1 Abstract

The Euclid space telescope is measuring the shapes and redshifts of galaxies to reconstruct the expansion history of the universe. Estimating its expected performance, particularly its predicted errors in cosmological parameters, is essential to make modifications that optimize the performance of the telescope. For this purpose, over the past few years the Euclid Collaboration has developed various codes that calculate these errors from observational probes, such as galaxy clustering (GC) from spectroscopy and photometric observations, as well as weak lensing (WL) data. GC measures the luminosity of galaxies to determine their distribution across the sky, while WL measures changes in the ellipticity of galaxies caused by the presence of large-scale structures between us and the objective.

The Fisher matrix formalism is a valuable tool for validating how data will be collected by Euclid. This matrix has been calculated based on various cosmological models and plays a crucial role in determining how well current or future experiments can constrain cosmological parameters. It is also used to define the confidence regions within which the data is validated.

The primary goal of this proposal is to develop a validation code aimed at calculating errors in cosmological parameters using the Fisher matrix formalism for the three afore mentioned cosmological probes. Two separate codes have been designed for this purpose, the first focusing on GC probes from spectroscopy observations, and the second working with WL observables. These have been constructed to operate within different cosmological models, each focusing on varying numbers of parameters. The next step is to develop a unified code that combines these two probes, along with GC photometric observables, allowing us to obtain three results, one from each probe. In this way, another objective is to make these codes user-friendly, so others can easily modify them to add new cosmological parameters, models, settings, or even change the survey range.

It is known that the Fisher matrix does not provide the best-fit values for parameters. Therefore, we aim to explore various possible fiducial cosmologies that could serve as the best-fit model for a given dataset. As an example, we can consider the Hu-Sawicki $f(R)$ cosmological model which introduces an universal modified gravity force in a scale-dependent way. Likewise, we can estimate how well future Euclid data will be able to constrain the extra parameter of the theory, f_{R0}, for the range in which this is still allowed by current observations. As can be seen, this procedure could be applied to other parameters within a cosmological model that aligns well with the data.

Another advancement that will be made in the project is the implementation of additional observational effects that can be modified to enhance data acquisition. To extract cosmological information from a cluster survey, we introduce the following functions: purity, which is the probability that a given detection corresponds to a real object, and completeness, the probability that an object in the real population will be detected in the survey.

During the development of this research, we use different programming paradigms in Python, such as Object-Oriented Programming and Procedural Programming. The process for obtaining the Fisher matrix varies for each observable, which shares the fact of having as input files matter densities obtained from CAMB (Code for Anisotropies in the Microwave Background). CAMB is a code for numerical calculations of cosmological functions based in a theoretical model. One of those functions is the matter Power Spectrum.

For spectroscopic GC, we modify the Power Spectrum of galaxies to account for observable changes, then compute an initial Fisher matrix for a first set of parameters. Afterward, we compute the final Fisher matrix by creating a new matrix that varies with each model. For WL, we compute a significant number of numerical integrals, building a mathematical tool that correlates the changed ellipticities of each galaxy observed by Euclid. This is called the Cosmic Shear Power Spectrum and serves as the main tool for obtaining the Fisher matrix. Once we have this last one, we invert it to obtain the covariance matrix, where the errors in cosmological parameters are represented in each of its elements. Lastly, for GC from photometric probes (GC_{ph}), the procedure is similar to the WL code, with the difference that we now correlate sectors of the universe affected by weak lensing and grouped into clusters. This yields a third Fisher matrix with the same characteristics as the others.

Considering this, the research will provide insights into how mathematical tools are applied to cosmological probes, how to import input files created with CAMB by providing the corresponding parameters, and how codes are typically developed for large sets of parameters. The expected results are cosmological errors that will be compared to the errors obtained by the Euclid Collaboration, and if they are sufficiently close, we will be able to validate our code within the collaboration.

1 Theoretical-conceptual foundations and state of the art

The calculation of errors in cosmological parameters began to be formalized as cosmology evolved into a modern scientific discipline. One of the most powerful tools for computing these errors is the Fisher matrix.

This study builds on the methodology used by the Euclid Collaboration in their forecasts to validate the Euclid cosmological probes prior to the launch of the telescope (Blanchard et al. 2020). The aim was to assess the impact of design decisions on the final results. For this purpose, the cosmological probes considered include GC, WL, and GC_{ph}.

The main objective of this forecast is to evaluate how effectively Euclid can distinguish the standard cosmological model from simple alternative dark energy scenarios, all based on the telescope's design specifications. Specifically, we follow the guidelines of the inter-science taskforce for forecasting (IST:F), which employs the Fisher Matrix formalism to compute errors for nine key cosmological parameters $\{ h, \Omega_{m,0}, \Omega_{DE,0}, \Omega_{b,0}, \sigma, n_s, \gamma, w_0, w_a \}$, from it fiducial values, which are shown in the following table.

