Practica 3 – SVM

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1 Tarea a)

1.1 Script R Usado para las Computaciones

```
# Importamos las Librerias necesarias
library (kernlab)
library(e1071)
source("lib.R")

# Creamos la funcion que dire a que clase pertenece cada punto


## APARTADO A
    ######################################################

# Creamos el conjunto de datos
dataA <- data.frame(
  x1 = c(0, 4),
  x2 = c(0, 4),
  y = c(1, -1)
)
# Indicamos que la columna y es la importante
dataA$y <- as.factor(dataA$y)

# Creamos el SVM con los datos del A con un kernel lineal
svmA <- svm(y~., dataA , kernel="linear")

#Vectores de soporte
vsA <- dataA[svmA$index,1:2]

# Calculamos los valores del kernel
x1=c(0,0)
x2=c(4,4)
KAA=t (x1) %*% x1
KAB=t (x1) %*% x2
KBB=t (x2) %*% x2

# Vector de pesos normal al hiperplano (W)
# Hacemos el CrosProduct entre los vectores soporte y el coe.
    de Lagrange
wA <- crossprod(as.matrix(vsA), svmA$coefs)

```

```r
# Calcular ancho del canal
widthA = 2/(sqrt(sum((wA)^2)))

# Calcular vector B
bA <- -svmA$rho

# Calcular la ecuacion del hiperplano y de los planos de soporte positivo
# y negativo
paste(c("[",wA,"]' * x + [",bA,"] = 0"), collapse=" ")
paste(c("[",wA,"]' * x + [",bA,"] = 1"), collapse=" ")
paste(c("[",wA,"]' * x + [",bA,"] = -1"), collapse=" ")

# Determinamos a la clase que pertenece cada uno
print_clasificacion(c(5, 6),wA, bA)
print_clasificacion(c(1, -4),wA, bA)
```

1.2 Los Vectores Soporte

$$\overrightarrow{v_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \qquad \overrightarrow{v_2} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$$

1.3 Los Valores del Kernel

$$K_{AA} = 0, \qquad K_{AB} = K_{BA} = 0, \qquad K_{BB} = 32,$$

1.4 El Ancho del Canal

$$ancho \approx 0.7071$$

1.5 El Vector $\overrightarrow{w}$

$$\overrightarrow{w} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

1.6 El Vector $\overrightarrow{b}$

$$\overrightarrow{b} = 0$$

1.7 La Ecuación del Hiperplano y de los Planos de Soporte Positivo y Negativo

$$\begin{pmatrix} -2 \\ -2 \end{pmatrix} * \overrightarrow{x} = 0, \qquad \begin{pmatrix} -2 \\ -2 \end{pmatrix} * \overrightarrow{x} = 1, \qquad \begin{pmatrix} -2 \\ -2 \end{pmatrix} * \overrightarrow{x} = -1$$

1.8 Determinación de la Clase

$$clase\left(\begin{pmatrix} 5 \\ 6 \end{pmatrix}\right) = -1, \qquad clase\left(\begin{pmatrix} 1 \\ -4 \end{pmatrix}\right) = 1$$

2 Tarea b)

2.1 Script R Usado para las Computaciones

```
library(kernlab)
library(e1071)
source("lib.R")

# Define data
data <- data.frame(
  x1 = c(2, 0, 1),
  x2 = c(0, 0, 1),
  y = c(1, -1, -1)
)

# 1. Determine support vectors
data$y <- as.factor(data$y)
svmB <- svm(y~., data, kernel="linear")
supportVectors <- data[svmB$index,1:2]
supportVectors
plot(supportVectors)

# 2. Kernel values

# Exclude last column, since it is only labels
kernel_matrix <- get_kernel_matrix(data[,-3], dot_product_kernel)
kernel_matrix

# 3. Width of street
wB <- crossprod(as.matrix(supportVectors), svmB$coefs)
widthB = 2/(sqrt(sum((wB)^2)))
widthB
# 4. Vector of weights, normal to hyperplane (W)
wB
# 5. Vector B
bB <- svmB$rho
bB
# 6. Hyperplane equation, negative and possitive support planes
paste(c("[",wB,"]' * x + [",bB,"] = 0"), collapse=" ")
paste(c("[",wB,"]' * x + [",bB,"] = 1"), collapse=" ")
paste(c("[",wB,"]' * x + [",bB,"] = -1"), collapse=" ")

# 7. Classification
print_clasificacion(c(5,6), wB, bB)
print_clasificacion(c(1,-4), wB, bB)
```

2.2 Los Vectores Soporte

$$\overrightarrow{v_1} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \qquad \overrightarrow{v_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \overrightarrow{v_3} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

