# Principles of Finance with Excel®

THIRD EDITION

Simon Benninga
Tal Mofkadi

OXFORD
UNIVERSITY PRESS

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with Excel®

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This book is dedicated to our families:

The Benninga family: Terry, Noah, Sara, and Zvi

The Mofkadi family: Lizzie, Danielle, Daphne, and Emma

# CONTENTS

# PREFACE

Finance is the study of financial decision making. Individuals and companies make financial decisions every day, and it's important to make them wisely. *Principles of Finance with Excel,* Third Edition (PFE3) will teach you how to make these decisions by providing both the theory and the implementation of wise financial decision making. It will also teach you how to express your decisions using Excel.

Learning to do finance with Excel serves two purposes: It teaches you an important academic and practical subject (finance), and it teaches you how to implement financial analysis using the most important tool (in most cases, the *only* tool) for financial analysis (Excel). Your knowledge of both finance and Excel will be enhanced by carefully working through the examples and exercises in each chapter.

Finance is a very practical discipline. Most readers of this book are studying finance not only to increase their understanding of the valuation process but also to get answers to practical problems. You will find that the extensive computation required in this book will not just enable you to get numerical answers to important problems (though that alone would justify the Excel-centered focus of this book)—it will also deepen your understanding of the concepts involved. The skills in this book are required no matter what your role in the corporation is.

## Changes in the Third Edition

This third edition incorporates a number of important changes. The structure of the book has been streamlined, so the reader goes straight into the heart of finance—time value of money and discounting. Examples have been updated and most now refer to the post-2008-crash financial world and the relatively low interest rate financial environment. We worked on making the discussion in each chapter as self-contained as possible and updated the content of each chapter. The end of chapter questions are labeled to enable more efficient and focused practice for each topic. As in the first and second editions, we include an Excel "primer" in the six chapters at the end of the book.

The third edition uses Excel 2016 throughout.[1] The chapter spreadsheets and exercise solution files for the end-of-chapter exercises may be found on the Benninga *Principles of Finance with Excel*, Third Edition websites (found at www.oup.com/us/benninga). Solutions are provided only on the instructor

[1] For readers using earlier versions of Excel—a Compatibility Pack is available from http://support.Microsoft.com. This pack, once installed, allows you to read all the spreadsheets with this book in earlier versions of Excel.

website. Extra questions with detailed solutions for self-practice are provided on the student website.

The third edition of the book is divided into five parts. The first part covers capital budgeting and basic valuation concepts. The second part discusses uncertainty, portfolio analysis, and asset pricing principles. The third part covers securities valuation such as stocks and bonds. Part Four is dedicated to the understanding of options. And the fifth part provides a primer on Excel techniques.

## Prerequisites—What Excel Background Is Required for *Principles of Finance with Excel*?

This book will teach you—alongside finance—all the Excel concepts needed for finance. However, you should not expect the book to be a complete Excel text. We expect that, before you start your finance course, you will know how to do the following things in Excel (just in case—many of these topics are covered in Chapter 21):

- Open and save an Excel workbook
- Use basic Excel functions, such as **Sum( )** ...
- Format numbers—here's an example of something that is usually not explained in the text:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | –$6,144.57 | <-- =PV(10%,10,1000) | | | |
| 2 | | | | | |
| 3 | –6,144.57 | <-- In many cases we prefer this format | | | |

- Use absolute and relative values in copying and formulas

## Somewhat More Advanced Excel Concepts

Chapters 22 to 26 cover a grab bag of other Excel concepts used in PFE3. You can refer to these chapters as you need them:

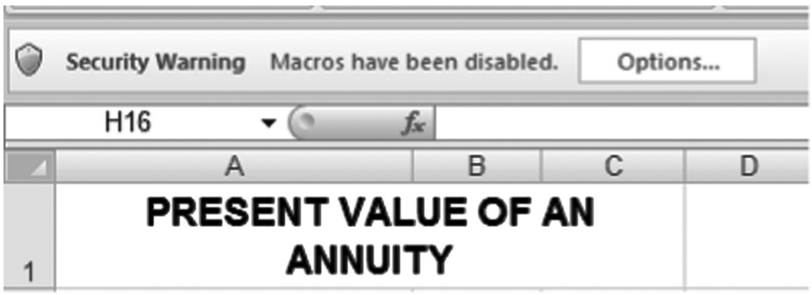
- **Charts in Excel:** Advanced charting techniques are explained in Chapter 22.
- **Excel functions:** Most of the Excel functions required for this book are explained the first time they occur. Chapter 23 is a compendium of these explanations and may be useful for reference.
- **Data tables:** "Data table" is Excel jargon for "sensitivity table." The data table technique is a little tricky, but it is well-worth learning (for some reason, data tables are often not covered in introductory Excel courses). Although the early chapters of PFE3 avoid the use of data tables, their use is required in later chapters of the book. Chapter 24 will teach you how to use data tables.

- **Goal Seek and Solver:** Excel's optimization tools are discussed in Chapter 25.
- **Dates in Excel:** Many finance computations require the use of dates. This topic is covered in Chapter 26.

## Data Files for PFE3

As noted above, each chapter is accompanied by two types of spreadsheet files which are provided on the PFE3 companion websites (http://global.oup.com/us/Benninga). The first type—named, for example, **PFE3_ch02.xlsm** or **PFE3_ch15.xlsm**—contains spreadsheets for all the examples covered in than chapter. The second type of spreadsheet provides the solutions to the end of chapter exercises in a given chapter—for example, **PFE3_ch02_solutions.xlsm** for chapter 2.

When you open a PFE3 spreadsheet, you may see the following message informing you that there is a macro attached to the spreadsheet:




This message refers to a little program (in Excel jargon: a "macro") which dynamically updates cell references, so that output like the following will automatically retain the correct cell references even if you move things around or add rows:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **CALCULATING PRESENT VALUES WITH EXCEL** | | | |
| 2 | | | | |
| 3 | X, future payment | 100 | | |
| 4 | n, time of future payment | 3 | | |
| 5 | r, interest rate | 6% | | |
| 6 | Present value, $X/(1+r)^n$ | 83.96 | <-- =B3/(1+B5)^B4 | |
| 7 | | | | |
| 8 | **Proof** | | | |
| 9 | Payment today | 83.96 | | |
| 10 | Future value in n years | 100 | <-- =B9*(1+B5)^B4 | |

Clicking on the **Options . . .** box gives you another box, in which you can safely click **Enable this content**.


You can safely enable this macro.[2]

## A Final Word

Professor Simon Benninga is the force behind this book—and is widely considered one of the "Fathers" of Financial Modeling. He and I worked together on the third edition, and he spent hours explaining to me why he presented material the way he did so that I could understand this book as well as he did. Sadly, after completing the third edition manuscript revisions, Professor Benninga passed away. Simon dedicated much of his life to research, teaching, and the dissemination of information about finance and financial analysis. He was an incredibly generous person and was in personal contact with thousands of instructors and students who used this book, including many who are mentioned in the Acknowledgements. His goals were always to help others teach financial analysis more effectively. I hope you find this book as inspiring as he was. The third edition is dedicated to the memory of Simon Benninga and to his great works. I hope you will feel free to contact me if you have any comments or suggestions about this edition.

Tal Mofkadi
mofkadi@mail.tau.ac.il

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[2] There is a document (**GetFormula.doc**) on the companion website showing you how to put this macro into any spreadsheet you will want to create.

# ACKNOWLEDGMENTS

This book has benefitted from the many wonderful comments from readers over the years. University instructors, financial professionals, and students have all chipped in to make PFE3 a better book. Students at a number of colleges and universities—including The Wharton School of the University of Pennsylvania, The Kellogg School of Management of Northwestern University, Tel Aviv University, Gonzaga University, Rutgers University, Rider University, Tulane University, and Copenhagen Business School—have been unwitting guinea pigs for the materials and we thank them as well.

We thank our families—The Benninga family: Terry, Noah, Zvi, and Sara and the Mofkadi family: Lizzie, Danielle, Daphne, and Emma—for their support and encouragement throughout the writing and development process. The weeks and months we invested in this book were surely taken away from them. We've tried to carefully note all the people who've been helpful along the way, and we apologize in advance for any people we have missed.

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Finally, our thanks go to the many people at Oxford University Press who saw the Third Edition through to completion, beginning with Editorial Director Patrick Lynch, Senior Editor Ann West, Senior Production Manager Micheline Frederick, Editorial Assistants Abigail Roberts and Alison Ball, copyeditor Bob Golden, and proofreader Wesley Morrison.

# PART 1

## The Time Value of Money

You'll learn in this book that cash flows are evaluated on three important dimensions. The first dimension is simply the sum of money—receiving more money is good and paying more money is bad. The second dimension is called "time value of money"—receiving money tomorrow is not as good as receiving it today. The third dimension is risk—risky cash flow is not as valuable as safe cash flow. You do not need us (or anyone else) to teach you about the first dimension. Part 1 of this book is designed to focus on the second dimension—the time value of money. Throughout this section, we'll assume that the cash flows are not risky or, when comparing cash flows, have the same risk. Chapter 1 presents the basics, Chapter 2 is key since it discusses the techniques of evaluating cash flows in different point of times, Chapter 3 discusses the implementation of the concepts to evaluate investment opportunities, Chapter 4 extends the understanding to analyzing loans, Chapter 5 teaches the proper way to analyze discount rates and infer the correct effective rate, and the final chapter in this part—Chapter 6—provides a comprehensive overview on calculating cash flow in reality.

In some curriculums, this material is called Finance Under Certainty. After reading the chapters in this part, you should understand much of the financial world around you and know how to analyze complex financial situations.

# 1 Introduction to Finance

Chapter Contents



## 1.1 What Is Finance?

Finance is the study of financial decision making. Individuals and companies make financial decisions every day, and it’s important to make them wisely. *Principles of Finance with Excel* discusses how to make these decisions. The book covers the theory and the implementation of wise financial decision making and how to express your decisions using Excel.

Learning to do finance with Excel serves two purposes: It teaches you an important academic and practical subject (finance), and it teaches you how to implement financial analysis using the most important tool (in most cases, the *only* tool) for financial analysis (Excel).

### Individual Financial Decision Making

People are constantly called on to make financial decisions in their personal lives. Here are examples of decisions we will examine in this book:

- How much should you save to attain a specific goal in the future? For example: You’re starting a savings plan today to save for your college education. How much should you put away each month in order for you to have the money to pay for your education?
- You’re thinking about buying a house and renting it out for the income. How should you evaluate this decision?

- You have some money saved from working, and you'd like to invest it. How should you choose your financial portfolio? Investors big and small have to decide whether to invest in stocks, bonds, or other assets such as real estate, art, and gold. They also have to decide how to choose the *investment proportions*: What percentage of your financial portfolio should you invest in stocks (and what percentage in *which* stocks), what percentage in bonds, what percentage in real estate, and so on.
- How should you finance a purchase, a project, or some other undertaking? Here are some examples: You're about to buy a new car. Should you borrow the money from the bank, or should you accept the car dealer's "zero interest loan" alternative? That piece of real estate you're buying—should you finance it with a mortgage? If so, how large should the mortgage be?
- What is financial risk, and how can it be measured? Financial risk can be measured using statistical tools. This book will show you which tools you need and how to apply them. When you're comfortable applying these tools, you will be better able to compare the riskiness of two assets or two investments. Comparing risks is critical to making optimal financial decisions.
- What is the fair value for stocks and bonds and other financial assets? This book will show you how to compute the value of stocks and bonds. It will also discuss the role of financial markets in incorporating available financial information into prices. If financial markets do this well, you may not need to determine these values yourself: You can let the financial markets tell you what the value should be.
- How can you value options? Options are securities which give you the right to buy a stock in the future. If you work in a corporate environment, your employers are likely, at some point, to offer you some options on the company's stock instead of a regular salary. If you're trying to regulate the risk of your financial portfolio, your investment advisor may try to sell you some options. In this book, you'll learn what an option is, how to use it to regulate financial risk, and how to value it.

As these examples show, the study of finance can benefit you in many areas of your personal life by enabling you to make better financial decisions.

## Financial Decisions in a Business Environment

You only have to turn on the TV, log onto the Internet, or read a newspaper to hear about the financial decisions made every day by businesses. Some of these financial decisions are huge and dramatic, like Comcast's failed 2015 bid to purchase Time Warner Cable for $45.2 billiion; some are smaller but nonetheless very important for the company, involving things like purchasing new equipment or building a new warehouse or distribution center.

Dramatic business decisions like mergers and acquisitions make the news, but "run-of-the-mill" business financial decisions that are critical to the financial health of the firm are made by all businesses, big or small. Here are some typical decisions that businesses make:

- A company wants to replace its current production line with a line of new, improved machines. The new machines cost more but are more efficient. Should the company buy the new machines or leave the old ones in place?
- A firm needs custom software to increase productivity. Should it buy off-the-shelf software and develop their own workarounds, or should they invest in a fully customized solution and platform that will be compatible with all their business needs?
- When a company wants to develop and produce a new product, how can it integrate the marketing forecasts for the new product with the financial requirements of the development and production processes? How can the company deal with the fact that the biggest costs of development and production will be incurred before any revenues have been realized from the sale of the product?
- How should the financial officers of a corporation plan for a new or existing business? A *financial planning model* can provide a systematic approach to making many of the financial decisions in a new or existing business. Perhaps you're thinking of setting up a laundromat on the corner of Main and Pine Streets. Perhaps you're starting a real estate business. Or perhaps you're trying to finance a new high-tech idea. In each case, your ability to get financing from financial institutions—whether banks or venture capital funds or your Uncle Joe—will depend on your ability to make a financial model for the new business. This financial model will show your thoughts about how the business will develop, how much equipment you'll need to purchase, and how you will finance sales. Most importantly, the financial model will project future earnings from the business.
- All companies must decide how to finance their activities. This is true for multi-national conglomerates, mom-and-pop convenience stores, and the new taxi company you're about to start with your cousin Sarah. In all cases, someone has to decide whether to borrow the money from others or whether to use shareholder funds (equity, in the terminology of finance) to finance the company.

## Wealth Maximization and Risk

This book is primarily about making *sensible financial decisions*. Sometimes a sensible financial decision is also an *optimal* financial decision. Optimal financial decisions make you better off than all the other relevant alternatives, including doing nothing at all. Economists call this property of optimal financial decisions *wealth maximization*. Not every case of money management boils down to making a wealth-maximizing decision; sometimes we will be able to only point to a *sensible set of financial alternatives* from which you can choose a final decision.

Making sensible or wealth-maximizing financial decisions always involves two elements.

- **Defining the parameters of the decision**: Financial decisions can always be defined in terms of numbers. The outcomes of a financial decision almost always depend on the *decision parameters*, the inputs which define the results of the financial decision.

Here's an example: You've been given $100 for your birthday, and you decide to save it toward your summer vacation next year. You have two choices: You can leave the money in your checking account, or you can put the money in a savings account. The two parameters of this decision are the amount you're saving ($100) and the interest paid on the account—the checking account pays 1% interest, whereas the savings account pays 4% interest. The *financial outcomes* are that 1 year from now, you will have $101 if you leave the money in your checking account and $104 if you put the money in a savings account. This decision is, of course, a no-brainer: You always prefer earning 4% to earning 1% on your money.[1]

This book will help you distinguish between the parameters of financial decisions and the outcomes of financial decisions.

- **Recognizing the risks of financial decisions**: Financial decisions should be made within a framework that takes into account the risks associated with them.

Let's go back to the $100 you intend to save for your summer vacation. In addition to the two alternatives (1% on your checking account and 4% on your savings account), your Uncle Joe suggests that you might want to buy shares in his new hot dog stand. Investors in Joe's previous hot dog stands have earned as much as 40% on their investment.

If you put your money in Uncle Joe's hot dog stand, you *might* have $140 at the end of the year, instead of $104, but if the hot dog stand does poorly, you could lose your $100 investment and end up with nothing. Uncle Joe's hot dog stand is *much more risky* than a bank account—although some investors have made as much as 40%, others have lost all their money with Joe. Comparing an investment in the hot dog stand with a deposit in a savings account must take into account the differences in their risks. This book will show you how to account for risks inherent in making financial decisions.

[1] Of course there may be other things going on: Checking account balances are always available, whereas perhaps the balance in your savings account needs to be there for some minimum period of time before you earn the interest.

**IT'S OFFICIAL: Intel is buying the autonomous-driving company Mobileye for $15.3 billion**

**Portia Crowe**
**Business Insider** March 13, 2017

A device, part of the Mobileye driving assist system, is seen on the dashboard of a vehicle during a demonstration for the media in Jerusalem October 24, 2012. REUTERS/Baz Ratner

(Mobileye technology.Thomson Reuters)

Intel is buying the Israeli autonomous-driving company Mobileye for $63.54 a share in cash, or about $15.3 billion.

Mobileye soared about 30% in premarket trading Monday after the Israeli newspaper Haaretz broke the news.

The Jerusalem-based company develops vision-based driver-assistance tools to provide warnings before collisions.

"Mobileye brings the industry's best automotive-grade computer vision and strong momentum with automakers and suppliers," Intel CEO Brian Krzanich said in a statement.

"Together, we can accelerate the future of autonomous driving with improved performance in a cloud-to-car solution at a lower cost for automakers."

Tesla began incorporating Mobileye's technology into Model S cars in 2015.

In January, it announced it was developing a test fleet of autonomous cars together with BMW and Intel.

Mobileye was cofounded in 1999 by Amnon Shashua, an academic, and Ziv Aviram, who is the CEO. Goldman Sachs and Morgan Stanley took it public in 2014.

Source: https://finance.yahoo.com/news/mobileyes-stock-soaring-report-intel-101337297.html

## 1.2 Microsoft Excel: Why This Book and Not Another?

There are dozens of introductory finance texts out there. Many of them are very good. So why this one? In a word: **Excel**. Finance is the study of financial decision making and is therefore inherently a topic requiring lots of computation. In this book, the computation is done in, and illustrated with, Excel, the premier business computational tool. Excel gives you the flexibility to change the elements of an example and to immediately get a new answer. We will use this flexibility extensively throughout *Principles of Finance with Excel*.

Finance is a very practical discipline. Most of you are studying finance not only to increase your understanding of the valuation process, but also to *get answers to practical problems*. You will find that the extensive computation required in this book will not just enable you to get numerical answers to important problems (though that alone would justify the Excel-centered focus of this book)—it will also deepen your understanding of the concepts involved.

Using Excel enables us to discuss many more real-life examples than is possible by using a calculator. Your knowledge of both finance and Excel will be enhanced by carefully working through the examples and exercises in each chapter.[2]

Most college students will be coming to a finance course after having taken an initial computing course which covers the basics of Excel used in this book. If you want an Excel review, the last six chapters of this book cover the essential Excel concepts that are used in this book. In addition, throughout the book you will find explanations of Excel functions and their application to financial problems. When things get really rough, you'll find little boxes called "Excel notes" which explain difficult concepts. Here is an example of such a box:

## EXCEL NOTE

### Using Sum to Compute Profit and Loss

The Excel function **Sum** can often be used to simplify calculations. Here's an example based on the computation of a profit and loss statement:

| | A | B | C |
|---|---|---|---|
| 1 | **USING SUM TO COMPUTE THE PROFIT AND LOSS** | | |
| 2 | Profit and loss | | |
| 3 | Sales | 1,000 | |
| 4 | Cost of goods sold | –500 | |
| 5 | Depreciation | –100 | |
| 6 | Interest | –35 | |
| 7 | Profit before taxes | 365 | <– =SUM(B3:B6) |
| 8 | Taxes (40%) | –146 | <– =–40%*B7 |
| 9 | Profit after taxes | 219 | <– =SUM(B7:B8) |

Cells B7 and B9 use the **Sum** function to add multiple cells. An alternative to using **Sum** in cell B7 would be to use the formula **=B3+B4+B5+B6**. As you can see, **Sum** is more concise.

[2] If you're a finance student at a college or university, this combination of Excel and finance will also enhance your employment opportunities. Excel is practically the only financial tool used by business today.

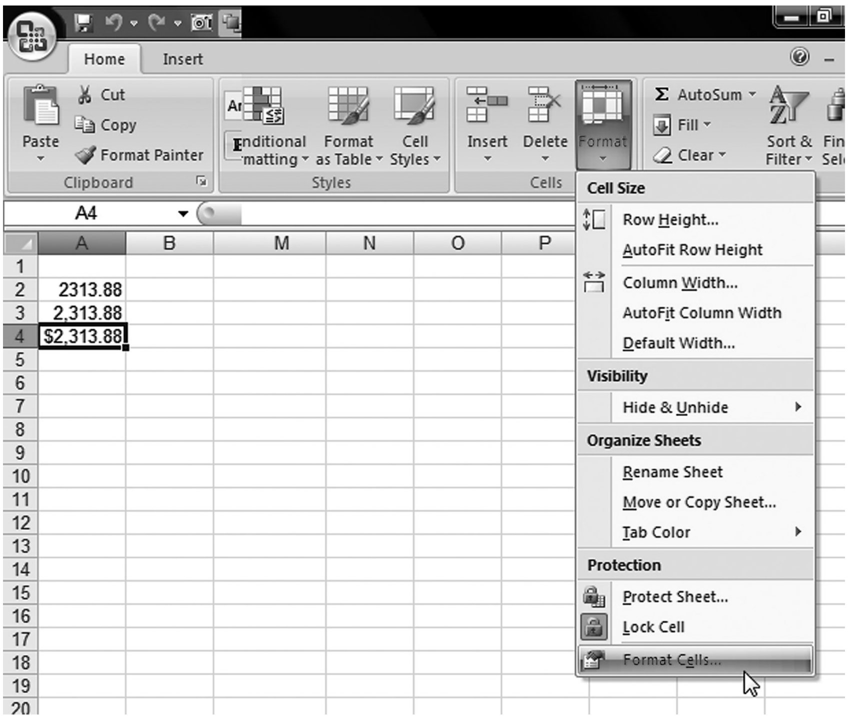
## Excel Versions

*Principles of Finance with Excel* illustrates all its examples using Excel 2016 for Windows, the spreadsheets are fully compatible with earlier versions of Excel.

## What Are the Excel Prerequisites for This Book?

You do not have to be an Excel expert to use this book. Almost all the Excel concepts needed to do finance are explained in the text itself. While this book will teach you the Excel concepts needed for finance, it is not a complete Excel text. Before you start Chapter 2, you should know how to do the following in Excel (all are covered in Chapter 24):

- Open and save an Excel notebook.
- **Format numbers:** You can make numbers appear in different forms. In the example below, the number 2,313.88 is shown in three different ways. You should know how to do this formatting. In this case, we've chosen an appropriate format from the drop-down list on the **Format** section of the **Home** tab of Excel and indicated the appropriate formatting.



- Use absolute and relative values in copying and formulas: When you copy in Excel, you can use either *relative* or *absolute* copying. As explained in Chapter 21, relative copying changes the cell reference addresses, whereas absolute copying leaves them the same.[3]
- Build basic Excel charts to graph data. You should know how to label axes, put in chart titles, format axes, and so on.

## 1.3 Eight Principles of Finance

In this section, we look at eight unifying principles of finance. At this stage, you may not understand them all or even find them convincing, but we introduce them here in order to give you an overview of what finance is all about. They will be more fully explained in the rest of *Principles of Finance with Excel*.

### Principle 1: Buy Assets that Add Value; Avoid Buying Assets that Don't

On the simplest level, making optimal financial decisions has to do with buying assets that add value and avoiding those that aren't. For example, you need to decide whether to keep using your old, inefficient photocopying machine or buy an expensive new one that works faster, doesn't break down as often, and uses less ink and energy. The finance question about these two alternatives is: Which—keeping the old photocopier or buying a new one—adds more value to your business? To make a determination about how valuable things (such as stocks, bonds, machines, and companies) are, you need to be sure that you are comparing apples with apples and oranges with oranges. This sounds like a simple principle to follow, but it can surprisingly tricky to implement!

### Principle 2: Cash Is King

The value of an asset is determined by the *cash flows it produces over its life*. The cash flow of an asset is the *after-tax cash* which the asset produces at a given point in time.

While it is too early in the book to give you the full flavor of the difference between a cash flow and a profit number, we can give a small example. Suppose your pizza parlor sells \$500 of pizzas on Tuesday night, and suppose the same day you bought \$300 worth of ingredients. Looking in the cash register at the end of the day, you expect to find \$200, but instead you're surprised to find \$300. The explanation: Of the \$500 of pizzas sold, you only collected \$400—the other \$100 were sold to a campus fraternity which maintains an account with you that

[3] If you find this sentence mysterious, look at Section 21.3.

they settle at the end of each month. Of the $300 of ingredients you bought, you only paid for $100—the other $200 will be billed to you for payment in ten days.

Cash flows are different from accounting profits or sales receipts. The pizza parlor's *accounting profit* for the day is $200, but its *cash flow* for the day is $300 (= $400 collected from sales minus $100 paid for supplies). The difference between the two is due to the *timing difference* between inflows and outflows. (Of course, 10 days from now the pizza parlor will have a negative cash flow of $200 as a result of paying for the ingredients.)

In finance, *cash flow is all-important*. Most corporate financial data come from accountants, who—despite the bad press they've gotten in the past few years—do a very good job at representing the economic realities of corporate activities. When making financial decisions, we have to translate the accounting data to their cash equivalents. Much of finance involves first translating accounting information into cash flows.[4]

## Principle 3: The Time Dimension of Financial Decisions Is Important

Many financial decisions have to do with comparing cash flows at different points in time. As an example: You pay for that new photocopy machine *today* (a cash *outflow*), but you save money in the *future* (a cash *inflow*). Finance has to do with correctly dealing with this time dimension of cash flows.

## Principle 4: Know How to Compute the Cost of Financial Alternatives

Financial alternatives are often bewildering: Is it more expensive to buy or lease a photocopier? When your credit card charges you "daily interest," is it more or less expensive than the bank loan which charges you "monthly interest"? In making financial decisions, you need to know how to compute the cost of two or more competing alternatives. This book will teach you how.

## Principle 5: Minimize the Cost of Financing

Many financial decisions have to do with choosing the right alternative. Should you finance that photocopier with a loan from the dealer or with a loan from the bank? Should you buy a new car or lease it?

Choosing the right financial alternative is, in many cases, a decision made separately from the investment decision: You've decided to purchase the copier (the investment decision), and now you have to choose whether to finance it through a bank loan or by accepting the dealer's "zero interest financing" (the financing decision).

---

[4] Not familiar with basic accounting? See the book's website http://pfe.wharton.upenn.edu for a review of basic accounting principles.

### Principle 6: Take Risk into Account

Many financial alternatives cannot be directly compared without taking into account their risk. Should you take money out of the bank and invest it in the stock market? On the one hand, people who invest in the stock market *on average* earn more than those who leave their money in the bank. On the other hand, a bank deposit is safe, whereas a stock market investment is much less safe (riskier).

"Risk" is the magic word in finance. This book will show you how to quantify risk so that you can compare financial alternatives.

### Principle 7: Markets Are Efficient and Deal Well with Information

Financial markets are awash in information. In making a financial decision, how can we possibly know or obtain all the information we need to make a sensible, well-informed decision? The bad news is that we probably can't incorporate all available information into our decision-making process. The good news is that we may not have to: The confluence of many market participants striving to make use of what information they have leads markets to work to eliminate riskless profitable opportunities. In many cases, financial markets work so well that we can't add anything to their information-gathering abilities. In short: It may well be that the stock market's valuation of XYZ stock is correct given all the information available about the stock. This *market efficiency* can simplify the way you think about assets and their prices when making financial decisions.

### Principle 8: Diversification Is Important

"Don't put all your eggs in one basket." The financial equivalent of this hackneyed expression is: Diversify the assets you hold; don't hold just a few stocks or bonds, buy a portfolio. *Principles of Finance with Excel* will show you both how to analyze portfolios of assets, and how to choose the individual assets in your portfolio wisely.

## 1.4 An Excel Note: Building Good Financial Models

We've chosen this place in the chapter to tell you a bit about financial modeling. A few simple rules will help you create better and neater Excel models.

**Modeling rule 1: Put all the variables which are important (the fashionable jargon is "value drivers") at the top of your spreadsheet.** In the "Saving for College" spreadsheet on page 13, the three value drivers—the interest rate, the annual deposit, and the annual cost of college—are in the top left-hand corner of the spreadsheet:

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | **SAVING FOR COLLEGE** | | | | | | | | |
| 2 | Interest rate | 8% | | | | | | | |
| 3 | Annual deposit | 12,000.00 | | | | | | | |
| 4 | Annual cost of college | 35,000 | | | | | | | |
| 5 | | | | | | | | | |
| 6 | **Birthday** | **In bank on birthday, before deposit/withdrawal** | **Deposit or withdrawal at beginning of year** | **End of year before interest** | **End of year with interest** | | | | |
| 7 | 10 | 0.00 | 12,000.00 | 12,000.00 | 12,960.00 | | | | |
| 8 | 11 | 12,960.00 | 12,000.00 | 24,960.00 | 26,956.80 | | | | |
| 9 | 12 | 26,956.80 | 12,000.00 | 38,956.80 | 42,073.34 | | | | |
| 10 | 13 | 42,073.34 | 12,000.00 | 54,073.34 | 58,399.21 | | | | |
| 11 | 14 | 58,399.21 | 12,000.00 | 70,399.21 | 76,031.15 | | | | |
| 12 | 15 | 76,031.15 | 12,000.00 | 88,031.15 | 95,073.64 | | | | |
| 13 | 16 | 95,073.64 | 12,000.00 | 107,073.64 | 115,639.53 | | | | |
| 14 | 17 | 115,639.53 | 12,000.00 | 127,639.53 | 137,850.69 | | | | |
| 15 | 18 | 137,850.69 | –35,000.00 | 102,850.69 | 111,078.75 | | | | |
| 16 | 19 | 111,078.75 | –35,000.00 | 76,078.75 | 82,165.05 | | | | |
| 17 | 20 | 82,165.05 | –35,000.00 | 47,165.05 | 50,938.25 | | | | |
| 18 | 21 | 50,938.25 | –35,000.00 | 15,938.25 | | | | | |
| 19 | | | | | | | | | |
| 20 | | NPV of all payments | 6,835.64 | <-- =C7+NPV(B2,C8:C18) | | | | | |

**Critical parameters (sometimes called "value drivers") are in the upper left-hand corner. The actual cost of saving for a college education is discussed in Chapter 2.**

**Modeling rule 2: Never use a number where a formula will also work.** Using formulas instead of "hard-wiring" numbers means that when you change a parameter value, the rest of the spreadsheet changes appropriately. As an example—cell C20 in the above spreadsheet contains the formula **=C7+NPV(B2,C8:C18)**. We could have written this as **=C7+NPV(8%,C8:C18)**. But this means that changing the entry in cell B2 will not go through the whole model.

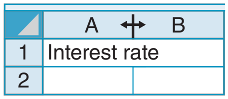
**Modeling rule 3: Avoid the use of blank columns to accommodate cell "spillovers."** Here's an example of a potentially bad model:

| | A | B | C |
|---|---|---|---|
| 1 | Interest rate | | 6% |

Because "Interest rate" has spilled over to column B, the author of this spreadsheet has decided to put the "6%" in column C. This could be confusing. A better way is to make column A wider and put the 6% in column B:

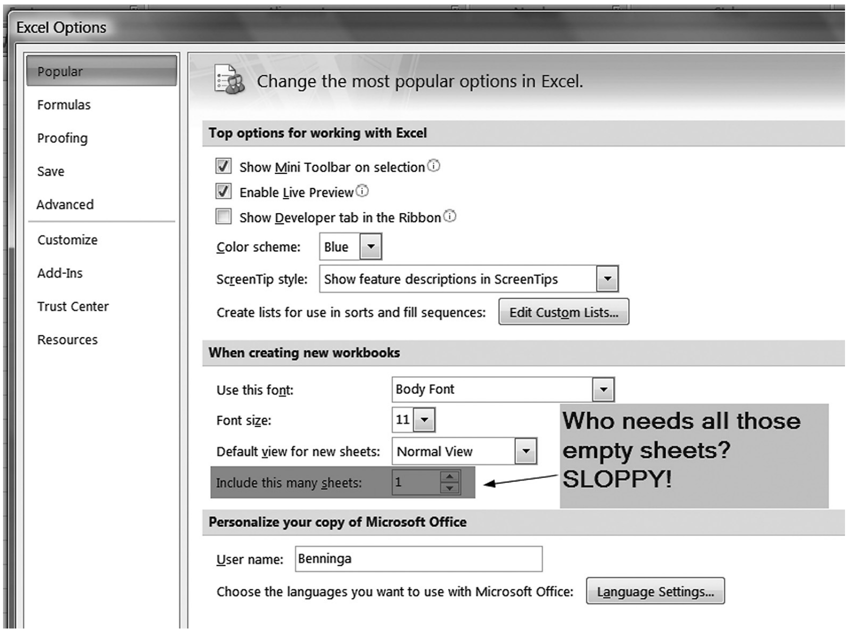
| | A | B |
|---|---|---|
| 1 | Interest rate | 6% |

Widening the column is simple: Put the cursor on the break between columns A and B:

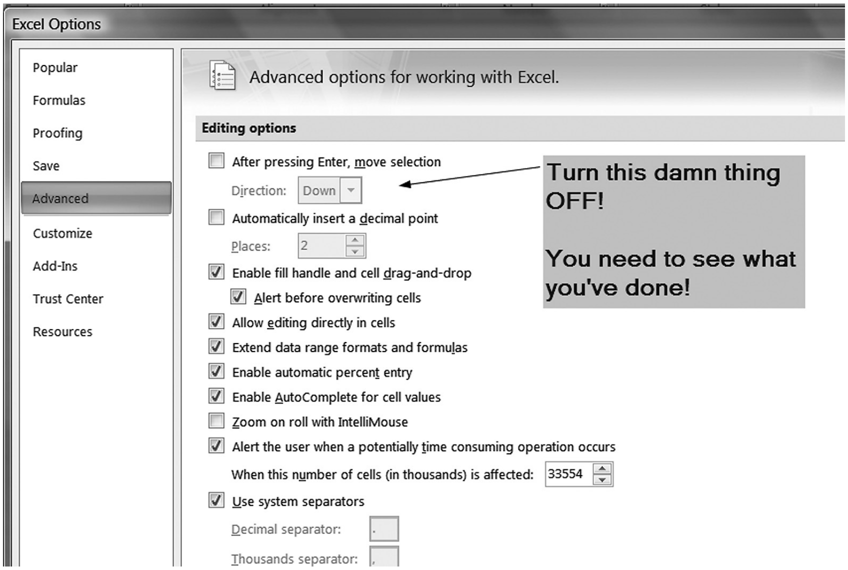



Double clicking the left mouse button will expand the column to accommodate the widest cell. You can also "stretch" the column by holding the left mouse button down and moving the column width to the right.

**Modeling rule 4: Make your Excel default one sheet.** Excel's default is to open notebooks with three spreadsheets, but 99% of the time you'll only need one sheet. So set your default to one sheet, and if you need more, you can always add. In Excel 2016, go to the **File ➔ Options ➔ General ➔ Popular**:




**Modeling Rule 5: Turn off the "auto jump down" feature.** The Excel default is that when you click **Enter**, the cursor jumps down to the next cell. But in financial modeling, we need to look at the formulas we've written to make sure that they make sense! So turn off this feature! Go to the **File ➔ Options ➔ Advanced**:




## 1.5 A Note About Excel Versions

This book uses Excel 2016, but the spreadsheets are compatible with earlier versions of Excel.

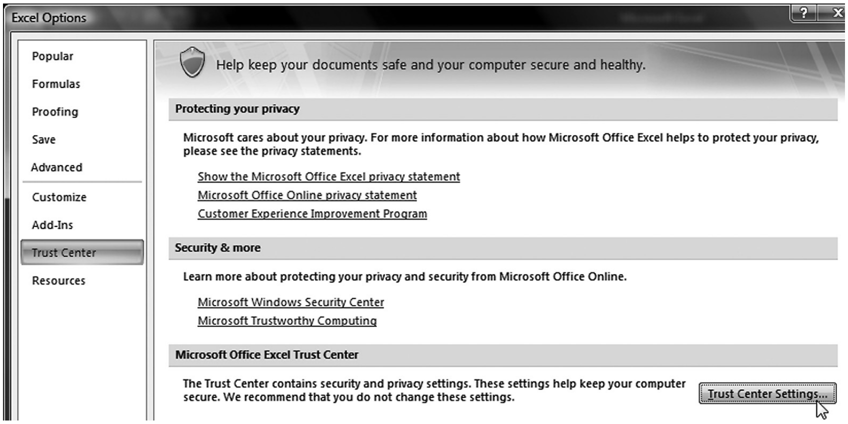
## 1.6 Adding "Getformula" to Your Spreadsheet

The spreadsheets which accompany *Principles of Finance with Excel* all include a short macro called **Getformula** which tracks the cell contents. I've found **Getformula** extraordinarily useful in my work—it allows me to explain (to myself and to my readers) what I've done in my spreadsheets. The macro is *dynamic*: When you change a spreadsheet by moving something (for example, by adding rows or columns), **Getformula** automatically updates the formulas.

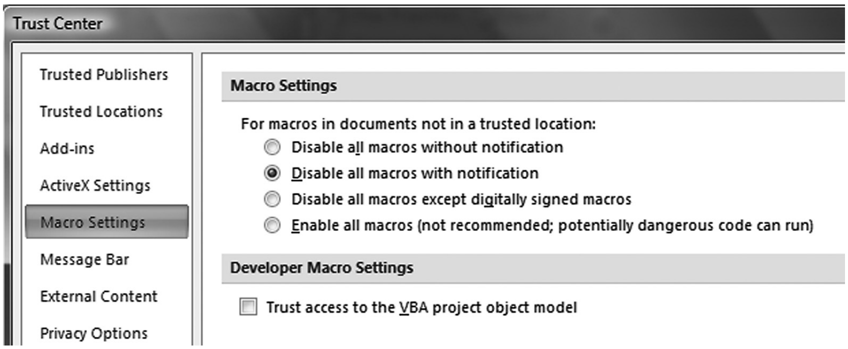
### Setting the Excel Security Level

In order to use **Getformula**, you have to first set the security level of Excel to medium. This allows you to choose to open (or not) macros in Excel. Doing this is a two-step procedure:

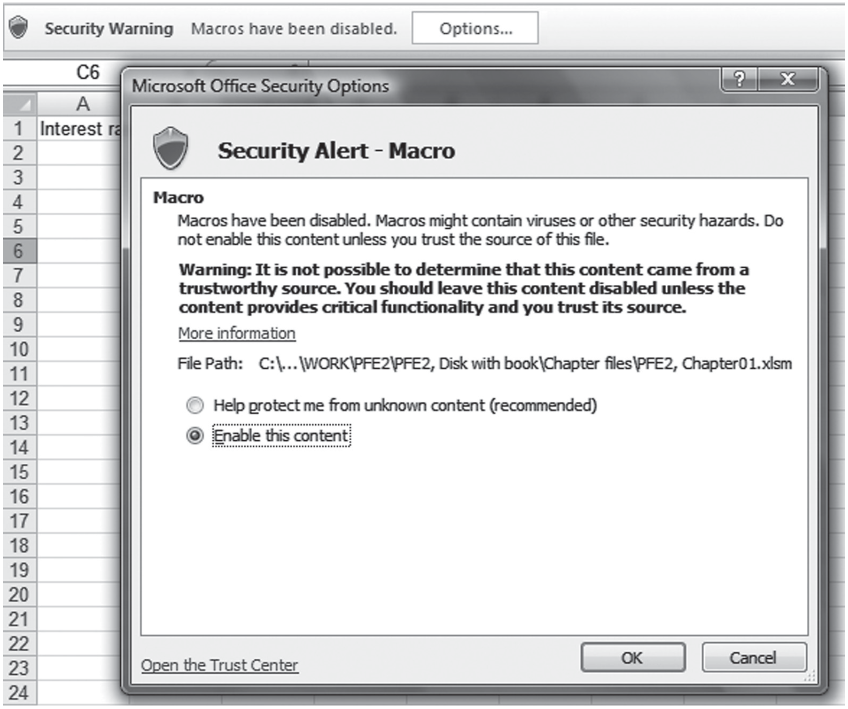
**Step 1:** In Excel 2016, go to the **File ➔ Options ➔ Trust Center**. Click on **Trust Center Settings:**




**Step 2:** Go to the **Macro Settings** tab, and click **Disable all macros with notification**. This is a backhanded way of saying that Excel will ask whether you want to open macros:



The above steps need be done only once. Now, when you open a spreadsheet from *Principles of Finance with Excel*, you will be asked whether to open the macros:




## Summary

The combination of finance concepts with an Excel implementation is a "killer app"! *Principles of Finance with Excel* is the only finance principles book on the market which contains this combination.

Enjoy!

# 2 The Time Value of Money

## Chapter Contents



## Overview

This chapter deals with the most basic concepts in finance: future value and present value. These concepts tell you how much your money will grow if deposited in a bank (future value) and how much promised future payments are worth today (present value).

Financial assets and financial planning always have a time dimension. Here are some simple examples:

- You put $100 in the bank today in a savings account. How much will you have in 3 years?
- You put $100 in the bank today in a savings account and plan to add $100 every year for the next 10 years. How much will you have in the account in 20 years?
- Your friend is planning for his retirement. He has just celebrated his 35th birthday, and he is planning to work until the age of 65. On retirement, he would like to withdraw $100,000 every year from the account until he reaches the age of 90. What is the annual deposit he should make every year until retirement?

This chapter discusses these and similar issues, all of which fall under the general topic of *time value of money*. You will learn how compound interest causes invested income to grow (*future value*), as well as how money to be received at future dates can be related to money in hand today (*present value*). The concepts of future value and present value underlie much of the financial analysis which will appear in the following chapters.

As always, we use Excel, the best financial analysis tool!

**Finance concepts discussed**

- Future value
- Present value
- Financial planning—pension and savings plans and other accumulation problems

**Excel functions used**

- **FV, PV, NPV, PMT, NPER, RATE, SUM**
- **Goal Seek**

## 2.1 Future Value

*Future value is the value at* some *future date of a payment (or payments) made before this future date. The future value includes the interest earned on the payments.*

Future value (FV) is a concept that relates the value in the future of money deposited in a bank account today and over time and left in the account to draw interest. Suppose, for example, that you put $100 in a savings account in your bank today and that the bank pays you 6% interest at the end of every year. If you leave the money in the bank for 1 year, you will have $106 after 1 year: $100 of the original savings balance + $6 in interest. The $106 is the *future value after 1 year of the initial deposit of $100 at 6% annual interest*.

Now suppose you leave the money in the account for a second year. At the end of this year, you will have:

| | |
|---|---|
| $106 | The savings account balance at the end of the first year |
| + | |
| 6%*$106 = $6.36 | The interest on this balance for the second year |
| = $112.36 | Total in account after 2 years |

The $112.36 is the *future value after 2 years of the initial deposit of $100 at 6% annual interest*. Another way to express this is $112.36 = $100*(1 + 6%)$^2$:

$$\underbrace{\underbrace{\underbrace{\$100}_{\text{Initial deposit}} * \underbrace{1.06}_{\text{Year 1's future value factor at 6\%}}}_{\text{Future value of \$100 after 1 year = \$100*1.06}} * \underbrace{1.06}_{\text{Year 2's future value factor}}}_{\text{Future value of \$100 after 2 years}} = \$100*\left(1+6\%\right)^2 = \$112.36$$

Notice that the future value uses the concept of *compound interest*: The interest earned in the first year ($6) itself earns interest in the second year. To sum up:

*The future value of $X deposited today in an account paying r% interest annually and left in the account for n years is* $FV = X*\left(1+r\right)^n$.

## Notation Note

In this book, we will often match our mathematical notation to that used by Excel. Since in Excel multiplication is indicated by a star (*), we will sometimes write 6%*$106 = $6.36. Similarly, we will sometimes write $(1.10)^3$ as 1.10 ^ 3.

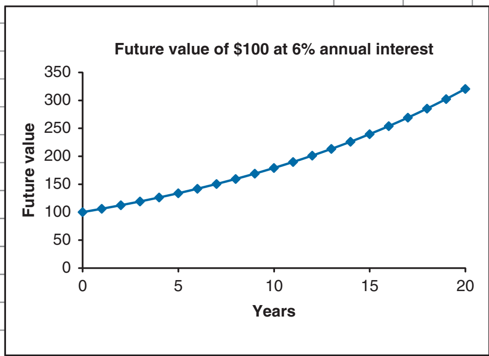
Future value calculations are easily done in Excel:

| | A | B | C |
|---|---|---|---|
| 1 | **CALCULATING FUTURE VALUES WITH EXCEL** | | |
| 2 | Initial deposit | 100 | |
| 3 | Interest rate | 6% | |
| 4 | Number of years, n | 2 | |
| 5 | | | |
| 6 | Account balance after n years | 112.36 | <-- =B2*(1+B3)^B4 |

Notice the use of the carat (^) to denote the exponent: In Excel, $\left(1+6\%\right)^2$ is written as (1 + B3)^B4, where cell B3 contains the interest rate and cell B4 contains the number of years.

We can use Excel to make a table of how the future value grows with the years and then use Excel's graphing abilities to graph this growth:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | THE FUTURE VALUE OF A SINGLE $100 DEPOSIT | | | | | | |
| 2 | Initial deposit | 100 | | | | | |
| 3 | Interest rate | 6% | | | | | |
| 4 | Number of years, n | 2 | | | | | |
| 5 | | | | | | | |
| 6 | Account balance after n years | 112.36 | <-- =B2*(1+B3)^B4 | | | | |
| 7 | | | | | | | |
| 8 | Year | Future value | | | | | |
| 9 | 0 | 100.00 | <-- =$B$2*(1+$B$3)^A9 | | | | |
| 10 | 1 | 106.00 | <-- =$B$2*(1+$B$3)^A10 | | | | |
| 11 | 2 | 112.36 | <-- =$B$2*(1+$B$3)^A11 | | | | |
| 12 | 3 | 119.10 | <-- =$B$2*(1+$B$3)^A12 | | | | |
| 13 | 4 | 126.25 | <-- =$B$2*(1+$B$3)^A13 | | | | |
| 14 | 5 | 133.82 | | | | | |
| 15 | 6 | 141.85 | | | | | |
| 16 | 7 | 150.36 | | | | | |
| 17 | 8 | 159.38 | | | | | |
| 18 | 9 | 168.95 | | | | | |
| 19 | 10 | 179.08 | | | | | |
| 20 | 11 | 189.83 | | | | | |
| 21 | 12 | 201.22 | | | | | |
| 22 | 13 | 213.29 | | | | | |
| 23 | 14 | 226.09 | | | | | |
| 24 | 15 | 239.66 | | | | | |
| 25 | 16 | 254.04 | | | | | |
| 26 | 17 | 269.28 | | | | | |
| 27 | 18 | 285.43 | | | | | |
| 28 | 19 | 302.56 | | | | | |
| 29 | 20 | 320.71 | | | | | |




# EXCEL NOTE

## Absolute References

Notice that the formula in cells B9:B29 in the table has $ signs on the cell references (for example: **=$B$2*(1+$B$3)^A9**). This use of the *absolute references* feature of Excel is explained in Chapter 21.

In the spreadsheet below, we present a table and graph that show the future value of $100 for three different interest rates: 0%, 6%, and 12%. As the spreadsheet shows, future value is *very* sensitive to the interest rate! Note that when the interest rate is 0%, the future value doesn't grow.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **FUTURE VALUE OF A SINGLE DEPOSIT AT DIFFERENT INTEREST RATES How $100 at time 0 grows at 0%, 6%, 12%** | | | | |
| 2 | Initial deposit | 100 | | | |
| 3 | Interest rate | 0% | 6% | 12% | |
| 4 | | | | | |
| 5 | **Year** | **FV at 0%** | **FV at 6%** | **FV at 12%** | |
| 6 | 0 | 100.00 | 100.00 | 100.00 | <-- =$B$2*(1+D$3)^$A6 |
| 7 | 1 | 100.00 | 106.00 | 112.00 | <-- =$B$2*(1+D$3)^$A7 |
| 8 | 2 | 100.00 | 112.36 | 125.44 | |
| 9 | 3 | 100.00 | 119.10 | 140.49 | |
| 10 | 4 | 100.00 | 126.25 | 157.35 | |
| 11 | 5 | 100.00 | 133.82 | 176.23 | |
| 12 | 6 | 100.00 | 141.85 | 197.38 | |
| 13 | 7 | 100.00 | 150.36 | 221.07 | |
| 14 | 8 | 100.00 | 159.38 | 247.60 | |
| 15 | 9 | 100.00 | 168.95 | 277.31 | |
| 16 | 10 | 100.00 | 179.08 | 310.58 | |
| 17 | 11 | 100.00 | 189.83 | 347.85 | |
| 18 | 12 | 100.00 | 201.22 | 389.60 | |
| 19 | 13 | 100.00 | 213.29 | 436.35 | |
| 20 | 14 | 100.00 | 226.09 | 488.71 | |
| 21 | 15 | 100.00 | 239.66 | 547.36 | |
| 22 | 16 | 100.00 | 254.04 | 613.04 | |
| 23 | 17 | 100.00 | 269.28 | 686.60 | |
| 24 | 18 | 100.00 | 285.43 | 769.00 | |
| 25 | 19 | 100.00 | 302.56 | 861.28 | |
| 26 | 20 | 100.00 | 320.71 | 964.63 | |
| 27 | | | | | |
| 28 | | | | | |
| 29 | | | | | |
| 30 | | | | | |
| 31 | | | | | |
| 32 | | | | | |
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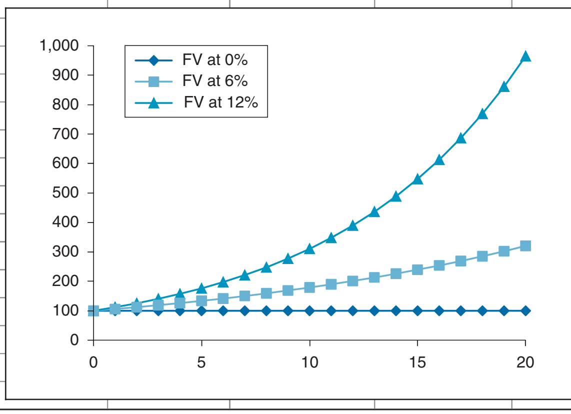



## Terminology: What's a Year? When Does it Begin?

While these questions may seem obvious, this is not the case. There's a lot of semantic confusion on this subject in finance courses and texts.

Throughout this book, we will use the following as synonyms:

| | | |
|---|---|---|
| **Year 0** | **Year 1** | **Year 2** |
| **Today** | **End of year 1** | **End of year 2** |
| **Beginning of year 1** | **Beginning of year 2** | **Beginning of year 3** |


To reiterate, the words "Year 0," "Today," and "Beginning of year 1" are synonyms. For example, "$100 at the beginning of year 2" is the same as "$100 at the end of year 1." If you're at loss to understand what someone means, ask for a drawing; better yet, ask for an Excel spreadsheet.

## Accumulation—Savings Plans and Future Value

In the previous example, you deposited $100 and left it in your bank. Suppose you intend to make 10 annual deposits of $100, with the first deposit made in year 0 (today) and each succeeding deposit made at the end of years 1, 2, . . . , 9. The *future value* of all these deposits at the end of year 10 tells you how much you will have accumulated in the account. If you are saving for the future (whether to buy a car at the end of your college years or to finance a pension at the end of your working life), this is obviously an important and interesting calculation.

