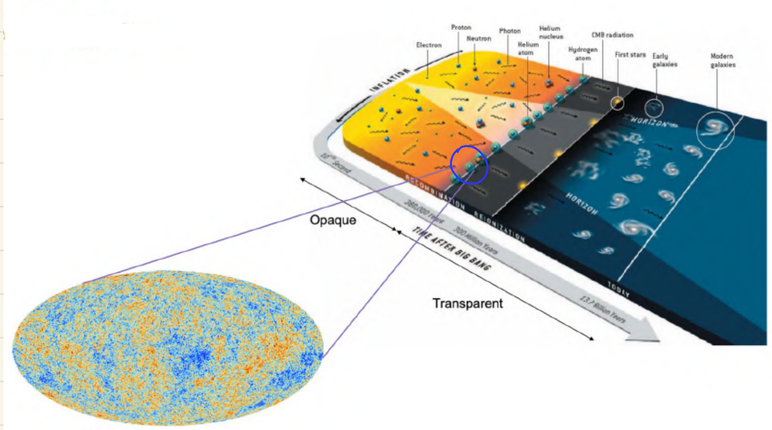


calientan el gas.

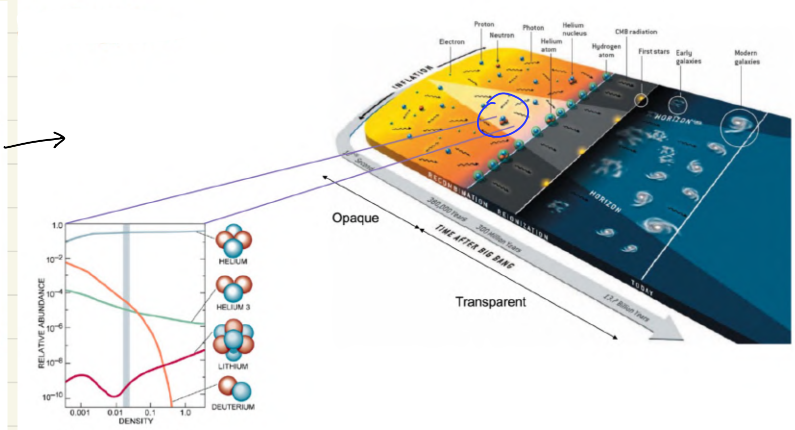
estrellas



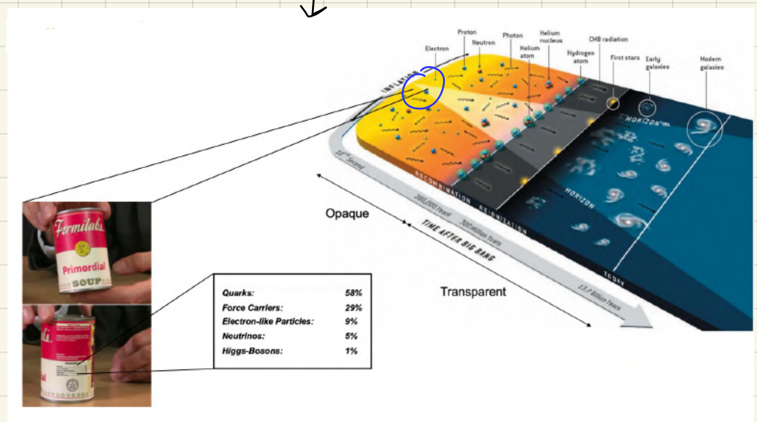
- Primeras estrellas ionizan el gas de H.



Cosmic Microwave Radiation



Big Bang Neuclosynthetis



Event	time t
Inflation	10^{-34} s (?)
Baryogenesis	?
EW phase transition	20 ps
QCD phase transition	20 μs
Dark matter freeze-out	?
Neutrino decoupling	1 s
Electron-positron annihilation	6 s
Big Bang nucleosynthesis	3 min
Matter-radiation equality	60 kyr
Recombination	260–380 kyr
Photon decoupling	380 kyr
Reionization	100–400 Myr
Dark energy-matter equality	9 Gyr
Present	13.8 Gyr

→ change from normal matter to quark-gluon plasma (QGP)

Inflation

Baryogenesis

Thermal History

Big Bang Nucleosynthesis

Cosmic Microwave Background Radiation

Cosmic Dawn
Large-Scale Structure
Observational Cosmology
Gravitational Waves
Open Problems

- No se puede ver más allá del CMB.

- Universo observable: Todo lo que se puede observar.
- Fotones relacionados con la densidad con la que se forman estructuras.
- Inflación cósmica → Distintos modelos. Se verá solo uno. → de energía oscura y gravedad modificada.

Principles:

- ppio. de equivalencia débil: $m_a = m_g$
- " " " fuerte: gravedad = aceleración.
- cosmological principle: homogeneo e isotrópico a gran escala → depende de cuánto es grande. (teo. central del límite)
- principio de relatividad: same equations everywhere.

Einstein eq.

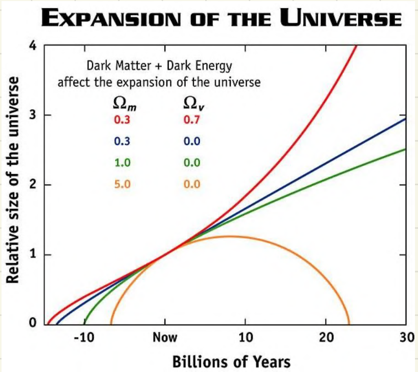
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \longrightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}.$$

Parámetro de Hubble: Tener en cuenta ≠ efectos:

- $H_0 \sim 70$ km/s/Mpc. → Velocidades peculiares: mov. de una galaxia dada por otra.
  H_0: present day expansion rate.

- ≠ colors ↔ ≠ fates (destino del universo)

- The "fireball" of the Big Bang: CMB.



- Más lejos estamos, más rápido se ve que se expande pero todos los puntos se expanden de la misma manera.

* radiaø : todas las componentes relativistas.

Aumenta temp.

· QED /QCD → partículas elementales.

· Dark matter freeze-out : Materia acoplada (muy caliente) → hay que esperar para que se junten $\bar{e}$ con protones.

· Matter-radiation equality: Estas 2 componentes son comparables entre sí.
↳ antes de esto domina la radiaø.

· Photon decoupling : e^- calientes emiten radiación. → CMB.
(fotones se desacoplan)
↓
fotón luego es absorbido por otro e^-.
(universo opaco)

35.000 años · universo opaco

· Early time cosmology : Antes de la CMB.

* EB limit : límite del espectro electromagnético

The fireball of the Big Bang :

Penzias & Wilson worked in 1965 at the Bell-Labs → investigating the radio emission from the Milky Way.

instrumentos de la guerra.

↓
measured a strange noise.
↓
CMB radiation → le dieron una explicaø física:
→ Dicke
→ Peebles
→ Roll
→ Wilkinson.

Expanding Universe → principle of redshift.

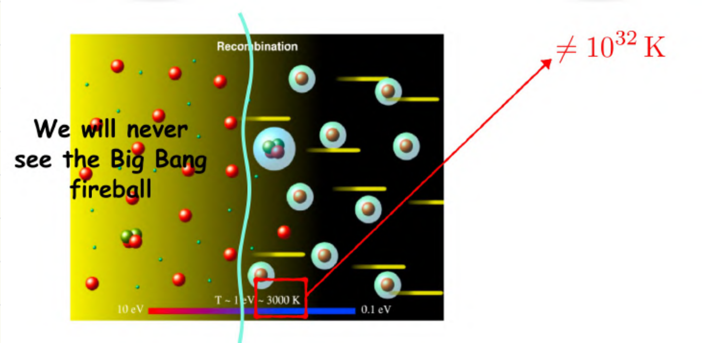
· Disminuye la energía → Disminuye la T°. → $\boxed{E = \frac{hc}{\lambda}}$

$T \approx 1.4 \cdot 10^{32}$ K ⇝ $T = 2.76$ K → We will never see the Big Bang fireball.
↳ barrera a los 3000K ≠ 10^{32} K

Big bang ← ; → Expansion to cool things

plasma primordial
↓
Fotones no pueden viajar libremente.
↳ last scattering
3000 K → $\boxed{T(a) = \frac{T_0}{a}}$

* En el desacople de los neutrinos :
$T_{CMB} = 10^{10}$ K



Cosmological Principle(s)

1. It assumes conventional physics, including general relativity theory and quantum physics.
    1. This yields a successful account of the origin of the light element.

2. In the large-scale average the universe is close to homogeneous, and has expanded in a near homogeneous way from a denser hotter state.

3. Structure formation on the scale of galaxies and larger was a result of the gravitational growth of small primeval departures from homogeneity, as described by general relativity in linear perturbation theory.

4. There's no preferred direction everywhere we look. → podemos simplificar el caracter vectorial

5. Only 4% of total matter is of a known kind, the 96% is unknown: the adiabatic cold dark matter (ΛCDM) model gives a fairly definite and strikingly successful prescription from the initial conditions to the gravitational instability.

All this growth and all this expansion depends on what the Universe has

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

* escalas grandes ~ 100 Mpc.

Todo en análisis de fourier
↳ vector de onda k y coord. r.

ΛCDM
Λ: energía oscura 4%
CDM: 96% — partículas pesadas, no tienen energía.

2006 - Smoot & Mather → discovery of the anisotropies of the CMB.

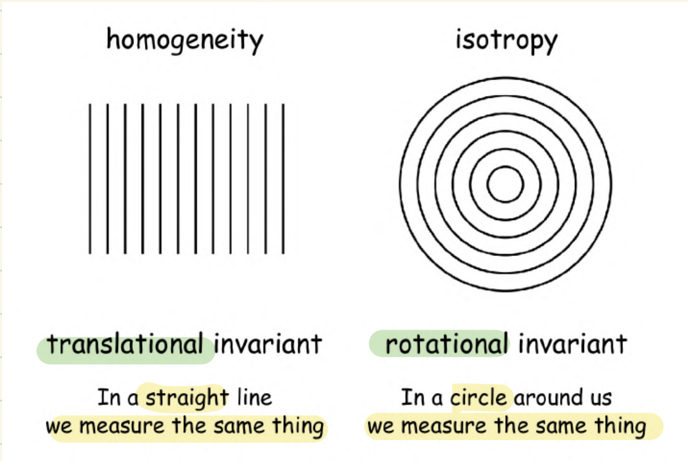
diferencias de T° de 10^{-5} K

· Pequeñas fluctuaciones primordiales.

+

Teoría de la gravedad que permita formar estructuras.

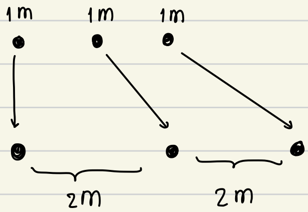
* Modificar la gravedad. → teorías.



with perturbation theory.

Análisis de background: Tomar el promedio.

· under homo. and isotropy, every point behaves the same.



asociada a la energía del vacío / componente "nueva de materia".

cosmological constant. (no cambia el caracter invariante de las ecuaciones).

Eq. Einstein:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

This part will depend on the metric $g_{\mu\nu}$

→ choose some form.

will depend on matter:

→ how much → con la densidad

→ how it "behaves". → presión de radiación.

· reemplaza el potencial gravitacional.
· coordenada localmente inercial.

sist. sin aceleración (mov. lineal uniforme)

The metric: $g_{\mu\nu}$
· De rango 2 (4x4 matrix).
· $g_{\mu\nu} = g_{\nu\mu}$
· Not degenerate, ie. $\det(g_{\mu\nu}) = |g_{\mu\nu}| = g \neq 0$

· Inverse $g^{\mu\nu} g_{\nu\sigma} = \delta^{\mu}{}_{\sigma}$.

Metric provides a notion of locally inertial frames → sense of "no rotation"

Línea de elemento o métrica. $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

↓

· curvature of the manifold.

$$\frac{\partial A_\mu}{\partial x^\nu} = \lim_{dx \to 0} \frac{A_\mu(x+dx) - A_\mu(x)}{dx^\nu}$$

Proper parallel transport: (derivada)

$$\frac{\partial A_\mu}{\partial x^\nu} = \lim_{dx\to 0} \frac{A_\mu(x+dx) - A_\mu(x) - \delta A_\mu}{dx^\nu} = \lim_{dx\to 0} \frac{A_\mu(x+dx) - A_\mu(x)}{dx^\nu} - \Gamma^\alpha{}_{\mu\beta} A_\alpha \frac{dx^\beta}{dx^\nu} =$$
$$= \frac{\partial A_\mu}{\partial x^\nu} - \Gamma^\alpha{}_{\mu\nu} A_\alpha$$

Covariant derivative $A_{\mu;\nu} = A_{\mu,\nu} - \Gamma^\alpha{}_{\mu\nu} A_\alpha$

Covariant derivative of a covariant vector $A_{\mu;\nu} = A_{\mu,\nu} - \Gamma^\alpha{}_{\mu\nu} A_\alpha$

There is also a covariant derivative of a controvariant vector $B^\mu{}_{;\nu} = B^\mu{}_{,\nu} + \Gamma^\mu{}_{\alpha\nu} B^\alpha$ → ortogonal al plano tangente.

the covariant derivative differs from the ordinary derivative by as many extra terms as there are indices. For example

$$\begin{aligned} B^{\mu\nu}{}_{;\nu} &= B^{\mu\nu}{}_{,\nu} + \Gamma^\mu{}_{\alpha\nu} B^{\alpha\nu} + \Gamma^\nu{}_{\mu\beta} B^{\mu\beta}, \\ A^{\mu\nu}{}_{\sigma;\nu} &= A^{\mu\nu}{}_{\sigma,\nu} + \Gamma^\mu{}_{\alpha\beta} A^{\alpha\nu}{}_\sigma + \Gamma^\nu{}_{\alpha\beta} A^{\mu\beta}{}_\sigma - \Gamma^\alpha{}_{\sigma\beta} A^{\mu\nu}{}_\alpha \end{aligned}$$

The covariant derivative follows the Leibniz chain formula $(A^\mu B^\nu)_{;\beta} = A^\mu{}_{;\beta} B^\nu + A^\mu B^\nu{}_{;\beta}$

But what is the Γ symbol?

