BHBdynamics

September 5, 2019

This manual describes BHBdynamics, a library to compute the number and eccentricity distribution of merging black hole (BH) binaries from an evolved star cluster model. The evolution of the cluster model must be computed separately and, preferably, using the package clusterBH[1]. Detailed information on both BHBdynamics and clusterBH can be found in Antonini and Gieles (2019).

Any queries that are not answered by this manual or by Antonini and Gieles (2019) can be addressed to: f.antonini@surrey.ac.uk and mgieles@icc.ub.edu.

1 Theory

The merger of a BH binary through (strong) binary-single encounters in a dense star cluster following its formation and dynamical hardening can occur in three different ways: (i) the merger occurs in between binary-single encounters while the binary is still bound to its parent cluster (*in-cluster inspirals*); (ii) a merger occurs during a binary-single (resonant) encounter as two BHs are driven to a short separation such that gravitational wave (GW) radiation will lead to their merger (*GW captures*); and (iii) the binary merges after being ejected from its parent cluster. BHBdynamics determines the rate and eccentricities of BH binary mergers produced by these three dynamical channels.

Given a time-dependent cluster model, BHBdynamics computes the number of BH binaries that merge within a look-back time τ by solving the following integral

$$
\begin{aligned}
\mathcal{N}_{\text{merge}} \quad &= \int_0^\tau \Gamma_{\text{bin}} \left[P_{\text{ex}} + P_{\text{GW}} + P_{\text{cap}} \right] d\tau' \\
&+ \int_\tau^{\tau_{\text{f}}} \Gamma_{\text{bin}} \left[P_{\text{ex}}(\tau') - P_{\text{ex}}(\tau' - \tau) \right] \, d\tau' \ ,
\end{aligned}
\tag{1}
$$

where τ_{f} is the look-back time at which the cluster is formed and Γ_{bin} is the formation rate of BH binaries in the cluster core; P_{ex}, P_{GW}, and P_{cap} are, respectively, the probability that a BH binary will merge after being ejected dynamically from its parent cluster, the probability that a binary merges inside the cluster in between two binary-single encounters, and the probability that the binary merges during a binary-single encounter via a GW capture.

---

[1] freely available at https://github.com/mgieles/clusterbh

Finally, BHBDYNAMICS computes the eccentricity distribution of the entire population of binary BHs that merge within a look-back time τ and at a GW frequency f_{GW}, by solving the following integral

$$\mathcal{N}_{<e} = \int_0^\tau \Gamma_{\mathrm{bin}} \left[\mathcal{P}_{\mathrm{ex}} + \mathcal{P}_{\mathrm{GW}} + \mathcal{P}_{\mathrm{cap}}\right] \mathrm{d}\tau'$$
$$+ \int_\tau^{\tau_\mathrm{f}} \Gamma_{\mathrm{bin}} \left[\mathcal{P}_{\mathrm{ex}}(\tau') - \mathcal{P}_{\mathrm{ex}}(\tau' - \tau)\right] \mathrm{d}\tau' , \tag{2}$$

where now $\mathcal{P}_{\mathrm{ex}}$, $\mathcal{P}_{\mathrm{GW}}$, and $\mathcal{P}_{\mathrm{cap}}$ are, respectively, the probability that an ejected binary, an in-cluster inspiral and a GW capture will have an eccentricity e at a given f_{GW}.

The first term in the right-hand-side of both Eq. (1) and Eq. (2) represents the binaries that are formed and merge at a look-back time $< \tau$, while the second term represents the BH binaries which are ejected at a look-back time $> \tau$ but merge within the observable volume.

For more details on the theoretical framework used to derive Eq. (1) and Eq. (2) and on how the corresponding probabilities are evaluated see §3 in Antonini and Gieles (2019).

