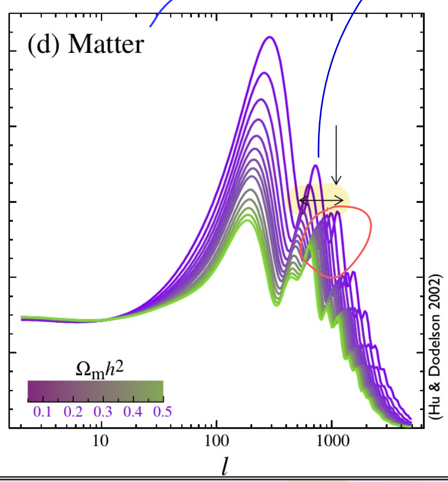
· Dark matter va a modificar el espectro de potencia → indicador del 2° y 3° peak.

↓

xq va a modificar el potencial gravitacional

• matter content – all matter

oscura



third peak separates dark matter from baryons => $\Omega_m h^2 \sim 0.3$

→ Anisotropías dependen del baryon-photon ratio (η)

→ DM se desacopló bastante antes de las anisotropías.

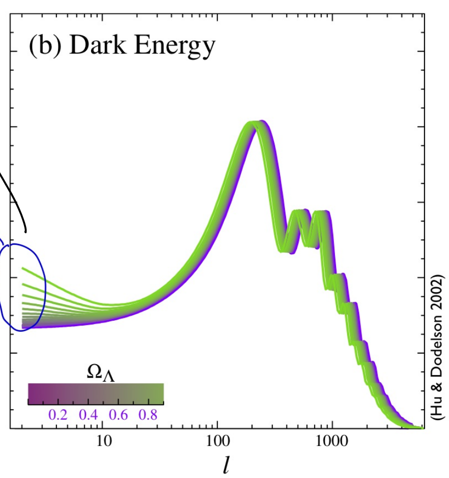
• matter content – dark energy

→ lo contrario a la curvatura.

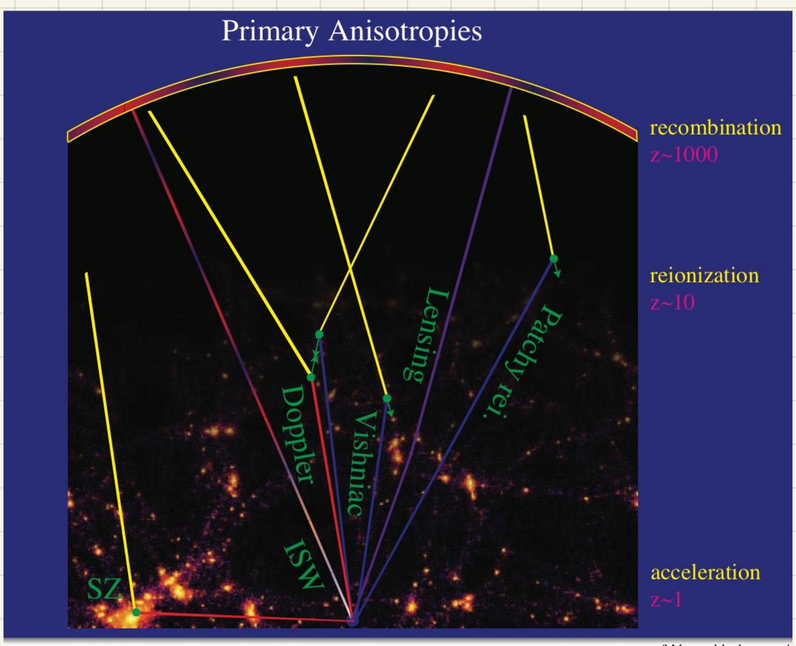
ISW effect

multipolos pequeños

COSMIC VARIANCE (se pueden hacer pocas mediciones)



$\boxed{\text{CMB fluctuations}}$ → secundarias.



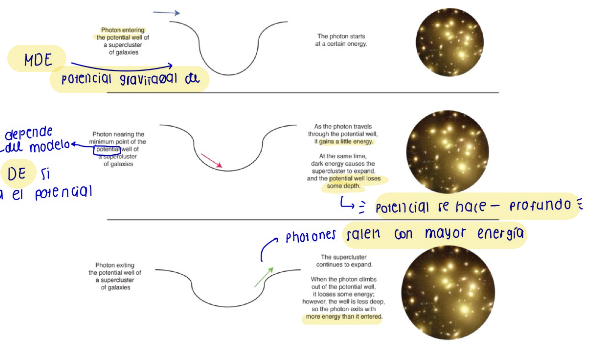
(Wayne Hu Lecture)

importantes a ≠ escalas.

▪ secondary fluctuations – where do they come from?

- integrated Sachs-Wolfe effect
- Rees-Sciama effect
- Sunyaev-Zeldovich effect (thermal & kinematic)
- Ostriker-Vishniac effect
- patchy reionisation of the Universe
- gravitational lensing

- integrated Sachs-Wolfe (ISW) effect
  - fluctuations due to **global** (time-varying) gravitational potential
  - caused by time-varying linear perturbations (e.g. superclusters)



MDE → potencial gravitacional de

depende del modelo

si hay DE si cambia el potencial

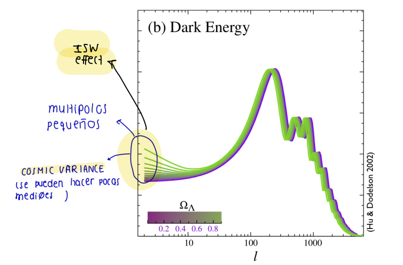
↳ ≡ potencial se hace − profundo ≡

photones salen con mayor energía

a lo ancho del potencial → Afecta en l pequeño.

≡ Tienen que ser escalas muy grandes ≡ para que le de tiempo al fotón de salir

- matter content – dark energy



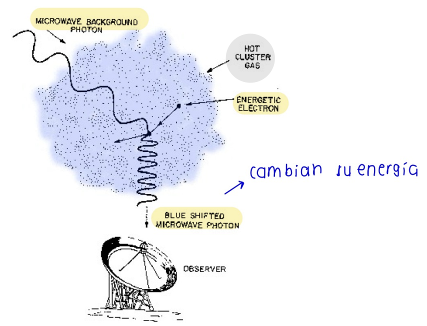
↳ Es difícil medir con la CMB la energía oscura.

- Rees-Sciama (RS) effect
  - fluctuations due to **local** (time-varying) gravitational potential
  - caused by time-varying non-linear perturbations (e.g. haloes)

→ A nivel de los halos.
↳ materia oscura alrededor de la galaxia para que se forme.

(escalas + pequeñas)

- Sunyaev-Zeldovich (SZ) effect



escalas pequeñas pero no tanto ←

↗ cambian su energía

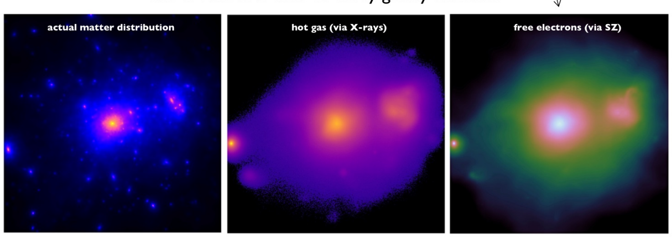
- Ostriker-Vishniac (OV) effect
  - higher order coupling between bulk flow of electrons and their density perturbations (outside virialized objects)

→ Escalas más pequeñas (localmente gas se pueden mover + rápido)

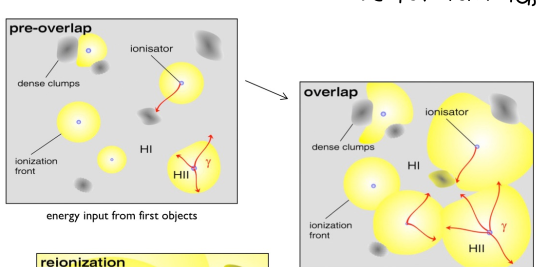
→ mov. muy rápidos a escalas muy pequeñas.

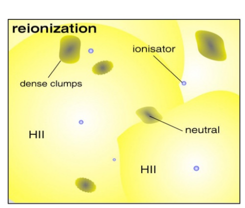
- Sunyaev-Zeldovich (SZ) effect
  - thermal: CMB photons scatter off the hot intra-cluster gas → gas caliente
  - kinetic: the cluster gas has a bulk motion with respects to the CMB and hence induces a Doppler shift → gas se mueve c/r a la CMB.
    ↳ cambia la frec.

the SZ effect is used to study galaxy clusters:



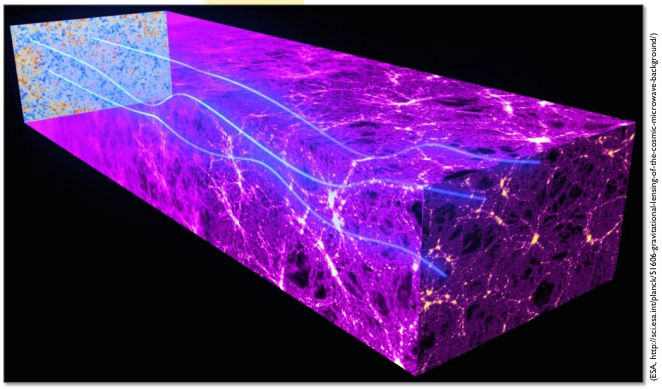
- patchy re-ionisation of the Universe → se forman las estrellas que emiten luz y el gas se ioniza
  ↓
  a muchas escalas

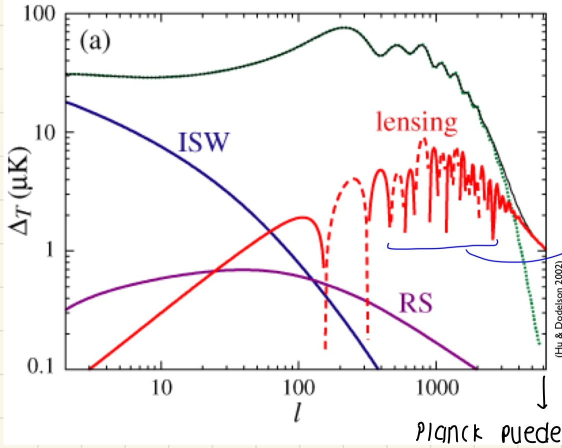




HII regions: free e^- available for scattering!

▪ gravitational lensing → fotones siguen las geodésicas. (cambia la dirección.





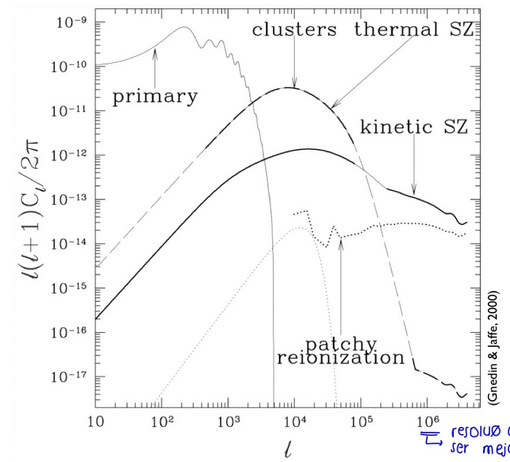
cuando potos de potencial son + grandes (a escalas pequeñas) ↓ materia más concentrada.

↓ Planck puede ver hasta 2000.

▪ relevance of secondary effects

for $l > 3000$ lensing and tSZ dominate anisotropies!

por la difracø debe aumentar la resoluø de la máquina para ver mayores ℓ.