2.3 Los Valores del Kernel

$$K_{AA} = 4, \qquad K_{AB} = K_{BA} = 0, \qquad K_{BB} = 0, \quad K_{BC} = K_{CB} = 0, \quad K_{CC} = 2$$

2.4 El Ancho del Canal

$$ancho \approx 1.898$$

2.5 El Vector $\overrightarrow{w}$

$$\overrightarrow{w} = \begin{pmatrix} 0.9995 \\ -0.3335 \end{pmatrix}$$

2.6 El Vector $\overrightarrow{b}$

$$\overrightarrow{b} = 0.3333$$

2.7 La Ecuación del Hiperplano y de los Planos de Soporte Positivo y Negativo

$$\begin{pmatrix} 0.9995 \\ -0.3335 \end{pmatrix} * \overrightarrow{x} + 0.3333 = 0, \qquad \begin{pmatrix} 0.9995 \\ -0.3335 \end{pmatrix} * \overrightarrow{x} + 0.3333 = -1$$

$$\begin{pmatrix} 0.9995 \\ -0.3335 \end{pmatrix} * \overrightarrow{x} + 0.3333 = -1$$

2.8 Determinación de la Clase

$$clase\left(\begin{pmatrix} 5 \\ 6 \end{pmatrix}\right) = 1, \qquad clase\left(\begin{pmatrix} 1 \\ -4 \end{pmatrix}\right) = 1$$

3 Tarea c)

TODO

3.1 Script R Usado para las Computaciones

```r
library(e1071)

source("lib.R")

# Define data
data <- data.frame(
  x1 = c(2, 2, -2, -2, 2, 2, -2, -2, 1, 1, -1, -1),
  x2 = c(2, -2, -2, 2, 2, -2, -2, 2, 1, -1, -1, 1),
  y = c(1, 1, 1, 1, 1, 1, 1, 1, -1, -1, -1, -1)
)

svmC <- svm(y~., data , kernel="linear")

# 1. Determine support vectors
supportVectors <- data[svmC$index,1:2]
supportVectors
plot(supportVectors)

# 2. Kernel values

# Exclude last column, since it is only labels
kernel_matrix <- get_kernel_matrix(data[,-3], dot_product_kernel)
kernel_matrix

# 3. Width of street
wC <- crossprod(as.matrix(supportVectors), svmC$coefs)
widthC = 2/(sqrt(sum((wC)^2)))
widthC

# 4. Vector of weights, normal to hyperplane (W)
wC

# 5. Vector B
bC <- svmC$rho
bC


# 6. Hyperplane equation, negative and possitive support planes
paste(c("[",wC,"]' * x + [",bC,"] = 0"), collapse=" ")
paste(c("[",wC,"]' * x + [",bC,"] = 1"), collapse=" ")
paste(c("[",wC,"]' * x + [",bC,"] = -1"), collapse=" ")

# 7. Classification
print_clasificacion(c(5,6), wC, bC)
print_clasificacion(c(1,-4), wC, bC)
```

3.2 Los Vectores Soporte

$$\overrightarrow{v_1} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \qquad \overrightarrow{v_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \overrightarrow{v_3} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

3.3 Los Valores del Kernel

$$K_{AA} = 4, \qquad K_{AB} = K_{BA} = 0, \qquad K_{BB} = 0, \quad K_{BC} = K_{CB} = 0, \quad K_{CC} = 2$$

3.4 El Ancho del Canal

$$ancho \approx 1.898$$

3.5 El Vector $\overrightarrow{w}$

$$\overrightarrow{w} = \begin{pmatrix} 0.9995 \\ -0.3335 \end{pmatrix}$$

3.6 El Vector $\overrightarrow{b}$

$$\overrightarrow{b} = 0.3333$$

3.7 La Ecuación del Hiperplano y de los Planos de Soporte Positivo y Negativo

$$\begin{pmatrix} 0.9995 \\ -0.3335 \end{pmatrix} * \overrightarrow{x} + 0.3333 = 0, \qquad \begin{pmatrix} 0.9995 \\ -0.3335 \end{pmatrix} * \overrightarrow{x} + 0.3333 = -1$$

$$\begin{pmatrix} 0.9995 \\ -0.3335 \end{pmatrix} * \overrightarrow{x} + 0.3333 = -1$$

3.8 Determinación de la Clase

$$clase\left(\begin{pmatrix} 5 \\ 6 \end{pmatrix}\right) = 1, \qquad clase\left(\begin{pmatrix} 1 \\ -4 \end{pmatrix}\right) = 1$$