Parameter	$\Omega_{m,0}$	$\Omega_{b,0}$	$\Omega_{DE,0}$	w_0	w_a	h	n_s	σ_8	γ
Fiducial value	0.32	0.05	0.68	-1	0	0.67	0.96	0.816	0.55

Table 1: Fiducial values of the cosmological parameters taken into account.

In our work, we aim to develop a code that achieves no more than a 10% deviation from the results of the Euclid Collaboration, with the goal of certifying our code as one of the collaboration's validated tools.

1.1 Spectroscopic galaxy clustering.

The matter power spectrum provides valuable information about the variance in the density contrast of matter across the universe. Euclid, through its spectroscopic galaxy clustering (GC_{spec}) observations, will derive the observed matter power spectrum of galaxies. To simulate this, we aim to construct the probe starting from the matter power spectrum computed with CAMB, incorporating various observational effects to produce the observed power spectrum. The five primary effects that need to be modeled are [4]:

1. The galaxy bias: Represents the relationship between the distribution of galaxies and the underlying dark matter distribution in the universe.
2. Anisortopies due to RSD: These arise due to the peculiar velocities of galaxies, causing distortions in their observed positions and introducing anisotropic features in the large-scale structure. This alters the apparent clustering of galaxies in redshift surveys.
3. The residual shot noise: Refers to the statistical noise resulting from the discrete sampling of galaxies, as opposed to observing a continuous matter density field.
4. The redshift uncertainty: Refers to errors in measuring galaxy redshifts, which impact the inferred positions of galaxies in cosmological surveys.
5. Distortions due to the AP effect: These occur when incorrect cosmological model assumptions distort the conversion of redshifts and angular positions into distances.

With these effects included, we construct the Fisher Matrix for an initial parameter set and then project it into another parameter space, tailored to the specific model being analyzed.

For this study, we create the Fisher Matrix for the most comprehensive parameter set, which corresponds to a non-flat model with w_0, w_a, γ. The results for this model, obtained using spectroscopic GC probes from the IST:F group under a pessimistic setting, are presented in the table below.

Parameter	$\Omega_{m,0}$	$\Omega_{b,0}$	$\Omega_{DE,0}$	w_0	w_a	h	n_s	σ_8	γ
Error	0.18	0.27	0.11	0.40	1.5	0.029	0.070	0.050	0.33

Table 2: Result for non-flat model with w_0, w_a, γ, for GC_{spec}. By IST:F group.

1.2 Weak Lensing

Measuring the errors in cosmological parameters from Weak Lensing (WL) probes requires the construction of a two-point correlation function called Cosmic Shear, which, with the matter power spectrum, provide information of the modify ellipticity of a galaxy with respect to another one.

The Euclid Collaboration decided to divide the three-dimensional space surveyed by the telescope into 10 bins, each separated to ensure an equal number of galaxies inside. Additionally, they considered another cosmological phenomenon that affects the original ellipticity of galaxies, known as Intrinsic Alignment (IA). IA refers to the alignment of galaxies or their structures (such as their shapes or spins) due to physical processes that occurred during their formation or evolution, rather than being a result of gravitational lensing or external forces.

As mentioned above, shear is the change in the ellipticity of the image of a background galaxy, caused by the lensing effect of large-scale structure along the line of sight. For an individual galaxy, this can be expressed by the following formula:

$$\varepsilon = \gamma + \varepsilon^I, \tag{1}$$

where ε is the intrinsic ellipticity of the galaxy, γ represents the change induced by gravitational lensing, and ε^I is the observed intrinsic ellipticity. The ellipticity is a spin-2 quantity with a non-zero two-point correlation function (or power spectrum), and its spherical harmonic transform is called the angular power spectrum [4]. To facilitate the computation of this function, the Limber approximation was used, and its final form can be seen in equation (12).

With this function, we are able to build the Fisher Matrix with de equation (13). Considering the same space of parameters than in the GC probe, the results from the IST:F group for the non-flat model with w_0, w_a, γ are shown in the following table.

Parameter	$\Omega_{m,0}$	$\Omega_{b,0}$	$\Omega_{DE,0}$	w_0	w_a	h	n_s	σ_8	γ
Error	0.055	7.8	0.037	0.25	1.9	0.23	0.034	0.029	1.1

Table 3: Result for non-flat model with w_0, w_a, γ, for WL probe. By IST:F group[4].

1.3 Photometric galaxy clustering

The last cosmological probe is photometric Galaxy Clustering (GC_{ph}) observations, where the variations in the brightness of galaxies are measured. In this case, we will focus on the combination of the last two probes, adding this one to complete the set of possible observations for Euclid.

To this end, we will construct a two-point correlation function of sectors of the universe, rather than individual galaxies. These sectors are affected by weak lensing (WL) and are combined with galaxy clustering. The latter will be divided into the two modes of observation. The final Fisher matrix can then be written as in equation (16).

The results for the same model considered with the last two cosmological probes can be seen in the following table.

Parameter	$\Omega_{m,0}$	$\Omega_{b,0}$	$\Omega_{DE,0}$	w_0	w_a	h	n_s	σ_8	γ
Error	0.012	0.036	0.033	0.044	0.30	0.0057	0.0058	0.0062	-

Table 4: Result for non-flat model with w_0, w_a, γ. By IST:F group [4].

