Condicionais completas:

A condicional completa para y_0 será:

$p(y_0 | y, \phi, \tau^{-1}) = p(y | \phi, y_0) p(y_0)$

Temos que:

$y_0 \sim N(1391.599, 12 \cdot 10^{-6})$

$\rightarrow \sigma^2 = \frac{1}{\sqrt{12 \cdot 10^{-5}}} = 200\sqrt{30}$

$$p(y_0) = \frac{1}{\sqrt{2\pi \cdot 200\sqrt{30}}} \cdot \exp\left\{ \frac{-1}{2 \cdot 200\sqrt{30}} \cdot (y_0 - 1391.599)^2 \right\}$$

$$p(y | \phi, y_0) \propto \exp\left\{ \frac{-1}{2\tau^{-1}} \sum_{i=1}^{t} (y_t - \phi y_{t-1})^2 \right\}$$

$$\rightarrow p(y_0 | \cdot) \propto \exp\left\{ \frac{-1}{2\tau^{-1}} \sum_{i=1}^{t} (y_t - \phi y_{t-1})^2 \right\} \cdot \exp\left\{ \frac{-1}{2 \cdot 200\sqrt{30}} (y_0 - 1391.599)^2 \right\}$$

$$p(y_0 | \cdot) \propto \exp\left\{ \frac{-1}{2\tau^{-1}} \sum_{i=1}^{t} (y_i - \phi y_{i-1})^2 + \frac{-1}{2 \cdot 200\sqrt{30}} (y_0 - 1391.599)^2 \right\}$$

$$p(y_0 | \cdot) \propto \exp\left\{ \underbrace{\cancel{\frac{-1}{2\tau^{-1}} \sum_{i=2}^{t} (y_i - y_{i-1})}}_{\text{constante}} + \left(\frac{-1}{2\tau^{-1}} (\underbrace{\cancel{y_1}}_{\text{cte}} - y_0) \right) + \left[\frac{-1}{400\sqrt{30}} \left(y_0^2 - 2 \cdot 1391.599 y_0 + \underbrace{\cancel{(1391.599)^2}}_{\text{cte}} \right) \right] \right\}$$

$$p(y_0 | \cdot) \propto \exp\left\{ \frac{-1}{2\tau^{-1}} - y_0) + \frac{-1}{400\sqrt{30}} (y_0^2 - 2 \cdot 1391.599 y_0) \right\}$$

$$\propto \exp\left\{ \frac{-1}{400\sqrt{30}} \left(y_0^2 - 2 \cdot 1391,599 y_0 - \frac{400\sqrt{30}}{2\tau^{-1}} y_0 \right) \right.$$

$$\propto \exp\left\{ \frac{1}{2 \cdot 200\sqrt{30}} \left(y_0 - \left(2783.198 - \frac{400\sqrt{30}}{2\tau^{-1}}\right) / 2 \right)^2 \right\}$$

$$\sim N\left(\frac{(2783.198 - 200\sqrt{30} \cdot \tau)}{2}, 200\sqrt{30} \right)$$