4 Tarea d)

4.1 Script R Usado para las Computaciones

```
library(kernlab)

source("lib.R")

transform <- function(x1, x2) {
  if (sqrt(x1^2 + x2^2) > 2) {
    transformed.x1 <- 4 - x2 + abs(x1 - x2)
    transformed.x2 <- 4 - x1 + abs(x1 - x2)
    return(c(transformed.x1, transformed.x2))
  }
  return(c(x1, x2))
}

apply_transform <- function(row) {
  transformed <- transform(row$x1, row$x2)
  row$x1 <- transformed[1]
  row$x2 <- transformed[2]
  return(row)
}

# Define data
data <- data.frame(
  x1 = c(2, 2, -2, -2, 2, 2, -2, -2, 1, 1, -1, -1),
  x2 = c(2, -2, -2, 2, 2, -2, -2, 2, 1, -1, -1, 1),
  y = c(1, 1, 1, 1, 1, 1, 1, 1, -1, -1, -1, -1)
)

# Apply the transformation to each row using by
transformed_data <- by(data, INDICES = seq_len(nrow(data)), FUN
     = apply_transform)

# Combine the result back into a data frame
data <- do.call(rbind, transformed_data)

svm <- ksvm(y~., data, type="C-svc",C = 100, kernel="vanilladot
    ", scaled=c())

# 1. Determine support vectors
supportVectors <- data[svm@SVindex, -3]
supportVectors

# 2. Kernel values

# Exclude last column, since it is only labels
kernel_matrix <- get_kernel_matrix(data[,-3], dot_product_
    kernel)
kernel_matrix

# 3. Width of street
w <- colSums(coef(svm)[[1]] * data[SVindex(svm),])
```

```r
# (Removes the 'y' column from w vector)
w <- w[-3]
b <- svm@b

widthB = 2/(sqrt(sum((w)^2)))
widthB

# 4. Vector of weights, normal to hyperplane (W)
w

# 5. Vector B
b

# Plot
plot(x2 ~ x1, data = data, col = ifelse(y == -1, "red", "blue")
    , pch = 19, main = "SVM Decision Boundary", xlab = "x1",
    ylab = "x2")
cat (-w[1]/w[2],"*x+",-b/w[2],"=1")
cat (-w[1]/w[2],"*x+",-b/w[2],"=-1")
cat (-w[1]/w[2],"*x+",-b/w[2],"=0")

abline(b/w[2],-w[1]/w[2])
abline((b+1)/w[2],-w[1]/w[2],lty=2)
abline((b-1)/w[2],-w[1]/w[2],lty=2)

# 6. Hyperplane equation, negative and possitive support planes
paste(c("[",w,"]' * x + [",b,"] = 0"), collapse=" ")
paste(c("[",w,"]' * x + [",b,"] = 1"), collapse=" ")
paste(c("[",w,"]' * x + [",b,"] = -1"), collapse=" ")

# 7. Point classification
print_clasificacion(c(8,8), w, b)
print_clasificacion(c(-2,-2), w, b)
```

4.2 Los Vectores Soporte

$$\overrightarrow{v_1} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \qquad \overrightarrow{v_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

4.3 Los Valores del Kernel

$$\begin{pmatrix}
8 & 32 & 24 & 32 & 8 & 32 & 24 & 32 & 4 & 0 & -4 & 0 \\
32 & 136 & 96 & 120 & 32 & 136 & 96 & 120 & 16 & 4 & -16 & -4 \\
24 & 96 & 72 & 96 & 24 & 96 & 72 & 96 & 12 & 0 & -12 & 0 \\
32 & 120 & 96 & 136 & 32 & 120 & 96 & 136 & 16 & -4 & -16 & 4 \\
8 & 32 & 24 & 32 & 8 & 32 & 24 & 32 & 4 & 0 & -4 & 0 \\
32 & 136 & 96 & 120 & 32 & 136 & 96 & 120 & 16 & 4 & -16 & -4 \\
24 & 96 & 72 & 96 & 24 & 96 & 72 & 96 & 12 & 0 & -12 & 0 \\
32 & 120 & 96 & 136 & 32 & 120 & 96 & 136 & 16 & -4 & -16 & 4 \\
4 & 16 & 12 & 16 & 4 & 16 & 12 & 16 & 2 & 0 & -2 & 0 \\
0 & 4 & 0 & -4 & 0 & 4 & 0 & -4 & 0 & 2 & 0 & -2 \\
-4 & -16 & -12 & -16 & -4 & -16 & -12 & -16 & -2 & 0 & 2 & 0 \\
0 & -4 & 0 & 4 & 0 & -4 & 0 & 4 & 0 & -2 & 0 & 2
\end{pmatrix}$$