It is worth noting that the last parameter (γ) is not included in the table. This is because CAMB cannot easily incorporate this parameter into its formulation of the matter power spectrum for this probe, and the Euclid Collaboration decided to omit it from the results.

1.4 New cosmological models

As the Fisher Matrix does not work with data, we can implement another cosmological models to test how close the parameters are to their fiducial values.

At the moment, we are considering nine cosmological models that are related to the non-flat model with w_0, w_a and γ. Each of these has a different number of parameters, and we are computing their associated errors with the goal of improving the use of Euclid. In this context, the Fisher matrix does not provide a best-fit function based on a given data set, but rather constrains the errors around the fiducial values of each parameter in the model under consideration. For this reason, there is nothing that prevents us from considering alternative cosmological models and computing the errors in their respective parameters.

In this context, we aim to consider the Hu-Sawicki model[1], a well-known extension of the Hilbert-Einstein action that introduces a universal modified gravity force in a scale-dependent manner. Specifically, this adds a new parameter f_{R0} which appears in the theory by the following relation:

$$S = \frac{c^4}{16\pi G_N} \int d^4x \sqrt{-g}[R + f(R)], \tag{2}$$

where

$$f(R) = -6\Omega_{DE,0} \frac{H_0^2}{c^2} + |f_{R0}| \frac{\bar{R}_0^2}{R}. \tag{3}$$

The equation (2) shows a modification of the Hilbert-Einstein action by adding an $f(R)$ term to the Ricci scalar R. We will focus on a specific model proposed by Hu & Sawicki, which is given by equation (3). The parameters appearing in this expression are part of the cosmological models we are considering, with the exception of f_{R0}, which is a new parameter introduced and is in the limit of $|f(R)| << 1$. In this way, the standard model is recovered when $f_{R0} \to 0$.

With this in mind, modifying the code to include this new cosmological parameter involves two important steps. For the spectroscopic probe of GC, we will adopt a phenomenological approach to account for the scale dependence of the growth of perturbations. On the other hand, for the photometric probes of weak lensing and the correlation between WL and GC, we will use a fitting formula developed in the literature that captures the modifications to the non-linear matter power spectrum induced by the $f(R)$ model.

1.5 Changing the accuracy of the power spectrum: Purity and completeness

Contamination by objects that do not belong to galaxy clusters (outliers) affects the accuracy of power spectrum estimates. The purity is the probability that a given detection corresponds to a real object, while the completeness is the probability that an object in the actual population will be detected in the survey. Since the assumed purity and completeness impact the effective volume and the observed power spectrum, specific cases with different values of purity and completeness can be used, including scenarios where one of these properties is fixed while the other is left as a free parameter.

2 Hypothesis or research questions and objectives

2.1 Questions

1. Can the Euclid telescope continue to be optimized? What aspects can be improved?
2. What parameters need to be considered when obtaining data with Euclid?
3. Is it possible to build a code that correctly calculates errors in cosmological parameters while running in optimal time?
4. What numerical methods can make the latter possible? Can this code be user-friendly, such that its functions are not difficult to use?

2.2 Objectives

1. GO: Develop a code to calculate errors in cosmological parameters and achieve an optimal minimum of these, based on numerically generated data with CAMB. With this, modify the physical considerations used when obtaining data from Euclid to improve its performance.

2. SO1: Create a user-friendly code that allows the modification of specifications when processing data.
3. SO2: Use efficient numerical methods to reduce the execution time of the code.
4. SO3: Add new cosmological models to compute the parameter errors. Specifically, consider the Modified Gravity model described by Hu-Sawicki.
5. SO4: Adjust the assumed accuracy of the power spectrum by varying the effective survey volume across different scenarios.

Hypothesis

It is possible to build a code capable of calculating errors in cosmological parameters that is user-friendly and time-efficient, with the aim of improving the performance of the Euclid telescope.

3 Methodology

3.1 Fisher matrix

As previously mentioned, the primary objective is to enhance the data collection capabilities of the Euclid mission. With this in mind, our goal is to develop a code that can effectively estimate the errors in our cosmological parameters without needing to map the universe to assess the telescope's performance. To achieve this, we have employed the Fisher Matrix, a mathematical tool that provides these error estimates without requiring direct data acquisition.

In this way, after data collection, the function that best fits the data is determined by minimizing the chi-squared statistic, as shown in the equation (4)

$$X^2(\{\lambda_a\}) = \sum_{l=1} \frac{[\hat{C}(l) - C^{theory}(l, \{\lambda_a\})]^2}{\mathrm{Var}[\hat{C}(l)]}. \tag{4}$$

This method is used to identify the model parameters that make the theoretical predictions align as closely as possible with the observed data. Essentially, it helps find the best fit of a function to a set of data points, which corresponds to the maximum of a likelihood function. This maximum is called the fiducial value of a cosmological parameter. It is important to note that there are several methods to obtain this minimum of chi-squared, such as the Markov Chain Monte Carlo (MCMC) or Metropolis-Hastings algorithms.