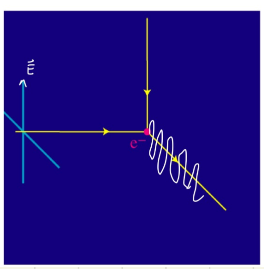
resoluø angular debe ser mejor.

Polarizaø

▪ Thomson scattering

• the scattered wave is polarised perpendicular to the incidence direction

• light cannot be polarised along direction of motion:
→ only one linear polarisation state gets scattered
↳ medimos la perpendicular a la dirección incidente.

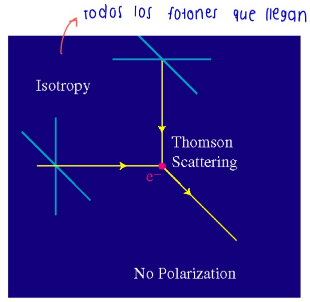


* campo eléctrico hace mover al e^-.

• the scattered wave is polarised perpendicular to the incidence direction

• incidence directions are isotropic → no net polarisation

todos los fotones que llegan son iguales? ↓ No

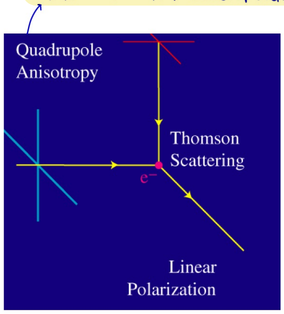


• the scattered wave is polarised perpendicular to the incidence direction

• incidence directions has quadrupole → net polarisation

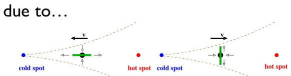
polarizaø neta ≠ 0

≡ Hay anisotropía cuadripolar ≡

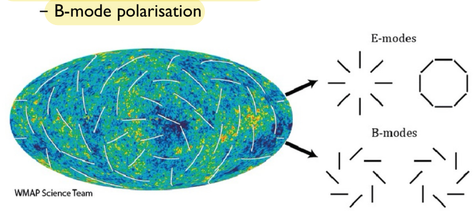


• quadrupole anisotropy in photon flux due to…

• scalar perturbations (density)
– E-mode polarisation



• tensor perturbations (grav. waves)
– B-mode polarisation



B modes:
↳ llegan de las ondas gravitaøales de la CMB.

extra ·

These terms are due to non-relativistic collisions between photons and electrons via Thomson scattering

(Left for exercise)

$$e^- + \gamma \rightleftharpoons e^- + \gamma.$$

4-vel. del electrón

$$\delta\tilde{C}^S_\gamma[f(\mathbf{p})] = -\bar{E}\frac{\partial f_\gamma}{\partial \bar{E}}\boxed{\sigma_T}\boxed{n_e}\left[\tilde{\Theta}^S_0(\mathbf{k},t) - \frac{1}{2}P_2(\mu)\tilde{\Theta}^S_2(\mathbf{k},t) - \tilde{\Theta}^S(\mathbf{k},\mathbf{p},t) + \boxed{\delta\tilde{\mathbf{u}}_e\cdot\hat{p}}\right].$$

Thomson scattering

Number of electrons

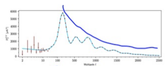
momento de los fotones

proyección: fotón que llega exita al electrón y la direcØ del electrón será paralela a $\hat{p}$.

Putting all together

$$\dot{\tilde{\Theta}}^S + i\frac{k}{a}P_1(\mu)\tilde{\Theta}^S - \boxed{\dot{\tau}}\tilde{\Theta}^S + \frac{1}{3}\left(\tilde{\psi} - k^2P_2(\mu)\dot{\tilde{E}}\right) = -\boxed{\dot{\tau}}\left[\tilde{\Theta}^S_0 - \frac{1}{2}P_2(\mu)\tilde{\Theta}^S_2 + i\frac{k}{a}\mu\,\delta\tilde{u}_e\right]$$

optical depth



Where we have the optical depth $\tau(t) \equiv \int_t^\infty \sigma_T\, n_e dt.$

→ Dumpling de las oscilaØes de la CMB.

Fotones chocan con el gas y pierde la fluctuaØ original.

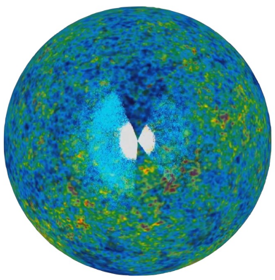
reionizaØ — CMB

↳ Estrellas calientan el gas → fotones libres → 2° scattering de Thomson.

Remind the interaction rate for photon $\sigma_T\, n_e = \Gamma_\gamma$

→ Hay 2 scattering de thomson
↳ en la CMB.
↳ en la fase de reionizaØ.

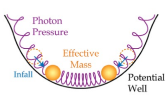
Perturbations in the CMB



The cosmic microwave background (CMB) radiation ~ a sphere of photons coming from everywhere. Photons last scattered with nuclei.

Their temperature, when scattered was ~ 3000K
They reach to us at T ~ 2.7 K due to expansion.

These photons left their corresponding potential wells.



Denser regions = hotter photons?
Yes and no.

Todos los pozos de potencial en los que cae el fotón.

llega de la superficie de scattering

$$\frac{\Delta T}{T}(\hat{n}) = \Theta(t_0, \mathbf{x}_0, -\hat{n}), \qquad \Theta(t,\mathbf{x},\hat{p}) = \int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}\tilde{\Theta}(t,\mathbf{k},\hat{p}),$$

$$[\Theta(\hat{n}) + \psi]_R = \underbrace{\Theta_0|_*}_{\text{temp.}} + \underbrace{\psi|_*}_{\text{gravity}} + \underbrace{e\cdot v_b|_*}_{\text{Doppler}} + \underbrace{\int_*^R(\dot{\psi}+\dot{\phi})\,d\eta}_{\text{ISW}}$$

Θ is Fourier transform of temp. fluct. from Boltzmann eqs.

iSW is late-time effect:
During MDE the potentials are constant
Only during DE-DE the potentials change

This means that iSW is sensitive to DE and so the CMB

$$C_{\rm iSW} = 4\pi\int\frac{dk}{k}\left[W^{\rm iSW}_\ell(k)\right]^2\frac{9}{25}\frac{k^3 P_{\mathcal{R}}}{2\pi^2}$$

espectro de potencia.

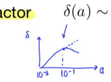
window function

más alta es la influencia de la D.E en la expansión

$$W^{\rm iSW}_\ell(k) = 2\int dz\boxed{\frac{dG}{dz}}j_\ell(k\,r(z))$$

$\propto \dot{\phi}$

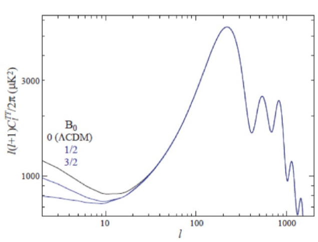
G is growth factor $\delta(a) \sim G(a)a$



$\delta = a \cdot G(a)$
↳ factoralizar la soluØ (decreci@ que hace la D.E.)

Transfer function

$$\frac{k^3P_{\mathcal{R}}}{2\pi^2} = A\left(\frac{k}{k_0}\right)^{n_s-1}T^2(k)$$



Behaviour of the matter power spectrum P(k)

1) Matter power spectrum is an important quantity, affecting the CMB

$$P(k) \equiv \langle|\delta_k|^2\rangle$$

↳ en el vector de onda.

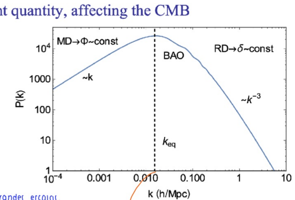
2) The potential can be written as

$$\Phi(k,a) = \Phi_p(k)\times T(k)\times\delta(a)$$

Initial value from inflation — Transfer function

$$\langle\Phi_p^2\rangle \sim k^{-3}k^{n_s-1} \qquad T(k) = \frac{\Phi(k,a_{\rm late})}{\Phi_{\rm large}(k,a_{\rm late})}$$

↳ 1 cuando consideramos grandes escalas.



$k_{eq} = 0.073\Omega_{m0}h\ (h/{\rm Mpc})$

peak periodo de equivalencia.

3) With these, express P(k) as

$$\begin{aligned} P(k) &= \langle\delta_k^2\rangle \\ &= k^4\langle\Phi_p^2\rangle T(k)^2\delta(a)^2 \\ &\sim k^4k^{-3}k^{n_s-1}T(k)^2 \\ &\sim k^{n_s}T(k)^2 \end{aligned}$$

$k >> k_{eq} \rightarrow \delta \sim const \rightarrow \Phi \sim 1/k^2 \rightarrow T \sim 1/k^2 \rightarrow P(k) \sim k^{-3}$

$k << k_{eq} \rightarrow \Phi \sim const \rightarrow \delta \sim k^2 \rightarrow T \sim 1 \rightarrow P(k) \sim k$

Behaviour of the matter power spectrum P(k)

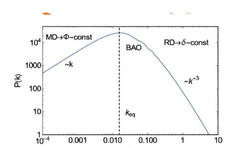
4) Behavior of transfer function

$$T(k) = \begin{cases} 1/k^2, & k >> k_{eq} \\ 1, & k << k_{eq} \end{cases}$$

→ a grandes escalas

5) Behavior of P(k)

$$P(k) = \begin{cases} 1/k^3, & k >> k_{eq} \\ k, & k << k_{eq} \end{cases}$$

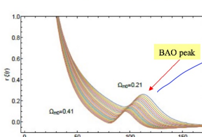


$k_{eq} = 0.073\Omega_{m0}h\ (h/{\rm Mpc})$

6) Fourier transform of P(k) is ξ(r), the correlation function (~prob of galaxies at r)

$$\xi(r) = \frac{1}{2\pi^2}\int_0^\infty P(k)j_0(kr)k^2dk$$

$$\xi(r) = r^{-n-3},\ n = (1,-3)$$



1 peak xq la transf. de Fourier del sen es una δ.

Large scale structure.

Let us take the perturbed metric:

$$\delta g_{\mu\nu} = a^2 \begin{pmatrix} -2\psi & E_{,i} \\ E_{,i} & 2\phi\delta_{ij} + D_{ij}B \end{pmatrix}$$

We can define the comoving curvature perturbation:

$$\mathcal{R} = \phi - \frac{1}{6}\nabla^2 B - H\delta u$$

$$\zeta = \phi - \frac{1}{6}\nabla^2 B - H\frac{\delta\rho}{\bar{\rho}}$$

We express perturbations in terms of a single perturbation R(x). How did R(x) acquire a specific spatially inhomogeneous profile?

One explanation: cosmic inflation. A pre-hot-Big-Bang phase, which thermalized universe with adiabatic perturbation.

But inflation is just a proposal to understand the origin of R and other physical mechanisms may have led to the existence of adiabatic perturbations.

First, it is not possible (not even in principle) to predict a particular realization of our inhomogeneous universe, that is, a particular spatial profile for R(x). Hence we need to treat it as a stochastic field.

$$\mathcal{R}(\mathbf{x}) = \{\mathcal{R}_1(\mathbf{x}), \mathcal{R}_2(\mathbf{x}), \ldots, \mathcal{R}_n(\mathbf{x})\}$$

Each time we reproduce the birth of our universe, a particular realization $R_i(x)$ will come up.

If we repeat N >> n some particular realization will be more frequent than other.

Thinking it as a stochastic variable, we can do statistics.