4.4 El Ancho del Canal

$$ancho \approx 1.414$$

4.5 El Vector $\overrightarrow{w}$

$$\overrightarrow{w} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

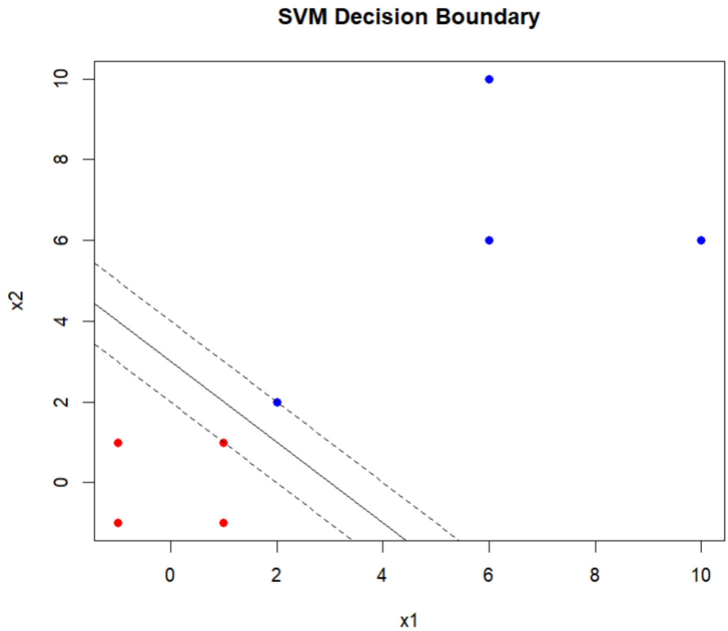
4.6 El Vector $\overrightarrow{b}$

$$\overrightarrow{b} = 3$$

4.7 La Ecuación del Hiperplano y de los Planos de Soporte Positivo y Negativo

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} * \overrightarrow{x} + 3 = 0, \qquad \begin{pmatrix} 1 \\ 1 \end{pmatrix} * \overrightarrow{x} + 3 = 1, \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix} * \overrightarrow{x} + 3 = -1,$$

4.8 Plot



5 Tarea e)

5.1 Script R Usado para las Computaciones

```r
library(kernlab)
source("lib.R")

transform <- function(x1, x2) {
  if (sqrt(x1^2 + x2^2) > 2) {
    transformed.x1 <- 4 - x2 + abs(x1 - x2)
    transformed.x2 <- 4 - x1 + abs(x1 - x2)
    return(c(transformed.x1, transformed.x2))
  }
  return(c(x1, x2))
}

apply_transform <- function(row) {
  transformed <- transform(row$x1, row$x2)
  row$x1 <- transformed[1]
  row$x2 <- transformed[2]
  return(row)
}

# Define data
data <- data.frame(
  x1 = c(3, 3, 6, 6, 1, 0, 0, -1),
  x2 = c(1, -1, 1, -1, 0, 1, -1, 0),
  y = c(1, 1, 1, 1, -1, -1 ,-1 ,-1)
)

# Apply the transformation to each row using by
transformed_data <- by(data, INDICES = seq_len(nrow(data)), FUN
    = apply_transform)

# Combine the result back into a data frame
data <- do.call(rbind, transformed_data)

svm <- ksvm(y~., data, type="C-svc",C = 100, kernel="vanilladot
  ", scaled=c())

# 1. Determine support vectors
supportVectors <- data[svm@SVindex, -3]
supportVectors

# 2. Kernel values

# Exclude last column, since it is only labels
kernel_matrix <- get_kernel_matrix(data[,-3], dot_product_
  kernel)
kernel_matrix

# 3. Width of street
w <- colSums(coef(svm)[[1]] * data[SVindex(svm),])
w <- w[-3]
```

```r
b <- svm@b

widthB = 2/(sqrt(sum((w)^2)))
widthB

# 4. Vector of weights, normal to hyperplane (W)
w

# 5. Vector B
b

# Plot
plot(x2 ~ x1, data = data, col = ifelse(y == -1, "red", "blue")
    , pch = 19, main = "SVM Decision Boundary", xlab = "x1",
    ylab = "x2")
cat (-w[1]/w[2],"*x+",-b/w[2],"=1")
cat (-w[1]/w[2],"*x+",-b/w[2],"=-1")
cat (-w[1]/w[2],"*x+",-b/w[2],"=0")

abline(b/w[2],-w[1]/w[2])
abline((b+1)/w[2],-w[1]/w[2],lty=2)
abline((b-1)/w[2],-w[1]/w[2],lty=2)

# 6. Hyperplane equation, negative and possitive support planes
paste(c("[",w,"]' * x + [",b,"] = 0"), collapse=" ")
paste(c("[",w,"]' * x + [",b,"] = 1"), collapse=" ")
paste(c("[",w,"]' * x + [",b,"] = -1"), collapse=" ")

# 7. Classify
print_clasificacion(c(4, 5), w, b)
```