2 Library

The BHBDYNAMICS package contains the following FORTRAN files:

BHBbinary.f – main routine. Calculates BH merger numbers and eccentricities and creates output data files.

read.f – routine that reads the cluster model evolution

functions.f – evaluates the merger rate and probabilities

cluster.f – generates a SIMPLE cluster model

mod.f – common block parameters

quadpack.f – a FORTRAN77 library which estimates integrals using numerical quadrature, by Piessens, deDoncker-Kapenga, Ueberhuber, and Kahaner. See http://www.netlib.org/quadpack

and

Makefile – gfortran compiler. Use command "make"

In BHBDYNAMICS (and in what follows) time, mass, radius and frequency are expressed in units of Gyr, $M_\odot$, pc and Hz, respectively.

2.1 Input/Output Files

2.1.1 Input files

input.ini initial parameters of the run: IMF, $input$, $\alpha, \tau_\mathrm{f}, \tau_\mathrm{e}, f_\mathrm{GW}$. These parameters are described below in § 2.2.

cluster.txt evolution of the cluster model, i.e., total mass in BHs , cluster mass and radius vs time. If $IMF=1$ and $input = 1$, it will also contain information about the evolution of the BH initial mass function (see § 2.2).

mbh.txt masses of all BHs initially in the cluster (used only when $input$=2).

2.1.2 Output files

Nmergers.txt total number of BH binary mergers vs look-back time. The total number of mergers that occur inside the cluster and among the ejected binaries are given separately.

Necc.txt cumulative eccentricity distribution of BH binaries that merge within a given look-back time, τ_e, at a GW frequency f_GW.

2.2 Run parameters

IMF = [0, 1] specifies how the BH mass function (following the natal kicks) will be generated.

IMF = 0; the BH initial mass function is not provided by the user. Thus, the BH masses are generated from a power-law with slope α.

IMF = 1; the BH initial mass function is given by the user according to the value of the $input$ parameter described below.

input = [0, 1, 2] refers to how the input cluster model is provided.

input = 0 or cluster.txt is not found; no cluster model is given. In this case a SIMPLE cluster model is generated, and written in cluster.txt (after which $input = 1$). The BH mass function is generated from a power-law with slope α.

input = 1; the cluster evolution is read from cluster.txt together with the current mass of the largest BH in the cluster. The file cluster.txt will have the following structure:

where M_BH, M_cl, r_h and m_max are the total mass in BHs, the total cluster mass, the cluster half mass radius and the mass of the heaviest BH in the cluster, respectively.

input = 2; as when $input = 1$, the evolution of the cluster model is read from cluster.txt, but now the BH masses are read from a separate file mbh.txt. This option can be used to explore the effect of the BH mass function on the merger rate and eccentricities of the BH mergers. The file mbh.txt contains a table with the mass of all the BHs that are initially in

3

Time	M_{BH}	M_{cl}	r_{h}	m_{max}
0.00	5000.00	100000.00	0.50	32.2
0.10	3314.80	98314.80	0.94	28.1
0.20	2311.04	97311.04	1.37	20.4
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
13.00	0.00	90470.81	20.77	3.2

the cluster, e.g.,

23.7
14.5
13.1
.
.
.
22.8

α; slope of the initial mass function of the BHs. Used if $IMF = 0$, or a SIMPLE model is generated.

τ_{f}; look-back time at which the cluster is formed.

τ_{e}; look-back time at which the cumulative eccentricity distribution, $\mathcal{N}(< e)$, is computed.

f_{GW}; GW frequency at which $\mathcal{N}(< e)$ is computed.