So how much will you have accumulated at the end of year 10? There's an Excel function for calculating this answer which we will discuss later; for the moment, we will set this problem up in Excel and do our calculation the long way, by showing how much we will have at the end of each year:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **FUTURE VALUE WITH ANNUAL DEPOSITS At beginning of year** | | | | | |
| 2 | Interest | 6% | | | | |
| 3 | =E5 | | | | | =(C6+B6)*$B$2 |
| 4 | **Year** | **Account balance, beg. year** | **Deposit at beginning of year** | **Interest earned during year** | **Total in account at end of year** | |
| 5 | 1 | 0.00 | 100.00 | 6.00 | 106.00 | <-- =B5+C5+D5 |
| 6 | 2 | 106.00 | 100.00 | 12.36 | 218.36 | <-- =B6+C6+D6 |
| 7 | 3 | 218.36 | 100.00 | 19.10 | 337.46 | |
| 8 | 4 | 337.46 | 100.00 | 26.25 | 463.71 | |
| 9 | 5 | 463.71 | 100.00 | 33.82 | 597.53 | |
| 10 | 6 | 597.53 | 100.00 | 41.85 | 739.38 | |
| 11 | 7 | 739.38 | 100.00 | 50.36 | 889.75 | |
| 12 | 8 | 889.75 | 100.00 | 59.38 | 1,049.13 | |
| 13 | 9 | 1,049.13 | 100.00 | 68.95 | 1,218.08 | |
| 14 | 10 | 1,218.08 | 100.00 | 79.08 | 1,397.16 | |
| 15 | | | | | | |
| 16 | | Future value using Excel's **FV** function | $1,397.16 | <-- =FV(B2,A14,–100,,1) | | |

For clarity, let's analyze a specific year: At the end of year 1 (cell E5), you've got $106 in the account. This is also the amount in the account at the beginning of year 2 (cell B6). If you now deposit another $100 and let the whole amount of $206 draw interest during the year, it will earn $12.36 interest. You will have $218.36 = (106+100)*1.06 at the end of year 2.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 6 | 2 | 106.00 | 100.00 | 12.36 | 218.36 |

Finally, look at rows 13 and 14: At the end of year 9 (cell E13), you have $1,218.08 in the account; this is also the amount in the account at the beginning of year 10 (cell B14). You then deposited $100, and the resulting $1,318.08 earns $79.08 interest during the year, accumulating to $1,397.16 by the end of year 10.

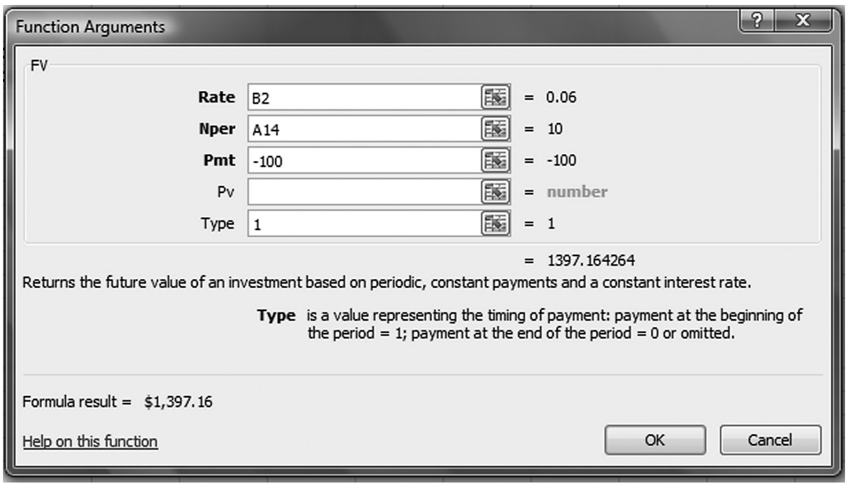
| | A | B | C | D | E |
|---|---|---|---|---|---|
| 13 | 9 | 1,049.13 | 100.00 | 68.95 | 1,218.08 |
| 14 | 10 | 1,218.08 | 100.00 | 79.08 | 1,397.16 |

## The Excel FV (Future Value) Function

The spreadsheet of the previous subsection illustrates in a step-by-step manner how money accumulates in a typical savings plan. To simplify this series of calculations, Excel has an **FV** function which computes the future value of any series of constant payments. This function is illustrated in cell C16:

| | B | C | D | E |
|---|---|---|---|---|
| 16 | Future value using Excel's **FV** function | $1,397.16 | <-- =FV(B2,A14,–100,,1) | |

The **FV** function and the inputs required can be computed using a dialog box—an important feature that comes with each Excel function. The dialog box for cell C16 is given below.



The **FV** function requires as inputs the **Rate** of interest, the number of periods **Nper**, and the annual payment **Pmt**. You can also indicate the **Type**, which tells Excel whether payments are made at the beginning of the period (type **1** as in our example) or at the end of the period (type **0**).

## EXCEL NOTE

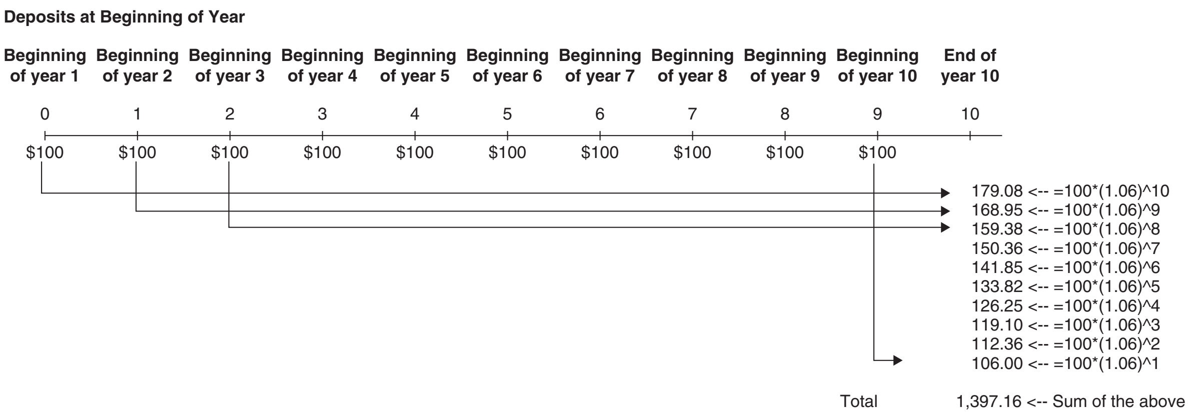
### Negative Values for PMT in the FV Function

Excel's **FV** function has a peculiarity: If we indicate positive payments for **PMT**, the **FV** function gives a negative future value. We prefer our **FV** computation to be positive, so we put in the **PMT** value as a negative number.

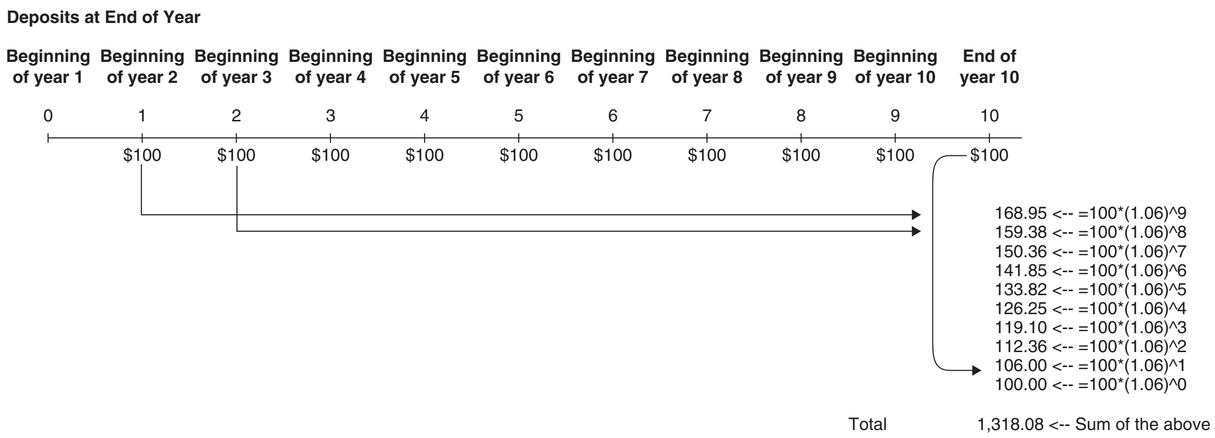
This peculiarity of the **FV** function is shared by a number of other Excel functions that will be discussed later, including **PV**, **PMT**, **IPMT**, and **PPMT**.

## Beginning Versus End of Period

In the example above, you make deposits of $100 at the *beginning* of each year. In terms of timing, your deposits are made at dates 0, 1, 2, 3, ..., 9. Here's a schematic way of looking at this, showing the future value of each deposit at the end of year 10:




Suppose you made 10 deposits of $100 at the *end of each year*. How would this affect the accumulation in the account at the end of 10 years? The schematic diagram below illustrates the timing and accumulation of the payments:

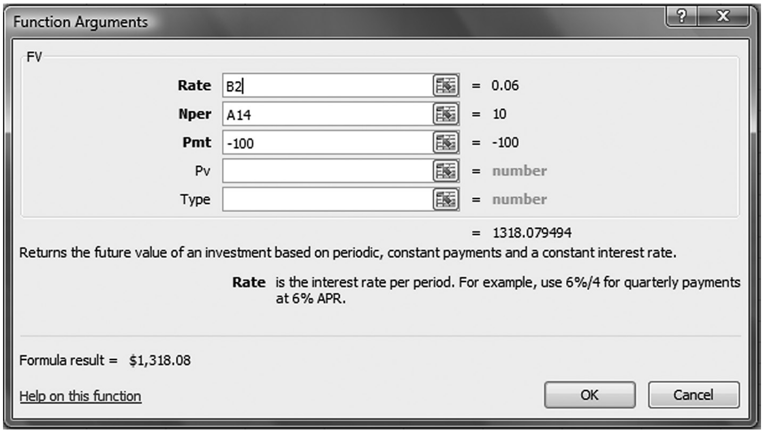



The account accumulation is less when you deposit at the end of each year than in the previous case, where you deposit at the beginning of the year. When you deposit at the end of each year, each deposit is in the account 1 year less and consequently earns 1 year's less interest. In a spreadsheet, this looks like:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **FUTURE VALUE WITH ANNUAL DEPOSITS At end of year** | | | | | |
| 2 | Interest | 6% | | | | |
| 3 | =E5 | | | | | =$B$2*B6 |
| 4 | **Year** | **Account balance, beg. year** | **Deposit at end of year** | **Interest earned during year** | **Total in account at end of year** | |
| 5 | 1 | 0.00 | 100.00 | 0.00 | 100.00 | <-- =B5+C5+D5 |
| 6 | 2 | 100.00 | 100.00 | 6.00 | 206.00 | <-- =B6+C6+D6 |
| 7 | 3 | 206.00 | 100.00 | 12.36 | 318.36 | |
| 8 | 4 | 318.36 | 100.00 | 19.10 | 437.46 | |
| 9 | 5 | 437.46 | 100.00 | 26.25 | 563.71 | |
| 10 | 6 | 563.71 | 100.00 | 33.82 | 697.53 | |
| 11 | 7 | 697.53 | 100.00 | 41.85 | 839.38 | |
| 12 | 8 | 839.38 | 100.00 | 50.36 | 989.75 | |
| 13 | 9 | 989.75 | 100.00 | 59.38 | 1,149.13 | |
| 14 | 10 | 1,149.13 | 100.00 | 68.95 | 1,318.08 | |
| 15 | | | | | | |
| 16 | | Future value | $1,318.08 | <-- =FV(B2,A14,–100) | | |

Cell C16 illustrates the use of the Excel **FV** formula to solve this problem. Here's the dialog box for the **FV** function in cell C16:

## Dialog Box for FV with End-Period Payments




In the example above we've omitted any entry in the **Type** box. We could have also put a 0 in the **Type** box and gotten the same result.

## Some Finance Jargon and the Excel FV Function

An *annuity* is a series of *equal, periodic* payments made over a specified amount of time. Examples of annuities are widespread:

- The allowance your parents give you ($1,000 per month, for your next 4 years of college) is a monthly annuity with 48 payments.
- Pension plans often give the retiree a fixed annual payment for as long as he lives. This is a bit more complicated annuity, since the number of payments is uncertain.
- Certain kinds of loans are paid off in fixed periodic (usually monthly, sometimes annual) installments. Mortgages and student loans are two examples.[1]

An annuity with payments at the end of each period is often called a *regular annuity*. As you've seen in this section, the future value of a regular annuity is calculated with **=FV(B2,A14,-100)**. An annuity with payments at the beginning of each period is often called an *annuity due* and its value is calculated with the Excel function **=FV(B2,A14,-100,,1)**.

[1] Loans are discussed in Chapter 4.

# EXCEL NOTE

## Functions and Dialog Boxes

Cell C16 of the previous example contains the function **FV(B2,A14,-100,,1)**. In this note, we illustrate the use of the dialog box for **FV** to generate this function.

The last part of this Excel note discusses why the payment of $100 is entered into this function as a negative number. This is a peculiarity of the **FV** function shared by many other Excel financial functions.

### Going Through the Function Wizard

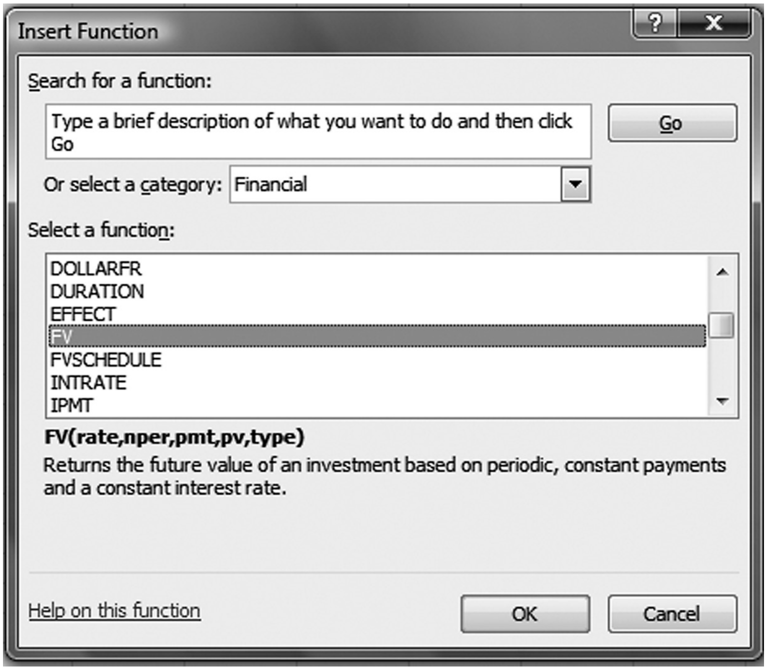
Suppose you're in cell C16 and you want to put the Excel function for future value in the cell. With the cursor in C16, you move your mouse to the *fx* icon on the tool bar:

Clipboard | Font | Alignment

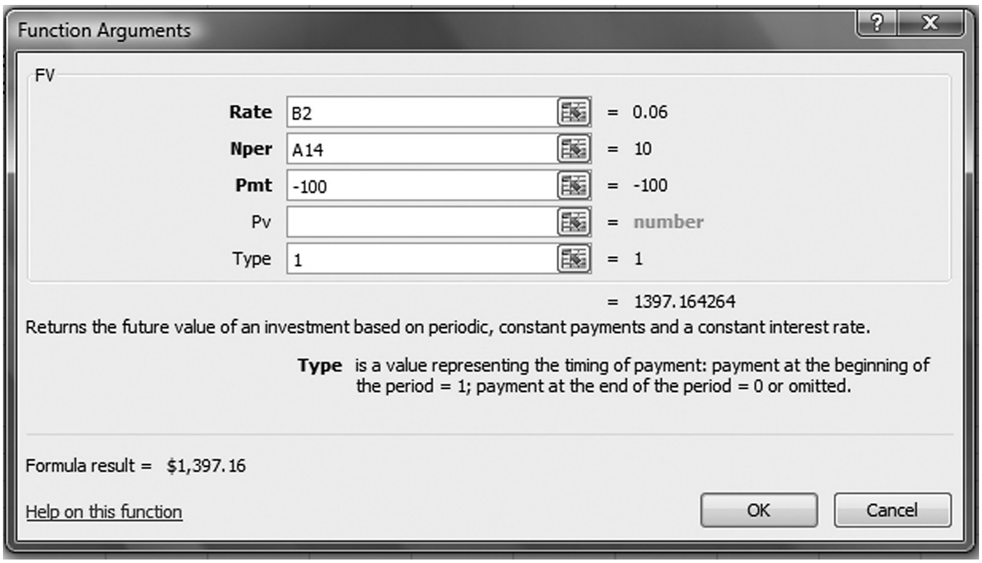
C16 *fx*

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | **FUTURE VA** | Insert Function<br>**at beginning** | **ANNUAL**<br>**of year** | **DEPOSITS** | |
| 2 | Interest | 6% | | | | |
| 3 | =E5 | | | | | = (C6+B6)*$B$2 |
| 4 | **Year** | **Account balance beg. year** | **Deposit at beginning of year** | **Interest earned during year** | **Total in account at end of year** | |
| 5 | 1 | 0.00 | 100.00 | 6.00 | 106.00 | <-- = B5+C5+D5 |
| 6 | 2 | 106.00 | 100.00 | 12.36 | 218.36 | <-- = B6+C6+D6 |
| 7 | 3 | 218.36 | 100.00 | 19.10 | 337.46 | |
| 8 | 4 | 337.46 | 100.00 | 26.25 | 463.71 | |
| 9 | 5 | 463.71 | 100.00 | 33.82 | 597.53 | |
| 10 | 6 | 597.53 | 100.00 | 41.85 | 739.38 | |
| 11 | 7 | 739.38 | 100.00 | 50.36 | 889.75 | |
| 12 | 8 | 889.75 | 100.00 | 59.38 | 1,049.13 | |
| 13 | 9 | 1,049.13 | 100.00 | 68.95 | 1,218.08 | |
| 14 | 10 | 1,218.08 | 100.00 | 79.08 | 1,397.16 | |
| 15 | | | | | | |
| 16 | | Future value using Excel's **FV** function | | | | |

Clicking on the *fx* icon brings up the dialog box below. We've chosen the **category** to be the **Financial** functions, and we've scrolled down in the next section of the dialog box to put the cursor on the **FV** function.




Clicking OK brings up the dialog box for the **FV** function, which can now be filled in as illustrated below:




Excel's function dialog boxes have room for two types of variables.

- **Boldfaced** variables must be filled in; in the **FV** dialog box these are the interest **Rate**, the number of periods **Nper**, and the payment **Pmt**. (Read on to see why we wrote a negative payment.)
- Variables which are not boldfaced are optional. For example, **Type** refers to when the payments are made and so has only two options: **1** when the payments are made at the beginning of the period and **0** when they made at the end of the period. In the example above we've indicated a **1** for the **Type**; this indicates (as shown in the dialog box itself) that the future value is calculated for payments made at the beginning of the period. Had we *omitted this variable* or put in **0**, Excel would compute the future value of a series of payments made at the end of the period; see the subsection of Section 2.1 entitled "Beginning Versus End of Period" for an illustration.

Notice that the dialog box already tells us (even before we click on **OK**) that the future value of $100 per year for 10 years compounded at 6% is $1,397.16.

## A Short Way to Get to the Dialog Box

If you know the name of the function you want, you can just write it in the cell and then click the *fx* icon on the tool bar. As illustrated below, you have to write

**=FV(**

and then click on the *fx* icon—note that we've written an **equal sign**, the **name of the function**, and the **opening parenthesis**.

Here's how the spreadsheet looks in this case:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | **FUTURE VALUE WITH ANNUAL DEPOSITS at beginning of year** | | | |
| 2 | Interest | 6% | | | | |
| 3 | =E5 | | | | | =$B$2*(C6+B6) |
| 4 | **Year** | **Account balance, beg. year** | **Deposit at beginning of year** | **Interest earned during year** | **Total in account at end of year** | |
| 5 | 1 | 0.00 | 100.00 | 6.00 | 106.00 | <-- =B5+C5+D5 |
| 6 | 2 | 106.00 | 100.00 | 12.36 | 218.36 | <-- =B6+C6+D6 |
| 7 | 3 | 218.36 | 100.00 | 19.10 | 337.46 | |
| 8 | 4 | 337.46 | 100.00 | 26.25 | 463.71 | |
| 9 | 5 | 463.71 | 100.00 | 33.82 | 597.53 | |
| 10 | 6 | 597.53 | 100.00 | 41.85 | 739.38 | |
| 11 | 7 | 739.38 | 100.00 | 50.36 | 889.75 | |
| 12 | 8 | 889.75 | 100.00 | 59.38 | 1,049.13 | |
| 13 | 9 | 1,049.13 | 100.00 | 68.95 | 1,218.08 | |
| 14 | 10 | 1,218.08 | 100.00 | 79.08 | 1,397.16 | |
| 15 | | | | | | |
| 16 | | Future value using Excel's **FV** function | =FV( | | | |
| 17 | | | FV(rate, nper, pmt, [pv], [type]) | | | |
| 18 | | | | | | |

Look in the text displayed by Excel below cell C16: As illustrated here, some versions of Excel show the format of the function when you type it in a cell.

### One Further Option

You don't have to use a dialog box! If you know the format of the function, then just type in its variables and you're all set. In the example of Section 2.1, you could just type **=FV(B2,A14,-100,,1)** in the cell. Hitting [Enter] would give the answer.

### Why Is the Pmt Variable a Negative Number?

In the **FV** dialog box, we've entered in the payment **Pmt** in as a negative number, as –100. The **FV** function has the peculiarity (shared by some other Excel financial functions) that a *positive* deposit generates a *negative* answer. We won't go into the (strange?) logic that produced this thinking; whenever we encounter it, we just put in a negative deposit.

## 2.2 Present Value

*The present value is the value today of a payment (or payments) that will be made in the future.*

Here's a simple example: Suppose that you anticipate getting $100 in 3 years from your Uncle Simon, whose word is as good as a bank's. Suppose that the bank pays 6% interest on savings accounts. *How much is the anticipated future payment worth today?* The answer is $\$83.96 = 100/(1.06)^3$; if you put $83.96 in the bank today at 6% annual interest, then in 3 years you would have $100 (see the "proof" in rows 8 and 9 below).[2] $83.96 is also called the *discounted or present value of $100 in 3 years at 6% interest*.

| | A | B | C |
|---|---|---|---|
| 1 | **SIMPLE PRESENT VALUE CALCULATION** | | |
| 2 | X, future payment | 100.00 | |
| 3 | n, time of future payment | 3 | |
| 4 | r, interest rate | 6% | |
| 5 | Present value, $X/(1 + r)^n$ | 83.96 | <-- =B2/(1+B4)^B3 |
| 6 | | | |
| 7 | **Proof** | | |
| 8 | Payment today | 83.96 | <-- =B5 |
| 9 | Future value in n years | 100.00 | <-- =B8*(1+B4)^B3 |

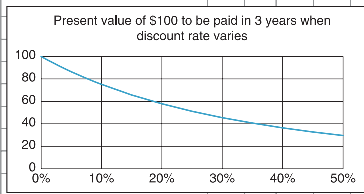
To summarize:

*The present value of $X to be received in n years when the appropriate interest rate is r% is* $\dfrac{X}{(1+r)^n}$.

[2] Actually, $\dfrac{100}{(1.06)^3} = 83.96193$, but we've used **Format|Cells|Number** to show only two decimals.

The interest rate $r$ is also called the *discount rate*. In the rest of this chapter, we will use interest rate and discount rate interchangeably. As you can see, higher discount rates make for lower present values:

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **THE PRESENT VALUE OF \$100 IN 3 YEARS**<br>**In this example, we vary the discount rate *r*** | | | | | | | |
| 2 | X, future payment | 100 | | | | | | |
| 3 | n, time of future payment | 3 | | | | | | |
| 4 | r, interest rate (discount rate) | 6% | | | | | | |
| 5 | Present value, $X/(1+r)^n$ | 83.96 | <-- =B2/(1+B4)^3 | | | | | |
| 6 | | | | | | | | |
| 7 | Table: How does the discount rate affect the present value? | | | | | | | |
| 8 | **Discount rate** | **Present value** | | | | | | |
| 9 | 0% | 100.00 | <-- =\$B\$2/(1+A9)^\$B\$3 | | | | | |
| 10 | 1% | 97.06 | <-- =\$B\$2/(1+A10)^\$B\$3 | | | | | |
| 11 | 2% | 94.23 | <-- =\$B\$2/(1+A11)^\$B\$3 | | | | | |
| 12 | 3% | 91.51 | | | | | | |
| 13 | 4% | 88.90 | | | | | | |
| 14 | 5% | 86.38 | | | | | | |
| 15 | 6% | 83.96 | | | | | | |
| 16 | 7% | 81.63 | | | | | | |
| 17 | 8% | 79.38 | | | | | | |
| 18 | 9% | 77.22 | | | | | | |
| 19 | 10% | 75.13 | | | | | | |
| 20 | 15% | 65.75 | | | | | | |
| 21 | 20% | 57.87 | | | | | | |
| 22 | 25% | 51.20 | | | | | | |
| 23 | 30% | 45.52 | | | | | | |
| 24 | 35% | 40.64 | | | | | | |
| 25 | 40% | 36.44 | | | | | | |
| 26 | 45% | 32.80 | | | | | | |
| 27 | 50% | 29.63 | | | | | | |



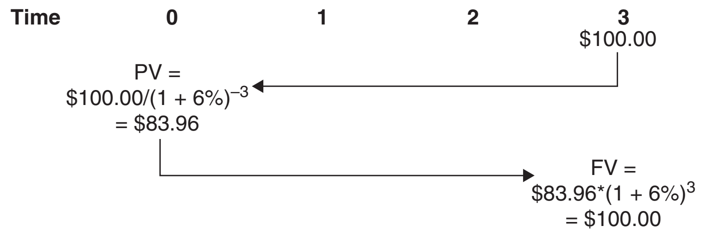

## Why Does PV Decrease as the Discount Rate Increases?

The Excel table above shows that the \$100 Uncle Simon promises you in 3 years is worth \$83.96 today if the discount rate is 6% but worth only \$40.64 if the discount rate is 35%. The mechanical reason for this is that taking the present value at 6% means dividing by a smaller denominator than taking the present value at 35%:

$$83.96 = \frac{100}{(1.06)^3} > \frac{100}{(1.35)^3} = 40.64$$

The economic reason relates to future values: If the bank is paying you 6% interest on your savings account, you would have to deposit \$83.96 today in order to have \$100 in 3 years. If the bank pays 35% interest, then \$40.64 today will grow to \$100 in 3 years, since $\$40.64 * (1.35)^3 = \$100$.

What this short discussion shows is that the *present value is the inverse of the future value*:




## Present Value of an Annuity

Recall that an *annuity* is a series of equal periodic payments. The *present value* of an annuity tells you *the value today* of all the future payments on the annuity.

*The present value of an annuity of $X to be received at the end of years 1, 2, 3, . . ., N when the appropriate interest rate is r% is*

$$\frac{X}{(1+r)}+\frac{X}{(1+r)^2}+\frac{X}{(1+r)^3}+\cdots+\frac{X}{(1+r)^N}$$

Here's an example: Suppose you've been promised $100 at the end of each of the next 5 years. Assuming that you can get 6% at the bank, this promise is worth $421.24 today:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **PRESENT VALUE OF AN ANNUITY WITH FIVE ANNUAL PAYMENTS OF $100 EACH** | | | |
| 2 | Annual payment | 100 | | |
| 3 | r, interest rate (discount rate) | 6% | | |
| 4 | | | | |
| 5 | **Year** | **Payment at end of year** | **Present value of payment** | |
| 6 | 1 | 100 | 94.34 | <-- =B6/(1+$B$3)^A6 |
| 7 | 2 | 100 | 89.00 | <-- =B7/(1+$B$3)^A7 |
| 8 | 3 | 100 | 83.96 | |
| 9 | 4 | 100 | 79.21 | |
| 10 | 5 | 100 | 74.73 | |
| 11 | | | | |
| 12 | | | | |
| 13 | **Present value of all payments** | | | |
| 14 | Summing the present values | | 421.24 | <-- =SUM(C6:C10) |
| 15 | Using Excel's **PV** function | | 421.24 | <-- =PV(B3,5,–B2) |
| 16 | Using Excel's **NPV** function | | 421.24 | <-- =NPV(B3,B6:B10) |

The example above shows three ways of getting the present value of $421.24:

- You can sum the individual discounted values. This is done in cell C13.
- You can use Excel's **PV** function, which calculates the present value of an annuity (cell C14).
- You can use Excel's **NPV** function (cell C16). This function calculates the present value of any series of periodic payments (whether they're flat payments, as in an annuity, or non-equal payments).

We devote separate subsections to the **PV** function and to the **NPV** function.

## The Present Value of a Perpetuity

A perpetuity is an annuity that goes on forever. In the appendix to this chapter, we show that:

*The present value of a perpetuity of $X to be received at the end of years 1, 2, 3, . . . when the appropriate interest rate is r% is*

$$\frac{X}{(1+r)}+\frac{X}{(1+r)^2}+\frac{X}{(1+r)^3}+\cdots=\frac{X}{r}$$

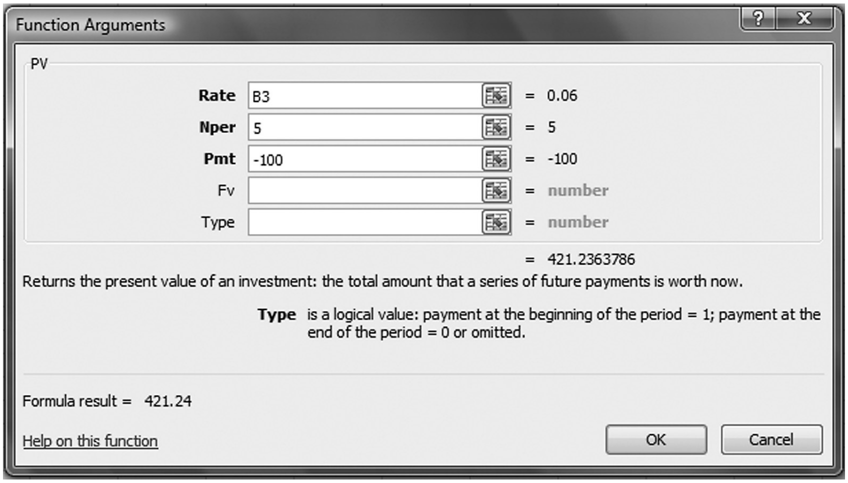
Suppose, for example, that you were offered an infinite stream of payment of $100 per year at the end of years 1, 2, 3, . . . . Suppose that the appropriate interest rate is $r = 5\%$. As the following spreadsheet shows, the present value of this perpetuity is $2,000:

| | A | B | C |
|---|---|---|---|
| 1 | **PRESENT VALUE OF AN ANNUITY** | | |
| 2 | Payment at end of each year | 100 | |
| 3 | Interest rate | 5% | |
| 4 | Perpetuity present value | 2,000 | <-- =B2/B3 |

## The Excel PV Function

The **PV** function calculates the present value of an *annuity* (a series of equal payments). It looks a lot like the **FV** discussed above, and like **FV**, it also has the peculiarity that positive payments give negative results (which is why we set **Pmt** equal to –100). As in the case of the **FV** function, **Type** denotes whether the payments are made at the beginning or the end of the year. Because end-year is the default, you can either enter **0** or leave the **Type** entry blank (if the payment is at the beginning of the period, you have to enter **1** in the **Type** box):

## Dialog Box for the PV Function

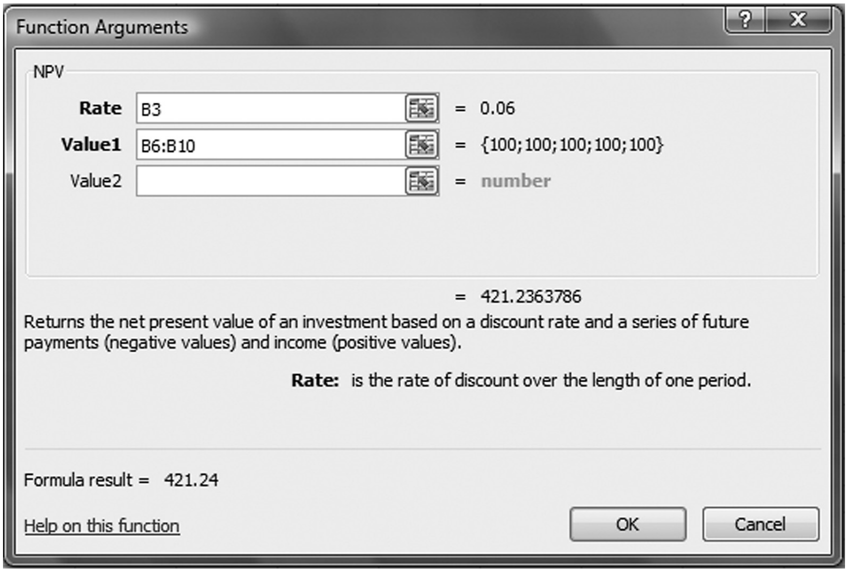



The "Formula result" in the dialog box shows that the answer is $421.24. Clicking **OK** in the dialog box inserts the function into the spreadsheet.

### The Excel NPV Function

The **NPV** function computes the present value of a series of payments. The payments need not be equal, though in the current example they are. The ability of the **NPV** function to handle non-equal payments makes it one of the most useful of all Excel's financial functions. We will make extensive use of this function throughout this book. In the current example, since the annual payments are equal, the result is the same ($421.24) whether we use the **PV** function or the **NPV** function.

## Dialog Box for the NPV Function




Excel's **NPV** function computes the present value of a series of payments. You can either enter the payments separately (as **Value1**, **Value2**, . . .), or—as illustrated above—you can enter a range of payments into the **Value1** box. Most of the time, you will prefer to put the range of cells to be discounted into **Value1** and not enter each value separately.

### Notation Note

Finance professionals use "NPV" to mean "net present value," a concept we explain in the next chapter. Excel's **NPV** function actually calculates the *present value* of a series of payments. Almost all finance professionals and textbooks would call the number computed by the Excel **NPV** function "PV." Thus the Excel use of "NPV" differs from the standard usage in finance which is explained in this chapter and the following one.

## Choosing a Discount Rate

We postpone a full discussion of this topic until Chapter 12. In this note, we'll just try to set out the most important dimensions of choosing a discount rate.

We've defined the present value of $X to be received in $n$ years as $\frac{X}{(1+r)^n}$. The interest rate $r$ in the denominator of this expression is also known as the *discount rate*. Why is 6% an appropriate discount rate for the money promised you by Uncle Simon? The basic principles are as follows:

1. Choose a discount rate that is appropriate to the *riskiness* and the duration of the cash flows being discounted. Uncle Simon's promise of $100 per year for 5 years is assumed to be as good as the promise of your local bank, which pays 6% on its savings accounts. Therefore 6% is an appropriate discount rate. A good rule of thumb is that the larger the riskiness of the cash flows, the larger the discount rate. Precise formulations of this rule of thumb will have to wait for Part 2 of this book, in particular Chapter 12.
2. $r$ and $n$ must be presented in the same "time frame." Make sure that if $n$ represents years, then $r$ represents the *annual discount* rate. Similarly, if $n$ represents months, then $r$ should be a *monthly discount* rate.

## The Present Value of Non-Annuity (Meaning: Non-Constant) Cash Flows

The present value concept can also be applied to non-annuity cash flow streams, meaning cash flows that are not the same every period. Suppose, for example, that your Aunt Terry has promised to pay you $100 at the end of year 1, $200 at the end of year 2, $300 at the end of year 3, $400 at the end of year 4, and $500 at the end of year 5. This is not an annuity, and so it cannot be accommodated by the **PV** function. But we can find the present value of this promise by using the **NPV** function:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **CALCULATING PRESENT VALUES WITH EXCEL** | | | |
| 2 | r, discount rate | 6% | | |
| 3 | | | | |
| 4 | **Year** | **Payment at end of year** | **Present value** | |
| 5 | 1 | 100 | 94.34 | <-- =B5/(1+$B$2)^A5 |
| 6 | 2 | 200 | 178.00 | <-- =B6/(1+$B$2)^A6 |
| 7 | 3 | 300 | 251.89 | |
| 8 | 4 | 400 | 316.84 | |
| 9 | 5 | 500 | 373.63 | |
| 10 | | | | |
| 11 | **Present value of all payments** | | | |
| 12 | Summing the present values | | 1,214.69 | <-- =SUM(C5:C9) |
| 13 | Using Excel's **NPV** function | | 1,214.69 | <-- =NPV(B2,B5:B9) |

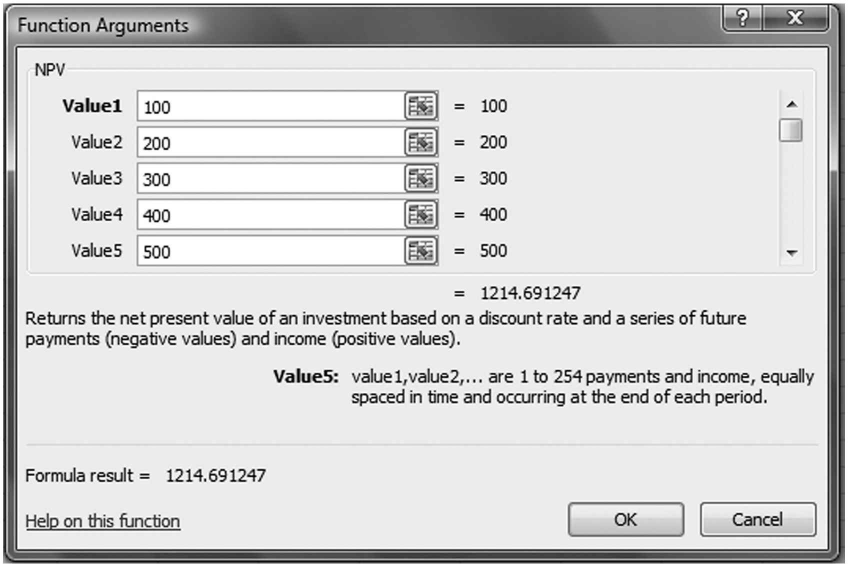
The example shows that the present value of Aunt Terry's promised series of payments over the next 5 years is $1,214.69:

$$PV_0 = \frac{\$100}{(1.06)^1} + \frac{\$200}{(1.06)^2} + \frac{\$300}{(1.06)^3} + \frac{\$400}{(1.06)^4} + \frac{\$500}{(1.06)^5} = \$1,214.69$$

## EXCEL NOTE

### NPV Function

Excel's **NPV** function allows you to input up to 29 payments directly in the function dialog box. Here's an illustration for the example above:

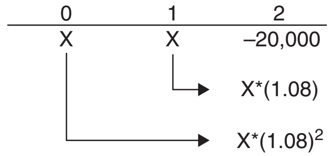



## 2.3 Saving for the Future—Buying a Car for Mario

Mario has his eye on a car that costs $20,000. He wants to buy a car in 2 years. He plans to open a bank account and to deposit $X today and $X in 1 year. Balances in the account will earn 8%. How much does Mario need to deposit so that he has $20,000 in 2 years? In this section, we'll show you that:

> *In order to finance future consumption with a savings plan, the future value of all the cash flows (positive and negative) has to be zero. In the jargon of finance—the future consumption plan is fully funded if the future value of all the cash flows is zero.*

In order to see this, start with a graphical representation of what happens:




In year 2, Mario will have accumulated $X*(1.08)+X*(1.08)^2$. This should finance the $20,000 car, so that

$$\underbrace{X*(1.08)+X*(1.08)^2}_{\substack{\uparrow\\ \text{Future value of}\\ \text{deposits in 2 years}}} = \underbrace{20{,}000}_{\substack{\uparrow\\ \text{Desired accumulation}}}$$

If you were to actually solve this equation, you would find that $X = \$8{,}903.13$. In order to fully fund the future purchase of the car, Mario has to deposit $8,903.13 today and another $8,903.13 one year from now. If he does this, the future value of all the payments is zero:

$$\underbrace{\underbrace{\$8{,}903.13*(1.08)^2}_{\substack{\uparrow\\ \text{The FV of the}\\ \text{deposit made}\\ \text{today}}} + \underbrace{\$8{,}903.13*(1.08)}_{\substack{\uparrow\\ \text{The FV of the}\\ \text{deposit made 1}\\ \text{year from now}}} - \underbrace{20{,}000}_{\substack{\uparrow\\ \text{The FV of the cost}\\ \text{of the car in 2 years}}}}_{\substack{\uparrow\\ \text{The FV of the 2 deposits}\\ \text{and the cost of the car in}\\ \text{year 2}}} = 0$$

## The Excel Solution to Mario's Problem

Of course, this same solution is easily reached using Excel:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | HELPING MARIO SAVE FOR A CAR | | | |
| 2 | Deposit, X | 8,903.13 | | | |
| 3 | Interest rate | 8.00% | | | |
| 4 | Year | In bank, before deposit | Deposit or withdrawal | Total at beginning of year | End of year with interest |
| 5 | 0 | - | 8,903.13 | 8,903.13 | 9,615.38 |
| 6 | 1 | 9,615.38 | 8,903.13 | 18,518.52 | 20,000.00 |
| 7 | 2 | 20,000.00 | (20,000.00) | - | - |
| 8 | | | | | |
| 9 | | **FV** of all deposits and payments | 0.00 | <-- =C5*(1+$B$3)^2+C6*(1+B3)^1+C7 | |

If Mario deposits $8,903.13 in years 0 and 1, then the accumulation in the account at the beginning of year 2 will be exactly $20,000 (cell B7). The FV of all the payments (cell C9) is zero.

In the next section, we discuss three methods for solving Mario's savings problem.

## 2.4 Solving Mario's Savings Problem—Three Solutions

We can solve Mario's savings problem by using one of three methods: trial and error, Excel's **Goal Seek**, and Excel's **PMT** function. Each of these three methods is illustrated in this section.

### Method 1: Trial and Error

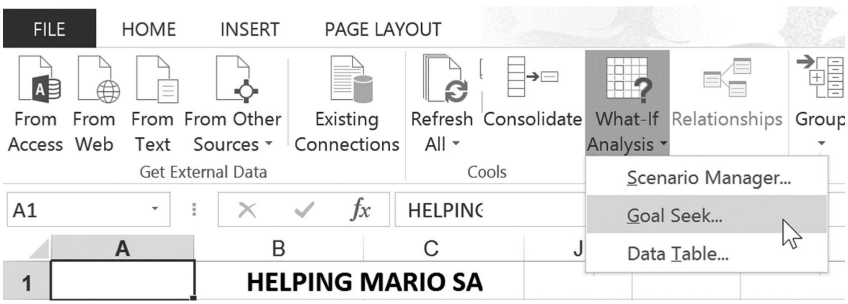
You can "play" with the spreadsheet, adjusting cell B2 until cell C9 equals zero. For example, if you put $5,000 into cell B2, you see that the Future value in cell C9 is negative, indicating that Mario is saving too little:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **HELPING MARIO SAVE FOR A CAR** | | | | |
| 2 | Deposit, X | 5,000.00 | | | |
| 3 | Interest rate | 8.00% | | | |
| 4 | **Year** | **In bank, before deposit** | **Deposit or withdrawal** | **Total at beginning of year** | **End of year with interest** |
| 5 | 0 | - | 5,000.00 | 5,000.00 | 5,400.00 |
| 6 | 1 | 5,400.00 | 5,000.00 | 10,400.00 | 11,232.00 |
| 7 | 2 | 11,232.00 | (20,000.00) | (8,768.00) | (9,469.44) |
| 8 | | | | | |
| 9 | | **FV** of all deposits and payments | –8,768.00 | <-- =C5*(1+$B$3)^2+C6*(1+B3)^1+C7 | |

If you put 10,000 into cell B2, cell C9 will be positive (2,464); this indicates that the answer is somewhere between 5,000 and 10,000. By trial and error, you can reach the correct solution.

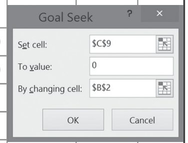
### Method 2: Using Excel's Goal Seek

**Goal Seek** is an Excel function that looks for a specific number in one cell by adjusting the value of a different cell (for a discussion of how to use **Goal Seek**, see Chapter 25). To solve Mario's problem, we can use **Goal Seek** to set cell C9 equal to 0. The Excel menu selection is **Data|Data tools|What-if analysis|Goal Seek**:



Having chosen **Goal Seek**, we fill in the dialog box as shown below:

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | HELPING MARIO SAVE FOR A CAR | | | | | | | |
| 2 | Deposit, X | 5,000.00 | | | | | | |
| 3 | Interest rate | 8.00% | | | | | | |
| 4 | Year | In bank, before deposit | Deposit or withdrawal | Total at beginning of year | End of year with interest | | | |
| 5 | 0 | - | 5,000.00 | 5,000.00 | 5,400.00 | | | |
| 6 | 1 | 5,400.00 | 5,000.00 | 10,400.00 | 11,232.00 | | | |
| 7 | 2 | 11,232.00 | (20,000.00) | (8,768.00) | (9,469.44) | | | |
| 8 | | | | | | | | |
| 9 | | **FV** of all deposits and payments | –8,768.00 | <-- =C5*(1+$B$3)^2+C6*(1+B3)^1+C7 | | | | |
| 10 | | | | | | | | |



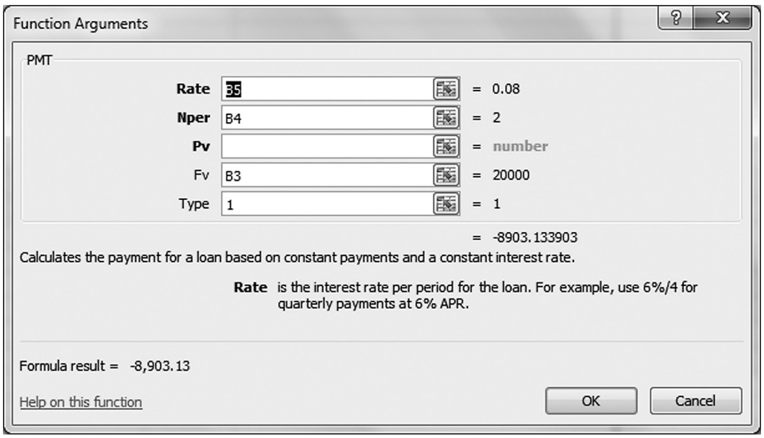

When we hit **OK**, **Goal Seek** will find the solution of 8,903.13.

## Method 3: Using the Excel PMT Function[3]

Excel's PMT function can solve Mario's problem directly, as illustrated by the following spreadsheet:

| | A | B | C |
|---|---|---|---|
| 1 | HELPING MARIO SAVE FOR A CAR | | |
| 2 | Using Excel PMT function | | |
| 3 | Goal | 20,000.00 | <-- The cost of the car |
| 4 | When to reach the goal? | 2.00 | <--The year in which Mario wants to buy the car |
| 5 | Interest rate | 8% | |
| 6 | Deposit, X | –8,903.13 | <-- =PMT(B5,B4,,B3,1) |

The dialog box for this function is given below:




[3]In the following examples in this chapter and in Chapter 3 we discuss other uses of the PMT function.

## 2.5 Saving for the Future—More Complicated Problems

In this section, we present two more complicated versions of Mario's problem from Section 2.4. We start by trying to determine whether a young girl's parents are putting enough money aside to save for her college education. Here's the problem:

- On her 10th birthday Linda Jones's parents decide to deposit $4,000 in a savings account for their daughter. They intend to put an additional $4,000 in the account each year on her 11th, 12th, ..., 17th birthdays.
- All account balances will earn 8% per year.
- On Linda's 18th, 19th, 20th, and 21st birthdays, her parents will withdraw $20,000 to pay for Linda's college education.

Is the $4,000 per year sufficient to cover the anticipated college expenses? We can easily solve this problem in a spreadsheet:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | **SAVING FOR COLLEGE** | | | |
| 2 | Interest rate | 8% | | | | |
| 3 | Annual deposit | 4,000.00 | | | | |
| 4 | Annual cost of collage | 20,000.00 | | | | |
| 5 | | | | | | |
| 6 | **Birthday** | **In bank on birthday, before deposit/withdrawal** | **Deposit or withdrawal at beginning of year** | **Total at beginning of year** | **Total at end of year with interest** | |
| 7 | 10 | 0.00 | 4,000.00 | 4,000.00 | 4,320.00 | <-- =D7*(1+$B$2) |
| 8 | 11 | 4,320.00 | 4,000.00 | 8,320.00 | 8,985.60 | <-- =D8*(1+$B$2) |
| 9 | 12 | 8,985.60 | 4,000.00 | 12,985.60 | 14,024.45 | <-- =D9*(1+$B$2) |
| 10 | 13 | 14,024.45 | 4,000.00 | 18,024.45 | 19,466.40 | <-- =D10*(1+$B$2) |
| 11 | 14 | 19,466.40 | 4,000.00 | 23,466.40 | 25,343.72 | <-- =D11*(1+$B$2) |
| 12 | 15 | 25,343.72 | 4,000.00 | 29,343.72 | 31,691.21 | <-- =D12*(1+$B$2) |
| 13 | 16 | 31,691.21 | 4,000.00 | 35,691.21 | 38,546.51 | <-- =D13*(1+$B$2) |
| 14 | 17 | 38,546.51 | 4,000.00 | 42,546.51 | 45,950.23 | <-- =D14*(1+$B$2) |
| 15 | 18 | 45,950.23 | –20,000.00 | 25,950.23 | 28,026.25 | <-- =D15*(1+$B$2) |
| 16 | 19 | 28,026.25 | –20,000.00 | 8,026.25 | 8,668.35 | <-- =D16*(1+$B$2) |
| 17 | 20 | 8,668.35 | –20,000.00 | –11,331.65 | –12,238.18 | <-- =D17*(1+$B$2) |
| 18 | 21 | –12,238.18 | –20,000.00 | –32,238.18 | –34,817.24 | <-- =D18*(1+$B$2) |
| 19 | | | | | | |
| 20 | | | **PV** of all payments | –13,826.40 | <-- =NPV(B2,C8:C18)+C7 | |

By looking at the end-year balances in column E, the $4,000 is *not* enough—Linda and her parents will run out of money somewhere between her 19th and 20th birthdays.[4] By the end of her college career, they will be $34,817 "in the

[4]At the end of Linda's 19th year (row 16), there is $8,668.35 remaining in the account. At the end of the following year, there is a negative amount in the account.

hole" (cell E18). Another way to see this is to look at the present value calculation in cell C20: As we saw in the previous section, a combination savings/withdrawal plan is fully funded when the PV of all the payments/withdrawals is zero. In cell C20, we see that the PV is negative—Linda's plan is *underfunded*.

How much should Linda's parents put aside each year? There are several ways to answer this question, which we explore below. These methods are basically the same as the three methods for solving Mario's problem presented in the previous section, but for completeness we present them again.

## Method 1: Trial and Error

Assuming that you have put the correct formulas in the spreadsheet, you can "play" with cell B3 until cell E18 or cell D20 equals zero. Doing this shows that Linda's parents should have planned to deposit $6,227.78 annually:

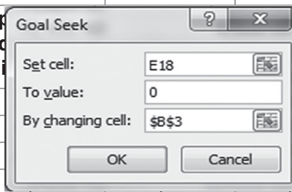
| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | SAVING FOR COLLEGE | | | |
| 2 | Interest rate | 8% | | | | |
| 3 | Annual deposit | 6,227.78 | | | | |
| 4 | Annual cost of collage | 20,000.00 | | | | |
| 5 | | | | | | |
| 6 | Birthday | In bank on birthday, before deposit/withdrawal | Deposit or withdrawal at beginning of year | Total at beginning of year | Total at end of year with interest | |
| 7 | 10 | 0.00 | 6,227.78 | 6,227.78 | 6,726.00 | <-- =D7*(1+$B$2) |
| 8 | 11 | 6,726.00 | 6,227.78 | 12,953.77 | 13,990.08 | <-- =D8*(1+$B$2) |
| 9 | 12 | 13,990.08 | 6,227.78 | 20,217.85 | 21,835.28 | <-- =D9*(1+$B$2) |
| 10 | 13 | 21,835.28 | 6,227.78 | 28,063.06 | 30,308.10 | <-- =D10*(1+$B$2) |
| 11 | 14 | 30,308.10 | 6,227.78 | 36,535.88 | 39,458.75 | <-- =D11*(1+$B$2) |
| 12 | 15 | 39,458.75 | 6,227.78 | 45,686.52 | 49,341.45 | <-- =D12*(1+$B$2) |
| 13 | 16 | 49,341.45 | 6,227.78 | 55,569.22 | 60,014.76 | <-- =D13*(1+$B$2) |
| 14 | 17 | 60,014.76 | 6,227.78 | 66,242.54 | 71,541.94 | <-- =D14*(1+$B$2) |
| 15 | 18 | 71,541.94 | –20,000.00 | 51,541.94 | 55,665.29 | <-- =D15*(1+$B$2) |
| 16 | 19 | 55,665.29 | –20,000.00 | 35,665.29 | 38,518.52 | <-- =D16*(1+$B$2) |
| 17 | 20 | 38,518.52 | –20,000.00 | 18,518.52 | 20,000.00 | <-- =D17*(1+$B$2) |
| 18 | 21 | 20,000.00 | –20,000.00 | 0.00 | 0.00 | <-- =D18*(1+$B$2) |
| 19 | | | | | | |
| 20 | | | PV of all payments | 0.00 | <-- =NPV(B2,C8:C18)+C7 | |

Notice that the present value of all the payments (cell C20) is zero when the solution is reached. The future payouts are fully funded when the NPV of all the cash flows is zero.