5.2 Los Vectores Soporte

$$\overrightarrow{v_1} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}, \qquad \overrightarrow{v_2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

5.3 Los Valores del Kernel

$$\begin{pmatrix} 34 & 60 & 49 & 75 & 5 & 3 & -3 & -5 \\ 60 & 106 & 87 & 133 & 9 & 5 & -5 & -9 \\ 49 & 87 & 73 & 111 & 8 & 3 & -3 & -8 \\ 75 & 133 & 111 & 169 & 12 & 5 & -5 & -12 \\ 5 & 9 & 8 & 12 & 1 & 0 & 0 & -1 \\ 3 & 5 & 3 & 5 & 0 & 1 & -1 & 0 \\ -3 & -5 & -3 & -5 & 0 & -1 & 1 & 0 \\ -5 & -9 & -8 & -12 & -1 & 0 & 0 & 1 \end{pmatrix}$$

5.4 El Ancho del Canal

$$ancho \approx 5$$

5.5 El Vector $\overrightarrow{w}$

$$\overrightarrow{w} = \begin{pmatrix} 0.32 \\ 0.24 \end{pmatrix}$$

5.6 El Vector $\overrightarrow{b}$

$$\overrightarrow{b} = 1.32$$

5.7 La Ecuación del Hiperplano y de los Planos de Soporte Positivo y Negativo

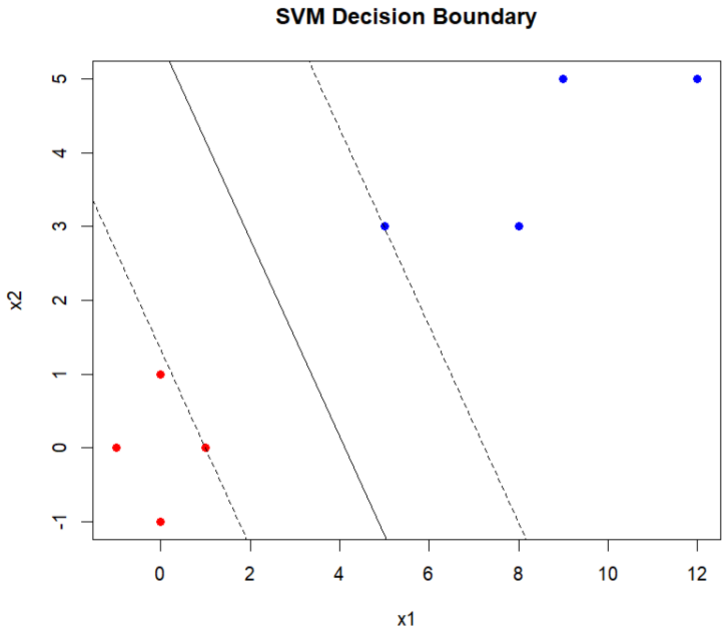
$$\begin{pmatrix} 0.32 \\ 0.24 \end{pmatrix} * \overrightarrow{x} + 1.32 = 0, \qquad \begin{pmatrix} 0.32 \\ 0.24 \end{pmatrix} * \overrightarrow{x} + 1.32 = 1,$$

$$\begin{pmatrix} 0.32 \\ 0.24 \end{pmatrix} * \overrightarrow{x} + 1.32 = -1$$

5.8 Determinación de la Clase

$$clase\left(\begin{pmatrix} 4 \\ 5 \end{pmatrix}\right) = 1$$

5.9 Plot



6 Tarea f)

6.1 Script R Usado para las Computaciones

```r
library(kernlab)

source("lib.R")

# Define data
data <- iris
svm <- ksvm(Species~., data, type="C-svc",C = 100, kernel="vanilladot")

# 1. Determine support vectors
supportVectors <- data[svm@SVindex, -3]
supportVectors

# 2. Kernel values

# Exclude last column, since it is only labels
kernel_matrix <- get_kernel_matrix(data[,-3], dot_product_kernel)
kernel_matrix

# 3. Width of street
w <- colSums(coef(svm)[[1]] * data[SVindex(svm),])
w <- w[-5]
b <- svm@b

widthB = 2/(sqrt(sum((w)^2)))
widthB

# 4. Vector of weights, normal to hyperplane (W)
w

# 5. Vector B
b

# 6. Hyperplane equation, negative and possitive support planes
paste(c("[",w,"]' * x + [",b,"] = 0"), collapse=" ")
paste(c("[",w,"]' * x + [",b,"] = 1"), collapse=" ")
paste(c("[",w,"]' * x + [",b,"] = -1"), collapse=" ")
```

