Statistical Methods in Physics- Parts of answers for Exercise 2&3

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Problem 2-1: The Gaussian distribution

$$f(v_i) = \left(\frac{m}{2\pi k_B T}\right)^{1/2} \exp\left(\frac{mv_i^2}{2k_B T}\right)$$

Gaussian(Normal) distribution:

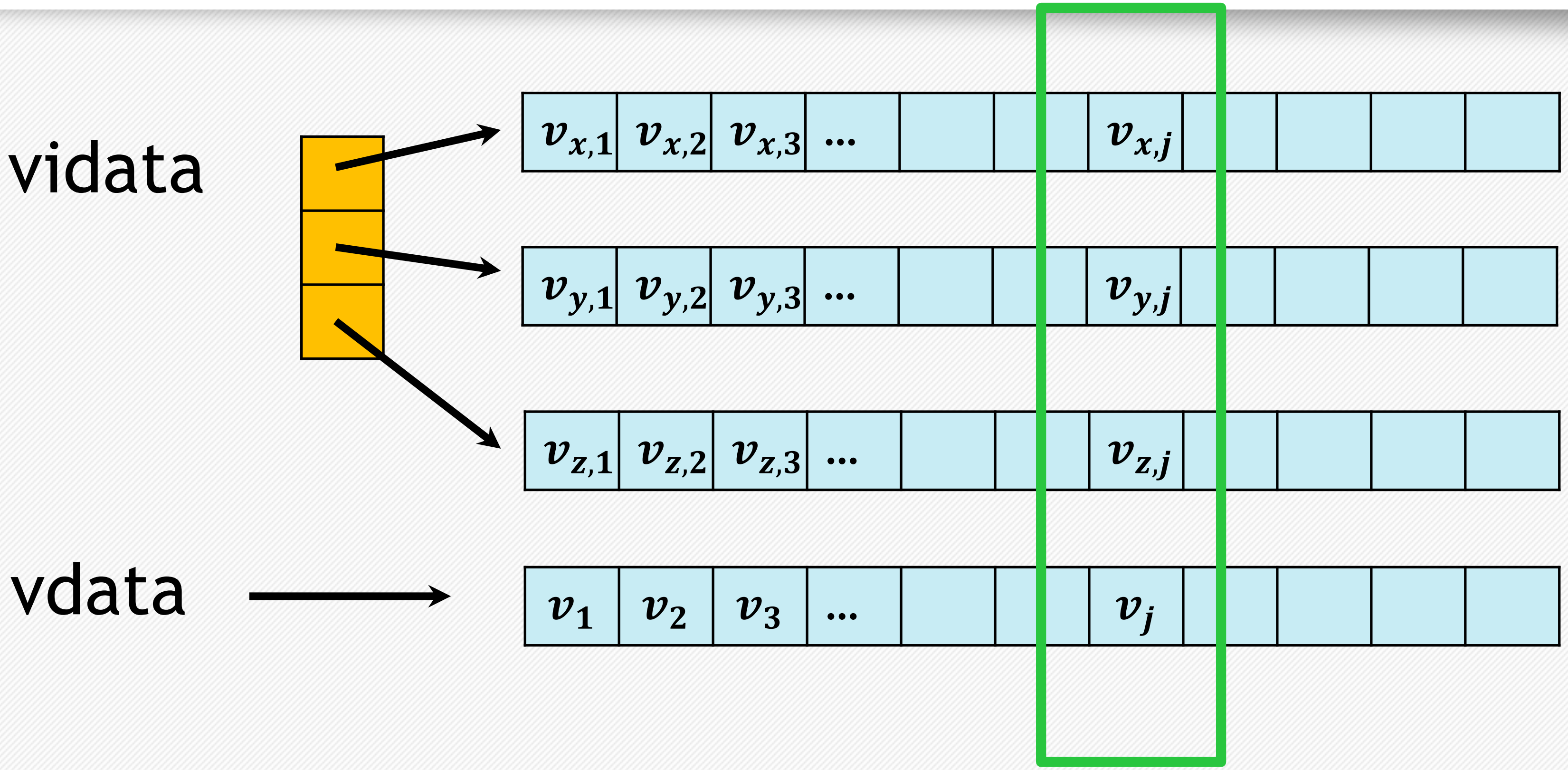
$$f(x) \ = \ N(\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

→ We find:

$$\mu = 0$$

$$\sigma = \sqrt{\frac{k_B T}{m}}$$

Problem 2-2: list creation



Information of particle j:

$$v_j = \sqrt{v_{x,j}^2 + v_{y,j}^2 + v_{z,j}^2}$$

Problem 2-2: $f(v_i) \to f(v)$

- Each dimension is independent with each other:

$$f\left(v_x, v_y, v_z\right) = f(v_x) f(v_y) f(v_z)$$

- Change of coordinate system:

$$f\left(v_x, v_y, v_z\right) \to f(v, \theta, \varphi)$$

- Integrated variables:

$$f(v) = \int d\theta \int d\varphi \; f(v, \theta, \varphi)$$

Problem 2-3: $f(v) \rightarrow f(E_k)$

- According to the rule for change of variables:

$$f(E_k) = \left(\frac{dE_k}{dv}\right)^{-1} f(v^{-1}(E_k))$$

- → $$\frac{dE_k}{dv} = mv = \sqrt{2mE_k} \quad , v = \sqrt{2E_k/m}$$
- → $$f(E_k) = \frac{1}{\sqrt{2mE_k}}\left(\frac{m}{2\pi k_B T}\right)^{3/2} \cdot 4\pi \cdot \frac{2E_k}{m} \cdot e^{-\frac{E_k}{k_B T}}$$
- → $$\boxed{f(E_k) = 2 \cdot \left(\frac{1}{k_B T}\right)^{\frac{3}{2}} \sqrt{\frac{E_k}{\pi}} e^{-\frac{E_k}{k_B T}}}$$