The average value $$\langle\mathcal{R}(\mathbf{x})\rangle = \frac{1}{N}\sum_{i=1}^{N} n_i\mathcal{R}_i(\mathbf{x})$$

The two point correlation function $$\langle\mathcal{R}(\mathbf{x})\mathcal{R}(\mathbf{y})\rangle = \frac{1}{N}\sum_{i=1}^{N} n_i\mathcal{R}_i(\mathbf{x})\mathcal{R}_i(\mathbf{y})$$

R(x) represent a perturbation to the metric, and that on average the metric must coincide with the background metric

$$\langle\mathcal{R}(\mathbf{x})\rangle = 0$$

If we add homogeneity, this implies that

$$\langle\mathcal{R}(\mathbf{x})\mathcal{R}(\mathbf{y})\rangle = \langle\mathcal{R}\mathcal{R}\rangle(|\mathbf{x}-\mathbf{y}|)$$

In Fourier space

$$\mathcal{R}(\mathbf{x}) = \int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}}\tilde{\mathcal{R}}(\mathbf{k}) \qquad \langle\mathcal{R}(\mathbf{k})\mathcal{R}(\mathbf{k}')\rangle \propto \delta^3(\mathbf{k}+\mathbf{k}') \qquad \langle\mathcal{R}(\mathbf{k})\mathcal{R}(\mathbf{k}')\rangle = (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')P_{\mathcal{R}}(k)$$

$$\tilde{R}(k) = \int d^3x \; e^{-ikx} R(x)$$

Only on k = |**k**| to preserve isotropy

Initial power spectrum

The power spectrum

$$\langle\mathcal{R}(\mathbf{k})\mathcal{R}(\mathbf{k}')\rangle = (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}')P_{\mathcal{R}}(k)$$

Defined as

$$\langle\mathcal{R}\mathcal{R}\rangle(|\mathbf{x}-\mathbf{y}|) = \int_{\mathbf{k}} P_{\mathcal{R}}(k)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$

Given that $P_R(k)$ has dimension of volume, we can rewrite it in terms of the so called dimensionless power spectrum $\Delta_R(k)$ as

$$P_{\mathcal{R}}(k) = \frac{2\pi^2}{k^3}\Delta_{\mathcal{R}}(k)$$

Which form? From inflation we want the scale invariant (or nearly), i.e. all perturbations out at the same scale

$$\boxed{\Delta_{\mathcal{R}}(k) = A_{\mathcal{R}}\left(\frac{k}{k_*}\right)^{n_s-1}}$$

Some pivot scale

Amplitude of primordial PS

$$A_{\mathcal{R}} = A_s \sim 10^{-9}$$

$$k_* = 0.05\,h/\mathrm{Mpc} \qquad \boxed{n_s = 0.96}$$

↳ indice espectral.

Matter power spectrum.

The large-scale structure of the universe started to grow after the epoch of the radiation-matter equality.

Since non-relativistic matter has a negligible pressure relative to its energy density, the gravitational attraction becomes stronger than the pressure repulsion in the matter-dominated epoch.

The perturbations of pressureless matter, especially the CDM perturbations, are responsible for the formation of galaxies.

We can quantify the matter distribution in the universe by measuring the correlation function or the power spectrum of the galaxies we observe in the sky.

Since inflation gave the initial potential, which sources everything, it is useful to study how the gravitational potential grows over time.

Which means to study the evolution from the beginning of RDE to now.

The separation small-large scale is set by the equality $k_{\mathrm{eq}} = a_{\mathrm{eq}} H(a_{\mathrm{eq}})$

Let us consider large scales model, i.e. k<< k_{eq} ;

We saw that the potential stays constant but we need to see the transition RDE -> MDE

$$3aH\left(\dot{\phi} + aH\phi\right) = -4\pi G a^2 \left(\rho_c \delta_c + 4\rho_r \Theta_{r,0}\right),$$
$$\dot{\delta}_c = 3\dot{\phi},$$
$$\dot{\Theta}_{r,0} = \dot{\phi}.$$

baryons and quadrupole neglected

From the last two we find:

$$\delta_c = 3\Theta_{r,0} + \cancel{C}$$

From adiabatic conditions $\delta_r = (4/3)\delta_m$ $\rho_r \propto T^4; \quad \rho_m \propto T^3$

Plugging into the first one:

$$y\frac{\mathrm{d}\phi}{\mathrm{d}y} + \phi = -\frac{3y+4}{6\,(y+1)}\delta_c$$

Where:

$\mathrm{d}y/\mathrm{d}\tau = aHy$ and $3H^2 = 8\pi G\rho_c(1 + 1/y)$ being $y = a/a_{\mathrm{eq}} = \rho_c/\rho_r$

Taking the derivative of $y\frac{\mathrm{d}\phi}{\mathrm{d}y} + \phi = -\frac{3y+4}{6\,(y+1)}\delta_c$

And eliminating the deltas $\frac{\mathrm{d}^2\phi}{\mathrm{d}y^2} + \frac{21y^2+54y+32}{2y(y+1)(3y+4)}\frac{\mathrm{d}\phi}{\mathrm{d}y} + \frac{1}{y(y+1)(3y+4)}\phi = 0$

Which has the solution:

$$\phi(y) = c_1\frac{\sqrt{1+y}}{y^3} + \frac{9y^3+2y^2-8y-16}{y^3}$$

$\phi_i = \phi(0)\,; \quad \phi_i' = 0 \quad \Longrightarrow c_1 = 16c_2 = (8/5)\phi(0)$

The final solution:

$$\phi(y) = \phi(0)\frac{9y^3+2y^2-8y-16+16\sqrt{1+y}}{10y^3}$$

The final solution: $\phi(y) = \phi(0)\frac{9y^3+2y^2-8y-16+16\sqrt{1+y}}{10y^3}$

$y = a/a_{eq} \to \infty$

$$\phi(y) = \frac{9}{10}\phi(0)$$

Larges scale is constant and reduced

In RDE

$$\ddot{\phi} + 3aH\left(c_s^2+1\right)\dot{\phi} + \left(k^2c_s^2 + 3(aH)^2c_s^2 + (aH)^2 + 2a\dot{H}\right)\phi = 0$$

In RDE

$c_s^2 \simeq 1/3\,; \quad \mathrm{d}(aH)/\mathrm{d}\tau \simeq -(aH)^2\,; \quad aH \simeq 1/\tau \longrightarrow \ddot{\phi} + \frac{4}{\tau}\dot{\phi} + \frac{k^2}{3}\phi = 0$

$$\phi(k,\tau) = 3\phi_I\frac{\sin(k\tau/\sqrt{3}) - (k\tau/\sqrt{3})\cos(k\tau/\sqrt{3})}{(k\tau/\sqrt{3})^3}$$

Larges scale — Constant: $\phi(k,\tau) \simeq \phi_I\left[1 - (k\tau)^2/10\right]$

Small scale — Decreases: $\phi(k,\tau) \simeq \phi_I/(k\tau)^2$

radiaø empuja todo (en RDE) pero materia quiere colapsar

se diluye más el potencial.

7 · nov.

Matter power spectrum.

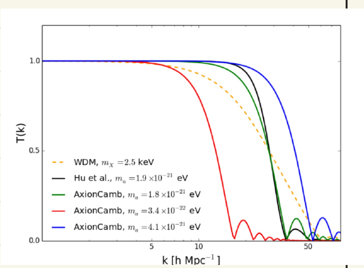
¿Cuando defino $k \ll k_{eq}$?
↓
para escalas grandes.

Transition RDE → MDE

· $y = a/a_{eq}$

¿Por qué hacemos todo c/r a a_{eq}?

Durante RDE perturbaciones no pueden crecer: se mantienen constantes hasta MDE. (radiación hace que se expanda).



Transfer function:

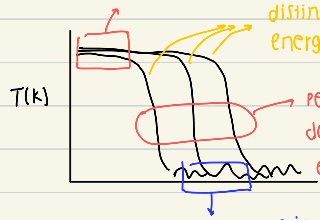
$$\boxed{T(k) = \frac{\phi(k, a_T)}{\phi_{LS}(k, a_T)}}$$

momento particular (normalmente en radiación)

potencial grav. a grandes escalas ⇒ ϕ cte.

$$\phi_{LS}(k, a_T) = \frac{9}{10}\phi(k, a_i)$$

valores de ϕ primordiales de inflación



* Las transfer function de CAMB son para ≠ componentes y la total.

* Silk damping: fotones pasan por el medio y la energía se disipa : $C(\ell)$. → por eso baja la altura de los peaks.

Crecimiento de estructuras : MDE!

* La solución de δ crece linealmente con a en MDE.

para $a > a_T$

$$\phi(k, a_0) = \frac{9}{10} T(k) \cdot \phi(k, a_i) \boxed{D(a_0)}$$

nos reescala en el tiempo.

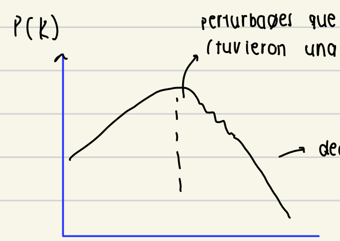
growth factor

· Potencial gravitacional

$\left(\frac{k}{H_0}\right)^{n_s}$

$$P_{\delta_m} = \langle|\delta_m(k, a_0)|^2\rangle \sim \dots \frac{\delta_H^2}{(\Omega_{m,0})^2}\left(\frac{k}{H_0}\right)^{\boxed{n_s}} T^2(k)\, D^2(a_0)\, H_0^{-3}.$$

0,9



When $a > a_T$ the gravitational potential remains constant during the matter era (if GR), but after the universe has entered the epoch of cosmic acceleration the potential is expected to vary. We introduce the growth function D(a) (sometimes G(a)):

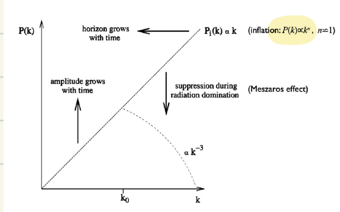
$$\phi(k, a_0) = \frac{9}{10}\phi(k, a_i)T(k)D(a_0)$$

But we want matter:

$$k^2\phi = -4\pi G a^2 \rho_m \delta_m \quad \longrightarrow \quad \delta_m(k, a) = -\frac{2k^2 a}{3\Omega_{m,0}H_0^2}\phi(k, a)$$

$$P_{\delta_m} = \langle|\delta_m(k, a_0)|^2\rangle = \frac{2\pi^2\delta_H^2}{(\Omega_{m,0})^2}\left(\frac{k}{H_0}\right)^{n_s} T^2(k)D^2(a_0)H_0^{-3}.$$

But why it has this form?



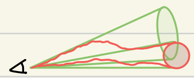
2 pt statistics

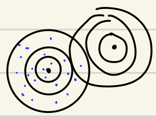
Galaxy clustering
↳ correlation of galaxy positions

Weak lensing
↳ of galaxy ellipticity.

diámetro del círculo dado por el vector de onda k.

Correlación a 2 puntos.





$r_1 - N_1$
$r_2 - N_2$
$r_3 - N_3$ → nº de galaxias

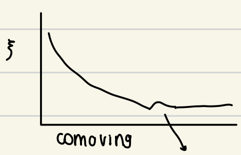
→ inhomogeneidades y anisotropías.