6.2 Los Vectores Soporte

Sepal.Length	Sepal.Width	Petal.Width	Species
5.1	3.3	0.5	setosa
4.5	2.3	0.3	setosa
4.9	2.4	1.0	versicolor
5.9	3.2	1.8	versicolor
6.3	2.5	1.5	versicolor
6.7	3.0	1.7	versicolor
6.0	2.7	1.6	versicolor
5.1	2.5	1.1	versicolor
4.9	2.5	1.7	virginica
6.0	2.2	1.5	virginica
6.2	2.8	1.8	virginica
6.1	3.0	1.8	virginica
7.2	3.0	1.6	virginica
6.3	2.8	1.5	virginica
6.0	3.0	1.8	virginica

6.3 Los Valores del Kernel

```
      [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10] [,11] [,12] [,13] [,14] [,15] [,16] [,17] [,18] [,19]
[1,] 39.30 36.53 36.21 35.35 39.14 42.27 36.42 38.44 33.63 36.86 41.53 37.42 36.00 33.45 44.62 45.55 42.27 39.32 43.43
[2,] 36.53 34.05 33.67 32.88 36.34 39.24 33.80 35.74 31.30 34.33 38.60 34.76 33.54 31.09 41.46 42.21 39.24 36.55 40.39
[3,] 36.21 33.67 33.37 32.58 36.06 38.94 33.56 35.42 31.00 33.97 38.26 34.48 33.18 30.83 41.10 41.95 38.94 36.23 40.01
[4,] 35.35 32.88 32.58 31.81 35.20 38.01 32.76 34.58 30.27 33.17 37.35 33.66 32.40 30.10 40.12 40.94 38.01 35.37 39.06
[5,] 39.14 36.34 36.06 35.20 39.00 42.12 36.30 38.28 33.48 36.68 41.36 37.28 35.82 33.32 44.44 45.42 42.12 39.16 43.24
[6,] 42.27 39.24 38.94 38.01 42.12 45.53 39.22 41.34 36.15 39.59 44.67 40.26 38.66 35.96 48.00 49.10 45.53 42.31 46.72
     [,20] [,21] [,22] [,23] [,24] [,25] [,26] [,27] [,28] [,29] [,30] [,31] [,32] [,33] [,34] [,35] [,36] [,37] [,38]
[1,] 40.37 40.48 40.04 37.10 38.66 37.42 37.04 38.48 39.81 39.46 36.21 36.37 40.52 41.89 43.79 36.88 37.74 41.34 38.61
[2,] 37.45 37.70 37.17 34.38 35.99 34.76 34.54 35.78 37.02 36.72 33.67 33.86 37.74 38.80 40.59 34.35 35.14 38.49 35.83
[3,] 37.19 37.30 36.89 34.18 35.63 34.48 34.14 35.46 36.68 36.36 33.37 33.52 37.34 38.58 40.33 33.99 34.78 38.09 35.57
[4,] 36.30 36.42 36.01 33.36 34.79 33.66 33.34 34.62 35.81 35.50 32.58 32.73 36.46 37.65 39.36 33.19 33.96 37.19 34.72
[5,] 40.24 40.28 39.90 37.00 38.48 37.28 36.84 38.32 39.64 39.28 36.06 36.20 40.32 41.78 43.66 36.70 37.56 41.14 38.48
[6,] 43.48 43.50 43.13 39.96 41.61 40.26 39.78 41.42 42.81 42.42 38.94 39.09 43.58 45.11 47.16 39.63 40.56 44.43 41.54
     [,39] [,40] [,41] [,42] [,43] [,44] [,45] [,46] [,47] [,48] [,49] [,50] [,51] [,52] [,53] [,54] [,55] [,56] [,57]
[1,] 33.98 38.95 38.81 32.06 34.68 38.87 40.39 36.04 40.35 35.70 41.02 38.09 49.18 46.14 48.34 38.36 45.25 41.13 46.00
[2,] 31.60 36.23 36.06 30.01 32.20 36.12 37.47 33.58 37.43 33.18 38.11 35.44 46.18 43.26 45.41 36.11 42.55 38.59 43.09
[3,] 31.32 35.89 35.76 29.57 31.96 35.82 37.21 33.22 37.17 32.90 37.79 35.10 45.42 42.62 44.65 35.47 41.81 38.01 42.49
[4,] 30.58 35.04 34.91 28.89 31.20 34.97 36.32 32.44 36.28 32.12 36.89 34.27 44.40 41.66 43.65 34.69 40.88 37.16 41.53
[5,] 33.84 38.78 38.66 31.84 34.56 38.72 40.26 35.86 40.22 35.56 40.86 37.92 48.80 45.82 47.96 38.04 44.88 40.84 45.70