Despite this, we are not doing this process but we are focussing on obtain how quickly chi-squared changes as a given cosmological parameter moves away from its fiducial value. Expanding chi-squared around its minimum we have,

$$X^2 = X^2(\bar{\lambda}) + \mathcal{F}(\lambda - \bar{\lambda})^2, \tag{5}$$

where

$$\mathcal{F} \equiv \frac{1}{2}\frac{\partial^2 \mathrm{X}^2}{\partial \lambda^2} \tag{6}$$

as the linear term vanishes since chi-squared is at a minimum at its fiducial value. We can notice that if the curvature F is small, the likelihood changes slowly and the data are not very constraining so, the resulting uncertainties on the parameter will be large. Since we are assuming that the likelihood for $C(l)$ is Gaussian, so that $\ln\mathcal{L} = -\mathrm{X}^2/2$. Therefore, the 1-sigma error on lambda is indeed simply $1/\mathcal{F}$.

Writing that equation, we have

$$\mathcal{F} = \sum_l \frac{1}{\mathrm{Var}\,\hat{C}(l)} \left[\left(\frac{\partial C_{\text{theory}}(l,\lambda)}{\partial \lambda} \right)^2 + \left(C_{\text{theory}}(l,\lambda) - \hat{C}(l) \right) \frac{\partial^2 C_{\text{theory}}(l,\lambda)}{\partial \lambda^2} \right], \tag{7}$$

and taking the expectation value,

$$\mathbf{F} \equiv \langle \mathcal{F} \rangle = \sum_l \frac{1}{\mathrm{Var}\,\hat{C}(l)} \left(\frac{\partial C_{\text{theory}}(l,\lambda)}{\partial \lambda} \right)^2. \tag{8}$$

Therefore, as $\mathbf{F}$ contains information of the average of curvature of the likelihood at its maximum, we can name it as the Fisher information which contains only "theory" quantities. The generalization of this quantity to many parameters is called the Fisher matrix [3] and is given by

$$\mathbf{F}_{\alpha\beta} = \sum_{l} \frac{1}{\mathrm{Var}\,\hat{C}(l)} \frac{\partial C_{\text{theory}}(l, \{\bar{\lambda}_\gamma\})}{\partial \bar{\lambda}_\alpha} \frac{\partial C_{\text{theory}}(l, \{\bar{\lambda}_\gamma\})}{\partial \bar{\lambda}_\beta}. \tag{9}$$

With this in mind, we can compute the Fisher matrix using various cosmological probes. We will focus on three of them: galaxy clustering from spectroscopy and photometric surveys GC_{spec} and GC_{ph}, respectively) and weak lensing observables (WL), which can be theoretically constructed using CAMB. Each of these probes will be observed by Euclid, with specific considerations. For instance, the survey area for GC_{spec} is divided into four equidistant bins, whereas for WL and GC_{ph}, there are ten logarithmically spaced bins designed to ensure a consistent number of galaxies.

The GC_{spec} probes gives us information about the Fourier coefficients δ_k of a galaxy distribution and their power spectrum calculated for a set of wave numbers k_i in some redshift bin. Using that, we can construct the Fisher matrix as

$$\mathbf{F}_{\text{bin},\alpha\beta}(z_i) = \frac{1}{8\pi^2} \int_{-1}^{1} d\mu \int_{k_{\min}}^{k_{\max}} k^2 dk \left[\frac{\partial \ln P_{\text{obs}}(k,\mu;z_i)}{\partial \alpha} \frac{\partial \ln P_{\text{obs}}(k,\mu;z_i)}{\partial \beta} \right] V_{\text{eff}}(z_i;k,\mu). \tag{10}$$

Where the full non-linear observed power spectrum is represented as,

$$P_{\text{obs}}(k_{\text{ref}}, \mu_{\text{ref}}; z) = \frac{1}{q_\perp^2 q_\perp} \left\{ \frac{\left[b\sigma_8(z) + f\sigma_8(z)\mu^2\right]^2}{1 + [f(z)k\mu\sigma_p(z)]^2} \right\} \frac{P_{\text{dw}}(k,\mu;z)}{\sigma_8^2(z)} F_z(k,\mu;z) + P_s(z). \tag{11}$$

The observed galaxy power spectrum relies on the de-wiggled matter power spectrum, which is the sum of two components: the linear matter power spectrum obtained from a Boltzmann code (CAMB) and the linear matter power spectrum without wiggles but exhibiting the same broad band as the linear one. Both components are modulated by a function that controls the non-linear damping of the BAO signal in all directions. The BAO signal is a key feature, representing the regular, periodic fluctuations in the density of visible baryonic matter in the universe. A detailed description of the terms in this equation can be found in [4].