↓

Para estudiar anisotropías
⇓
armónicos esféricos

Correlation function ← *no es tan simple.*

* redshift space distortion $(1+f\mu^2)$



BAO : en un círculo de 100 Mpc, se ven + galaxias

Galaxias
· OII → Galaxias con líneas de emisión muy alta → + presió.
H_α

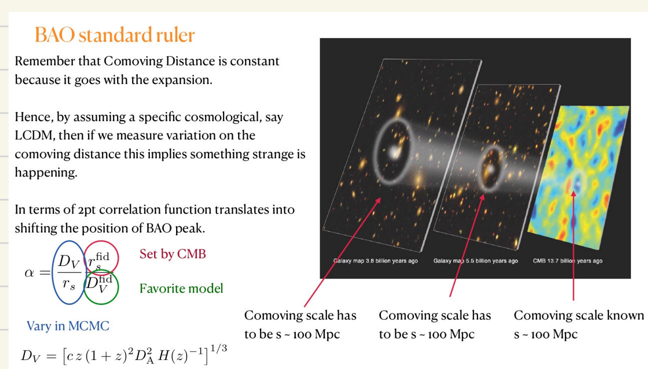
BAO standard ruler:

distance volume. (volumen de la esfera)

$$\alpha = \frac{D_V}{r_s} \cdot \frac{r_s^{fid}}{D_V^{fid}}$$

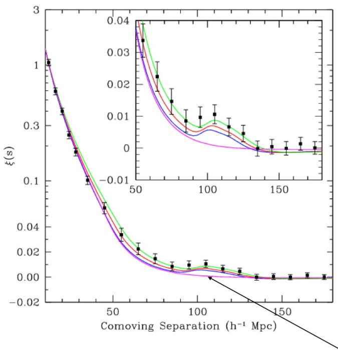
r_s^{fid} → set by CMB.
D_V^{fid} ↳ fav. model
vary in MCMC.

→ Si $\alpha = 1$: la cosmología de referencia es correcta
→ si α cambia con el redshift, nuestro modelo de expansión es raro.
↳ se expande de forma ≠ a ≠ escalas.



Funø de correlaø de 2 pts → una densidad.

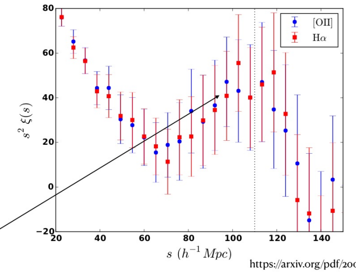
2 pt statistics in galaxies



Correlation function

$$\boxed{\xi_l(s)} = \frac{i^l}{2\pi^2}\int_0^{+\infty} P_l(k) j_l(ks) k^2 \mathrm{d}k$$

$$P_l(k) = \frac{2l+1}{2}\int_{-1}^{+1}(1+f\,\mu^2)P(k)L_l(\mu)\mathrm{d}\mu$$



BAO peak

https://arxiv.org/pdf/2004.1343

$\boxed{P_{obs}}$

Haven't you noticed an extra function on the 2pt function?

At the moment we talked about the matter power spectrum, basically only the DM power spectrum... but galaxies are made of baryons and they just fall into DM halo. Furthermore galaxies move due to gravity.

→ se usa μ

Galaxy power spectrum

Effective galaxy bias.

The idea is: baryons fall into DM wells, hence there's a proportion

$$P_g(k;\,z) = b^2\,P_m(k;\,z)$$

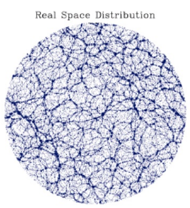
$$\delta_g = b\delta_m = \begin{cases} b(k) \\ b(z) \\ b(k,z) \\ b_{red} \neq b_{blue} \\ \dots \end{cases}$$

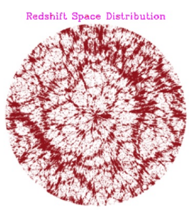
Anisotropies due to RSD.

We do not observe in real space but in redshift space because we measure photons.

$$1 + z_{obs} = (1+z)(1+\frac{v_\parallel}{c})$$

Velocity along line-of-sight





Two terms:

Overall expansion & peculiar velocity

Hubble — Motion of galaxies in gravity

$$P_{zs}(k,\,\mu;\,z) = \left[b\sigma_8(z) + f(z)\sigma_8(z)\mu^2\right]^2 \frac{P_m(k\ z)}{\sigma_8^2(z)}$$

Kaiser 1987

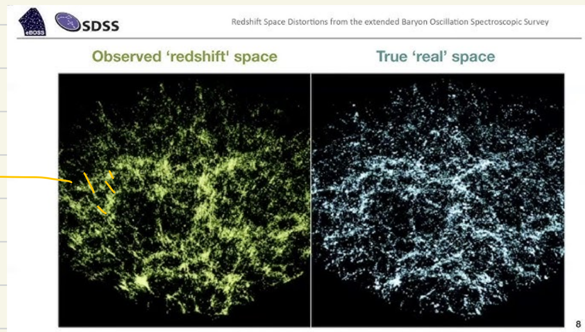
*Redshift space distortions → $(1 + f\mu^2)$

· Velocidades peculiares: mov. de la galaxia debido al cluster.
· Galaxia se aleja además por la expansión.

vel. peculiar: vel. along line-of-sight.

$$1 + z_{obs} = (1 + z)\left(1 + \frac{v_{\parallel}}{c}\right)$$

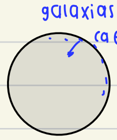
vel. peculiares



b: bias → En estadística se marginaliza.

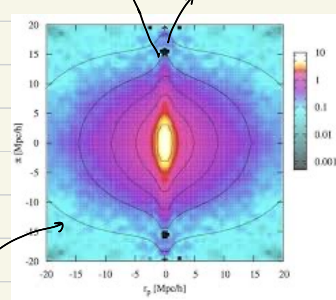
En el espacio de redshift:

galaxias quieren caer al pozo de potencial



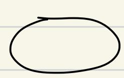
"Finger of God" effect.

se opone a la expansión del universo



kaiser effect

los.

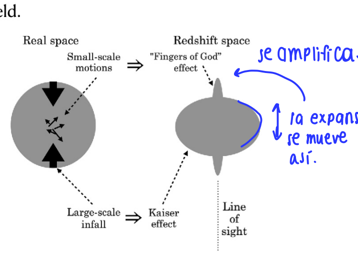


Redshift space distortion

The galaxy distances are measured through their redshifts.
The redshift includes the peculiar velocity of the galaxies, so there is an error in the distances.

The small scale peculiar velocities cause the fingers-of-god effect: galaxies in a cluster acquire an additional random redshift that distorts the cluster distribution, making it appear elongated along the line of sight.

On large scales, the galaxies tend to fall toward concentrations, so that the velocity field is coupled to the density field.



se amplifica.

la expansión se mueve así.

Given a peculiar velocity v of a source at position r , the projection of the velocity along the line-of-sight

$$u(r) = \vec{v} \cdot \frac{\vec{r}}{r}$$

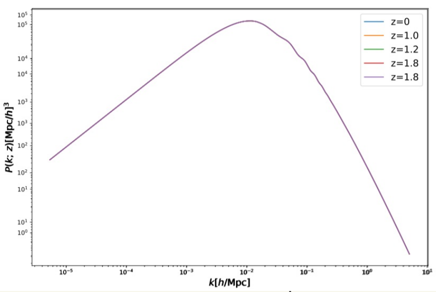
The coordinate transformation from real space (r) to z-space (s) is

$$\vec{s} = \vec{r}\left[1 + \frac{u(r) - u(0)}{r}\right]$$

We saw the equation of perturbation for matter: 2nd ODE but not k.
We are in linear theory and we said that each k evolves independently, hence if there were a k appearing, then we fix the k and we evaluate the ODE.
So the solution is just a function of time. What does this mean for the P(k)?

$$\delta(k;\, z) = \delta(k)\delta(z) \quad \Rightarrow \quad \begin{array}{l} \delta(k) \to T(k) \\ \delta(z) \to \text{growing solution} \end{array}$$

$$\delta(z) = \frac{G(z)}{1+z}$$ Growth factor



$$f(z) = -\frac{\mathrm{d}\ln G(z)}{\mathrm{d}\ln(1+z)}$$ Growth rate

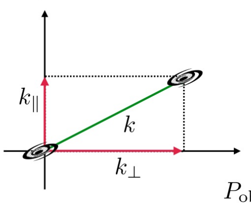
rms density fluctuation in a sphere

$$\sigma_8 = \frac{1}{2\pi^2}\int \mathrm{d}k\, k^2 P_m(k;\, z=0)\, |W(k\, R_8)|^2$$

integral en un vol. nos da la masa.

Alcock-Paczynski effect:
The measurement of the galaxy power spectrum requires the assumption of a reference cosmology to transform the observed redshifts into distances. Assuming an incorrect cosmology leads to a rescaling of components

sólo tenemos fotones, no distancias (obligados a asumir una cosmología de referencia).



$$k_\parallel = \frac{H(z)}{H_{\rm ref}(z)} k_{\parallel,\rm ref} \qquad k = \sqrt{k_\parallel^2 + k_\perp^2} \quad \& \quad \mu = \frac{k_\parallel}{k}$$

$$k_\perp = \frac{D_{\rm A,ref}(z)}{D_{\rm A}(z)} k_{\perp,\rm ref}$$

$$P_{\rm obs}(k_{\rm ref},\, \mu_{\rm ref};\, z) = \frac{D^2_{\rm A,ref}(z)}{D^2_{\rm A}(z)}\frac{H(z)}{H_{\rm ref}(z)}\left[b\sigma_8(z) + f(z)\sigma_8(z)\mu^2\right]^2 \frac{P_m(k\, z)}{\sigma_8(z)}$$

Incertidumbre en la transformación de redshift a distancia.

Redshift uncertainty:
accounts for the smearing of the galaxy density field along the line of sight direction

$$P_{\rm obs}(k_{\rm ref},\, \mu_{\rm ref};\, z) = \frac{D^2_{\rm A,ref}(z)}{D^2_{\rm A}(z)}\frac{H(z)}{H_{\rm ref}(z)}\left[b\sigma_8(z) + f(z)\sigma_8(z)\mu^2\right]^2 \frac{P_m(k\, z)}{\sigma_8(z)} F_z(k,\, \mu;\, z) \qquad F_z(k,\, \mu;\, z) = e^{-\left[\frac{k^2\mu^2 c^2}{H^2(z)}(1+z)^2\sigma^2_{0,z}\right]}$$

perturbed photon propagation

12 · nov.

- λ parámetro afín : no se puede integrar
- A background dτ = dr → elemento de línea 0.
- r : line of sight.

Gathering and dividing by $k^0 = \mathrm{d}\tau/\mathrm{d}\lambda_s$

$$\frac{\mathrm{d}k^0}{\mathrm{d}\lambda_s} = -2aH(k^0)^2 - (k^0)^2\left(\frac{\partial\psi}{\partial\tau} - \frac{\partial\phi}{\partial\tau} + 2\psi_{,r}\right)$$

$$\frac{1}{a^2k^0}\frac{\mathrm{d}(a^2k^0)}{\mathrm{d}\tau} = -\left(\frac{\partial\psi}{\partial\tau} - \frac{\partial\phi}{\partial\tau} + 2\psi_{,r}\right)$$

Now we can split the momentum $k^0 = \hat{k}^0 + \delta k^0$ realizing that from the geodesic $\mathrm{d}(a^2\hat{k}^0)/\mathrm{d}\tau = 0$

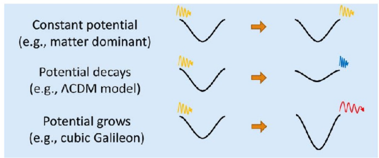
We can simply substitute $\frac{1}{a^2k^0}\frac{\mathrm{d}(a^2k^0)}{\mathrm{d}\tau} \to \frac{\mathrm{d}(\delta k^0/k^0)}{\mathrm{d}\tau}$

And we finally have: $$\frac{\mathrm{d}(\delta k^0/k^0)}{\mathrm{d}\tau} = -\left(\frac{\partial\psi}{\partial\tau} - \frac{\partial\phi}{\partial\tau} + 2\psi_{,r}\right)$$

SW effect

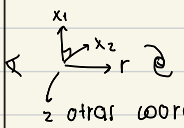
- SW effect es un observable excelente para teoría de grav. modificada. Importan las estructuras muy grandes : más que una galaxia

$$\frac{\mathrm{d}(\delta k^0/k^0)}{\mathrm{d}\tau} = -\left(\frac{\partial\psi}{\partial\tau} - \frac{\partial\phi}{\partial\tau} + 2\psi_{,r}\right)$$



SW

ecuaø de la frec. perturbada.



otras coordenadas

$$\frac{\mathrm{d}^2x^i}{\mathrm{d}\lambda_s^2} + 2aH\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_s}\frac{\mathrm{d}x^i}{\mathrm{d}\lambda_s} = -\left(\frac{\mathrm{d}\tau}{\mathrm{d}\lambda_s}\right)^2(\phi+\psi)_{,i}$$

This equation lead to the lensing effect: deviation of light ray passing through gravitational potentials.