3 SIMPLE

If $input = 0$ or if the cluster.txt file is not in the working directory, a SIMPLE cluster model will be generated. The user will be required to enter the initial value of cluster mass, radius and the initial mass fraction in BHs. SIMPLE will then solve the following evolution equations:

$$\dot{M}_{\mathrm{BH}} = -\beta \frac{M_{\mathrm{cl}}}{t_{\mathrm{rh}}} \tag{3}$$

$$\dot{M}_{\mathrm{cl}} = \dot{M}_{\mathrm{BH}} \tag{4}$$

$$\dot{r}_{\mathrm{h}} = \zeta \frac{r_{\mathrm{h}}}{t_{\mathrm{rh}}} + 2 \frac{\dot{M}_{\mathrm{cl}}}{M_{\mathrm{cl}}} r_{\mathrm{h}} . \tag{5}$$

The half-mass relaxation time-scale is the average relaxation time-scale within $r_{\rm h}$, which is given by

$$t_{\rm rh} = 0.138\sqrt{\frac{M_{\rm cl}r_{\rm h}^3}{G}} \frac{1}{\langle m_{\rm all}\rangle\psi\ln\Lambda} \ .$$

(6)

Here $\langle m_{\rm all}\rangle$ is the mean mass of the stars and all stellar remnants, which is set to $\langle m_{\rm all}\rangle = 0.638\,{\rm M}_\odot$; $\ln\Lambda$ is the Coulomb logarithm, which depends weakly on N, and we therefore use a constant $\ln\Lambda = 10$; the quantity ψ depends on the mass spectrum within $r_{\rm h}$ and it is assumed here to be a constant $\psi = 5$; and $\beta = 2.55 \times 10^{-3}$.

SIMPLE is a simplified version of CLUSTERBH, as the latter code also takes into account the mass-loss due to stellar evolution and the evolution of ψ due to the narrowing of the BH mass spectrum, that are neglected here. As shown in Antonini and Gieles (2019), these are two important ingredients in order to reproduce the cluster evolution observed in N-body simulations. Thus, SIMPLE should only be intended as a test base model for BHBDYNAMICS, while CLUSTERBH should be used for the production runs.

4 Examples

In the directory "./EAMPLES" there are a few example runs which are also briefly described in what follows.

Example 1 the following set of parameters generates a SIMPLE model formed at a look-back time $\tau = 13$Gyr and with $\alpha = -1$. After the initial cluster parameters are entered, BHBDYNAMICS computes the number of merging BHs $vs\ \tau$, and the cumulative eccentricity distribution of mergers occurring at $\tau < 2.5$Gyr and at 10Hz

input	IMF	α	$\tau_{\rm f}$	$\tau_{\rm e}$	$f_{\rm GW}$
0	0	-1.0	13.0	2.5	10.0

Example 2 here $input = 1$ and $IMF = 1$, so that both the evolution of the cluster properties and of $m_{\rm max}(t)$ are read from cluster.txt. This option can be used to import a model from CLUSTERBH that also includes the evolution of the BH mass function. Then BHBDYNAMICS sets the cluster formation time at $\tau = 5$Gyr and computes the number of merging BHs $vs\ \tau$ and their eccentricity distribution at $\tau < 3$Gyr and at 1Hz

input	IMF	α	$\tau_{\rm f}$	$\tau_{\rm e}$	$f_{\rm GW}$
1	1	-0	5.0	3.	1.

Example 3 in this case $input = 2$ and $IMF = 1$. Thus, the evolution of the cluster properties is read from cluster.txt while the BH masses are read from mbh.txt. This choice of parameters can be used to import a cluster model from an N-body or Monte-Carlo simulation of a star cluster for which the masses of the individual BHs are known. Then BHBDYNAMICS sets the cluster formation time at $\tau = 5\mathrm{Gyr}$ and computes the number of merging BHs $vs\ \tau$ and the cumulative eccentricity distribution of mergers occurring at $\tau < 1\mathrm{Gyr}$ and at $0.01\mathrm{Hz}$

input	IMF	α	τ_f	τ_e	f_GW
2	1	-0	5.0	1	0.01

Example 4 this is similar to Example 3, but now the BH masses are generated from a simple power-law model with slope $\alpha = -1$

input	IMF	α	τ_f	τ_e	f_GW
2	0	-1	5.0	1	0.01

References

[Antonini, & Gieles(2019)] Antonini, F., & Gieles, M. 2019, arXiv e-prints, arXiv:1906.11855