February 7, 2025

Probabilistic Programming

Final Exam

Question 1 (0.5 points). Let us assume a probability distribution for the time between events in a process in which events occur continuously and independently at a constant average rate. Which of the following types of distribution would we choose for modelling:

a) Binomial distribution
b) Exponential distribution
c) Poisson distribution

Question 2 (0.5 points). From the following distribution which one is a conjugate prior for the Poisson distribution (likelihood):

a) Gamma distribution
b) Beta distribution
c) Normal distribution

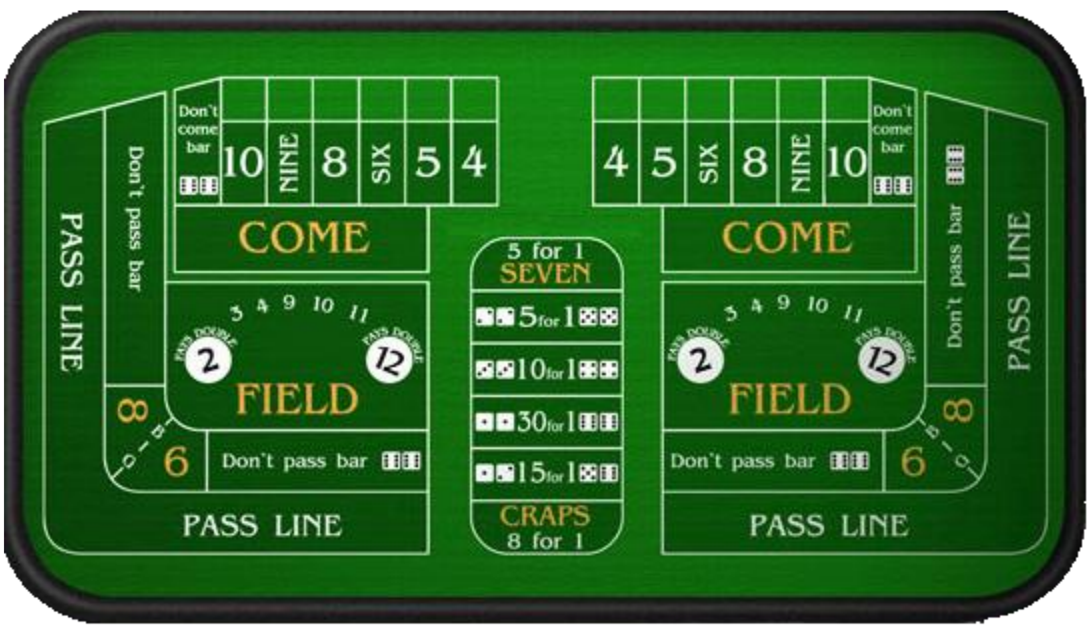
Problem 1 (1.5 points). The most primitive lots (a generic term for physical random number generators) were binary in operation. They usually consisted of a handful of natural objects which, when thrown and allowed to fall at random, would each come to rest with either of two distinctive faces uppermost. Several binary lots would be thrown together to give a reasonable number of different outcomes. Thus, the eleven outcomes of two six-sided dice are paralleled by the outcomes of ten binary lots, which when thrown together may fall with 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, or 0 marked or unmarked sides uppermost. “Marked’ and ‘unmarked’ sides are equivalent to the ‘1’and ‘0’of binary arithmetic. Adding 2 to these outcomes give exactly the same outcomes as the outcomes of two six-sided dice:

10 binary lots:	0	1	2	3	4	5	6	7	8	9	10	+2
2x6-sided dice:	2	3	4	5	6	7	8	9	10	11	12	

Are these two methods equivalent? That means, have the two methos produced the same distribution of the outcomes? Justify / prove your answer.

Problem 2 (1.5 points). “Crown and Anchor” is a simple dice game, traditionally played for gambling purposes by sailors in the Royal Navy as well as those in the British merchant and fishing fleets. It is still played as a carnival game in some casinos. The game is played with three standard six-sided dice, between a player and a banker. The bettor places stakes on a board with six numbered spaces, labelled 1 through 6, inclusive. He receives a 1:1 payout if the number bet on appears once, a 2:1 payout if the number appears twice, and a 3:1 payout if the number is rolled all 3 times. Otherwise (that is, if the specified outcome appears on none of the three dice), the player loses. If player bets 1$, write a single PyMC stochastic variables that can represent the stochastic output of the game: 0 (losing), 1 (winning 1$)), 2 (winning 2$)), 3 (winning 3$).

Problem 3 (2.5 points). Of the various dice games enlisted by the gaming casinos, the most prevalent, by far, is Craps. Today the cabalistic signs of the Craps layout can be seen in every major casino of the world.



Craps is played with two conventional dice. The types of wagers are displayed in the betting layout shown in the figure above. The basic bet (in which we are interested) is the “pass line,” which wins unconditionally when the player initially throws a 7 or 11 (“naturals”) and loses unconditionally when the initial outcome is a 2, 3, or 12 (referred to as “craps”). The remaining outcomes—4, 5, 6, 8, 9, 10—are each known as a “point.” When a point is set, the player continues to roll the dice; the pass-line bettor then wins if that point is rolled again before the appearance of a 7 and loses if the 7 appears first. The latter event, known as a “seven-out,” ends the player’s tenure with the dice.

Write a (complete) PyMC program that estimates the probability of winning of a specified “point” (4, 5, 6, 8, 9, 10).

Problem 4 (2.5 points). Write a (complete) PyMC program to solve the following problem (from Aubrey Clayton's book *Bernoulli's Fallacy*):

Your friend rolls a six-sided die and secretly records the outcome; this number becomes the target T. You then put on a blindfold and roll the same six-sided die over and over. You're unable to see how it lands, so each time your friend […] tells you only whether the number you just rolled was greater than, equal to, or less than **T**.

Suppose in one round of the game we had this sequence of outcomes, with **G** representing a greater roll, **L** a lesser roll, and **E** an equal roll:

G, **G**, **L**, **E**, **L**, **L**, **L**, **E**, **G**, **L**

Based on this data, what is the posterior distribution of **T**?