The quantity $(A^\mu B^\nu g_{\mu\nu})_{,\beta} = 0$ is a scalar and it's derivatives is zero.
The same is for it's covariant derivative. This will give: $g_{\mu\nu;\beta} = 0$
From this, we can construct cyclically the Christoffel symbol:

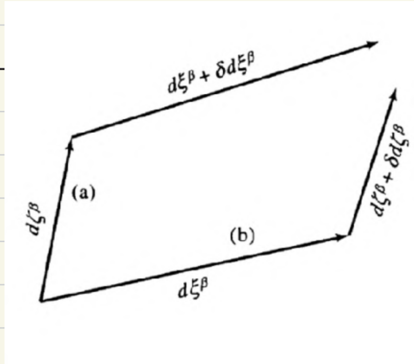
$$\Gamma^\alpha{}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(\frac{\partial g_{\beta\mu}}{\partial x^\nu} + \frac{\partial g_{\nu\beta}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta} \right) = \frac{1}{2} g^{\alpha\beta} \left(g_{\beta\mu,\nu} + g_{\nu\beta,\mu} - g_{\mu\nu,\beta} \right)$$

It is easy to verify that the covariant derivative of the inverse of the metric tensor is also identically zero $g^{\mu\nu}{}_{;\alpha} = 0$

It follows that the operation of raising or lowering of indices commutes with covariant differentiation.

$$(g^{\mu\nu} A_\nu)_{;\alpha} = g^{\mu\nu} A_{\nu;\alpha}.$$

parallel transport.



paths 1 and 2 connecting P and P'. Since it is curved space, the difference is not zero

$$\Delta A^\alpha = (-\Gamma^\alpha{}_{\beta\mu,\nu} + \Gamma^\alpha{}_{\beta\nu,\mu} + \Gamma^\alpha{}_{\sigma\mu}\Gamma^\sigma{}_{\beta\nu} - \Gamma^\alpha{}_{\sigma\nu}\Gamma^\sigma{}_{\beta\mu})A^\beta d\xi^\nu d\zeta^\mu$$

This shows that the result of parallel transport is path dependent

$$\Delta A^\alpha = R^\alpha{}_{\beta\mu\nu}A^\beta d\xi^\nu dx^\mu$$

Riemann tensor, which has 4x4x4x4 component (only 20 are independent)

$$R^\alpha{}_{\beta\mu\nu} = -\Gamma^\alpha{}_{\beta\mu,\nu} + \Gamma^\alpha{}_{\beta\nu,\mu} + \Gamma^\alpha{}_{\sigma\mu}\Gamma^\sigma{}_{\beta\nu} - \Gamma^\alpha{}_{\sigma\nu}\Gamma^\sigma{}_{\beta\mu}$$

└→ describe la curvatura < gravedad (derivada del símbolo de Christoffel).
efectos no lineales

* la doble derivada de la métrica → curvatura
↳ tiene sentido con el tensor de Riemann.

· la doble derivada de la métrica corresponde a una "aceleraø"
"desplaza@"
└→ gravedad
└ deriv. cov. del símbolo de Christoffel.

Riemann tensor $R^\alpha{}_{\beta\mu\nu} = -\Gamma^\alpha{}_{\beta\mu,\nu} + \Gamma^\alpha{}_{\beta\nu,\mu} + \Gamma^\alpha{}_{\sigma\mu}\Gamma^\sigma{}_{\beta\nu} - \Gamma^\alpha{}_{\sigma\nu}\Gamma^\sigma{}_{\beta\mu}$

We can also lower indices $\boxed{R_{\alpha\beta\mu\nu} = g_{\alpha\sigma}R^\sigma{}_{\beta\mu\nu}}$

Important identies

$$R_{\alpha\beta\mu\nu} = -R_{\beta\alpha\mu\nu}$$
$$R_{\alpha\beta\mu\nu} = -R_{\alpha\beta\nu\mu}$$
$$R_{\alpha\beta\mu\nu} = R_{\mu\nu\alpha\beta}$$
$$R_{\alpha\beta\mu\nu} + R_{\alpha\mu\nu\beta} + R_{\alpha\nu\beta\mu} = 0$$

We can have an extra useful identity: $R^\alpha{}_{\beta\mu\nu;\sigma} + R^\alpha{}_{\beta\nu\sigma;\mu} + R^\alpha{}_{\beta\sigma\mu;\nu} = 0$ ⟶ si se mueve en todas direcciones, volverá a la misma dirección.

if we introduce geodesic coordinates then Christoffel symbols are zero, so the Riemann tensor reads:

$R^\alpha{}_{\beta\mu\nu} = -\Gamma^\alpha{}_{\beta\mu,\nu} + \Gamma^\alpha{}_{\beta\nu,\mu}$ Which reduces to $-\Gamma^\alpha{}_{\beta\mu,\nu,\sigma} + \Gamma^\alpha{}_{\beta\nu,\mu,\sigma} - \Gamma^\alpha{}_{\beta\nu,\sigma,\nu} + \Gamma^\alpha{}_{\beta\sigma,\nu,\mu} - \Gamma^\alpha{}_{\beta\sigma,\mu,\nu} + \Gamma^\alpha{}_{\beta\mu,\sigma,\nu} = 0$

Which is trivial

By contracting the first and last indices of $R^\alpha{}_{\beta\mu\nu}$

We have the Ricci tensor $R_{\beta\mu} = R^\alpha{}_{\beta\mu\alpha}$ which is symmetric

Contracting again the two indices we have the Ricci scalar $R = R^\beta{}_\beta = R^{\alpha\beta}{}_{\beta\alpha}$

From the Bianchi's identity we find that this composition is $\left(R^\mu{}_\nu - \frac{1}{2}\delta^\mu_\nu R\right)_{;\mu} = 0$

What does the Riemann tensor tell us mathematically?

The commutative rule fails

It can be shown that: $A_{\beta;\mu;\nu} - A_{\beta;\nu;\mu} = R^\alpha{}_{\beta\mu\nu}A_\alpha$

Formal: starting with an action and derive the corresponding equations of motion.

Newton $\nabla^2\Phi = 4\pi G\rho \longrightarrow T_{\mu\nu}$ with $T_{\mu\nu;\,\mu} = 0$

materia se conserva

D' alambertiano:

deriv. cov. de $g_{\mu\nu} = 0$

$$[\nabla^2 g]_{\mu\nu} \rightarrow \boxed{\Box g_{\mu\nu}} \rightarrow R^{\rho}{}_{\sigma\mu\nu} \rightarrow R_{\mu\nu}$$

no es tensor

$$R_{\mu\nu} = F(\Gamma, \Gamma_{,\mu}) = F(g, g_{,\mu}, g_{,\mu\nu})$$

$$R_{\rho\sigma\mu\nu;\lambda} + R_{\sigma\lambda\mu\nu;\rho} + R_{\lambda\rho\sigma\mu\nu;\sigma} = 0$$

$$0 = g^{\nu\sigma} g^{\mu\lambda} \left(R_{\rho\sigma\mu\nu;\lambda} + R_{\sigma\lambda\mu\nu;\rho} + R_{\lambda\rho\sigma\mu\nu;\sigma}\right) = \nabla^{\mu} R_{\rho\mu} - \nabla_{\rho} R + \nabla^{\nu} R_{\rho\nu} \quad \Longrightarrow \quad \left[R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right]_{;\mu} = 0$$

xq el universo es homogéneo e isotrópico.

Since we want to describe the average expanding dynamics of space-time of a homogenous and isotropic Universe

General metric: $$\mathrm{d}s^2 = g_{00}\mathrm{d}t^2 + 2g_{0i}\mathrm{d}x^i\mathrm{d}t - \sigma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

isotropy ⇒ $g_{0i} = 0$

recalcular el tiempo.

synchronization ⇒ $g_{00} = -1$

spatial part to expand uniformly ⇒

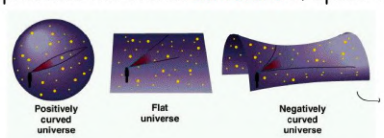
Friedman-Lemaitre-Robertson-Walker metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \underbrace{a^2(t)}\left[\frac{\mathrm{d}r^2}{1 - kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right]$$

factor de escala (espacio que se expande)

universo homogéneo e isótropo en expansión.

k represents the overall curvature of space-time



hiperbólico.

· Asumimos $k \sim 0$ (casi plano)

slide 03 cosmo:

Cosmology: metric

Metric in general: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ where $x^\mu = (x,\, y,\, z) = \text{vector}$

In 3D Euclidean space: $ds^2 = dx^2 + dy^2 + dz^2 \implies g_{\mu\nu} = 1$

In curved space-time: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ where $x^\mu = (ct,\, x^i)$ 4-vector

$$g_{\mu\nu} = \begin{pmatrix} \boxed{-1} & 0 & 0 & 0 \\ 0 & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ 0 & \gamma_{21} & \gamma_{22} & \gamma_{23} \\ 0 & \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix} = \text{metric tensor}$$

Any non-trivial g_{00} component can be absorbed redefining time $dt' = \sqrt{g_{00}}\, dt$

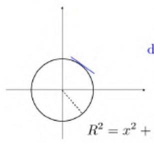
$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ 0 & \gamma_{21} & \gamma_{22} & \gamma_{23} \\ 0 & \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix} = \text{metric tensor}$$

goi = 0 -> if time depended on space, measurement of time could distinguish one place from another...

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \gamma_{11} & \gamma_{12} & \gamma_{13} \\ 0 & \gamma_{21} & \gamma_{22} & \gamma_{23} \\ 0 & \gamma_{31} & \gamma_{32} & \gamma_{33} \end{pmatrix} = \text{metric tensor}$$

It should describe a homogeneous and isotropic 3D space, i.e. maximal symmetry for the 4D hypersurface described by the vector x

1D space = hypersurface in 2D space



$dl^2 = dx^2 + dy^2$ ➡ 1D distance on 2D hypersurface

$R^2 = x^2 + y^2$ ➡ 2D hypersurface

We would like to get rid of the 2nd dimension (i.e. y) as we live in 1D space (i.e. x)

$dl^2 = dx^2 + dy^2$

$R^2 = x^2 + y^2 \xrightarrow{\text{derivative}} 0 = x dx + y dy \qquad dy = -x dx / y$

$R^2 = x^2 + y^2 \xrightarrow{\text{rearrange}} y^2 = R^2 - x^2$

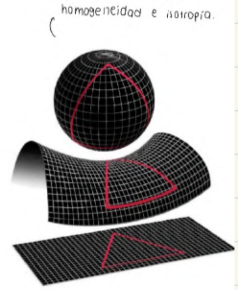
$$dl^2 = dx^2 + \underbrace{\frac{x^2 dx^2}{R^2 - x^2}}_{dy^2 \text{ a través de las ecs. de la circunferencia}} \quad (R \to +\infty \implies dl = dx)$$

no hay curvatura en $R \to \infty$ (euclídeos)

$$\gamma_{11} = 1 + \frac{x^2}{R^2 - x^2}$$

3D space = hypersurface in 4D space

bajo la asumpción de homogeneidad e isotropía



Spherical 3D space

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + dz^2$$
$$R^2 = x_1^2 + x_2^2 + x_3^2 + z^2$$

→ isotropía, no hay términos mezclados ($x_1 x_2 \ldots$).

Hyperbolic 3D space

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + dz^2$$
$$-R^2 = x_1^2 + x_2^2 + x_3^2 + z^2$$

↳ única diferencia

Euclidean 3D space

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2$$

→ desaparece una coordenada

These are the only possible cases for homogenous and isotropic metrics!

Spherical 3D space

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + dz^2$$
$$\boxed{R^2 = x_1^2 + x_2^2 + x_3^2 + z^2}$$

↳ es la que usamos para definirla en función de las otras dimensiones (nuestro y).

$\gamma_{ij} = ?$

differentiate

$$0 = x_1 dx_1 + x_2 dx_2 + x_3 dx_3 + z dz$$

combine

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + \underbrace{\frac{(x_1 dx_1 + x_2 dx_2 + x_3)^2}{R^2 - (x_1^2 + x_2^2 + x_3^2)}}_{\text{depende de las demás.}}$$

Spherical 3D space

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + \frac{(x_1 dx_1 + x_2 dx_2 + x_3)^2}{R^2 - (x_1^2 + x_2^2 + x_3^2)}$$

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \sin\theta \cos\phi \\ r \sin\theta \sin\phi \\ r \cos\theta \end{pmatrix}$$

$$dx_1^2 + dx_2^2 + dx_3^2 = g_r dr^2 + g_\theta d\theta^2 + g_\phi d\phi^2$$

$$g_r = \left|\frac{\partial \vec{r}}{\partial r}\right| = 1, \quad g_\theta = \left|\frac{\partial \vec{r}}{\partial \theta}\right| = r, \quad g_\phi = \left|\frac{\partial \vec{r}}{\partial \phi}\right| = r \sin\theta$$

$$dl^2 = dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + \frac{r^2 dr^2}{R^2 - r^2}$$

$$dl^2 = \underbrace{\boxed{\left(\frac{R^2}{R^2 - r^2}\right)}}_{\text{se escala el dr (variar la componente radial)}} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

General 3D space

$$dl^2 = \left(\frac{R^2}{R^2 - r^2}\right) dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Using rescaled (comoving) coordinate

$$r = Rx \qquad dr = R\, dx$$

Hiper esfera

$$dl^2 = R^2 \left[\left(\frac{1}{1 - x^2}\right) dx^2 + x^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

en el libro "gravitation" se calcula este paso.

$$dl^2 = R^2 \left[\left(\frac{1}{1 - Kx^2}\right) dx^2 + x^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

$K = +1$: spherical
$K = 0$: Euclidean
$K = -1$: hyperbolic

We add time

R varía la distancia entre 2 puntos.

$$ds^2 = -dt^2 + R^2 \left[\left(\frac{1}{1 - Kx^2}\right) dx^2 + x^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

Nuestro a.

We arrive to the FLRW metric with R(t) -> a(t)

diciendo que el universo se expande

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - Kr^2} + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$$

comp. angular no importa (homogéneo e isótropo)

Compact version (as before)

$$x^\mu = (ct,\, x^i)$$
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

where

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & \frac{a^2}{1 - Kr^2} & 0 & 0 \\ 0 & 0 & a^2 r^2 & 0 \\ 0 & 0 & 0 & a^2 r^2 \sin^2\theta \end{pmatrix}$$

nos queda

$$0 = -c^2 dt^2 + a^2(t) \frac{dr^2}{1 - \cancel{Kr^2}} \implies c\, dt = a(t)\, dr$$

And with this we can do all the calculations:

Redshift
Geodesic
Friedman equations
Distances

From the FLRW metric, if we assign the coordinates r, θ, φ at a given time t_0, the function a(t) acts as an overall factor in the expansion or contraction.

nos movemos en el elemento de línea de cómo se mueve la luz.

The physical distance measured along a null geodesic ds=0 (i.e. along a light beam) is

$$D = c\, dt = a(t)\, dr \qquad \dot{D} = \dot{a}\, dr = H\, D \qquad H = \frac{\dot{a}}{a}$$

Hubble law

↳ espacio tiempo que se expande.

Hubble law applies to any system that expands (or contracts) in a homogeneous and isotropic way

comoving distance

Distance

$$r = \int \frac{dr'}{\sqrt{1 - k r'^2}}$$

→ distancia comovente: se mueve con la expansión

are fixed on space and time and "move" with it. These are therefore called comoving distances.

The physical distances D = a(t)r vary instead with the expansion.

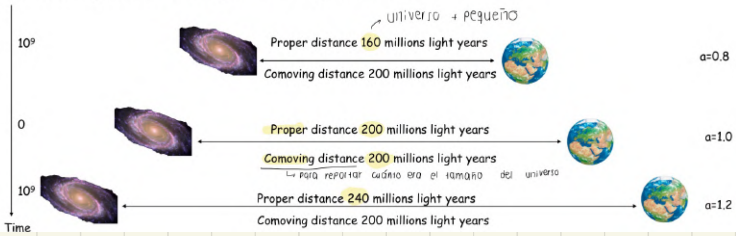
The first distance

distancia cambia entre galaxias según cuanto se expande el universo entre ellas.