On the other hand, WL is a physical phenomena in which the ellipticities of the galaxies are change because the lensing of large-scale-structures. Measuring the correlation between two galaxies separated by spherical-harmonics, allow us to construct the Cosmic Shear Power Spectrum ($C_{ij}^{\epsilon\epsilon}$), a two-point statistic function that describes the change of the ellipticity of one galaxy with respect to another one. The theoretical structure of this function is the following

$$C_{ij}^{\epsilon\epsilon} = \frac{c}{H_0} \left[\int dz \frac{W_i^\gamma W_j^\gamma}{E(z) r^2(z)} P_{\delta\delta} + \int dz \frac{W_i^\gamma W_j^{IA} + W_i^{IA} W_j^\gamma}{E(z) r^2(z)} P_{I\delta} + \int dz \frac{W_i^{IA} W_j^{IA}}{E(z) r^2(z)} P_{II} \right] + N_{ij}^{\epsilon}, \tag{12}$$

where l is the angular separation of the galaxies and $N_{ij}^{\epsilon\epsilon}$ denotes the Poisson noise resulting from observing galaxies as point sources in the sky.

The additional functions presented in equations (10), (11), and (12) have theoretical meaning provided in [4]. These represent another cosmological processes, such as the expansion of the universe and the growth of large-scale structures.

The $P_{\delta\delta}\left[z, \frac{l+1/2}{r(z)}\right]$ is called the matter power spectrum, a two-point statistic function that describes the matter density in different scales of the universe. As in the case of GC_{spec}, we construct it from CAMB, giving some fiducial values to the parameters considered. While $P_{\delta I}$ and P_{II} are variation of it, that give us information about another processes that causes modifications in the ellipticities[1].

Finally, considering all the bins, the Fisher Matrix can take the form of

[1]The main process behind these characteristics is called Intrinsic Alignment (IA), and it is related to the intrinsic deviations of galaxy shapes due to the initial conditions of their growth or perturbations at the beginning of the universe. We will not explain these effects in more detail due to the scope of this work, but for further information, please refer to the reference [4].

$$F_{\alpha\beta} = \sum_{l=l_{min}}^{l_{max}} \sum_{ij,mn} \frac{\partial C_{ij}^{\epsilon\epsilon}}{\partial \theta_\alpha} [\Delta C^{\epsilon\epsilon}(l)]^{-1}{}_{jm} \frac{\partial C_{mm}^{\epsilon\epsilon}(l)}{\partial \theta_\beta} [\Delta C^{\epsilon\epsilon}(l)]^{-1}{}_{ni}, \tag{13}$$

where the error on the observed $C_{ij}^{\epsilon\epsilon}$ take the following structure

$$\Delta C_{ij}^{\epsilon\epsilon}(l) = \sqrt{\frac{2}{(2l+1)\cdot \Delta l \cdot f_{sky}}} C_{ij}^{\epsilon\epsilon}(l).$$

Ultimately, the third probe from which we can construct the Fisher Matrix is GC_{ph}. The procedure for constructing this matrix is similar to that of WL, with the key difference being that this probe correlates sectors of the universe influenced by WL and clustering, not individual galaxies. These physical phenomena are represented in the following cosmic shear power spectra, where L denotes the lensing effect and G represents the clustering of galaxies

$$C_{ij}^{AB} = c\left[\int dz \frac{W_i^L W_j^L}{H(z)r^2(z)} P_{\delta\delta} + \int dz \frac{W_i^G W_j^L}{H(z)r^2(z)} P_{\delta\delta} + \int dz \frac{W_i^G W_j^G}{H(z)r^2(z)} P_{\delta\delta}\right] + N_{ij}^{LL} + N_{ij}^{GG}. \tag{14}$$

In this case, the Fisher Matrix is:

$$F_{\alpha\beta}^{XC} = \frac{1}{2} \sum_{l=l_{min}}^{l_{max}} (2l+1) \sum_{ABCD} \sum_{ij,mn} \frac{\partial C_{ij}^{AB}}{\partial \theta_\alpha} [\Delta C^{BC}(l)]^{-1}{}_{jm} \frac{\partial C_{mn}^{CD}(l)}{\partial \theta_\beta} [\Delta C^{DA}(l)]^{-1}{}_{ni}, \tag{15}$$

where A, B, C, D run over the combined probes L and G.

The final ojective is build a general Fisher Matrix with the three probes:

$$F_{\alpha\beta} = F_{\alpha\beta}^{\text{spec}} + F_{\alpha\beta}^{XC}. \tag{16}$$

In summary, the final Fisher Matrix will serve as a mathematical tool that integrates three important cosmological probes (WL, GC_{spec}, GC_{ph}) to provide insights into the deviations of parameters from their fiducial values. By adjusting certain aspects of the telescope, such as the number of bins or the survey area, we aim to minimize these deviations. This approach will optimize the use of Euclid in terms of funding and survey time, allowing us to extract the maximum amount of data possible. Ultimately, this will enhance our understanding of significant unknown parameters of the universe, such as its expansion driven by Dark Energy and the distribution of Dark Matter.

