

## Abstract

A parity inversion consists of changing the sign of spatial coordinates. While many physical laws conserve this symmetry, phenomena such as the weak interaction demonstrate that parity is not a universal property, revealing a fundamental asymmetry in nature. Under this premise, this work explores the possible violation of parity in Primordial Gravitational Waves (PGWs) generated during inflation—the period of accelerated expansion at the beginning of the universe. These waves are spacetime perturbations whose orientation is described by their helicity. A parity violation would imply that the wave helicities (+ and ×) propagate differently, creating an asymmetry in their evolution. This footprint remains imprinted on the polarization of the Cosmic Microwave Background (CMB). Since observations suggest hints of parity violation in the CMB, this study analyzes how such an anomaly could be evidence of parity-violating processes in the early universe. To this end, the evolution of Non-Chiral PGWs (which conserve parity) was analyzed in the stages prior to the CMB to model their signature on this radiation. This scenario will then be contrasted with the case of Chiral Gravitational Waves in future work to identify how parity violation alters the observational signals of the early universe.

## Introduction

**Context and Problem:** In standard cosmology, the universe is assumed to be symmetric under parity. This symmetry implies that certain correlations between light polarization patterns—specifically E-modes and B-modes—must be zero. However, recent data has revealed signals where these correlations are non-zero, suggesting the existence of parity-violating mechanisms at a cosmic scale.

**Physical Mechanisms:** These signatures may arise from cosmic birefringence, where photon coupling to pseudoscalar fields rotates the polarization planes, or from a background of chiral gravitational waves (CGWs) [1]. Unlike the standard case, CGWs exhibit an amplitude asymmetry between circular helicity states, imprinting a unique signature on CMB polarization.

**Objective:** To discern the origin of this parity violation, it is essential to accurately model the evolution of gravitational waves and characterize their theoretical contribution to  $E$  and  $B$  modes. This work establishes the framework for analyzing such evolution across different expansion regimes to identify chiral traces in the early universe [2].

**Current Work and Outlook:** In this work, we analyze the evolution of non-chiral tensor perturbations across radiation- and matter-dominated eras, establishing the necessary framework to extend this analysis to chiral scenarios. This study provides the foundation for our upcoming research, which will characterize the specific impact of chiral signatures on the net CMB polarization.

## Methodology

### 1. Linearized Action

We consider tensor perturbations to the second order in  $\gamma_{ij}$  in an FLRW background [3]. Expanding the Einstein-Hilbert action to quadratic order in the ADM formalism, we isolate the tensor action:

$$S_\gamma = \frac{m_{\text{Pl}}^2}{8} \int d^3x \int dt a^3(t) \left( \dot{\gamma}_{ij} \dot{\gamma}_{ij} - \frac{1}{a^2(t)} \partial_k \gamma_{ij} \partial_k \gamma_{ij} \right). \quad (1)$$

### 2. Canonical Field Redefinition

To simplify the dynamics, we decompose  $\gamma_{ij}$  into helicity states  $a \in \{+, \times\}$  and perform the canonical redefinition  $u_a = \frac{m_{\text{Pl}}}{2} a(\tau) \gamma_a$ . In conformal time  $\tau$ , the action transforms into:

$$S_\gamma = \frac{1}{2} \sum_a \int d^3x \int d\tau \left[ (u'_a)^2 - (\nabla u_a)^2 + (\mathcal{H}^2 + \mathcal{H}') u_a^2 \right]. \quad (2)$$

### 3. Evolution across Cosmological Epochs

Varying the action, we can obtain the equation of motion for specific expansion regimes:

- **Inflation (Quasi-de Sitter):** With  $a(\tau) = -1/(H\tau)$ , the evolution follows:

$$u''_{ak} + \left[ k^2 - \frac{1}{\tau^2} (2 + 3\epsilon) \right] u_{ak} = 0. \quad (3)$$

where  $\epsilon$  is the slow-roll parameter.