The physical (proper) distance $\lambda = a(t)\underbrace{\lambda_0}_{\text{distancia ahora.}}$ → 0 en el pasado y 1 ahora

how it was in the past — what we measure today

For convenience, we often define the present distances such that λ_0 is what we measure today.
The comoving distance (moving with the expansion) is fixed.



Consider a wave source at rest. The interval between two crests is $\lambda_0 = c\,\mathrm{d}t$

If in the same dt the source moves away from the observer with velocity –v, the interval between two crests stretches by the distance traveled by the source, that is D = v dt

$\lambda_e = c\,\mathrm{d}t + \underbrace{v}_{\text{se aleja con esta velocidad el espacio}}\mathrm{d}t$ — relative difference $\dfrac{\mathrm{d}\lambda}{\lambda} = \dfrac{\lambda_e - \lambda_0}{\lambda_0} = \dfrac{v}{c}$ efecto Doppler.

Moving according to Hubble $\dfrac{\mathrm{d}\lambda}{\lambda} = \dfrac{v}{c} = \dfrac{HD}{c} = -H\mathrm{d}t = -\dfrac{\mathrm{d}a}{a}$ adimensional. $\dot{D} = v$. * z y a son adimensionales.

Because $\mathrm{d}t = t_{\text{emission}} - \underbrace{t_{observed}}_{\text{es ahora: mayor a lo demás.}} < 0$

Integrating and normalizing $\displaystyle\int_{\lambda_0}^{\lambda_e} \frac{\mathrm{d}\lambda}{\lambda} = -\int_{a_0=1}^{a} \frac{\mathrm{d}a}{a} \implies \frac{\lambda_e}{\lambda_0} = a^{-1}$ — longitud de onda emitido. — lo observado. — a(t) puede ir de 0 a 1.

Defining the redshift as $z = \dfrac{\lambda_e - \lambda_0}{\lambda_0} \implies \lambda_e = (1+z)\lambda_0 \implies (1+z) = \dfrac{1}{a}$

↳ redshift tiene relación con λ
↳ a con una frecuencia
↳ energía del fotón: hf

Cosmology: dynamics

The Christoffel symbols are

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\alpha}\left(g_{\alpha\nu,\lambda} + g_{\alpha\lambda,\nu} - g_{\nu\lambda,\alpha}\right)$$

→ T depende de la métrica

But the metric is $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$ $\quad \gamma_{11} = \dfrac{1}{1-Kr^2}, \quad \gamma_{22} = r^2, \quad \gamma_{33} = r^2\sin^2\theta$

And $g_{00} = -1$

$$\Gamma^0_{00} = \frac{1}{2}g^{0\alpha}\left[g_{\alpha 0,0} + g_{0\alpha,0} - g_{00,\alpha}\right] = -\frac{1}{2}g^{00}g_{00,0} = 0$$

And then one by one, the non-null Christoffel symbols are:

$$\begin{aligned}
\Gamma^0_{ij} &= a^2 H\gamma_{ij}, \quad \Gamma^i_{0j} = \Gamma^i_{j0} = H\delta^i_j, \\
\Gamma^1_{11} &= \frac{Kr}{1-Kr^2}, \quad \Gamma^1_{22} = -r\left(1-Kr^2\right), \quad \Gamma^1_{33} = -r\left(1-Kr^2\right)\sin^2\theta, \\
\Gamma^2_{33} &= -\sin\theta\cos\theta, \quad \Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \quad \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta,
\end{aligned}$$

Geodesic equation

Geodesic equation — cómo se mueve la part. en caída libre
Generalization of the "straight line";
world-line of a particle free of external forces:
i.e. freely moving or falling particle

$$\underbrace{\frac{\mathrm{d}^2x^\mu}{\mathrm{d}s^2} + \Gamma^\mu{}_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s} = 0}_{\text{ec. para weak lensing.}}$$

4-velocity $u^\mu = \dfrac{\mathrm{d}x^\mu}{\mathrm{d}s}$ ➡ $\dfrac{\mathrm{d}u^\mu}{\mathrm{d}s} + \Gamma^\mu{}_{\alpha\beta}u^\alpha u^\beta = 0$

From India to Alaska in a straight line

The μ = 0 component

$\dfrac{\mathrm{d}u^0}{\mathrm{d}s} + \Gamma^0{}_{\alpha\beta}u^\alpha u^\beta = 0$ because $\Gamma^0{}_{ij} = a^2 H\delta_{ij}$

$$\frac{\mathrm{d}u^0}{\mathrm{d}s} + \Gamma^0{}_{ij} = \frac{\mathrm{d}u^0}{\mathrm{d}s} + a^2 H\gamma_{ij}u^iu^j = \frac{\mathrm{d}u^0}{\mathrm{d}s} + H(u^0)^2$$

$$0 = g_{\mu\nu}u^\mu u^\nu = -(u^0)^2 + a^2\gamma_{ij}u^iu^j$$

massless particle

$$\frac{\mathrm{d}}{\mathrm{d}s} = \frac{\mathrm{d}x^0}{\mathrm{d}s}\frac{\mathrm{d}}{\mathrm{d}x^0}$$

$$\boxed{\frac{\mathrm{d}u^0}{\mathrm{d}t} = -\frac{\mathrm{d}a}{a}} \implies u^0 \propto \frac{1}{a}$$

por el volumen y la expansión del universo.

densidad de radiac. decrece como $\frac{1}{a^4(t)}$

The energy decreases with a(t)

↳ densidad de E es m/vol
y vol ∝ a^3

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Ricci tensor

$$R_{\mu\nu} = \Gamma^\alpha_{\mu\nu,\alpha} - \Gamma^\alpha_{\mu\alpha,\nu} + \Gamma^\beta_{\mu\nu}\Gamma^\alpha_{\alpha\beta} - \Gamma^\alpha_{\beta\nu}\Gamma^\beta_{\mu\alpha}$$

Ricci scalar

$$R = g^{\mu\nu}R_{\mu\nu}$$

Christoffel symbols

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\alpha}\left(g_{\alpha\nu,\lambda} + g_{\alpha\lambda,\nu} - g_{\nu\lambda,\alpha}\right)$$

How to proceed? Basically one by one with the indices and find the non-null solutions.
Remind: 0 is time and i=1,2,3 is space

$$\begin{aligned}
R_{00} &= \Gamma^\alpha_{00,\alpha} - \Gamma^\alpha_{0\alpha,0} + \Gamma^\beta_{00}\Gamma^\alpha_{\alpha\beta} - \Gamma^\alpha_{\beta 0}\Gamma^\beta_{0\alpha} = \\
&= \cancel{\Gamma^0_{00,0}} + \Gamma^i_{00,i} - \cancel{\Gamma^0_{00,0}} - \Gamma^i_{0i,0} + \Gamma^0_{00}\Gamma^\alpha_{\alpha 0} + \Gamma^i_{00}\Gamma^\alpha_{\alpha i} - \Gamma^\alpha_{00}\Gamma^0_{0\alpha} - \Gamma^\alpha_{i0}\Gamma^i_{0\alpha} \\
&= \Gamma^i_{00,i} - \Gamma^i_{0i,0} + \cancel{\Gamma^0_{00}}\Gamma^0_{00} + \Gamma^0_{00}\Gamma^i_{i0} + \cancel{\Gamma^i_{00}}\Gamma^0_{0i} + \Gamma^i_{00}\Gamma^j_{ji} - \cancel{\Gamma^0_{00}}\Gamma^0_{00} - \cancel{\Gamma^i_{00}}\Gamma^0_{0i} - \Gamma^0_{i0}\Gamma^i_{00} - \Gamma^j_{i0}\Gamma^i_{0j}
\end{aligned}$$

The Ricci tensor is a combination of the Christoffel symbols

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\alpha}\left(g_{\alpha\nu,\lambda} + g_{\alpha\lambda,\nu} - g_{\nu\lambda,\alpha}\right)$$

Good procedure is to list them $g_{00} = -1$; $g_{ij} = a^2(t)\delta_{ij}$

$$\Gamma^0_{00} = \frac{1}{2}g^{0\alpha}\left[g_{\alpha 0,0} + g_{0\alpha,0} - g_{00,\alpha}\right] = \frac{1}{2}g^{00}g_{00,0} = 0$$

$$\Gamma^0_{ij} = \frac{1}{2}g^{0\alpha}\left[g_{\alpha j,i} + g_{i\alpha,j} - g_{ij,\alpha}\right] = -\frac{1}{2}g^{00}g_{ij,0} = \frac{\dot a}{a}\delta_{ij} \quad \text{and} \quad \Gamma^j_{i0} = \Gamma^j_{0i} = \frac{\dot a}{a}\delta^j_i$$

All the others, for this particular metric, are 0!

Watch: we assumed that k=0

The Ricci tensor is a combination of the Christoffel symbols

$$\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2} g^{\mu\alpha}\left(g_{\alpha\nu,\lambda} + g_{\alpha\lambda,\nu} - g_{\nu\lambda,\alpha}\right)$$

But the metric is $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$ $\quad \gamma_{11} = \dfrac{1}{1-Kr^2}, \quad \gamma_{22} = r^2, \quad \gamma_{33} = r^2\sin^2\theta$

$$\begin{aligned}
\Gamma^0_{ij} &= a^2 H\gamma_{ij}, \quad \Gamma^i_{0j} = \Gamma^i_{j0} = H\delta^i_j, \\
\Gamma^1_{11} &= \frac{Kr}{1-Kr^2}, \quad \Gamma^1_{22} = -r\left(1-Kr^2\right), \quad \Gamma^1_{33} = -r\left(1-Kr^2\right)\sin^2\theta, \\
\Gamma^2_{33} &= -\sin\theta\cos\theta, \quad \Gamma^2_{12} = \Gamma^2_{21} = \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \quad \Gamma^3_{23} = \Gamma^3_{32} = \cot\theta,
\end{aligned}$$

característica del espacio

The components are $R_{00} = -3\left(H^2 + \dot H\right), \quad R_{0i} = R_{i0} = 0, \quad R_{ij} = a^2\left(3H^2 + \dot H + \frac{2K}{a^2}\right)\gamma_{ij}$

$$R = 6\left(2H^2 + \dot H + \frac{K}{a^2}\right).$$

Dynamics depends on matter

cte de gravitación

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi \boxed{G}}{c^4} T_{\mu\nu}$$

→ para todas las componentes del universo.

We need to define what to add in our Universe which is homogenous and isotropic.

We need a fluid that is homogeneous and isotropic in the same rest frame where the universe is experienced to be homogeneous and isotropic. Such fluids are said to be perfect.

→ Fluido perfecto: No hay traspaso de flujo de calor.

solo depende de t

$$T_{00} = \rho(t), \quad T_{i0} = T_{0j} = 0, \quad T_{ij} = p(t) a^2(t) \gamma_{ij},$$

→ sino, habría una dirección privilegiada.

In a compact form introducing the 4-velocity $u^\mu = \{-1, 0, 0, 0\}$ → Nos movemos con el fluido.

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + p g_{\mu\nu}$$

What is a perfect fluid?
Real fluids are "sticky" and contain (and conduct) heat. Perfect fluids are idealized models in which these possibilities are neglected. Specifically, perfect fluids have no shear stresses, viscosity, or heat conduction.

↑

Densidad no depende de la posición sino que del tiempo.

→ · Igual en todos los puntos.
· Si me pongo lejos o afuera del fluido, las componentes espaciales son 0 también xq no se mueven.

We have all the materials to work out the equations that describe the whole dynamics

$$G_{00} = R_{00} - \frac{1}{2} g_{00} R = 8\pi G\, T_{00} \qquad 3\left(H^2 + \frac{K}{a^2}\right) = 8\pi G \rho$$

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = 8\pi G\, T_{ij} \qquad -a^2\left(3H^2 + 2\dot{H} + \frac{K}{a^2}\right)\delta_{ij} = 8\pi G a^2 p\, \delta_{ij}$$

The Friedmann equations:

densidades (sin curvatura).

The Hubble equation: $H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}$ → Velocity of expansion

The Raychaudhuri equation: $3H^2 + 2\dot{H} = -8\pi G p - \frac{K}{a^2}$ → Acceleration of expansion

We constructed the Einstein equations such that

materia está conservada.

$$G_{\mu\nu;\mu} = 0 \iff T_{\mu\nu;\mu} = 0$$

purely geometric   matter is conserved

This means that we can find a conservation law for matter component

$$\dot{\rho} + 3H(\rho + p) = 0$$

For adiabatic fluid whose pressure is a function only of the density
And we introduced a new parameter w = equation of state parameter

$$(p = w\rho)$$

$$\left[\rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)}\right]$$

a redshift 0

We are assuming that w is constant. ρ_0 and a_0 extremes of integration.
The EOS w will tell us how that particular matter behaves.

Ec. de conservación
$\dot{\rho} + 3H(\rho + p)$
$\xrightarrow{\text{Fluido adiabático: } p = w\rho}$ $\rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)}$

slide 04-05

Materia relativista: → fotones, neutrinos relat, ... $\boxed{T \gg m}$

$w = 1/3$

$$\rho_r = \rho_{r,0}\left(a/a_0\right)^{-4}$$

Materia no-relativista:

$w = 0$

$$\rho_m = \rho_{m,0}\left(a/a_0\right)^{-3}$$ como el volumen.

Dark energy:

$w = -1$ ⇒ p negativo → $\rho_{DE} = \rho_{DE,0}\left(a/a_0\right)^{-3(1+w)} = \rho_{DE,0}$.

curvature can be considered as a fluid:

$w = -1/3$ $\rho_K = \rho_{K,0}\left(a/a_0\right)^{-2}$ → No es un fluido! (sólo para escribirlo)

It is useful to define the critical density as $\rho_{\text{crit}} = \frac{3H^2}{8\pi G}$ → densidad tal que la curvatura es igual a 0

And the density parameter for a specie (normalized) $\Omega_s = \frac{\rho_s}{\rho_{\text{crit}}}$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \sum_s \Omega_{s,0}\left(\frac{a}{a_0}\right)^{-3(1+w_s)}$$

evaluada al t actual ⇒ sale un H_0

The final Hubble parameter (you've been waiting for)

fully radiation dominated era (cuando sólo tomamos esta componente).

$$H^2 = H_0^2\left[\underbrace{\Omega_{r,0}a^{-4}}_{\text{radiaø}} + \underbrace{\Omega_{m,0}a^{-3}}_{\text{no relativista (masa)}} + \underbrace{\Omega_{K,0}a^{-2}}_{\text{curvatura}} + \underbrace{\Omega_{DE,0}a^{-3(1+w)}}_{\text{DE.}}\right]$$

Important issue in cosmology: $\Omega_K = -K_0/H_0^2$ makes the total $\Omega = \Sigma_s \Omega_s = 1$ at any given time.

In cosmology we want to measure $\sum_{s\neq K} \Omega_s = 1 - \Omega_K = 1 - \Omega_{r,0} - \Omega_{m,0} - \Omega_{DE,0}$ $+\Omega_K$ → suma de densidades es 1.

Big bang
↑
0 NOW
inflaø. —•————————|→ t

10^x ←————————| 2,7 K — T

10^{-9} •————————→ R → Radio del universo observable.