Central Limit Theorem

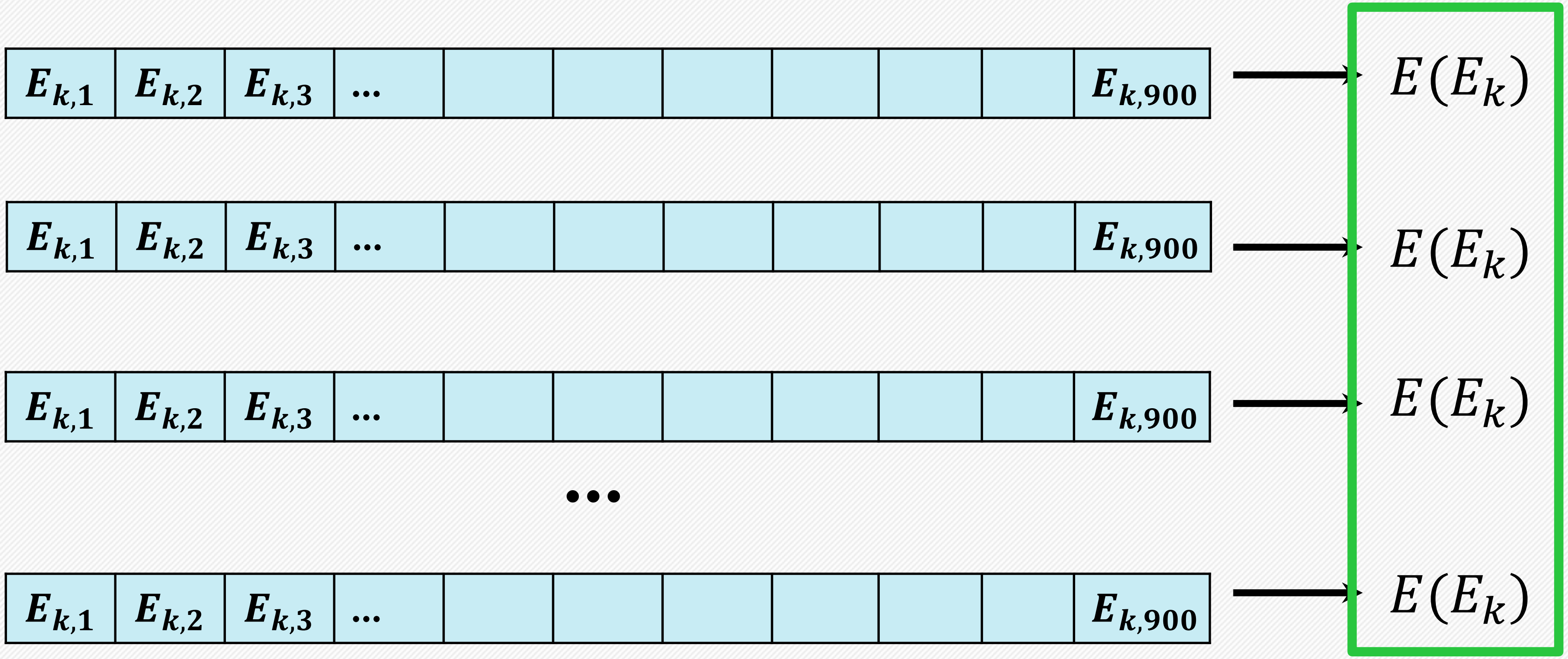
- → Lecture note Sec. 4.5
- Assume a probability function $f(X)$ with expectation value μ and variance σ^2
- We take samples $X_1, X_2, \dots, X_n$ and compute the sample average $\bar{X}$
- When $n \to +\infty$, we expect $\bar{X}$ will follows a Gaussian with expectation value $\mu' = \mu$ and standard variance $\sigma' = \sigma/\sqrt{n}$

→ For the kinetic energy distribution we considered in this exercise: $f(E_k) = 2 \cdot \left(\frac{1}{k_B T}\right)^{\frac{3}{2}} \sqrt{\frac{E_k}{\pi}}\, e^{-\frac{E_k}{k_B T}}$

It has $\mu = 1.5 k_B T$ and $\sigma = 1.5 (k_B T)^2$

Problem 2-6: Central Limit Theorem

$N_{exp} = 400$ experiments



Gaussian

$$\sigma_{E(E_k)} = \frac{\sigma_{E_k}}{\sqrt{N}}$$

$\dfrac{\sigma_{E_k}}{20}$ or $\dfrac{\sigma_{E_k}}{30}$?