- **Radiation-Dominated (RD):** For  $a(\tau) \propto \tau$ , the evolution follows:

$$u''_{ak} + k^2 u_{ak} = 0. \quad (4)$$

- **Matter-Dominated (MD):** For  $a(\tau) \propto \tau^2$ , the equation becomes:

$$u''_{ak} + \left[ k^2 - \frac{2}{\tau^2} \right] u_{ak} = 0. \quad (5)$$

- **Matter and Radiation Dominated Era:** The scale factor considering both components is  $a(\tau) = a_{\text{eq}}[(\frac{\tau}{\tau_0} + 1)^2 - 1]$ . There, the mode function is

$$u''_{ak} + \left[ k^2 - \frac{2c}{c\tau^2 + 2\tau} \right] u_{ak} = 0. \quad (6)$$

## Results

### 1. Inflation

Imposing the Bunch–Davies vacuum conditions ( $\beta_k = 0, \alpha_k = 1$ ), the primordial evolution of gravitational waves is expressed in terms of Hankel functions of the first kind [4]:

$$\gamma_{a,k}(\tau) = -\frac{1}{m_{\text{Pl}}} \sqrt{\pi} \sqrt{-\tau^3} H e^{i\pi(\nu/2+1/4)} H_\nu^{(1)}(-k\tau) \quad (7)$$

### 2. Radiation-Dominated Era (RDE)

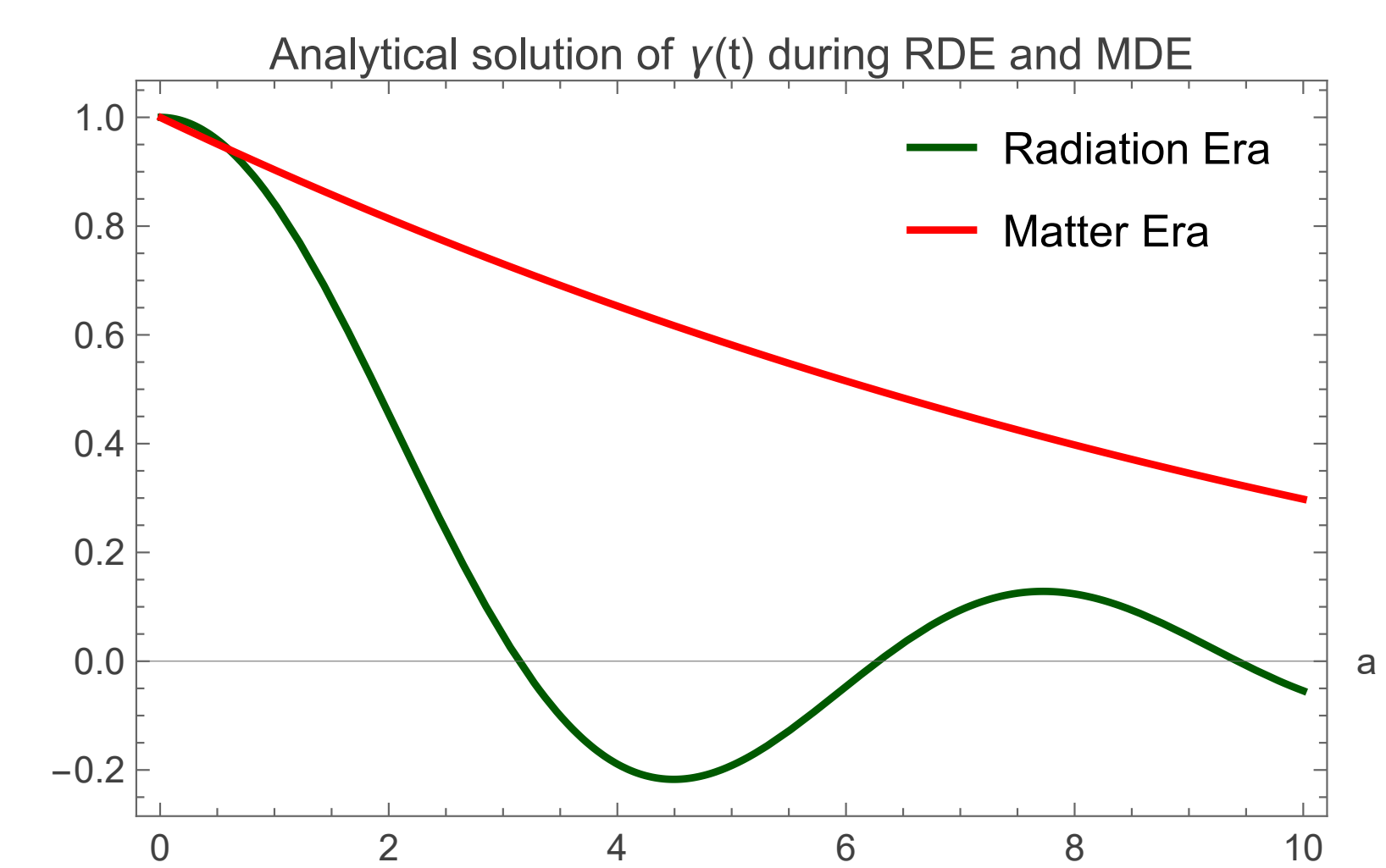
By requiring the perturbation to remain finite in the superhorizon limit ( $k\tau \rightarrow 0$ ), the solution simplifies to a spherical Bessel function of order zero, representing a damped oscillation:

$$\gamma_{a,k}(\tau) = \gamma_a(0) \frac{\sin(k\tau)}{k\tau}. \quad (8)$$

### 3. Matter-Dominated Era (MDE)

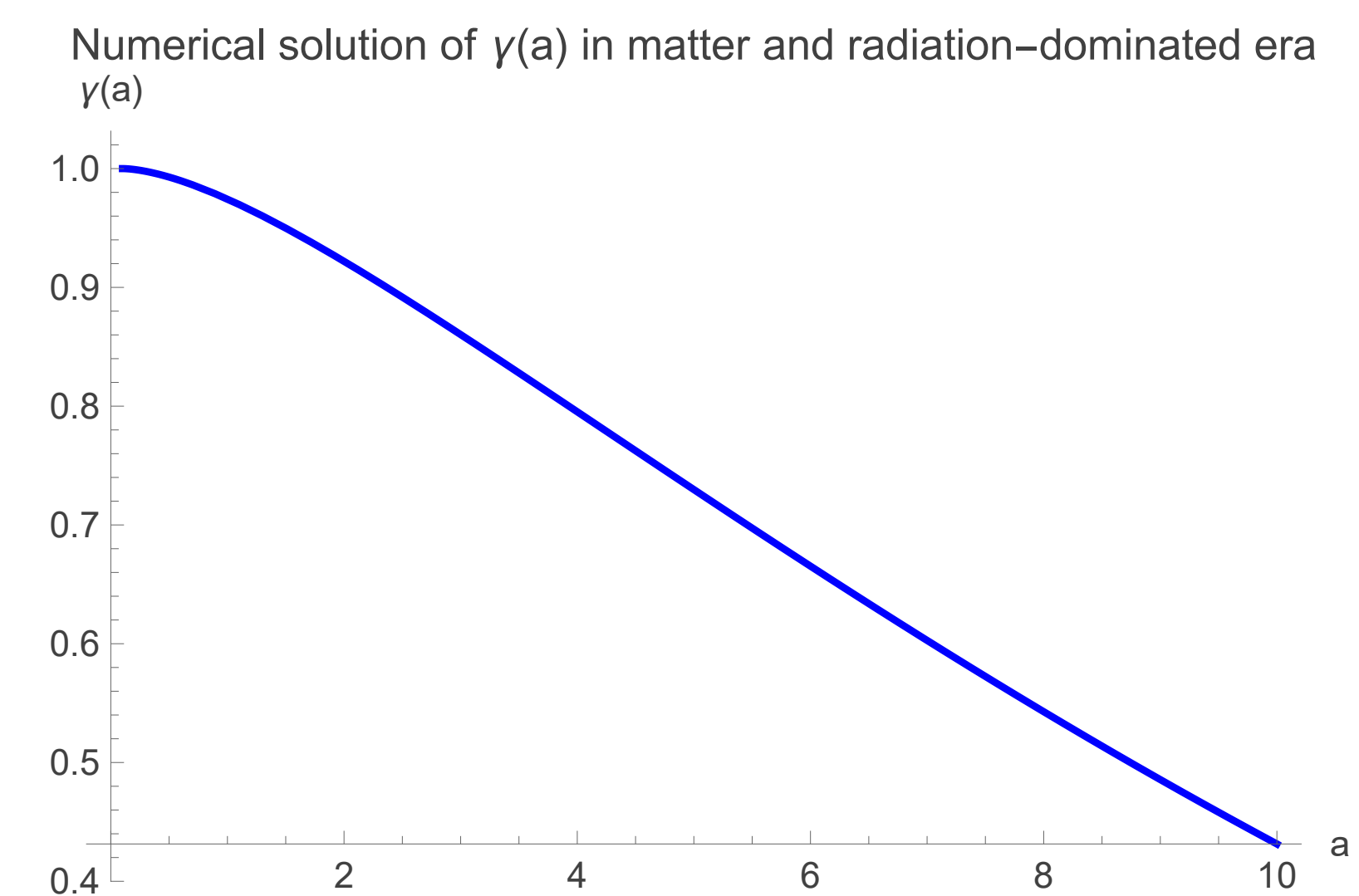
In the MDE, the physical requirement for superhorizon modes to remain constant leads to a solution governed by the spherical Bessel function of the first kind  $J_{3/2}$ :