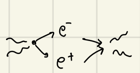
· $T_{\mu\nu;\mu} = 0$ → No hay perdida de materia!

$\gamma + \gamma \to e^- + e^+$ → se preserva la carga

interacción fotón fotón. electrón y positrón

nunca estas partículas están libres para poder desacoplarse.

$\gamma + DM \xrightarrow{\text{no interactúa}} \gamma + DM$ ∼ materia oscura



decoupling e^-

e^- → 1001
e^+ → 1000
→ e^-

aniquilación

Hay mecanismos para que se genere un poco más de las partículas que vemos. → exceso de electrones en comparaø con positrones.

* Inflaø: Qué pudo crear materia?
↓
debido a un campo : Inflatón
↓ decae y crea
Higgs boson.

* Part. livianas sin masa sienten más la T° de su entorno
⇓
se desacoplan antes las partículas más pesadas.

se desacoplan por completo los fotones ⇒ CMB

* siempre es un sistema aislado → conservaø de energía. → No puede salir la E afuera del universo ya que la información tendría que viajar más rápido que la $\vec{v}$ de la luz.

Matter species

They are broadly classified into:
relativistic particles;
non-relativistic matter;
dark energy;
a scalar field (?) dominating inflation in the early universe.

We are "sure" we know

No clue yet

Some info from GR: a particles as a momentum and a mass $\{\vec{p},\, m\}$ with $|\vec{p}| = p$ and its energy is $E = \sqrt{m^2 + p^2}$

The phase-space occupancy of a specie in equilibrium at a temperature T follows
└ cómo todas las partículas se comportan en este gas.

$$f(\vec{p}) = \frac{1}{\exp\left[(E-\mu)/T\right] \pm 1}$$

Fermi-Dirac

Bose-Einstein

→ Nos dice cómo se distribuyen las partículas en el espacio de momento y la coord. x. (cómo se portan en este gas)

watch: the above equation depends only on the module of p due to homogeneity and isotropy

Since the minimum volume of phase space in terms of x and p is given by $(2\pi\hbar)^3$ due to Heisenberg's principle, the number of phase space elements is $d^3x d^3p/(2\pi\hbar)^3$

→ para que sea correcto completa# hay que considerar las colisiones.

Materia relativista $T \gg m$
- Bosones → Fotones → $g = 2$
  - } $\omega = 1/3$
- Fermiones → Neutrinos → $g = 6$ (3 · 2 helicidades)
  - Electron
  - Positron } $g = 2$ (spin)

Materia no relativista
- → Bariones
- CDM
  - } $\omega = 0$

The energy density and the pressure per unit volume (no integration on x) are

nos da los grados de libertad intrínsecos de la partícula

$$\rho = g_* \int \frac{d^3p}{(2\pi\hbar)^3} E(p) f(p) = \frac{\boxed{g_*}}{2\pi^2} \int_m^\infty dE \frac{\left(E^2 - m^2\right)^{1/2}}{\exp\left(E-\mu\right)/T \pm 1} E^2 \,,$$

$$p = g_* \int \frac{d^3p}{(2\pi\hbar)^3} \frac{pv}{3} f(p) = g_* \int \frac{d^3p}{(2\pi\hbar)^3} \frac{p^2}{3E} f(p) = \frac{g_*}{3\pi^2} \int_m^\infty dE \frac{\left(E^2 - m^2\right)^{3/2}}{\exp\left(E-\mu\right)/T \pm 1} E^2 \,.$$

grados de libertad de la partícula (spin)

→ distinto por cada componente de especie.

remind in special relativity that

$E = mc^2\gamma$

$$E = m\,c^2/\sqrt{1 - v^2/c^2}\,; \quad p = m\,v/\sqrt{1 - v^2/c^2}$$

Relativistic component

$$\rho = \begin{cases} (\pi^2/30)\, g_* T^4, & \text{(Bosons)} \\ (7/8)(\pi^2/30) g_* T^4, & \text{(Fermions)} \end{cases}$$

(Bosons) → Fotones. (2 polarizaciones)

$T \gg m \implies m \to 0$

temp mucho más grande que la masa (como los fotones)

$$\boxed{p = \rho/3,} \to p = \omega\rho \Rightarrow \frac{\rho}{3} = \omega\rho \Rightarrow \omega = \frac{1}{3}$$ para las comp. relativistas

And for non-degenerate particles T>> μ

With integrals $\displaystyle\int_0^\infty dx\, x^3/(e^x - 1) = \pi^4/15\,;\quad \int_0^\infty dx\, x^3/(e^x + 1) = 7\pi^4/120$

photons

are bosons and have two spin states: g=2

grados de libertad internos.

also $\mu/T < 9 \times 10^{-5}$

$$\rho_\gamma = \frac{\pi^2 k_B^4 T^4}{15\hbar^3 c^5}$$

temp. de la CMB.

permite obtener la relación $K^4 \propto \frac{kg}{m^3}$

The Planck[1] satellite measured the present temperature of CMB photons to be T = 2.725 ± 0.002K

$$\Omega_{\gamma,0} = \frac{\rho_{\gamma,0}}{\rho_{crit}}$$

densidad tq. nuestro universo sea plano.

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} = \frac{3\,(100\,h)^2}{8\pi G} \frac{\text{km}}{\text{s Mpc}}$$

distintos valores le ponemos a h (nº entre 0 y 1)

$$1\,\text{pc} = 3.26\,\text{l.y.} = 3.26 \times \left(9.461 \times 10^{12}\right)\,\text{km}$$

$$G = 6.674 \times 10^{-11}\text{m}^3/\text{kg/s}^2$$

$$\rho_{crit} = 1.88 \times 10^{-26}\,h^2\,\text{kg/m}^3$$

With some magic conversion $1\,\text{K}^4 = 1.279 \times 10^{-32}\text{kg/m}^3$

And with the temperature measured by Planck

$$\rho_{\gamma,0} = 4.641 \times 10^{-31}\text{kg/m}^3 \longrightarrow \Omega_{\gamma,0} = 2.469 \times 10^{-5}\,h^{-2}$$

→ trabajamos con su fracción.

Which depends on the measured h

Relativistic matter

decoupling
↓
neutrinos → electron positron. → BBN

neutrinos

are fermions

But their energy density is given by

Neutrinos, partículas con masa ~0. → Si la partícula tiene una masa liviana, hay que esperar a que baje la T°. (para que se desacoplen).

$$\rho_\nu = N_{\rm eff}\frac{7\pi^2}{120}T_\nu^4$$

Where N_{eff} is the effective number of neutrinos, which in the standard model is = 3

procesos relativistas

However, Planck measured N_{eff} = 3.15. Where does this come from? To allow extra relativistic degree of freedom[1].

The Universe is filled with boson and fermions, which they decouple at different times from the thermal bath.

The Big Bang Nucleosynthesis (BBN) occurred around the energy scale ~ 0.1MeV to form light elements such as deuterium and helium. The decoupling of neutrinos from the rest of the cosmic plasma, immediately followed by the annihilation of electrons (e^-) and positrons (e^+), occurred earlier than the BBN epoch. The presence of extra relativistic degrees of freedom changes the amount of the light elements predicted by the BBN, which allows to put a bound on Neff.

The non integer of Neff comes from the cooling down of the Universe and different species decouple at different times

neutrinos and ~ 1MeV

coef. que aparece de la ≠ de bosones y fermiones.

3 · 2 helicidades

2 spin

$$\rho_{\rm rel} = \underset{\text{dof.}}{\boxed{g_*}}\frac{\pi^2}{30}T^4 \quad \text{and} \quad g_* = g_{\rm bos} + \boxed{\frac{7}{8}}\rho_{\rm fer} \qquad g_* = g_\gamma + \frac{7}{8}(g_\nu + g_{e^+} + g_{e^-}) = 2 + \frac{7}{8}(6+2+2) = 10.75$$

3 neutrinos with 2 helicity, and 2 spins for electrons and positrons

The only interactions keeping neutrinos coupled are

$$\nu_e + \bar\nu_e \rightleftharpoons e^+ + e^-,$$
$$\nu_\mu + \bar\nu_\mu \rightleftharpoons e^+ + e^-,$$
$$\nu_\tau + \bar\nu_\tau \rightleftharpoons e^+ + e^-,$$

When do they start to decouple? When the interaction rate ~ expansion rate ($H^2 \sim \rho_{rad}$)

T° del entorno baje

$$\frac{\boxed{\Gamma_{{\rm int},\nu}}}{H} \simeq \left(\frac{T}{1\,{\rm MeV}}\right)^4$$

It happens at ~ 1 MeV

cuando el universo se expande, es imposible que las partículas se junten de nuevo.

From that point on, the distribution of neutrinos freezes, and evolves as if it were a thermal bath with its own temperature.

neutrinos and ~ 0.5 MeV

Now, the neutrinos are decoupled and the thermal bath is composed by

$$\rho_{\rm rel} = g_*^{rest}\frac{\pi^2}{30}T^4 + g_*^\nu\frac{7}{8}\frac{\pi^2}{30}T_\nu^4$$

with

$$g_*^{rest} = \underset{\text{fotón}}{2} + \frac{7}{8}\times\underset{e^- \text{ y } e^+}{4} \quad \text{and} \quad g_*^\nu = \frac{7}{8}\times\underset{\text{helicidades}}{6}$$

No se desacoplan antes los neutrinos?

with cooling positron and electrons annihilate

$$g_*^{rest} = g_*^\gamma = 2 \quad \text{and} \quad g_*^\nu = \frac{7}{8}\times 6$$

sólo quedan los fotones.

This change is what makes things right, but for this we need the Entropy!

densidad de energía por el volumen

The change of density in a volume implies a change of density

$$TdS = {\rm d}(\rho V) + p{\rm d}V = V{\rm d}\rho + (p+\rho){\rm d}V$$

deriv. parcial c/r a la T°.

$$\underset{\text{depende de la T° y el volumen.}}{{\rm d}S} = \frac{V}{T}\frac{{\rm d}\rho}{{\rm d}T}{\rm d}T + \frac{(p+\rho)}{T}{\rm d}V \implies \partial_V S = (\rho+p)/T \quad \text{and} \quad \partial_T S = (V/T){\rm d}\rho/{\rm d}T$$

Using the continuity eq.

$$\partial_T\partial_V S = \partial_V\partial_T S \implies \frac{{\rm d}p}{{\rm d}T} = \frac{(\rho+p)}{T} \qquad \underset{\text{entropía total}}{{\rm d}S} = {\rm d}\left[\frac{V(\rho+p)}{T}\right] = 0 \qquad {\rm d}\rho + (\rho+p)\frac{{\rm d}V}{V} = 0$$

(no densidad de entropía)

Adiabaticidad.

↳ conservation eq. $\dot{\rho} + 3H(\rho + p) = 0$

2° ley termo. $T\,dS = dU + p\,dV$

$dV = d(a^3)$

$dU = d(a^3\rho)$

$$\frac{dU}{dt} + p\frac{dV}{dt} = 3a^2\dot{a}\rho + a^3\dot{\rho} + 3pa^2\dot{a} = 0$$

→ fluido barotrópico.
↳ presión de.
↳ + que un fluido perfecto

$$T\,dS = 0$$

2° ley termo. $T\,dS = dU + p\,dV = 0$

expansión es adiabática:

To barotropic fluid: $p = w\rho \longrightarrow \rho a^{3(1+w)} = cte$

Relativistic matter

This condition $dS = d\left[\frac{\overbrace{V(\rho+p)}^{cte.}}{T}\right] = 0$ implies that during transitions

valor inicial / final

$$\frac{V_i(\rho_i+p_i)}{T_i} = \frac{V_f(\rho_f+p_f)}{T_f}$$

$g^i = g_*^{rest} = 2 + \frac{7}{8} \times 4$ (fotones; 4 → pareja de e^- y e^+)

$$T_f = \left(\frac{5.5}{2}\right)^{1/3} \frac{a_i}{a_f} T_i = \left(\frac{11}{4}\right)^{1/3} \frac{a_i}{a_f} T_i \longleftarrow T_f = \left(\frac{g_*^i}{g_*^f}\right)^{1/3} \frac{a_i}{a_f} T_i \xleftarrow{\text{For radiation}} \frac{a_i^3(\rho_i+p_i)}{\boxed{T_i}} = \frac{a_f^3(\rho_f+p_f)}{\boxed{T_f}}$$

término de expansión

de las ecuaciones de Boltzman.

$g^f = g_*^{rest} = 2$ → se aniquilan e^- y e^+ → en la fase de expansión

throughout the annihilation the temperature of neutrinos continued evolving as ~ a^{-1}

$$T_f = \frac{a_i}{a_f} T_i$$

This implies that after annihilation, the temperature of the neutrino bath can be expressed in terms of temperature T of the true bath as

$$\underset{\boxed{T_i^\circ}}{T_\nu} = \left(\frac{4}{11}\right)^{1/3} \underset{\boxed{T_f^\circ}}{T}$$

→ T° de los neutrinos que se desacoplaron del baño térmico.
se desacoplaron antes que e^+ y e^-

sin término de expansión (antes)

Neutrinos se quedan con la T° antes del desacople y luego, hay T° final del baño.

$$\rho_{rel} = g_*^\gamma \frac{\pi^2}{30} T^4 + g_*^\nu \frac{7}{8} \frac{\pi^2}{30} T_\nu^4 = \left[2 + 6 \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3}\right] \frac{\pi^2}{30} T^4$$

3.363

And the density of the neutrinos $\rho_\nu = 3 \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$

$$\rho_\nu = \boxed{N_{\rm eff}} \times \frac{7}{8} \times \left(\frac{4}{11}\right)^{4/3} \rho_\gamma$$

densidad de los neutrinos.

Non-relativistic component

$T \ll m$

$$\rho = g_* m \left(\frac{mT}{2\pi}\right)^{3/2} \exp\left[-(m-\mu)/T\right],$$

$$p = g_* T \left(\frac{mT}{2\pi}\right)^{3/2} \exp\left[-(m-\mu)/T\right] = \frac{T}{m}\rho, \qquad \ll 1 \sim 0$$

$$E = \sqrt{m^2+p^2} \sim m + \frac{p^2}{2m}$$

Non-relativistic component: baryons

- During the Big Bang Nucleosynthesis (BBN) epoch deuterium and helium were formed from neutrons and protons.
- most of the neutrons decayed to protons (through β-decay: $n \to p^+ + e^- + \bar{\nu}_e$) before the formation of deuterium (1p + 1n),
- and neutrons that did not decay to protons were eventually trapped in helium (2p + 2n),
- if we increase the baryon density, the process of the BBN occurs faster and hence more neutrons remain without decaying to protons,
- this leads to an increase of the abundance of helium, whereas the abundance of deuterium decreases,
- thus the amount of light elements produced during the BBN epoch is sensitive to the baryon density.

n → decaen a p → deuterio
n → atrapados en helio

+ Ω_b
↓
BBN más rápido
↓
+ helio, − deuterio.

$$\Omega_{b,0}h^2 = 0.020 \pm 0.002 \qquad \text{(BBN constraint)},$$

https://arxiv.org/abs/astro-ph/0010171 → según la abundancia que tenemos hoy

$$\Omega_{b,0}h^2 = 0.02230 \pm 0.00014 \qquad \text{(Planck constraint)},$$

Non-relativistic component: CDM

$$\Omega_{c,0}h^2 = 0.1188 \pm 0.0010 \qquad \text{(Planck constraint)},$$

Isn't there missing something? Wasn't total Ω ~ 1?