[6,] 36.54 41.88 41.77 34.39 37.32 41.89 43.52 38.74 43.44 38.40 44.13 40.95 52.84 49.64 51.95 41.19 48.62 44.22 49.53
     [,58] [,59] [,60] [,61] [,62] [,63] [,64] [,65] [,66] [,67] [,68] [,69] [,70] [,71] [,72] [,73] [,74] [,75] [,76]
[1,] 35.59 46.07 38.25  34.7 42.89 40.50 43.54 40.97 47.30 41.36 41.23 41.62 39.53 43.65 43.17 43.18 43.15 45.05 46.44
[2,] 33.41 43.30 35.86  32.7 40.21 38.20 40.87 38.40 44.41 38.74 38.72 39.28 37.16 40.87 40.55 40.67 40.53 42.32 43.62
[3,] 32.91 42.56 35.36  32.1 39.63 37.44 40.23 37.86 43.69 38.22 38.10 38.48 36.54 40.33 39.89 39.91 39.87 41.62 42.90
[4,] 32.18 41.61 34.57  31.4 38.74 36.62 39.33 37.01 42.71 37.36 37.25 37.64 35.73 39.42 39.00 39.03 38.98 40.69 41.94
[5,] 35.34 45.70 38.00  34.4 42.60 40.12 43.22 40.70 46.94 41.10 40.92 41.22 39.22 43.38 42.84 42.80 42.82 44.70 46.08
[6,] 38.22 49.47 41.17  37.2 46.16 43.38 46.81 44.07 50.83 44.54 44.25 44.66 42.43 47.06 46.38 46.37 46.34 48.39 49.90
     [,77] [,78] [,79] [,80] [,81] [,82] [,83] [,84] [,85] [,86] [,87] [,88] [,89] [,90] [,91] [,92] [,93] [,94] [,95]
[1,] 46.76 47.01 43.05 40.37 38.67 38.65 41.27 42.37 40.34 44.82 47.32 42.44 41.32 39.06 39.39 43.89 40.92 35.75 40.27
[2,] 44.00 44.17 40.40 37.93 36.37 36.35 38.76 39.82 37.76 41.92 44.43 40.03 38.70 36.71 36.99 41.17 38.46 33.60 37.80
[3,] 43.20 43.43 39.78 37.31 35.75 35.73 38.14 39.16 37.28 41.40 43.71 39.23 38.18 36.11 36.41 40.55 37.82 33.06 37.22
[4,] 42.24 42.46 38.89 36.48 34.96 34.94 37.29 38.29 36.44 40.46 42.73 38.37 37.32 35.31 35.60 39.64 36.98 32.33 36.39
[5,] 46.36 46.64 42.74 40.06 38.36 38.34 40.96 42.04 40.10 44.56 46.96 42.04 41.06 38.76 39.10 43.58 40.60 35.48 39.98
[6,] 50.20 50.56 46.31 43.32 41.50 41.46 44.33 45.57 43.46 48.30 50.87 45.51 44.46 41.97 42.32 47.20 43.94 38.37 43.29
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[1,] 41.81 41.48 44.03 36.98  41.13  47.18  42.41  50.13  45.64  47.09  52.68  37.08  50.74  46.28  52.82  47.75  45.47
[2,] 39.17 38.89 41.34 34.71  38.59  44.27  39.90  47.21  42.93  44.29  49.66  34.85  47.83  43.69  49.58  44.85  42.84
[3,] 38.63 38.33 40.68 34.19  38.01  43.67  39.28  46.39  42.25  43.59  48.74  34.37  46.95  42.85  48.86  44.19  42.10
[4,] 37.76 37.47 39.77 33.43  37.16  42.71  38.43  45.38  41.33  42.64  47.68  33.63  45.93  41.93  47.78  43.22  41.19
[5,] 41.54 41.20 43.70 36.72  40.84  46.88  42.10  49.72  45.30  46.74  52.22  36.84  50.30  45.86  52.46  47.42  45.10
[6,] 44.96 44.61 47.31 39.73  44.22  50.89  45.61  53.88  49.05  50.68  56.58  39.89  54.45  49.65  56.92  51.38  48.85
     [,113] [,114] [,115] [,116] [,117] [,118] [,119] [,120] [,121] [,122] [,123] [,124] [,125] [,126] [,127] [,128]
[1,]  48.60  41.22  42.86  47.30  47.01  56.01  51.83  41.60  49.85  41.76  52.47  44.94  49.14  51.28  44.78  44.97