## Method 2: Using Excel's Goal Seek

We can use **Goal Seek** to set E20 equal to zero. After hitting **Data|What-if Analysis|Goal Seek**, we fill in the dialog box:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | **SAVING FOR COLLEGE** | | | |
| 2 | Interest rate | 8% | | | | |
| 3 | Annual deposit | 4,000.00 | | | | |
| 4 | Annual cost of collage | 20,000.00 | | | | |
| 5 | | | | | | |
| 6 | **Birthday** | **In bank on birthday, before deposit/withrawal** | **Dep withd beginni** | | **al at f year iterest** | |
| 7 | 10 | 0.00 | | | 320.00 | <-- =D7*(1+$B$2) |
| 8 | 11 | 4,320.00 | | | 85.60 | <-- =D8*(1+$B$2) |
| 9 | 12 | 8,985.60 | | | 24.45 | <-- =D9*(1+$B$2) |
| 10 | 13 | 14,024.45 | | | 66.40 | <-- =D10*(1+$B$2) |
| 11 | 14 | 19,466.40 | 4,000.00 | 23,466.40 | 25,343.72 | <-- =D11*(1+$B$2) |
| 12 | 15 | 25,343.72 | 4,000.00 | 29,343.72 | 31,691.21 | <-- =D12*(1+$B$2) |
| 13 | 16 | 31,691.21 | 4,000.00 | 35,691.21 | 38,546.51 | <-- =D13*(1+$B$2) |
| 14 | 17 | 38,546.51 | 4,000.00 | 42,546.51 | 45,950.23 | <-- =D14*(1+$B$2) |
| 15 | 18 | 45,950.23 | –20,000.00 | 25,950.23 | 28,026.25 | <-- =D15*(1+$B$2) |
| 16 | 19 | 28,026.25 | –20,000.00 | 8,026.25 | 8,668.35 | <-- =D16*(1+$B$2) |
| 17 | 20 | 8,668.35 | –20,000.00 | –11,331.65 | –12,238.18 | <-- =D17*(1+$B$2) |
| 18 | 21 | –12,238.18 | –20,000.00 | –32,238.18 | –34,817.24 | <-- =D18*(1+$B$2) |
| 19 | | | | | | |
| 20 | | | **PV** of all payments | –13,826.40 | <-- =NPV(B2,C8:C18)+C7 | |




When we hit "OK," Goal Seek looks for the solution. The result is the same as before: $6,227.78.

## Method 3: Using the Excel PV and PMT Functions

We can use the Excel **PV** and **PMT** functions to solve this problem directly, as illustrated in the following spreadsheet screen:

| | A | B | C |
|---|---|---|---|
| 1 | **SAVING FOR COLLEGE<br>Using PV and PMT functions** | | |
| 2 | Interest rate | 8% | |
| 3 | Linda's age today | 10 | |
| 4 | Age at starting college | 18 | |
| 5 | Years of college | 4 | |
| 6 | | | |
| 7 | Annual cost of college | 20,000 | |
| 8 | | | |
| 9 | **PV** of college at 18 | 71,541.94 | <-- =PV(B2,B5,–B7,,1) |
| 10 | Annual payment | 6,227.78 | <-- =PMT(B2,B4–B3,,–B9,1) |

**Explanation:** Cell B9 is the present value of the college tuitions at the start of the 18th year. The **PMT** function computes the annual payment required so that the future value of the payments (compounded at 8% for 8 years) will be equal to $71,541.94.

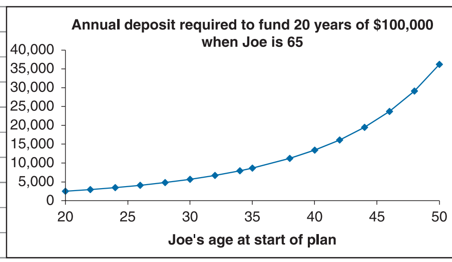
We can, of course, integrate the **PV** function into the **PMT** function, so that the result is even simpler:

| | A | B | C |
|---|---|---|---|
| 1 | **SAVING FOR COLLEGE<br>PV function is inside the PMT function** | | |
| 2 | Interest rate | 8% | |
| 3 | Linda's age today | 10 | |
| 4 | Age at starting college | 18 | |
| 5 | Years of college | 4 | |
| 6 | | | |
| 7 | Annual cost of college | 20,000 | |
| 8 | | | |
| 9 | Annual payment | 6,227.78 | <-- =PMT(B2,B4–B3,,PV(B2,B5,B7,,1),1) |

## Pension Plans

The savings problem of Linda's parents is exactly the same as that faced by an individual who wishes to save for his retirement. Suppose that Joe is 20 today and wishes to start saving so that when he's 65 he can have 20 years of $100,000 annual withdrawals. Adapting the previous spreadsheet, we get:

| | A | B | C |
|---|---|---|---|
| 1 | **SAVING FOR RETIREMENT** | | |
| 2 | Joe's age today | 20 | |
| 3 | Joe's age at last deposit | 64 | |
| 4 | Number of deposits | 45 | <-- =B3–B2+1 |
| 5 | Number of withdrawals | 20 | |
| 6 | Annual withdrawal from age 65 | 100,000 | |
| 7 | Interest rate | 8% | |
| 8 | | | |
| 9 | Annual deposit | 2,540.23 | <-- =PMT(B7,B4,,PV(B7,B5,B6)) |
| 10 | | | |
| 11 | **Joe's age** | **Annual amount deposited** | |
| 12 | 20 | 2,540.23 | <-- =PMT($B$7,$B$3–A12+1,,PV($B$7,$B$5,$B$6)) |
| 13 | 22 | 2,978.96 | <-- =PMT($B$7,$B$3–A13+1,,PV($B$7,$B$5,$B$6)) |
| 14 | 24 | 3,496.73 | <-- =PMT($B$7,$B$3–A14+1,,PV($B$7,$B$5,$B$6)) |
| 15 | 26 | 4,109.02 | |
| 16 | 28 | 4,834.85 | |
| 17 | 30 | 5,697.73 | |
| 18 | 32 | 6,727.03 | |
| 19 | 34 | 7,959.85 | |
| 20 | 35 | 8,666.90 | |
| 21 | 38 | 11,239.91 | |
| 22 | 40 | 13,430.03 | |
| 23 | 42 | 16,123.53 | |
| 24 | 44 | 19,471.60 | |
| 25 | 46 | 23,688.86 | |
| 26 | 48 | 29,090.61 | |
| 27 | 50 | 36,159.79 | |
| 28 | | | |



In the table in rows 12–27 you see the power of compound interest: If Joe starts saving at age 20 for his retirement, an annual deposit of $2,540.23 will grow to provide him with his retirement needs of $100,000 per year for 20 years at age 65. On the other hand, if he starts saving at age 35, it will require $8,666.90 per year.

## Summary

In this chapter we have covered the basic concepts of the time value of money:

- Future value (FV): The amount you will accumulate at some future date from deposits made in the present.
- Present value (PV): The value today of future anticipated cash flows.
- The number of periods to pay off the investment (NPER).

We have also shown you the Excel functions **FV**, **PV**, **NPV**, **NPER** which do these calculations and discussed some of their peculiarities. Finally, we have shown you how to do these calculations using formulas.

## Exercises

**Note:** The data for these exercises can be found on the Benninga, *Principles of Finance with Excel, Third Edition* companion website (www.oup.com/us/Benninga).

1. (FV single cash flow) You just put $600 in the bank, and you intend to leave it there for 10 years. If the bank pays you 15% interest per year, how much will you have at the end of 10 years?
2. (FV single cash flow, finding *r*) Five years ago you made a deposit of $50. The value of this deposit today is $70.13. What was the annual return earned on the deposit?
    a. Use the template in this problem to arrive at the answer with trial and error.
    b. In Section 2.1, we show that the accumulation grows by a factor of $(1+r)^n$. Solve the problem again by solving $70.13 = 50*(1+r)^5$.
3. (Savings plans) You plan to open a savings account by depositing $1,000 in the bank today. You also plan to deposit $1,000 in the bank in 1 year, 2 years, . . . , 9 years. If the bank pays interest of 3% per year, how much will you have in the bank 10 years from now?
4. (FV annuity) Your parents have just opened a savings account for you. They plan to make monthly deposits of $200 for the next 10 years (120 deposits) where the first deposit starts today. Assume that the account earns 0.5% interest per month. What will be the value of your saving account after 10 years?

5. (PV single cash flow) Your friend comes to you with a $2,000 post-dated check. The check is due 2 years from today. If the interest rate is 5%, what is the value of the check today?

6. (PV single cash flow, finding *r*) If you deposit $25,000 today, Union Bank offers to pay you $50,000 at the end of 10 years. What is the annual interest rate offered by the bank?

7. (PV annuity) Your uncle has just announced that he's going to give you $10,000 per year at the end of each of the next 4 years. If the relevant interest rate is 7%, what's the value today of this promise?

8. (PV growing annuity) "Starving artists" is an organization that supports painters by giving them a guaranteed annual income. Majd has been promised an annual payment from Starving Artists. The 10 payments start today ($10,000) and increase by 2% per year. Assume that the interest rate is 14%. What is the value today of the Starving Artist grant?

9. (Annuity, calculating *r*) Screw-'Em-Good Corp. (SEG) has just announced a revolutionary security: If you pay SEG $1,000 now, you will get back $150 at the end of each of the next 15 years. What is the internal rate of return (IRR) of this investment?

10. (FV single cash flow, two interest rates) You have just received a $15,000 signing bonus from your new employer and decide to invest it for 2 years. Your banker suggests two alternatives, both of which require a commitment for the full 2 years. The first alternative will earn 8% per year for both years. The second alternative earns 6% for the first year and 10% for the second year. Interest compounds annually. Which alternative should you choose?

11. (FV of annuity) In the spreadsheet, below we calculate the future value of 5 deposits of $100, with the first deposit made at time 0. As shown in Section 2.1, this calculation can also be made using the Excel function **=FV(interest,periods,-amount,,1)**.

    a. Show that you can also compute this by **=FV(interest,periods, -amount)*(1+interest)**.

    b. Can you explain why **FV(r,5,-100,,1)=FV(r,5,-100)*(1+r)**?

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | | **FUTURE VALUE** | | |
| 2 | Interest | 6% | | | | |
| 3 | | | | | | |
| 4 | **Year** | **Account balance, beginning of year** | **Deposit at beginning of year** | **Interest earned during year** | **Total in account at end of year** | |
| 5 | 1 | 0.00 | 100.00 | 6.00 | 106.00 | <-- =B5+C5+D5 |
| 6 | 2 | 106.00 | 100.00 | 12.36 | 218.36 | <-- =B6+C6+D6 |
| 7 | 3 | 218.36 | 100.00 | 19.10 | 337.46 | |
| 8 | 4 | 337.46 | 100.00 | 26.25 | 463.71 | |
| 9 | 5 | 463.71 | 100.00 | 33.82 | 597.53 | |

**12.** (FV annuity) Abner and Maude are both in their eighties. They're thinking of selling their house for $500,000 and moving into an apartment complex for seniors. The senior complex will cost $50,000 per year, payable in full at the beginning of each year. This payment covers all of Abner and Maude's costs (food, rental, entertainment, and medical).

- **a.** If they can earn 6% annually on the proceeds from their house and if they live for 10 more years, how much will they be able to leave to their children as an inheritance?
- **b.** What is the longest they can live from the apartment proceeds before the money runs out?
- **c.** Find the answers to parts a and b above assuming that the interest rate is 7%.

**13.** (FV annuity) Michael is considering his consumption habits, trying to figure out how to save money. He realizes that he could save $10 every week by ordering regular coffee instead of latte at the local coffee shop. Since he buys a cup of coffee every work day (5 days a week), every week (52 weeks a year), this works out to be quite a sum.

- **a.** If Michael is 25 today and retires at age 65, how much money will he have accumulated from savings on coffee versus latte? Assume that the weekly interest rate is 0.1% and that the savings occur at the end of each week.
- **b.** Michael was astounded at the answer to part a of this problem. He realized he had more wasteful habits, and he made a list of possible savings to see how much richer he could be at age 65. What are the rewards to Michael's frugality?

| | A | B | C | D |
|---|---|---|---|---|
| 10 | **Exercise 13 b** | | | =FV($B$11,52,–B15) |
| 11 | Weekly interest rate | 0.10% | | |
| 12 | | | | |
| 13 | **Item** | **Weekly savings** | **Yearly savings** | **Future value at age 65** |
| 14 | | | | |
| 15 | Latte versus regular coffee | 10.00 | 533.48 | |
| 16 | Deli versus brown bag lunch | 25.00 | | |
| 17 | Excess alcohol | 10.00 | | |
| 18 | Cigarettes | 11.00 | | |
| 19 | Candy | 5.00 | | |
| 20 | Excess junk food | 10.00 | | |
| 21 | Cell phone (chat vs. needed calls) | 6.00 | | |
| 22 | Wasted groceries | 7.00 | | |
| 23 | Restaurant: fast food vs. eat at home | 30.00 | | |
| 24 | Wasted energy: heat, AC, lights | 12.00 | | |
| 25 | Movies versus books | 10.00 | | |
| 26 | Expensive cable TV | 13.00 | | |
| 27 | Wasted gasoline on excessive trips, etc. | 8.00 | | |
| 28 | Wasteful spending at mall | 10.00 | | |
| 29 | **Money saved by time Michael is 65** | **167.00** | | |

**14.** (PV annuity) Your annual salary is \$100,000. You are offered two options for a severance package. Option 1 pays you 6 months' salary now. Option 2 pays you and your heirs \$6,000 per year forever (first payment at the end of the current year). If your required return is 11%, which option should you choose?

**15.** (PV of annuity, finding *r*) You have just invested \$10,000 in a new fund that pays \$1,500 at the end of each of the next 10 years. What is the compound rate of interest being offered in the fund?

**16.** (PV single cash flow + annuities) Assuming that the interest rate is 5%, which of the following is more valuable?

- **a.** \$5,000 today
- **b.** \$10,000 at the end of 5 years
- **c.** \$9,000 at the end of 4 years
- **d.** \$300 a year in perpetuity (meaning: forever), with the first payment at the end of this year

**17.** (PV perpetuity, finding *r*) A fund of \$10,000 is set up to pay \$250 at the end of each year indefinitely. What is the fund's rate? (There's no Excel function that answers this question—use some logic!)

**18.** (PV growing cash flows) You are the CFO of Termination, Inc. Your company has 40 employees, each earning \$40,000 per year. Employee salaries grow at 4% per year. Starting from next year, and every second year thereafter, eight employees retire and no new employees are recruited. Your company has in place a retirement plan that entitles retired workers to an annual pension which is equal to their annual salary at the moment of retirement. Life expectancy is 20 years after retirement, and the annual pension is paid at year-end. The return on investment is 10% per year. What is the total value of your pension liabilities?

**19.** (Financial planning, finding pmt) Anuradha Dixit just turned 55. Anuradha is planning to retire in 10 years, and she currently has \$500,000 in her pension fund. Based on the longevity pattern of her family, she assumes that she will live 20 years past her retirement age; during each of these years, she desires to withdraw \$100,000 from her pension fund. If the interest rate is 5% annually, how much will Anuradha have to save annually for the next 10 years? Assume that the first deposit to her pension fund will be today, followed by nine more annual deposits, and that the annual withdrawals from age 65 will occur at the beginning of each year.

Use the following spreadsheet (the numbers are not correct) and **Goal Seek** to find an answer:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **SAVING FOR THE FUTURE** | | | | | |
| 2 | Annual desired pension payout | 100,000 | | | | |
| 3 | Annual payment | 10,000 | <-- start with a number | | | |
| 4 | Interest rate | 5% | | | | |
| 5 | | | | | | |
| 6 | **Your age** | **Account balance at beginning of year** | **Deposit or withdrawal at beginning of year** | **Interest earned during year** | **Total in account end of year** | |
| 7 | 55 | 500,000 | 10,000 | 25,500 | 535,500 | <-- =D7+C7+B7 |
| 8 | 56 | 535,500 | 10,000 | 27,275 | 572,775 | <-- =D8+C8+B8 |
| 9 | 57 | 572,775 | 10,000 | 29,139 | 611,914 | |
| 10 | 58 | 611,914 | 10,000 | 31,096 | 653,009 | |
| 11 | 59 | 653,009 | 10,000 | 33,150 | 696,160 | |
| 12 | 60 | 696,160 | 10,000 | 35,308 | 741,468 | |
| 13 | 61 | 741,468 | 10,000 | 37,573 | 789,041 | |
| 14 | 62 | 789,041 | 10,000 | 39,952 | 838,993 | |
| 15 | 63 | 838,993 | 10,000 | 42,450 | 891,443 | |
| 16 | 64 | 891,443 | 10,000 | 45,072 | 946,515 | |
| 17 | 65 | 946,515 | (100,000) | 42,326 | 888,841 | |
| 18 | 66 | 888,841 | (100,000) | 39,442 | 828,283 | |
| 19 | 67 | 828,283 | (100,000) | 36,414 | 764,697 | |
| 20 | 68 | 764,697 | (100,000) | 33,235 | 697,932 | |
| 21 | 69 | 697,932 | (100,000) | 29,897 | 627,829 | |
| 22 | 70 | 627,829 | (100,000) | 26,391 | 554,220 | |
| 23 | 71 | 554,220 | (100,000) | 22,711 | 476,931 | |
| 24 | 72 | 476,931 | (100,000) | 18,847 | 395,778 | |
| 25 | 73 | 395,778 | (100,000) | 14,789 | 310,566 | |
| 26 | 74 | 310,566 | (100,000) | 10,528 | 221,095 | |
| 27 | 75 | 221,095 | (100,000) | 6,055 | 127,150 | |
| 28 | 76 | 127,150 | (100,000) | 1,357 | 28,507 | |
| 29 | 77 | 28,507 | (100,000) | (3,575) | (75,068) | |
| 30 | 78 | (75,068) | (100,000) | (8,753) | (183,821) | |
| 31 | 79 | (183,821) | (100,000) | (14,191) | (298,012) | |
| 32 | 80 | (298,012) | (100,000) | (19,901) | (417,913) | |
| 33 | 81 | (417,913) | (100,000) | (25,896) | (543,808) | |
| 34 | 82 | (543,808) | (100,000) | (32,190) | (675,999) | |
| 35 | 83 | (675,999) | (100,000) | (38,800) | (814,799) | |
| 36 | 84 | (814,799) | (100,000) | (45,740) | (960,539) | |

**20.** (Financial planning, finding pmt) Solve the previous problem using functions **PV** and **PMT** and the template below:

| | A | B | C |
|---|---|---|---|
| 1 | **SAVING FOR THE FUTURE** | | |
| 2 | Pension savings today | 500,000 | |
| 3 | Annual desired pension payout | 100,000 | |
| 4 | Number of years until retirement | 10 | |
| 5 | Number of payout years after retirement | 20 | |
| 6 | Interest rate | 5% | |
| 7 | | | |
| 8 | Present value today of all future retirement payments | | |
| 9 | Annual payment until retirement | | |

**21.** (Financial planning) Today is your 40th birthday. You expect to retire at age 65, and actuarial tables suggest that you will live to be 100. You want to move to Hawaii when you retire. You estimate that it will cost you $200,000 to make the move (on your 65th birthday). Starting on your 65th birthday and ending on your 99th birthday, your annual living expenses will be $25,000 a year. You expect to earn an annual return of 7% on your savings.

**a.** How much will you need to have saved by your retirement date?

**b.** For this part, assume that you already have $50,000 in savings today. How much would you need to save today and at ages 41 to 64 to be able to afford this retirement plan?

**c.** If you did not have any current savings and did not expect to be able to start saving money for the next 5 years (that is, your first savings payment will be made on your 45th birthday), how much would you have to set aside each year after that to be able to afford this retirement plan?

**22.** (Financial planning) John is turning 13 today. His birthday resolution is to start saving toward the purchase of a car that he wants to buy on his 18th birthday. The car costs $15,000 today, and he expects the price to grow at 2% per year.

John has heard that a local bank offers a savings account which pays an interest rate of 5% per year. He plans to make six contributions of $1,000 each to the savings account (the first contribution to be made today); he will use the funds in the account on his 18th birthday as a down payment for the car, financing the balance through the car dealer.

He expects the dealer to offer the following terms for financing: seven equal yearly payments (with the first payment due 1 year after he takes possession of the car); an annual interest rate of 7%.

**a.** How much will John need to finance through the dealer?

**b.** What will be the amount of his yearly payment to the dealer?

(**Hint:** This is similar to the college savings problem discussed in Section 2.5.)

**23.** (Financial planning) Mary has just completed her undergraduate degree from Northwestern University and is already planning to enter an MBA program 4 years from today. The MBA tuition will be $50,000 per year for 2 years, paid at the beginning of each year. In addition, Mary would like to retire 15 years from today and receive a pension of $60,000 every year for 20 years, with the first pension payment paid out 15 years from today. Mary can borrow and lend as much as she likes at a rate of 7%, compounded annually. In order to fund her expenditures, Mary will save money at the end of years 0–3 and at the end of years 6–14.

- Calculate the constant annual dollar amount that Mary must save at the end of each of these years to cover all of her expenditures (tuition and retirement). (It might be helpful to use **Goal Seek**.)

**Note**: Just to remove all doubts, here are the cash flows:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | Mary's Financial Planning | | | | | |
| 2 | Year | Balance at beginning of year before withdrawal | Withdrawal beginning of year | Net balance beginning of year | Savings at end of year | Account end of year |
| 3 | 0 | 0.00 | | | $X | |
| 4 | 1 | | | | $X | |
| 5 | 2 | | | | $X | |
| 6 | 3 | | | | $X | |
| 7 | 4 | | –50,000.00 | | | |
| 8 | 5 | | –50,000.00 | | | |
| 9 | 6 | | | | $X | |
| 10 | 7 | | | | $X | |
| 11 | 8 | | | | $X | |
| 12 | 9 | | | | $X | |
| 13 | 10 | | | | $X | |
| 14 | 11 | | | | $X | |
| 15 | 12 | | | | $X | |
| 16 | 13 | | | | $X | |
| 17 | 14 | | | | $X | |
| 18 | 15 | | –60,000.00 | | | |
| 19 | 16 | | –60,000.00 | | | |
| 20 | 17 | | –60,000.00 | | | |
| 21 | 18 | | –60,000.00 | | | |
| 22 | 19 | | –60,000.00 | | | |
| 23 | 20 | | –60,000.00 | | | |
| 24 | 21 | | –60,000.00 | | | |
| 25 | 22 | | –60,000.00 | | | |
| 26 | 23 | | –60,000.00 | | | |
| 27 | 24 | | –60,000.00 | | | |
| 28 | 25 | | –60,000.00 | | | |
| 29 | 26 | | –60,000.00 | | | |
| 30 | 27 | | –60,000.00 | | | |
| 31 | 28 | | –60,000.00 | | | |
| 32 | 29 | | –60,000.00 | | | |
| 33 | 30 | | –60,000.00 | | | |
| 34 | 31 | | –60,000.00 | | | |
| 35 | 32 | | –60,000.00 | | | |
| 36 | 33 | | –60,000.00 | | | |
| 37 | 34 | | –60,000.00 | | | 0 |

**24.** (Financial planning) You are 30 years old today and are considering studying for an MBA. You have just received your annual salary of $50,000 and expect it to grow by 3% per year. MBAs typically earn $80,000 upon graduation, with salaries growing by 4% per year.

The MBA program you're considering is a full-time, 2-year program that costs $30,000 per year, payable at the end of each study year. You want to retire on your 65th birthday. The relevant discount rate is 8%.[5] Is it worthwhile for you to quit your job in order to do an MBA (ignore income taxes)?

**25.** (Financial planning) You're 55 years old today, and you wish to start saving for your pension. Here are the parameters:

- You intend to make a deposit today and at the beginning of each of the next 9 years (that is, on your 55th, 56th, ..., 64th birthdays).

[5] **Meaning:** Your MBA is an investment like any other investment. On other investments, you can earn 8% per year; the MBA has to be judged against this standard.

- Starting from your 65th birthday until your 84th, you would like to withdraw $50,000 per year (no plans for after that).
- The interest rate is 12%.

**a.** How much should you deposit in each of the initial years in order to fully fund the withdrawals?

**b.** How much should you deposit in each of the initial years in order to fully fund the withdrawals if you start saving at age 60?

**c.** (More challenging) Set up the formula for the savings amount so that you can solve for various starting ages. Do a sensitivity analysis which shows the amount you need to save as a function of the age at which you start saving.

**26.** (Financial planning) Section 2.5. of this chapter discusses the problem of Linda Jones's parents, who wish to save for Linda's college education. The setup of the problem implicitly assumes that the bank will let the Jones's borrow from their savings account and will charge them the same 8% it was paying on positive balances. This is unlikely!

In this problem, you are asked to program the spreadsheet that follows. In it, you will assume that the bank pays Linda's parents 8% on positive account balances but charges them 10% on negative balances.

If Linda's parents can only deposit $4,000 per year in the years preceding college, how much will they owe the bank at the beginning of year 22 (the year after Linda finishes college)?

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **SAVING FOR COLLEGE** | | | | |
| 2 | Interest rates | 8% | | | |
| 3 | On positive balances | 10% | | | |
| 4 | On negative balances | 4,000.00 | | | |
| 5 | Annual deposit | 20,000 | | | |
| 6 | Annual cost of college | | | | |
| 7 | | | | | |
| 8 | **Birthday** | **In bank on birthday, before deposit/withdrawal** | **Deposit or withdrawal at beginning of year** | **Total** | **End of year with interest** |
| 9 | 10 | | 4,000.00 | | |
| 10 | 11 | | 4,000.00 | | |
| 11 | 12 | | 4,000.00 | | |
| 12 | 13 | | 4,000.00 | | |
| 13 | 14 | | 4,000.00 | | |
| 14 | 15 | | 4,000.00 | | |
| 15 | 16 | | 4,000.00 | | |
| 16 | 17 | | 4,000.00 | | |
| 17 | 18 | | –20,000.00 | | |
| 18 | 19 | | –20,000.00 | | |
| 19 | 20 | | –20,000.00 | | |
| 20 | 21 | | –20,000.00 | | |
| 21 | 22 | | | | |
| 22 | | | | | |

# APPENDIX 2.1: Algebraic Present Value Formulas

Most of the computations in the chapter can also be done with one basic bit of high-school algebra relating to the sum of a geometric series. Suppose you want to find the sum of a geometric series of $n$ numbers $a + aq + aq^2 + aq^3 + \cdots + aq^{n-1}$. In the jargon of geometric series:

$a$ is the *first term*

$q$ is the *ratio* between terms (the number by which the previous term is multiplied to get the next term)

$n$ is the *number of terms*

Denote the sum of the series by $S$: $S = a + aq + aq^2 + aq^3 + \cdots + aq^{n-1}$. In high school, you learned a trick to find the value of $S$:

1. Multiply $S$ by $q$:

$$qS = aq + aq^2 + aq^3 + \cdots + aq^{n-1} + aq^n$$

2. Subtract $qS$ from $S$:

$$\begin{aligned} S &= a + aq + aq^2 + aq^3 + \cdots + aq^{n-1} \\ -qS &= -\left( aq + aq^2 + aq^3 + \cdots + aq^{n-1} + aq^n \right) \\ (1-q)S &= a - aq^n \Rightarrow S = \frac{a(1-q^n)}{1-q} \end{aligned}$$

In the remainder of this appendix, we apply this formula to a variety of situations covered in the chapter.

### The Future Value of a Constant Payment

This topic is covered in Section 2.1. The problem there is to find the value of $100 deposited annually over 10 years, with the first payment today:

$$S = 100 * (1.06)^{10} + 100 * (1.06)^9 + \cdots + 100 * (1.06) = ???$$

For this geometric series:

$$a = First\ term = 100 * 1.06^{10}$$

$$q = Ratio = \frac{1}{1.06}$$

$$n = Number\ of\ terms = 10$$

The formula gives

$$S = \frac{a\left(1-q^n\right)}{1-q} = \frac{100*1.06^{10}\left(1-\left(\frac{1}{1.06}\right)^{10}\right)}{1-\frac{1}{1.06}} = 1397.16$$

where we have done the calculation in Excel:

| | A | B | C |
|---|---|---|---|
| 1 | **FUTURE VALUE FORMULA** | | |
| 2 | First term, a | 179.0848 | <– =100*1.06^10 |
| 3 | Ratio, q | 0.943396 | <– =1/1.06 |
| 4 | Number of terms, n | 10 | |
| 5 | | | |
| 6 | Sum | 1,397.16 | <– =B2*(1–B3^B4)/(1–B3) |
| 7 | Excel **PV** function | 1,397.16 | <– =FV(6%,B4,–100,,1) |

Substituting symbols for the numerical values, we get

$$\begin{array}{c}\text{Future value of } n \text{ payments}\\ \text{at end of year } n,\text{ at interest } r\\ \text{first payment today}\end{array} = \frac{Payment*(1+r)^n\left(1-\left(\frac{1}{1+r}\right)^n\right)}{1-\frac{1}{1+r}} = \underbrace{\mathrm{FV}(r,n,-1,,1)}_{\text{The Excel function}}$$

### Present Value of an Annuity

We can also apply the formula to find the present value of an annuity. Suppose, for example, that we want to calculate the present value of an annuity of \$150 per year for 5 years:

$$\frac{150}{(1.06)}+\frac{150}{(1.06)^2}+\frac{150}{(1.06)^3}+\frac{150}{(1.06)^4}+\frac{150}{(1.06)^5}$$

For this annuity:

$$a = First\ term = \frac{150}{1.06}.^{\dagger}$$

$$q = Ratio = \frac{1}{1.06}$$

$$n = Number\ of\ terms = 5$$

† If you're like most of the rest of humanity, you (mistakenly) thought that the first term was $a$ = 150. But look at the series—the first term actually is $\frac{150}{1.06}$. So there you are.

Thus the present value of the annuity becomes

$$S = \frac{a\left(1-q^n\right)}{1-q} = \frac{\frac{150}{1.06}\left(1-\left(\frac{1}{1.06}\right)^5\right)}{1-\frac{1}{1.06}} = 631.85 = \underbrace{\text{PV}(6\%,5,-150)}_{\text{The Excel function}}.$$

We can work this out in a spreadsheet:

| | A | B | C |
|---|---|---|---|
| 1 | **ANNUITY FORMULAS** | | |
| 2 | First term, a | 141.509434 | <– =150/1.06 |
| 3 | Ratio, q | 0.943396226 | <– =1/1.06 |
| 4 | Number of terms, n | 5 | |
| 5 | | | |
| 6 | Sum | 631.85 | <– =B2*(1–B3^B4)/(1–B3) |
| 7 | Excel **PV** function | 631.85 | <– =PV(6%,5,–150) |

## Cleaning Up the Formula (a Bit)

Standard textbooks often manipulate the annuity formula to make it look "better." Here's an example of something you might see in a textbook:

$$S = \frac{a\left(1-q^n\right)}{1-q} = \frac{\frac{Annual\ \ payment}{(1+r)}\left(1-\left(\frac{1}{1+r}\right)^n\right)}{1-\frac{1}{1+r}}$$
$$= \frac{Annual\ \ payment}{r}\left(1-\left(\frac{1}{1+r}\right)^n\right)$$

This is not a different annuity formula—it's just an algebraic simplification of the formula we've been using. If you put it in Excel, you'll get the same answer (and in our opinion, there's no point in the simplification).

## The Present Value of Series of Growing Payments

Suppose we're trying to apply the formula to the following series:

$$\frac{150}{(1.06)} + \frac{150*(1.10)}{(1.06)^2} + \frac{150*(1.10)^2}{(1.06)^3} + \frac{150*(1.10)^3}{(1.06)^4} + \frac{150*(1.10)^4}{(1.06)^5}$$

Here there are five payments, the first of which is \$150; this payment grows at an annual rate of 10%. We can apply the formula:

$$a = First\ term = \frac{150}{1.06}$$

$$q = Ratio = \frac{1.10}{1.06}$$

$$n = Number\ of\ terms = 5$$

In the following spreadsheet, you can see that the formula and the Excel **NPV** function give the same answer for the present value:

| | A | B | C |
|---|---|---|---|
| 1 | **CONSTANT GROWTH CASH FLOW** | | |
| 2 | First term, a | 141.5094 | <– =150/1.06 |
| 3 | Ratio, q | 1.037736 | <– =1.1/1.06 |
| 4 | Number of terms, n | 5 | |
| 5 | | | |
| 6 | Sum | 763.00 | <– =B2*(1–B3^B4)/(1–B3) |
| 7 | | | |
| 8 | **Year** | **Payment** | |
| 9 | 1 | 150.00 | |
| 10 | 2 | 165.00 | <– =B9*1.1 |
| 11 | 3 | 181.50 | <– =B10*1.1 |
| 12 | 4 | 199.65 | |
| 13 | 5 | 219.62 | |
| 14 | | | |
| 15 | Present value | 763.00 | <– =NPV(6%,B9:B13) |

Notice that the formula in cell B6 is more compact than Excel's **NPV** function. **NPV** requires you to list all the payments, whereas the formula in cell B6 requires only several lines (think about finding the present value of a very long series of growing payments—clearly the formula is more efficient).

### The Present Value of a Constant Growth Annuity

An annuity is a series of annual payments; a constant growth annuity is an annuity whose payments grow at a constant rate. Here's an example of such a series:

$$\frac{20}{(1.10)} + \frac{20*(1.05)}{(1.10)^2} + \frac{20*(1.05)^2}{(1.10)^3} + \frac{20*(1.05)^3}{(1.10)^4} + \frac{20*(1.05)^4}{(1.10)^5} + \cdots$$

We can fit this into our formula:

$$a = First\ term = \frac{20}{1.10}$$

$$q = Ratio = \frac{1.05}{1.10}$$

$$n = Number\ of\ terms = \infty$$

The formula gives

$$S = \frac{a\left(1-q^n\right)}{1-q} = \frac{\frac{20}{1.10}\left(1-\left(\frac{1.05}{1.10}\right)^n\right)}{1-\frac{1.05}{1.10}}$$

When $n \to \infty$, $\left(\frac{1.05}{1.10}\right)^n \to 0$, so that

$$S = \frac{a\left(1-q^n\right)}{1-q} = \frac{\frac{20}{1.10}\left(1-\left(\frac{1.05}{1.10}\right)^n\right)}{1-\frac{1.05}{1.10}} = \frac{\frac{20}{1.10}}{1-\frac{1.05}{1.10}} = \frac{20}{0.10-0.05} = 400$$

**Warning**: You have to be careful! This version of the formula only works because the growth rate of 5% is smaller than the discount rate of 10%. The discounted sum of an infinite series of constantly growing payments only exists when the growth rate $g$ is less than the discount rate $r$.

Here's a general formula:

$$\begin{array}{c}\text{Sum of}\\ \text{constant growth}\\ \text{annuity}\end{array} = \frac{CF}{(1+r)} + \frac{CF*(1+g)}{(1+r)^2} + \frac{CF*(1+g)^2}{(1+r)^3} + \cdots = \frac{\frac{CF}{(1+r)}\left(1-\left(\frac{1+g}{1+r}\right)^{\infty}\right)}{1-\frac{1+g}{1+r}}$$

$$= \begin{cases} \dfrac{CF}{r-g} & \text{when } |g| < |r| \\ \text{undefined} & \text{otherwise} \end{cases}$$

To summarize:

*The present value of a constant growth annuity—a series of cash flows with first term CF which grows at rate g—that is discounted at rate r is* $\frac{CF}{r-g}$, *provided that* $g < r$.

We use this formula in Chapter 16, when we discuss the valuation of stocks using discounted dividends (the "Gordon dividend model").

# APPENDIX 2.2: Annuity Formulas in Excel

As presented in Appendix 2.1, the regular annuity formula is

$$S = \frac{Annual\ \ payment}{r}\left(1-\left(\frac{1}{1+r}\right)^{n}\right)$$

If we also assume that we get one additional cash flow at the end of the annuity (period $n$), the formula will be

$$S = \frac{Annual\ \ payment}{r}\left(1-\left(\frac{1}{1+r}\right)^{n}\right)+\frac{Single\ \ payment_{n}}{(1+r)^{n}}$$

This formula has five arguments: *S, Annual payment, r, n, Single payment*$_n$. So given any four arguments, we can solve for the fifth (the unknown).

Excel uses the following form of the above formula:

$$0 = PV + \frac{PMT}{RATE}\left(1-\left(\frac{1}{1+RATE}\right)^{NPER}\right)+\frac{FV}{(1+RATE)^{NPER}}$$

Keeping this in mind enables us to understand the following:

1. The reason we get a negative PV value when we solve for PV and PMT is entered in positive number.
2. We can solve for the present value using the last four arguments (or omit FV and Excel will plug "0" for FV).
3. We can solve for the future value using the other four arguments (or omit PV and Excel will plug "0" for PV).
4. We can solve for the periodic payment using the PMT function.
5. We can solve for the interest rate of the annuity using the RATE function.
6. We can solve for the number of periods using the NPER function.

# 3 Measures for Evaluation of Investment Opportunities

## Chapter Contents



## Overview

This chapter deals with the practical implications of the "time value of money" concept. In this chapter, we discuss the most commonly used measures of evaluating and prioritizing investment opportunities (or projects): net present value (NPV), internal rate of return (IRR), hurdle rate, equivalent annual cash flow (EAC), and profitability index (PI). These concepts tell you what an investment is worth (net present value), what percentage rate of return you're getting on your investments (internal rate of return), what is the range of returns in which a project is profitable (hurdle rate), how to compare mutually exclusive projects with unequal lives (EAC), and how to allocate your funds in projects that are not mutually exclusive.

Financial assets and financial planning always have a time dimension. Here are some simple examples:

- Your Aunt Sara is considering making an investment. The investment costs \$1,000 and will pay back \$50 per month in each of the next 36 months.

- o Should she do this, or should she leave her money in the bank, where it earns 5%?
- o What is the annual return that your aunt Sara is getting on the investment?

- Should we purchase expensive machinery with a cost of $1M and a lifetime of 10 years, or should we purchase cheap machinery with a cost of $0.5M and a lifetime of 4 years when the alternative cost of capital is 10%?
- We are offered five good projects, and we can invest in more than one. We would like to prioritize our investments efficiently. How should we do it?

**Finance concepts discussed**

- Net present value (NPV)
- Internal rate of return (IRR)
- Modified IRR (MIRR)
- Equivalent annual cash flows (EAC)
- Profitability index (PI)

**Excel functions used**

- **FV, PV, NPV, IRR, MIRR, PMT, RANK, IF**

## 3.1 Net Present Value

*The net present value (NPV) of a series of future cash flows is their present value minus the initial investment required to obtain the future cash flows. The NPV equals the present value of future cash flows minus the initial investment. The NPV of an investment represents the increase in wealth which you get if you make the investment.*

Here's an example:

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | **NPV EXAMPLE** | | | | |
| 2 | Discount rate, r | 6% | | | | | | |
| 3 | | | | | | | | |
| 4 | **Year** | **0** | **1** | **2** | **3** | **4** | **5** | |
| 5 | Cash flow | –1,500.00 | 100.00 | 200.00 | 300.00 | 400.00 | 500.00 | |
| 6 | Present value | –1,500.00 | 94.34 | 178.00 | 251.89 | 316.84 | 373.63 | <-- =G5/(1+$B$2)^G4 |
| 7 | | | | | | | | |
| 8 | **Present value of future cash flows** | | | | | | | |
| 9 | | 1,214.69 | <-- =SUM(C6:G6) | | | | | |
| 10 | | 1,214.69 | <-- =NPV(B2,C5:G5) | | | | | |
| 11 | | | | | | | | |
| 12 | **Net present value (NPV)** | | | | | | | |
| 13 | | –285.31 | <-- =SUM(B6:G6) | | | | | |
| 14 | | –285.31 | <-- =B5+NPV(B2,C5:G5) | | | | | |

Would you pay \$1,500 today to get the series of future cash flows in cells C6:G6? Certainly not—they're worth only \$1,214.69, so why pay \$1,500? If asked to pay \$1,500, the NPV of the investment would be

$$NPV = \underbrace{-\$1,500}_{\substack{\uparrow \\ \text{Cost of the} \\ \text{investment}}} + \underbrace{\frac{100}{1.06} + \frac{200}{(1.06)^2} + \frac{300}{(1.06)^3} + \frac{400}{(1.06)^4} + \frac{500}{(1.06)^5}}_{\substack{\uparrow \\ \text{Present value of} \\ \text{investment's future} \\ \text{cash flows at discount} \\ \text{rate of 6\%}}} = \underbrace{-\$285.31}_{\substack{\uparrow \\ \text{Net present value}}}$$

$$= -\$1,500 + \qquad \$1,214.69$$

If you paid \$1,500 for this investment, you would be overpaying \$285.31 for the investment, and you would be poorer by the same amount. That's a bad deal!

On the other hand, if you were offered the same future cash flows for \$1,000, you'd snap up the offer, since you would be paying \$214.69 less for the investment than its worth:

$$NPV = \underbrace{-\$1,000}_{\substack{\uparrow \\ \text{Cost of the} \\ \text{investment}}} + \underbrace{\$1,214.69}_{\substack{\uparrow \\ \text{Present value of} \\ \text{investment's future} \\ \text{cash flows at discount} \\ \text{rate of 6\%}}} = \underbrace{\$214.69}_{\substack{\uparrow \\ \text{Net present value}}}$$

In Excel:

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | | | | **NPV EXAMPLE** | | | | |
| 2 | Discount rate, r | 6% | | | | | | |
| 3 | | | | | | | | |
| 4 | **Year** | **0** | **1** | **2** | **3** | **4** | **5** | |
| 5 | Cash flow | –1,000.00 | 100.00 | 200.00 | 300.00 | 400.00 | 500.00 | |
| 6 | Present value | –1,000.00 | 94.34 | 178.00 | 251.89 | 316.84 | 373.63 | <-- =G5/(1+$B$2)^G4 |
| 7 | | | | | | | | |
| 8 | **Present value of future cash flows** | | | | | | | |
| 9 | | 1,214.69 | <-- =SUM(C6:G6) | | | | | |
| 10 | | 1,214.69 | <-- =NPV(B2,C5:G5) | | | | | |
| 11 | | | | | | | | |
| 12 | **Net present value (NPV)** | | | | | | | |
| 13 | | 214.69 | <-- =SUM(B6:G6) | | | | | |
| 14 | | 214.69 | <-- =B5+NPV(B2,C5:G5) | | | | | |

In this case, the investment would make you \$214.69 richer. As we said before, the NPV of an investment represents the increase in your wealth if you make the investment.

To summarize:

*The net present value (NPV) of a series of cash flows is used to make investment decisions: An investment with a positive NPV is a good investment, and an investment with a negative NPV is a bad investment. An investment with a zero NPV is a "fair game"—the future cash flows of the investment exactly compensate you for the investment's initial cost.*

Net present value (NPV) is a basic tool of financial analysis. It is used to determine whether a particular investment ought to be undertaken; in cases where we can make only one of several investments (the investment opportunities are "mutually exclusive"), it is the tool of choice to determine which investment to undertake.

Here's another NPV example: You've found an interesting investment—If you pay $800 today to your local pawnshop, the owner promises to pay you $100 at the end of year 1, $150 at the end of year 2, $200 at the end of year 3, ..., $300 at the end of year 5. You feel that the pawnshop owner is as reliable as your local bank, which is currently paying 5% interest. The following spreadsheet shows the NPV of this $800 investment:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **CALCULATING NET PRESENT VALUE (NPV) WITH EXCEL** | | | |
| 2 | r, interest rate | 5% | | |
| 3 | | | | |
| 4 | **Year** | **Payment** | **Present value** | |
| 5 | 0 | –800 | –800.00 | |
| 6 | 1 | 100 | 95.24 | <-- =B6/(1+$B$2)^A6 |
| 7 | 2 | 150 | 136.05 | <-- =B7/(1+$B$2)^A7 |
| 8 | 3 | 200 | 172.77 | |
| 9 | 4 | 250 | 205.68 | |
| 10 | 5 | 300 | 235.06 | |
| 11 | | | | |
| 12 | **NPV** | | | |
| 13 | Summing the present values | | 44.79 | <-- =SUM(C5:C10) |
| 14 | Using Excel's **NPV** function | | 44.79 | <-- =NPV($B$2,B6:B10)+C5 |

The spreadsheet shows that the value of the investment—the *net present value (NPV)* of its payments, including the initial payment of –$800—is $44.79:

$$NPV = -800 + \underbrace{\frac{100}{(1.05)} + \frac{150}{(1.05)^2} + \frac{200}{(1.05)^3} + \frac{250}{(1.05)^4} + \frac{300}{(1.05)^5}}_{\substack{\text{The } \textit{present value} \text{ of the future payments:} \\ \text{Calculated with Excel } \textbf{NPV} \text{ function } = 844.79}} = 44.79$$

At a 5% discount rate, you should make the investment, since its NPV is $44.79, which is positive.

# EXCEL NOTE

## NPV

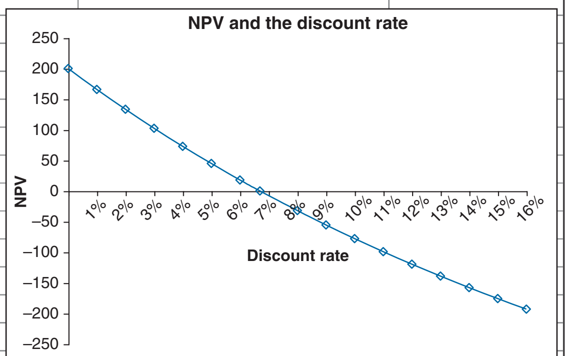
As noted in Chapter 2 (Section 2.2, subsection entitled "The Excel **NPV** Function"), the Excel **NPV** function's name does **not correspond** to the standard finance use of the term "net present value."[1] In finance, "present value" usually refers to the value today of future payments (in the previous example, the present value is $\frac{100}{(1.05)}+\frac{150}{(1.05)^2}+\frac{200}{(1.05)^3}+\frac{250}{(1.05)^4}+\frac{300}{(1.05)^5}=844.79$). Finance professionals use *net present value* (NPV) to mean the *present value* of future payments *minus the cost of the initial payment*; in the previous example, this is \$844.79 – \$800 = \$44.79. In this book, we use the term "net present value" (NPV) to mean its true finance sense. The Excel function **NPV** will always appear in boldface. We trust that you will rarely be confused.

### NPV Depends on the Discount Rate

Let's revisit the pawnshop example on page 63, and use Excel to create a table which shows the relation between the discount rate and the NPV. As the graph below shows, the higher the discount rate, the lower the net present value of the investment:

[1] There's a long history to this confusion, and it doesn't start with Microsoft. The original spreadsheet—Visicalc—(mistakenly) used "NPV" in the sense which Excel still uses today; this misnomer has been copied ever since by all other spreadsheets: Lotus, Quattro, and Excel.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **CALCULATING NET PRESENT VALUE (NPV) WITH EXCEL** | | | | |
| 2 | r, interest rate | 5% | | | |
| 3 | | | | | |
| 4 | **Year** | **Payment** | **Present value** | | |
| 5 | 0 | –800 | –800.00 | | |
| 6 | 1 | 100 | 95.24 | <-- =B6/(1+$B$2)^A6 | |
| 7 | 2 | 150 | 136.05 | <-- =B7/(1+$B$2)^A7 | |
| 8 | 3 | 200 | 172.77 | | |
| 9 | 4 | 250 | 205.68 | | |
| 10 | 5 | 300 | 235.06 | | |
| 11 | | | | | |
| 12 | **NPV** | | | | |
| 13 | Summing the present values | | 44.79 | <-- =SUM(C5:C10) | |
| 14 | Using Excel's **NPV** function | | 44.79 | <-- =NPV($B$2,B6:B10)+C5 | |
| 15 | | | | | |
| 16 | **Discount rate** | **NPV** | | | |
| 17 | 0% | 200.00 | <-- =NPV(A17,$B$6:$B$10)+$B$5 | | |
| 18 | 1% | 165.86 | <-- =NPV(A18,$B$6:$B$10)+$B$5 | | |
| 19 | 2% | 133.36 | <-- =NPV(A19,$B$6:$B$10)+$B$5 | | |
| 20 | 3% | 102.41 | | | |
| 21 | 4% | 72.92 | | | |
| 22 | 5% | 44.79 | | | |
| 23 | 6% | 17.96 | | | |
| 24 | 6.6965% | 0.00 | | | |
| 25 | 8% | –32.11 | | | |
| 26 | 9% | –55.48 | | | |
| 27 | 10% | –77.83 | | | |
| 28 | 11% | –99.21 | | | |
| 29 | 12% | –119.67 | | | |
| 30 | 13% | –139.26 | | | |
| 31 | 14% | –158.04 | | | |
| 32 | 15% | –176.03 | | | |
| 33 | 16% | –193.28 | | | |




Note that we've highlighted a special discount rate: When the discount rate is 6.6965%, the net present value of the investment is zero. The 6.6965% rate is referred to as the *internal rate of return (IRR)*. For discount rates less than the IRR, the net present value is positive, and for discount rates greater than the IRR, the net present value is negative. We discuss the IRR in more detail in Section 3.2.