Time and scale factor

se dieron cuenta que no estaba desacelerando.

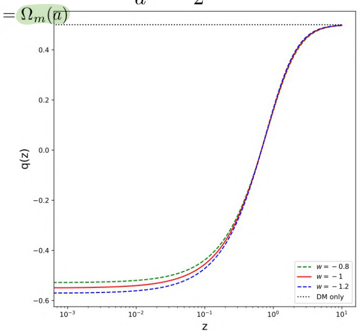
parámetro de deceleración (cuánto está desacelerando el universo)

Once we have the Friedmann equations we can evaluate the dynamics of space-time

Let's give an example assuming the Universe is filled by only one form of general matter

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = H_0^2\Omega_0\left(\frac{a}{a_0}\right)^{-3(1+w)} \qquad \frac{\rho_m(a)}{\rho_{crit}(a)} = \frac{\Omega_{m,0}\,a^{-3}}{H^2(a)/H_0^2} = \Omega_m(a)$$

$$q \stackrel{\text{def}}{=} -\frac{\ddot a\, a}{\dot a^2} = \frac{1}{2}(1+3w)$$



For the specific flat LCDM model (and ignoring radiation)

añadimos esta componente

$$q = \frac{1}{2}\sum_i \Omega_i(1+3w_i) = \Omega_r(a) + \frac{1}{2}\Omega_m(a) + \frac{1+3w_{DE}}{2}\Omega_{DE}(a)$$

Debe ser negativo

only 4% for 2% variation in DE eos!
Can be measured?

We measure H(z) but... $q = -1 - \frac{\dot H}{H}$

$$\int_{t_0}^{t} dt' = \int_{a_0}^{a}\frac{da'}{a\,H} \qquad a(t) = a_0\left[1+\frac{3}{2}(1+w)H_0(t-t_0)\right]^{\frac{2}{3(1+w)}}$$

Both time and scale factor depend critically on the matter specie(s) in the Universe.

We can work out a more direct relation between the scale factor and time.

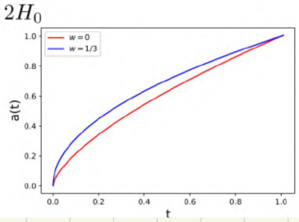
Estudiar los parámetros según las épocas dominadas.

If we set t=0 and a=0, we find

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}} \quad \text{with} \quad t_0 = \frac{1}{2H_0}$$

For matter, w=0 $\quad a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{2}{3}}$

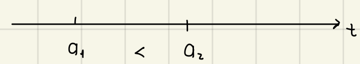
For radiation, w=1/3 $\quad a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{1}{2}}$



término dominante en una época.

$$H^2 = H_0^2\left\{\Omega_{r,0}\,a^{-4} + \Omega_{m,0}\,a^{-3} + \Omega_{DE,0}\,a^{-3(1+w)}\right\}$$

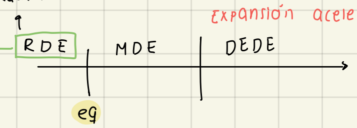
$a \in (0, -1)$



Radiation

perturbaciones no pueden crecer, T° muy caliente y no se forman estructuras

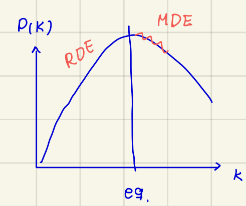
Expansión acelerada.



↳ equivalencia entre $\rho_r = \rho_m$

RDE $\quad a \sim t^{1/2}$

MDE $\quad a \sim t^{2/3}$



Early time Universe

Let us study the early universe, at times much earlier than the Λ-dominated era, i.e. $\Lambda \to 0$

$$H^2 = H_0^2\left[\Omega_{r,0}a^{-4} + \Omega_{m,0}a^{-3}\right] \rightarrow \text{los 2 componentes son dominantes.} \longrightarrow H_0 dt = \frac{a\,da}{\sqrt{\Omega_{r,0} + \Omega_{m,0}a}} \qquad \Big/ \int_{t_0}^{t}$$

$$H_0(t-t_0) = \frac{2}{3\Omega_{m,0}^2}(a\,\Omega_{m,0} - 2\Omega_{r,0})\sqrt{a\,\Omega_{m,0} + \Omega_{r,0}}\,\Bigg|_{a_0}^{a}$$

C.I.

Setting $t_0 = 0 = a_0$. And defining radiation density = matter density

$$\rho_r(a) = \rho_m(a)$$

$$\Omega_{r,0}a_{eq}^{-4} = \Omega_{m,0}a_{eq}^{-3}$$

$$a_{eq} = \Omega_{r,0}/\Omega_{m,0}$$

en qué tiempo estas densidades son iguales.

$$t = \frac{\sqrt{2}t_{eq}}{\sqrt{2}-1}\left[1-\left(1-\frac{a}{2\,a_{eq}}\right)\sqrt{1+\frac{a}{a_{eq}}}\right]$$

with

* CMB: 300.000 años después fue el CMB

Medido con CAMB ≠ Medido con local measurements.

$$t_{eq} = \frac{2\sqrt{2}}{3\,H_0\Omega_{m,0}^{1/2}}a_{eq}^{3/2}(\sqrt{2}-1) \begin{array}{ll} = 49491\,\text{years} & \text{if } h = 0.67 \\ = 37110\,\text{years} & \text{if } h = 0.72 \end{array}$$

$$\Omega_{\gamma,0} = 2.469\times 10^{-5}\,h^{-2}$$

$$\Omega_{r,0} = \Omega_{\gamma,0} + \Omega_{\nu,0} = \Omega_{\gamma,0}\left[1 + N_{\text{eff}}\times\frac{7}{8}\times\left(\frac{4}{11}\right)^{4/3}\right]$$

$t_{eq} \approx 50.000$ años

CMB ≈ 300.000 años ⟶ CMB después de eq.

Late time Universe

Let us study the late universe, at times in Λ-dominated era

$$H = H_0\sqrt{\Omega_{m,0}a^{-3} + \Omega_\Lambda} \qquad\qquad H_0 \mathrm{d}t = \frac{\sqrt{a}\,\mathrm{d}a}{\sqrt{\Omega_{m,0} + \Omega_\Lambda a^3}}$$

$$\rho_\Lambda(a) = \rho_m(a)$$

$$t - t_0 = \frac{2a_{\Lambda M}^{3/2}}{3H_0\Omega_{M,0}^{1/2}}\left(\operatorname{arcsinh}\left[\left(\frac{a(t)}{a_\Lambda}\right)^{3/2}\right] - \operatorname{arcsinh}\left[a_\Lambda^{-3/2}\right]\right) \qquad a_\Lambda = (\Omega_m/\Omega_\Lambda)^{1/3}$$

↳ matter-DE.

To determine t0 we consider the evolution of the universe throughout the three ages (rad-mat-DE) -> we introduce an intermediate time t_* during which the universe was matter dominated, such that $a_\Lambda \gg a_* \gg a_{eq}$ → Factor de escala intermedio.

$$\boxed{H_0 t_0} = \int_{a_*}^{1}\frac{da}{[\Omega_{M,0}a^{-1} + \Omega_\Lambda a^2]^{1/2}} + \int_0^{a_*}\frac{da}{[\Omega_{M,0}a^{-1} + \Omega_{R,0}a^{-2}]^{1/2}}.$$

→ para separar la integral en las componentes.

→ calculamos toda la edad del universo.

And we find

$$\frac{1}{\Omega_{M,0}^{1/2}}\left[\frac{2}{3}a_\Lambda^{3/2}\operatorname{arcsinh}\left(a_\Lambda^{-3/2}\right) - \frac{2}{3}a_*^{3/2}\right] + \frac{4a_{\mathrm{eq}}^2}{3\Omega_{R,0}^{1/2}}\left[1 + \frac{1}{2}\left(\frac{a_*}{a_{\mathrm{eq}}}\right)^{\frac{3}{2}}\right] = H_0 t_0$$

Given $\Omega_{m,0} = a_{eq}\,\Omega_{r,0}$, in the previous expression the terms containing a_* cancel out, and one is left with

$$t_0 = \frac{2a_{\mathrm{eq}}^2}{3H_0\Omega_{R,0}^{1/2}}\left[\left(\frac{a_\Lambda}{a_{\mathrm{eq}}}\right)^{3/2}\operatorname{arcsinh}\left(a_\Lambda^{-3/2}\right) + 2\right]$$

$\Omega_{\mathrm{b},0}h^2 = 0.02230$

$\Omega_{\mathrm{c},0}h^2 = 0.1188$

$\Omega_{\mathrm{r},0}h^2 = 4.1511 \times 10^{-5}$

$h = 0.67$

$\Longrightarrow \quad t_0 = 1.36 \times 10^{10}$ years slide 06 cosmo

Distances

- Physical (proper) distance: cuán distante están 2 puntos entre ellos. Depende del tiempo con el factor de escala.
- comoving distance: Distancia que nunca cambia en el tiempo. se mide con la expansión del t.

$$* \quad ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Light beam travels in:

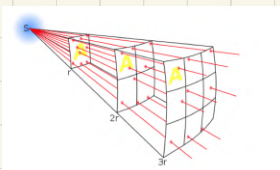
$$\frac{dt}{a(t)} = \frac{-dr}{\sqrt{1-kr^2}} \longrightarrow$$ cuánto viajaron los fotones.

$$\cdot \int_{t_e}^{t_0} \frac{dt}{a(t)} = S_k(r) \quad , \quad S_k(r) = \int_0^r \frac{dr}{\sqrt{1-kr^2}} \quad , \quad r = S_k(\chi) = \begin{cases} \sin(\chi) & k=1 \\ \chi & k=0 \\ \sinh(\chi) & k=-1 \end{cases}$$

depende de la curvatura.

con la inversa.

Luminosity distance



Flux of a source is what we receive.

$$d_L = \sqrt{\frac{L_e}{4\pi F}} \longrightarrow$$ Luminosidad emitida

$$\boxed{F = \frac{L_0}{4\pi S_k^2(r)}} \longrightarrow$$ Luminosidad intrínseca al tiempo 0. (observada)

Flujo depende de la superficie

cuánto viajó la luz.

luminosidad emitida

$$d_L = \sqrt{\frac{L_e}{L_0}}\, S_k(r) = \sqrt{\frac{\Delta E\, \Delta t_0}{\Delta t_e\, \Delta E_0}}\, S_k(r)$$

$$= \sqrt{\underbrace{\frac{\lambda_0}{\lambda_e}}_{1+z}\,\underbrace{\frac{\Delta t_0}{\Delta t_e}}_{1+z}}\, S_k(r)$$

$$= (1+z)\, S_k(r)$$

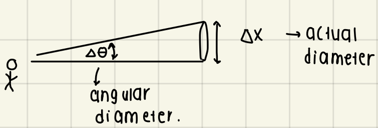
Luminosidad: Energía en un tiempo.

* En el tiempo en que la luz es emitida en comparación con el tiempo en el que llega, el universo se expandió.

$$\cdot \quad d_L = \frac{c(1+z)}{H_0\sqrt{-\Omega_k}} \sinh\left(\sqrt{-\Omega_k}\int_0^z \frac{H_0}{H(z')}dz'\right) \longrightarrow$$ Forma compacta de las ecs. anteriores.

↳ para relacionar el flujo que recibimos con la distancia.

Angular diameter distance



$$\cdot \, d_A = \frac{\Delta x}{\Delta \theta}$$

$$\rightarrow \Delta x = a(t_1)\, S_k(r)\, \Delta\theta$$

$$\Rightarrow d_A = a(t_1)\, S_k(r) = \frac{a_0}{1+z} S_k(r) = \frac{1}{1+z}\cdot\frac{c}{H_0\sqrt{\Omega_k}}\sinh\left(\sqrt{\Omega_k}\int_0^z \frac{H}{H(z')}dz'\right)$$

$a_0 \to 1$

Distance duality

$$d_L = (1+z)^2 d_A(z)$$

→ No hay pérdida de fotones.

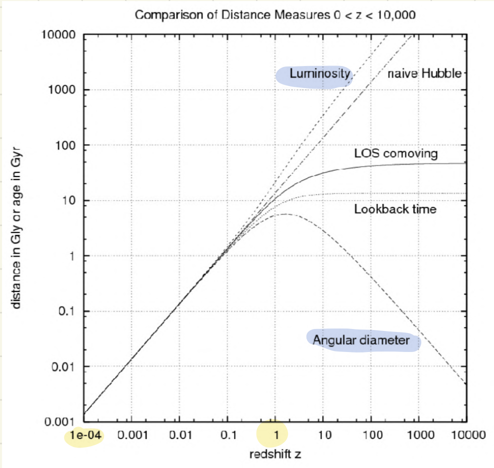
→ Fotones viajan en geodésicas nulas.

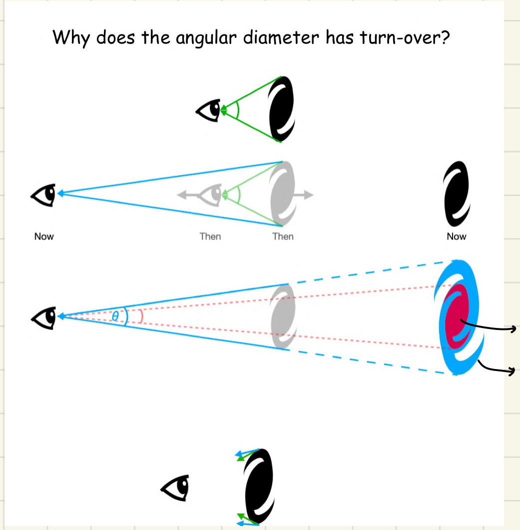
* $\boxed{d_L}$ + lejos la fuente, más viajan los fotones, + grande el efecto de estiramiento.