With all of this completed, the result of the work should be a quadratic Fisher Matrix, with dimensions corresponding to the number of parameters considered. For instance, we consider nine cosmological parameters: $\{h$, $\Omega_{m,0}$, $\Omega_{DE,0}$, $\Omega_{b,0}$, σ, n_s, γ, w_0, and $w_a\}$. These represent a range of cosmological characteristics, such as the presence of a dynamic dark energy and a non-flat geometry. Consequently, the final result is a matrix with dimensions of 9×9.

Finally, by utilizing the Fisher Matrix, we can obtain the 1-sigma errors of the cosmological parameters by inverting it and computing the square root of the diagonal elements:

$$\sigma_\lambda = \sqrt{(\mathbf{F}^{-1}{}_{\lambda\lambda})}. \tag{17}$$

3.2 New cosmological models

In order to compute a Modify Gravity model, we need to change some functions in the spectroscopic and photometric probes.

For the spectroscopic GC, we will add a space dependence. The terms in Equation (11) that would change are those related to the growth rate $f(z)$, which includes the growth rate itself. This modifies its scale independence, and now we consider $f(z,k)$. Additionally, the two phenomenological parameters related to the velocity dispersion also change. Specifically, the parameters σ_v and σ_p, which account for the damping of the BAO features and the Finger of God effect, respectively. Now, these expressions can be written as:

$$\sigma_v^2(z,\mu_\theta) = \frac{1}{6\pi^2}\int dk P_{\delta\delta}\{1-\mu_\theta^2+\mu_\theta^2[1+f(k,z)^2]\}, \tag{18}$$

$$\sigma_p^2(z) = \frac{1}{6\pi^2}\int dk P_{\delta\delta}(k,z) f^2(k,z). \tag{19}$$

Then, the de-wiggled power spectrum is computed with the function $g_\mu = \sigma_v(z,\mu_\theta)$, by the following formula:

$$P_{dw}(k,\mu;z) = P_{\delta\delta}(k;z)e^{g_\mu k^2} + P_{nw}(1-e^{-g_\mu k^2}). \tag{20}$$

The main difference with the standard model is that, in that case, $\sigma_v = \sigma_p$, due to the scale independence of $f(z)$. Now, we will evaluate these two parameters in each redshift bin, but we will keep them fixed in the Fisher matrix analysis.

For the photometric probes, the observables we will compute are WL, GC_{ph}, and their cross-correlation XC_{ph}. When moving away from general relativity, the window functions $W_i^X(z)$[2] in Equation (14) must be modified in order to account for changes in the evolution of cosmological perturbations. The background evolution remains the same as in the standard model, since we are in the limit of $|f(R)| \ll 1$. Thus, we introduce a new term to the matter power spectrum:

$$P_{\varphi+\psi}(k,z) = \left[-3\Omega_{m,0}\left(\frac{H_0}{c}\right)^2(1+z)\Sigma(k,z)\right]^2 P_{\delta\delta}. \tag{21}$$

This modification affects the clustering of matter and, consequently, the GC_{ph} probe.

On the other hand, for the WL observables, the change is more explicit in the window function, as can be seen in the following formula:

$$W_i^L(k,z) = \frac{3}{2}\Omega_{m,0}\left(\frac{H_0}{c}\right)^2(1+z)r(z)\Sigma(k,z)\cdot\int_z^{z_{\max}} dz'\frac{n_i(z)}{\bar{n}_i}\frac{r(z'-z)}{r(z')} + W_i^{IA}(k,z), \tag{22}$$

where $\frac{n_i(z)}{\bar{n}_i}$ and $b_i(k,z)$ represent the galaxy distribution and the galaxy bias in the i-th redshift bin, respectively. As in the standard case, the term W_i^{IA} accounts for processes, different from weak lensing, that change the ellipticities of the galaxies. Now, it also takes into account the dependence on the scale factor k, as discussed in [5].

Finally, the function Σ, appearing in Equation (22), relates the matter power spectrum to the perturbation in the metric of the standard model, and can be computed as:

$$\Sigma(a) = \frac{1}{1+f_R(a)}, \tag{23}$$

where $f_R(a)$ is given by equation (3), and its dependence on the scale factor a is derived from the Ricci scalar R.

3.3 Changing the accuracy of the power spectrum: Purity and completeness

Finally, if we want to change the accuracy of the power spectrum, we need to evaluate the effects of purity and completeness into the observed power spectrum.

The number density of clustering objects n_C is,

$$n_C = n_{\text{true}}(1-f_{\text{inc}}). \tag{24}$$

For the non-clustering objects, we have:

$$n_{\text{NC}} = \frac{n_C f_{\text{out}}}{1-f_{\text{out}}} = n_{\text{true}}\frac{f_{\text{out}}(1-f_{\text{inc}})}{1-f_{\text{out}}}, \tag{25}$$

[2]These represent the weight of gravitational lensing and galaxy clustering in each bin.

where $f_{\text{out}} = 1 - \text{purity}$ and $f_{\text{inc}} = 1 - \text{completeness}$, with f_{out} representing the percentage of non-clustering objects (random catastrophic outliers).