## Using NPV to Choose Between Investments

In the examples discussed thus far, we've used NPV only to choose whether to undertake a particular investment or not. But NPV can also be used to choose between investments. Look at the following spreadsheet: You have \$800 to invest, and you've been offered the choice between Investment A and Investment B. The spreadsheet below shows that at an interest rate of 15%, Investment A has an NPV = \$219.06 and Investment B has an NPV = \$373.75. If the investments are not mutually exclusive, you would want to invest in both A and B, since they each have a positive NPV. But if you are forced to choose only one investment, you should choose Investment B because it has a higher net present value. Investment A will increase your wealth by \$219.06, whereas Investment B increases your wealth by \$373.75.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **USING NPV TO CHOOSE BETWEEN INVESTMENTS** | | | |
| 2 | Discount rate | 15% | | |
| 3 | | | | |
| 4 | **Year** | **Investment A** | **Investment B** | |
| 5 | 0 | –800 | –800 | |
| 6 | 1 | 250 | 600 | |
| 7 | 2 | 500 | 200 | |
| 8 | 3 | 200 | 100 | |
| 9 | 4 | 250 | 500 | |
| 10 | 5 | 300 | 300 | |
| 11 | | | | |
| 12 | **NPV** | 219.06 | 373.75 | <-- =NPV(B2,C6:C10)+C5 |

To summarize:

*In using the NPV to choose between two mutually exclusive positive-NPV investments, we choose the investment with the higher NPV.*

## Terminology—Is It a Discount Rate or an Interest Rate?

In some of the examples above, we've used *discount rate* instead of *interest rate* to describe the rate used in the net present value calculation. As you will see in further chapters of this book, the rate used in the NPV has several synonyms: Discount rate, interest rate, cost of capital, opportunity cost—these are but a few of the names for the rate that appears in the denominator of the NPV:

$$\frac{\textit{Cash flow in year } t}{(1+r)^t}$$

$\uparrow$

*Discount rate*
*Interest rate*
*Cost of capital*
*Opportunity cost*

## 3.2 The Internal Rate of Return (IRR)

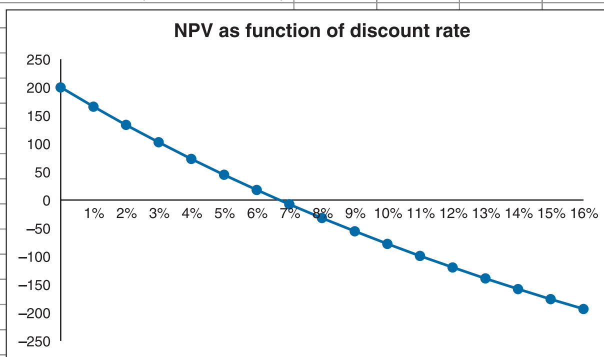
*The IRR of a series of cash flows is the discount rate that sets the net present value of the cash flows equal to zero.*

Before we explain in depth (in the next section) why you want to know the IRR, we explain how to compute it. Let's go back to the example on page 63: If you pay \$800 today to your local pawnshop, the owner promises to pay you \$100 at the end of year 1, \$150 at the end of year 2, \$200 at the end of year 3, \$250 at the end of year 4, and \$300 at the end of year 5. Discounting these cash flows at rate $r$, the NPV can be written:

$$NPV = -800 + \frac{100}{(1+r)} + \frac{150}{(1+r)^2} + \frac{200}{(1+r)^3} + \frac{250}{(1+r)^4} + \frac{300}{(1+r)^5}$$

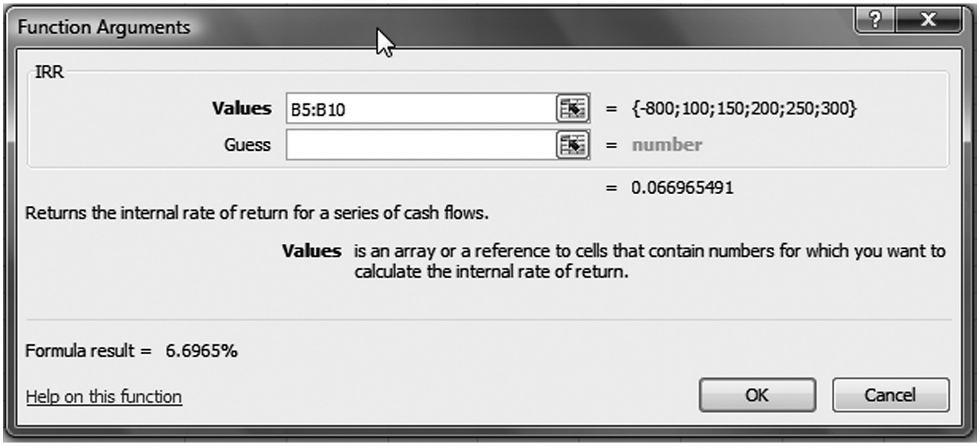
In cells B16:B32 of the spreadsheet below, we calculate the NPV for various discount rates. As you can see, somewhere between $r$ = 6% and $r$ = 7%, the NPV becomes negative.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **CALCULATING THE IRR WITH EXCEL** | | | | | | |
| 2 | **Year** | **Payment** | | | | | |
| 3 | 0 | –800 | | | | | |
| 4 | 1 | 100 | | | | | |
| 5 | 2 | 150 | | | | | |
| 6 | 3 | 200 | | | | | |
| 7 | 4 | 250 | | | | | |
| 8 | 5 | 300 | | | | | |
| 9 | | | | | | | |
| 10 | **IRR** | 6.6965% | <-- =IRR(B3:B8) | | | | |
| 11 | **Check: The NPV using IRR as discount rate** | | | | | | |
| 12 | | 0.00 | <-- =B3+NPV(B10,B4:B8) | | | | |
| 13 | | | | | | | |
| 14 | **Discount rate** | **NPV** | | | | | |
| 15 | 0% | 200.00 | <-- =\$B\$3+NPV(A15,\$B\$4:\$B\$8) | | | | |
| 16 | 1% | 165.86 | <-- =\$B\$3+NPV(A16,\$B\$4:\$B\$8) | | | | |
| 17 | 2% | 133.36 | | | | | |
| 18 | 3% | 102.41 | | | | | |
| 19 | 4% | 72.92 | | | | | |
| 20 | 5% | 44.79 | | | | | |
| 21 | 6% | 17.96 | | | | | |
| 22 | 7% | –7.65 | | | | | |
| 23 | 8% | –32.11 | | | | | |
| 24 | 9% | –55.48 | | | | | |
| 25 | 10% | –77.83 | | | | | |
| 26 | 11% | –99.21 | | | | | |
| 27 | 12% | –119.67 | | | | | |
| 28 | 13% | –139.26 | | | | | |
| 29 | 14% | –158.04 | | | | | |
| 30 | 15% | –176.03 | | | | | |
| 31 | 16% | –193.28 | | | | | |
| 32 | | | | | | | |



In cell B10, we use Excel's **IRR** function to calculate the exact discount rate at which the NPV becomes 0. The answer is 6.6965%; at this interest rate, the NPV of the cash flows equals zero (look at cell B12). We can use the dialog box for the Excel **IRR** function:

## Dialog Box for IRR Function




Notice that we haven't used the second option ("Guess") to calculate our IRR. We discuss this option in Section 3.5.

### What Does the IRR Mean?

Suppose you could get 6.6965% interest at the bank, and suppose you wanted to save today to provide yourself with the future cash flows of the example on page 63:

- To get \$100 at the end of year 1, you would have to put the present value $100/(1.06965) = 93.72$ in the bank today.
- To get \$150 at the end of year 2, you would have to put its present value $150/(1.06965)^2 = 131.76$ in the bank today.
- And so on . . . (see the picture below)

The total amount you would have to save is \$800, exactly the cost of this investment opportunity. This is what we mean when we say that: *The internal rate of return is the compound interest rate you earn on an investment.*

| Time | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Save for time 1's \$100<br>\$100/(1+6.6965%) | 93.72 | FV=93.72*(1+6.6965%)<br>=\$100.00 | | | | |
| Save for time 1's \$150<br>\$150/(1+6.6965%)$^2$ | 131.76 | | FV=131.76*(1+6.6965%)$^2$<br>=\$150.00 | | | |
| Save for time 3's \$200<br>\$200/(1+6.6965%)$^3$ | 164.66 | | | FV=164.66*(1+6.6965%)$^3$<br>=\$200.00 | | |
| Save for time 4's \$250<br>\$250/(1+6.6965%)$^4$ | 192.90 | | | | FV=192.90*(1+6.6965%)$^4$<br>=\$250.00 | |
| Save for time 5's \$300<br>\$300/(1+6.6965%)$^5$ | 216.95 | | | | | FV=216.95*(1+6.6965%)$^5$<br>=\$300.00 |
| **Total saving at time 0** | **800.00** | | | | | |

## Using IRR to Make Investment Decisions

The IRR is often used to make investment decisions. Suppose your Aunt Sara has been offered the following investment by her broker: For a payment of $1,000, a reputable finance company will pay her $300 at the end of each of the next 4 years. Aunt Sara is currently getting 5% on her bank savings account. Should she withdraw her money from the bank to make the investment? To answer the question, we compute the IRR of the investment and compare it to the bank interest rate:

| | A | B | C |
|---|---|---|---|
| 1 | USING IRR TO MAKE INVESTMENT DECISIONS | | |
| 2 | Year | Cash flow | |
| 3 | 0 | –1,000 | |
| 4 | 1 | 300 | |
| 5 | 2 | 300 | |
| 6 | 3 | 300 | |
| 7 | 4 | 300 | |
| 8 | | | |
| 9 | IRR | 7.71% | <-- =IRR(B3:B7) |

The IRR of the investment, 7.71%, is greater than the 5% Sara can earn on her alternative investment (the bank account). Thus she should make the investment.

Summarizing:

*In using the IRR to make investment decisions, an investment with an IRR greater than the alternative rate of return is a good investment, and an investment with an IRR less than the alternative rate of return is a bad investment.*

## Using IRR to Choose Between Two Investments

We can also use the internal rate of return to choose between two investments. Suppose you've been offered two investments. Both Investment A and Investment B cost $1,000, but they have different cash flows. If you're using the IRR to make the investment decision, then you would choose the investment with the *higher* IRR. Here's an example:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | USING IRR TO CHOOSE BETWEEN INVESTMENTS | | | |
| 2 | Year | Investment A cash flows | Investment B cash flows | |
| 3 | 0 | –1,000 | –1,000 | |
| 4 | 1 | 450 | 550 | |
| 5 | 2 | 425 | 300 | |
| 6 | 3 | 350 | 457 | |
| 7 | 4 | 450 | 200 | |
| 8 | | | | |
| 9 | IRR | 24.74% | 21.65% | <-- =IRR(C3:C7) |

We would choose Investment A, which has the higher IRR.

Summarizing:

*In using the IRR to choose between two comparable investments, we choose the investment which has the higher IRR. This assumes the following: (1) Both investments have an IRR greater than the alternative rate. (2) The investments are of comparable risk.*[2]

## Using NPV and IRR to Make Investment Decisions

In this chapter we have now developed two tools, NPV and IRR, for making investment decisions. We've also discussed two kinds of investment decisions. Here's a summary:

| | **"Yes or No": Choosing whether to undertake a single investment** | **"Investment ranking": Comparing two investments which are mutually exclusive** |
|---|---|---|
| **NPV criterion** | The investment should be undertaken if its NPV > 0. | Investment A is preferred to investment B if NPV(A) > NPV(B) |
| **IRR criterion** | The investment should be undertaken if its IRR > $r$, where $r$ is the appropriate discount rate. | Investment A is preferred to investment B if IRR(A) > IRR(B) |

## Understanding the Meaning of IRR

In the previous section, we gave a simple illustration of what we meant when we said that *the internal rate of return (IRR) is the compound interest rate that you earn on an investment or project*. This short sentence underlies a slew of finance applications: When finance professionals discuss the "rate of return" on an investment or the "effective interest rate" on a loan, they are almost always referring to the IRR. In this section, we explore some meanings of the IRR. Almost the whole of Chapter 5 is devoted to this topic.

## A Simple Example

Suppose you buy an asset for \$200 today, and suppose that the asset will pay you \$300 in 1 year. Then the asset's IRR is 50%. To see this, recall that the IRR is the interest rate which makes the NPV zero. Since the investment NPV equals $-200+\dfrac{300}{1+r}$, this means that the NPV is zero when $1+r=\dfrac{300}{200}=1.5$. Solving this equation gives $r = 50\%$.

[2] In Sections 3.4–3.6, we discuss various issues that arise in the interpretation of the IRR. The upshot of this discussion is that whereas IRR is a wonderful and intuitive tool to judge investments, you have to be very careful in interpreting its meaning. **Our rule of thumb:** Even if you want to use IRR, also use the NPV to make sure that you're correctly interpreting the IRR. (Who said that finance is simple?)

Here's another way to think about this investment and its 50% IRR:

- At time 0, you pay $200 for the investment.
- At time 1, the $300 investment cash flow repays the initial $200. The remaining $100 represents a 50% return on the initial $200 investment. This is the IRR.

*The IRR is the rate of return on an investment; it is the rate that repays, over the life of the asset, the initial investment in the asset and that pays interest on the outstanding investment balances.*

## A More Complicated Example

We now give a more complicated example, which illustrates the same point. This time, you buy an asset costing $200. The asset's cash flow are $130.91 at the end of year 1 and $130.91 at the end of year 2. Here's our IRR analysis of this investment:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **THE IRR AS A RATE OF RETURN ON AN INVESTMENT** | | | | | |
| 2 | **IRR** | 20.00% | <-- =IRR({–200,130.91,130.91}) | | | |
| 3 | **Year** | **Investment at beginning of year** | **Payment at end of year** | **Part of payment which is interest** | **Part of payment which is repayment of principal** | |
| 4 | 1 | 200.00 | 130.91 | 40.00 | 90.91 | |
| 5 | 2 | 109.09 | 130.91 | 21.82 | 109.09 | |
| 6 | 3 | 0.00 | | | | |
| 7 | | | | | | |
| 8 | =B4–E4 | | | =$B$2*B4 | =C4–D4 | |
| 9 | | | | | | |
| 10 | | | =B5–E5 | | | =C5–D5 |
| 11 | | | | | =$B$2*B5 | |

- The IRR for the investment is 20.00%. Note how we calculated this—we simply typed into cell B2 the formula **=IRR({–200,130.91,130.91})**. (If you're going to use this method of calculating the IRR in Excel, you have to put the cash flows in the curly brackets.)
- Using the 20% IRR, $40.00 (=20%*$200) of the first year's payment is interest, and the remainder—$90.91—is repayment of principal. Another way to think of the $40.00 is to consider that to buy the asset, you gave the seller the $200 cost of the asset. When he pays you $130.91 at the end of the year, $40 (=20%*$200) is interest—your payment for allowing someone else to use your money. The remainder, $90.91, is a partial repayment of the money lent out.

- This leaves the outstanding principal at the beginning of year 2 as $109.09. Of the $130.91 paid out by the investment at the end of year 2, $21.82 (=20%*109.09) is interest, and the rest (*exactly* $109.09) is repayment of principal.
- The outstanding principal at the beginning of year 3 (the year *after* the investment finishes paying out) is *zero*.

As in the first example of this section, the IRR is the rate of return on the investment—defined as the rate that repays, over the life of the asset, the initial investment in the asset and that pays interest on the outstanding investment balances.

## 3.3 NPV vs. IRR

In this section, we discuss the differences between NPV and IRR. Much of what we say is a repetition of the previous section—we again discuss how to both judge (the "yes–no" decision for a project) and how to rank ("is Project A better than Project B").

### The NPV Rule for Judging Investments and Projects

In preceding sections, we introduced the basic NPV and IRR concepts and their application to capital budgeting. We start off this section by summarizing each of these rules: the NPV rule in this section and the IRR rule in the following section.

Here's a summary of the decision criteria for investments implied by the net present value:

**The NPV rule for deciding whether or not a specific project is worthwhile:** Suppose we are considering a project which has cash flows $CF_0, CF_1, CF_2, \ldots, CF_N$. Suppose that the appropriate discount rate for this project is $r$. Then the NPV of the project is

$$NPV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \ldots + \frac{CF_N}{(1+r)^N} = CF_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t}$$

**Rule:** A project is worthwhile by the NPV rule if its NPV is greater than 0.

**The NPV rule for deciding between two mutually exclusive projects:** Suppose you are trying to decide between two Projects, A and B, each of which can achieve the same objective. For example: Your company needs a new widget machine, and the choice is between widget machine A or machine B. You will buy either A or B, or perhaps neither machine, but you will certainly not buy both machines. In finance jargon these projects are "mutually exclusive."

Suppose that Project A has cash flows $CF_0^A, CF_1^A, CF_2^A, ..., CF_N^A$ and that Project B has cash flows $CF_0^B, CF_1^B, CF_2^B, \ldots, CF_N^B$.

**Rule:** Project A is preferred to Project B if

$$NPV(A) = CF_0^A + \sum_{t=1}^{N} \frac{CF_t^A}{(1+r)^t} > CF_0^B + \sum_{t=1}^{N} \frac{CF_t^B}{(1+r)^t} = NPV(B)$$

The logic of both the NPV rules presented above is that the *present value* of a project's cash flows, $PV = \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t}$, is the economic value today of the project. Thus, if we have correctly chosen the discount rate $r$ for the project, the PV is what we ought to be able to sell the project for in the market.[3] The net present value is the *wealth increment* produced by the project, so that NPV > 0 means that a project adds to our wealth:

$$NPV = \underbrace{CF_0}_{\substack{\uparrow \\ \text{Initial cash} \\ \text{flow required} \\ \text{to implement} \\ \text{the project.} \\ \text{This is usually} \\ \text{a negative number.}}} + \underbrace{\sum_{t=1}^{N} \frac{CF_t}{(1+r)^t}}_{\substack{\uparrow \\ \text{Market value} \\ \text{of future cash} \\ \text{flows.}}}$$

## An Initial Example

To set the stage, let's assume that you're trying to decide whether to undertake one of two projects. Project A involves buying expensive machinery which produces a better product at a lower cost. The machines for Project A cost \$1,000 and, if purchased, you anticipate that the project will produce cash flows of \$500 per year for the next 5 years. Project B's machines are cheaper, costing \$800, but they produce smaller annual cash flows of \$420 per year for the next 5 years. We'll assume that the correct discount rate is 12%.

Suppose we apply the NPV criterion to Projects A and B:

[3] This assumes that the discount rate is "correctly chosen," by which we mean that it is appropriate with regard to the riskiness of the project's cash flows. For the moment, we fudge the question of how to choose discount rates; this topic is discussed in Chapter 5.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **TWO PROJECTS** | | | |
| 2 | Discount rate, r | 12% | | |
| 3 | | | | |
| 4 | **Year** | **Project A** | **Project B** | |
| 5 | 0 | –1,000 | –800 | |
| 6 | 1 | 500 | 420 | |
| 7 | 2 | 500 | 420 | |
| 8 | 3 | 500 | 420 | |
| 9 | 4 | 500 | 420 | |
| 10 | 5 | 500 | 420 | |
| 11 | | | | |
| 12 | **NPV** | 802.39 | 714.01 | <-- =C5+NPV($B$2,C6:C10) |

Both projects are worthwhile, since each has a positive NPV. If we have to choose between the projects, then Project A is preferred to Project B because it has the higher NPV.

## The IRR Rule for Judging Investments

An alternative to using the NPV criterion for capital budgeting is to use the internal rate of return (IRR). Recall that the IRR is defined as the discount rate for which the NPV equals zero. It is the compound rate of return which you get from a series of cash flows.

Here are the two decision rules for using the IRR in capital budgeting:

**The IRR rule for deciding whether a specific investment is worthwhile:** Suppose we are considering a project that has cash flows $CF_0$, $CF_1$, $CF_2$, ...., $CF_N$. *IRR is an interest rate such that*

*IRR is an interest rate such that*

$$CF_0 + \frac{CF_1}{(1+IRR)} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_N}{(1+IRR)^N} = CF_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+k)^t} = 0$$

***Rule***: If the appropriate discount rate for a project is $r$, you should accept the project if its $IRR > r$ and reject it if its $IRR < r$.

The logic behind the IRR rule is that the IRR is the compound return you get from the project. Since $r$ is the project's required rate of return, it follows that if $IRR > r$, you get more than you require.

Applying the IRR rule to our Projects A and B, we get:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | TWO PROJECTS | | | |
| 2 | Discount rate, r | 12% | | |
| 3 | | | | |
| 4 | **Year** | **Project A** | **Project B** | |
| 5 | 0 | –1,000 | –800 | |
| 6 | 1 | 500 | 420 | |
| 7 | 2 | 500 | 420 | |
| 8 | 3 | 500 | 420 | |
| 9 | 4 | 500 | 420 | |
| 10 | 5 | 500 | 420 | |
| 11 | | | | |
| 12 | **IRR** | 41.04% | 44.03% | <-- =IRR(C5:C10) |

Both Project A and Project B are worthwhile, since each has an IRR > 12%, which is our relevant discount rate. If we have to choose between the two projects by using the IRR rule, Project B is preferred to Project A because it has a higher IRR.

## Interim Summary

We can sum up the NPV and the IRR rules as follows:

| | **"Yes or No": Choosing whether to undertake a single project** | **"Project ranking": Comparing two mutually exclusive projects** |
|---|---|---|
| **NPV criterion** | The project should be undertaken if its NPV > 0. | Project *A* is preferred to project B if NPV(A) > NPV(B) |
| **IRR criterion** | The project should be undertaken if its IRR > *r*, where *r* is the appropriate discount rate. | Project *A* is preferred to project B if IRR(A) > IRR(B). |

Both the NPV rules and the IRR rules look logical. In many cases, your investment decision—to undertake a project or not, or which of two competing projects to choose—will be the same whether or not you use NPV or IRR. There are some cases, however (such as that of Projects A and B illustrated above), where NPV and IRR give different answers. In our present value analysis, Project A won out because its NPV was greater than Project B's. In our IRR analysis of the same projects, Project B was chosen because it had the higher IRR. In such cases, we should always use the NPV to decide between projects. The logic is that if individuals are interested in maximizing their wealth, they should use NPV, which measures the incremental wealth from undertaking a project.

## The "Yes–No" Criterion: When Do IRR and NPV Give the Same Answer?

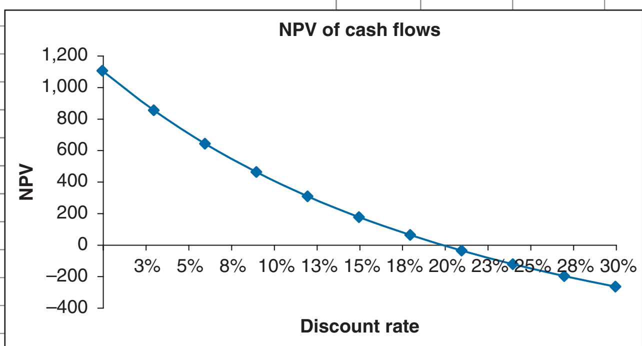
Consider the following project: The initial cash flow of –$1,000 represents the cost of the project today, and the remaining cash flows for years 1–6 are projected future cash flows. The discount rate is 15%:

| | A | B | C |
|---|---|---|---|
| 1 | **SIMPLE CAPITAL BUDGETING EXAMPLE** | | |
| 2 | Discount rate | 15% | |
| 3 | | | |
| 4 | **Year** | **Cash flow** | |
| 5 | 0 | –1,000 | |
| 6 | 1 | 100 | |
| 7 | 2 | 200 | |
| 8 | 3 | 300 | |
| 9 | 4 | 400 | |
| 10 | 5 | 500 | |
| 11 | 6 | 600 | |
| 12 | | | |
| 13 | **PV** of future cash flows | 1,172.13 | <-- =NPV(B2,B6:B11) |
| 14 | **NPV** | 172.13 | <-- =B5+NPV(B2,B6:B11) |
| 15 | **IRR** | 19.71% | <-- =IRR(B5:B11) |

The NPV of the project is $172.13, meaning that the present value of the project's future cash flows ($1,172.13) is greater than the project's cost of $1,000.00. Thus the project is worthwhile.

If we graph the project's NPV, we can see that the IRR—the point where the NPV curve crosses the *x*-axis—is very close to 20%. As you can see in cell B15, the actual IRR is 19.71%.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 18 | **Discount rate** | **NPV** | | | | | |
| 19 | 0% | 1,100.00 | <-- =$B$5+NPV(A19,$B$6:$B$11) | | | | |
| 20 | 3% | 849.34 | <-- =$B$5+NPV(A20,$B$6:$B$11) | | | | |
| 21 | 6% | 637.67 | | | | | |
| 22 | 9% | 457.83 | | | | | |
| 23 | 12% | 304.16 | | | | | |
| 24 | 15% | 172.13 | | | | | |
| 25 | 18% | 58.10 | | | | | |
| 26 | 21% | –40.86 | | | | | |
| 27 | 24% | –127.14 | | | | | |
| 28 | 27% | –202.71 | | | | | |
| 29 | 30% | –269.16 | | | | | |
| 30 | | | | | | | |
| 31 | | | | | | | |
| 32 | | | | | | | |
| 33 | | | | | | | |
| 34 | | | | | | | |



## Accept or Reject? Should We Undertake the Project?

It is clear that the above project is worthwhile:

- Its NPV is greater than 0, so that by the NPV criterion the project should be accepted.
- Its IRR of 19.71% is greater than the project discount rate of 15%, so that by the IRR criterion the project should be accepted.

## A General Principle

We can derive a general principle from this example:

*For conventional projects, projects with an initial negative cash flow and subsequent non-negative cash flows ($CF_0 < 0, CF_1 \geq 0, CF_2 \geq 0, \ldots, CF_N \geq 0$), the NPV and IRR criteria lead to the same "Yes–No" decision: If the NPV criterion indicates a "Yes" decision, then so will the IRR criterion (and vice versa).*

## Do NPV and IRR Produce the Same Project Rankings?

In the previous section, we saw that for conventional projects, NPV and IRR give the same "Yes–No" answer about whether to invest in a project. In this section, we'll see that NPV and IRR do not necessarily *rank* projects the same, even if the projects are both conventional.

Suppose we have two projects and can choose to invest in only one. The projects are *mutually exclusive*: They are both ways to achieve the same end, and thus we would choose only one. In this section, we discuss the use of NPV and IRR to rank the projects. To sum up our results before we start:

- Ranking projects by NPV and IRR can lead to possibly contradictory results. Using the NPV criterion may lead us to prefer one project, whereas using the IRR criterion may lead us to prefer the other project.
- Where a conflict exists between NPV and IRR, the project with the larger NPV is preferred. That is, the NPV criterion is the correct criterion to use for capital budgeting. This is not to impugn the IRR criterion, which is often very useful. However, NPV is preferred over IRR because it indicates the *increase in wealth* which the project produces.

## An Example

Below we show the cash flows for Project A and Project B. Both projects have the same initial cost of $500 but have different cash flow patterns. The relevant discount rate is 15%.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **RANKING PROJECTS WITH NPV AND IRR** | | | |
| 2 | Discount rate, r | 15% | | |
| 3 | | | | |
| 4 | **Year** | **Project A** | **Project B** | |
| 5 | 0 | –500 | –500 | |
| 6 | 1 | 100 | 250 | |
| 7 | 2 | 100 | 250 | |
| 8 | 3 | 150 | 200 | |
| 9 | 4 | 200 | 100 | |
| 10 | 5 | 400 | 50 | |
| 11 | | | | |
| 12 | **NPV** | 74.42 | 119.96 | <-- =C5+NPV($B$2,C6:C10) |
| 13 | **IRR** | 19.77% | 27.38% | <-- =IRR(C5:C10) |

**Comparing the projects using IRR:** If we use the IRR rule to choose between the projects, then B is preferred to A, since the IRR of Project B is higher than that of Project A.

**Comparing the projects using NPV:** Here the choice is more complicated. When the discount rate is 15% (as illustrated above), the NPV of Project B is higher than that of Project A. In this case, the IRR and the NPV agree: Both indicate that Project B should be chosen. Now suppose that the discount rate is 8%; in this case, the NPV and IRR rankings conflict:

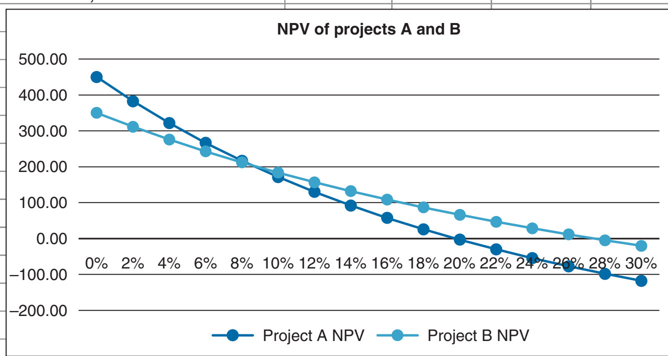
| | A | B | C | D |
|---|---|---|---|---|
| 1 | **RANKING PROJECTS WITH NPV AND IRR** | | | |
| 2 | Discount rate | 8% | | |
| 3 | | | | |
| 4 | **Year** | **Project A** | **Project B** | |
| 5 | 0 | –500 | –500 | |
| 6 | 1 | 100 | 250 | |
| 7 | 2 | 100 | 250 | |
| 8 | 3 | 150 | 200 | |
| 9 | 4 | 200 | 100 | |
| 10 | 5 | 400 | 50 | |
| 11 | | | | |
| 12 | **NPV** | 216.64 | 212.11 | <-- =C5+NPV(B2,C6:C10) |
| 13 | **IRR** | 19.77% | 27.38% | <-- =IRR(C5:C10) |

In this case, we have to resolve the conflict between the ranking on the basis of NPV (A is preferred) and ranking on the basis of IRR (B is preferred). As we stated in the introduction to this section, the solution to this question is that you should choose on the basis of NPV. We explore the reasons for this later on, but first we discuss a technical question.

## Why Do NPV and IRR Give Different Rankings?

Below we build a table and graph that show the NPV for each project as a function of the discount rate:

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **RANKING PROJECTS WITH NPV AND IRR** | | | | | | | |
| 2 | Discount rate, r | 15% | | | | | | |
| 3 | | | | | | | | |
| 4 | **Year** | **Project A** | **Project B** | | | | | |
| 5 | 0 | –500 | –500 | | | | | |
| 6 | 1 | 100 | 250 | | | | | |
| 7 | 2 | 100 | 250 | | | | | |
| 8 | 3 | 150 | 200 | | | | | |
| 9 | 4 | 200 | 100 | | | | | |
| 10 | 5 | 400 | 50 | | | | | |
| 11 | | | | | | | | |
| 12 | **NPV** | 74.42 | 119.96 | <-- =C5+NPV($B$2,C6:C10) | | | | |
| 13 | **IRR** | 19.77% | 27.38% | <-- =IRR(C5:C10) | | | | |
| 14 | | | | | | | | |
| 15 | **Data table: NPV and discount rates** | | | | | | | |
| 16 | | | | | | | | |
| 17 | | **Project A NPV** | **Project B NPV** | | | | | |
| 18 | | 74.42 | 119.96 | <-- =C12, data table header | | | | |
| 19 | 0% | 450.00 | 350.00 | | | | | |
| 20 | 2% | 382.57 | 311.53 | | | | | |
| 21 | 4% | 321.69 | 275.90 | | | | | |
| 22 | 6% | 266.60 | 242.84 | | | | | |
| 23 | 8% | 216.64 | 212.11 | | | | | |
| 24 | 10% | 171.22 | 183.49 | | | | | |
| 25 | 12% | 129.85 | 156.79 | | | | | |
| 26 | 14% | 92.08 | 131.84 | | | | | |
| 27 | 16% | 57.53 | 108.47 | | | | | |
| 28 | 18% | 25.86 | 86.57 | | | | | |
| 29 | 20% | –3.22 | 66.00 | | | | | |
| 30 | 22% | –29.96 | 46.66 | | | | | |
| 31 | 24% | –54.61 | 28.45 | | | | | |
| 32 | 26% | –77.36 | 11.28 | | | | | |
| 33 | 28% | –98.39 | –4.93 | | | | | |
| 34 | 30% | –117.87 | –20.25 | | | | | |




From the graph, you can see why contradictory rankings occur:

- Project B has a higher IRR (27.38%) than project A (19.77%). (Remember that the IRR is the point at which the NPV curve cuts the *x*-axis.)
- When the discount rate is low, Project A has a higher NPV than Project B, but when the discount rate is high, Project B has a higher NPV. There is a crossover point (in the next subsection, you will see that this point is 8.51%) that marks the disagreement/agreement range.

- Project A's NPV is more sensitive to changes in the discount rate than Project B. The reason for this is that Project A's cash flows are more spread out over time than those of Project B; another way of saying this is that Project A has substantially more of its cash flows at later dates than Project B.

Summing up:

| | **Discount rate < 8.51%** | **Discount rate = 8.51%** | **Discount rate > 8.51%** |
|---|---|---|---|
| **NPV criterion** | A preferred:<br>NPV(A) > NPV(B) | Indifferent between A and B:<br>NPV(A) = NPV(B) | B preferred:<br>NPV(B) > NPV(A) |
| **IRR criterion** | B always preferred to A, since IRR(B) > IRR(A) | | |

## Calculating the Crossover Point

The crossover point—which we claimed above was 8.51% — is the discount rate at which the NPV of the two projects is equal. A bit of formula manipulation will show you that *the crossover point is the IRR of the differential cash flows*. To see what this means, go back to our previous example:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **CALCULATING THE CROSSOVER POINT FOR A AND B** | | | | |
| 2 | **Year** | **Project A** | **Project B** | **Differential cash flows: cash flow(A) – cash flow (B)** | |
| 3 | 0 | –500 | –500 | 0 | <-- =B3–C3 |
| 4 | 1 | 100 | 250 | –150 | <-- =B4–C4 |
| 5 | 2 | 100 | 250 | –150 | |
| 6 | 3 | 150 | 200 | –50 | |
| 7 | 4 | 200 | 100 | 100 | |
| 8 | 5 | 400 | 50 | 350 | |
| 9 | | | | | |
| 10 | | | **IRR** | 8.51% | <-- =IRR(D3:D8) |

Column D in the above example contains the differential cash flows—the difference between the cash flows of Project A and Project B. In cell D43, we use the Excel **IRR** function to compute the crossover point.

A bit of theory (can be skipped): To see why the crossover point is the IRR of the differential cash flows, suppose that for some rate $r$, NPV(A) = NPV(B):

$$NPV(A) = CF_0^A + \frac{CF_1^A}{(1+r)} + \frac{CF_2^A}{(1+r)^2} + \cdots + \frac{CF_N^A}{(1+r)^N}$$
$$= CF_0^B + \frac{CF_1^B}{(1+r)} + \frac{CF_2^B}{(1+r)^2} + \cdots + \frac{CF_N^B}{(1+r)^N} = NPV(B)$$

Subtracting and rearranging shows that $r$ must be the IRR of the differential cash flows:

$$CF_0^A - CF_0^B + \frac{CF_1^A - CF_1^B}{(1+r)} + \frac{CF_2^A - CF_2^B}{(1+r)^2} + \cdots + \frac{CF_N^A - CF_N^B}{(1+r)^N} = 0$$

## What to Use? NPV or IRR?

Let's go back to the initial example and suppose that the discount rate is 8%:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **RANKING PROJECTS WITH NPV AND IRR** | | | |
| 2 | Discount rate | 8% | | |
| 3 | | | | |
| 4 | **Year** | **Project A** | **Project B** | |
| 5 | 0 | –500 | –500 | |
| 6 | 1 | 100 | 250 | |
| 7 | 2 | 100 | 250 | |
| 8 | 3 | 150 | 200 | |
| 9 | 4 | 200 | 100 | |
| 10 | 5 | 400 | 50 | |
| 11 | | | | |
| 12 | **NPV** | 216.64 | 212.11 | <-- =C5+NPV(B2,C6:C10) |
| 13 | **IRR** | 19.77% | 27.38% | <-- =IRR(C5:C10) |

In this case, we know there is disagreement between the NPV (which would lead us to choose Project A) and the IRR (by which we choose Project B). Which is correct?

The answer to this question is that we should—for the case where the discount rate is 8%—choose using the NPV (that is, choose Project A). This is just one example of the general principal that *using the NPV is always preferred*, since the NPV is the additional *wealth* that you get, whereas IRR is the compound rate of return. The economic assumption is that consumers maximize their wealth, not their rate of return.

# 3.4 Problems with IRR (I): You Can't Always Tell Good Projects from Bad Ones

In this and the next section, we discuss some problems with the IRR concept. Understanding these problems will give you insight into the compound return assumption and will also help you understand both why finance professionals prefer NPV over IRR.

Sometimes it's hard to tell from the IRR whether a project is good or bad. Here's a simple example: You've decided to buy a car; the list price is $11,000, and the dealer has offered you two purchase options:

- You can pay the dealer cash and get a $1,000 discount, thus paying only $10,000.
- You can pay $5,000 now and pay $2,000 in each of the next 3 years. The dealer calls this his "zero-interest car loan" plan. The bank is giving car loans at 9% interest, so the dealer claims that his plan is much cheaper.

Which offer is better? Having learned a bit of finance, you set up the following Excel spreadsheet:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | | BUYING A CAR | | |
| 2 | List price of car | 11,000 | | | |
| 3 | Downpayment | 5,000 | | | |
| 4 | Cash cost of car | 10,000 | | | |
| 5 | Bank rate of interest | 9% | | | |
| 6 | | | | | |
| 7 | **Year** | **Payment in cash** | **Payment with credit** | **Differential dealer cash flow** | |
| 8 | 0 | –10,000 | –5,000 | 5,000 | <-- =C8–B8 |
| 9 | 1 | | –2,000 | –2,000 | <-- =C9–B9 |
| 10 | 2 | | –2,000 | –2,000 | |
| 11 | 3 | | –2,000 | –2,000 | |
| 12 | | | | | |
| 13 | Internal rate of return | | | 9.70% | <-- =IRR(D8:D11) |
| 14 | | | | | |
| 15 | **NPV** of cash saved | | | –62.59 | <-- =D8+NPV(B5,D9:D11) |

The critical element in the spreadsheet is column D, which compares the annual cash outlays of the credit plan with those of the cash-payment plan. Column D shows that if you pay with the credit plan instead of paying cash, you'll spend $5,000 *less* in year 0. On the other hand, you'll spend $2,000 *more* in years 1, 2, and 3. The IRR of this column is 9.70%. Since the bank is lending money at 9%, you should take a bank loan instead of using the dealer's credit plan.

To understand this further, note that the pattern of the cash flows in column D is like the pattern of cash flows from taking a loan. When you take a loan, there is an initial positive cash flow (this is when you get the loan) and subsequent negative cash flows (the loan repayments). When you buy the car using the dealer's credit plan, the cash flow pattern is the same: There is an initial positive cash flow (the savings from paying only $5,000 instead of $10,000) and subsequent negative cash flows (the additional $2,000 annual cost of the credit plan). Thus the IRR of 9.70% represents the *cost* of the dealer's credit plan. Since the bank lends at a cost of 9%, it is cheaper to borrow through the bank.

What if you don't have the $10,000 cash for the cash-payment plan? Then you should take a bank loan (read on for details).

Cell B15 discounts the differential payment flow of column D at the bank interest rate. This shows that this flow has a negative NPV, another indication that you shouldn't undertake this project: You should opt for the cash-payment plan.

## How Will You Pay for the Car?

So you're better off paying the dealer cash. If you don't have the $10,000 cash, you could borrow $5,000 from the bank. This plan would have cash flows (assuming equal annual payments of principal and interest, calculated using Excel's **PMT** function) of:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 18 | **Borrowing the money from the bank** | | | | |
| 19 | **Year** | **Payment in cash** | **Bank loan cash flows** | **Total cash flows to car owner** | |
| 20 | 0 | –10,000.00 | 5,000.00 | –5,000.00 | <-- =B20+C20 |
| 21 | 1 | | –1,975.27 | –1,975.27 | <-- =B21+C21 |
| 22 | 2 | | –1,975.27 | –1,975.27 | |
| 23 | 3 | | –1,975.27 | –1,975.27 | |

The cash flows in cells D20:D23 are an improvement over those in cells C7:C10, which shows (again) that it's better to buy the car in cash and borrow the money from the bank than to take the dealer's financing offer.

## The Dealer's Cash Flows

To see how confusing the IRR can be, consider the dealer's cash flows. He's offered you the choice of paying $10,000 in cash or $5,000 down with 3 equal payment of $2,000:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **IRR VERSUS NPV—THE DEALER'S PROBLEM** | | | | |
| 2 | List price of car | 11,000 | | | |
| 3 | Downpayment | 5,000 | | | |
| 4 | Cash cost of car | 10,000 | | | |
| 5 | Bank rate of interest | **9%** | | | |
| 6 | | | | | |
| 7 | **Year** | **Payment in cash** | **Payment with credit** | **Differential dealer cash flow** | |
| 8 | 0 | –10,000 | 5,000 | –5,000 | <-- =C8-B8 |
| 9 | 1 | | 2,000 | 2,000 | <-- =C9-B9 |
| 10 | 2 | | 2,000 | 2,000 | |
| 11 | 3 | | 2,000 | 2,000 | |
| 12 | | | | | |
| 13 | Internal rate of return | | | 9.70% | <-- =IRR(D8:D11) |
| 14 | | | | | |
| 15 | **NPV** of cash saved | | | 62.59 | <-- =D8+NPV(B5,D9:D11) |

Column D shows that between the two plans, the dealer has a negative cash flow of $5,000 in year 0, but then has a positive cash flow of $2,000 in each of the 3 subsequent years. Effectively, the dealer is acting like a bank giving a loan, and the 9.70% represents the interest earned by the dealer on the loan; if he can borrow the $5,000 in cell D8 from the bank at 9%, he's better off—his NPV on the loan is $62.59.

### What's the Point?

The dealer's IRR and your IRR are the same. But this turns out to mean that the payment plan is bad for you and good for the dealer: The IRR of the dealer's cash flows represents the interest he earns on the loan he's giving you; the IRR of your cash flows is the cost of the loan you're taking. To tell whether you're getting a good deal or a bad deal, use the NPV of the differential payments discounted at the bank's loan rate; this NPV clearly shows that the payment plan is bad for you (negative NPV of $62.59) and good for the dealer (positive NPV of $62.59).

## 3.5 Problems with IRR (II): Multiple Internal Rates of Return

A project has a "conventional cash flow pattern" when all the positive and negative cash flows are bunched together. If this condition is not met, then we'll call the cash flow pattern of the project "nonconventional." Here are some examples of conventional and nonconventional cash flows:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **CONVENTIONAL AND NONCONVENTIONAL CASH FLOW PATTERNS** | | | | | | |
| 2 | **Year** | **Cash flow Project A** | **Cash flow Project B** | **Cash flow Project C** | **Cash flow Project D** | **Cash flow Project E** | **Cash flow Project F** |
| 3 | 0 | –100 | –100 | 100 | 25 | –25 | –250 |
| 4 | 1 | 200 | –50 | 55 | 35 | 80 | 35 |
| 5 | 2 | 500 | 60 | 35 | –200 | –100 | 145 |
| 6 | 3 | 50 | 80 | 50 | 33 | 200 | 330 |
| 7 | 4 | 60 | 99 | –100 | 55 | 55 | 55 |
| 8 | 5 | 35 | 100 | –35 | 155 | –250 | –250 |
| 9 | | ↑ Conventional cash flow pattern | ↑ Conventional cash flow pattern | ↑ Conventional cash flow pattern | ↑ Nonconventional cash flow pattern | ↑ Nonconventional cash flow pattern | ↑ Nonconventional cash flow pattern |
| 10 | | Initial negative cash flow followed by positive cash flows | Two initial negative cash flows followed by positive cash flows | Initial positive cash flows followed by negative cash flows | Two positive cash flows, then negative, then three positive cash flows | Initial negative cash flow, then positive, then negative, positive, negative cash flows | Negative cash flows at beginning and end, other cash flows positive |

In Section 3.3, we show that for projects with conventional cash flows, the NPV and the IRR criteria give the same answers to the "yes–no" capital budgeting question (the question of whether a particular project is worthwhile). In this section, we discuss the IRR of projects with nonconventional cash flows. Such projects often have multiple IRRs, which makes our analysis of nonconventional project using the IRR confusing. We will ultimately conclude that NPV is a better decision tool.

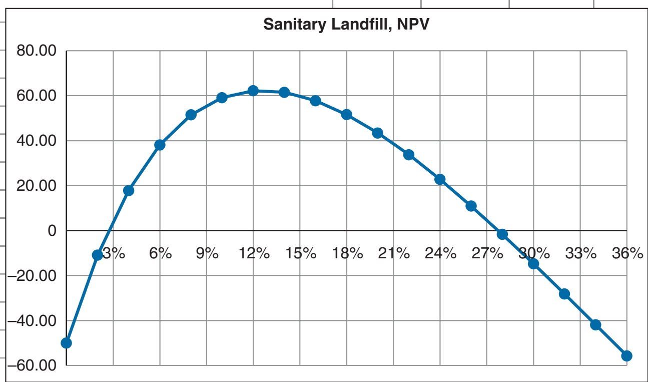
Consider the case of a company that operates sanitary landfills. A "landfill" is basically a big hole in the ground where lots of garbage is dumped until the hole is filled in.

Here are the cash flows anticipated by the company for a new landfill:

- The initial cost of the landfill is $800,000: This covers the expense of digging the hole, fencing it, and providing appropriate truck access.
- The annual net cash inflows from the landfill are $450,000. These represent the fees the company collects in return for giving trash collection companies the right to dump their trash in the landfill. These cash inflows are net of any costs incurred by the landfill company.
- After 5 years, the landfill will be full. The costs of closing the land fill, incurred at the end of year 6, are $1,500,000. This includes the costs of abiding by various ecological regulations, etc.

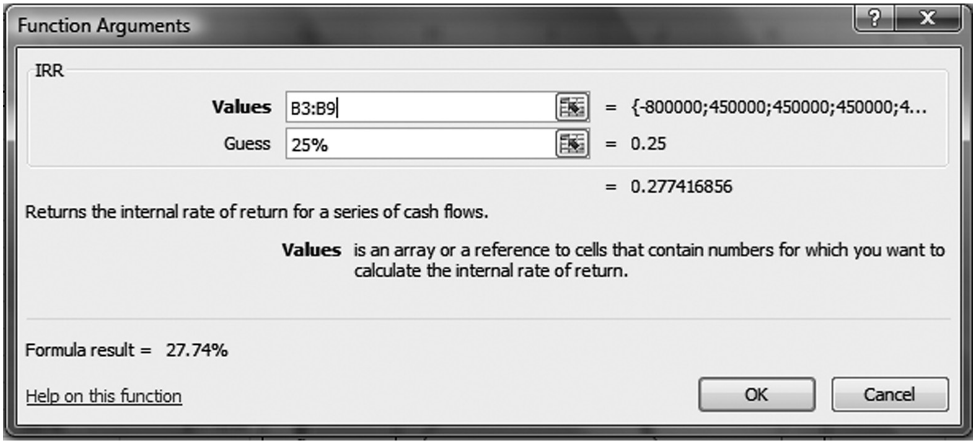
In the spreadsheet below, the cash flows for the landfill are given in cells B3:B9. The **Data Table** computes the net present value of these cash flows at various discount rates. The graph shows that the cash flows have *two* internal rates of return: These are the two points at which the graph cuts the *x*-axis.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **SANITARY LANDFILL, INC.** | | | | | | |
| 2 | **Year** | **Cash flow** | | | | | |
| 3 | 0 | –800 | | | | | |
| 4 | 1 | 450 | | | | | |
| 5 | 2 | 450 | | | | | |
| 6 | 3 | 450 | | | | | |
| 7 | 4 | 450 | | | | | |
| 8 | 5 | 450 | | | | | |
| 9 | 6 | –1,500 | | | | | |
| 10 | | | | | | | |
| 11 | First **IRR** | 2.68% | <-- =IRR(B3:B9,0) | | | | |
| 12 | Second **IRR** | 27.74% | <-- =IRR(B3:B9,25%) | | | | |
| 13 | | | | | | | |
| 14 | **Data Table: NPV of cash flows** | | | | | | |
| 15 | **Discount rate** | **NPV** | | | | | |
| 16 | 0% | –50.00 | <-- =$B$3+NPV(A16,$B$4:$B$9) | | | | |
| 17 | 2% | –10.90 | <-- =$B$3+NPV(A17,$B$4:$B$9) | | | | |
| 18 | 4% | 17.85 | <-- =$B$3+NPV(A18,$B$4:$B$9) | | | | |
| 19 | 6% | 38.12 | <-- =$B$3+NPV(A19,$B$4:$B$9) | | | | |
| 20 | 8% | 51.47 | | | | | |
| 21 | 10% | 59.14 | | | | | |
| 22 | 12% | 62.20 | | | | | |
| 23 | 14% | 61.51 | | | | | |
| 24 | 16% | 57.77 | | | | | |
| 25 | 18% | 51.58 | | | | | |
| 26 | 20% | 43.43 | | | | | |
| 27 | 22% | 33.72 | | | | | |
| 28 | 24% | 22.79 | | | | | |
| 29 | 26% | 10.92 | | | | | |
| 30 | 28% | –1.66 | | | | | |
| 31 | 30% | –14.76 | | | | | |
| 32 | 32% | –28.22 | | | | | |
| 33 | 34% | –41.91 | | | | | |
| 34 | 36% | –55.73 | | | | | |
| 35 | | | | | | | |




In cells B11 and B12, we identify both of these IRRs, using Excel's **IRR** function. We have used the **Guess** option for this function. This option allows you to identify the *approximate* IRR (we used the graph to identify this number); Excel then computes an IRR close to this approximation. In the spreadsheet above, we use 25% as a **Guess** in cell B11. Excel's **IRR** function then shows that the actual IRR which is close to this **Guess** is 27.74%.

### Dialog Box for IRR Function, Showing Use of *Guess*




**Note:** If you entered a lower **Guess** (say 0% or 3%), Excel would find the IRR of 2.68%. If you do not enter a **Guess**, Excel looks for the IRR closest to zero.

## Two IRRs: What Does This Mean?

This business of two IRRs is confusing! Suppose we're trying to decide whether to undertake the landfill project. As you saw in this chapter, there are two traditional rules for accepting or rejecting a project:

- **NPV rule:** A project is acceptable if its NPV > 0. In the case of the sanitary landfill, the NPV rule says that the project is acceptable if the discount rate is larger than 2.68% and smaller than 27.74%.
- **IRR rule:** A project is acceptable if its IRR is greater than the appropriate discount rate. Because there are two IRRs in this case, the IRR rule is impossible to apply. In practical terms, this means that when a project has more than one IRR, you should determine its attractiveness only by the NPV rule.

## How Many IRRs Are There?

For a given set of cash flows, there are potentially as many IRRs as there are changes in sign of the cash flow. The cash flow pattern of a conventional project has an initial negative cash flow and thereafter only positive cash flows; there is only one change of sign (from negative to positive) and hence only one possible IRR. The previous cash flow example has two changes in sign (and hence two possible IRRs): from –800,000 in year 0 to 450,000 in year 1 and then again from 450,000 in year 5 to –1,500,000 in year 6.[4]

## 3.6 Problems with IRR (III): The Reinvestment Rate Assumption

The IRR assumes that *intermediate cash flows are reinvested at the IRR*. To give meaning to this strange sentence, consider the project below (columns A and B): The Excel **IRR** function in cell B11 shows that the internal rate of return is 8.36%:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **IRR ASSUMES REINVESTMENT OF INTERMEDIATE CASH FLOW AT IRR** | | | | |
| 2 | **Year** | **Cash flow** | | **Cash flow reinvested at IRR** | |
| 3 | 0 | –1,000 | | –1,000.00 | <-- =B3 |
| 4 | 1 | 200 | | 275.71 | <-- =B4*(1+$B$11)^(5–A4) |
| 5 | 2 | 300 | | 381.67 | <-- =B5*(1+$B$11)^(5–A5) |
| 6 | 3 | 450 | | 469.64 | |
| 7 | 4 | 200 | | 216.71 | |
| 8 | 5 | 150 | | 150.00 | <-- =B8*(1+$B$11)^(5–A8) |
| 9 | | | | | |
| 10 | | | | 1,493.73 | <-- =SUM(D4:D8) |
| 11 | **IRR** | 8.36% | <-- =IRR(B3:B8) | 8.36% | <-- =(D10/–D3)^(1/5)–1 |

In column D, we assume that each year's cash flow is reinvested at the IRR until the end of year 5. Thus, for example,

- The year 1 cash flow of 200, if reinvested at the IRR, has a future value of $200 * (1 + 8.36\%) ^\wedge 4 = 275.41$ in year 5.
- The year 2 cash flow of 300 becomes 381.67 in year 5 if reinvested at the IRR.
- The future year 5 value of the years 1–5 cash flows when reinvested at the IRR is 1,493.73.

[4] Exercises 9-11 at the end of this chapter show examples with three IRRs.