Our starting point is: $$\frac{\mathrm{d}(\delta k^0/k^0)}{\mathrm{d}\tau} = -\left(\frac{\partial\psi}{\partial\tau} - \frac{\partial\phi}{\partial\tau} + 2\psi_{,r}\right)$$

We can find the following relation

$$\hat{k}^0\left(\frac{\partial\psi}{\partial\tau} + \psi_{,r}\right) = \frac{\partial\psi}{\partial\tau}\frac{\partial\tau}{\partial\lambda_s} + \frac{\partial\psi}{\partial r}\frac{\partial r}{\partial\lambda_s} = \frac{\mathrm{d}\psi}{\mathrm{d}\lambda_s} \quad\to\quad \frac{\partial\psi}{\partial\tau} + \psi_{,r} = \frac{\mathrm{d}\psi}{\mathrm{d}\tau}$$

Using the relations: $\hat{k}^0 = \mathrm{d}\tau/\mathrm{d}\lambda_s$ $\quad\mathrm{d}\tau = \mathrm{d}r\quad$ Null-condition

And our starting point becomes

$$\frac{\mathrm{d}(\delta k^0/k^0)}{\mathrm{d}\tau} = -2\frac{\mathrm{d}\psi}{\mathrm{d}\tau} + \left(\frac{\partial\psi}{\partial\tau} + \frac{\partial\phi}{\partial\tau}\right)$$

Integrate SW

SW

variaø de la frec. de los fotones desde que emite la fuente hasta el t observado.

$$\frac{\mathrm{d}(\delta k^0/k^0)}{\mathrm{d}\tau} = -2\frac{\mathrm{d}\psi}{\mathrm{d}\tau} + \left(\frac{\partial\psi}{\partial\tau} + \frac{\partial\phi}{\partial\tau}\right) \xrightarrow{\text{Integrating}} \left.\frac{\delta k^0}{k^0}\right|_E^O = -2\,\psi|_E^O + \int_E^O\left(\frac{\partial\psi}{\partial\tau} + \frac{\partial\phi}{\partial\tau}\right)\mathrm{d}\tau$$

O = instant of observation

E = instant of emission

$$\psi|_E^O = \psi_O - \psi_E$$

- Fluctuaøes de T° de los fotones debido a que salen de zonas de potencial.

sobredensidades → zonas con menos T°
→ SW effect
→ fotón tiene que perder mucha energía para salir de un gran potencial

Efecto doppler ←

· Temperature: $-k^\mu u_\mu = \hat{\nu}$

$$\frac{T_O}{T_E} = \frac{(k^\mu u_\mu)_O}{(k^\mu u_\mu)_E} \longrightarrow \left.\frac{\delta T}{T}\right|_O = \left.\frac{\delta T}{T}\right|_E + \left.\frac{\delta\left(k^\mu u_\mu\right)}{k^\mu u_\mu}\right|_O - \left.\frac{\delta\left(k^\mu u_\mu\right)}{k^\mu u_\mu}\right|_E$$

· En la direcc radial: $d\tau = dr$. → viaja la partícula.

$k^\mu u_\mu = k^0 u_0$

sólo parte perturbada espacial

$u_\mu = [-a(1+\psi), a v_i]$

$$\left.\frac{\delta T}{T}\right|_O = \left.\frac{\delta T}{T}\right|_E + \left.\frac{\delta\left(k^\mu u_\mu\right)}{k^\mu u_\mu}\right|_O - \left.\frac{\delta\left(k^\mu u_\mu\right)}{k^\mu u_\mu}\right|_E$$

are written as

$$\frac{k^0 \delta u^0}{k^\mu u_\mu} = \frac{\delta u_0}{u_0} = \psi$$

$$\frac{k^i \delta u^i}{k^\mu u_\mu} = \frac{k^i \delta u^i}{k^0 u_0} = -\boxed{\frac{k^i}{k^0}} v_i = \boxed{e^i} v_i$$

cuadrimomento normalizado por su energía

vector unitario.

direcc del momento de los fotones

Collecting everything

$$\left.\frac{\delta T}{T}\right|_O = \left.\frac{\delta T}{T}\right|_E + e^i v_i|_E^O - \psi|_E^O + \int_E^O \left(\frac{\partial\psi}{\partial\tau} + \frac{\partial\phi}{\partial\tau}\right) d\tau$$

* En la CMB las fluctuaciones de densidad no nos importan ya que todos los fotones tendrán el mismo efecto.

Weak lensing.

· Queremos como cambian las componentes a lo largo de r

· $\Psi = -\phi - \psi$ = potencial grav. de lente.

· Si x^1, x^2 pequeñas

Let us consider the spatial part

$$\frac{d^2 x^i}{d\lambda_s^2} + 2aH \frac{d\tau}{d\lambda_s}\frac{dx^i}{d\lambda_s} = -\left(\frac{d\tau}{d\lambda_s}\right)^2 (\phi+\psi)_{,i}$$

With the help of $\hat{k}^0 = \frac{d\tau}{d\lambda_s} \propto a^{-2}$

The geodesic becomes

$$\frac{d^2 x^i}{dr^2} = \Psi_{,i} \qquad \text{where} \qquad \Psi = -\phi - \psi$$

Since the displacement vector $\vec{x} = (x^1, x^2)$ is small, we can write $x^i = r\theta^i$

$$\frac{d^2}{dr^2}\left(r\theta^i\right) = \Psi_{,i}$$

x_1 y x_2 (no r).

If the light ray reaches the observer located at $r = 0$ through the direction $\theta_0^i = (\theta_0^1, \theta_0^2)$ the integration

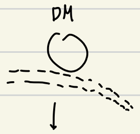
$$\theta^i = \theta_0^i + \frac{1}{r}\int_0^r dr'' \int_0^{r''} dr' \Psi_{,i}\left(r'\theta_0^1, r'\theta_0^2, r'\right)$$

Validity $\quad 0 < r'' < r, \quad 0 < r' < r'' \rightarrow \quad r' < r'' < r, \quad 0 < r' < r$

posició de la lente grav.

Posición de la fuente.

DM



Estos fotones se toman muy cerca y se desvían por la F de marea.

Integrating out the r'' in the region r' < r'' < r, it follows that

$$\theta^i = \theta_0^i + \int_0^r dr' \left(1 - \frac{r'}{r}\right) \Psi_{,i}\left(r'\theta_0^1, r'\theta_0^2, r'\right)$$

Two light rays separated by a small interval Δx will obey the equation

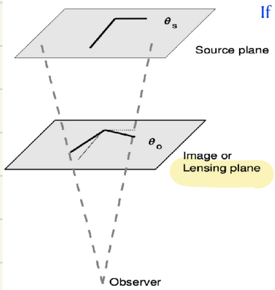
$$\Delta\theta^i = \Delta\theta_0^i + \Delta\theta_0^j \int_0^r dr' \left(1 - \frac{r'}{r}\right) r'\Psi_{,ij} \left(r'\theta_0^1, r'\theta_0^2, r'\right)$$

comes by taking the variation of potential with respect to the angles

· como varía la posición de la fuente según lo que observamos

· κ_{wl} → convergence: vemos la fuente + luminosa (fotones viajando convergen en un punto)

shear → distorsión de la imagen.



If the separation is in the source plane

we have an equation that connects the source plane with the observation plane, which is described with the matrix

$$A_{ij} \equiv \frac{\partial \theta_s^i}{\partial \theta_0^i} = \delta_{ij} + D_{ij}$$

Distortion tensor

$$D_{ij} = \int_0^{r_s} \mathrm{d}r' \left(1 - \frac{r'}{r_s}\right) r' \Psi_{,ij} = \begin{pmatrix} -\kappa_{\rm wl} - \gamma_1 & -\gamma_2 \\ -\gamma_2 & -\kappa_{\rm wl} + \gamma_1 \end{pmatrix}$$

Convergence: describe the magnification of the source image.

$$\kappa_{\rm wl} = -\frac{1}{2} \int_0^{r_s} \mathrm{d}r' \left(1 - \frac{r'}{r_s}\right) r' \left(\Psi_{,11} + \Psi_{,22}\right)$$

Shear: describe the distortion of the source image.

$$\gamma_1 = -\frac{1}{2} \int_0^{r_s} \mathrm{d}r' \left(1 - \frac{r'}{r_s}\right) r' \left(\Psi_{,11} - \Psi_{,22}\right) ,$$

$$\gamma_2 = -\int_0^{r_s} \mathrm{d}r' \left(1 - \frac{r'}{r_s}\right) r' \Psi_{,12}$$

12. nov.

correlation function and power spectrum.

Estadística descriptiva: cómo se distribuyen los datos.

" de estimación: derivar info. de los parámetros.

· Bariones no importan mucho pero es lo único que observamos → conectarlo con Dark matter.

No es suficiente estadística xq esto es uniforme y queremos calcular las perturbaciones. ←

· $\rho_0 = \frac{N}{V}$ → N objetos, → volumen } → densidad en número

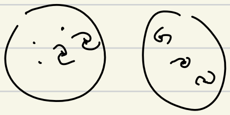
media estadística del nº.

$dN_{ab} = \langle n_a n_b \rangle$ the average number of *pairs* in the two volumes separated by a distance r_{ab}.

· si el nº de objetos de un vol. no depende del otro: $\xi = 0$

nos interesa saber la dist. entre 2 pts.

gravedad hace que las 2 esferas estén correlacionadas



$$\mathrm{d}N_{ab} = \langle n_a n_b \rangle = \rho_0^2 \mathrm{d}V_a \mathrm{d}V_b \left[1 + \boxed{\xi(r_{ab})}\right]$$ con $\boxed{r_{ab} > 0}$ los volumen no coinciden

The average is expressed as <>

There are two types of average:

Ensemble average: the two volumes come from two different realizations (e.g. N-body simulations). ⟶ para ≠ modelos.

Sample average: the two volumes come form the same realizations. → consideran la media de alguna cant. particular. → bias según el modelo

The last one, if the volume are "well" separated that they be considered uncorrelated -> **ensemble** = **sample**

If the distribution comes from randomly distributed particles:

$$\langle n_a n_b \rangle = \langle n_a \rangle \langle n_b \rangle = \rho_0^2 \mathrm{d}V_a \mathrm{d}V_b \quad \Longrightarrow \quad \xi = 0$$

Inversely, if $\xi \neq 0$ particles are correlated

Then the correlation function can be written as a spatial average of the product of the density contrast

¿despejando

$$\delta(r_a) = \frac{n_a}{\rho_0 \mathrm{d}V_a} - 1 \quad \longrightarrow \quad \xi(r_{ab}) = \frac{\mathrm{d}N_{ab}}{\rho_0^2 \mathrm{d}V_a \mathrm{d}V_b} - 1 = \langle \delta(r_a)\delta(r_b) \rangle$$

→ Contraste de densidad entre el punto a y el punto b.