[2,]  45.74  38.83  40.30  44.42  44.21  52.57  48.99  39.30  46.87  39.24  49.53  42.33  46.15  48.24  42.14  42.25
[3,]  44.98  38.19  39.70  43.78  43.51  51.79  47.97  38.54  46.13  38.68  48.55  41.61  45.47  47.44  41.46  41.63
[4,]  44.00  37.37  38.84  42.82  42.56  50.64  46.94  37.72  45.12  37.84  47.50  40.71  44.47  46.40  40.56  40.72
[5,]  48.22  40.90  42.56  46.98  46.66  55.62  51.32  41.22  49.48  41.48  51.98  44.58  48.80  50.88  44.44  44.66
[6,]  52.26  44.33  46.20  50.96  50.52  60.28  55.64  44.58  53.66  44.96  56.30  48.27  52.89  55.08  48.12  48.36
     [,129] [,130] [,131] [,132] [,133] [,134] [,135] [,136] [,137] [,138] [,139] [,140] [,141] [,142] [,143] [,144]
[1,]  45.86  50.54  50.92  56.99  45.88  45.23  43.49  53.23  47.51  46.85  44.46  49.46  48.50  49.50  42.41  49.34
[2,]  43.18  47.60  48.04  53.51  43.20  42.57  40.97  50.19  44.55  44.02  41.76  46.53  45.61  46.57  39.90  46.38
[3,]  42.46  46.76  47.12  52.69  42.48  41.87  40.27  49.25  43.97  43.36  41.16  45.77  44.89  45.81  39.28  45.66
[4,]  41.54  45.74  46.10  51.52  41.56  40.96  39.40  48.18  43.00  42.41  40.26  44.77  43.91  44.81  38.43  44.66
[5,]  45.50  50.12  50.46  56.58  45.52  44.88  43.14  52.76  47.22  46.52  44.16  49.08  48.14  49.12  42.10  48.98
[6,]  49.32  54.22  54.64  61.28  49.36  48.54  46.64  57.20  51.24  50.37  47.82  53.19  52.23  53.27  45.61  53.12
     [,145] [,146] [,147] [,148] [,149] [,150]
[1,]  49.22  48.13  44.26  47.05  46.98  43.95
[2,]  46.23  45.29  41.75  44.25  44.04  41.27
[3,]  45.55  44.55  40.99  43.55  43.48  40.69
[4,]  44.55  43.58  40.11  42.60  42.52  39.80
[5,]  48.88  47.76  43.88  46.70  46.70  43.66
[6,]  53.05  51.80  47.53  50.60  50.66  47.28
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6.4 El Ancho del Canal

$$ancho \approx 0.3397$$

6.5 El Vector $\overrightarrow{w}$

$$\overrightarrow{w} = \begin{pmatrix} -5.469 \\ 0.2363 \\ 1.962 \\ 0.9185 \end{pmatrix}$$

Sepal.Length	Sepal.Width	Petal.Length	Petal.Width
-5.4691320	0.2362545	1.9623159	0.9175388

6.6 El Vector $\overrightarrow{b}$

$$\overrightarrow{b} = \begin{pmatrix} -1.474 & -0.2958 & 10.58 \end{pmatrix}$$

6.7 La Ecuación del Hiperplano y de los Planos de Soporte Positivo y Negativo

$$\begin{pmatrix} -5.4691 \\ 0.2363 \\ 1.9623 \\ 0.9175 \end{pmatrix} * \overrightarrow{x} + \begin{pmatrix} -1.474 & -0.2958 & 10.58 \end{pmatrix} = 0,$$

$$\begin{pmatrix} -5.4691 \\ 0.2363 \\ 1.9623 \\ 0.9175 \end{pmatrix} * \overrightarrow{x} + \begin{pmatrix} -1.474 & -0.2958 & 10.58 \end{pmatrix} = 1$$

$$\begin{pmatrix} -5.4691 \\ 0.2363 \\ 1.9623 \\ 0.9175 \end{pmatrix} * \overrightarrow{x} + \begin{pmatrix} -1.474 & -0.2958 & 10.58 \end{pmatrix} = -1$$