This future value computation shows us another way of computing the IRR:

$$\textit{Alternative IRR computation} = \left(\frac{\textit{Future value, year 5}}{\textit{Initial cost}}\right)^{(1/5)} - 1$$

$$= \left(\frac{1{,}493.73}{1{,}000}\right)^{(1/5)} - 1 = 8.36\%$$

## What if the Reinvestment Rate Is Different from the IRR?

Now suppose that the years 1–5 cash flows from the project are reinvested at a rate different from the IRR. This could easily happen—if the firm making the investment decides that all intermediate investment cash flows will be deposited in a bank account earning 5% per year, the IRR calculation should be modified as follows:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **REINVESTING INTERMEDIATE CASH FLOWS AT A RATE DIFFERENT FROM THE IRR** | | | | |
| 2 | Reinvestment rate | 5% | | | |
| 3 | **Year** | **Cash flow** | | **Cash flow reinvested at IRR** | |
| 4 | 0 | –1,000 | | –1,000.00 | <-- =B4 |
| 5 | 1 | 200 | | 243.10 | <-- =B5*(1+$B$2)^(5–A5) |
| 6 | 2 | 300 | | 347.29 | <-- =B6*(1+$B$2)^(5–A6) |
| 7 | 3 | 400 | | 441.00 | |
| 8 | 4 | 200 | | 210.00 | |
| 9 | 5 | 150 | | 150.00 | <-- =B9*(1+$B$2)^(5–A9) |
| 10 | | | | | |
| 11 | | | | 1,391.39 | <-- =SUM(D5:D9) |
| 12 | **IRR** | 8.36% | <-- =IRR(B4:B9) | 6.83% | <-- =(D11/–D4)^(1/5)–1 |
| 13 | | | | 6.83% | <-- =MIRR(B4:B9,0,B2) |

## MIRR with Financing and Reinvestment Rate

Suppose that some of the cash flows are negative and some are positive, and suppose that the financing rate for the negative cash flows is different from the reinvestment rate for the positive cash flows. Excel's **MIRR** function can take care of this situation:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | MIRR WITH DIFFERENT FINANCING AND REINVESTMENT RATES | | | | | |
| 2 | Financing rate | 8% | | | | |
| 3 | Reinvestment rate | 5% | | | | |
| 4 | Year | Cash flow | | FV of cash flow reinvested | PV of cash flow borrowed | |
| 5 | 0 | –1,000 | | –1,000.00 | –1,000.00 | <-- =IF(B5<=0,B5/(1+$B$2)^A5,0) |
| 6 | 1 | 800 | | 972.41 | 0.00 | <-- =IF(B6<=0,B6/(1+$B$2)^A6,0) |
| 7 | 2 | –300 | | 0.00 | –257.20 | <-- =IF(B7<=0,B7/(1+$B$2)^A7,0) |
| 8 | 3 | –200 | | 0.00 | –158.77 | <-- =IF(B8<=0,B8/(1+$B$2)^A8,0) |
| 9 | 4 | 900 | | 945.00 | 0.00 | |
| 10 | 5 | 150 | | 150.00 | 0.00 | |
| 11 | | | | | | |
| 12 | Modified investment | | | –1,415.97 | <-- =SUM(E5:E10) | |
| 13 | Modified future value | | | 2,067.41 | <-- =SUM(D6:D10) | |
| 14 | **IRR** | 11.55% | <-- =IRR(B5:B10) | 7.86% | <-- =(D13/–D12)^(1/5)–1 | |
| 15 | | | | 7.86% | <-- =MIRR(B5:B10,B2,B3) | |

The MIRR assumes that negative cash flows are borrowed at a financing rate of 8% and added to the initial investment of 1,000; this gives a total present value of investment of 1,415.97. The future value of the positive cash flows is 2,067.41, making the **MIRR**:

$$\begin{aligned} \textit{Modified internal rate of return} &= \left( \frac{\textit{Future value of positive cash flows}}{\textit{Present value of negative cash flows}} \right)^{(1/5)} - 1 \\ &= MIRR(\textit{all cash flows, financing rate, reinvestment rate}) \\ &= 7.86\% \end{aligned}$$

## 3.7 Choosing Between Projects with Different Life Spans

Sometimes our capital budgeting choices involve projects with different life spans. Suppose your company is considering buying one of two tank trucks to haul high-tech liquid materials. The company is trying to decide between two alternatives:

- Truck A is a relatively cheap truck. It costs $100,000 and has a 6-year life, during which it will produce an annual cash flow of $150,000.
- Truck B is much more expensive. It costs $250,000 and has only a 3-year life, after which it has to be replaced. However, truck B is much more efficient than truck A, and during each of the 3 years of its life, it produces a cash flow of $300,000.

If your company's discount rate is 12%, which truck should it choose? Here's a simple (and, as it turns out, misleading) way of doing the analysis.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **DIFFERENT LIFE SPANS** | | | |
| 2 | Discount rate | 12% | | |
| 3 | | | | |
| 4 | **Year** | **Track A** | **Track B** | |
| 5 | 0 | –100 | –250 | |
| 6 | 1 | 150 | 300 | |
| 7 | 2 | 150 | 300 | |
| 8 | 3 | 150 | 300 | |
| 9 | 4 | 150 | | |
| 10 | 5 | 150 | | |
| 11 | 6 | 150 | | |
| 12 | | | | |
| 13 | **NPV** | 516.71 | 470.55 | <– =C5+NPV($B$2,C6:C11) |

Using this analysis, you might conclude that truck A is preferable to truck B, since its NPV is higher. But because the two trucks have different life spans, there's a problem concluding that A is preferred to B. To make them comparable, we assume that at the end of year 3 we will replace truck B with another, similar truck. This makes the year 3 cash flow:

$$\textit{Year 3 cash flow}: \underbrace{300}_{\substack{\text{Year 3 cash flow}\\\text{from truck}}} - \underbrace{250}_{\substack{\text{Purchase price}\\\text{of new truck}}} = 50$$

Once we've replaced truck B in year 3, the cash flows in years 4, 5, and 6 will be $300. We can put this into a spreadsheet:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **DIFFERENT LIFE SPANS**<br>**At end of year 3, truck B is replaced** | | | |
| 2 | Discount rate | 12% | | |
| 3 | | | | |
| 4 | **Year** | **Cash flow (A)** | **Cash flow (B)** | |
| 5 | 0 | –100 | –250 | |
| 6 | 1 | 150 | 300 | |
| 7 | 2 | 150 | 300 | |
| 8 | 3 | 150 | 50 | <– =300-250 |
| 9 | 4 | 150 | 300 | |
| 10 | 5 | 150 | 300 | |
| 11 | 6 | 150 | 300 | |
| 12 | | | | |
| 13 | **NPV** | 516.71 | 805.48 | <– =C5+NPV($B$2,C6:C11) |

As you can see in cells B13 and C13, the NPV from the 2 (now comparable) projects indicates that B is preferred to A.

There's another way to reach this same conclusion: Look at the following calculations:

$$NPV(\text{A}) = -100 + \frac{150}{(1.12)} + \frac{150}{(1.12)^2} + \frac{150}{(1.12)^3} + \frac{150}{(1.12)^4} + \frac{150}{(1.12)^5} + \frac{150}{(1.12)^6} = 516.71$$
$$= \sum_{t=1}^{6} \frac{125.68}{(1.12)^t}$$
$$NPV(\text{B}) = -250 + \frac{300}{(1.12)} + \frac{300}{(1.12)^2} + \frac{300}{(1.12)^3} = 470.55$$
$$= \sum_{t=1}^{3} \frac{195.91}{(1.12)^t}$$

What these calculations show is that truck A is equivalent to getting a constant cash flow of \$125.68 per year for each of the 6 years of its life, whereas truck B is equivalent to getting a constant cash flow of \$195.91 for each of its 3 years of life. We call these cash flow the *equivalent annuity cash flow* (EAC). Since every time you buy truck B you get \$195.91 per year and every time you buy truck A you get \$125.68 per year, it is clear that truck B is preferred.

The EAC is easy to compute. It is defined as a constant future cash flow whose present value is equal to the net present value of the project:

$$NPV = CF_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t} = \sum_{t=1}^{N} \frac{Equivalent\ annuity\ cash\ flow\ (EAC)}{(1+r)^t}$$

where $N$ is the project life. We rearrange this equation a bit and use the Excel function **PMT** to compute the EAC:

$$EAC = \frac{NPV = CF_0 + \sum_{t=1}^{N} \frac{CF_t}{(1+r)^t}}{\sum_{t=1}^{N} \frac{1}{(1+r)^t}} = \mathbf{-PMT}(r, N\ periods, NPV)$$

We implement this formula in the following spreadsheet:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **DIFFERENT LIFE SPANS**<br>**Computing the equivalent annuity cash flow (EAC)** | | | |
| 2 | Discount rate | 12% | | |
| 3 | | | | |
| 4 | **Year** | **Cash flow (A)** | **Cash flow (B)** | |
| 5 | 0 | –100 | –250 | |
| 6 | 1 | 150 | 300 | |
| 7 | 2 | 150 | 300 | |
| 8 | 3 | 150 | 300 | |
| 9 | 4 | 150 | | |
| 10 | 5 | 150 | | |
| 11 | 6 | 150 | | |
| 12 | | | | |
| 13 | **NPV** | 516.71 | 470.55 | <– =C5+NPV($B$2,C6:C11) |
| 14 | EAC—Equivalent annuity cash flow | 125.68 | 195.91 | <-- =–PMT(B2,3,C13) |
| 15 | | | | |
| 16 | =–PMT(B2,6,B13) | | | |

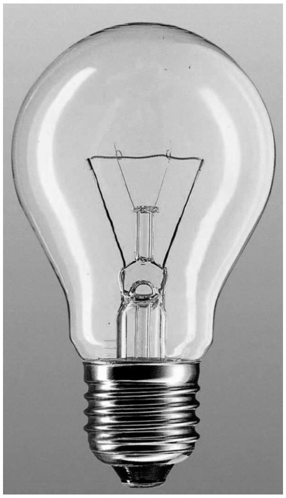
## A Nontrivial Example of Different Life Spans

This business of the EAC may seem somewhat academic and ethereal, but it's not. In this section, we offer a real-life example which can only be solved using the EAC.

You're considering replacing the light bulbs in a hotel you own. Currently you're using 100-watt incandescent bulbs, which cost \$1 each and have an average lifetime of 1,000 hours. You're thinking of replacing them with compact fluorescent bulbs. These are much more expensive, costing \$5 each. But they produce the same luminescence, use only 15 watts, and last for 10,000 hours. Here are some additional facts:

- A kilowatt of electricity costs \$0.10.
- You tend to burn a light bulb 250 hours per month.
- The interest rate is 8%. In the computations below, we translate this to a monthly interest rate of $0.643\% = (1+8\%)^{1/12} - 1$.

Should you replace the bulbs?



Standard incandescent bulb—cheap to buy, expensive to operate, short life.


Energy-saving fluorescent bulb—expensive to buy, cheap to operate, long life.

**FIGURE 3.1:** Standard bulbs versus energy-saving fluorescents.

This problem can be readily solved using the equivalent annuity cash flow (EAC):

| | A | B | C |
|---|---|---|---|
| 1 | **LIGHT BULBS**<br>**Choosing between cheap incandescents and expensive fluorescents** | | |
| 2 | Annual discount rate | 8% | |
| 3 | Monthly discount rate | 0.643% | <-- =(1+B2)^(1/12)–1 |
| 4 | Electricity cost per kilowatt<br>(a kilowatt = 1,000 watts) | 0.10 | |
| 5 | | | |
| 6 | **Incandescent bulb** | | |
| 7 | Watts | 100 | |
| 8 | Cost | $1.00 | |
| 9 | Hours per month used | 250 | |
| 10 | Lifetime of bulb (hours) | 1,000 | |
| 11 | Lifetime in months | 4 | |
| 12 | Monthly cost | 2.50 | <-- =B9*$B$4*B7/1,000 |
| 13 | **NPV** of lifetime use | 10.84 | <-- =B8+PV(B3,B11,–B12) |
| 14 | Monthly equivalent annuity cash flow<br>(EAC) for cheap incandescent | 2.75 | <-- =-PMT(B3,B11,B13) |
| 15 | | | |
| 16 | **Equivalent fluorescent bulb** | | |
| 17 | Watts | 15 | |
| 18 | Cost | $5.00 | |
| 19 | Hours per month used | 250 | |
| 20 | Lifetime of bulb (hours) | 10,000 | |
| 21 | Lifetime in months | 40 | |
| 22 | Monthly cost | 0.38 | <-- =B19*$B$4*B17/1,000 |
| 23 | **NPV** of lifetime use | 18.19 | <-- =B18+PV(B3,B21,–B22) |
| 24 | Monthly equivalent annuity cash flow<br>(EAC) for expensive fluorescent | 0.52 | <-- =-PMT(B3,B21,B23) |

This spreadsheet requires some additional explanation:

- An incandescent bulb costs \$1.00 to buy and \$2.50 per month to operate. As shown in cell B13, the NPV of buying and operating one incandescent bulb during its 4-month life is

$$1.00+\frac{2.50}{1+0.643\%}+\frac{2.50}{(1+0.643\%)^2}+\frac{2.50}{(1+0.643\%)^3}+\frac{2.50}{(1+0.643\%)^4}=10.84$$

- A fluorescent bulb costs \$5.00 to buy and costs \$0.38 per month to operate. As shown in cell B23, the NPV of buying and operating one fluorescent bulb during its 40-month life is

$$5.00+\frac{0.38}{1+0.643\%}+\frac{0.38}{(1+0.643\%)^2}+\cdots+\frac{0.38}{(1+0.643\%)^{40}}=18.19$$

- To find the monthly equivalent annuity cash flow (EAC) of each bulb, we divide the NPV of the bulb's cost and operation by the appropriate PV factor:

$$\textit{Incandescent EAC}=\frac{10.84}{\sum_{t=1}^{4}\frac{1}{(1.00684)^t}}=-\mathbf{PMT}(0.643\%,4,10.84)=2.75/\text{month}$$

$$\textit{Fluorescent EAC}=\frac{18.19}{\sum_{t=1}^{40}\frac{1}{(1.00684)^t}}=-\mathbf{PMT}(0.643\%,40,18.19)=0.52/\text{month}$$

- As you can see, the monthly equivalent annuity cash flow of the incandescent light bulb is \$2.75, whereas the monthly equivalent annuity cash flow of the fluorescent bulb is \$0.52. The EAC tells you that it's *much cheaper* to switch to the fluorescent!

## 3.8 Prioritizing Projects Using the Profitability Index

The NPV rule and the EAC rule are suitable for mutually exclusive projects. We will use the rule when we need to choose a project from a set of projects that are offered to us or when we are facing a "yes–no" investment decision.

However, in many cases, we face a set of projects that can be taken at the same time, meaning they are not mutually exclusive. In this case, we would like to prioritize the projects since often we face a resource constraint regarding those projects. Usually the resource constraint is a budget constraint, but it may also result from limited management availability, manufacturing capacity, and other such constraints.

In this case, we would like to maximize the profit from the limiting resource. The ranking should be according to the profitability index:

$$Profitability\ Index\ (PI) = \frac{NPV}{Required\ resource}$$

The result is the profitability in present value for every unit of resource. So if, for example, we face three projects as follows (assume $r = 10\%$):

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **PROFITABILITY INDEX EXAMPLE** | | | | | |
| 2 | Discount rate, r | 10% | | | | |
| 3 | | | | | | |
| 4 | **Project** | **Investment (t=0)** | **t=1** | **t=2** | **NPV** | |
| 5 | A | –100 | 150 | 170 | 176.86 | <-- =B5+NPV($B$2,C5:D5) |
| 6 | B | –120 | 200 | 140 | 177.52 | |
| 7 | C | –150 | 300 | 300 | 370.66 | |
| 8 | | | | | | |

Now assume we have a resource limit, namely, management attention. We only have 50 hours per week, Project A requires 30 management hours per week, Project B requires 20 hours per week, and Project C requires 50 hours per week. How should we allocate the management attention to the projects? Using the profitability index measure, we get that Project A generates $5.90 per hour invested, Project B generates $8.88 per hour, and Project C generates $7.41 per hour. It is clear that the ranking of the projects should be: B, C, A. In order to maximize our welfare, we should allocate the first 20 hours per week to Project B, and then the remaining 30 hours to Project C. Since project C requires 50 hours, we need to find a partner to invest another 20 hours in the project, and we will actually get 30/50 = 60% of Project C and our partner will get 40% (20/50) of the project.

## PI and Budget Constraint

In the case of a budget constraint, we get

$$\textit{Profitability index (PI)} = \frac{NPV}{\textit{Required budget (investment)}}$$

So in our example, assuming we have a budget constraint of \$200, we get:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | | | **PROFITABILITY INDEX EXAMPLE** | | | |
| 2 | Discount rate, r | 10% | | | | |
| 3 | | | | | | |
| 4 | **Project** | **Investment (t=0)** | **t=1** | **t=2** | **NPV** | |
| 5 | A | –100 | 150 | 170 | 176.86 | <-- =B5+NPV($B$2,C5:D5) |
| 6 | B | –120 | 200 | 140 | 177.52 | |
| 7 | C | –150 | 300 | 300 | 370.66 | |
| 8 | | | | | | |
| 9 | | | <-- =E5 | | | |
| 10 | **Project** | **NPV** | **Resource requirement (hours/week)** | **PI** | **Rank** | |
| 11 | A | 176.86 | 30 | \$5.90 | 3 | <-- =RANK(D11,$D$11:$D$13) |
| 12 | B | 177.52 | 20 | \$8.88 | 1 | <-- =RANK(D12,$D$11:$D$13) |
| 13 | C | 370.66 | 50 | \$7.41 | 2 | <-- =RANK(D13,$D$11:$D$13) |
| 14 | | <-- =B11/C11 | | | | |
| 15 | | | | | | |

Using the profitability index measure, we get that Project A generates \$1.77 per dollar invested, Project B generates \$1.48 per dollar invested, and Project C generates \$2.47 per dollar invested. It is clear that the ranking of the projects should be C, A, B. In order to maximize our welfare, we should invest the first \$150 in Project B, and then the remaining \$50 in Project A. Since Project A requires \$100, we need to find a partner to invest another \$50 in the project, and we will actually get 50/1,000 = 50% of Project A and our partner will get 50% (50/100) of the project.

## Summary

The two basic ways of evaluating investments are NPV and IRR. NPV, which measures the *wealth increment* that an investment gives to the investor, is our instrument of choice. However, in many situations, IRR, which measures the compound return of the project, is a convenient method of evaluation.

This chapter shows that in many cases, for the “yes–no” decision—the question of whether a particular investment should be undertaken—the NPV and the IRR both give the same answers. However, the *ranking* of projects using NPV and IRR can be contradictory, and in this case, we recommend using the NPV criterion as having more economic logic than the IRR.

The chapter also examines various problems with the IRR: The IRR for a borrowing and a lending project is the same, and projects with cash flow changes can produce multiple IRRs. A final problem is that the IRR implicitly assumes that all intermediate cash flows are reinvested at the project's IRR. To this final problem, we have offered Excel's **MIRR** function as a solution—this function allows us to specify the reinvestment rate for the intermediate cash flow.

Finally, the chapter investigates two additional issues in project valuation: (a) the case of comparing projects with different life spans and (b) the profitability index.

## Exercises

**Note:** The data for these exercises can be found on the Benninga, *Principles of Finance with Excel,* Third Edition companion website (www.oup.com/us/Benninga).

**1.** (NPV and IRR) You are offered the following project:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | Discount rate | 6% | | | | | |
| 2 | | | | | | | |
| 3 | Time | 1 | 2 | 3 | 4 | 5 | 6 |
| 4 | Cash flow | –1,000 | 100 | 200 | 300 | 300 | 300 |

**a.** What is the NPV of the project at a discount rate $r = 6\%$?

**b.** Do you want to invest in the project at $r = 6\%$?

**c.** Would you change your mind if $r = 4\%$?

**d.** What is the IRR of the project?

**2.** (NPV and IRR) You are considering a project whose cash flows are given below:

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Discount rate, r | 25% | | | | | | | |
| 2 | | | | | | | | | |
| 3 | Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 4 | Cash flow | –1,000 | 100 | 200 | 300 | 400 | 500 | 600 | 700 |

**a.** Calculate the present value of the future cash flows of the project.

**b.** Calculate the project's net present value.

**c.** Calculate the project's internal rate of return.

**d.** Should you undertake the project?

**3.** (NPV or IRR) You are considering buying an asset that has a 3-year life and costs $15,000. As an alternative to buying the asset, you can lease it for four payments of $4,000. The first payment is due on the day you sign the lease, and the next three payments are due at the end of each following year. If you can borrow from your bank at 10%, should you lease or buy?

**4.** (NPV or IRR) You are considering buying an asset that has a 3-year life and costs $2,000. As an alternative to buying the asset, you can lease it for $600 per year; the first payment is due today and the following three payments at the end of the next years. Your bank is willing to lend you money for 15%.

**a.** Should you lease or purchase the asset?

**b.** What is the largest lease payment you would be willing to make?

**5.** (NPV and IRR) You work in a company that sells furniture. The company is considering a new marketing campaign. The marketing campaign cost is $1M to be paid immediately. You expect that as a result of the campaign, the company will increase its market share and will generate additional annual cash flows of $150,000 forever, starting 1 year from now.

**a.** If your company's cost of capital (the discount rate) is 10%, should it undertake the marketing campaign? Explain.

**b.** What is the marketing campaign's IRR?

**6.** (NPV and IRR) You are offered the following three projects:

| | A | B | C |
|---|---|---|---|
| 2 | **Project** | **Cash Flow Today ($)** | **Cash Flow Next Year** |
| 3 | A | –20 | 40 |
| 4 | B | 30 | –15 |
| 5 | C | 7 | 10 |

Assume that the cost of capital (interest rate) is 12%.

**a.** Find the NPV of each project?

**b.** If you can choose only one project, which one should you choose?

**c.** How would your answer to question b change if the cost of capital is 5%?

**d.** What is the range of cost of capital in which you will invest in Project A?

**7.** (NPV, IRR, IRR, crossover point) Your firm is considering two projects with the following cash flows:

| | A | B | C |
|---|---|---|---|
| 2 | Discount rate | 12% | |
| 3 | | | |
| 4 | **Year** | **Project A** | **Project B** |
| 5 | 0 | –500 | –500 |
| 6 | 1 | 180 | 200 |
| 7 | 2 | 175 | 250 |
| 8 | 3 | 160 | 170 |
| 9 | 4 | 100 | 25 |
| 10 | 5 | 100 | 30 |

**a.** If the appropriate discount rate is 12%, rank the two projects.

**b.** Which project is preferred if you rank by IRR?

**c.** Calculate the crossover rate, namely, the discount rate $r$ at which the NPVs of both projects are equal.

**d.** Should you use NPV or IRR to choose between the two projects? Give a brief discussion.

**8.** (NPV and IRR with alternative costs) Mr. Fox has just bought an old apartment for $1,000,000. Mr. Fox plans to rent out the apartment. The apartment in its current condition can be rented out for $3,500 at the end of each month forever.

An alternative to renting out the apartment in its current condition is to do a renovation. The renovation is expected to last 12 months (during this period Fox cannot rent the apartment), and the renovation cost is expected to be $50,000 today and an additional $2,000 at the end of each month for 12 months. After the renovation Mr. Fox can rent out the apartment for $4,000 per month.

Assume that Mr. Fox's cost of capital is 0.5% monthly.

**a.** Should Mr. Fox invest in the apartment renovation?

**b.** What is the yearly alternative cost of capital that will make Mr. Fox indifferent between doing, or not doing, the renovation?

**c.** What is the monthly rent after the renovation that will make Mr. Fox indifferent between doing, or not doing, the renovation?

**9.** (Multiple IRRs) You observe the following project in the market. Your cost of capital is 10%:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | Discount rate, r | 10% | | | |
| 2 | | | | | |
| 3 | Year | 0 | 1 | 2 | 3 |
| 4 | Cash flow | –150 | 1,000 | –1,000 | 100 |

**a.** Should you take the project or reject it? Explain.

**b.** What is the range of cost of capital for which you should take the project? Explain. (**Hint:** Start looking for IRRs from 10%—the default starting point in Excel—and from 1,000%).

**10.** (MIRR) For the question above, what is the modified IRR (MIRR) of the project assuming the "reinvestment or financing rate" is also the discount rate (10%)?

**11.** (Multiple IRRs) You would like to open an investment company named Rip'em. The cost of building up the company is $750,000. You offer your clients the following deal: For five annual deposits of $500,000 for the next 5 years, you will pay them $2 million at year 6. Your cash flow is expected to be as follows:

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | Year | **0** | **1** | **2** | **3** | **4** | **5** | **6** |
| 2 | Cash flow | –750,000 | 500,000 | 500,000 | 500,000 | 500,000 | 500,000 | –2,000,000 |

**a.** If your cost of capital is 5%, should you make the investment?

**b.** If your cost of capital is 10%, should you make the investment?

**c.** If your cost of capital is 45%, should you make the investment?

**d.** For what range of cost of capital should you accept the project? Explain.

**12.** (MIRR) For the question above, if your "reinvestment or financing rate" is 15%, what is the modified IRR?

**13.** (Mutually exclusive projects) The following projects are offered to you. The annual cash flows are expected to continue forever.

| | A | B | C |
|---|---|---|---|
| 1 | **Project** | **Investment** | **Annual perpetual cash flow** |
| 2 | A | –1,000 | 100 |
| 3 | B | –600 | 80 |

a. What is the IRR of each project?

b. If the cost of capital is 7%, which one will you choose?

c. If the cost of capital is 4%, which one will you choose?

d. What is the cost of capital at which you are indifferent between the two projects?

e. What is the range of interest rate in which you will prefer Project B over A? and what is the range in which you will prefer Project A over B?

f. What is the range of interest rate in which you will reject both projects?

**14.** (IRR) You are offered the following two projects:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | **Year 0** | **Year 1** | **Year 2** | **Year 3** |
| 2 | Project A | –100 | 0 | 0 | 172.8 |
| 3 | Project B | 50 | –75 | 0 | 0 |

a. What is the IRR of Project A?

b. What is the IRR of Project B?

c. If your cost of capital is 10%, which project should you choose?

**15.** (NPV curves) A company would like to invest in one of two mutually exclusive projects (A or B). The projects have negative cash flow at time 0 and positive cash flows later on. The project's IRRs are 15% for Project A and 20% for Project B. Additionally, their NPV charts intersect.

Determine whether each of the following statements is true or false and provide an explanation:

a. An investor who requires a 0% rate of return will be indifferent regarding the choice of project.

b. An investor who requires a 10% rate of return will necessarily prefer Project A.

c. An investor who requires a 17% rate of return will necessarily prefer Project B.

d. An investor who requires a 22% rate of return will shy away from both projects.

e. It cannot be determined which project an investor who requires a 15% rate of return will choose without knowing the projects NPV.

**16.** (Lease vs. purchase) You're considering leasing or purchasing an asset with the following cash flows:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **LEASE VERSUS PURCHASE WITH RESIDUAL VALUE** | | | |
| 2 | Asset cost | 20,000 | | |
| 3 | Annual lease payment | 5,500 | | |
| 4 | Residual value, year 3 | 4,000 | <-- Value of asset at end year 3 | |
| 5 | Bank rate | 15% | | |
| 6 | | | | |
| 7 | **Year** | **Purchase cash flow** | **Lease cash flow** | |
| 8 | 0 | 20,000 | 5,500 | |
| 9 | 1 | | 5,500 | |
| 10 | 2 | | 5,500 | |
| 11 | 3 | –4,000 | 5,500 | |

**a.** Calculate the present value of the lease versus the purchase. Which is preferable?

**b.** What is the largest annual lease payment you would be willing to pay that will make the leasing more attractive?

**17.** (Lease vs. purchase) You intend to buy a new laptop. Its price at electronic shops is \$2,000, but your next-door neighbor offered to lease you the same computer for monthly payments of \$70 for a 24-month period, with the first payment made today. Assuming you can sell the computer at the end of 2 years at \$500, and assuming that the annual interest rate in the market is 20%, should you buy or lease?

**18.** (Car leasing) You're considering leasing or purchasing a car. The details of each method of financing are given below:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **AUTO LEASE VERSUS PURCHASE** | | | | | | |
| 2 | MSRP | 50,000.00 | <-- Manufacturer's suggested retail price | | | | |
| 3 | Capitalized cost | 45,000.00 | <-- Negotiated price, the actual price you pay for the car | | | | |
| 4 | Destination charge | 415.00 | <-- Paid both by the lessee and the buyer | | | | |
| 5 | Acquisition fee | 450.00 | <-- Paid only by the lessee | | | | |
| 6 | Security deposit | 450.00 | <-- Refunded at end of lease | | | | |
| 7 | | | | | | | |
| 8 | Payment due at signing | 1,315.00 | | | | | |
| 9 | Monthly payment | 600.00 | <-- Dealer's lease offer | | | | |
| 10 | | | | | | | |
| 11 | Residual value after 2 years as % of MSRP | 60% | | | | | |
| 12 | Lease residual value after 2 years | 30,000.00 | <-- =B11*B2 | | | | |
| 13 | Your estimated residual value | 35,000.00 | <-- Your guess | | | | |
| 14 | | | | | | | |
| 15 | Bank loan cost (annual) | 7% | | | | | |

**a.** The lease is for 24 months. What should you do?

**b.** What will be your answer if the estimated market value of the car at the end of the lease is \$10,000? Show a data table with your answer for market value of the car at the end of the lease between \$10,000 and \$18,000 jumping by \$500.

**19.** (NPV and PI) You are offered the following four single projects (there is only one of each project):

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **Project** | **0** | **1** | **2** | **3** |
| 2 | A | –200 | 0 | 0 | 500 |
| 3 | B | –50 | 40 | 40 | 40 |
| 4 | C | –75 | 45 | 45 | 70 |
| 5 | D | –100 | 0 | 0 | 133.1 |

Assume that the company is indifferent between taking and rejecting Project D.

**a.** Calculate the IRR of Project D.

**b.** Assume that the projects can be done *only* once. If you can choose *only* one of the projects above (i.e., the projects are mutually exclusive), which one will you choose?

**c.** Assume that you can take more than one project at the same time and you have no budget constraints (and the projects are independent of each other). What will you do?

**d.** Assume you can take more than one project at the same time. Also assume that you can invest in a fraction of a project (you do not have to buy the entire project). You have only \$200. What will you do?

**20.** (EAC) Your friend is not sure whether she should buy or lease a car. She presents you the following financial information:

- The car of her dreams costs \$20,000. The car is expected to be replaced every 5 years. At the end of the fifth year, the car is expected to be sold at \$5,000. Since she likes the car so much, she will replace it with an exactly similar car at the same cost.
- She can lease the same car. The leasing cost is \$350 to be paid at the end of each month and is expected to stay at the same level forever.
- The cost of capital is 0.5% per month.

What should you recommend your friend (buying or leasing)? Explain.

**21.** (NPV, EAC) Emma, the CFO of a company, would like to purchase a new car. She intends to replace the car every few years. The annual expected dollar costs of the car are given below:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | Car cost | 10,000 | | | | |
| 2 | Cost of capital | 5% | | | | |
| 3 | | | | | | |
| 4 | | **Year 1** | **Year 2** | **Year 3** | **Year 4** | **Year 5** |
| 5 | Registration and city taxes | 100 | 100 | 100 | 100 | 100 |
| 6 | Insurance | 600 | 500 | 450 | 450 | 450 |
| 7 | Fuel | 600 | 650 | 700 | 750 | 800 |
| 8 | Repairs (first 3 years are paid by the dealer) | 0 | 0 | 0 | 500 | 600 |
| 9 | Market value of car at the end of the year | 8,500 | 7,500 | 6,700 | 6,000 | 5,400 |

Also assume:

- All costs are incurred at the end of the year.
- A new car costs $10,000.
- The above figures are expected to remain constant in real value.
- Emma's annual real cost of capital is 5%.
- You can ignore the effects of taxes in your analysis.

Answer the following questions:

**a.** How often should Emma replace the car?

**b.** A leasing company offers Emma the same car under a leasing agreement. What is the highest yearly lease that Emma is willing to pay?

**22.** (NPV, IRR, EAC, PI, challenging) Your company is considering an investment in the following projects (cash flows are in thousand dollars).

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **Project** | **Year 0** | **Year 1** | **Year 2** | **Year 3** | **Year 4** |
| 2 | A | –800 | 200 | ? | 200 | 200 |
| 3 | B | –600 | 500 | 350 | 0 | 150 |
| 4 | C | ? | 250 | 150 | 0 | 0 |
| 5 | D | –400 | ? | 0 | 0 | 0 |
| 6 | E | –500 | 0 | 450 | 300 | 0 |

The CFO of your company has calculated the following values for the projects:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 9 | **Project** | **NPV** | **IRR** | **PI** | **EAV** |
| 10 | A | –229 | ? | 0.29 | ? |
| 11 | B | ? | 36.11% | ? | ? |
| 12 | C | ? | 69.30% | ? | ? |
| 13 | D | ? | ? | 0.3 | 140 |
| 14 | E | ? | ? | ? | ? |

Assume the following:

- All cash flows presented in the table are received on the last day of the year.
- You can invest in fraction of projects—for example, you can invest half of the cost and receive half of the cash flows.
- All the projects carry the same risk and should be discounted with the same discount rate.

Answer the following questions:

**a.** Use Project D to show that the discount rate (cost of capital) is 16.67% per annum.

**b.** Complete the missing values from the table.

**c.** Which project should the company choose if the projects are mutually exclusive?

**d.** Which project should the company choose if the projects are not mutually exclusive and the company faces a $1,000K budget constraint?

# 4 Loans and Amortization Tables

## Chapter Contents



## Overview

This chapter discusses loans. A loan is a debt agreement between a borrower and a lender. Here is the relevant terminology:

- **Principal:** The amount lent.
- **Term:** The number of years or periods over which the loan is made.
- **Loan terms:** Specify when payments of interest and repayments of principal are made.
- **Loan covenants:** The lender may specify restrictions on the behavior of the borrower until the loan is fully repaid. *For example:* The lender may specify that the borrower cannot take out any additional loans.
- **Loan amortization table:** A table specifying, for each payment date, the amount paid off on the loan and the split of this amount between interest and repayment of principal.

In Sections 4.8 and 4.9, we contrast and compare two additional concepts:

- **Loan contractual value:** This is the principal of the loan. Since a loan is a contract between the borrower and lender, its contractual value is the amount lent. When a loan is paid off over time, the contractual value declines. The loan contractual value at time *t* is the present value of the future payments of the loan where the discount rate is the loan interest rate.
- **Loan market value:** When interest rates change, the value of the loan may change. The loan's market value at time *t* is the present value of the future payments of the loan where the discount rate is the time-*t* prevailing market interest rate.

| Finance concepts discussed | Excel functions used |
|---|---|
| • Loans | • **PMT** |
| • Amortization tables | • **PV** |
| • Outstanding loan balance and payment of a loan | • Amortization table template |
| • Loan types ("interest-only loan," "equal amortization term loan," "mortgage loan," "balloon loan," and "bullet loan") | • **NPV** |
| • Loan contractual value | • **IPMT** |
| • Loan market value | • **PPMT** |
| • Refinancing a loan | • **SUM** |

## 4.1 Why Take a Loan?

There are two main reasons to use loans:

1. A loan entails the transfer of funds throughout time. If the borrower is more "time impatient" than the lender, meaning that the borrower would prefer the funds today more than the fund plus interest tomorrow and the lender would prefer the funds plus interest tomorrow more than the funds today. It is better for both of them to enter into a loan agreement.
2. Taking a loan (or, in financial jargon, "leveraging") can amplify the outcome of the uncertain transaction (for better or worse). Here is a simple example:

   Assume you are interested in buying a box and selling it 1 year later. The price of the box today is \$100 and is expected to be \$110 when you sell it. Your expected profit is \$10, which represents 10% return on your investment.

| | A | B | C | D |
|---|---|---|---|---|
| 2 | **No loan (unlevered)** | **Action** | **Cash flow** | |
| 3 | Today (t=0) | Purchase a box | –100.00 | |
| 4 | One year from now (t=1) | Sell the box | 110.00 | |
| 5 | | Cash flow in 1 year | 110.00 | <-- =C4 |
| 6 | | Profits as % of initial investment | 10.00% | <-- =C5/–C3–1 |

Now assume you can take a 1-year loan of $60 at 5%. This means that you can lower your own investment to $40. As the computation below shows, taking the loan (in the jargon of finance, "levering the transaction") raises your return to 17.5% (almost doubled compared to the unlevered investment).

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **LEVERAGE AMPLIFIES RETURNS<br>In this example: good outcomes become great with a loan** | | | |
| 8 | **$60 loan (levered)** | **Action** | **Cash flow** | |
| 9 | Today (t=0) | Purchase 1 box with own capital | –100.00 | |
| 10 | | Borrow 60 at 5% to finance the transaction | 60.00 | |
| 11 | | Net investment | –40.00 | <-- =C9+C10 |
| 12 | One year from now (t=1) | Sell the box | 110.00 | |
| 13 | | Pay back the loan | –63.00 | <-- =–C10*(1+5%) |
| 14 | | Cash flow in 1 year | 47.00 | <-- =SUM(C12:C13) |
| 15 | | Profit as % of initial investment | 17.50% | <-- =C14/–C11–1 |

## A Variation on the Leveraged Transaction

In the previous example, you used the loan to decrease your own investment from $100 to $40. Perhaps you want to use the leverage to increase the size of the investment: You're going to take a $900 loan and invest $100 of your own capital into the box purchase. This way you will be able to buy 10 boxes instead of 1. As the computation below shows, this raises your return to 55% (more the five times as much compared to the unlevered investment):

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **LEVERAGE AMPLIFIES RETURNS<br>In this example: good outcomes become great with a loan** | | | |
| 17 | **$900 loan (levered)** | **Action** | **Cash flow** | |
| 18 | Today (t=0) | Purchase 1 box with own capital | –100.00 | |
| 19 | | Borrow $900 to purchase 9 more boxes | 900.00 | |
| 20 | One year from now (t=1) | Sell the 10 boxes | 1,100.00 | |
| 21 | | Pay back the loan | –945.00 | <-- =–C19*(1+5%) |
| 22 | | Cash flow in 1 year | 155.00 | <-- =SUM(C20:C21) |
| 23 | | Profits as % of initial investment | 55.00% | <-- =C22/–C18–1 |

## A Word of Caution

Taking a loan "amplifies the outcome of the transaction": A loan will generally make a good transaction better and make a bad transaction worse. In the Excel examples below, we examine the bad results of two kinds of leveraged transactions:

- In the first leveraged transaction you borrow $60 to decrease your own investment in the box from $100 to $40. You had planned to sell the box in 1 year for $110, but in fact, the sale price turns out to be $90. The profit as a percentage of the initial investment is –32.5%.

- In the second leveraged transaction you invest \$100 of your own money plus \$900 of borrowed funds to buy 10 boxes. When the transaction goes bad (the sale price of the boxes is \$90 per box), your profit percentage is –145%.

**The message:** More leverage causes more extreme profit margins, both for good and for bad outcomes. Here is the computation of the returns, without leverage and with leverage:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **LEVERAGE AMPLIFIES RETURNS<br>In these examples: bad outcomes become worse with a loan** | | | |
| 2 | **No loan (unlevered)** | **Action** | **Cash flow** | |
| 3 | Today (t=0) | Purchase a box | –100.00 | |
| 4 | One year from now (t=1) | Sell the box | 90.00 | |
| 5 | | Cash flow in 1 year | 90.00 | <-- =C4 |
| 6 | | Profit as % of initial investment | –10.00% | <-- =C5/–C3–1 |
| 7 | | | | |
| 8 | **\$60 loan (levered)** | **Action** | **Cash flow** | |
| 9 | Today (t=0) | Purchase 1 box with own capital | –100.00 | |
| 10 | | Borrow 60 at 5% to finance the transaction | 60.00 | |
| 11 | | Net investment | –40.00 | <-- =C9+C10 |
| 12 | One year from now (t=1) | Sell the box | 90.00 | |
| 13 | | Pay back the loan | –63.00 | <-- =-C10*(1+5%) |
| 14 | | Cash flow in 1 year | 27.00 | <-- =SUM(C12:C13) |
| 15 | | Profit as % of initial investment | –32.50% | <-- =C14/–C11–1 |
| 16 | | | | |
| 17 | **\$900 loan (levered)** | **Action** | **Cash flow** | |
| 18 | Today (t=0) | Purchase 1 box with own capital | –100.00 | |
| 19 | | Borrow \$900 to purchase 9 more boxes | 900.00 | |
| 20 | One year from now (t=1) | Sell the 10 boxes | 900.00 | |
| 21 | | Pay back the loan | –945.00 | <-- =–C19*(1+5%) |
| 22 | | Cash flow in 1 year | –45.00 | <-- =SUM(C20:C21) |
| 23 | | Profits as % of initial investment | –145.00% | <-- =C22/–C18–1 |

## 4.2. Amortization Table

Amortization refers to the process of paying off a loan over time. A portion of each periodic payment is for interest, while the remaining amount is applied toward the loan's principal balance. The amortization table determines the breakdown of each payment between interest and the repayment of the loan principal.

Every amortization table has the following characteristics:

1. The loan balance at the beginning of the first year of the loan is the initial loan amount.
2. The periodic interest payment is calculated as

$$\left(Interest\ payment\right)_t = r \times \left(Current\ year's\ beginning\ balance\ of\ loan\right)_t$$

3. The total payment is the sum of the interest payment and the principal payment.
4. The ending balance of the loan is calculated as

$$(Ending\ balance)_t = (Beginning\ balance)_t - (Principal\ repayment)_t$$

5. The beginning balance of any other payment is the ending balance of the loan at the previous year. Formally:

$$(Beginning\ balance)_t = (Ending\ balance)_{t-1}$$

6. The ending balance of the loan after the last payment should equal zero. The loan was fully amortized.

## The Loans Discussed

Different kinds of loans have different patterns of amortization. In succeeding sections we build amortization tables for the following loans:

- **Interest-only loan:** In every period, the borrower pays the periodic interest on the loan. The principal on an interest-only loan is repaid at the end of the terminal year.
- **Equal amortization term loan:** In every period, the borrower repays an equal amount of the loan principal. The interest in each period is paid on the outstanding principal at the beginning of the period.
- **Term loan:** This loan is also called a "mortgage loan" or "an equal payment term loan." An ordinary term loan has equal payments in every period. The breakdown of the payments between interest and repayment of principal varies by period.
- **Balloon loan:** Until the last payment of principal, the periodic payments on a balloon loan are relatively small. The last payment of principal is large. Interest is computed on the initial loan principal balance of every year.
- **Bullet loan:** In this type of loan, the borrower does not pay any interest or make any repayment of the loan principal until the last period.

For each of these types of loans, we ask the following questions:

1. What is the loan amortization table for the loan?
2. What is the loan's remaining balance after *n* payments?
3. What is the $n^{th}$ payment of the loan, and what is the interest and principal portion of this payment?

Throughout we focus on the same example: Your bank has agreed to give you a \$100,000 loan at 8% interest. The loan is to be repaid over 10 years with annual payments. We examine the payment pattern for this example for different types of loans.

## 4.3 An Interest-Only Loan

In an interest-only loan, the borrower repays the loan principal only in the last payment. In each period, the borrower pays only the interest on the outstanding principal at the beginning of the period.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | AMORTIZATION TABLE: INTEREST-ONLY LOAN | | | | | | |
| 2 | Loan amount | \$ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Number of payments per year | 1 | | | | | |
| 5 | Loan period in years | 10 | | | | | |
| 6 | | | | | | | |
| 7 | **Payment number** | **A. Loan balance at beginning of year** | **B. Interest payment = r*A** | **C. Principal repayment** | **D. Total payment = B+C** | **E. Loan principal balance at end of year = A–C** | |
| 8 | 1 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | <-- =B8–D8 |
| 9 | 2 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 10 | 3 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 11 | 4 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 12 | 5 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 13 | 6 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 14 | 7 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 15 | 8 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 16 | 9 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 17 | 10 | 100,000.00 | 8,000.00 | 100,000.00 | 108,000.00 | 0.00 | |
| 18 | | | | | | | |
| 19 | =B2 | | | =\$B\$3*B8 | | =C8+D8 | |
| 20 | | | | | | | |

### What Is the Remaining Balance of a Loan After *n* Payments?

After every payment the remaining balance of the loan is \$100,000. This is because we do not repay any principal until the last payment.

### What Is the $n^{th}$ Payment of the Loan?

This is a simple question in this loan type, since in every period 1–9 the borrower pays only the interest: 8% of \$100,000 = \$8,000. The last payment is constructed of \$8,000 of interest and the \$100,000 loan principal for a total of \$108,000.

## 4.4 An Equal Amortization Term Loan

In an equal amortization term loan, the principal is repaid in equal amounts over the life of the loan, and the appropriate amount interest is then added on to each principal repayment.

In our example, the equal principal payment is $\frac{\$100,000}{10} = \$10,000.$

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | AMORTIZATION TABLE: EQUAL AMORTIZATION TERM LOAN | | | | | | |
| 2 | Loan amount | $ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Number of payments per year | 1 | | | | | |
| 5 | Loan period in years | 10 | | | | | |
| 6 | | | | | | | |
| 7 | Year | A. Loan balance at beginning of year | B. Interest payment = r*A | C. Principal repayment | D. Total payment = B+C | E. Loan principal balance at end of year = A–C | |
| 8 | 1 | 100,000.00 | 8,000.00 | 10,000.00 | 18,000.00 | 90,000.00 | <-- =B8–D8 |
| 9 | 2 | 90,000.00 | 7,200.00 | 10,000.00 | 17,200.00 | 80,000.00 | <-- =B9–D9 |
| 10 | 3 | 80,000.00 | 6,400.00 | 10,000.00 | 16,400.00 | 70,000.00 | <-- =B10–D10 |
| 11 | 4 | 70,000.00 | 5,600.00 | 10,000.00 | 15,600.00 | 60,000.00 | |
| 12 | 5 | 60,000.00 | 4,800.00 | 10,000.00 | 14,800.00 | 50,000.00 | |
| 13 | 6 | 50,000.00 | 4,000.00 | 10,000.00 | 14,000.00 | 40,000.00 | |
| 14 | 7 | 40,000.00 | 3,200.00 | 10,000.00 | 13,200.00 | 30,000.00 | |
| 15 | 8 | 30,000.00 | 2,400.00 | 10,000.00 | 12,400.00 | 20,000.00 | |
| 16 | 9 | 20,000.00 | 1,600.00 | 10,000.00 | 11,600.00 | 10,000.00 | |
| 17 | 10 | 10,000.00 | 800.00 | 10,000.00 | 10,800.00 | 0.00 | |
| 18 | | | | | | | |
| 19 | =B2 | =$B$3*B8 | =$B$2/10 | | | =C8+D8 | |

### What Is the Remaining Balance of an Equal Amortization Loan After $n$ Payments?

In every payment, the borrower repays $10,000 of principal. You can see the decline in the loan's principal balance (labeled "E" in the above amortization table) by this amount.

### What Is the $n^{th}$ Payment of an Equal Amortization Loan?

Suppose we want to compute the total payment on the loan in the beginning of year 7. To find the total payment, we start by calculating its components, namely, the principal payment and the interest payment.

- The principal repayment in this type of loans for each year is $100,000/10 = $10,000. Thus, at the beginning of year 7, the loan principal balance$_7$ = 100,000–6*10,000 = $40,000.

- The interest payment in each period is the 8% interest times the loan principal balance$_t$. Thus the interest payment in year 7 is 8%*40,000 = $3,200.
- The total payment at the end of year 7 is $10,000 + $3,200 = $13,200.

## 4.5 An Equal Payment Term Loan (Mortgage)

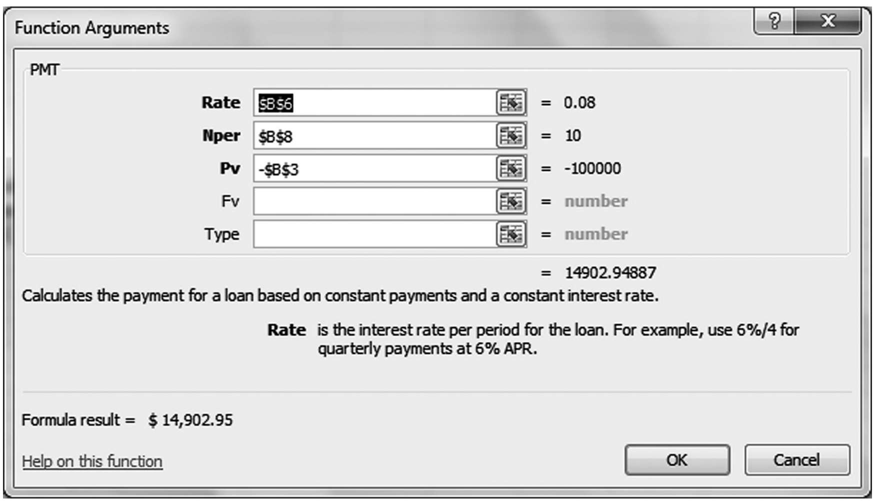
Housing is most often the largest personal asset an individual owns. Financing housing with a mortgage is something almost every reader of this book will do in his or her lifetime. Calculating and understanding the details of a mortgage is thus a useful exercise.

A mortgage is an example of an ordinary term loan, sometimes called an equal payment term loan. As the name indicates, an equal payment loan is repaid in a series of identical payments. When we calculate the amortization table for an equal payment loan, we see that the breakdown between interest and principal repayment varies with each payment. The reason an equal-payment loan is often attractive to borrowers is because the total cash outflow in each period is the same, so an individual can better plan for the future and manage his or her income vs. expenses.

The first step in any equal payment term loan is to calculate the periodic payment. Using our example of a $100,000 mortgage with 8% interest, the annual payment is $14,902.95, using Excel's **PMT** function: The **PMT** function calculates an annuity payment (a constant periodic payment) that pays off a loan:

$$\$100{,}000 = \frac{PMT}{(1+8\%)^1} + \frac{PMT}{(1+8\%)^2} + \cdots + \frac{PMT}{(1+8\%)^{10}} \Rightarrow PMT = \$14{,}902.95$$

**Dialog Box for the PMT Function**



The dialog box for Excel's **PMT** function: **Rate** is the interest rate on the loan, **Nper** is the number of repayment periods, and **Pv** is the loan principal. As discussed in Chapter 2, if the loan principal is written as a positive number, Excel presents the payment as a negative number; to avoid this, we write **Pv** as a negative number.

Below is the amortization table for a mortgage with equal annual payments:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **AMORTIZATION TABLE: MORTGAGE (EQUAL PAYMENTS)** | | | | | | |
| 2 | Loan amount | $ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Loan period in years | 1 | | | | | |
| 5 | Annual payment | $14,902.95 | <-- =PMT(B3,B4,–B2) | | | | |
| 6 | | | | | | | |
| 7 | **Year** | **A. Loan balance at beginning of year** | **B. Total payment** | **C. Interest payment = r*A** | **D. Principal repayment = B–C** | **E. Loan balance at end of year = A–D** | |
| 8 | 1 | 100,000.00 | 14,902.95 | 8,000.00 | 6,902.95 | 93,097.05 | <-- =B8–D8 |
| 9 | 2 | 93,097.05 | 14,902.95 | 7,447.76 | 7,455.18 | 85,641.87 | |
| 10 | 3 | 85,641.87 | 14,902.95 | 6,851.35 | 8,051.60 | 77,590.27 | |
| 11 | 4 | 77,590.27 | 14,902.95 | 6,207.22 | 8,695.73 | 68,894.54 | |
| 12 | 5 | 68,894.54 | 14,902.95 | 5,511.56 | 9,391.39 | 59,503.15 | |
| 13 | 6 | 59,503.15 | 14,902.95 | 4,760.25 | 10,142.70 | 49,360.46 | |
| 14 | 7 | 49,360.46 | 14,902.95 | 3,948.84 | 10,954.11 | 38,406.34 | |
| 15 | 8 | 38,406.34 | 14,902.95 | 3,072.51 | 11,830.44 | 26,575.90 | |
| 16 | 9 | 26,575.90 | 14,902.95 | 2,126.07 | 12,776.88 | 13,799.03 | |
| 17 | 10 | 13,799.03 | 14,902.95 | 1,103.92 | 13,799.03 | 0.00 | |
| 18 | | | | | | | |
| 19 | =F8 | =$B$5 | =$B$3*B9 | =C9–D9 | | | |
| 20 | | | | | | | |

Note that in this type of loan, we know the total payment ($14,902.95) and the principal payment is the difference between the total payment and the interest payment.