$\delta = \rho/\rho_0$

With the condition $\langle \delta(r_a) \rangle = \langle \delta(r_b) \rangle = 0$

If this average is taken to be the sample average, then it means we have to average over all possible positions:

$$\xi(\mathbf{r}) = \frac{1}{V}\int \delta(\mathbf{y})\delta(\mathbf{y}+\mathbf{r})\mathrm{d}V_y$$

<u>Homogeneous</u>: if it depends only on the separation **r** and not on $\mathbf{r}_a$ and $\mathbf{r}_b$.
(Strictly, also all the higher moments should have the same property).
<u>Ergodic</u> (or *fair* sample): if the ensemble average = sample average.

Usually, the correlation function is evaluated considering one of the two volumes with uniform distribution

$$\rho_0 \mathrm{d}V_a = 1 \quad \Longrightarrow \quad \mathrm{d}N_b = \rho_0 \mathrm{d}V_b\,[1 + \xi(r_b)]$$

So, the correlation function is:

$$\xi(r) = \frac{\mathrm{d}N(r)}{\rho_0 \mathrm{d}V} - 1 = \frac{\langle \rho_c \rangle}{\rho_0} - 1$$

In a finite volume:

> 0 ⇒ more particles than in a uniform distribution

$$\int \xi(r)\mathrm{d}V = \frac{1}{\rho_c}\int \frac{\mathrm{d}N}{\mathrm{d}V}\mathrm{d}V - V = \frac{N}{\rho_0} - V = 0$$

< 0 ⇒ less particles than in a uniform distribution

· Hay que crear catálogos que tengan la misma tipología de galaxias que quiero observar.
↳ D → real (datos)
↳ R → randoms.

The simplest way to measure ξ is to compare the real catalog to an artificial random catalog with exactly the same boundaries and the same selection function. Then the estimator can be written as

$$\xi(r) = \frac{DD}{RR} - 1$$

Peebles & Hauser

DD = the number of galaxies at distance r counted by an observer centered on a real galaxy (data D).
RR = the number of galaxies at the same distance but in the random catalog.

The most common one is the Landy & Szalay

$$\xi(r) = \frac{DD - 2\boxed{DR} + RR}{RR}$$

→ Posición de la galaxia real y lo pongo en un mapa de catálogo random.

In Fourier space, any real quadratic function of a perturbation variable is called a *power spectrum*.

· En fourier cada modo de la perturbación es indep.

$$f(\mathbf{x}) = \frac{V}{(2\pi)^3}\int f_{\mathbf{k}}\, e^{i\mathbf{k}\cdot\mathbf{x}}\, \mathrm{d}^3k$$

$$f_{\mathbf{k}} = \frac{1}{V}\int f_{\mathbf{x}}\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, \mathrm{d}^3x$$

These conventions are used so they both have the same dimensions. Important pre-factor, only $(2\pi)^{-3}$.

$$\delta_D(\mathbf{x}) = (2\pi)^{-3}\int e^{i\mathbf{k}\cdot\mathbf{x}}\mathrm{d}^3k \qquad \int \delta_D(\mathbf{k})\mathrm{d}^3k = \int \delta_D(\mathbf{x})\mathrm{d}^3x = 1$$

$$\delta_D(\mathbf{k}) = (2\pi)^{-3}\int e^{i\mathbf{k}\cdot\mathbf{x}}\mathrm{d}^3x$$

The Fourier transform of the density contrast of a density field $\delta_k = \frac{1}{V}\int \delta(\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}}\mathrm{d}V$

The power spectrum is defined as $P(\mathbf{k}) = V\,|\delta_k|^2 = V\langle \delta_k \delta_k^* \rangle$

Hence the power spectrum has dimensions of volume, and it follows

$$P(\mathbf{k}) = \frac{1}{V}\int \delta(\mathbf{x})\delta(\mathbf{y})e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\mathrm{d}V_x\,\mathrm{d}V_y$$

dist. relativa entre las 2 zonas.

With $\mathbf{r} = \mathbf{x} - \mathbf{y} \quad \longrightarrow \quad P(\mathbf{k}) = \int \xi(\mathbf{r})e^{-\boxed{i\mathbf{k}\cdot\mathbf{r}}}\mathrm{d}V$

→ Bajo isotropía, el prod. escalar no nos importa, solo el cos θ ya que no importa desde donde miramos.

Where $\xi(\mathbf{r}) = \langle \delta(\mathbf{y}+\mathbf{r})\delta(\mathbf{y}) \rangle = \frac{1}{V}\int \delta(\mathbf{y}+\mathbf{r})\delta(\mathbf{y})\mathrm{d}V_y$

the power spectrum is the Fourier transform of the correlation function

Assuming spatial isotropy, i.e. that the correlation function depends only on the modulus r, the spectrum depends only on the modulus of k:

$$P(k) = \int \xi(r) r^2 \, \mathrm{d}r \int_0^\pi e^{-ikr\cos\theta} \sin\theta \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\phi = 4\pi \int \xi(r) \frac{\sin kr}{kr} r^2 \, \mathrm{d}r$$

A more general definition considers the ensemble average rather then the volume average; considere the ensemble:

$$V \langle \delta_{\mathbf{k}} \delta^*_{\mathbf{k}'} \rangle = \frac{1}{V} \int \langle \delta(\mathbf{x}) \delta(\mathbf{y}) \rangle e^{-i\mathbf{k}\cdot\mathbf{x} + i\mathbf{k}'\cdot\mathbf{y}} \mathrm{d}V_x \, \mathrm{d}V_y$$
$$= \frac{1}{V} \int \langle \delta(\mathbf{y}) \delta(\mathbf{y} + \mathbf{r}) \rangle e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{y} - i\mathbf{k}\cdot\mathbf{r}} \mathrm{d}V_r \, \mathrm{d}V_y.$$

$$V \langle \delta_{\mathbf{k}} \delta^*_{\mathbf{k}'} \rangle = \frac{1}{V} \int \xi(\mathbf{r}) e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{y} - i\mathbf{k}\cdot\mathbf{r}} \mathrm{d}V_r \, \mathrm{d}V_y$$
$$= \frac{1}{V} \int e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{y}} \mathrm{d}V_y \int e^{-i\mathbf{k}\cdot\mathbf{r}} \xi(\mathbf{r}) \mathrm{d}V_r$$
$$= \frac{(2\pi)^3}{V} P(\mathbf{k}) \delta_D (\mathbf{k} - \mathbf{k}').$$

↳ Es no nulo cuando $k = k'$. Modos del mismo tamaño.

To perform ensemble averages, we need to fix a position and take the average over the ensemble of realizations. Then the average can enter the integration and acts only over the random variables, i.e. δ.

ensemble average: debería ser la media entre contrastes de densidad sobre ≠ universos → no se puede → se toma sample average (la media).

1) modes at different wavelengths are uncorrelated if the field is statistically homogeneous = if ξ does not depend on the position y but only on the separation r.

2) Since δ is a real function $\Rightarrow$ $\delta(\mathbf{k}) = \delta^*(-\mathbf{k}) \implies V \langle \delta_{\mathbf{k}} \delta_{\mathbf{k}'} \rangle = (2\pi)^3 V^{-1} P(\mathbf{k}) \delta_D (\mathbf{k} + \mathbf{k}')$

14. nov.

· Materia oscura domina y hay que considerar la distribuø de las galaxias.

· Distribuø: Poisson.

· N partículas en posiø x_i en un vol. V: $\int W(x) dV = 1$

↓ window function: un valor dentro de un vol. y fuera otro.

* Top-hat window function: como la funø escalón.

Correlation function and P.S.

Inside the survey $W(\mathbf{x}) = 1/V$

The real quantity $\delta_s = \delta(\mathbf{x}) V W(\mathbf{x})$

We express the field as a sum of Dirac functions $\rho(\mathbf{x}) = \sum_i \delta_D (\mathbf{x} - \mathbf{x}_i)$ → densidad no nula en una galaxia.

promedio (background)

$$\delta_s(\mathbf{x}) = \left[\frac{\rho(\mathbf{x})}{\rho_0} - 1 \right] V W(\mathbf{x}) = \frac{V}{N} \sum_i w_i \delta_D (\mathbf{x} - \mathbf{x}_i) - V W(\mathbf{x})$$

$\rho_0 = N/V$ ↳ vol. en el que hacemos los cálculos.

$w_i = V W(\mathbf{x}_i)$ ↳ se calcula en la posiø de la galaxia

The Fourier transform is

$$\delta_k = \frac{1}{V} \int \left(\frac{V}{N} \sum_i w_i \delta_D (\mathbf{x} - \mathbf{x}_i) - V W(\mathbf{x}) \right) e^{-i\mathbf{k}\cdot\mathbf{x}} \mathrm{d}V = \frac{1}{N} \sum_i w_i e^{-i\mathbf{k}\cdot\mathbf{x}_i} - W_k$$

↔ ; $\delta_k = \sum \frac{g_i}{V^{1/2}} e^{-ik\cdot r_i}$

¿Qué es? ⊛

The spherical top-hat window function is defined:

$$W(x) = 1/V, \quad \text{inside a spherical volume } V \text{ of radius } R,$$
$$W(x) = 0, \quad \text{outside.}$$

In Fourier space is

$$W_k = V^{-1} \int_V e^{-ik\cdot x} \, \mathrm{d}V = \frac{3}{R^3} \int_0^R \frac{r \sin kr}{k} \, \mathrm{d}r = \frac{3(\sin kR - kR \cos kR)}{(kR)^3},$$

modo.

Radio del volumen (~8 Mpc normalª) ↓ radio en el cual la raíz cuadrática media es cerca a 1.

Note: the function declines rapidly as k -> π/ R.

squaring and averaging $\delta(\mathbf{k})$ by separating the i=j terms from the others, we find

$$\langle \Delta^2(\mathbf{k}) \rangle \equiv V \langle \delta_{\mathbf{k}} \delta^*_{\mathbf{k}} \rangle = P(\mathbf{k}) + P_n \longrightarrow \text{ruido}$$

The true PS

$$\langle \Delta^2(\mathbf{k}) \rangle \equiv V \langle \delta_{\mathbf{k}} \delta^*_{\mathbf{k}} \rangle = P(\mathbf{k}) + P_n$$

Where each term is (note the sum)

$$P(\boldsymbol{k}) = \frac{V}{N^2} \sum_{i \neq j} \langle w_i w_j \rangle \, e^{-i\boldsymbol{k}\cdot(\boldsymbol{x}_i - \boldsymbol{x}_j)} - V W_k^2,$$

$$P_n = \frac{V}{N^2} \sum_i w_i^2 = \frac{V}{N},$$

Only if w is 0 or 1.

The noise spectrum:

- negligible only for large densities, ρ = N / V -> ∞
- It is the power spectrum of a distribution with no intrinsic correlation, i.e. obtained by throwing the particles at random, i.e. the power spectrum of a Poissonian distribution.

Since the galaxy distributions are often sparse, the noise is not always negligible and has to be subtracted.