* $\boxed{d_A}$

Redshift 0,3 ⤳ 1 ⤳
· Momento en que la energía oscura comienza a dominar.

$$a_{eq} = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3}$$



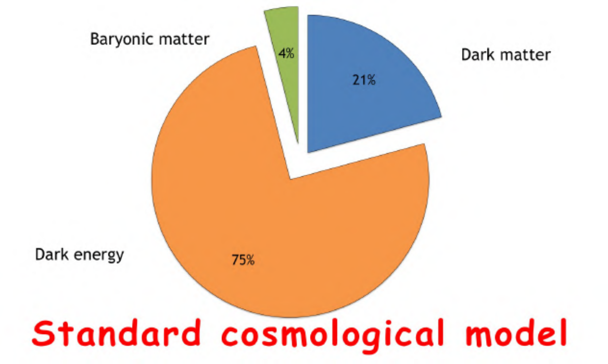


→ el real es menor.
→ tamaño que vemos no es real

· d_L → standard candles — intrinsic luminosity known
IA supernovas

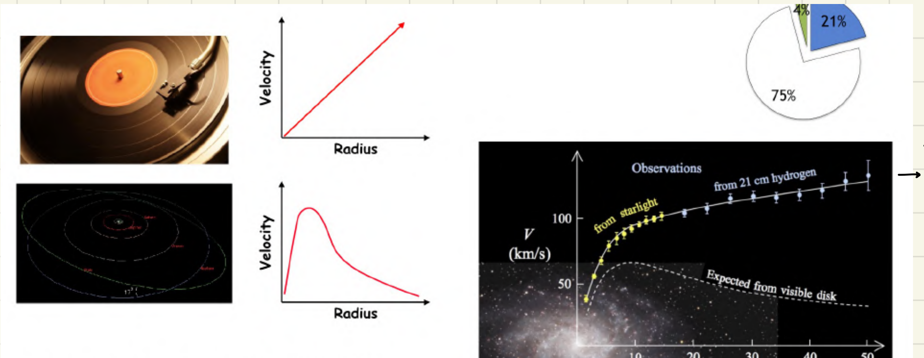
d_A → " rulers → BAO → medir del parámetro de Hubble. desde la CMB.
↘ intrinsic size. known

* curvas de luminosidad: forma de la fuente.



Materia Bariónica

Materia oscura
→

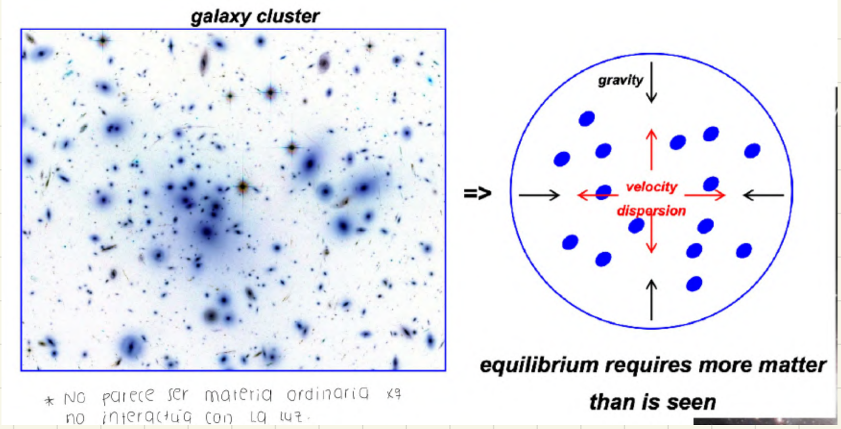


"Dark Matter" already proposed by Fritz Zwicky in 1933:

"Dark Matter" needed to explain Vera Rubin's galactic rotation curves in 1975/80

· velocidad de las estrellas al alejarse de las → galaxias crece más de lo que debería.
· se puede si hay materia que no se ve.

→

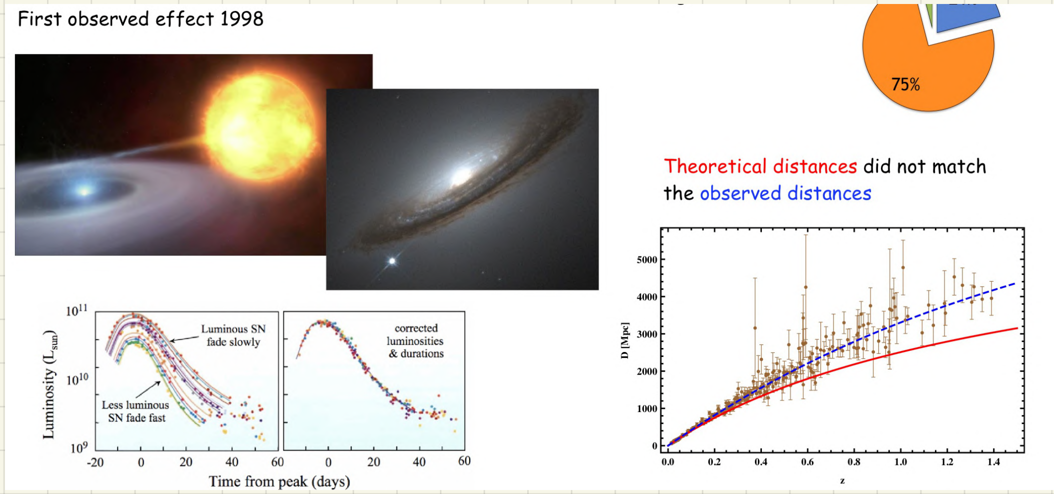


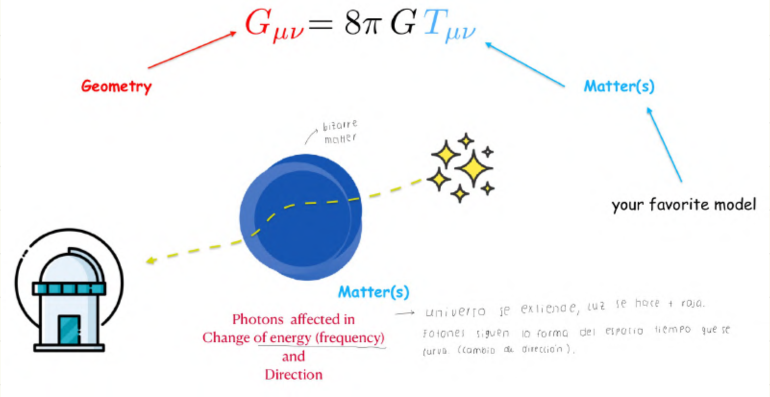
* No parece ser materia ordinaria xq no interactúa con la luz.

→ Para que se junten más galaxias en cúmulos de galaxias debe haber DM.

→ También se obtiene de la CMB.

Energía oscura





→ universo se extiende, luz se hace + roja.
Fotones siguen la forma del espacio tiempo que se curva. (cambio de dirección).

Si comprimimos la materia bariónica comienza a actuar la presión.

· Presión: cómo se propaga info. de un pto. a otro.

The ΛCDM model

· Ct. cosmológica no cambia c/ el espacio-tiempo

Radiation:

Photons the totality come from CMB when they decoupled form matter — $\Omega_{\gamma,0} = 4.48 \times 10^{-5}$

Neutrinos they decoupled much earlier than photons — $\Omega_{\nu,0} = 3.4 \times 10^{-5}$

Baryons — $\Omega_{b,0} = 0.044 \pm 0.003$

CMB

→ medidas precisas para los 1° 3 parám.

Dark matter — $\Omega_{c,0} = 0.258 \pm 0.015$ ← CMB buen estimador.

Curvature — $\Omega_{K,0} = 0.001 \pm 0.002$

Dark energy — $\Omega_{DE,0} = 0.6889 \pm 0.0056$

CMB + late

Más sensible a objetos que dominan ahora!

Hubble parameters

tensión of the Hubble parám.

$H_0 = 67.66 \pm 0.42\,\mathrm{km/s/Mpc}$ CMB

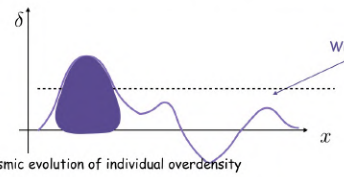
$H_0 = 73.66 \pm 1.68\,\mathrm{km/s/Mpc}$ Late

supernovas IA

Slide 07-08 cosmo

Growth of matter perturbations

We were interested on how the energy densities evolve in time



Cosmic evolution of individual overdensity

Density contrast

$$\delta = \frac{\delta\rho}{\bar{\rho}} = \frac{\rho(t,x) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

Density at background

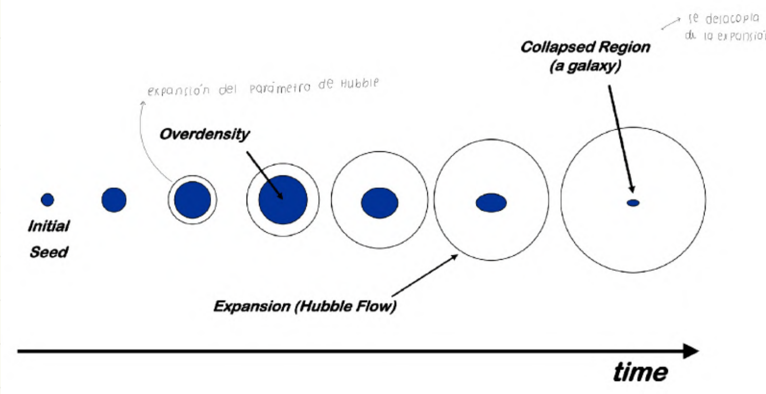
ciertas zonas se desacoplan de la expansión
↳ colapsa xq F grav. es más fuerte.

$$-1 \leq \delta < +\infty \quad \text{and} \quad <\delta>= 0$$

The idea is to say: at the very beginning the overdensities are really comparable to the background density with just small variations

$$\rho(t,x) \sim \bar{\rho}(t) \quad \Longrightarrow \quad \delta \ll 1$$

$\boxed{\text{We start cosmological perturbation theory}}$

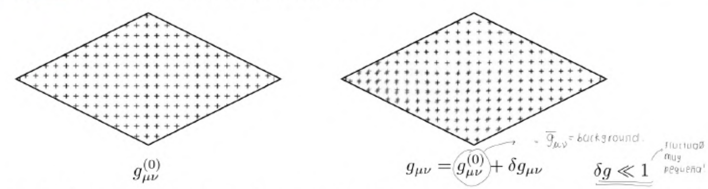


Cosmological perturbations theory

1º orden perturbativo lineal

$\delta \ll 1$ is not a priority of the theory but rather to linearize our equations, i.e.: linear perturbation theory

A metric that deviates from the FLRW spacetime can be written as the sum of an unperturbed FLRW part plus something else, that we can generally call "perturbed" metric.



In GR the field equations are invariant under a general coordinate change. This means that the split between a background metric and a perturbed one is not unique.
We would like to keep the FLRW metric as "the" background whenever we make a general transformation.
These are called Gauge transformations

Métrica FLRW (background) → Gauge → Newtonian → ya dentro del horizonte causal. ↳ dentro de la perturbación. — no vemos campos de $\vec{v}$.

Gauge → Synchronous → encima de la perturbación (Early time cosmology) ↳ cuando las estructuras comienzan a estar dentro del horizonte causal.

The choice of gauge is not an easy task, it depends on what you want/need.
In cosmology we use two main gauges:

- Newtonian gauge: attach the "observer" to the unperturbed frame. — the observers will detect a velocity field of particles falling into the clumps of matter and will measure a gravitational potential
- Synchronous gauge: attach the "observer" to the perturbed particle. — they do not see any velocity field and there is no gravitational potential

Which one to use?

Newtonian is used for perturbations inside the horizon: late time cosmology

Synchronous is used for perturbations outside the horizon: early time cosmology

Newtonian (or longitudinal) gauge

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$$

Most general metric

simétrica: homogeneidad — cant. vectorial

$$\boxed{\delta g_{\mu\nu} = a^2 \begin{pmatrix} -2\psi & w_i \\ w_i & 2\phi\delta_{ij} + h_{ij} \end{pmatrix}}$$

escalar — tensor 3x3.

Here ψ and φ are space scalars
wi is a 3-vector
hij is a traceless 3-tensor
sin traza

Remind that we need to "play" with the equations and we need up indices things. We use the condition:

bajar y subir índice.

$$g_{\alpha\mu} g^{\mu\beta} = \delta^{\beta}_{\alpha} \quad ➡ \quad \delta g^{\mu\nu} = -\delta g_{\alpha\beta}\, g^{(0)\,\alpha\mu}\, g^{(0)\,\beta\nu}$$

↳ en orden lineal xq los términos al cuadrado se eliminan

From Helmholtz's theorem one can decompose the vector wi into a longitudinal and a transverse component

$$\vec{w_i} = \vec{w_i}^{\parallel} + \vec{w_i}^{\perp}$$

rotor — gauss

by construction

$$\boxed{\nabla \cdot w_i^{\perp} = \nabla \wedge w_i^{\parallel} = 0}$$

rotor

Being $\nabla \wedge w_i^{\parallel} = 0$ then $w_i^{\parallel} = \nabla w_s$

vector como el grad. de un escalar.

escalar!

But which to take?

When we derive the Einstein equations for the (0i) components, we will have therefore longitudinal and transverse terms.

Taking the curl of the equations, we are left with only the transverse equations.

Taking the divergence, we are left with the longitudinal ones.

gauge newtoniano o gauge longitudinal.

Therefore, the two components can be treated separately.

The density perturbation δ is a scalar quantity, only the longitudinal terms, which can be derived from a scalar quantity and it couples to the density perturbations.

The same discussion applies to h_{ij}

We can decompose $h_{ij} = h_{ij}^{\parallel} + h_{ij}^{\perp} + h_{ij}^{T}$, and $\partial^i h_{ij}^{\parallel}$, $\partial^i h_{ij}^{\perp}$, h_{ij}^{T}

vectors long. & transv. vectors transv.

Taking $\epsilon_{ijk}\partial_i\partial_k h_{ij}^{\parallel} = 0\,,\quad \partial_i\partial_j h_{ij}^{\perp} = 0\,,\quad \partial_i h_{ij}^{T} = 0$

Curl Divergence

Now, since $\partial_i h^{\parallel}_{ij}$ is curl-free, it can be written in terms of a scalar function B

$$h_{ij}^{\parallel} = \left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right) B = D_{ij} B\,,$$

representa la diagonal

On the contrary, the perturbations $h^{\perp}_{ij}$, h^{T}_{ij} cannot be derived from a scalar function.

The first one is a vector giving rise to rotational velocity perturbations.

The second one is a tensor giving rise to gravitational waves.

We need to take into account only the part of w_i and h_{ij} derived from scalars. This may be done by introducing two new scalar functions, E and B, that produce the vector $E_{,i}$ and the tensor $D_{ij}B$

$$\delta g_{\mu\nu} = a^2 \begin{pmatrix} -2\psi & \cancel{E_{,i}}^{\,0} \\ \cancel{E_{,i}}^{\,0} & 2\phi\delta_{ij} + \cancel{D_{ij}B}^{\,0} \end{pmatrix}$$

* componente vectorial se hace 0 xq no podemos tener direcoes privilegiadas.

Out of the four scalar functions ψ, φ, E, B, one can construct gauge-invariant quantities, that is, combinations that remain invariant at first-order under a general coordinate infinitesimal transformation.

The situation can be much simplified if one works in a specific gauge. This can be done by imposing up to four conditions on the metric, which corresponds to the four gauge coordinate transformations. Here we choose them to be $w_i = 0$ (from which $E = 0$) and $B = 0$, and we are left:

FLRW background.

$$ds^2 = a^2(\tau)\left[-(1+2\psi)\,d\tau^2 + (1-2\phi)\,dx_i dx^i\right]$$

parám. de Hubble al tiempo propio.