## Computing Principal and Interest Directly: The IPMT and PPMT Functions

In the above example, we used **PMT** to compute the total payment and then split this total payment into its two components: interest and repayment of principal. The Excel **IPMT** and **PPMT** functions directly compute the interest and principal for each year, as illustrated in the example below:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **AMORTIZATION TABLE: MORTGAGE USING IPMT AND PPMT** | | | | | |
| 2 | Loan amount | 100,000 | | | | |
| 3 | Annual interest rate, r | 8% | | | | |
| 4 | Loan period in years | 10 | | | | |
| 5 | Annual payment | $14,902.95 | | | | |
| 6 | | | | | | |
| 7 | **Year** | **A. Loan balance at beginning of year** | **C. Interest payment = r*A** | **D. Principal repayment = B–C** | **E. Loan balance at end of year = A–D** | |
| 8 | 1 | 100,000.00 | 8,000.00 | 6,902.95 | 93,097.05 | <-- =B8–D8 |
| 9 | 2 | 93,097.05 | 7,447.76 | 7,455.18 | 85,641.87 | |
| 10 | 3 | 85,641.87 | 6,851.35 | 8,051.60 | 77,590.27 | |
| 11 | 4 | 77,590.27 | 6,207.22 | 8,695.73 | 68,894.54 | |
| 12 | 5 | 68,894.54 | 5,511.56 | 9,391.39 | 59,503.15 | |
| 13 | 6 | 59,503.15 | 4,760.25 | 10,142.70 | 49,360.46 | |
| 14 | 7 | 49,360.46 | 3,948.84 | 10,954.11 | 38,406.34 | |
| 15 | 8 | 38,406.34 | 3,072.51 | 11,830.44 | 26,575.90 | |
| 16 | 9 | 26,575.90 | 2,126.07 | 12,776.88 | 13,799.03 | |
| 17 | 10 | 13,799.03 | 1,103.92 | 13,799.03 | 0.00 | |
| 18 | | | | | | |
| 19 | =E8 | =IPMT($B$3,A9, $B$4,–$B$2) | =PPMT($B$3,A9, $B$4,–$B$2) | | | |
| 20 | | | | | | |

## What Is the Remaining Balance of a Term Loan After *n* Payments?

We return to the previous spreadsheet. Is there some way to compute the remaining loan balance directly? The answer is yes:

*In any year, the loan balance is the present value of all the future loan payments, with the discount rate being the loan rate itself. This can be computed using either the Excel* ***NPV*** *function or—if the future payments are the same (as in the case of the mortgage)—by using the Excel* ***PV*** *function.*[1]

Here's an illustration:

[1] Recall from Chapters 2 and 3 that the Excel **NPV** function is misnamed: This function computes the present value of a series of future payments, not its net present value.

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **AMORTIZATION TABLE: REMAINING LOAN BALANCE = Present value of future loan payments at the loan rate For a term loan, use either Excel's NPV or PV functions** | | | | | | | |
| 2 | Loan amount | 100,000 | | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | | |
| 4 | Loan period in years | 10 | | | | | | |
| 5 | Annual payment | 14,902.95 | | | | | | |
| 6 | | | | | | | | |
| 7 | | | | | | | | |
| 8 | | | | | | | **Loan balance at end-year is the present value of future loan payments** | |
| 9 | **Year** | **A. Loan balance at beginning of year** | **Payment on loan at end-year** | **C. Interest payment = r*A** | **D. Principal repayment = B–C** | **E. Loan balance at end of year = A–D** | **NPV =NPV($B$3,C11:$C$19)** | **PV =PV($B$3, $B$4–A10,–$B$5)** |
| 10 | 1 | 100,000.00 | 14,902.95 | 8,000.00 | 6,902.95 | 93,097.05 | 93,097.05 | 93,097.05 |
| 11 | 2 | 93,097.05 | 14,902.95 | 7,447.76 | 7,455.18 | 85,641.87 | 85,641.87 | 85,641.87 |
| 12 | 3 | 85,641.87 | 14,902.95 | 6,851.35 | 8,051.60 | 77,590.27 | 77,590.27 | 77,590.27 |
| 13 | 4 | 77,590.27 | 14,902.95 | 6,207.22 | 8,695.73 | 68,894.54 | 68,894.54 | 68,894.54 |
| 14 | 5 | 68,894.54 | 14,902.95 | 5,511.56 | 9,391.39 | 59,503.15 | 59,503.15 | 59,503.15 |
| 15 | 6 | 59,503.15 | 14,902.95 | 4,760.25 | 10,142.70 | 49,360.46 | 49,360.46 | 49,360.46 |
| 16 | 7 | 49,360.46 | 14,902.95 | 3,948.84 | 10,954.11 | 38,406.34 | 38,406.34 | 38,406.34 |
| 17 | 8 | 38,406.34 | 14,902.95 | 3,072.51 | 11,830.44 | 26,575.90 | 26,575.90 | 26,575.90 |
| 18 | 9 | 26,575.90 | 14,902.95 | 2,126.07 | 12,776.88 | 13,799.03 | 13,799.03 | 13,799.03 |
| 19 | 10 | 13,799.03 | 14,902.95 | 1,103.92 | 13,799.03 | 0.00 | | |

## Shorter Method

Now that we understand the principles of a loan amortization table, we can shorten the process. For loans that have a constant annual payment (like a mortgage), we can create the following spreadsheet (no amortization table!):

| | A | B | C |
|---|---|---|---|
| 1 | **COMPUTING THE MORTGAGE PAYMENTS AND BALANCE IN A GIVEN PERIOD t** | | |
| 2 | Loan amount | 100,000 | |
| 3 | Annual interest rate, r | 8% | |
| 4 | Loan period in years | 10 | |
| 5 | Annual payment | 14,902.95 | <-- =PMT(B3,B4,–B2) |
| 6 | | | |
| 7 | Period t | 2 | |
| 8 | Interest payment | 7,447.76 | <-- =IPMT(B3,B7,B4,–B2) |
| 9 | Repayment of principal | 7,455.18 | <-- =PPMT(B3,B7,B4,–B2) |
| 10 | Total payment | 14,902.95 | <-- =B9+B8 |
| 11 | Remaining balance after period t payment | 85,641.87 | <-- =PV(B3,B4–B7,–B5) |

## 4.6 A Balloon Loan

A balloon loan is a loan which does not fully amortize over the period of the loan. This means that the final loan payment is relatively large compared to the intermediate loan payments.

Consider the example of this chapter: a $100,000 loan for 10 years bearing an annual rate of 8%. We will assume that lender payments in years 1–9 are $10,000 dollars. The amortization table for this loan is given below:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **AMORTIZATION TABLE: BALLOON LOAN** | | | | | | |
| 2 | Loan amount | $ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Loan period in years | 10 | | | | | |
| 5 | | | | | | | |
| 6 | **Year** | **A. Loan balance at beginning of year** | **B. Total payment** | **C. Interest payment = r*A** | **D. Principal repayment = B–C** | **E. Loan principal balance at end of year = A–D** | |
| 7 | 1 | 100,000.00 | 10,000.00 | 8,000.00 | 2,000.00 | 98,000.00 | <-- =B7–E7 |
| 8 | 2 | 98,000.00 | 10,000.00 | 7,840.00 | 2,160.00 | 95,840.00 | |
| 9 | 3 | 95,840.00 | 10,000.00 | 7,667.20 | 2,332.80 | 93,507.20 | |
| 10 | 4 | 93,507.20 | 10,000.00 | 7,480.58 | 2,519.42 | 90,987.78 | |
| 11 | 5 | 90,987.78 | 10,000.00 | 7,279.02 | 2,720.98 | 88,266.80 | |
| 12 | 6 | 88,266.80 | 10,000.00 | 7,061.34 | 2,938.66 | 85,328.14 | |
| 13 | 7 | 85,328.14 | 10,000.00 | 6,826.25 | 3,173.75 | 82,154.39 | |
| 14 | 8 | 82,154.39 | 10,000.00 | 6,572.35 | 3,427.65 | 78,726.74 | |
| 15 | 9 | 78,726.74 | 10,000.00 | 6,298.14 | 3,701.86 | 75,024.88 | |
| 16 | 10 | 75,024.88 | 81,026.88 | 6,001.99 | 75,024.88 | 0.00 | |

### Balloon Loans with Negative Principal Repayments

Suppose in the previous example the borrower wants to limit her annual repayment in years 1–9 to $6,000. This does not even cover the annual interest, resulting in negative principal repayments. What this means is that the loan principal ("A. Loan balance at beginning of year" above) increases over time:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **AMORTIZATION TABLE: BALLOON LOAN**<br>**Some or all of the principal repayments are negative** | | | | | | |
| 2 | Loan amount | $ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Loan period in years | 10 | | | | | |
| 5 | | | | | | | |
| 6 | **Year** | **A. Loan balance at beginning of year** | **B. Total payment** | **C. Interest owed = r*A** | **D. Principal repayment = B–C** | **E. Loan principal balance at end of year = A–D** | |
| 7 | 1 | 100,000.00 | 6,000.00 | 8,000.00 | –2,000.00 | 102,000.00 | <-- =B7–E7 |
| 8 | 2 | 102,000.00 | 6,000.00 | 8,160.00 | –2,160.00 | 104,160.00 | |
| 9 | 3 | 104,160.00 | 6,000.00 | 8,332.80 | –2,332.80 | 106,492.80 | |
| 10 | 4 | 106,492.80 | 6,000.00 | 8,519.42 | –2,519.42 | 109,012.22 | |
| 11 | 5 | 109,012.22 | 6,000.00 | 8,720.98 | –2,720.98 | 111,733.20 | |
| 12 | 6 | 111,733.20 | 6,000.00 | 8,938.66 | –2,938.66 | 114,671.86 | |
| 13 | 7 | 114,671.86 | 6,000.00 | 9,173.75 | –3,173.75 | 117,845.61 | |
| 14 | 8 | 117,845.61 | 6,000.00 | 9,427.65 | –3,427.65 | 121,273.26 | |
| 15 | 9 | 121,273.26 | 6,000.00 | 9,701.86 | –3,701.86 | 124,975.12 | |
| 16 | 10 | 124,975.12 | 134,973.12 | 9,998.01 | 124,975.12 | 0.00 | |

Note that we have made a subtle change in the labeling of the columns of the amortization table. Instead of "C. Interest payment = r*A", we have written "C. Interest owed = r*A". In each year, the borrower owes an interest payment of r*A, but the actual payment does not cover this interest, resulting in an increase in the ending loan principal balance.[2]

## 4.7 A Bullet Loan

Suppose the borrower makes a zero annual payment on her mortgage. This is a special case of a balloon loan often called a "bullet loan." We can easily adapt the previous spreadsheet to this situation:

[2] A bit of "financial philosophy": One interpretation of interest is that it is a charge for using someone else's money. In any period *t*, the "interest usage charge" = $r^*\text{Principal}_{\text{beginning of period } t}$. If the borrower doesn't fully pay this charge, then the difference between the actual payment and interest usage charge increases the loan principal.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | AMORTIZATION TABLE: BULLET LOAN<br>Payments in years 1–9 are zero | | | | | | |
| 2 | Loan amount | $ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Loan period in years | 10 | | | | | |
| 5 | | | | | | | |
| 6 | Year | A. Beginning balance of loan | B. Total payment | C. Interest owed = r*A | D. Principal repayment = B–C | E. Loan principal balance at end of year = A–D | |
| 7 | 1 | 100,000.00 | 0.00 | 8,000.00 | –8,000.00 | 108,000.00 | <-- =B7–E7 |
| 8 | 2 | 108,000.00 | 0.00 | 8,640.00 | –8,640.00 | 116,640.00 | |
| 9 | 3 | 116,640.00 | 0.00 | 9,331.20 | –9,331.20 | 125,971.20 | |
| 10 | 4 | 125,971.20 | 0.00 | 10,077.70 | –10,077.70 | 136,048.90 | |
| 11 | 5 | 136,048.90 | 0.00 | 10,883.91 | –10,883.91 | 146,932.81 | |
| 12 | 6 | 146,932.81 | 0.00 | 11,754.62 | –11,754.62 | 158,687.43 | |
| 13 | 7 | 158,687.43 | 0.00 | 12,694.99 | –12,694.99 | 171,382.43 | |
| 14 | 8 | 171,382.43 | 0.00 | 13,710.59 | –13,710.59 | 185,093.02 | |
| 15 | 9 | 185,093.02 | 0.00 | 14,807.44 | –14,807.44 | 199,900.46 | |
| 16 | 10 | 199,900.46 | 215,892.50 | 15,992.04 | 199,900.46 | 0.00 | |

It is easy to confirm that the last year's total payment on a balloon loan = $(Loan\ amount)*(1+r)^N$. This is shown in the following spreadsheet:

| | A | B | C | D |
|---|---|---|---|---|
| 18 | COMPUTING THE LAST YEAR'S PAYMENT ON A BULLET LOAN | | | |
| 19 | Initial loan amount | 100,000 | <-- =B2 | |
| 20 | Year | 10 | | |
| 21 | Interest rate | 8% | <-- =B3 | |
| 22 | Total payment at end of year 10 | 215,892.50 | <-- =B19*(1+B21)^B20 | |

## 4.8 The Market Value vs. The Contractual Value of a Loan

Suppose you borrow $100,000 from the bank at 8%. In previous sections, we show through an amortization table how the payments of the loan determine the remaining loan principal. We call this the *contractual value* of the loan. In this section, we compare the contractual value with the *market value* of the loan:

- The contractual value of a loan is the loan's outstanding unpaid principal. In our example below, we repeat some of the calculations from earlier sections and show that the loan contractual value can be computed either through an amortization table or as the present value of the remaining loan payments, where the present value is computed at the loan's interest rate.

- The market value of a loan is the value of the loan at the current market interest rate. The market value of the loan is the present value (PV) of the remaining loan payments, where the present value is computed at the market interest rate.

## Example 1: Market vs. Contractual Value for an Interest-Only Loan

Consider the case of a 10-year, 8%, interest-only loan of $100,000 (see Section 4.3). As you can see from the amortization table below, the contractual value of the loan in each of the years is $100,000.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **AMORTIZATION TABLE: INTEREST-ONLY LOAN** | | | | | | |
| 2 | Loan amount | $ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Number of payments per year | 1 | | | | | |
| 5 | Loan period in years | 10 | | | | | |
| 6 | | | | | | | |
| 7 | **Payment number** | **A. Loan balance at beginning of year** | **B. Interest payment = r*A** | **C. Principal repayment** | **D. Total payment = B+C** | **E. Loan principal balance at end of year = A–D** | |
| 8 | 1 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | <-- =B8–D8 |
| 9 | 2 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 10 | 3 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 11 | 4 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 12 | 5 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 13 | 6 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 14 | 7 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 15 | 8 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 16 | 9 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 17 | 10 | 100,000.00 | 8,000.00 | 100,000.00 | 108,000.00 | 0.00 | |
| 18 | | | | | | | |
| 19 | =B2 | | | =$B$3*B8 | | =C8+D8 | |
| 20 | | | | | | | |

Now suppose that after 2 years, with eight more payments left on the loan, the market rate for similar loans decreases from 8% to 5%. Then the market value of the loan is

$$\left(\textit{Market value of loan}\right)_2 = \frac{8{,}000}{(1+5\%)^1} + \frac{8{,}000}{(1+5\%)^2} + \cdots + \frac{108{,}000}{(1+5\%)^8} = \$119{,}389.64$$

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **AMORTIZATION TABLE: INTEREST-ONLY LOAN<br>Market Value vs Contractual Value, end of year 2** | | | | | | |
| 2 | Loan amount | $ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Number of payments per year | 1 | | | | | |
| 5 | Loan period in years | 10 | | | | | |
| 6 | | | | | | | |
| 7 | **Payment number** | **A. Loan balance at beginning of year** | **B. Interest payment = r*A** | **C. Principal repayment** | **D. Total payment = B+C** | **E. Loan principal balance at end of year = A–D** | |
| 8 | 1 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | <-- =B8–D8 |
| 9 | 2 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 10 | 3 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 11 | 4 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 12 | 5 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 13 | 6 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 14 | 7 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 15 | 8 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 16 | 9 | 100,000.00 | 8,000.00 | 0.00 | 8,000.00 | 100,000.00 | |
| 17 | 10 | 100,000.00 | 8,000.00 | 100,000.00 | 108,000.00 | 0.00 | |
| 18 | | | | | | | |
| 19 | **Loan market value** | | | | | Future loan payments at end of year 2 (highlighted) | |
| 20 | Year, t | 2 | | | | | |
| 21 | Market interest rate | 5% | | | | | |
| 22 | Loan market value = **PV** of future loan payments discounted at market interest rate | 119,389.64 | <-- =NPV(B21,E10:E17) | | | | |

## Become a Financial Strategist: Refinance Your Loan

You've taken a $100,000, 10-year, 8%, interest-only loan. After 2 years of payments, you still owe $100,000 in outstanding principal. Now you realize that the interest rate on similar loans has decreased to 5%. Being the wily finance type that you are, this suggests to you the following refinancing strategy:

- Borrow $100,000 as an interest-only, 8-year loan with 5% interest. The annual payments on this loan are $5,000 instead of the $8,000 on the previous loan.
- Use the $100,000 you borrowed at 5% to repay the bank the outstanding $100,000 on the original loan.
- You are now better off by $3,000 per year! As the spreadsheet below shows, the present value of your gain is 19,389.64, which is exactly the difference between loan's contractual value and the market value we computed above.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **REFINANCING THE LOAN** | | | | |
| 2 | Loan amount | $ 100,000 | | | |
| 3 | Old interest rate | 8% | | | |
| 4 | New interest rate | 5% | | | |
| 5 | Loan period in years | 8 | | | |
| 6 | | | | | |
| 7 | **Year** | **Payment on old loan** | **Payment on new loan** | **Gain** | |
| 8 | 3 | 8,000.00 | 5,000.00 | 3,000.00 | <-- =B8–C8 |
| 9 | 4 | 8,000.00 | 5,000.00 | 3,000.00 | |
| 10 | 5 | 8,000.00 | 5,000.00 | 3,000.00 | |
| 11 | 6 | 8,000.00 | 5,000.00 | 3,000.00 | |
| 12 | 7 | 8,000.00 | 5,000.00 | 3,000.00 | |
| 13 | 8 | 8,000.00 | 5,000.00 | 3,000.00 | |
| 14 | 9 | 8,000.00 | 5,000.00 | 3,000.00 | |
| 15 | 10 | 108,000.00 | 105,000.00 | 3,000.00 | |
| 16 | | | | | |
| 17 | Present value of the gain | 19,389.64 | <-- =NPV(B4,D8:D15) | | |

**Note:** You've made a lot of money by switching out of the expensive loan to a cheaper loan, but of course, we didn't tell you the small print:

- We've assumed that your original 8% mortgage allows prepayment at any time.
- We've assumed that the prepayment is costless—as you'll see in the next section, many loans have prepayment costs.

## What Are the Principles of Refinancing?

Here's what we've learned so far:

- The contractual value of a loan at the end of year $t$ is the remaining principal from the loan's amortization table. This is also the present value of the loan's payments in years $t + 1, t + 2, \ldots, N$ discounted at the loan interest rate.
- The market value of a loan at the end of year $t$ is the present value of the remaining total payments of the loan's payments in years $t + 1, t + 2, \ldots, N$ discounted at the market interest rate.
- If the contractual value is less than market value, consider refinancing the loan. You may have to consider refinancing costs (we've ignored them thus far, but consider them in Section 4.9).

## Example 2: Market vs. Contractual Value for a Term Loan

We illustrate the principles of refinancing by repeating considering a term loan. Recall that most mortgages are term loans. Our previous example (Section 4.5) was the following:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **AMORTIZATION TABLE: TERM LOAN (EQUAL PAYMENTS)** | | | | | | |
| 2 | Loan amount | 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Loan period in years | 10 | | | | | |
| 5 | Annual payment | 14,902.95 | <-- =PMT(B3,B4,–B2) | | | | |
| 6 | | | | | | | |
| 7 | **Year** | **A. Loan balance at beginning of year** | **B. Total payment** | **C. Interest payment = r*A** | **D. Principal repayment = B–C** | **E. Loan principal balance at end of year = A–D** | |
| 8 | 1 | 100,000.00 | 14,902.95 | 8,000.00 | 6,902.95 | 93,097.05 | <-- =B8–E8 |
| 9 | 2 | 93,097.05 | 14,902.95 | 7,447.76 | 7,455.18 | 85,641.87 | |
| 10 | 3 | 85,641.87 | 14,902.95 | 6,851.35 | 8,051.60 | 77,590.27 | |
| 11 | 4 | 77,590.27 | 14,902.95 | 6,207.22 | 8,695.73 | 68,894.54 | |
| 12 | 5 | 68,894.54 | 14,902.95 | 5,511.56 | 9,391.39 | 59,503.15 | |
| 13 | 6 | 59,503.15 | 14,902.95 | 4,760.25 | 10,142.70 | 49,360.46 | |
| 14 | 7 | 49,360.46 | 14,902.95 | 3,948.84 | 10,954.11 | 38,406.34 | |
| 15 | 8 | 38,406.34 | 14,902.95 | 3,072.51 | 11,830.44 | 26,575.90 | |
| 16 | 9 | 26,575.90 | 14,902.95 | 2,126.07 | 12,776.88 | 13,799.03 | |
| 17 | 10 | 13,799.03 | 14,902.95 | 1,103.92 | 13,799.03 | 0.00 | |
| 18 | | | | | | | |
| 19 | =F8 | =$B$5 | =$B$3*B9 | =C9–D9 | | | |

Now assume that the market interest rate on the term loan at the end of year 2 drops from 8% to 5%. The contractual value of the term loan is

$$\textit{Term loan contractual value, end year 2} = \frac{14{,}902.95}{1.08} + \frac{14{,}902.95}{(1.08)^2} + \cdots + \frac{14{,}902.95}{(1.08)^8} = \$85{,}641.87$$

The market value of the term loan is

$$\textit{Term loan market value, end year 2} = \frac{14{,}902.95}{1.05} + \frac{14{,}902.95}{(1.05)^2} + \cdots + \frac{14{,}902.95}{(1.05)^8} = \$96{,}320.93$$

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **TERM LOAN (EQUAL PAYMENTS)**<br>**Market vs. contractual value, end of year 2** | | | | | | |
| 2 | Loan amount | $ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Loan period in years | 10 | | | | | |
| 5 | Annual payment | 14,902.95 | <-- =PMT(B3,B4,–B2) | | | | |
| 6 | | | | | | | |
| 7 | **Year** | **A. Loan balance at beginning of year** | **B. Total payment** | **C. Interest payment = r*A** | **D. Principal repayment = B–C** | **E. Loan principal balance at end of year = A–D** | |
| 8 | 1 | 100,000.00 | 14,902.95 | 8,000.00 | 6,902.95 | 93,097.05 | <-- =B8–E8 |
| 9 | 2 | 93,097.05 | 14,902.95 | 7,447.76 | 7,455.18 | 85,641.87 | |
| 10 | 3 | 85,641.87 | 14,902.95 | 6,851.35 | 8,051.60 | 77,590.27 | |
| 11 | 4 | 77,590.27 | 14,902.95 | 6,207.22 | 8,695.73 | 68,894.54 | |
| 12 | 5 | 68,894.54 | 14,902.95 | 5,511.56 | 9,391.39 | 59,503.15 | |
| 13 | 6 | 59,503.15 | 14,902.95 | 4,760.25 | 10,142.70 | 49,360.46 | |
| 14 | 7 | 49,360.46 | 14,902.95 | 3,948.84 | 10,954.11 | 38,406.34 | |
| 15 | 8 | 38,406.34 | 14,902.95 | 3,072.51 | 11,830.44 | 26,575.90 | |
| 16 | 9 | 26,575.90 | 14,902.95 | 2,126.07 | 12,776.88 | 13,799.03 | |
| 17 | 10 | 13,799.03 | 14,902.95 | 1,103.92 | 13,799.03 | 0.00 | |
| 18 | | | | | | | |
| 19 | **Loan contractual value** | | | | | | |
| 20 | Year, t | 2 | | | Future loan payments at end of year 2 (highlighted) | | |
| 21 | Loan interest rate | 8% | | | | | |
| 22 | Loan contractual value = **PV** of future loan payments discounted at loan interest rate | 85,641.87 | <-- =NPV(B21,C10:C17) | | | | |
| 23 | | | | | | | |
| 24 | | | | | | | |
| 25 | **Loan market value** | | | | | | |
| 26 | Year, t | 2 | | | | | |
| 27 | Market interest rate | 5% | | | | | |
| 28 | Loan market value = **PV** of future loan payments discounted at market interest rate | 96,320.93 | <-- =NPV(B27,C10:C17) | | | | |

**Refinancing:** As in Example 1 of this section, you refinance your loan by borrowing the remaining principal at the end of year 2 ($85,641.87) at the new lower rate of 5%. Let's assume that the new loan is also a term loan, with maturity of 8 years to match the remaining maturity of your original mortgage. Below we compare the new and the old amortization tables and show that the present value of your gain is the difference between the loan's market value and its contractual value:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **REFINANCING THE TERM LOAN AT THE END OF YEAR 2** | | | | |
| 2 | | | | | |
| 3 | **Original loan: 10 years, 8%, term loan** | | | | |
| 4 | Loan amount | 100,000 | | | |
| 5 | Annual interest rate, r | 8% | | | |
| 6 | Loan period in years | 10 | | | |
| 7 | Annual payment | 14,902.95 | <-- =PMT(B5,B6,–B4) | | |
| 8 | Contractual value, end of year 2 | 85,642 | <-- =PV(B5,8,–B7) | | |
| 9 | Market value, end of year 2 | 96,321 | <-- =PV(B13,8,–B7) | | |
| 10 | | | | | |
| 11 | **New loan: 8 years, 5%, term loan** | | | | |
| 12 | Loan amount | 85,642 | <-- =B8 | | |
| 13 | Annual interest rate, r | 5% | | | |
| 14 | Loan period in years | 8 | | | |
| 15 | Annual payment | 13,250.66 | <-- =PMT(B13,B14,–B12) | | |
| 16 | | | | | |
| 17 | Present value of refinancing savings | 10,679.06 | <-- =NPV(B13,D21:D28) | | |
| 18 | Check: Market value minus contractual value | 10,679.06 | <-- =B9–B8 | | |
| 19 | | | | | |
| 20 | **Year** | **Payment on original loan** | **Payment on new loan** | **Savings** | |
| 21 | 3 | 14,902.95 | 13,250.66 | 1,652.28 | <-- =B21–C21 |
| 22 | 4 | 14,902.95 | 13,250.66 | 1,652.28 | |
| 23 | 5 | 14,902.95 | 13,250.66 | 1,652.28 | |
| 24 | 6 | 14,902.95 | 13,250.66 | 1,652.28 | |
| 25 | 7 | 14,902.95 | 13,250.66 | 1,652.28 | |
| 26 | 8 | 14,902.95 | 13,250.66 | 1,652.28 | |
| 27 | 9 | 14,902.95 | 13,250.66 | 1,652.28 | |
| 28 | 10 | 14,902.95 | 13,250.66 | 1,652.28 | |

## 4.9 Costly Refinancing

In the previous section we drove home the following point:

***The principle of refinancing:*** *When interest rates change, discount the remaining loan payments at the loan rate (the contractual value of the loan) and at the new market interest rate (the market value of the loan). If the loan's market value is greater than the loan's contractual value, refinance the loan at the new market rate.*

The refinancing principle ignores the costs of switching out of the old loan to the new loan. Many lenders impose costs on borrowers wishing to refinance. These costs go by lots of names: "Prepayment penalty," "refinancing costs," and "make whole provisions" are just some of the terms we have encountered. Taking into account prepayment penalties, we can restate our Principle of Refinancing as follows (we've marked the change in **bold**):

***The principle of refinancing with prepayment penalties:*** *When interest rates change, discount the remaining loan payments at the loan rate (the contractual value of the loan) and at the new market interest rate (the market value of the loan). If the* ***loan's market value exceeds the loan's contractual value by more than the prepayment penalty****, refinance the loan at the new market rate.*

## Prepayment Penalties at Work: A Balloon Loan

Suppose you've borrowed $100,000 at 8% for 10 years in a balloon loan with annual payment in years 1–9 of $10,000 (see Section 4.6):

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **AMORTIZATION TABLE: BALLOON LOAN** | | | | | | |
| 2 | Loan amount | $ 100,000 | | | | | |
| 3 | Annual interest rate, r | 8% | | | | | |
| 4 | Loan period in years | 10 | | | | | |
| 5 | | | | | | | |
| 6 | **Year** | **A. Loan balance at beginning of year** | **B. Total payment** | **C. Interest payment = r*A** | **D. Principal payment = B–A** | **E. Loan principal balance at end of year = A–D** | |
| 7 | 1 | 100,000.00 | 10,000.00 | 8,000.00 | 2,000.00 | 98,000.00 | <-- =B7–D7 |
| 8 | 2 | 98,000.00 | 10,000.00 | 7,840.00 | 2,160.00 | 95,840.00 | |
| 9 | 3 | 95,840.00 | 10,000.00 | 7,667.20 | 2,332.80 | 93,507.20 | |
| 10 | 4 | 93,507.20 | 10,000.00 | 7,480.58 | 2,519.42 | 90,987.78 | |
| 11 | 5 | 90,987.78 | 10,000.00 | 7,279.02 | 2,720.98 | 88,266.80 | |
| 12 | 6 | 88,266.80 | 10,000.00 | 7,061.34 | 2,938.66 | 85,328.14 | |
| 13 | 7 | 85,328.14 | 10,000.00 | 6,826.25 | 3,173.75 | 82,154.39 | |
| 14 | 8 | 82,154.39 | 10,000.00 | 6,572.35 | 3,427.65 | 78,726.74 | |
| 15 | 9 | 78,726.74 | 10,000.00 | 6,298.14 | 3,701.86 | 75,024.88 | |
| 16 | 10 | 75,024.88 | 81,026.88 | 6,001.99 | 75,024.88 | 0.00 | |

At the end of year 2, the market interest rate drops from 8% to 5%. The potential gain from refinancing the loan is $16,865.91:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | REFINANCING THE BALLOON LOAN<br>End of year 2 | | | | | |
| 2 | | | | | | |
| 3 | Original loan: 10 years, 8%, balloon loan | | | | | |
| 4 | Contractual value<br>end of year 2 | 95,840.00 | | | | |
| 5 | Annual interest rate, r | 8% | | | | |
| 6 | Loan period in years | 10.00 | | | | |
| 7 | Annual payment, years 3–10 | 10,000.00 | | | | |
| 8 | | | | | | |
| 9 | | | | | | |
| 10 | New loan: 8 years, 5%, balloon loan | | | | | |
| 11 | Market value<br>end of year 2 | 112,705.91 | <-- =NPV(B12,C19:C26) | | | |
| 12 | Annual interest rate, r | 5% | | | | |
| 13 | Loan period in years | 8 | | | | |
| 14 | Annual payment, years 3–10 | 10,000 | | | | |
| 15 | Potential refinancing gain | 16,865.91 | <-- =B11–B4 | | | |
| 16 | | | | | | |
| 17 | | Original loan: Amortization table, years 3–10 | | | | |
| 18 | Year | A. Loan balance at beginning of year | B. Total payment | C. Interest payment = 8%*A | D. Principal payment = B–C | E. Loan principal balance at end of year = A–D |
| 19 | 3 | 95,840.00 | 10,000.00 | 7,667.20 | 2,332.80 | 93,507.20 |
| 20 | 4 | 93,507.20 | 10,000.00 | 7,480.58 | 2,519.42 | 90,987.78 |
| 21 | 5 | 90,987.78 | 10,000.00 | 7,279.02 | 2,720.98 | 88,266.80 |
| 22 | 6 | 88,266.80 | 10,000.00 | 7,061.34 | 2,938.66 | 85,328.14 |
| 23 | 7 | 85,328.14 | 10,000.00 | 6,826.25 | 3,173.75 | 82,154.39 |
| 24 | 8 | 82,154.39 | 10,000.00 | 6,572.35 | 3,427.65 | 78,726.74 |
| 25 | 9 | 78,726.74 | 10,000.00 | 6,298.14 | 3,701.86 | 75,024.88 |
| 26 | 10 | 75,024.88 | 81,026.88 | 6,001.99 | 75,024.88 | 0.00 |
| 27 | | | | | | |
| 28 | | New loan: Amortization table, years 3–10 | | | | |
| 29 | Year | A. Loan balance at beginning of year | B. Total payment | C. Interest payment = 5%*A | D. Principal payment = B–C | E. Loan principal balance at end of year = A–D |
| 30 | 3 | 95,840.00 | 10,000 | 4,792.00 | 5,208.00 | 90,632.00 |
| 31 | 4 | 90,632.00 | 10,000 | 4,531.60 | 5,468.40 | 85,163.60 |
| 32 | 5 | 85,163.60 | 10,000 | 4,258.18 | 5,741.82 | 79,421.78 |
| 33 | 6 | 79,421.78 | 10,000 | 3,971.09 | 6,028.91 | 73,392.87 |
| 34 | 7 | 73,392.87 | 10,000 | 3,669.64 | 6,330.36 | 67,062.51 |
| 35 | 8 | 67,062.51 | 10,000 | 3,353.13 | 6,646.87 | 60,415.64 |
| 36 | 9 | 60,415.64 | 10,000 | 3,020.78 | 6,979.22 | 53,436.42 |
| 37 | 10 | 53,436.42 | 56,108.24 | 2,671.82 | 53,436.42 | 0.00 |

We can confirm the refinancing gain by directly computing the difference between the payments on the original loan versus those of the new loan:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 40 | | Confirming the refinancing gain | | | |
| 41 | **Year** | **Payment on original loan** | **Payment on new loan** | **Gain** | |
| 42 | 3 | 10,000.00 | 10,000 | 0.00 | <-- =B42–C42 |
| 43 | 4 | 10,000.00 | 10,000 | 0.00 | |
| 44 | 5 | 10,000.00 | 10,000 | 0.00 | |
| 45 | 6 | 10,000.00 | 10,000 | 0.00 | |
| 46 | 7 | 10,000.00 | 10,000 | 0.00 | |
| 47 | 8 | 10,000.00 | 10,000 | 0.00 | |
| 48 | 9 | 10,000.00 | 10,000 | 0.00 | |
| 49 | 10 | 81,026.88 | 56,108 | 24,918.63 | |
| 50 | | | | | |
| 51 | Potential refinancing gain | 16,865.91 | <-- =NPV(B12,D42:D49) | | |

## Refinancing Costs

Now suppose that your bank imposes a \$10,000 refinancing cost. Simple logic suggests that the potential refinancing gain is \$16,865.91 – \$10,000 = \$6,865.91. One way to confirm this is to redo the previous calculation, but to assume that instead of borrowing the loan’s contractual value of \$95,840, you borrow \$105,850:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **REFINANCING THE BALLOON LOAN**<br>**End of year 2, refinancing costs: $10,000** | | | | | |
| 2 | | | | | | |
| 3 | **Original loan: 10 years, 8%, balloon loan** | | | | | |
| 4 | Contractual value end of year 2 | 95,840.00 | | | | |
| 5 | Annual interest rate, **r** | 8% | | | | |
| 6 | Loan period in years | 10.00 | | | | |
| 7 | Annual payment, years 3–10 | 10,000.00 | | | | |
| 8 | | | | | | |
| 9 | **New loan: 8 years, 5%, balloon loan** | | | | | |
| 10 | Refinancing costs | 10,000.00 | | | | |
| 11 | Need to borrow to refinance | 105,840.00 | <-- Loan contractual value + refinancing costs | | | |
| 12 | Annual interest rate, r | 5% | | | | |
| 13 | Loan period in years | 8 | | | | |
| 14 | Annual payment, years 3–10 | 10,000 | | | | |
| 15 | | | | | | |
| 16 | | **Original loan: Amortization table, years 3–10** | | | | |
| 17 | **Year** | **A. Loan balance at beginning of year** | **B. Total payment** | **C. Interest payment = 8%*A** | **D. Principal payment = B–C** | **E. Loan principal balance at end of year = A–D** |
| 18 | 3 | 95,840.00 | 10,000.00 | 7,667.20 | 2,332.80 | 93,507.20 |
| 19 | 4 | 93,507.20 | 10,000.00 | 7,480.58 | 2,519.42 | 90,987.78 |
| 20 | 5 | 90,987.78 | 10,000.00 | 7,279.02 | 2,720.98 | 88,266.80 |
| 21 | 6 | 88,266.80 | 10,000.00 | 7,061.34 | 2,938.66 | 85,328.14 |
| 22 | 7 | 85,328.14 | 10,000.00 | 6,826.25 | 3,173.75 | 82,154.39 |
| 23 | 8 | 82,154.39 | 10,000.00 | 6,572.35 | 3,427.65 | 78,726.74 |
| 24 | 9 | 78,726.74 | 10,000.00 | 6,298.14 | 3,701.86 | 75,024.88 |
| 25 | 10 | 75,024.88 | 81,026.88 | 6,001.99 | 75,024.88 | 0.00 |
| 26 | | | | | | |
| 27 | | **New loan: Amortization table, years 3–10** | | | | |
| 28 | **Year** | **A. Loan balance at beginning of year** | **B. Total payment** | **C. Interest payment = 5%*A** | **D. Principal payment = B–C** | **E. Loan principal balance at end of year = A–D** |
| 29 | 3 | 105,840.00 | 10,000 | 5,292.00 | 4,708.00 | 101,132.00 |
| 30 | 4 | 101,132.00 | 10,000 | 5,056.60 | 4,943.40 | 96,188.60 |
| 31 | 5 | 96,188.60 | 10,000 | 4,809.43 | 5,190.57 | 90,998.03 |
| 32 | 6 | 90,998.03 | 10,000 | 4,549.90 | 5,450.10 | 85,547.93 |
| 33 | 7 | 85,547.93 | 10,000 | 4,277.40 | 5,722.60 | 79,825.33 |
| 34 | 8 | 79,825.33 | 10,000 | 3,991.27 | 6,008.73 | 73,816.59 |
| 35 | 9 | 73,816.59 | 10,000 | 3,690.83 | 6,309.17 | 67,507.42 |
| 36 | 10 | 67,507.42 | 70,882.80 | 3,375.37 | 67,507.42 | 0.00 |

You can confirm your intuition that the benefits of refinancing have decreased by the $10,000 refinancing cost:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 39 | | **Confirming the refinancing gain** | | | |
| 40 | **Year** | **Payment on original loan** | **Payment on new loan** | **Gain** | |
| 41 | 3 | 10,000.00 | 10,000 | 0.00 | <-- =B41–C41 |
| 42 | 4 | 10,000.00 | 10,000 | 0.00 | |
| 43 | 5 | 10,000.00 | 10,000 | 0.00 | |
| 44 | 6 | 10,000.00 | 10,000 | 0.00 | |
| 45 | 7 | 10,000.00 | 10,000 | 0.00 | |
| 46 | 8 | 10,000.00 | 10,000 | 0.00 | |
| 47 | 9 | 10,000.00 | 10,000 | 0.00 | |
| 48 | 10 | 81,026.88 | 70,883 | 10,144.08 | |
| 49 | | | | | |
| 50 | Potential refinancing gain | 6,865.91 | <-- =NPV(B12,D41:D48) | | |

## Summary

This chapter discussed various types of loans and their repayment patterns. Loans allow the borrower to put less of her own money into a transaction and magnify the returns to the equity investment. This "magnification" (the effect of leverage) swings both ways: As you increase the leverage, good returns become better and bad returns become worse.

The most important analytical tool in this chapter is the amortization table, which splits each loan payment into an interest component and a repayment of principal.

The last two sections of the chapter discussed the refinancing of loans. Provided that the costs of refinancing are not too large, when the market interest rate decreases, it is advantageous to switch out of your original loan and to refinance at the new, lower market rate.

## Exercises

**Note:** The data for these exercises can be found on the Benninga, *Principles of Finance with Excel,* Third Edition companion website (www.oup.com/us/Benninga).

**1.** (Leverage) Your friend presents you with the following deal: You can buy a box for $9, decorate it at a cost of $1, and sell it in 1 year for $11.

- **a.** You have $100 to invest in the boxes (and decorations). What are your profits assuming the deal goes through? What is the return in percentage on your investment?
- **b.** Suppose that in addition to your own $100, you can borrow $900 at 5% interest. The whole amount of $1,000 will be invested in boxes.

What are your profits assuming the deal goes through? What is the return in percentage on your investment?

**c.** Assume you borrowed the $900 carrying a 5% interest rate for the deal period, bought the boxes and decorated them, but you have just found out that the market price of the decorated boxes has dropped to $8.5. What are your profits/losses in dollars? What is the return in percentage on your investment?

**2.** (Interest-only loan, amortization table) You are taking an interest-only $10,000 loan. The loan is to be paid in 2 years and carries monthly interest payments of 0.5%.

**a.** Show an amortization table for the loan.

**b.** What is the outstanding balance of the loan at the end of any month?

**3.** (Interest-only loan) Your friend took a $50,000 loan. The loan is a 15-year, 5% "interest-only" loan (paying interest in the loan period and the principal in one installment at the end).

**a.** Show the loan amortization table.

**b.** What is the outstanding balance of the loan after 7 years?

**4.** (Balloon loan) Christine would like to take a loan. The loan carries a 1% monthly interest rate. Christine can only pay back $800 every month.

**a.** What is the highest loan that Christine can take? (**Hint:** Since the loan amount is increasing with the maturity, answer assuming the loan is perpetual.)

**b.** What is the highest loan that Christine can take assuming the monthly interest rate on the loan is 0.75%?

**5.** (Interest-only loan, refinancing) Five years ago, you took an interest-only loan. The loan carries a 1% monthly interest rate, and the loan principal is $150,000. The loan has two more years (24 monthly payments to be paid at the end of every month).

**a.** What is the monthly payment on the loan?

**b.** A financial advisor approaches you and offers to refinance the loan for a consultancy fee of $8,000. The new loan has the same characteristics as the current loan but carries a 0.75% monthly rate. Should you refinance the loan?

**6.** (An equal payment loan, refinancing) Omer is the CFO of ABC Corp. The firm just took a $1M loan with 20 equal annual payments and an annual interest rate of 10%.

- **a.** What is the annual payment on the loan?
- **b.** Since the interest payment is tax deductible, Omer would like to know what the interest rate portion of the first payment is. Can you provide an answer?
- **c.** Show an amortization table for the loan.
- **d.** What is the outstanding balance of the loan after 8 years?
- **e.** After 8 years, when the market interest rate has dropped to 8%, the company would like to refinance the loan. What is the "refinancing fine" the bank would charge the firm if it would like to be compensated for the cost of the refinancing?

**7.** (An equal term loan, amortization table) Ruth and Moab have just purchased their dream apartment. They have financed the purchase with a term loan of $150,000 (mortgage). The loan term is for 10 years, with 6% stated annual interest (0.5% monthly interest payment).

- **a.** Show an amortization table for the loan.
- **b.** Break down the 10th payment to the principal and interest portion.
- **c.** Four years after taking the loan, Ruth and Moab want to repay the loan. What is the outstanding balance of the loan?

**8.** (An equal payment loan, finding *n*) You would like to borrow $200,000 to finance your new house. The bank charges a monthly interest rate of 0.75%. What is the loan term in years if you would like to pay $2,000 every month to the bank and the loan is an equal payment mortgage?

**9.** (Mortgage, finding PV) Jane would like to purchase a new car. She has $5,000 in cash. She is also willing to take a car loan and pay a monthly amount of $200 for the next 5 years. Assuming that Jane can lend or borrow at 0.3% per month, what is the most expensive car she can purchase?

**10.** (Mortgage, finding *n*) ABC Corp. would like to borrow $10M to fund a new manufacturing plant. According to the projected business plan, ABC feels comfortable with an equal annual payment of $1,550,000. What is the term of the loan that ABC Corp. can take assuming the alternative cost of capital is 9% per year?

**11.** (Mortgage, finding *r*, payment breakdown, and outstanding balance) You are interviewing with one of the largest mortgage brokerage firms. Your future boss asks you to analyze a $150,000 mortgage with 10 years to maturity and monthly payments of $2,000 (120 payments).

a. What is the monthly rate on the loan?

b. Break down the 10th payment to principal payment and interest payment.

c. What is the outstanding balance of the loan after 4 years?

12. (Mortgage, amortization table) “Iris” company took a 5-year, \$150,000 loan carrying a 10% annual interest rate. The loan will be paid back in five equal annual installments.

a. What is the fixed amount to be paid in each installment?

b. Show an amortization table for the loan.

c. What is the outstanding loan balance after the third installment?

d. What is the principal payment on the fourth payment?

13. (Mortgage, refinancing a loan) You financed the purchase of a \$300,000 apartment with a down payment in cash of 20% of the purchase price. The remaining 80% is financed with a mortgage with a 1% monthly interest rate over the next 20 years. The mortgage is repaid with equal monthly installments.

a. Compute the monthly installments on the mortgage.

b. What is the outstanding principal balance of the mortgage after 5 years (i.e., after 60 installments)?

c. (Challenging) Ten years later (after 120 installments), your bank manager offers to refinance your mortgage with a new loan carrying a 0.9% interest rate for a one-time commission. What is the maximal commission you would be willing to pay? (**Hint:** Think about the “fair market value of the loan” and the “contractual value” of the loan.)

14. (Equal principal payment loan) You took a 5-year, \$100,000 loan. The loan has equal principal payments. The loan carries a 6% annual interest rate and is paid back in annual payments.

a. What is the outstanding balance of the loan after 3 years?

b. Compute an amortization table for the loan.

c. What is the interest payment on the fourth installment?

# 5 Effective Interest Rates

## Chapter Contents



## Overview

In Chapters 2 and 3 we introduced the basic tools of financial analysis—present value (PV), net present value (NPV), and internal rate of return (IRR). In this chapter, we use these tools to answer two basic types of questions:

- **What is it worth?** Presented with an asset—this could be a stock, a bond, a real estate investment, a computer, or a used car—we would like to know *how to value* the asset. The finance tools used to answer this question are mostly related to the concept of PV and NPV. The basic principle is that the value of an asset is the present value of its future cash flows. Comparing this present value to the asset's price tells us whether we should buy it or not. We introduced PV and NPV in Chapters 2 and 3, returned.
- **What does it cost?** This sounds like an innocuous question—after all, you usually know the price of the stock, bond, real estate investment, or used car which you're trying to value. But many interesting questions of *financing alternatives* depend on the relative interest costs of each alternative. For example, should you pay cash for a car or borrow money to pay for it (and hence make a series of payments over time)? Should you lease that new computer you want or buy it outright? Or perhaps borrow money from the bank

to buy it? It's all clearly a question of *cost*—you'd like to pick the alternative that costs the least.

The tools used for the second question—What does it cost?—are mostly derived from the concept of internal rate of return (IRR). This concept—introduced in Chapter 3—measures compound rate of return of a series of cash flows. In this chapter, we show you that rate of return, when properly used, can be used to measure the cost of financing alternatives. The main concept presented in this chapter is the *effective annual interest rate* (EAIR), a concept based on the annualized IRR that you can use to compare financing alternatives.

Much of the discussion in this chapter relates to calculating the EAIR and showing its relation to the IRR. We show that the EAIR is a much better gauge of the financing costs than the *annualized percentage rate* (APR), the financing cost often quoted by many lenders such as banks and credit card companies. We show you how to apply this concept to credit card borrowing, mortgages, and auto leasing.

| Finance concepts discussed | Excel functions used |
|---|---|
| • Effective annual interest rate (EAIR) | • **IRR** |
| • Internal rate of return (IRR) | • **PMT, IPMT,** and **PPMT** |
| • Annual percentage rate (APR) | • **Rate** |
| • Loan tables | • **NPV** |
| • Mortgage points | • **PV** |
| • Lease versus purchase | • **Max** |
| | • **Exp** |
| | • **Ln** |
| | • **If** |
| | • **Data Table** |

## 5.1 Don't Trust the Quoted Interest Rate—Three Examples

In order to set the stage for the somewhat more complicated examples in the rest of the chapter, we start with three simple examples. Each example shows why *quoted interest rates* are not necessarily representative of costs.

We use the three examples in this section to introduce the concept of *effective annual interest rate* (EAIR):

*The effective annual interest rate (EAIR) is the annualized internal rate of return (IRR) of the cash flows of a particular credit arrangement or security.*

### Example 1: Borrowing from a Bank

In finance, "cost" often refers to an interest rate: "I'm taking a loan from the East Hampton Bank because it's cheaper—West Hampton Bank charges 8%, and East

Hampton Bank charges 6% interest." This is a sentence we all understand—6% interest results in lower payments than 8% interest.

But now consider the following alternatives. You want to borrow $100 for 1 year, and you've investigated both the West Hampton Bank and East Hampton Bank:

- West Hampton Bank is lending at 8% interest. If you borrow $100 from them today, you'll have to repay them $108 in 1 year.
- The East Hampton Bank is willing to lend you any amount you want at a 6% rate. BUT: East Hampton Bank has a "loan initiation charge" of 4%. What this means is that for each $100 you borrow, you'll get only $96, even though you'll pay interest on the full $100.[1]

Obviously the cost of West Hampton's loan is 8%. But is this cheaper or more expensive than the East Hampton loan? You reason as follows: To actually get $100 in your hands from East Hampton, you'll have to borrow $104.17; after they deduct their 4% charge, you'll be left with $100 in hand, which is exactly what you need ((1–4%)*104.17 = 100). At the end of a year, you'll owe East Hampton Bank 104.17*(1+6%) = $110.42. So the actual interest rate they're charging you (we'll call it the *effective annual interest rate*, EAIR) is calculated as follows:

$$0 = 100 - \frac{110.42}{(1+EAIR)^{t=1}} \Rightarrow EAIR = \frac{110.42}{100} - 1 = 10.42\%$$

This makes everything easier—West Hampton's 8% loan (EAIR = 8%) is actually cheaper than East Hampton's "6%" loan (EAIR = 10.42%).

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **CHEAPER LOAN: WEST HAMPTON OR EAST HAMPTON?** | | | |
| 2 | | **West Hampton** | **East Hampton** | |
| 3 | Quoted interest rate | 8% | 6% | |
| 4 | Initial charges | 0% | 4% | |
| 5 | Amount borrowed to get $100 today | 100.00 | 104.17 | <-- =100/(1–C4) |
| 6 | | | | |
| 7 | **Date** | **Cash flow** | **Cash flow** | |
| 8 | Date 1, get loan | 100.00 | 100.00 | |
| 9 | Date 2, pay it back | –108.00 | –110.42 | <-- =–C5*(1+C3) |
| 10 | Effective annual interest rate, EAIR | 8.00% | 10.42% | <-- =IRR(C8:C9) |

[1] Such charges are common in many kinds of bank loans, especially mortgages. They're obviously a way to increase the cost of the loan and to befuddle the customer.

Notice in this example that the EAIR is just an IRR, adjusted for the cost of taking the loan from East Hampton. EAIR is *always an interest rate*, but usually with some kind of adjustment.

**The lesson of Example 1:** When calculating the cost of financial alternatives, *you must include the fees*, even if the lender (in our case, East Hampton Bank) fudges this issue.

## Example 2: Monthly Versus Annual Interest

You want to buy a computer for $1,000. You don't have any money, so you'll have to finance the computer by taking out a loan for $1,000. You've got two financing alternatives:

- Your bank will lend you the money for 15% annual interest. When you ask the bank what this means, they assure you that they will give you $1,000 today and ask you to repay $1,150 = 1,000*(1 + 15%) at the end of 1 year.
- Loan Shark Financing Company will also lend you the $1,000. Their ads say "14.4% annual percentage rate (APR) on a monthly basis." When you ask them what this means, it turns out that Loan Shark charges 1.2% *per month* (they explain to you that $\frac{14.4\%}{12} = 1.2\%$). This means that each month Loan Shark adds 1.2% to the loan balance outstanding at the end of the previous month:

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 1 | **HOW LOAN SHARK CHARGES: 14.4% PER YEAR ON A MONTHLY BASIS = 1.2% PER MONTH** | | | | | | | |
| 2 | | | | | | | | |
| 3 | **Loan balance outstanding at the end of each month** | | | | | | | |
| 4 | **Month 0** | **Month 1** | **Month 2** | **Month 3** | **Month 4** | **Month 5** | **Month 6** | |
| 5 | $ 1,000.00 | | | | | | | |
| 6 | | $ 1,012.00 | <-- =A5*(1+1.2%) | | | | | |
| 7 | | | $ 1,024.14 | <-- =B6*(1+1.2%) | | | | |
| 8 | | | | $ 1,036.43 | <-- =C7*(1+1.2%) | | | |
| 9 | | | | | $ 1,048.87 | <-- =D8*(1+1.2%) | | |
| 10 | | | | | | $ 1,061.46 | <-- =E9*(1+1.2%) | |
| 11 | | | | | | | $ 1,074.19 | <-- =F10*(1+1.2%) |
| 12 | | | | | | | | |
| 13 | **Month 7** | **Month 8** | **Month 9** | **Month 10** | **Month 11** | **Month 12** | | |
| 14 | $ 1,087.09 | <-- =G11*(1+1.2%) | | | | | | |
| 15 | | $ 1,100.13 | <-- =A14*(1+1.2%) | | | | | |
| 16 | | | $ 1,113.33 | <-- =B15*(1+1.2%) | | | | |
| 17 | | | | $ 1,126.69 | <-- =C16*(1+1.2%) | | | |
| 18 | | | | | $ 1,140.21 | <-- =D17*(1+1.2%) | | |
| 19 | | | | | | $ 1,153.89 | <-- =E18*(1+1.2%) | |

By the end of the year, you will owe Loan Shark $1,153.89.

$$\$1{,}153.89 = \$1{,}500 * (1 + \underbrace{\frac{14.4\%}{12}}_{\substack{\uparrow \\ \text{Loan Shark's loan} \\ \text{is "compounded} \\ \text{monthly." This means} \\ \text{that the 14.4\% annual} \\ \text{interest translates} \\ \text{to 1.2\% per month.}}})^{12}$$

Since this is more than the $1,150 you will owe the bank, you should prefer the bank loan.