The true estimator of the power spectrum

$$\hat{P}(\boldsymbol{k}) = \underbrace{\Delta^2(\boldsymbol{k})}_{\text{matter P.S.} + P_n} - P_n$$

the power spectrum does not characterize a distribution completely, unless we know the distribution has some specific property, e.g., Gaussian, or Poisson, etc. In particular, if we assume the fluctuations to be Gaussian, we can derive the variance of the power spectrum

$$\sigma_P^2 \equiv \left\langle [\hat{P}(\boldsymbol{k}) - P(\boldsymbol{k})]^2 \right\rangle = \langle \Delta^4(\boldsymbol{k}) \rangle - \langle \Delta^2(\boldsymbol{k}) \rangle^2$$

where $P(\boldsymbol{k}) \equiv \langle \hat{P}(\boldsymbol{k}) \rangle = \langle \Delta^2(\boldsymbol{k}) \rangle - P_n$

We need to evaluate $\Delta^4(\boldsymbol{k})$ we proceed as by neglecting the window (= large volume) ⇒ $\delta_k = \sum_i \frac{g_i}{V^{1/2}} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_i}$

And gi are Gaussian random variable, so we have

$$\Delta^2(\boldsymbol{k}) = V \langle \delta_{\boldsymbol{k}} \delta^*_{\boldsymbol{k}} \rangle = \sum_i g_i^2$$

como nuestras perturbaçøes son lineales no pueden oscilar tanto y k+k' es fuerte# oscilatoria.

We want to evaluate:

$$\langle \Delta^2(\boldsymbol{k}) \Delta^2(\boldsymbol{k}') \rangle = V^2 \langle \delta_{\boldsymbol{k}} \delta^*_{\boldsymbol{k}} \delta_{\boldsymbol{k}'} \delta^*_{\boldsymbol{k}'} \rangle = \sum_{ijmn} \langle g_i g_j g_m g_n \rangle \, e^{-i\left(\boldsymbol{k}\cdot\boldsymbol{r}_i - \boldsymbol{k}\cdot\boldsymbol{r}_j + \boldsymbol{k}'\cdot\boldsymbol{r}_m - \boldsymbol{k}'\cdot\boldsymbol{r}_n\right)}$$

Now, the oscillating terms in the sum are negligible except when i=j=m=n or when the indices are equal in pairs. Then we can write

i=m , j=n.

$$\langle \Delta^2(\boldsymbol{k}) \Delta^2(\boldsymbol{k}') \rangle = \sum_i \langle g_i^4 \rangle + \sum_i \sum_{j \neq i} \langle g_i^2 \rangle \langle g_j^2 \rangle \left[1 + e^{-i\left(\boldsymbol{k} + \cancel{\boldsymbol{k}'}\right)\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)} + e^{-i\left(\boldsymbol{k} - \boldsymbol{k}'\right)\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)} \right].$$

goes away for k~k' since it's a fast oscillation

g is a Gaussian variable:

- all odd moments are zero → momentos impares son 0.
- and even moments can be written in terms of the variance; in particular $\langle g_i^4 \rangle = 3 \langle g_i^2 \rangle^2$ → para una distrib. gaussiana.

$$\langle \Delta^2(\boldsymbol{k}) \Delta^2(\boldsymbol{k}') \rangle = \sum_i \sum_j \langle g_i^2 \rangle \langle g_j^2 \rangle \left[1 + e^{-i\left(\boldsymbol{k} - \boldsymbol{k}'\right)\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)} \right],$$

proporcional a la varianza de nuestros campos.

And it becomes

$$\langle \Delta^2(\boldsymbol{k}) \Delta^2(\boldsymbol{k}') \rangle = \langle \Delta^2(\boldsymbol{k}) \rangle \langle \Delta^2(\boldsymbol{k}') \rangle + V^2 \langle \delta_{\boldsymbol{k}} \delta^*_{\boldsymbol{k}'} \rangle^2 .$$

And using (already seen)

$$\sigma_P^2 \equiv \left\langle [\hat{P}(\boldsymbol{k}) - P(\boldsymbol{k})]^2 \right\rangle = \langle \Delta^4(\boldsymbol{k}) \rangle - \langle \Delta^2(\boldsymbol{k}) \rangle^2 \qquad \langle \Delta^2(\mathbf{k}) \rangle \equiv V \langle \delta_{\mathbf{k}} \delta^*_{\mathbf{k}} \rangle = P(\mathbf{k}) + P_n$$

We find

$$\sigma_P^2(\boldsymbol{k}) = V^2 \langle \delta_{\boldsymbol{k}} \delta^*_{\boldsymbol{k}} \rangle^2 = (P(\boldsymbol{k}) + P_n)^2 \quad \Longrightarrow \quad \frac{\sigma_P^2(\boldsymbol{k})}{P^2(\boldsymbol{k})} = \left(1 + \frac{1}{nP(\boldsymbol{k})}\right)^2$$

In general we consider the shell-averaged spectrum, i.e. the spectrum for all modes k lying in the shell Δk of volume V_k

↳ modos que tengan un cierto tamaño.

$$P(k) = \frac{1}{V_k} \int_{\Delta k} P(\boldsymbol{k}') \, \mathrm{d}^3 k'$$

If the survey has a volume $V_s = L^3$, the lowest wavenumber we have is = $2\pi/L$. Then the number of independent k-modes in a volume V_k is

$$N_k = \frac{V_k}{k^3_{\text{min}}} = \frac{V_k V_s}{(2\pi)^3}$$

* no podemos considerar escalas mayores que nuestro cielo.

Therefore the error on the shell-averaged spectrum P(k) is reduced by the factor $1/N_k$ and we obtain

$$\frac{\sigma_P^2(k)}{P^2(k)} \simeq \frac{(2\pi)^3}{V_k V_s} \left(1 + \frac{1}{nP}\right)^2$$

→ + grande la survey, mejor nuestra observaø.

Another way of looking at this equation is to say that the effective k-volume resolution $k^3_{\text{min}} = 2\pi/L$ degrades due to the shot noise to $k^3_{\text{min}}(1 + 1/nP)^2$

so that there are effectively less independent k-volumes to average over.

The general formula is

$$\frac{\sigma_P^2(k)}{P^2(k)} = \frac{(2\pi)^3 \int_{V_s} \mathrm{d}^3 r n^4 w^4 [1 + 1/(nP(k))]^2}{V_k \left[\int_{V_s} \mathrm{d}^3 r n^2 w^2\right]^2}$$

where:

- n = n(r) is the average density
- w = w(r) is a weighting function to minimize variance itself

If both are constante, we end up to the previous one. → se reduce a la anterior.

From P.S. to moments

The P(k) is often the basic outcome of structure formation theories, it is convenient to express all the other quantities in terms of it. Here we find the relation between the power spectrum and the moments of the counts in random cells.

Consider a finite cell. Divide it into infinitesimal cells with counts n_i either zero or unity, so that for any positive power m we have

$$\sum_i \langle n_i^m \rangle = N_0 \, (= \rho_0 V)$$

Being N_0 the counting average. We have, by definition of the correlation function:

$$\langle n_i n_j \rangle = \rho_0^2 \, (1 + \underbrace{\xi_{ij}}_{\text{funç du correlaç}}) \, \mathrm{d}V_i \; \mathrm{d}V_j \qquad \text{for} \quad i \neq j$$

→ media entre galaxias de 2 regiones.

The count in cell $N = \sum_i n_i$ and the variance (second-order moment) is

$$M_2 = \frac{1}{N_0^2} \left\langle (\Delta N)^2 \right\rangle = \frac{1}{N_0^2} \left(\left\langle N^2 \right\rangle - N_0^2 \right) \qquad \text{with} \qquad \Delta N = N - N_0$$

The expected value of

$$\left\langle N^2 \right\rangle = \left\langle \sum n_i \sum n_j \right\rangle = \sum_i \left\langle n_i^2 \right\rangle + \sum_{i \neq j} \langle n_i n_j \rangle = N_0 + N_0^2 \int W_i W_j \, (1 + \xi_{ij}) \, \mathrm{d}V_i \; \mathrm{d}V_j$$

Where W is the window function

$$\sigma^2 = \int W_1 W_2 \xi_{12} \; \mathrm{d}V_1 \; \mathrm{d}V_2 \qquad \text{with} \qquad \int W_i \; \mathrm{d}V = 1$$

varialanza en el n° de objetos

Then we find

$$\left\langle N^2 \right\rangle = N_0 + N_0^2 \left(1 + \sigma^2 \right)$$

And by definition

$$\xi_{12} = (2\pi)^{-3} \int P(\boldsymbol{k}) e^{i\boldsymbol{k} \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2)} \mathrm{d}^3 k$$

We arrive

bajo isotropía se va.

$$\sigma^2 = (2\pi)^{-3} \int P(\boldsymbol{k}) e^{i\boldsymbol{k} \cdot (\boldsymbol{r}_1 - \boldsymbol{r}_2)} W_1 W_2 \; \mathrm{d}^3 k \; \mathrm{d}^3 r_1 \; \mathrm{d}^3 r_2$$

For spherical cells of radius R, integrating over the angles, this reduces to

$$\sigma_R^2 = \frac{1}{2\pi^2} \int P(k) W_R^2(k) \, k^2 \; \mathrm{d}k$$

→ se obtiene σ_8.

And if the cell has a radius of 8 Mpc/h, the previous one is close to unity.
We also can find:

$$M_2 = N_0^{-1} + \sigma^2$$

The first and the second terms correspond to the noise and the count variance in the continuous limit.
The third moment is:

$$M^3 = N_0^{-2} + \int W_i W_j W_k \, (1 + \xi_{ijk}) \, \mathrm{d}V_i \; \mathrm{d}V_j \; \mathrm{d}V_k$$

similar relations can be found at any order. Non-zero higher-order moments are useful to quantify the deviation from Gaussianity of the matter and galaxy distribution.

19. nov.

ver Jupiter

· Datos se distribuyen de manera random.
- ↳ cuando se toman datos tenemos muchos errores que no controlamos.
- ↳ 2 tipos de errores → sistemático
  - ↳ estadístico
- ↳ variables random tienen una distribución
  - ↳ Gaussiana
  - ↳ Poisson.

  } más comunes

· variables random.
- · Discretas: funø de masa de prob.
- · continuas: PDF.

· Mean and variance:
- · si $b = E(\hat{\theta}) - \theta = 0$ → estimaø es no bias.
- · bias es un error sistemático ya que es la elección que yo tomo (no error de mediø).

* x: mediøes
  θ: parámetros (conjunto)

$$Var[\hat{\theta}] = E[(\hat{\theta} - E[\hat{\theta}])^2] = \int (\hat{\theta} - E[\hat{\theta}])^2 f(x_1;\theta)\, f(x_2;\theta) \ldots$$

xq las mediøes son indep.
(podemos factorializar)

Var. de la media:

$$Var(\hat{x}) = \frac{\sigma^2}{n} \quad \longrightarrow \text{ estimador bias.}$$

Estadística Bayesiana

PDF: $f(x;\theta)$

↳ diferenciar entre datos y parámetros.

probabilidad condicional de tener tal dato x dado un parámetro θ.

↳ suponemos un modelo.

. Funø de distribuø binomial:

$P(x|\theta)$ *

Likelihood for a fair coin $\theta = 0{,}5 \longrightarrow P(x=7 \mid \theta = 0{,}5) = 0{,}1172$

" " " biased coin $\theta = 0{,}7 \rightarrow P(x=7 \mid \theta = 0{,}7) = 0{,}2668$.

$\boxed{m_{th}} = 5 \log_{10} d_L(z;\Omega_{m,0}, \Omega_{\Lambda,0}) + \mathcal{M}$

↳ m : magnitud aparente distribuida como una variable Gaussiana.