Then, the total number density is:

$$n_{\text{tot}} = n_C + n_{\text{NC}} = \frac{n_C}{1 - f_{\text{out}}} = n_{\text{true}} \frac{1 - f_{\text{inc}}}{1 - f_{\text{out}}}. \tag{26}$$

Then, the estimated galaxy power spectrum will be

$$\tilde{P}_{\text{estimated}}(z; k, \mu) = P_{\text{meas}}(z; k, \mu) - \frac{1}{n_{\text{tot}}} = [1 - f_{\text{out}}]^2 P_{\text{ideal}}(z; k, \mu). \tag{27}$$

where P_{ideal} is the true galaxy power spectrum of an ideal continuous density distribution.

The effective volume will be:

$$\tilde{V}_{\text{eff}} = V \left[\frac{n_{\text{tot}} \tilde{P}_{\text{estimated}}}{1 + n_{\text{tot}} \tilde{P}_{\text{estimated}}} \right]^2 == V \left[\frac{n_{\text{true}}(1 - f_{\text{inc}})(1 - f_{\text{out}}) P_{\text{ideal}}}{1 + n_{\text{true}}(1 - f_{\text{inc}})(1 - f_{\text{out}}) P_{\text{ideal}}} \right]^2. \tag{28}$$

Due to the above, we can note that the main objective is for the code to allow changing the number of measured objects, $n(z)$, according to the assumed observational effects.

3.4 Numerical tools

As previously mentioned, the construction of the Fisher Matrix must rely exclusively on theoretical expressions. For this reason, we utilize the CAMB library to generate functions such as $P_{\delta\delta}$. We provide CAMB with fiducial values for the cosmological parameters under consideration. Using this information, the library generates a set of $P_{\delta\delta}(z, k)$ values for given z and k (k is the wave number, which can be expressed in terms of the angular separation as $k = \frac{l+1/2}{r(z)}$, where $r(z)$ is the comoving distance between galaxies). Subsequently, we employ SciPy and NumPy to interpolate a complete function for $P_{\delta\delta}$ with z and k as inputs. This procedure is identical for the derivatives of $P_{\delta\delta}$ with respect to the cosmological parameters.

3.5 Extensions of this work

After computing the parameter errors, we can measure the accuracy with which the experiment constrains specific parameters. To do this, we will use the Figure of Merit (FoM). The FoM is proportional to the inverse of the area of the 2-σ contour in the marginalized parameter plane for two parameters, θ_α and θ_β. This implies the FoM can be computed from the marginalized Fisher submatrix, $\tilde{F}_{\alpha\beta}$, for these two parameters.

$$FoM_{\alpha\beta} = \sqrt{det(\tilde{F}_{\alpha\beta})}. \tag{29}$$

In the models considered, we postulate a dynamical Dark Energy (DE) that is responsible for the cosmic acceleration. In this context, its redshift-dependent equation of state can be written as:

$$w_{\text{DE}} = w_0 + w_a \frac{z}{z + 1}. \tag{30}$$

Thus, the parameters w_0 and w_a describe the Dark Energy and can be used to calculate a FoM that quantifies the accuracy of the experiment. The equation to compute this quantity is:

$$\text{FoM}_{w_0, w_a} = \sqrt{\det(\tilde{F}_{w_0, w_a})}. \tag{31}$$

In summary, we can choose two parameters that describe some aspect of the universe and compute its FoM. This will provide us with the tools to assess how well Euclid constrain the parameter set.

4 Work Plan

After computing the errors of cosmological parameters in models close to the standard cosmology, ΛCDM, we have two projects in mind to complete within. The first one involves assuming a different cosmology, in this case, a modified gravity model, and constraining the errors of the new parameters involved. On the other hand, we can also modify some observational effects to optimize the error constraints and, thereby, improve the use of Euclid. In this context, the two observational effects used are "Purity" and "Completeness", which play a role in the observation of spectroscopic galaxy clustering (GC) probes.

4.1 Project 1

The work plan involves developing a comprehensive validation code to calculate errors in cosmological parameters using the Fisher matrix formalism across three cosmological probes: galaxy clustering (GC) from spectroscopy, weak lensing (WL), and GC from photometry.

As the first project, we will build a Fisher matrix that takes into account three cosmological probes. This matrix will provide the errors of the cosmological parameters for the nine models considered. Two separate codes have already been created, one for GC spectroscopy and another for WL observations, each tailored to different cosmological models and parameter sets. The next objective is to create a unified code that integrates these probes, allowing for combined analysis and individual results for each probe. This unified framework will also be designed to be user-friendly, enabling easy modifications to include new cosmological parameters, models, settings, or survey ranges.

The computation and optimization of the matrix is a large project, which will be done in the first five months of the year. The good performance of this code will give us the opportunity to certify our work in the Euclid Collaboration, as one of the correct forecasts for Euclid.

4.2 Project 2

In this second project, we will investigate the modified gravity model proposed by Hu & Sawicki [1]. The primary objective is to modify the original code to accommodate both spectroscopic and photometric probes.