The effective annual interest rate (EAIR) of each loan is the *annualized interest rate* charged by the loan. The bank charges you 15% annually, and Loan Shark charges you 15.39% annually:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **THE BANK OR LOAN SHARK?** | | | |
| 2 | | **Bank** | **Loan Shark** | |
| 3 | Quoted interest rate | 15.0% | 14.4% | |
| 4 | Borrow today | 1,000.00 | 1,000.00 | |
| 5 | Repay in 1 year | –1,150.00 | –1,153.89 | <-- =–C4*(1+C3/12)^12 |
| 6 | Effective annual interest rate, EAIR | 15.00% | 15.39% | <-- =–C5/C4–1 |
| 7 | | | | |
| 8 | **A second way to compute the EAIR** | | | |
| 9 | Monthly interest rate | | 1.20% | <-- =C3/12 |
| 10 | EAIR Annualized monthly rate | | 15.39% | <-- =(1+C9)^12–1 |

Cells C9 and C10 show another way to compute the 15.39% EAIR charged by the loan shark. Cell C9 computes the monthly rate charged by the loan shark as 1.20%, and cell C10 annualizes this rate $\left(1+\frac{14.4\%}{12}\right)^{12} - 1 = 15.39\%$. Thus there are two ways to compute the EAIR:

$$EAIR = 15.39\% = \begin{cases} \dfrac{Payment\ at\ end\ of\ year}{Loan\ taken\ out\ beginning\ of\ year} - 1 = \dfrac{\$1{,}153.89}{\$1{,}000.00} - 1 & \leftarrow \text{Cell } C6 \\ \left(1+\dfrac{APR}{m}\right)^{m} - 1 = \left(1+\dfrac{14.4\%}{12}\right)^{12} - 1 & \leftarrow \text{Cell } C10 \end{cases}$$

where $m$ represents the number of compounding periods within a year.

**The lesson of Example 2:** Annual percentage rate (APR) does not always correctly reflect the costs of borrowing. To compute the true cost, calculate the effective annual interest rate (EAIR).

### Example 3: An "Interest Free" Loan

You're buying a used car. The Junkmobile your heart desires has a price tag of \$2,000. You have two financing options:

- The dealer explains that if you pay cash you'll get a 15% discount. In this case, you'll pay \$1,700 for the car today. Since you don't have any money now, you intend to borrow the \$1,700 from your Uncle Frank, who charges 10% interest.
- On the other hand, the dealer will give you "0% financing": You don't pay anything now, and you can pay the dealer the full cost of the Junkmobile at the end of the year.

Thus you have two choices: the dealer's 0% financing and Uncle Frank's 10% rate. Which is cheaper?

A little thought will show that the dealer is actually charging you an effective annual interest rate (EAIR) of 17.65%. His "0% financing" essentially involves a loan to you of \$1,700 with an end-year repayment of \$2,000:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **FINANCING THE JUNKMOBILE** | | | | |
| 2 | **Year** | **Pay cash** | **Dealer's "0% financing"** | **Differential cash flow** | |
| 3 | 0 | –1,700 | 0 | 1,700 | <-- =C3–B3 |
| 4 | 1 | | –2,000 | –2,000 | <-- =C4–B4 |
| 5 | | | | | |
| 6 | Effective annual interest rate (EAIR) charged by dealer | | | 17.65% | <-- =IRR(D3:D4) |

The general idea here is the concept of comparing the two offers. Taking the dealer's offer means that we do not pay \$1,700 today but rather pay \$2,000 next year. Which is a different way to say that when comparing the two alternatives, we save \$1,700 today and pay \$2,000 next year.

Uncle Frank's EAIR is 10%: He will loan you \$1,700 and have you repay only \$1,870. So you're better off borrowing from him.

**The lesson of Example 3:** Free loans are usually not free! To compute the cost of a "free" loan, calculate the EAIR of the differential cash flows.

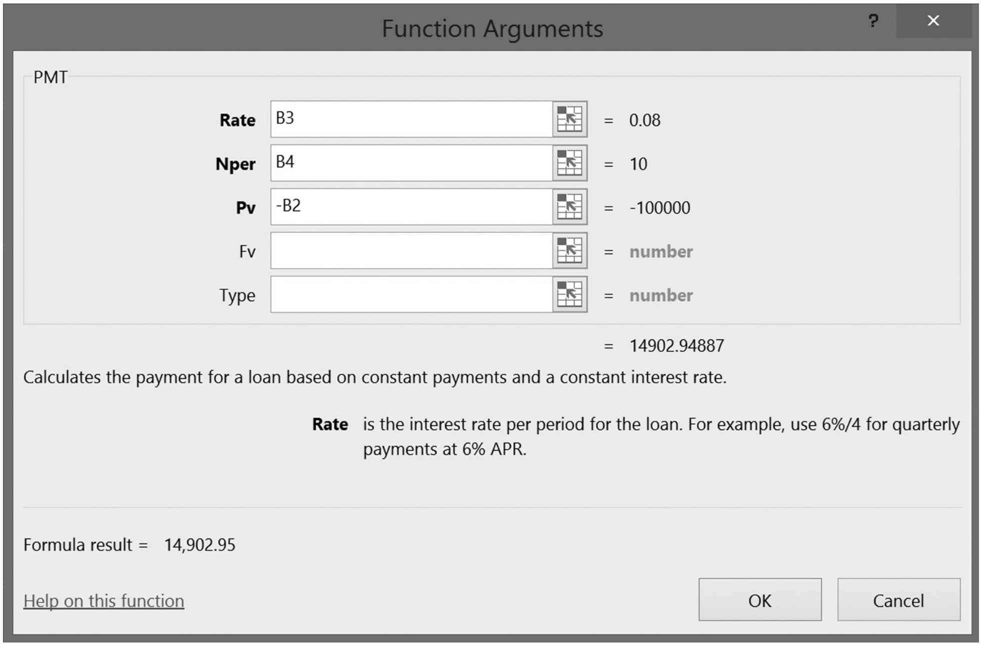
## 5.2 Calculating the Cost of a Mortgage

Now that we've set the stage, we'll proceed to a series of somewhat more complicated examples. We start with a mortgage. Housing is most often the largest personal asset an individual owns. Financing housing with a mortgage is something almost every reader of this book will do in his or her lifetime. Calculating the cost of a mortgage is thus a useful exercise. In this chapter, mortgages will be one of the examples we use to illustrate the problems encountered in computing the cost of financial assets. The reader can easily extend the analysis to other types of loans (presented in Chapter 4 of this book).

## A Simple Mortgage

We start with a simple example. Your bank has agreed to give you a $100,000 mortgage, to be repaid over 10 years at 8% interest. For simplicity, we assume that the payments on the mortgage are annual.[2] The bank calculates the annual payment as $14,902.95, using Excel's **PMT** function:

### Dialog Box for PMT Function




The dialog box for Excel's **PMT** function: **Rate** is the interest rate on the loan; **Nper** is the number of repayment periods; **Pv** is the loan principal. As discussed in Chapter 2, if the loan principal is written as a positive number, Excel presents the payment as a negative number; to avoid this, we write **Pv** as a negative number.

[2] In the real world, payments are probably monthly; see the example in Section 5.3.

The **PMT** function calculates an annuity payment (a constant periodic payment) which pays off a loan:

$$100{,}000 = \sum_{t=1}^{10} \frac{14{,}902.95}{(1.08)^t} = \frac{14{,}902.95}{(1.08)} + \frac{14{,}902.95}{(1.08)^2} + \frac{14{,}902.95}{(1.08)^3} + \cdots + \frac{14{,}902.95}{(1.08)^{10}}$$

We can summarize all of this in an Excel spreadsheet:

| | A | B | C |
|---|---|---|---|
| 1 | **A SIMPLE MORTGAGE** | | |
| 2 | Mortgage principal | 100,000 | |
| 3 | Interest rate | 8% | |
| 4 | Mortgage term (years) | 10 | |
| 5 | Annual payment | $14,902.95 | <-- =PMT(B3,B4,–B2) |
| 6 | | | |
| 7 | **Year** | **Mortgage cash flow** | |
| 8 | 0 | 100,000.00 | |
| 9 | 1 | –14,902.95 | <-- =–$B$5 |
| 10 | 2 | –14,902.95 | |
| 11 | 3 | –14,902.95 | |
| 12 | 4 | –14,902.95 | |
| 13 | 5 | –14,902.95 | |
| 14 | 6 | –14,902.95 | |
| 15 | 7 | –14,902.95 | |
| 16 | 8 | –14,902.95 | |
| 17 | 9 | –14,902.95 | |
| 18 | 10 | –14,902.95 | |
| 19 | | | |
| 20 | Effective annual interest rate (EAIR) | 8.00% | <-- =IRR(B8:B18) |

The *effective annual interest rate* (EAIR) of this particular mortgage is simply the internal rate of return of its payments. Because the payments on the mortgage are annual, the IRR in cell B20 is already in annual terms.

## The Bank Charges "Mortgage Points"

As in the previous example, you've asked the bank for a $100,000 mortgage. They've agreed to give you this mortgage, and they've explained that you'll be asked to repay $14,902.95 per year for the next 10 years. However, when you get to the bank, you learn that the bank has deducted "1.5 points" from your mortgage. What this means is that you only get $98,500 ($100,000 minus 1.5%). Your payments, however, continue to be based on a principal of $100,000.[3] You realize

[3] Some banks and mortgage brokers also charge an "origination fee," defined as a payment to cover the initial cost of processing the mortgage. The net effect of "points" and the "origination fee" is the same: You are charged interest on more money than you actually get in hand.

immediately that this mortgage is more expensive than the mortgage discussed in the previous subsection. The question is: By *how much* is it more expensive? We can answer this question by calculating the *effective annual interest rate (EAIR)* on the mortgage. The calculation below shows that you're actually paying 8.34% interest annually.

| | A | B | C |
|---|---|---|---|
| 1 | **A MORTGAGE WITH POINTS** | | |
| 2 | Mortgage principal | 100,000 | |
| 3 | "Points" | 1.50% | |
| 4 | Quoted interest | 8.00% | |
| 5 | Mortgage term (years) | 10 | |
| 6 | Annual payment | $14,902.95 | <-- =PMT(B4,B5,–B2) |
| 7 | | | |
| 8 | **Year** | **Mortgage cash flow** | |
| 9 | 0 | 98,500.00 | <-- =B2*(1–B3) |
| 10 | 1 | –14,902.95 | <-- =–$B$6 |
| 11 | 2 | –14,902.95 | |
| 12 | 3 | –14,902.95 | |
| 13 | 4 | –14,902.95 | |
| 14 | 5 | –14,902.95 | |
| 15 | 6 | –14,902.95 | |
| 16 | 7 | –14,902.95 | |
| 17 | 8 | –14,902.95 | |
| 18 | 9 | –14,902.95 | |
| 19 | 10 | –14,902.95 | |
| 20 | | | |
| 21 | Effective annual interest rate | 8.34% | <-- =IRR(B9:B19) |

Notice that the effective annual interest rate (EAIR) of 8.34% is the internal rate of return of the stream of payments consisting of the actual loan amount ($98,500) versus the actual payments you're making ($14,902.95 annually). Here's the calculation:

$$98,500 = \sum_{t=1}^{10} \frac{14,902.95}{(1.0834)^t} = \frac{14,902.95}{(1.0834)} + \frac{14,902.95}{(1.0834)^2} + \frac{14,902.95}{(1.0834)^3} + \cdots + \frac{14,902.95}{(1.0834)^{10}}$$

At the end of each year, you will report to the Internal Revenue Service the amount of interest paid on the mortgage. Because this interest is an expense for tax purposes, it's important to get it right. To calculate this interest, we need a loan table, which allocates the each year's payment made between interest and repayment of principal (see Section 4.2, page 111). This table is often called an "amortization table" ("amortize" means to repay with a series of periodic payments):

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 23 | MORTGAGE AMORTIZATION TABLE | | | | | |
| 24 | Year | Mortgage principal at beginning of year | Payment at end of year | Part of payment that is interest (expense for taxes!) | Part of payment that is repayment of principal (not an expense for tax purposes) | |
| 25 | 1 | 98,500.00 | $14,902.95 | $8,211.41 | 6,691.54 | <-- =C25–D25 |
| 26 | 2 | 91,808.46 | $14,902.95 | $7,653.58 | 7,249.37 | |
| 27 | 3 | 84,559.09 | $14,902.95 | $7,049.23 | 7,853.71 | |
| 28 | 4 | 76,705.38 | $14,902.95 | $6,394.51 | 8,508.44 | |
| 29 | 5 | 68,196.94 | $14,902.95 | $5,685.21 | 9,217.74 | |
| 30 | 6 | 58,979.20 | $14,902.95 | $4,916.78 | 9,986.17 | |
| 31 | 7 | 48,993.03 | $14,902.95 | $4,084.28 | 10,818.66 | |
| 32 | 8 | 38,174.37 | $14,902.95 | $3,182.39 | 11,720.56 | |
| 33 | 9 | 26,453.81 | $14,902.95 | $2,205.31 | 12,697.64 | |
| 34 | 10 | 13,756.17 | $14,902.95 | $1,146.78 | 13,756.17 | |
| 35 | | | | | | |
| 36 | =B25–E25 | | =$B$21*B25 | | | |

Column D of the table gives the interest expense for tax purposes. If you report interest payments on your tax return, this is the payment you'd be allowed to report. Notice that the interest portion of the annual $14,902.95 payments gets smaller over the years, while the repayment of principal portion (which is not deductible for tax purposes) gets larger.

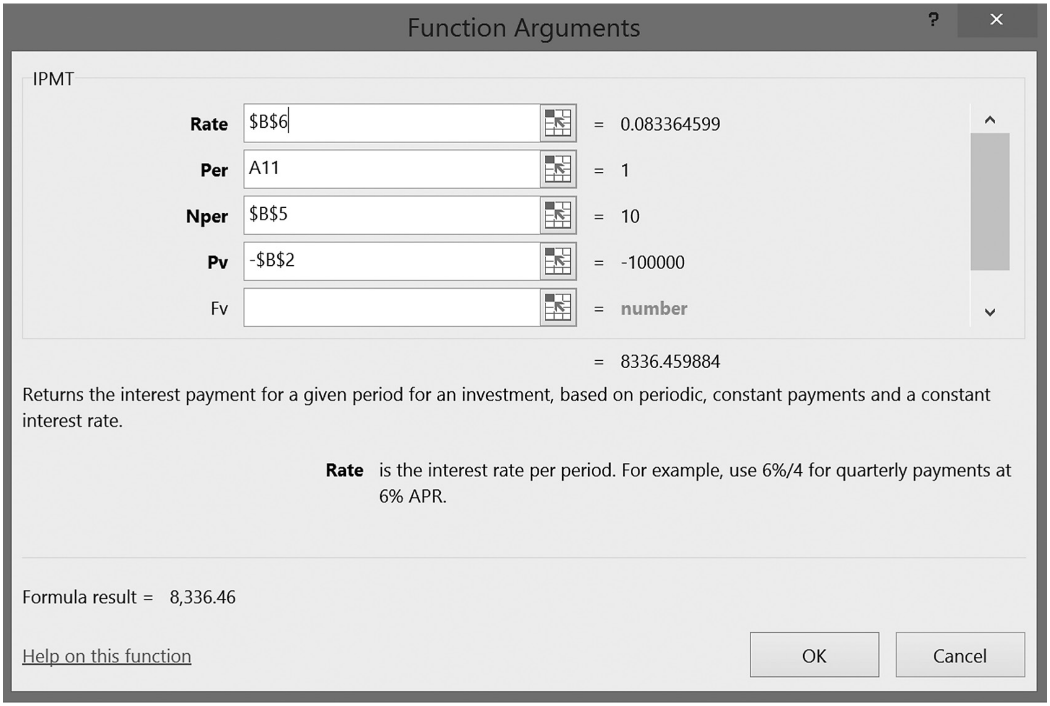
## Calculating the Individual Payments with IPMT and PPMT

The above spreadsheet gives the intuition behind the loan table and the split between interest and principal of each payment. The interest and repayment-of-principal payments can be computed directly using the Excel functions **IPMT** and **PPMT**. This is illustrated below.[4]

[4] Note that **IPMT** and **PPMT** work only when the loan payments are equal.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **COMPUTING THE MORTGAGE PAYMENTS USING IPMT AND PPMT** | | | | | | |
| 2 | Loan principal | 100,000 | | | | | |
| 3 | Points | 1.50% | | | **The interest rate from the previous spreadsheet has been formatted to show only two decimal places. The actual interest rate is 8.336459884%.** | | |
| 4 | Quoted interest | 8.00% | | | | | |
| 5 | Term (years) | 10 | | | | | |
| 6 | Effective interest rate | 8.34% | <-- ='Mortgage with points'!B21 | | | | |
| 7 | Annual payment | 14,902.95 | <-- =PMT(B6,B5,–B2*(1–B3)) | | | | |
| 8 | | | | | | | |
| 9 | **Mortgage amortization table** | | | | | | |
| 10 | **Year** | **Mortgage principal at beginning of year** | **Payment at end of year** | **Part of payment that is interest** | **Part of payment that is repayment of principal** | | |
| 11 | 1 | 98,500.00 | 14,902.95 | 8,211.41 | 6,793.44 | <-- =PPMT($B$6,A11,$B$5,–$B$2) | |
| 12 | 2 | 91,706.56 | 14,902.95 | 7,653.58 | 7,359.77 | | |
| 13 | 3 | 84,346.79 | 14,902.95 | 7,049.23 | 7,973.31 | | |
| 14 | 4 | 76,373.48 | 14,902.95 | 6,394.51 | 8,638.01 | | |
| 15 | 5 | 67,735.47 | 14,902.95 | 5,685.21 | 9,358.11 | | |
| 16 | 6 | 58,377.36 | 14,902.95 | 4,916.78 | 10,138.24 | | |
| 17 | 7 | 48,239.12 | 14,902.95 | 4,084.28 | 10,983.42 | | |
| 18 | 8 | 37,255.70 | 14,902.95 | 3,182.39 | 11,899.04 | | |
| 19 | 9 | 25,356.66 | 14,902.95 | 2,205.31 | 12,891.00 | | |
| 20 | 10 | 12,465.66 | 14,902.95 | 1,146.78 | 13,965.66 | | |
| 21 | | | | | | | |
| 22 | | =IPMT($B$6,A11,$B$5,–$B$11) | | | | | |

Here is the dialog box for **IPMT** in cell D9 (the syntax of **PPMT** is similar). Notice that **Per** specifies the specific period for which the interest is calculated:




## 5.3 Mortgages with Monthly Payments

We continue with the mortgage examples from Section 4.5. This time, we introduce the concept of monthly payments. Suppose you get a $100,000 mortgage with an 8% interest rate, payable monthly, and suppose you have to pay the mortgage back over 1 year (12 months).[5] Many banks interpret the combination of 8% annual interest and "payable monthly" to mean that the monthly interest on the mortgage is $\frac{8\%}{12} = 0.667\%$. This is often referred to as "monthly compounding," although the usage of this term is not uniform. To compute the monthly repayment on the mortgage, we use Excel's **PMT** function:

[5] Most mortgages are, of course, for much longer term. But 12 months enables us to fit the example comfortably within a page. Later we'll consider longer terms, but the principles will be the same.

| | A | B | C |
|---|---|---|---|
| 1 | **MORTGAGE WITH MONTHLY PAYMENTS** | | |
| 2 | Loan principal | 100,000 | |
| 3 | Loan term (years) | 1 | |
| 4 | Quoted interest rate | 8% | |
| 5 | | | |
| 6 | **Month** | **Cash flow** | |
| 7 | 0 | 100,000.00 | |
| 8 | 1 | –8,698.84 | <-- =PMT($B$4/12,$B$3*12,$B$2) |
| 9 | 2 | –8,698.84 | |
| 10 | 3 | –8,698.84 | |
| 11 | 4 | –8,698.84 | |
| 12 | 5 | –8,698.84 | |
| 13 | 6 | –8,698.84 | |
| 14 | 7 | –8,698.84 | |
| 15 | 8 | –8,698.84 | |
| 16 | 9 | –8,698.84 | |
| 17 | 10 | –8,698.84 | |
| 18 | 11 | –8,698.84 | |
| 19 | 12 | –8,698.84 | |
| 20 | | | |
| 21 | Monthly **IRR** | 0.667% | <-- =IRR(B7:B19) |
| 22 | Effective annual interest rate, EAIR | 8.30% | <-- =(1+B21)^12–1 |

The EAIR on the mortgage in the example is computed by using Excel's **IRR** function (cell B21). In our case, the **IRR** function will give a monthly interest rate of 0.667% (we already knew this, since $\frac{8\%}{12} = 0.667\%$). Annualizing this gives $8.30\% = \left(1+\frac{8\%}{12}\right)^{12} - 1$ (cell B22).

## Mortgages: A More Complicated Example

As we saw in section 5.2, many mortgages in the United States have "origination fees" or "discount points" (the latter are often just called "points"). All of these fees reduce the initial amount given to you by the bank, *without reducing* the principal on which the bank computes its payments (sounds misleading, doesn't it?).

As an example, consider the above 12-month mortgage with an 8% annual rate, payable monthly, but with an origination fee of 0.5% and 1 point. This means that you actually get \$98,500 (\$100,000 minus \$500 for the origination fee and \$1,000 for the point) but that your monthly repayment remains \$8,698.84:

| | A | B | C |
|---|---|---|---|
| 1 | **MORTGAGE EXAMPLE WITH POINTS AND ORIGINATION FEE** | | |
| 2 | Loan principal | 100,000.00 | |
| 3 | Loan term (years) | 1 | |
| 4 | Quoted interest rate | 8% | |
| 5 | Discount points | 1 | |
| 6 | Origination fee | 0.5% | |
| 7 | | | |
| 8 | **Month** | **Cash flow** | |
| 9 | 0 | 98,500.00 | <-- =B2*(1–B5/100–B6) |
| 10 | 1 | –8,698.84 | <-- =PMT($B$4/12,$B$3*12,$B$2) |
| 11 | 2 | –8,698.84 | |
| 12 | 3 | –8,698.84 | |
| 13 | 4 | –8,698.84 | |
| 14 | 5 | –8,698.84 | |
| 15 | 6 | –8,698.84 | |
| 16 | 7 | –8,698.84 | |
| 17 | 8 | –8,698.84 | |
| 18 | 9 | –8,698.84 | |
| 19 | 10 | –8,698.84 | |
| 20 | 11 | –8,698.84 | |
| 21 | 12 | –8,698.84 | |
| 22 | | | |
| 23 | Monthly **IRR** | 0.9044% | <-- =IRR(B9:B21) |
| 24 | EAIR | 11.41% | <-- =(1+B23)^12–1 |
| 25 | | | |
| 26 | Monthly **IRR** using Excel's **Rate** function | 0.9044% | <-- =RATE(12,8698.84,–98500) |

The monthly IRR (cell B23) is the interest rate that sets the present value of the monthly payments equal to the initial $98,500 received:

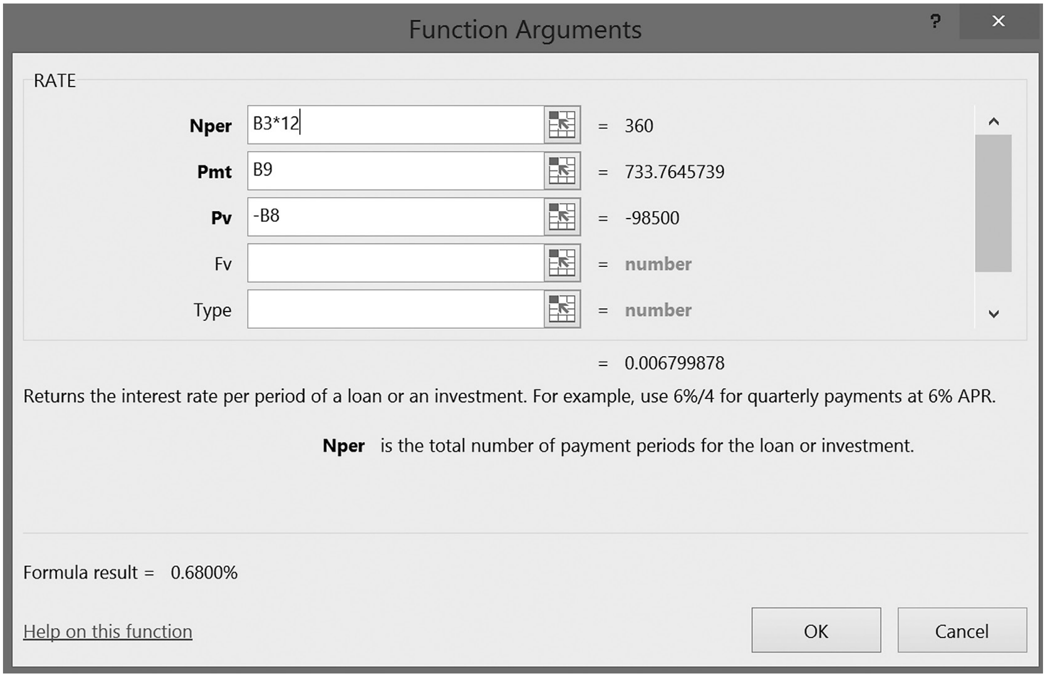
$$\$98,500 = \frac{\$8,698.94}{(1+0.9044\%)} + \frac{\$8,698.94}{(1+0.9044\%)^2} + \frac{\$8,698.94}{(1+0.9044\%)^3} + \cdots + \frac{\$8,698.94}{(1+0.9044\%)^{12}}$$

The effective annual interest rate $EAIR = 11.41\% = (1+0.9044\%)^{12} - 1$ is the annualized cost of the mortgage payments.

As you can see in cell B26, Excel's **Rate** function will also calculate the monthly IRR that we've calculated in cell B23:

EXCEL NOTE

## Calculating the Monthly IRR with Rate




The **Rate** function computes the IRR of a series of constant (the financial jargon is "flat" or "even") payments so that the discounted value equals the **PV** indicated. Notice that in the **Rate** function the signs of the payments (indicated by **Pmt**) and the present value **PV** of these payments must be different. This is a feature which **Rate** shares with Excel functions like **PMT** and **PV** discussed in Chapter 2.

## Longer-Term Mortgages

Suppose the mortgage in the previous example has a 30-year term (meaning: 360 = 30*12 repayments). Each repayment would be $733.76, and the EAIR would be 8.4721%:

| | A | B | C |
|---|---|---|---|
| 1 | **30-YEAR MORTGAGE With points and origination fee** | | |
| 2 | Loan principal | 100,000.00 | |
| 3 | Loan term (years) | 30 | |
| 4 | Quoted interest rate | 8% | |
| 5 | Discount points | 1 | |
| 6 | Origination fee | 0.5% | |
| 7 | | | |
| 8 | Initial amount of loan, net of fees | 98,500.00 | <-- =B2*(1–B5/100–B6) |
| 9 | Monthly repayment | 733.76 | <-- =PMT(B4/12,B3*12,–B2) |
| 10 | | | |
| 11 | **Calculating the EAIR** | | |
| 12 | Monthly interest rate | 0.6800% | <-- =RATE(B3*12,B9,–B8) |
| 13 | Effective annual interest rate (EAIR) | 8.4721% | <-- =(1+B12)^12–1 |

We've used **PMT** to calculate the payment and **Rate** to compute the monthly interest rate. The EAIR is computed in the usual manner—by compounding the monthly payments (cell B13).

Note that the effect of the initial mortgage fees on mortgage EAIR declines when the mortgage is longer term:

- For the 1-year mortgage discussed previously, the 1.5% initial fee increased the EAIR of the mortgage from 8% to 11.41%.
- For the 30-year mortgage discussed above, the same initial fees increase the EAIR from 8% to 8.4721%.
- The reason that the fees have a smaller effect for the second mortgage is that they are spread out over a much longer term.

## 5.4 Lease or Purchase?

This section uses the concepts of present value and internal rate of return to explore the relative advantages of leasing versus buying an asset. As you will see, the choice between leasing and buying basically comes down to choosing the cheaper of two methods of financing.

Here's our terminology: A *lease* is a rental agreement; in our examples, leases will usually be for equipment (we discuss a computer lease and a car lease), but the analysis for real estate is virtually the same. The party that rents the asset and uses it is called the *lessee*, and the owner of the asset is called the *lessor*.

### A Simple Lease Example

You need a new computer, but you can't decide whether you should buy it or lease it. The computer costs $4,000. The lessor is your neighborhood computer leasing company, which offers to lease you the computer for $1,500 per year. The lessor's conditions are that you make four payments of $1,500: the first payment at the start of the lease (time 0) and subsequent payments at the end of years 1, 2, and 3. Based on past experience, you know that you will keep your new computer for about 3 years. One additional fact: You can borrow from your bank at 15%.

Here is a spreadsheet with the cash flows for the lease and for the purchase:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **BASIC LEASE VERSUS PURCHASE** | | | |
| 2 | Asset cost | 4,000.00 | | |
| 3 | Annual lease payment | 1,500.00 | | |
| 4 | Bank rate | 15% | | |
| 5 | | | | |
| 6 | **Year** | **Purchase cash flow** | **Lease cash flow** | |
| 7 | 0 | 4,000.00 | 1,500.00 | |
| 8 | 1 | | 1,500.00 | |
| 9 | 2 | | 1,500.00 | |
| 10 | 3 | | 1,500.00 | |
| 11 | | | | |
| 12 | PV of costs | 4,000.00 | 4,924.84 | <-- =C7+NPV($B$4,C8:C10) |
| 13 | Lease or purchase? | purchase | | <-- =IF(B12<C12,"purchase","lease") |

To decide whether the lease is preferable, we discount the cash flows from both the lease and the purchase at the 15% bank lending rate. We write the outflows as positive numbers, so that the PV in row 12 is the *present value of the costs*. As you can see in cell C12, the present value of the lease costs is $4,924.84, which is more than the $4,000 cost of purchasing the computer. Thus you prefer the purchase, which is less costly.

There's another way of doing this same calculation. We compute the IRR of the *differential* cash flows by subtracting the lease cash flow from the purchase cash flow in each of the years:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **BASIC LEASE VERSUS PURCHASE The differential cash flows** | | | | |
| 2 | Asset cost | 4,000 | | | |
| 3 | Annual lease payment | 1,500 | | | |
| 4 | Bank rate | 15% | | | |
| 5 | | | | | |
| 6 | **Year** | **Purchase cash flow** | **Lease cash flow** | **Differential cash flow** | |
| 7 | 0 | 4,000 | 1,500 | 2,500 | <-- =B7–C7 |
| 8 | 1 | | 1,500 | –1,500 | <-- =B8–C8 |
| 9 | 2 | | 1,500 | –1,500 | <-- =B9–C9 |
| 10 | 3 | | 1,500 | –1,500 | <-- =B10–C10 |
| 11 | | | | | |
| 12 | **IRR** of differential cash flows | | | 36.31% | <-- =IRR(D7:D10) |
| 13 | Lease or purchase? | | | purchase | <-- =IF(D12>B4,"purchase","lease") |
| 14 | | | | | |
| 15 | Explanation: The lease is like a loan—you save 2,500 in year 0 and pay back 1,500 in each of years 1–3. The IRR of this "loan" is 36.31%. | | | | |

The computer lease is equivalent to paying $1,500 for the computer in year 0 and taking a loan of $2,500 from the computer leasing company. The computer leasing "loan" has three equal repayments of $1,500 and an internal rate of return (IRR) of 36.31%. Because you can borrow money from the bank at 15%, you would prefer to purchase the computer with borrowed money from the bank (at 15%) rather than "borrowing" $2,500 from the leasing company which charges 36.31%.

In the spreadsheet below, you can see another way to make this same point. If you borrowed $2,500 from the bank at 15%, you'd have to pay back $1,094.94 per year for each of the next 3 years (assuming the bank asked for a flat repayment schedule). This is substantially less than the $1,500 that the computer lessor asks for on the same loan.

$$2,500 = \frac{1,094.94}{1.15} + \frac{1,094.94}{(1.15)^2} + \frac{1,094.94}{(1.15)^3}$$

The conclusion is that if you decide to borrow $2,500 to purchase the computer, you should do so from the bank rather than from the computer leasing company. In the spreadsheet, we've used the Excel **PMT** function to compute the repayment:

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 17 | **What if you borrowed $2,500 from the bank?** | | | | |
| 18 | **Year** | **Money saved by leasing** | | **Same amount from bank** | |
| 19 | 0 | 2,500 | | 2,500.00 | |
| 20 | 1 | –1,500 | | –1,094.94 | <-- =PMT($B$4,3,$D$19) |
| 21 | 2 | –1,500 | | –1,094.94 | |
| 22 | 3 | –1,500 | | –1,094.94 | |

## What Have We Assumed About Leasing Versus Purchasing?

The leasing example we've considered above illustrates the spirit of lease/purchase analysis. The example makes some simplifying assumptions which are worth noting:

- **No taxes:** When corporations lease equipment, the lease payments are expenses for tax purposes; when these corporations buy assets, the depreciation on the asset is an expense for tax payments. Taxes complicate the analysis somewhat; the case of leasing with taxes is considered in Chapter 6.
- **Operational equivalence of lease and purchase:** In our analysis we don't ask whether you need a computer—we assume that you've already answered this question positively, so that only the method of acquisition is in question. Our analysis also assumes that any maintenance or repairs that need to

be done on the computer will be done by you, whether you lease or buy the computer.

- **No residual value:** We've assumed that the asset (in this case, the computer) is worthless at the end of the lease term.

We explore the last point briefly. Suppose you think that the computer will be worth $800 at the end of year 3. Then—as shown below—the purchase cash flows change, so that owning the computer gives you an inflow of $800 in year 3.[6] The cost of purchasing the computer is reduced (the present value of the purchase is now $3,304), and the lease alternative becomes even less attractive than the purchase. Another way of seeing this is to look at the IRR of the differential cash flows, which is now 45.07% (cell B23).[7]

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **LEASE VERSUS PURCHASE WITH RESIDUAL VALUE** | | | |
| 2 | Asset cost | 4,000.00 | | |
| 3 | Annual lease payment | 1,500.00 | | |
| 4 | Residual value, yr. 3 | 800 | <-- Value of computer at end of year 3 | |
| 5 | Bank rate | 15% | | |
| 6 | | | | |
| 7 | **Year** | **Purchase cash flow** | **Lease cash flow** | |
| 8 | 0 | 4,000.00 | 1,500.00 | |
| 9 | 1 | 0.00 | 1,500.00 | |
| 10 | 2 | 0.00 | 1,500.00 | |
| 11 | 3 | –800.00 | 1,500.00 | |
| 12 | | | | |
| 13 | PV of costs | 3,473.99 | 4,924.84 | <-- =C8+NPV($B$5,C9:C11) |
| 14 | Lease or purchase? | | purchase | <-- =IF(B13<C13,"purchase","lease") |
| 15 | | | | |
| 16 | **Calculating the IRR of the differential cash flow** | | | |
| 17 | **Year** | | **Money saved by leasing** | |
| 18 | 0 | | 2,500.00 | |
| 19 | 1 | | –1,500.00 | |
| 20 | 2 | | –1,500.00 | <-- =B8–C8 |
| 21 | 3 | | –2,300.00 | <-- =B9–C9 |
| 22 | | | | |
| 23 | **IRR** of differential | | 45.07% | <-- =IRR(C18:C21) |
| 24 | Lease or purchase? | | purchase | <-- =IF(C23>B5,"purchase","lease") |

[6] Note that since we're writing *outflows* (like the cost of the computer) as *positive numbers*, we have to write the inflows as negative numbers.

[7] A caveat is in order here: We're treating the computer's residual value as if it has the same certainty as the rest of the cash flows, whereas clearly it is less certain. The finance literature has a technical solution to this: We find the *certainty equivalent* of the residual value. For example, it may be that we expect the residual value to be $1,200 but that—recognizing the uncertainty of getting this value—we treat this as equivalent to getting an $800 residual with certainty.

## 5.5 Auto Lease Example

Here's a slightly more realistic (and more complicated) example of leasing: You've decided to get a new car. You can either lease the car or buy it; if you decide to buy the car, you can finance with a 6% bank loan. The relevant facts are given in the spreadsheet below, but we'll summarize them here:

- The cash cost of the car is \$20,000. This price represents your alternative purchase cost if you decide to buy instead of lease the car.
- The dealer has offered you the following lease terms:
  - You pay \$2,000 at the signing of the lease.
  - Starting next month, you pay \$400 per month for the next 24 months.
  - You guarantee that the car will have a residual value of \$10,000 at the end of the lease. What this means is that if the car is worth less than \$10,000 at the end of the 24th month, the lessee (you) will make up the difference.[8] The end-lease payment associated with this residual can be written as

$$\text{Lease-end residual payment} = \begin{cases} 10{,}000 - \text{market value} & \text{if market value} < 10{,}000 \\ 0 & \text{otherwise} \end{cases}$$

Another way of writing this payment is $Max(10{,}000 - market\ value, 0)$. The *max(A,B)* notation means that you pay the larger of *A* or *B*. Conveniently, **Max** is also a function in Excel.

The residual value turns out to be an important factor in the way you view leasing versus purchase. We'll devote more time to it later. For the moment, let's assume that you think the car will actually be worth \$15,000 at the end of 2 years, so your last payment on the lease is zero:

$$\begin{aligned} \text{Lease-end residual payment} &= Max(10{,}000 - market\ value, 0) \\ &= Max(10{,}000 - 15{,}000, 0) = Max(-5000, 0) = 0 \end{aligned}$$

[8] According to www.edmunds.com: "The lease-end fees are generally reasonable, unless the car has 100,000 miles on it, a busted-up grille and melted chocolate smeared into the upholstery. Dealers and financial institutions want you to buy or lease another car from them, and can be rather lenient regarding excess mileage and abnormal wear. After all, if they hit you with a bunch of trumped-up charges you're not going to remain a loyal customer, are you? . . . But keep in mind that if you take your business elsewhere, you're going to be facing a bill for items like worn tires, paint chips, door dings, and the like."

On the other hand, if you think the car will be worth $8,000 at the end of 2 years, you will be looking at an additional payment of $2,000:

$$\begin{aligned} \textit{Lease-end residual payment} &= Max\left(10{,}000 - \textit{market value}, 0\right) \\ &= Max\left(10{,}000 - 8{,}000, 0\right) = Max\left(2000, 0\right) = 2{,}000 \end{aligned}$$

In the spreadsheet below, we have listed all the lease parameters and cash flows; as usual, we have written costs as negative numbers and inflows (for example, the terminal value of the car, if you buy it) as positive cash flows. All of the lease costs are listed in column B. To evaluate these costs, compare them to column C, which shows the costs associated with buying the car; there are only two of these: (a) the initial purchase price of the car ($20,000) and (b) what you anticipate will be the market value of the car at the end of the lease term (in the example below, you think the car will actually be worth $8,000).

This last number bears some examination: If you lease, your last payment is

$$\begin{aligned} \textit{Last lease payment} &= \textit{last month's rental} + \textit{end-of-lease residual payment} \\ &= 400 + Max\left(10{,}000 - \textit{market value}, 0\right) \end{aligned}$$

If you're right and the actual market value of the car is $8,000, then your last "payment" is $2,400.

Column D in the spreadsheet subtracts the lease from the purchase cash flows. Initially, the lease saves you $18,000; in months 1–23, the lease costs you $400 more than the purchase; and at the end of month 24, the lease costs you $10,400 more than the purchase. Thus leasing versus purchase is like getting a loan from the car leasing company. To evaluate the lease, we compute its effective annual interest rate (EAIR = (1 + monthly IRR)^12 – 1, as shown in cell B11).

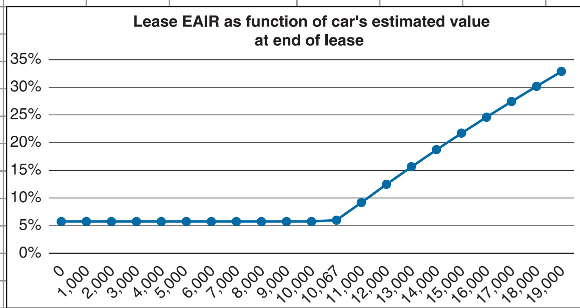
| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | | **ANALYZING A CAR LEASE** | | |
| 2 | Car purchase price | 20,000 | | | |
| 3 | Initial lease payment | 2,000 | | | |
| 4 | Monthly lease payment | 400 | | | |
| 5 | Lessee guaranteed terminal value | 10,000 | | | |
| 6 | Lessee estimated terminal value | 8,000 | | | |
| 7 | Lessee estimated final lease payment | 2,000 | <-- =MAX(B5–B6,0) | | |
| 8 | | | | | |
| 9 | **IRR of differential cash flows** | | | | |
| 10 | Monthly effective rate | 0.47% | <-- =IRR(D16:D40) | | |
| 11 | EAIR, annualized IRR | 5.76% | <-- =(1+B10)^12–1 | | |
| 12 | Bank interest rate | 6.00% | | | |
| 13 | Lease or purchase? | lease | <-- =IF(B11<B12,"lease","purchase") | | |
| 14 | | | | | |
| 15 | **Month** | **Lease payments** | **Purchase payments** | **Differential cash flows: Lease minus purchase** | |
| 16 | 0 | –2,000 | –20,000 | 18,000 | <-- =B16–C16 |
| 17 | 1 | –400 | | –400 | |
| 18 | 2 | –400 | | –400 | |
| 19 | 3 | –400 | | –400 | |
| 20 | 4 | –400 | | –400 | |
| 21 | 5 | –400 | | –400 | |
| 22 | 6 | –400 | | –400 | |
| 23 | 7 | –400 | | –400 | |
| 24 | 8 | –400 | | –400 | |
| 25 | 9 | –400 | | –400 | |
| 26 | 10 | –400 | | –400 | |
| 27 | 11 | –400 | | –400 | |
| 28 | 12 | –400 | | –400 | |
| 29 | 13 | –400 | | –400 | |
| 30 | 14 | –400 | | –400 | |
| 31 | 15 | –400 | | –400 | |
| 32 | 16 | –400 | | –400 | |
| 33 | 17 | –400 | | –400 | |
| 34 | 18 | –400 | | –400 | |
| 35 | 19 | –400 | | –400 | |
| 36 | 20 | –400 | | –400 | |
| 37 | 21 | –400 | | –400 | |
| 38 | 22 | –400 | | –400 | |
| 39 | 23 | –400 | | –400 | |
| 40 | 24 | –2,400 | 8,000 | –10,400 | <-- =B40–C40 |

Should you buy, or should you lease? It depends on your alternative cost of financing. If you can finance at a bank for less than 5.76%, then you should buy the car; otherwise, the lease looks like a good deal. In our case, you can finance at the bank for 6% (cell B12), so you should buy the car with a bank loan instead of leasing it.

## The Role of the Residual

The residual value of the car is very important in determining the cost of the lease. To illustrate this, we use a **Data Table** (see Chapter 24) to run a sensitivity table which shows the EAIR and the lease/buy decision as a function of your estimated end-lease market value of the car:

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 43 | **Data table: Lease IRR vs car terminal value** | | | | | | | | | |
| 44 | | | | | | | | | | |
| 45 | **Estimated terminal value** | **Lease IRR** | **Lease or buy?** | | | | | | | |
| 46 | | | | <-- Data table headers are hidden | | | | | | |
| 47 | 0 | 5.76% | lease | | | | | | | |
| 48 | 1,000 | 5.76% | lease | | | | | | | |
| 49 | 2,000 | 5.76% | lease | | | | | | | |
| 50 | 3,000 | 5.76% | lease | | | | | | | |
| 51 | 4,000 | 5.76% | lease | | | | | | | |
| 52 | 5,000 | 5.76% | lease | | | | | | | |
| 53 | 6,000 | 5.76% | lease | | | | | | | |
| 54 | 7,000 | 5.76% | lease | | | | | | | |
| 55 | 8,000 | 5.76% | lease | | | | | | | |
| 56 | 9,000 | 5.76% | lease | | | | | | | |
| 57 | 10,000 | 5.76% | lease | | | | | | | |
| 58 | 10,067 | 6.00% | lease | | | | | | | |
| 59 | 11,000 | 9.18% | purchase | | | | | | | |
| 60 | 12,000 | 12.47% | purchase | | | | | | | |
| 61 | 13,000 | 15.65% | purchase | | | | | | | |
| 62 | 14,000 | 18.72% | purchase | | | | | | | |
| 63 | 15,000 | 21.71% | purchase | | | | | | | |
| 64 | 16,000 | 24.61% | purchase | | | | | | | |
| 65 | 17,000 | 27.43% | purchase | | | | | | | |
| 66 | 18,000 | 30.18% | purchase | | | | | | | |
| 67 | 19,000 | 32.86% | purchase | | | | | | | |




As the **Data Table** shows, leasing is preferable if you think that the actual market value at the end of the lease term will be low relative to the guaranteed lease-residual of $10,000. The lease is based on your "reselling" the car to the dealer for $10,000; if you think that the actual market value of the car will be much higher, then you're better off buying the car and reselling it yourself.[9] The breakeven market value—the estimated market value for which you are indifferent between leasing the car or financing it at the bank at 6%—is $10,067; at this price, the EAIR of the lease is 6%, which is equal to the cost of the alternative financing.

[9] Some leases actually give you the option of buying the car for the residual value at the end of the lease term. This effectively locks in the lease EIAR, since if the car is worth more than the lease residual value, you can always buy it for the residual and resell the car on the open market.

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **DETERMINING THE BREAKEVEN ESTIMATED TERMINAL VALUE** | | | | | | | | | |
| 2 | Car purchase price | 20,000 | | | | | | | | |
| 3 | Initial lease payment | 2,000 | | | | | | | | |
| 4 | Monthly lease payment | 400 | | | | This sheet computes the lessee's estimated terminal value (cell B6) for which the EAIR (cell B11) is equal to the bank lending rate (cell B12). This value ($10,067) is the breakeven value for leasing: If the lessee thinks that the terminal value of the car < $10,067, she should lease; otherwise she should buy the car. | | | | |
| 5 | Lessee guaranteed terminal value | 10,000 | | | | | | | | |
| 6 | Lessee estimated terminal value | 10,067 | | | | | | | | |
| 7 | Lessee estimated final lease payment | 0 | <-- =MAX(B5–B6,0) | | | | | | | |
| 8 | | | | | | | | | | |
| 9 | **IRR of differential cash flows** | | | | | | | | | |
| 10 | Monthly | 0.49% | <-- =IRR(D16:D40) | | | **Note:** Because of the way we have defined cell B13, at a terminal value of 10,067, it says "lease" and at a terminal value of $10,068 it says "purchase". | | | | |
| 11 | EAIR, annualized IRR | 6.00% | <-- =(1+B10)^12–1 | | | | | | | |
| 12 | Bank interest rate | 6.00% | | | | | | | | |
| 13 | Lease or purchase? | lease | <-- =IF(B11<B12,"lease","purchase") | | | | | | | |

## 5.6 More-Than-Once-a-Year Compounding and the Effective Annual Interest Rate (EAIR)

Suppose you are charged interest on a monthly basis but you want to compute the annual interest cost. Here's an example: XYZ Bank says that it charges an annual percentage rate (APR) of 18% on your credit card balances, with "interest computed monthly." Suppose that what the bank means is that it charges 1.5% per month on the outstanding balance at the beginning of the month. To determine what this means in practice, you should ask yourself "If I have a credit balance of \$100 outstanding for 12 months, how much will I owe at the end of the 12-month period?" If we set this up in Excel, we get:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **MONTHLY COMPOUNDING OF CREDIT CARD BALANCES** | | | |
| 2 | "Annual" rate | 18% | | |
| 3 | Monthly rate | 1.5% | <-- =B2/12 | |
| 4 | | | | |
| 5 | **Month** | **Balance at beginning of month** | **Interest for month** | **Balance at end of month** |
| 6 | 1 | 100.00 | 1.50 | 101.50 |
| 7 | 2 | 101.50 | 1.52 | 103.02 |
| 8 | 3 | 103.02 | 1.55 | 104.57 |
| 9 | 4 | 104.57 | 1.57 | 106.14 |
| 10 | 5 | 106.14 | 1.59 | 107.73 |
| 11 | 6 | 107.73 | 1.62 | 109.34 |
| 12 | 7 | 109.34 | 1.64 | 110.98 |
| 13 | 8 | 110.98 | 1.66 | 112.65 |
| 14 | 9 | 112.65 | 1.69 | 114.34 |
| 15 | 10 | 114.34 | 1.72 | 116.05 |
| 16 | 11 | 116.05 | 1.74 | 117.79 |
| 17 | 12 | 117.79 | 1.77 | 119.56 |
| 18 | | | | |
| 19 | Effective annual interest rate (EAIR) | 19.56% | <-- =D17/B6–1 | |
| 20 | | 19.56% | <-- =(1+B3)^12–1 | |

At the end of 12 months, you would owe \$119.56—the initial \$100 balance plus \$19.56 in interest. Cells B19 and B20 show two ways of calculating the effective annual interest rate:

- In cell B19, we take the end-year balance that results from the initial \$100 credit card balance and divide it by the initial balance to calculate the interest rate:

$$EAIR = \frac{End\text{-}year\ balance}{Initial\ balance} - 1$$

$$= \frac{119.56}{100} - 1 = 19.56\%$$

- In cell B20, we take the monthly interest rate and compound it:

$$\begin{aligned} EAIR &= (1 + Monthly\ rate)^{12} - 1 \\ &= (1.015)^{12} - 1 = 19.56\% \end{aligned}$$

*When the annual interest rate r is compounded n times per year, the* $EAIR = \left(1 + \frac{r}{n}\right)^n - 1.$

## APR and EAIR

By an act of Congress ("The Federal Truth in Lending Act"), lenders are required to specify the *annual percentage rate* (APR) charged on loans. Unfortunately, the Truth in Lending Act does not specify how the APR is to be computed, and the use of the term by lenders is not uniform. Although "APR" is legal terminology designed to help the consumer understand the true cost of borrowing, in practice the APR is not well-defined and may not represent the actual cost of borrowing. Sometimes the APR is the actual effective annual interest rate (EAIR), but in other cases—like the credit card example of this section—the APR is something else. The result is much convoluted wording and a lot of confusion.[10]

## The EAIR and the Number of Compounding Periods per Year *n*

In the above example, the credit card company takes its 18% "annual" interest rate charge and turns it into a 1.5% monthly interest rate. As we saw, the resulting effective annual interest rate (EAIR) is 19.56%.

In Figure 5.1, we compute the effect of the number of compounding periods on the EAIR:

[10] A case which is presented in this book gives three actual APR examples and the resulting EAIR. In each case, the definition of APR used by the lender is different. In only one of the three cases does the APR correspond to the EAIR.

| THE EFFECTIVE ANNUAL INTEREST RATE AND THE NUMBER OF COMPOUNDING PERIODS<br>The stated annual interest rate is 18% | | |
|---|---|---|
| **Number of compounding periods per year** | **Effective annual interest rate formula** | **Effective annual interest rate (EAIR)** |
| 1 | $(1+18\%)-1$ | 18.00% |
| 2<br>(semi-annual compounding) | $\left(1+\frac{18\%}{2}\right)^2-1$ | 18.81% |
| 4<br>(quarterly compounding) | $\left(1+\frac{18\%}{4}\right)^4-1$ | 19.252% |
| 12<br>(monthly compounding) | $\left(1+\frac{18\%}{12}\right)^{12}-1$ | 19.562% |
| 24<br>(semi-monthly compounding) | $\left(1+\frac{18\%}{24}\right)^{24}-1$ | 19.641% |
| 52<br>(weekly compounding) | $\left(1+\frac{18\%}{52}\right)^{52}-1$ | 19.685% |
| 365<br>(daily compounding) | $\left(1+\frac{18\%}{365}\right)^{365}-1$ | 19.716% |

**FIGURE 5.1:** Effective annual interest rate (EAIR) when an annual interest rate of 18% is compounded for various times per year.