Where we used the conformal time $dt = a d\tau$ which changes $H = \frac{da}{a dt} = \frac{da}{a^2 d\tau} = \frac{\mathcal{H}_\tau}{a}$

$$\mathcal{H} = aH = \frac{da}{d\tau}\cdot\frac{1}{a} = \frac{da}{dt}\frac{dt}{d\tau}\frac{1}{a} = \left(\frac{\dot a}{a}\right) a$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$G_{\mu\nu}^{(0)} + \delta G_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(0)} + 8\pi G\, \delta T_{\mu\nu}$$

When working with perturbations, just think of them as a "derivative operator". For instance

$$\delta R_{\mu\nu} = \delta\Gamma^{\alpha}_{\mu\nu,\alpha} - \delta\Gamma^{\alpha}_{\mu\alpha,\nu} + \delta\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} + \Gamma^{\alpha}_{\mu\nu}\delta\Gamma^{\beta}_{\alpha\beta} - \delta\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\alpha\nu} - \Gamma^{\alpha}_{\mu\beta}\delta\Gamma^{\beta}_{\alpha\nu}$$

Which also implies

$$\delta\Gamma^{\mu}_{\nu\lambda} = \frac{1}{2}\delta g^{\mu\alpha}\left(g_{\alpha\nu,\lambda} + g_{\alpha\lambda,\nu} - g_{\nu\lambda,\alpha}\right) + \frac{1}{2}g^{\mu\alpha}\left(\delta g_{\alpha\nu,\lambda} + \delta g_{\alpha\lambda,\nu} - \delta g_{\nu\lambda,\alpha}\right)$$

Doing the calculations (a lot), the only non-zero terms are

Remind we changed variable, the dot refers to conformal time

$$\begin{aligned}
\delta\Gamma^{0}_{00} &= \dot\psi\,, \\
\delta\Gamma^{0}_{ij} &= -\delta_{ij}\left[2(\phi+\psi)\mathcal{H} + \dot\phi\right] \\
\delta\Gamma^{0}_{0i} &= \delta\Gamma^{0}_{i0} = \delta\Gamma^{0}_{00} = \psi_{,i}\,, \\
\delta\Gamma^{i}_{j0} &= \delta\Gamma^{i}_{0j} = -\delta^{i}_{j}\dot\phi
\end{aligned}$$

We want to work in Fourier space.

It's equal to assume that the perturbation variables φ, ψ (and also those of matter) are the sum of plane waves; since the equations are linear, each plane wave obey the same equations with a different k:

$$\begin{aligned}
\phi(x,\tau) &\;\to\; e^{ikr}\phi_k(\tau) \\
\nabla\phi(x,\tau) &\;\to\; i\,e^{ikr}k\,\phi_k(\tau) \\
\nabla^2\phi(x,\tau) = \nabla_i\nabla_j\phi(x,\tau) &\;\to\; -e^{ikr}k^2\phi_k(\tau)
\end{aligned}$$

The Fourier modes e^{ikr} can be dropped out, since the equations are linear and therefore they decoupled.

$$\begin{aligned}
\delta G^{0}_{0} &= \frac{2}{a^2}\left[k^2\phi + 3\mathcal{H}\left(\mathcal{H}\psi + \dot\phi\right)\right] \\
\delta G^{0}_{i} &= -\delta G^{i}_{0} = -\frac{2k}{a^2}\left(\mathcal{H}\psi + \dot\phi\right) \\
\delta G^{i}_{i} &= \frac{2}{a^2}\left\{k^2(\phi-\psi) + 3\psi\left(\mathcal{H}^2 + 2\dot{\mathcal{H}}\right) + 3\left[\mathcal{H}\left(2\dot\phi + \dot\psi\right) + \ddot\phi\right]\right\}, \\
\delta G^{i}_{j} &= -\frac{k^2}{a^2}(\phi-\psi)\,, \qquad \text{when } i \neq j\,.
\end{aligned}$$

ψ1 a τ.

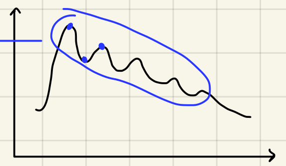
No es una ec. dinámica

constrain entre los potenciales.

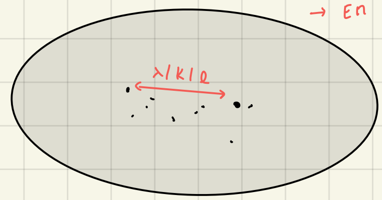
· Funø de correlaø a 2 puntos :

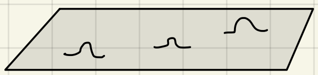
CMB $C(\ell)$ → Angular P.S. $\sim \left\langle \frac{\delta T}{T_0} \frac{\delta T}{T_0} \right\rangle \sim \mu K$ → media de T°.
correlaø c/ respecto a la media.

BAO ←



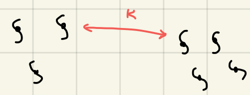
→ En fourier.





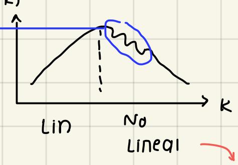
$$\rho(t,\vec{x}) = \bar{\rho}(t) + \delta\rho(t,\vec{x})$$
$$= \bar{\rho}(t)\left[1 + \frac{\delta\rho}{\bar{\rho}}\right]$$

$$\delta = \frac{\delta\rho}{\bar{\rho}} \ll 1$$



$P(k) \sim \langle \delta\delta \rangle$ → la media de las dos fluctuaciones.

BAO ←



· Baile entre fotones y bariones.

· zonas con sobre densidad

$\sim 1/\lambda$

10^{-4} 10^{-2}

→ No funciona $\rho = \bar{\rho} + \delta\rho$.

→ Newtonian Gauge: 2 potenciales gravitaøales.

$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$ = escalar! (invariante)

$$= \begin{pmatrix} -a^2(1+2\Psi) & & 0 & \\ & g_{11} & & \\ 0 & & g_{22} & \\ & & & g_{33} \end{pmatrix} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

$g_{ij} = a^2(1-2\phi)$

porque es de 1° orden perturbativo.

hay que aprenderlos para el examen!

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$
$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}\left[g_{\mu\beta,\nu} + g_{\beta\nu,\mu} - g_{\mu\nu,\beta}\right]$$
$$\Gamma^0_{00} = \underbrace{\bar{\Gamma}^0_{00}}_{\text{background}} + \underbrace{\delta\Gamma^0_{00}}_{\text{parte perturbada.}}$$

$$G_{00} = \cancel{\bar{G}_{00}} + \delta G_{00} = 8\pi G(\cancel{\bar{T}_{00}} + \delta T_{00}) \longrightarrow \delta G_{00} = 8\pi G\, \delta T_{00}$$

Friedmann
$H^2 = \frac{8\pi G}{3}\bar{\rho}$

- Pasar al espacio de fourier.
- Cualquier onda la podemos representar como una suma infinita de términos.

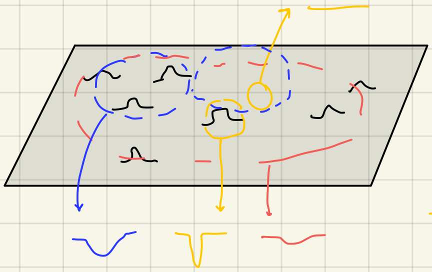
desaparece el prod. escalar entre ambos vectores por la isotropía.

$$\phi(x,\tau) \longrightarrow e^{ikr}\,\phi_k(\tau)$$

$$\nabla\phi(x,\tau) \longrightarrow i\,e^{ikr}\,k\,\phi_k(\tau)$$

$$\nabla^2\phi = \nabla_i\nabla_j\,\phi(x,\tau) \rightarrow -e^{ikr}k^2\phi_k(\tau)$$

- Eventos astronómicos son sistemático $\rightarrow$ tener en consideraø efectos sistemáticos.
- Marginalizar sobre todos los valores de cada parámetro.



Hay que hacer una media de los modos indep.

↳ Hay muchos modos independientes

↳ Menos

Pozo de potencial.

- Un espacio más grande $\Rightarrow$ efectos gravitacionales son + grandes.

Clase 2

Ec. Einstein:

$$\delta G^{\mu}{}_{\nu} = 8\pi G\,\delta T^{\mu}{}_{\nu}$$

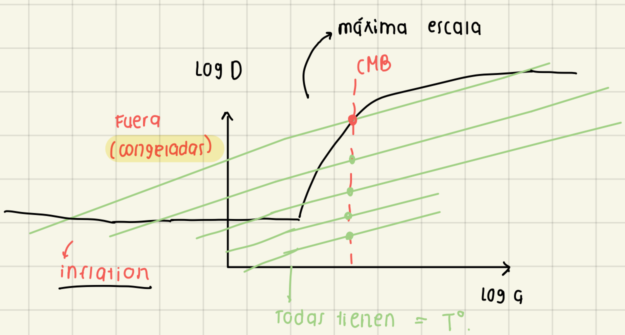
$$\delta G^{i}{}_{j} = k^2(\phi - \psi) = 8\pi G T^{i}{}_{j}$$

$$\rightarrow T^{\mu}{}_{\nu} = (\rho + P)\,u^{\mu}u_{\nu} + P\delta^{\mu}{}_{\nu} + \pi^{\mu}{}_{\nu}$$

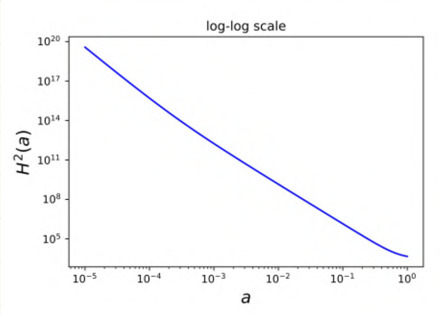
$T^{i}{}_{j} = 0 + \pi^{i}{}_{j} \longrightarrow$ ninguna componente contribuye

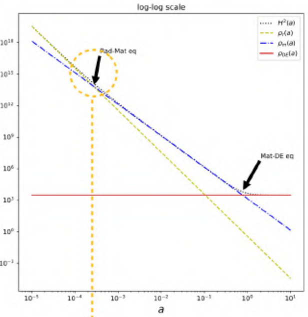
viscosidades o efectos particulares del fluido.

* CAMB $\longrightarrow$ toma un k y evalúa la ec. diferencial
CLASS



$$D \sim \int \frac{1}{a^2 H} \longrightarrow \text{distancia comovente}$$





Radiation-Matter equality

$$\rho_r(a) = \rho_m(a) \implies \Omega_{r,0}a_{eq}^{-4} = \Omega_{m,0}a_{eq}^{-3}$$

T° baja y la materia empieza a dominar antes del desacople. (CMB)

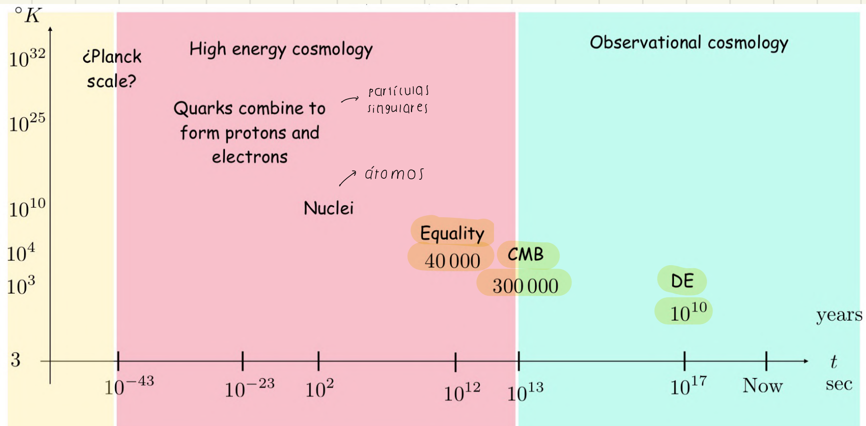
$$a_{eq} = \frac{\Omega_{r,0}}{\Omega_{m,0}} = 2.61 \times 10^{-4}$$

Matter-DE equality

$$\rho_m(a) = \rho_{DE}(a) \implies \Omega_{m,0}a_{eq}^{-3} = \Omega_{DE,0}a_{eq}^{-3(1+w)}$$

$$a_{eq} = \left(\frac{\Omega_{m,0}}{\Omega_{DE,0}}\right)^{\frac{1}{3}} = 0.76$$

Important for structure formation and it is the "real" equality



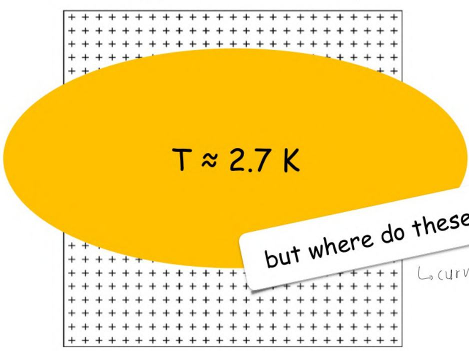
Where and how structures form?

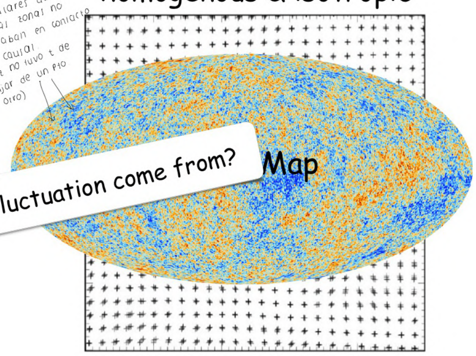
Inflación estaba en contacto causal

homogenous & isotropic

son muy similares aunque estas zonas no estaban en contacto causal (luz no tuvo t de viajar de un pto a otro)

homogenous & isotropic





but where do these matter fluctuation come from?

↳ curvatura ~ 0.

background

w/ small perturbations

$$\frac{\Delta T}{T} \sim 10^{-5}$$

What about the matter component? We do exactly the same thing.

viscosidad del fluido

$$T_{\mu\nu} = \underbrace{(\rho+p)\,u_\mu u_\nu + p\,g_{\mu\nu}}_{\text{usual terms}} + \underbrace{[q_\mu u_\nu + q_\nu u_\mu + \pi_{\mu\nu}]}_{\text{new terms}} \neq 0$$

- para fotones antes de la CMB
- Neutrinos

· alguna componente del fluido se escapa, pierde energía

q = heat vector

π = sheer stress

If they are zero ➡ perfect fluid
(new terms)

We assume this

métrica perturbada, aprox. lineal (taylor)

$$\begin{aligned} u^\mu &= \frac{dx^\mu}{ds} = \left\{\frac{1}{a\,(1+2\psi)}, \frac{dx^i}{a\,d\tau}\right\} = \left\{\frac{1}{a}(1-\psi)\,, \frac{v^i}{a}\right\} \\ u_\mu &= g_{\mu\nu}u^\nu = \left\{-a\,(1+\psi)\,, a\,v_i\right\} \\ u_\mu u^\mu &= -1 \end{aligned}$$

con respecto a un invariante

adimensional

$$\delta = \frac{\delta\rho}{\rho},$$

contraste de densidad

$$\theta = \nabla_i v^i = ik_i v^i$$

divergencia de la $\vec{v}$

And we perturbed the EM tensor of the fluid

?

$$\delta T^\mu_\nu = (\delta\rho + \delta p)\,u_\nu u^\mu + (\rho+p)\,(\delta u_\nu u^\mu + u_\nu \delta u^\mu) + \delta p\, g^\mu_\nu + p\,\delta g^\mu_\nu$$

Barotropic fluid

$$\delta p = \boxed{c_s^2}\delta\rho = \frac{dp}{d\rho}\delta\rho = \frac{\dot{p}}{\dot{\rho}}\delta\rho = w\delta\rho$$

For constant w

vel. del sonido

Admitimos que dependa sólo del tiempo y sea adiabático

The perturbed Einstein equations

0 - 0	$k^2\phi + 3\mathcal{H}\left(\mathcal{H}\psi + \dot{\phi}\right) = -4\pi G a^2 \delta\rho$
0 - i	$\mathcal{H}\psi + \dot{\phi} = 4\pi G a^2 (1+w)\rho\theta$
i - i	$\frac{1}{3}k^2(\phi-\psi) + \psi\left(\mathcal{H}^2 + 2\dot{\mathcal{H}}\right) + \mathcal{H}\left(2\dot{\phi} + \dot{\psi}\right) + \ddot{\phi} = -4\pi G a^2 c_s^2 \delta\rho$
i - j	$\phi = \psi$

δp

→ c.l. de las otras 2 ecs.