For the spectroscopic probes, we aim to incorporate the scale dependence of perturbation growth. This directly impacts the parametrization of the $P_{\rm obs}$, as the integral must include the $f(z,k)$ component. Additionally, the parameters σ_v and σ_p are also affected by this modification.

For the photometric probes, we account for nonlinear modifications to the matter power spectrum. These changes introduce new expressions into the window function, due to the space-time modifications described by the function $f(R)$. As a result, the effects of this modification are directly incorporated into the WL probes. For galaxy clustering in photometric surveys, the changes are reflected in the matter power spectrum itself.

The work will focus on integrating these modifications into the core code to assess the experiment's ability to constrain the new parameter f_{R0}. To achieve this, we will implement a new $P_{\delta\delta}$ using MGCAMB, a variant of CAMB that incorporates these perturbations within the Hilbert-Einstein action. Consequently, the updated Fisher matrix will include additional parameters, offering users the flexibility to adjust the cosmological model of their choice.

4.3 Project 3

Finally, in the third project, we will focus on the contamination by outliers, objects that do not belong to galaxy clusters, which affects the accuracy of power spectrum estimates. To address this, the analysis focuses on two key parameters: purity, which measures the likelihood that a detected object is real, and completeness, which reflects the probability of detecting all objects in the population.

The planned work focuses on developing a robust computational framework to account for the effects of observational contamination on galaxy clustering analyses. Specifically, the code will allow for dynamic adjustments to the number of detected objects in a galaxy survey, considering the influence of purity and completeness, two critical parameters that describe the accuracy of the detection process.

The code will simulate various scenarios by allowing users to modify purity and completeness levels. This includes cases where one parameter is fixed while the other is allowed to vary, as well as situations where both parameters are adjusted simultaneously. This flexibility will make it possible to study the full range of potential observational effects, including extreme cases of high contamination or low detection efficiency. In practice, the code will model how the presence of outliers impacts the observed clustering signal. By incorporating equations that describe the number densities of clustering and non-clustering objects, the tool will calculate how these densities combine to affect the total population of detected objects. Furthermore, the code will compute the effective survey volume as a function of these parameters, providing a detailed understanding of how purity and completeness jointly alter cosmological measurements.

4.4 Gantt chart

We will divide the future work as can be seen in the following Gantt chart.

	Jan - Mar	Mar-May	May-Jul	Jul-Sept	Sept-Nov	Nov-Dec
Project 1: Compute Fisher matrix for spectroscopy and photometric probes.						
Project 1: Optimize Fisher matrix.						
Project 2: Modify Gravity for spectroscopic probes.						
Project 2: Modify Gravity for photometric probes.						
Project 3: Code that includes purity and completeness.						
Project 3: Setting values for purity and completeness and searching for observational effects.						

5 Background information to assess the capacity of the team to implement the proporsal

5.1 Relevant experience of the team

The research team consists of two fourth-year undergraduate physics students who have been working under the supervision of their advisor for a year, researching and developing the Fisher matrix. The final Fisher matrix for each probe can be refined to achieve smaller parameter errors and optimized to develop a fast, comprehensive code that can be validated. These considerations are crucial, given the team's expertise and experience in developing this code. Additionally, they completed a summer internship under the same advisor, where they studied genetic algorithms and their application in estimating cosmological parameters, successfully developing a useful code for this purpose.

The professor leading this research demonstrates strong analytical and numerical skills and has published work on topics such as statistics, Monte Carlo simulations, Bayesian statistics, and machine learning.

5.2 Available infrastructure and resources

The team has access to personal computers for each member, as well as common rooms with computers at the university, which can be used for this work. The students also have access to public areas at the university for collaborative work, in addition to the advisor's office, where progress is shared collectively. This research has the advantage of not requiring complex infrastructure or specific laboratories.

References

[1] Hu, W. & Sawicki, I. 2007, Phys. Rev., D76, 064004

[2] C. Carbone, D. Sapone, V. Cardone Purity and Completeness effects on FoM , 2022.

[3] S. Dodelson. Modern Cosmology. Academic Press, Elsevier Science, 2003.

[4] A. Blanchard et al. Euclid preparation: VII. Forecast validation for Euclid cosmological probes. Astronomy & Astrophysics, 642:A191, 2020. DOI: 10.1051/0004-6361/202038071. Available at: http://dx.doi.org/10.1051/0004-6361/202038071.

[5] S. Casas et al. Euclid: Constraints on f(R) cosmologies from the spectroscopic and photometric primary probes. arXiv preprint arXiv:2306.11053, 2023. Available at: https://arxiv.org/abs/2306.11053.

[6] S. Gallo, M. Douspis, E. Soubrié, and L. Salvati. Characterising galaxy clusters' completeness function in Planck with hydrodynamical simulations. Astronomy & Astrophysics, 686:A15, 2024. DOI: 10.1051/0004-6361/202347678. Available at: http://dx.doi.org/10.1051/0004-6361/202347678.