→ $\Omega_{m,0} + \Omega_{\Lambda,0} + \Omega_{kp} = 1$

↓

Modelo Non-flat ΛCDM

↳ Hubble parameter: $\frac{H^2(z)}{H_0^2} = \Omega_{m,0} \cdot (1+z)^3 + (\Omega_{\Lambda,0}) + (1 - \Omega_{m,0} - \Omega_{\Lambda,0})(1+z)^2$.

· Reducir errores estadísticos

★Los sistemáticos son los humanos o astronómicos (ej: la explosión de la supernova que no conocemos)

↓

Hay que marginalizar sobre este parámetro.

Prob. conjunta (join prob.):

· $x_1 \in [x_1 - dx_1, x_1 + dx_1]$

· $x_2 \in$ ...

$\boxed{PDF}$

$f(x_i;\theta)d^n x = \prod_i f_i(x_i;\theta)\,dx_i$

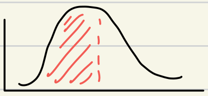
Queremos encontrar los θ que maximizan la prob. conjunta

Maximum likelihood estimation (MLE)

$\frac{\partial f}{\partial \theta_\alpha} = 0 \quad , \quad \alpha = 1, \ldots, m \qquad \longrightarrow \hat{\theta}_\alpha$ maximum likelihood estimator.

· Estimador θ es random xq dependen de los datos.

· Prob. de que $x < 1$:



Intervalo de confianza

↓

a 1σ, 2σ, 3σ

Bayes

* No es frecuentista xq no podemos repetir el experimento muchas veces (en muchos universos).

· Estimadores muy bias ⟶ Red Luminous galaxies son viejas.
↳ dependiendo de las galaxias que tomo, el bias es $\neq$.

· Nos apoyamos de la info. previa

· Dadas las mediciones, qué puedo decir del modelo:

likelihood   prior : info previa

$$\boxed{P(T, D)} = \frac{\boxed{P(D;T)}\,\boxed{P(T)}}{\boxed{P(D)}}$$

posterior.   ↳ evidencia

↓
teo. de Bayes.

posterior o likelihood

$$L(\theta_\alpha, x_i) = \frac{f(x_i;\theta_\alpha)\, p(\theta_\alpha)}{g(x_i)}$$

Func de distribuc normalizada a 1:

$$\int L\, d^n\theta_\alpha = 1 = \frac{\int f(x_i;\theta_\alpha)\, p(\theta_\alpha)\, d^n\theta_\alpha}{\boxed{g(x_i)}} \to \text{evidencia}$$

$$\to \int f(x_i;\theta)\, p(\theta_\alpha)\, d^n\theta_\alpha = g(x_i)$$ ⟶ Integral multidimensional
↳ No sirve para estimar los parámetros xq no depende de θ.

Prior: $P(\theta_\alpha)$ → Info. a priori
↳ si no tenemos info. de θ, prior es $= 1$ ⟶ El parámetro puede tomar cualquier valor de $-\infty$ a ∞ $(\Omega_m \in [0,1])$
↳ si se tiene a priori que $\Omega_{m,0} \not< 0{,}1$ $\Rightarrow$ $P(\Omega_{m,0} < 0{,}1) = 0$.
↳ si se tiene que $h = 0{,}72 \pm 0{,}08$ $\Rightarrow$ $p(h)$ es una Gaussiana con media 0,72 y desviación estándar 0,08.

From PDF to posterior:

$$\frac{\partial f}{\partial \theta_\alpha} = 0 \longrightarrow \frac{\partial L(\theta_i)}{\partial \theta_i} = 0 \qquad, i = 1, \dots, n$$

Confidence regions.

$$\int_{R(\alpha)} L(\theta_i)\, d^n\theta = \alpha$$

Marginalization $\longrightarrow$ $m_{th} = 5\log_{10} d_L + C$

$$L(\Omega_{m,0}, \Omega_{L,0}) = \int L(C, \Omega_{m,0}, \Omega_{L,0})\, dC$$

Marginalizar de manera analítica, $d_L \propto 1/H_0$

si $\alpha = M + 25 - 5\log_{10} H_0$

degenerado (hay que marginalizar para ambos.

· $L(\theta_j) = N \int d\alpha \exp\left[-\frac{1}{2}\sum_i \frac{(m_i - \mu_i - \alpha)^2}{\sigma^2}\right]$

· Si fijamos M podemos obtener H_0

* $\theta_i - \theta_i^{(B)} \longrightarrow$ Best-fit.

$$L(\theta_i) = \prod$$

Nuevos parámetros

$$= L_{max} \prod_i \frac{1}{\sqrt{2\pi\sigma^2_{p,i}}} \exp\left[-\frac{(\theta_i - \theta_i^*)^2}{2\sigma^2_{i*}}\right] \exp\left[\frac{(\theta_i^B - \theta_i^P)^2}{\dots}\right.$$

Evidencia: Nos dice si un modelo está más favorecido o no.

$\rightarrow$ $L_{max} = e^{-\frac{\chi^2_{min}}{2}}$

mayor $\Rightarrow$ modelo fitea mejor los datos

$$E = L_{max} \prod \frac{\sigma_{i*}}{\sigma_{p,i}} \exp\left\{-\frac{1}{2}\left[\left(\frac{\theta_i^B}{\sigma_{B,i}}\right)^2 + \left(\frac{\theta_i^P}{\sigma_{p,i}}\right)^2 - \left(\frac{\theta_i^*}{\sigma_{i*}}\right)^2\right]\right\}$$

error sobre los priors.

no penaliza su estadística.

· si mi análisis da 10 (best-fit) y el prior da 1, cualquiera de los 2 puede estar mal.

· Nuisance parámeters

MCMC

- Se mueve hacia el punto en que L es más alta (no se mueve por todo el espacio de parámetros)
- ≠ algoritmos $\longrightarrow$ ej: Metropolis - Hastings.
- Cuidado con el máx local!

· Distribuø estaøaria $\pi(x)$

$$\underbrace{\pi(x)}_{\text{única}} P(x'|x) = \pi(x') P(x|x') \longrightarrow \text{Transiø reversible}$$

única:

$$\frac{P(x'|x)}{P(x|x')} = \frac{P(x')}{P(x)}$$

$$\cdot\ P(x'|x) = \underbrace{g(x'|x)}_{\text{proposal distribution}} \underbrace{A(x',x)}_{\text{acceptance ratio}}$$

$$\cdot\ A(x',x) = \min\left(1,\ \frac{P(x')}{P(x)} \frac{g(x|x')}{g(x'|x)}\right) \longrightarrow \text{Buscar el min entre los valores}$$

$$A = \exp\Big(\underbrace{\text{proposal } \ln(L)}_{\chi^2} - \underbrace{\text{current } \ln(L)}_{\text{negativo.}}\Big)$$

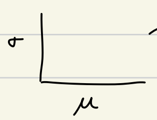
· Lo comparamos con un nº random $u \in [0,1]$.

· Si $u < A$ ⟶ aceptamos ese punto $x_{t+1} = x'$

para explorar varios puntos en el espacio y a veces aceptar nº random aunque tenga mayor χ^2.

26· nov.

Markov chain:



→ 2 parámetros
Busca el máximo.

· Metropolis - Hastings: accept or reject based on a prob. ratio.

· Gibbs sampling:

· Hamiltonian ...

· permite ir en otra direcø el random.

Fisher Matrix

Cramér - Rao Bound

Fisher Matrix es la mejor estimación.

$$\mathrm{var}(\hat{\theta}_\alpha) \geq \frac{1}{F_{\alpha\alpha}}$$

regla de la cadena

· Si la likelihood no es Gaussiana, se puede encontrar una gaussiana alrededor del peak que lo aproxima.

General Fisher Matrix:

$$L(\theta) = \frac{1}{(2\pi)^{n/2}\sqrt{\det(C)}} \exp\left[-\frac{1}{2} (x-\bar{x})^T C^{-1} (x-\bar{x}) \right] \quad / \log .$$

$$* \quad \frac{\partial \log \det(C)}{\partial \theta_\alpha} = Tr\left(C^{-1} \frac{\partial C}{\partial \theta_\alpha} \right)$$

$$\rightarrow \frac{\partial \log L(\theta)}{\partial \theta_\alpha} = -\frac{1}{2} Tr\left(C^{-1} \frac{\partial C}{\partial \theta_\alpha} \right) - (x-\bar{x})^T C^{-1} \frac{\partial \bar{x}}{\partial \theta_\alpha} + \frac{1}{2} (x-\bar{x})^T C^{-1} \frac{\partial C}{\partial \theta_\alpha} C^{-1} (x-\bar{x}) .$$

(deriv. segunda

(val de espectaø:

$$\rightarrow F_{\alpha\beta} = \underbrace{\frac{1}{2} Tr\left(C^{-1} \frac{\partial C}{\partial \theta_\alpha} C^{-1} \frac{\partial C}{\partial \theta_\beta} \right)} + \underbrace{\frac{\partial \bar{x}}{\partial \theta_\alpha} C^{-1} \frac{\partial \bar{x}}{\partial \theta_\beta}} .$$

¿por qué en nuestro caso no está el data vector en la Gaussiana?

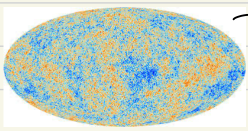
★ Sachs-Wolfe effect

· Corrimiento al rojo gravitacional que experimentan los fotones en el CMB al atravesar pozos de potencial gravitatorio generado por fluctuaciones de densidad en el universo temprano

SW En last scattering, cuando los fotones dejan de interactuar fuertemente con la materia y viajan libre#. Salen de una región de mayor densidad y pierden energía ⇒ corrim al rojo.

ISW (Sachs-Wolfe integrado) variación temporal de los pozos de potencial desde last-scattering hasta nosotros.

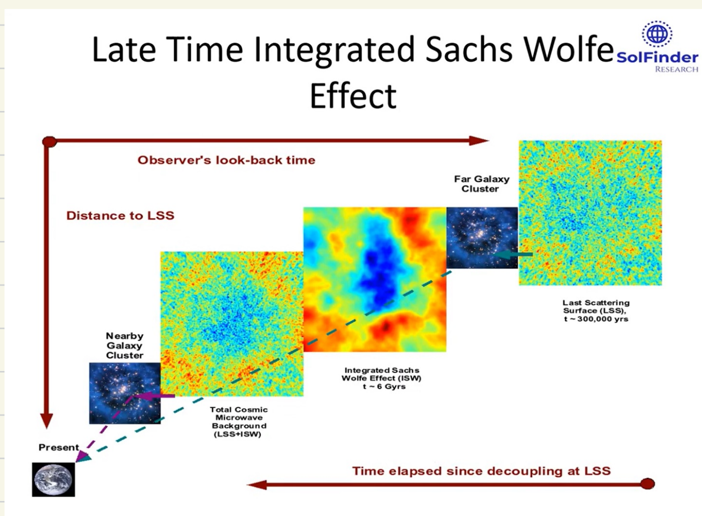
· pozo se profundiza ⇒ redshift
· " " hace menos profundo ⇒ blueshift.



Primary anisotropies in the CMB arise from:

① Intrinsic temperature fluctuations in the electron-nucleon-photon plasma at the time of last scattering
② Doppler effect due to velocity fluctuations in the plasma at last scattering.
③ SW effect
④ ISW effect

SW ⟶ domina en escalas grandes. (Multipolos bajos ($\ell < 40$))
ISW ⟶ anisotropías en multipolos bajos ($\ell < 20$)



EXTRA.

- ★ Curva de rotación de las galaxias → Observable de materia oscura.
  Supernovas tipo IA → " de energía oscura.

- ★ Fractional difference between the cosmologically interesting C_ℓ and the observed C_ℓ^{obs} is known as the cosmic variance.