The perturbed fluid equations… we want the fluid to be conserved $T_{\mu\nu;\mu} = 0$

$$\delta T^\mu_{\nu;\mu} = \delta T^\mu_{\nu,\mu} - \delta\Gamma^\alpha_{\nu\beta}T^\beta_\alpha - \Gamma^\alpha_{\nu\beta}\delta T^\beta_\alpha + \delta\Gamma^\beta_{\beta\alpha}T^\beta_\nu + \Gamma^\beta_{\beta\alpha}\delta T^\beta_\nu = 0$$

ν = 0 for our density contrast

$$\dot{\delta} = -(1+w)\left(\theta - 3\dot{\phi}\right) - \mathcal{H}\left(\frac{\delta p}{\rho} - w\delta\right)$$

ec. dinámica c/r a δ.

Dependen de los campos grav. del fluido.

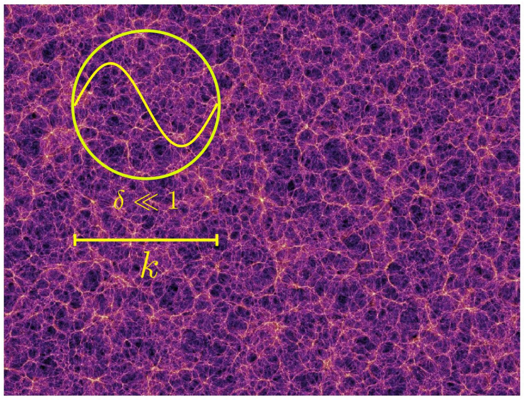
Espacio-tiempo modifica la materia.

Todo conectado

ν = i for our velocity component

$$\dot{\theta} = -\mathcal{H}(1-3w)\,\theta - \frac{\dot{w}}{1+w}\theta + \frac{1}{1+w}\frac{k^2\delta p}{\rho} + k^2\psi$$

We used the wave number k in our equations but what does it mean practically?



This is how matter distributes: cosmic web

Let's do some numbering

$$\Omega_{m,0} = 0.32$$

$$\rho_{\text{crit}} = \frac{3\,H_0^2}{8\pi\,G} \sim 10^{-26}\text{kg/m}^3$$

$$\rho_{m,0} \sim 10^{-26}\text{kg/m}^3$$ → densidad de background

$$\rho_{\text{galaxy}} \sim 10^{-19}\text{kg/m}^3$$ → perturbac. (galaxia).

$$\delta \sim 10^7 \not\ll 1$$ → $\frac{\rho_m}{\rho_{galaxy}} = \delta$ ✱

By saying we consider perturbations of a certain scale means considering certain volumes.

importante

But the scale k = $2\pi/\lambda$ and λ is a distance, which is measured in Mpc. λ = 100Mpc k = 0.062 Mpc^{-1}.

The equations of perturbations depend on the scale k; hence the density field δ (and all the other perturbed quantities) will grow differently at different scales.

There exist typical scales above or below which perturbations can grow, stay constant or even decrease.

· En inflación todas las perturbaciones estaban dentro del horizonte causal.

↱ distancia la cual viajó el fotón

Causal horizon & sound horizon.

↱ cambia en el tiempo. + t le dan al fotón de viajar ⇒ + espacio puede viajar

Causal horizon:

↳ límite observativo (hasta donde medimos)

the scale above which there's no communication

↱ la perturbación

$$k_{\rm ch} = a\,H(a)$$

It's a scale that evolves with time.

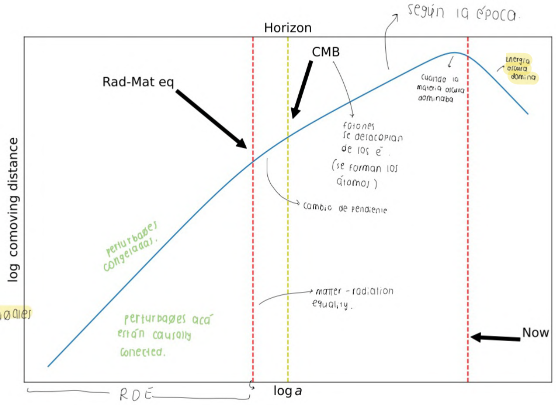
For perturbations to grow they have to be causally connected, i.e. inside this horizon

↳ tienen que comunicarse de alguna manera

perturbaciones tienen que estar dentro de h_c causal para que los efectos gravitacionales se puedan sentir.

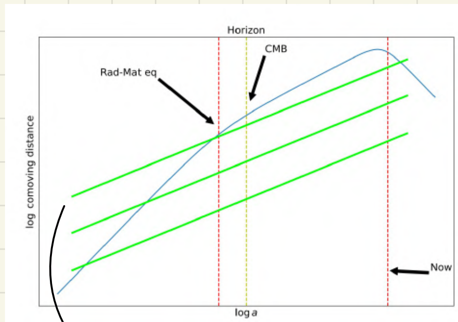
$$\lambda = a\,\lambda_0 \quad \text{or} \quad k \sim 1/\lambda$$

escala en el pasado — el valor medido ahora

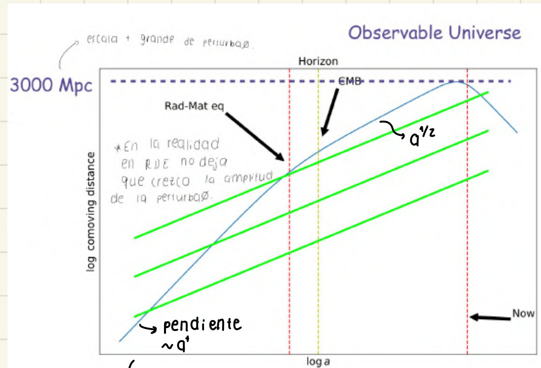


* k y λ es una escala fija (cada línea verde) pero lo que evoluciona es δ.

· Inflación permite que entren las perturbaciones.



perturbaciones que entran después tienen menos tiempo para evolucionar



comoving distance $\sim \int \frac{1}{a^2 H}$

Different perturbations enter the horizon at different epoch, depending when they will grow at a certain rate.

Sound horizon:

the scale above which there's communication but below which the pressure support is too strong

$$k_{\rm sh} = a\,H(a)/c_s$$

≠ para ≠ componentes de materia.

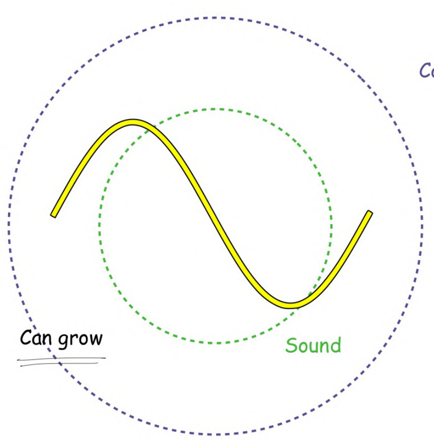
It's a scale that also evolves with time.

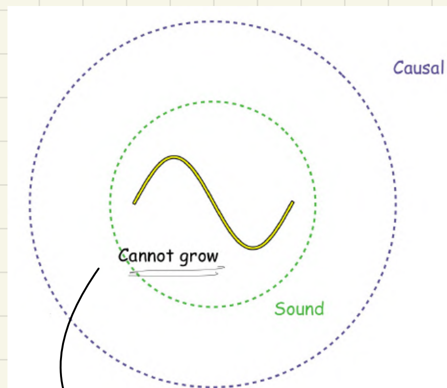
Look that c_s < 1 (=1 light)

➡ $k_{sh} > k_{ch}$

It depends on the particular fluid we are considering:

- radiation c_s = 1/3 — radiación hace que no colapsen las perturbaciones
- matter c_s = 0
- DE c_s ~ 1 ⟶ perturbaciones no pueden crecer



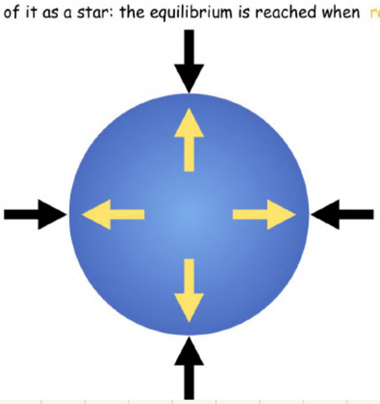


pueden colapsar.

· Si c_s = 1, las pert. recién entraron al horizonte causal así que no han podido crecer.

What is this pressure support for a fluid?

Think of it as a star: the equilibrium is reached when radiation pressure balances gravity



But what c_s = 0 means?

Matter perturbations at any scale can grow indefinitely

↱ si los comprimimos mucho se colapsan

Is it unrealistic? Maybe for baryons since they interact each other with other forces...

Remember: we're considering cosmological scales. Very small scales cannot be treated with linear theory

* CDM → cold: no tiene un término de presión importante podemos comprimirlo mucho.

Scale larger than the horizon k << aH

Ecs. generales para cualquier componente.

$$\dot\delta = -(1+w)\left(\theta - 3\dot\phi\right) - \mathcal{H}\cancel{\left(\frac{\delta p}{\rho} - w\delta\right)}$$

$$\dot\theta = -\mathcal{H}(1-3w)\,\theta - \cancel{\frac{\dot w}{1+w}\theta} + \cancel{\frac{1}{1+w}\frac{k^2\delta p}{\rho}} + \cancel{k^2\psi}$$

Assume perfect barotropic fluid $\delta p = w\,\delta\rho$

If you want to solve the eqs just replace the epoch and the fluid

Assume also ϕ = const

$$\ddot\delta + 3aH(1-3w)\dot\delta = 0$$

δ = constant

δ = decaying → xq no hay un término de fuente proporcional a δ. (x eso δ no aumenta)

$$k^2\phi + 3aH(\cancel{\dot\phi + aH\psi}) = -4\pi G a^2 \rho\delta$$

Perturbations outside the horizon do not grow

Scale smaller than the horizon k >> aH

$$\dot\delta = -(1+w)\left(\theta - 3\dot\phi\right) - \mathcal{H}\left(\frac{\delta p}{\rho} - w\delta\right)$$

$$\dot\theta = -\mathcal{H}(1-3w)\,\theta - \frac{\dot w}{1+w}\theta + \frac{1}{1+w}\frac{k^2\delta p}{\rho} + k^2\psi$$

→Pressurless matter, w=0 but with some c_s

$$\begin{aligned}\dot\delta &= -\theta + \cancel{3\dot\phi} - \cancel{\mathcal{H}c_s^2\delta}\,,\\ \dot\theta &= -\mathcal{H}\theta + c_s^2k^2\delta + k^2\phi\end{aligned}$$

k >> aH → ec. de Einstein

$\phi \sim \delta/k^2$

$$\delta'' + \mathcal{H}\delta' + (c_s^2k^2 + k^2\phi) = 0$$

$$k^2\phi = -4\pi G a^{-1}\delta$$

We find $\ddot\delta + \mathcal{H}\dot\delta + \boxed{\left(c_s^2k^2 - \frac{3}{2}\mathcal{H}^2\right)}\delta = 0$

$$c_s^2k^2 - \frac{3}{2}\mathcal{H}^2 \to \begin{cases}\geq 0 & \text{oscillating}\\ <0 & \text{growing}\end{cases}$$

$2\pi/\lambda$

Jeans scale

$$\lambda_J = c_s\sqrt{\frac{\pi}{G\rho}}$$

→ Jean scale es básicamente la $\vec{v}$ del sonido. (perturbaø más pequeña que puede crecer).

↳ A escalas más pequeñas, la perturbación no crece.

In MDE

Pure CDM w=cs=0

→ Pure CDM: pudo desacoplarse antes que la radiación.

$$\ddot\delta + \mathcal{H}\dot\delta - \frac{3}{2}\mathcal{H}^2\delta = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau} = a^2H\frac{\mathrm{d}}{\mathrm{d}a}$$

$$\delta'' + \left(\frac{3}{a} + \frac{H'}{H}\right)\delta' - \frac{3}{2\,a^2}\delta = 0$$

The presence of H might lead to different growth: we can replace any dominant component, but it's not that easy. Remind that the gravitational potential is the sum of all the components, hence it will take the dominant component at that time.

cuando a la materia le cuesta acoplarse, δ disminuye un poco xq la expansión va muy rápido.

$$H = H_0\sqrt{\Omega_{m,0}a^{-3}} \qquad \delta'' + \frac{3}{2a}\delta' - \frac{3}{2a^2}\delta = 0 \qquad \delta = c_1\,a + c_2\,a^{-3/2}$$

sol. que crece   sol. que decrece.

During the epoch? They'll grow slower. So MDE is the most interesting epoch for LSS.

Extra info: we said that in linear theory each wave evolves independently and what we did is

$$\delta = \delta(a,k) = \delta(a)\delta(k)$$

→ xq cualquier escala evoluciona independiente de las otras

In RDE

Pure CDM w=cs=0

$$\dot\delta = -\left(\theta - 3\dot\phi\right)$$

$$\dot\theta = -\mathcal{H}\theta + k^2\psi$$

para CDM (comparamos $\delta_r \ll \delta_m$)

$$\frac{\mathrm{d}}{\mathrm{d}\tau} = a^2H\frac{\mathrm{d}}{\mathrm{d}a}$$

· CDM se pudo desacoplar del baño térmico mucho antes que los bariones.

And the potential is:

$$k^2\phi = -4\pi G a^2\sum_s \rho_s\left[\delta_s + \frac{3aH}{k^2}(1+w_s)\theta_s\right]$$

→ Ahora tenemos que incluir todas las componentes! → componente que domina es la radiaø.

And we saw that for pure CDM in MDE the potential had only CDM. If we assume we are in RDE (and still small scales) then

$$k^2\phi = -4\pi G a^2\rho_r\delta_r \sim 0$$

→ fluctuaø pequeña con respecto a la CDM.

Small wrt CDM

$$\delta'' + \left(\frac{3}{a} + \frac{H'}{H}\right)\delta' = 0 \qquad \text{And with:} \quad H = H_0\sqrt{\Omega_{r,0}a^{-4}} \implies \delta = \text{const.} + \ln(a)$$

→ no crece la perturbaø.