$$\gamma_{a,k}(\tau) = 3\gamma_a(0) \sqrt{\frac{\pi}{2}} \frac{J_{3/2}(-k\tau)}{(-k\tau)^{3/2}}. \quad (9)$$



### 4. Matter-Radiation Dominated Era

The numerical solution accounts for the coupled dynamics of the transition regime, ensuring continuity between the RD and MD analytical limits.



The solutions exhibit the characteristic transition at horizon entry: modes remain constant at large scales and undergo damped oscillations once they enter the causal horizon ( $k\tau > 1$ ).

## Future Work

The natural extension of this framework is the study of parity-violating interactions in the early universe. We consider an effective action modified by a Chern-Simons gravitational term coupled to a field  $\Phi$  [1]:

$$\mathcal{L}_{\text{int}} = f(\Phi) R^\lambda_{\sigma\mu\nu} \tilde{R}^{\sigma\mu\nu}_\lambda. \quad (10)$$

While the background cosmology remains unaffected, the time evolution of the scalar field breaks parity in the tensor sector. This induces a modification in the linearized equations of motion, leading to an amplitude asymmetry between left- and right-handed circular polarization states ( $h_L \neq h_R$ ).

### Research Outlook:

- **Chiral Evolution:** We will apply the formalism established in this work to solve the modified equations of motion for CS-gravity during the radiation and matter eras.
- **CMB Signatures:** By quantifying the net circular polarization, we aim to predict the specific contribution to  $C_\ell^{EB}$  spectra.

## References

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- [4] G. B. Arfken, H. J. Weber, and F. E. Harris, "Chapter 14 - Bessel functions," in *Mathematical Methods for Physicists (Seventh Edition)* (G. B. Arfken, H. J. Weber, and F. E. Harris, eds.), pp. 643–713, Boston: Academic Press, seventh edition ed., 2013.