The EAIR grows with the number of compounding periods. The EAIR is

$$EAIR = \left(1+\frac{Stated\ annual\ interest\ rate}{Number\ of\ annual\ compounding\ periods}\right)^{Number\ of\ compounding\ periods\ per\ year} - 1$$

When we do this in Excel, we see that the EAIR grows as the number of compounding periods increases. For a very large number of compounding periods, the EAIR approaches a limit of 19.722% (cell C20 below):

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | **EAIR AND NUMBER OF COMPOUNDING PERIODS** | | | | |
| 2 | "Annual" rate | 18% | | | |
| 3 | | | | | |
| 4 | **Number of compounding periods per year** | **Rate per period** | **EAIR** | | |
| 5 | 1 | 18.00% | 18.000% | <-- =(1+B5)^A5–1 | |
| 6 | 2 | 9.00% | 18.810% | <-- =(1+B6)^A6–1 | |
| 7 | 3 | 6.00% | 19.102% | <-- =(1+B7)^A7–1 | |
| 8 | 4 | 4.50% | 19.252% | | |
| 9 | 6 | 3.00% | 19.405% | | |
| 10 | 12 | 1.50% | 19.562% | | |
| 11 | 24 | 0.75% | 19.641% | | |
| 12 | 36 | 0.50% | 19.668% | | |
| 13 | 52 | 0.35% | 19.685% | | |
| 14 | 100 | 0.18% | 19.702% | | |
| 15 | 150 | 0.12% | 19.709% | | |
| 16 | 200 | 0.09% | 19.712% | | |
| 17 | 250 | 0.07% | 19.714% | | |
| 18 | 300 | 0.06% | 19.715% | | |
| 19 | 365 | 0.05% | 19.716% | | |
| 20 | infinite | | 19.722% | <-- =EXP(B2)–1 | |
| 21 | | | | | |
| 22 | | | | | |
| 23 | | | | | |
| 24 | | | | | |
| 25 | | | | | |
| 26 | | | | | |
| 27 | | | | | |
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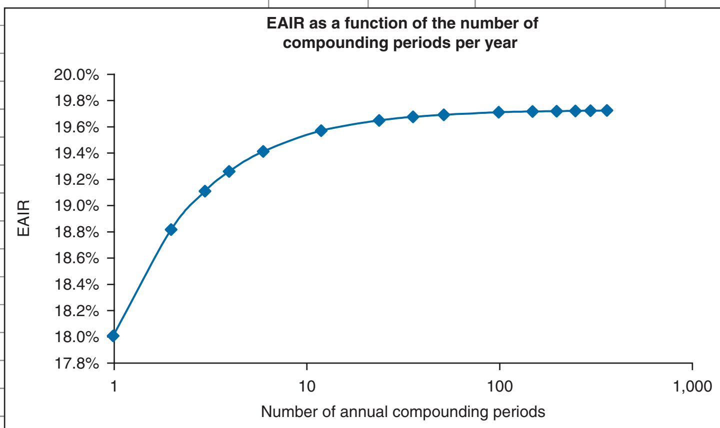
| 38 | | | | | |




There are two important things to notice about the EAIR computation:

- As the number of compounding periods per year $n$ increases, the $EAIR = \left(1+\frac{r}{n}\right)^n - 1$ gets higher.
- The rate at which the EAIR increases gets smaller as the number of annual compounding periods gets larger. There is very little difference between the EAIR when interest is compounded 36 times per year (EAIR = 19.668%) and the EAIR when we compound 365 times per year (EAIR = 19.716%).

# 5.7 Continuous Compounding and Discounting (Advanced Topic)

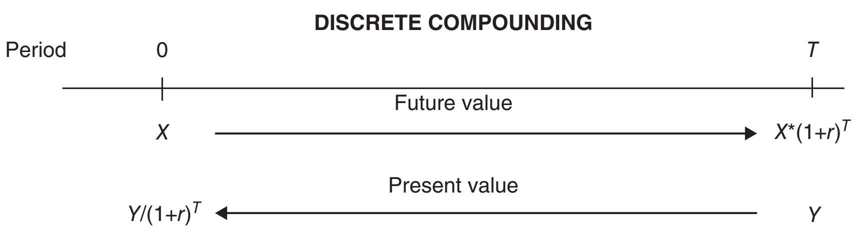
In cell C20, we compute the limit of the EAIR when the number of compounding periods gets very large. This limit is called *continuous compounding*. For $n$ annual compounding periods per year, the *EAIR* equals $\left(1+\frac{r}{n}\right)^n - 1$. When the number of annual compounding periods $n$ gets very large, the EAIR becomes close to $e^r - 1$. The number $e$ = 2.71828182845904 is the base of natural logarithms, and is included in Excel as the function **Exp( )**. In the jargon of finance, $e^{rT}$ is called the *continuously compounded future value after T years at annual interest rate r*. In the spreadsheet below, you can see the difference between the *discretely compounded* future value and the *continuously compounded* future value:

| | A | B | C |
|---|---|---|---|
| 1 | **CONTINUOUS COMPOUNDING** | | |
| 2 | "Annual" rate | 18% | |
| 3 | Number of compounding periods per year | 250 | |
| 4 | Number of years, T | 3 | |
| 5 | Effective annual interest rate, EAIR | | |
| 6 | | 19.71% | |
| 7 | Discretely compounded future value after *t* years =(1+EAIR)$^T$ | 1.7157 | <-- =(1+B2/B3)^(B3*B4) |
| 8 | Continuously compounded future value =e$^{rT}$ | 1.7160 | <-- =EXP(B2*B4) |

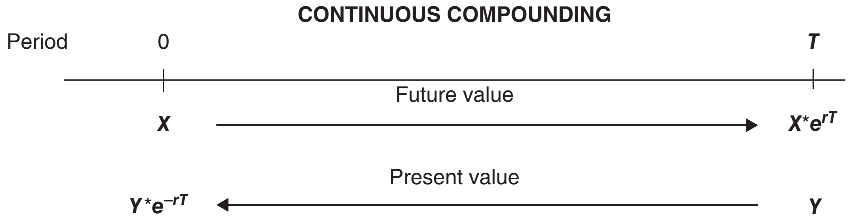
When the number of compounding periods gets very large, the difference between the discrete and continuous interest rates becomes very small.

## The Continuously Compounded Discount Factor

In Chapter 2, we saw that future value and present value are closely related:



A similar relation holds for continuous compounding:




The following spreadsheet summarizes these relations:

| | A | B | C |
|---|---|---|---|
| 1 | **DISCRETE AND CONTINUOUS COMPOUNDING** | | |
| 2 | Interest rate | 10% | |
| 3 | Initial amount, X | 100 | |
| 4 | Terminal date, T | 3 | |
| 5 | Discretely compounded future value, $X*(1+r)^T$ | 133.100 | <-- =B3*(1+B2)^B4 |
| 6 | Continuously compounded future value, $X*e^{rT}$ | 134.986 | <-- =B3*EXP(B2*B4) |
| 7 | | | |
| 8 | Interest rate | 10% | |
| 9 | Terminal amount, Y | 100 | |
| 10 | Terminal date, T | 3 | |
| 11 | Discretely compounded present value, $Y/(1+r)^T$ | 75.131 | <-- =B9/(1+B8)^B10 |
| 12 | Continuously compounded present value, $Y*e^{-rT}$ | 74.082 | <-- =B9*EXP(–B10*B8) |

## An Actual Credit Card Example

Continuously compounded interest may seem like an ethereal concept—highly theoretical but not very useful. The example in this subsection shows how useful continuously compounded interest can actually be. Consider a hypothetical university called "We Teach You Best University." We Teach You Best University offers a credit card charging a penalty annual percentage rate (APR) of 10.99%.[11] The company is actually charging 0.03011% *per day* on outstanding balances. This rate is calculated by taking 10.99% and dividing it by the number of days per year: $0.03011\% = \dfrac{10.99\%}{365}$.

[11] What is the penalty rate? For most credit cards "If you are late making *any* payment, the penalty rate applies to all existing balances.

If you carried a \$100 balance throughout the year, you would owe $100*(1+0.03011\%)^{365} = 111.615$ at the end of the year.[12] As the spreadsheet shows, this translates into a 11.615% EAIR (cell B5):

| | A | B | C |
|---|---|---|---|
| 1 | WE TEACH YOU BEST UNIVERSITY | | |
| 2 | APR | 10.99% | |
| 3 | Daily | 0.03011% | <-- =B2/365 |
| 4 | | | |
| 5 | Effective annual interest rate (EAIR) | 11.615% | <-- =(1+B3)^365–1 |
| 6 | Continuously compounded interest | 11.617% | <-- =EXP(B2)–1 |

As you can see in cell B6, essentially the same interest rate can be computed by using continuous compounding. From a computational point of view, using continuous compounding is simpler than discretely compounding a daily interest rate.

## Continuous Compounding in This Book

We rarely use continuous compounding in this book, except in the options chapters (Chapters 17–20). In other cases, we use only discrete compounding, although we may occasionally point out in a footnote the continuously compounded counterpart of a discretely compounded calculation.

[12] 10.99%/365 = 0.031096%. We Teach You Best University's card takes the extra 0.0000004% As they say: "Every little bit helps."

## Computing the Continuous Return

In the past subsections, we have measured discrete versus continuous compounding. In this section, we show how to measure discrete versus continuous rates of return. Suppose that an investment grows from $X$ at time 0 to $Y$ at time $T$. Then the *discretely compounded annual return* on the investment is $r = \left(\frac{Y}{X}\right)^{1/T} - 1$. The *continuously compounded annual return* is given by $r = \frac{1}{T}\mathrm{Ln}\left(\frac{Y}{X}\right)$. In the example below, an investment of $100 grows to $200 over a period of 4 years.[13] The spreadsheet below computes both the discretely compounded and the continuously compounded annual returns:

| | A | B | C |
|---|---|---|---|
| 1 | **COMPUTING DISCRETE AND CONTINUOUS RETURNS** | | |
| 2 | Investment at time 0 | 100 | |
| 3 | Value at time T | 200 | |
| 4 | T (years) | 4 | |
| 5 | | | |
| 6 | Discretely compounded annual return | 18.92% | <-- =(B3/B2)^(1/B4)–1 |
| 7 | Continuously compounded annual return | 17.33% | <-- =(1/B4)*Ln(B3/B2) |
| 8 | | | |
| 9 | **Why is this true: Compounding the initial investment to get the future value** | | |
| 10 | Discrete compounding | 200 | <-- =B2*(1+B6)^B4 |
| 11 | Continuous compounding | 200 | <-- =B2*EXP(B7*B4) |

The discretely compounded annual return (cell B6) is 18.92%; this is proved in cell B10, where we show that $200 = 100 * (1 + 18.92\%)^4$. In other words, over 4 years the discretely compounded future value of $100 at 18.92% annually is $200.

The continuously compounded annual return is 17.32% (cell B7); this is proved in cell B11, where we show that $200 = 100 * \exp(17.32\% * 4)$. Over 4 years, the continuously compounded future value of $100 at 17.32% annually is $200.

Note that you would be indifferent between an annual discretely compounded return of 18.92% and an annual continuously compounded return of 17.32%. Over any given time frame, both rates make an initial investment grow to the same final result!

[13] The Excel function **Ln** computes the natural logarithm of a number. If $\mathrm{Ln}(a) = b$, then $e^b = a$.

## Summary

In this chapter, we've applied the time value of money (PV, NPV, and IRR) to a number of relevant problems:

- **Finding the effective annual interest rate (EAIR):** This is the compound annual interest rate implicit in a specific financial asset; another way to think about this is that it's the annualized IRR. We've given a number of examples—leases, mortgages, credit cards—all of which illustrate that the only way to evaluate the financing cost is by calculating the EAIR.
- **The effect of non-annual compounding periods:** Many interest rates are calculated on a monthly or even a daily basis. The EAIR demands that we annualize these interest rates so that we can compare them. When the number of compounding periods gets very large (like our university credit cards example), we have EAIR = $e^r$, where $e$ = 2.71828182845905 (computed by **=Exp()** in Excel) and $r$ is the stated interest rate.

## Exercises

**Note:** The data for these exercises can be found on the Benninga, *Principles of Finance with Excel,* Third Edition companion website (www.oup.com/us/Benninga).

**1.** (APR and EAIR) Your local bank has offered you a mortgage of $100,000. There are no points, no origination fees, and no extra initial costs (meaning: you get the full $100,000). The mortgage is to be paid back over 10 years in monthly payments, and the bank charges 12% annual interest computed monthly (1%, this rate is "stated" or APR).

  **a.** Calculate the monthly mortgage payments.

  **b.** What is the mortgage EAIR?

**2.** (APR vs. EAIR) You've been offered three credit cards:

  - Credit card 1 charges 19% annually, on a monthly basis.
  - Credit card 2 charges 19% annually, on a weekly basis
  - Credit card 3 charges 18.90% annually, on a daily basis

  Rank the cards on the basis of EAIR.

**3.** (APR vs. EAIR) A recent notice from your credit card company included the following statement:

> ***Annual Percentage Rate for Cash Advances:***
> *Your annual percentage rate for cash advances is the U.S. Prime Rate plus 14.99%, but such cash advance rate will never be lower than 19.99%. As of August 1, 2014, this cash advance ANNUAL PERCENTAGE RATE is 19.99%, which corresponds to a daily periodic rate of 0.0548%. A daily periodic rate is the applicable annual percentage rate divided by 365.*

As of August 1, 2014, what is the effective annual interest rate (EAIR) charged by the credit card on cash advances?

**4.** (EAIR for 1 year) You are considering buying the latest stereo system model. The dealer in "The Stereo World" store has offered you two payment options. You can either pay $10,000 now, or you can take advantage of their special deal and "buy now and pay a year from today," in which case you will pay $11,100 in 1 year. Calculate the effective annual interest rate (EAIR) of the store's special deal.

**5.** (EAIR annuity) You have two options of paying for your new dishwasher: You can either make a single payment of $400 today, or you can pay $70 for each of the next 6 months, with the first payment made today. What is the effective annual interest rate (EAIR) of the second option?

**6.** (EAIR annuity) Your lovely wife has decided to buy you a vacuum cleaner for your birthday (she always supports you in your hobbies. . .). She called your best friend, a manager of a vacuum cleaner store, and he has suggested one of two payment plans: She can either pay $100 now or make 12 monthly payments of $10 each, starting from today. What are the monthly IRR and the EAIR of the payment-over-time plan?

**7.** (Average compounding rate) WindyRoad is an investment company which has two mutual funds. The WindyRoad Dull Fund invests in boring corporate bonds, and its Lively Fund invests in "high-risk, high-return" companies. The returns for the two funds in the 5-year period 2011–2015 are given below.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **DULL FUND OR LIVELY FUND?** | | | |
| 2 | **Year** | **Dull Fund return** | **Lively Fund return** | |
| 3 | 2011 | 8.00% | 11.50% | |
| 4 | 2012 | 5.20% | –14.50% | |
| 5 | 2013 | 4.30% | –20.00% | |
| 6 | 2014 | 3.30% | 42.50% | |
| 7 | 2015 | 7.00% | 14.00% | |
| 8 | | | | |
| 9 | Average return | 5.56% | 6.70% | <-- =AVERAGE(C3:C7) |

**a.** Suppose you had invested $100 in each of the two funds at the beginning of 2011. How much would you have at the end of 2015?

**b.** What was the effective annual interest rate (EAIR) paid by each of the funds over the 5-year period 2011–2015?

**c.** Is there a conclusion you can draw from this example?

**(Advanced section)**

**d.** Compute the annual continuous returns for Dull Fund and Lively Fund for each of the years 2011–2015. What is the average continuous return $r_{average}^{continuous}$ for each fund (reminder: $value_t = value_{t-1} * e^{r_t^{continuous}}$)?

**e.** Suppose you had invested $100 in each of the two funds at the beginning of 2011. Show that the total amount you would have in each fund (see part a) can be written as $\$100 * e^{5 * r_{average}^{continuous}}$. Notice that this makes computations much simpler.

**8.** (EAIR with processing fees) Your local bank has offered you a 5-year, $100,000 mortgage. The bank is charging 1.2 points, and the processing fee to be paid immediately is $600. If the interest rate is 12% annually with one mortgage payment per year, calculate the EAIR of the loan.

**9.** (Mortgage + EAIR + processing fee) Your local bank has offered you a 20-year, $100,000 mortgage. The bank is charging 1.5 points, with "processing" costs of $750; both points and processing costs are deducted from the mortgage when it is given. The mortgage carries a 10% annual interest rate and is paid in 20 equal annual payments. Note that the annual payments on the mortgage are calculated on the full amount of the mortgage (that is, $100,000).

**a.** Calculate the annual mortgage payment.

**b.** Calculate the EAIR.

**c.** Present an amortization table which shows the amount of effective interest you pay each year.

**10.** (Mortgage + EAIR) Your local bank has offered you a 20-year, $100,000 mortgage. The bank is charging 1.5 points, with "processing" costs of $750; both points and processing costs are deducted from the mortgage when it is given. The mortgage carries a 10% annual interest rate and is paid in equal monthly payments. Note that the monthly payments on the mortgage are calculated on the full amount of the mortgage (that is, $100,000).

**a.** Calculate the monthly payment on the mortgage, show the amortization table, and compute the EAIR.

**b.** Will the EAIR of the mortgage change if the loan period is 6 years?

**c.** Compute the total interest paid in each year of the mortgage. You can base your answer on the amortization table or investigate the Excel function **Cumipmt**.

**11.** (Mortgage + EAIR) You just bought the first floor of the famous "Egg-Plant" Building for $250,000. You plan to rent the space to convenience stores. Your banker has offered you a mortgage with the following terms:

- The mortgage is for the full amount of $250,000.
- The mortgage will be repaid in equal monthly payments over 36 months, starting 1 month from now.
- The annual interest rate on the mortgage is 8%, compounded monthly (meaning: 8%/12 = 2/3% per month).
- You have to pay the bank an initiation charge of $1,500 and 1 point.

**a.** What is the monthly payment you will pay the bank?

**b.** What is the EAIR of the mortgage?

**c.** Compute an amortization table which shows the amount of interest pay each month.

**12.** (EAIR in loans) You're considering buying a new top-of-the-line luxury car. The car's list price is $99,000. The dealer has offered you two alternatives for purchasing the car:

- You can buy the car for $90,000 in cash and get a $9,000 discount in the bargain.
- You can buy the car for the list of $99,000. In this case, the dealer is willing to take $39,000 as an initial payment. The remainder of $60,000 is a "zero-interest loan" to be paid back in equal installments over 36 months.

Alternatively, your local bank is willing to give you a car loan at an annual interest rate of 10%, compounded monthly (that is, 10%/12 per month).

Decide how to finance the car: Bank loan or zero-interest loan with the dealer, or cash payment.

**13.** (FV calculation with stated rate) You plan to put $1,000 in a savings plan and leave it there for 5 years. You can choose between various alternatives. How much will you have in 5 years under each alternative?

**a.** Bellon Bank is offering 12% stated annual interest rate, compounded once a year.

**b.** WNC Bank is offering 11% stated annual interest rate, compounded twice a year.

**c.** Plebian Bank is offering 10% stated annual interest rate, compounded monthly.

**d.** Byfus Bank is offering 11.5% stated annual interest rate, compounded continuously.

**14.** (PV with stated rate) Assuming that the interest rate is 5%, compounded semi-annually, which of the following is more valuable?

**a.** \$5,000 today

**b.** \$10,000 at the end of 5 years

**c.** \$9,000 at the end of 4 years

**d.** \$400 at the end of each year (in perpetuity) commencing in 1 year

**15.** (FV with stated rate) You plan to put \$10,000 in a savings plan for 2 years. How much will you have at the end of 2 years with each of the following options?

**a.** Receive 12% stated annual interest rate, compounded monthly.

**b.** Receive 12.5% stated annual interest rate, compounded annually.

**c.** Receive 11.5% stated annual interest rate, compounded daily.

**d.** Receive 10% stated annual interest rate in the first year and 15% stated annual interest rate in the second year, compounded annually.

**16.** (Refinancing mortgage, EAIR) Michael Smith was in trouble: He was unemployed and living on his monthly disability pay of \$1,200. His credit card debts of \$19,000 were threatening to overwhelm this puny income. Every month in which he delayed paying the credit card debt cost him 1.5% on the remaining balance. His only asset was his house, on which he had a \$67,000 mortgage.

Then Michael got a phone call from Uranus Financial Corporation: The corporation offered to refinance Michael's mortgage. The Uranus representative explained to Michael that, with the rise in real estate values, Michael's house could now be re-mortgaged for \$90,000. This amount would allow Michael to repay his credit card debts and even leave him with some money.

Here are some additional facts:

- The new mortgage would be for 25 years and would have an annual interest rate of 9.23% (this is APR, i.e., stated rate). The mortgage would be repayable in equal monthly payments over this term, at a monthly interest rate of $9.23\%/12 = 0.76917\%$. The fees on the new mortgage are \$8,000 to be paid today.
- There are no penalties involved in repaying the \$67,000 existing mortgage.

Answer the following questions:

**a.** What will Michael's monthly payments be on the new mortgage?

**b.** After repaying his credit card debts, how much money will Michael have left?

**c.** What is the effective annual interest rate on the Uranus mortgage?

**17.** (EAIR on a lease) Capital Star Motors of Canberra, Australia, has the following lease offer for a Smart car:

- Lease term: 48 months
- No initial deposit, $8,995 balloon payment at end of lease
- Daily payment: $9.95 per day, payable monthly
- Smart car cash cost: $18,800

**a.** Assuming 30 days per month, compute the effective annual interest rate (EAIR) in the lease.

**b.** Compute the balloon payment that gives a 7% EAIR.

# 6

# Capital Budgeting: Valuing Business Cash Flows

## Chapter Contents



## Overview

*Capital budgeting* is finance jargon for the process of deciding whether to undertake an investment project. There are two standard concepts used in capital budgeting: net present value (NPV) and internal rate of return (IRR). Both of these concepts were introduced in Chapter 3; here, we discuss their application to capital budgeting.

In this chapter, we expand on the discussion of capital budgeting started in Chapter 3 and examine a number of issues that often cause confusion. The capital budgeting decisions we examined in Chapter 3 were all pretty cut and dried: The NPV and IRR criteria indicated which investment was worthwhile for the individual or the company. As you might expect, in real life the decisions of where and how to spend your investment dollars are not always so clear cut.

Here are some of the topics covered in this chapter:

- Sunk costs: How should we account for costs incurred in the past?
- The cost of foregone opportunities.

- Salvage values and terminal values.
- Incorporating taxes into the valuation decision.
- Discounting cash flows that don't occur at year end ("mid-year discounting").
- Incorporating inflation into the capital budgeting decision.

**Finance concepts discussed: Capital budgeting principles**

- Principle 1: Cash is king
- Principle 2: Ignore sunk costs and concentrate on controllable future cash flows
- Principle 3: Take taxes into account
- Principle 4: Don't ignore foregone opportunities
- Principle 5: Take into account the timing of the cash flows
- Principle 6: Don't forget inflation

**Excel functions used**

- **NPV**
- **IRR**
- **Data Table**
- **XNPV, XIRR, SUM,** and **Goal Seek**

## 6.1 Capital Budgeting Principle 1: Cash Is King

The point of this principle is that there may be important differences between the accounting profits and the cash flows produced by an asset. The value of an asset is determined by the *cash flows it produces over its life*. The cash flow of an asset is the *after-tax cash* that the asset produces at a given point in time.

To illustrate the "cash is king" principle, we start with a small example: Remember from Chapter 1 that your pizza parlor sells $500 of pizzas on Tuesday night, and suppose that on the same day you bought $300 worth of ingredients. Looking in the cash register at the end of the day, you expect to find $200, but instead you're surprised to find $300. The explanation: Of the $500 of pizzas sold, you only collected $400—the other $100 were sold to a campus fraternity which maintains an account with you that they settle at the end of each month. Of the $300 of ingredients you bought, you only paid for $100—the other $200 will be billed to you for payment in ten days. Your cash flow for the day:

| | A | B | C |
|---|---|---|---|
| 1 | **PIZZA PARLOR CASH FLOW** | | |
| 2 | Pizza sold | 500 | |
| 3 | Cost of ingredients | 300 | |
| 4 | Profit | 200 | <-- =B2–B3 |
| 5 | | | |
| 6 | Collected on sales | 400 | |
| 7 | Paid for ingredients | 100 | |
| 8 | Net in cash register | 300 | <-- =B6–B7 |

Cash flows are different from accounting profits or sales receipts. The pizza parlor's *accounting profit* for the day is $200, but its *cash flow* for the day is $300 (= $400 collected from sales minus $100 paid for supplies). The difference between the two is due to the *timing difference* between inflows and outflows. (Of course, 10 days from now the pizza parlor will have a negative cash flow of $200 as a result of paying for the ingredients.)

In finance, *cash flow is all-important*. Most corporate financial data come from accountants, who—despite the bad press they've gotten in the past few years—do a very good job at representing the economic realities of corporate activities. When making financial decisions, we have to translate the accounting data to their cash equivalents. Much of finance involves first translating accounting information into cash flows.[1] Here's the way the accountant would see our pizza parlor story:

| | A | B | C |
|---|---|---|---|
| 1 | **PIZZA PARLOR CASH FLOW**<br>**As seen by the accountant** | | |
| 2 | Pizza sold | 500 | |
| 3 | Cost of ingredients | 300 | |
| 4 | Profit | 200 | <-- =B2–B3 |
| 5 | | | |
| 6 | Increase in accounts receivable | 100 | <-- Unpaid bills by customers |
| 7 | Increase in accounts payable | 200 | <-- Unpaid bills to suppliers |
| 8 | Increase in net working capital | 100 | <-- =–B6+B7 |
| 9 | | | |
| 10 | Cash flow for the day:<br>Profit + change in NWC | 300 | <-- =B4+B8 |

It is well known in practice that accounting profits are subjected to manipulations and timing decisions. This is why the practitioners say that cash is a fact, profit is an opinion. At the most basic level, profit is the difference between sales of products or services (also called "revenues") and what you spend: salaries, rent, raw materials, and the like (also called "costs" or "expenses"). So, when that difference is positive, you have "made money," right? Profit is money, right? Wrong! This is delusional. If customers, for example, "buy" but don't pay, there comes a moment when you run out of actual cash.

Profit isn't unimportant, certainly, but it's not as important as cash. Profit is just a number. You can't put it in your pocket and spend it as you see fit. To repeat: **Profit is not unimportant, but it's not as important as cash.**

[1] Not familiar with basic accounting? A primer on accounting is can be found on the book's website: www.oup.com/us/Benninga.

## 6.2 Capital Budgeting Principle 2: Ignore Sunk Costs and Consider Only Marginal Cash Flows

This is an important principle of capital budgeting and project evaluation: Ignore the cash flows you can't control and look only at the *marginal cash flows*—the outcomes of financial decisions you can still make. In the jargon of finance: Ignore *sunk costs*, costs that have already been incurred and are thus not affected by future capital budgeting decisions.

**Here's an example:** You recently bought a plot of land and built a house on it. Your intention was to sell the house immediately, but it turns out that the house is very badly built and cannot be sold in its current state.[2] The house and land cost you $100,000, and a friendly local contractor has offered to make the necessary repairs, which will cost $20,000. Your real estate broker estimates that even with these repairs you'll never sell the house for more than $90,000. What should you do? There are two approaches to answering this question:

- "My father always said 'Don't throw good money after bad.'" If this is your approach, you won't do anything. This attitude is typified in column B below, which shows that if you make the repairs you will have lost 25% on your money.
- "My mother was a finance professor, and she said 'Don't cry over spilt milk. Look only at the marginal cash flows.'" These turn out to be pretty good. In column C below, you see that making the repairs will give you a 350% return on your $20,000.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **IGNORE SUNK COSTS** | | | |
| 2 | House cost | 100,000 | | |
| 3 | Fix up cost | 20,000 | | |
| 4 | | | | |
| 5 | **Year** | **Include sunk costs: WRONG!** | **Marginal cash flow: RIGHT!** | |
| 6 | 0 | –120,000 | –20,000 | |
| 7 | 1 | 90,000 | 90,000 | |
| 8 | **IRR** | –25% | 350% | <-- =IRR(C6:C7) |

Of course your father was wrong and your mother right (this often happens): Even though you made some disastrous mistakes (you never should have built the house in the first place), you should—at this point—ignore the sunk cost of $100,000 and make the necessary repairs.

[2] This example assumes that sale of the house in its current state is not an option. One of the end-of-chapter exercises for this chapter discusses the alternative of sale as is.

## 6.3 Capital Budgeting Principle 3: Don't Forget the Effects of Taxes

No one needs to be told that taxes are very important.[3] In this section, we discuss the capital budgeting problem faced by Sally and Dave, two business-school grads who are considering buying a condominium apartment and renting it out for the income. We use Sally and Dave and their condo to emphasize the place of taxes in the capital budgeting process.

Sally and Dave—fresh out of business school with a little cash to spare—are considering buying a nifty condo as a rental property. The condo will cost $100,000, and they're planning to buy it with all cash. Here are some additional facts:

- Sally and Dave assume they can rent out the condo for $24,000 per year. They'll have to pay property taxes of $1,500 annually, and they're figuring on additional miscellaneous expenses of $1,000 per year.
- All the income from the condo has to be reported on their annual tax return. Currently, Sally and Dave have a tax rate of 30%, and they think this rate will continue for the foreseeable future.
- Their accountant has explained to them that they can depreciate the full cost of the condo over 10 years—each year they can charge $10,000 depreciation $\left( = \dfrac{\textit{Condo cost}}{\textit{10-year depreciable life}} \right)$ against the income from the condo.[4] This means that they can expect to pay $3,450 in income taxes per year if they buy the condo and rent it out and have net income from the condo of $8,050:

| | A | B | C |
|---|---|---|---|
| 1 | **SALLY & DAVE'S CONDO** | | |
| 2 | Cost of condo | 100,000 | |
| 3 | Sally & Dave's tax rate | 30% | |
| 4 | | | |
| 5 | **Annual reportable income calculation** | | |
| 6 | Rent | 24,000 | |
| 7 | Expenses | | |
| 8 | Property taxes | –1,500 | |
| 9 | Miscellaneous expenses | –1,000 | |
| 10 | Depreciation | –10,000 | |
| 11 | Reportable income | 11,500 | <-- =SUM(B6:B10) |
| 12 | Taxes (rate = 30%) | –3,450 | <-- =-B3*B11 |
| 13 | Net income | 8,050 | <-- =B11+B12 |

[3] Will Rogers: "The difference between death and taxes is death doesn't get worse every time Congress meets."

[4] You may want to read the box feature on depreciation before going on.

## What Is Depreciation?

In computing the taxes they owe, Sally and Dave get to subtract expenses from their income. Taxes are computed on the basis of the *income before taxes* (= income – expenses – depreciation – interest). When Sally and Dave get the rent from their condo, this is *income*—money earned from their asset. When Sally and Dave pay to fix the faucet in their condo, this is an *expense*—a cost of doing business.

The $100,000 cost of the condo is neither income nor an expense. It's a *capital investment*—money paid for an asset that will be used over many years. Tax rules specify that each year part of the capital investments can be taken off the income ("expensed," in the accounting jargon). This reduces the taxes paid by the owners of the asset and takes account of the fact that the asset has a limited life.

There are many depreciation methods in use. The simplest method is *straight-line depreciation*. In this method, the asset's annual depreciation is a percentage of its initial cost. In the case of Sally and Dave, for example, we've specified that the asset is depreciated over 10 years. This results in annual depreciation charges of

$$\textit{Straight-line depreciation} = \frac{\textit{Initial asset cost}}{\textit{Depreciable life span}} = \frac{\$100{,}000}{10} = \$10{,}000 \textit{ annually}$$

In some cases, depreciation is taken on the asset cost minus its salvage value: If you think that the asset will be worth $20,000 at the end of its life (this is the salvage value), then the annual straight-line depreciation might be $8,000:

$$\begin{array}{c}\textit{Straight-line depreciation}\\ \textit{with salvage value}\end{array} = \frac{\textit{Initial asset cost} - \textit{Salvage value}}{\textit{Depreciable life span}} = \frac{\$100{,}000 - \$20{,}000}{10} = \$8{,}000 \textit{ annually}$$

### Accelerated Depreciation

Although historically depreciation charges are related to the life span of the asset, in many cases this connection has been lost. Under United States tax rules, for example, an asset classified as having a 5-year depreciable life (trucks, cars, and some computer equipment is in this category) will be depreciated over 6 years (yes, *six*) at 20%, 32%, 19.2%, 11.52%, 11.52%, 5.76% in each of the years 1, 2, . . . , 6. Notice that this method *accelerates* the depreciation charges—more than one-sixth of the depreciation is taken annually in years 1–3 and less in later years. Since—as we show in the text—depreciation ultimately saves taxes, this benefits the asset's owner, who now gets to take more of the depreciation in the early years of the asset's life.

## Two Ways to Calculate the Cash Flow

In the previous spreadsheet, you saw that Sally and Dave's net income was $8,050. In this section, you'll see that the *cash flow produced by the condo* is more that this amount. It all has to do with depreciation: Because the depreciation is an expense for tax purposes but not a cash expense, the *cash flow* from the condo rental is different. So even though the net income from the condo is $8,050, the annual cash flow is $18,050—you have to add back the depreciation to the net income to get the cash flow generated by the property.

| | A | B | C |
|---|---|---|---|
| 16 | **Cash flow, method 1<br>Add back depreciation** | | |
| 17 | Net income | 8,050 | <-- =B13 |
| 18 | Add back depreciation | 10,000 | <-- =–B10 |
| 19 | Cash flow | 18,050 | <-- =B17+B18 |

In the above calculation, we've added the depreciation back to the net income to get the cash flow.

*An asset's* ***cash flow*** *(the amount of cash produced by an asset during a particular period) is computed by taking the asset's net income (also called profit after taxes or sometimes just "income") and adding back non-cash expenses like depreciation.*

## Tax Shields

There's another way of calculating the cash flow which involves a discussion of *tax shields*. A tax shield is a tax saving that results from being able to report an expense for tax purposes. In general, a tax shield just reduces the cash cost of an expense. In the above example, since Sally and Dave's property taxes of $1,500 are an expense for tax purposes, the after-tax cost of the property taxes is

$$(1-30\%)*\$1,500 = \$1,500 - \underbrace{30\%*1,500}_{\substack{\uparrow \\ \text{This \$450 is the} \\ \text{tax shield}}} = \$1,050$$

The tax shield of $450 (= 30% * $1,500) has reduced the cost of the property taxes.

Depreciation is a special case of a *non-cash expense* which generates a tax shield. A little thought will show you that the $10,000 depreciation on the condo generates $3,000 of cash. Because depreciation reduces Sally and Dave's reported income, each dollar of depreciation saves them $0.30 (30 cents) of taxes, without actually costing them anything in out-of-pocket expenses (the $0.30 comes from the fact that Sally and Dave's tax rate is 30%). Thus $10,000 of depreciation is worth $3,000 of cash. This $3,000 *depreciation tax shield* is a cash flow for Sally and Dave.

In the spreadsheet below we calculate the cash flow in two stages:

- We first calculate Sally and Dave's net income ignoring depreciation. If depreciation were not an expense for tax purposes, Sally and Dave's net income would be $15,050.
- We then add to this figure the depreciation tax shield of $3,000. The result (cell B32) gives the cash flow for the condo.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 21 | **Cash flow, method 2<br>Compute after-tax income without depreciation, then add depreciation tax shield** | | | | | |
| 22 | Rent | 24,000 | | | | |
| 23 | Expenses | | | | | |
| 24 | Property taxes | –1,500 | | This is what the net income would have been if depreciation were not an expense for tax purposes. | | |
| 25 | Miscellaneous expenses | –1,000 | | | | |
| 26 | Depreciation | 0 | | | | |
| 27 | Reportable income | 21,500 | <-- =SUM(B22:B26) | | | |
| 28 | Taxes (rate = 30%) | –6,450 | <-- =–B3*B27 | | | |
| 29 | Net income without depreciation | 15,050 | <-- =SUM(B27:B28) | | | |
| 30 | | | | Effect of depreciation is to add a $3,000 tax shield. | | |
| 31 | Depreciation tax shield | 3,000 | <-- =B3*10000 | | | |
| 32 | Cash flow | 18,050 | <-- =B29+B31 | | | |
| 33 | | | | | | |

## Is Sally and Dave's Condo Investment Profitable? A Preliminary Calculation

At this point, Sally and Dave can make a preliminary calculation of the net present value and internal rate of return on their condo investment. Assuming a discount rate of 12% and that they only hold the condo for 10 years, the NPV of the condo investment is $1,987, and its IRR is 12.48%:

| | A | B | C |
|---|---|---|---|
| 1 | **SALLY & DAVE'S CONDO—PRELIMINARY VALUATION** | | |
| 2 | Discount rate | 12% | |
| 3 | | | |
| 4 | **Year** | **Cash flow** | |
| 5 | 0 | –100,000 | |
| 6 | 1 | 18,050 | |
| 7 | 2 | 18,050 | |
| 8 | 3 | 18,050 | |
| 9 | 4 | 18,050 | |
| 10 | 5 | 18,050 | |
| 11 | 6 | 18,050 | |
| 12 | 7 | 18,050 | |
| 13 | 8 | 18,050 | |
| 14 | 9 | 18,050 | |
| 15 | 10 | 18,050 | |
| 16 | | | |
| 17 | Net present value, **NPV** | 1,987 | <-- =+B5+NPV(B2,B6:B15) |
| 18 | Internal rate of return, **IRR** | 12.48% | <-- =IRR(B5:B15) |

## Is Sally and Dave's Condo Investment Profitable? Incorporating Terminal Value

A little thought about the previous spreadsheet reveals that we've left out an important factor: the value of the condo at the end of the 10-year horizon. In finance, an asset's value at the end of the investment horizon is called the asset's *salvage value* or *terminal value*. In the above spreadsheet, we've assumed that the terminal value of the condo is zero, but this assumption implausible.

To make a better calculation about their investment, Sally and Dave will have to make an assumption about the condo's terminal value. Suppose they assume that at the end of the 10 years they'll be able to sell the condo for $80,000. The taxable gain relating to the sale of the condo is the difference between the condo's sale price and its book value at the time of sale—the initial price minus the sum of all the depreciation since Sally and Dave bought it. Since Sally and Dave have been depreciating the condo by $10,000 per year over a 10-year period, its book value at the end of 10 years will be zero.

In cell E10 below, you can see that the sale of the condo for $80,000 will generate a cash flow of $56,000:

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **SALLY & DAVE'S CONDO: PROFITABILITY AND TERMINAL VALUE** | | | | | |
| 2 | **Cost of condo** | 100,000 | | | | |
| 3 | **Sally & Dave's tax rate** | 30% | | | | |
| 4 | | | | | | |
| 5 | **Annual reportable income calculation** | | | **Terminal value** | | |
| 6 | Rent | 24,000 | | Estimated resale value, year 10 | 80,000 | |
| 7 | Expenses | | | Book value | 0 | |
| 8 | Property taxes | –1,500 | | Taxable gain | 80,000 | <-- =E6–E7 |
| 9 | Miscellaneous expenses | –1,000 | | Taxes | 24,000 | <-- =B3*E8 |
| 10 | Depreciation | –10,000 | | Net after-tax cash flow | | |
| 11 | Reportable income | 11,500 | <-- =SUM(B6:B10) | from terminal value | 56,000 | <-- =E8–E9 |
| 12 | Taxes (rate = 30%) | –3,450 | <-- =–B3*B11 | | | |
| 13 | Net income | 8,050 | <-- =B11+B12 | | | |
| 14 | | | | | | |
| 15 | **Cash flow, method 1 Add back depreciation** | | | | | |
| 16 | Net income | 8,050 | <-- =B13 | | | |
| 17 | Add back depreciation | 10,000 | <-- =–B10 | | | |
| 18 | Cash flow | 18,050 | <-- =B17+B16 | | | |

To compute the rate of return of Sally and Dave's condo investment, we put all the numbers together:

| | A | B | C | D |
|---|---|---|---|---|
| 20 | Discount rate | 12% | | |
| 21 | | | | |
| 22 | **Year** | **Cash flow** | | |
| 23 | 0 | –100,000 | | |
| 24 | 1 | 18,050 | <-- =B18, Annual cash flow from rental | |
| 25 | 2 | 18,050 | | |
| 26 | 3 | 18,050 | | |
| 27 | 4 | 18,050 | | |
| 28 | 5 | 18,050 | | |
| 29 | 6 | 18,050 | | |
| 30 | 7 | 18,050 | | |
| 31 | 8 | 18,050 | | |
| 32 | 9 | 18,050 | | |
| 33 | 10 | 74,050 | <-- =B32+E10 | |
| 34 | | | | |
| 35 | **NPV** of condo investment | 20,017 | <-- =B23+NPV(B20,B24:B33) | |
| 36 | **IRR** of investment | 15.98% | <-- =IRR(B23:B33) | |

Assuming that the 12% discount rate is the correct rate, the condo investment should be undertaken: Its NPV is positive, and its IRR exceeds the discount rate.[5]

[5] When we say that a discount rate is "correct," we usually mean that it is appropriate to the riskiness of the cash flows being discounted. We postpone the discussion of how to compute the "correct" or "risk-adjusted" discounted rate until Part II. For the moment, let's assume that the 12% discount rate is appropriate regarding the riskiness of the condo's cash flows.

## Book Value Versus Terminal Value

The *book value* of an asset is its initial purchase price minus the accumulated depreciation. The *terminal value* of an asset is its assumed market value at the time you "stop writing down the asset's cash flows." This sounds like a weird definition of terminal value, but often when we do present value calculations for a long-lived asset (like Sally and Dave's condo, or in the company valuations we discuss in Part III), we write down only a limited number of cash flows.

Sally and Dave are reluctant to make predictions about condo rents and expenses beyond a 10-year horizon. Past this point, they're worried about the accuracy of their guesses. So they project 10 years of cash flows; the terminal value is their best guess of the condo's value at the end of year 10. Their thinking is "let's examine the profitability of the condo if we hold on to it for 10 years and sell it."

This is what we mean when we say that "the terminal value is what the asset is worth when we stop writing down the cash flows."

**Taxes:** If Sally and Dave are right in their terminal value assumption, they will have to take account of taxes. The tax rules for selling an asset specify that the tax bill is computed on the *gain over the book value*. So, in the example of Sally and Dave:

$$\begin{aligned}Terminal\ value - Taxes\ on\ gain\ over\ book &= Terminal\ value - Tax\ rate * (Terminal\ value - Book\ value)\\ &= 80{,}000 - 30\% * (80{,}000 - 0) = 56{,}000\end{aligned}$$

## Doing Some Sensitivity Analysis (Advanced Topic)

A sensitivity analysis can show how the IRR of the condo investment varies as a function of the annual rent and the terminal value. Using Excel's **Data Table** (see Chapter 24), we build a sensitivity table:

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| 38 | **Data table—Condo IRR as function of annual rent and terminal value** | | | | | | | |
| 39 | | | | | **Rent** | | | |
| 40 | | 15.98% | **18,000** | **20,000** | **22,000** | **24,000** | **26,000** | **28,000** |
| 41 | **Terminal value -->** | **50,000** | 9.72% | 11.45% | 13.15% | 14.82% | 16.47% | 18.10% |
| 42 | | **60,000** | 10.26% | 11.93% | 13.59% | 15.22% | 16.84% | 18.44% |
| 43 | | **70,000** | 10.77% | 12.40% | 14.01% | 15.61% | 17.19% | 18.76% |
| 44 | =B36 | **80,000** | 11.25% | 12.84% | 14.42% | 15.98% | 17.54% | 19.08% |
| 45 | | **90,000** | 11.71% | 13.27% | 14.81% | 16.34% | 17.87% | 19.38% |
| 46 | | **100,000** | 12.15% | 13.67% | 15.19% | 16.69% | 18.19% | 19.68% |
| 47 | | **110,000** | 12.58% | 14.06% | 15.55% | 17.02% | 18.50% | 19.96% |
| 48 | | **120,000** | 12.98% | 14.44% | 15.90% | 17.35% | 18.80% | 20.24% |
| 49 | | **130,000** | 13.37% | 14.80% | 16.23% | 17.66% | 19.09% | 20.51% |
| 50 | | **140,000** | 13.75% | 15.15% | 16.56% | 17.96% | 19.37% | 20.78% |
| 51 | | **150,000** | 14.11% | 15.49% | 16.87% | 18.26% | 19.65% | 21.03% |
| 52 | | **160,000** | 14.46% | 15.82% | 17.18% | 18.55% | 19.91% | 21.28% |
| 53 | | | | | | | | |
| 54 | | **Note:** The data table above computes the IRR of the condo investment for combinations of rent (from $18,000 to $26,000 per year) and terminal value (from $50,000 to $160,000). Data tables are very useful, though not trivial to compute. See Chapter 24 for more information. | | | | | | |
| 55 | | | | | | | | |
| 56 | | | | | | | | |

The calculations in the data table aren't that surprising: For a given rent, the IRR is higher when the terminal value is higher, and for a given terminal value, the IRR is higher given a higher rent.

## Mini Case

A mini case for this chapter looks at Sally and Dave's condo once more—this time under the assumption that they take out a mortgage to buy the condo. Highly recommended! See the course website of this book: Benninga, Principles of Finance with Excel, Third Edition companion website (www.oup.com/us/Benninga).

## 6.4 More on Salvage Values

In the Sally–Dave condo example, we've focused on the effect of non-cash expenses on cash flows: Accountants and the tax authorities compute earnings by subtracting certain kinds of expenses from sales, even though these expenses are *non-cash expenses*. In order to compute the cash flow, we add back these non-cash expenses to accounting earnings. We showed that these non-cash expenses create *tax shields*—they create cash by saving taxes.

In this section's example, we consider a capital budgeting example in which a firm sells its asset before it is fully depreciated. We show that the asset's book value at the date of the terminal value creates a tax shield and we look at the effect of this tax shield on the capital budgeting decision.

Here's the example. Your firm is considering buying a new machine. Here are the facts:

- The machine costs $800.
- Over the next 8 years (the life of the machine), the machine will generate annual sales of $1,000.
- The annual cost of the goods sold (COGS) is $400 per year, and other costs—selling, general, and administrative expenses (GS&A)—are $300 per year.
- Depreciation on the machine is straight-line over 8 years (that is, $100 per year).
- At the end of 8 years, the machine's salvage value (or terminal value) is zero.
- The firm's tax rate is 40%.
- The firm's discount rate for projects of this kind is 15%.

Should the firm buy the machine? Here's the analysis in Excel:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **BUYING A MACHINE—NPV ANALYSIS** | | | | | | |
| 2 | Cost of the machine | 800 | | | | | |
| 3 | Annual anticipated sales | 1,000 | | | | | |
| 4 | Annual COGS | 400 | | | | | |
| 5 | Annual SG&A | 300 | | | **NPV analysis** | | |
| 6 | Annual depreciation | 100 | | | Year | Cash flow | |
| 7 | | | | | 0 | –800 | <-- =–B2 |
| 8 | Tax rate | 40% | | | 1 | 220 | <-- =$B$23 |
| 9 | Discount rate | 15% | | | 2 | 220 | |
| 10 | | | | | 3 | 220 | |
| 11 | **Annual profit and loss (P&L)** | | | | 4 | 220 | |
| 12 | Sales | 1,000 | | | 5 | 220 | |
| 13 | Minus COGS | –400 | | | 6 | 220 | |
| 14 | Minus SG&A | –300 | | | 7 | 220 | |
| 15 | Minus depreciation | –100 | | | 8 | 220 | |
| 16 | Profit before taxes | 200 | <-- =SUM(B12:B15) | | | | |
| 17 | Subtract taxes | –80 | <-- =–B8*B16 | | **NPV** | 187 | <-- =F7+NPV(B9,F8:F15) |
| 18 | Profit after taxes | 120 | <-- =B16+B17 | | | | |
| 19 | | | | | | | |
| 20 | **Calculating the annual cash flow** | | | | | | |
| 21 | Profit after taxes | 120 | | | | | |
| 22 | Add back depreciation | 100 | | | | | |
| 23 | Cash flow | 220 | | | | | |

Notice that we first calculate the profit and loss (P&L) statement for the machine (cells B12:B18) and then turn this P&L into a cash flow calculation (cells B21:B23). The annual cash flow is $220. Cells F7:F15 show the table of cash flows, and cell F17 gives the NPV of the project. The NPV is positive, and we would therefore buy the machine.

## Salvage Value—A Variation on the Theme

Suppose the firm can sell the machine for $300 at the end of year 8. To compute the cash flow produced by this salvage value, we must make the distinction between *book value* and *market value*:

| | |
|---|---|
| Book value | **An accounting concept:** The book value of the machine is its initial cost minus the accumulated depreciation (the sum of the depreciation taken on the machine since its purchase). In our example, the book value of the machine in year 0 is $800, in year 1 it is $700, . . . , and at the end of year 8 it is zero. |
| Market value | The market value is the price at which the machine can be sold. In our example the market value of the machine at the end of year 8 is $300. |
| Taxable gain | The taxable gain on the machine at the time of sale is the difference between the market value and the book value. In our case the taxable gain is positive ($300), but it can also be negative (see an example at the end of this chapter). |

Here's the NPV calculation including the salvage value:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **BUYING A MACHINE—NPV ANALYSIS With salvage value** | | | | | | |
| 2 | Cost of the machine | 800 | | | | | |
| 3 | Annual anticipated sales | 1,000 | | | | | |
| 4 | Annual COGS | 400 | | | | | |
| 5 | Annual SG&A | 300 | | | **NPV analysis** | | |
| 6 | Annual depreciation | 100 | | | Year | Cash flow | |
| 7 | | | | | 0 | –800 | <-- =–B2 |
| 8 | Tax rate | 40% | | | 1 | 220 | <-- =$B$23 |
| 9 | Discount rate | 15% | | | 2 | 220 | |
| 10 | | | | | 3 | 220 | |
| 11 | **Annual profit and loss (P&L)** | | | | 4 | 220 | |
| 12 | Sales | 1,000 | | | 5 | 220 | |
| 13 | Minus COGS | –400 | | | 6 | 220 | |
| 14 | Minus SG&A | –300 | | | 7 | 220 | |
| 15 | Minus depreciation | –100 | | | 8 | 400 | <-- =$B$23+B30 |
| 16 | Profit before taxes | 200 | <-- =SUM(B12:B15) | | | | |
| 17 | Subtract taxes | –80 | <-- =–B8*B16 | | **NPV** | 246 | <-- =F7+NPV(B9,F8:F15) |
| 18 | Profit after taxes | 120 | <-- =B16+B17 | | | | |
| 19 | | | | | | | |
| 20 | **Calculating the annual cash flow** | | | | | | |
| 21 | Profit after taxes | 120 | | | | | |
| 22 | Add back depreciation | 100 | | | | | |
| 23 | Cash flow | 220 | | | | | |
| 24 | | | | | | | |
| 25 | **Calculating the cash flow from salvage value** | | | | | | |
| 26 | Machine market value, year 8 | 300 | | | | | |
| 27 | Book value, year 8 | 0 | | | | | |
| 28 | Taxable gain | 300 | <-- =B26–B27 | | | | |
| 29 | Taxes paid on gain | 120 | <-- =B8*B28 | | | | |
| 30 | Cash flow from salvage value | 180 | <-- =B26–B29 | | | | |

Note the calculation of the cash flow from the salvage value (cell B30) and the change in the year 8 cash flow (cell F15).

## One More Example

Suppose we change the example slightly:

- The annual sales, SG&A, COGS, and depreciation are still as specified in the original example. The machine will still be depreciated on a straight-line basis over 8 years.
- However, we think we will sell the machine at the *end of year 7* for an estimated salvage value of $450. At the end of year 7, the book value of the machine is $100.

Here's how our calculations look now:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **BUYING A MACHINE—NPV ANALYSIS With salvage value of machine sold at end of year 7** | | | | | | |
| 2 | Cost of the machine | 800 | | | | | |
| 3 | Annual anticipated sales | 1,000 | | | | | |
| 4 | Annual COGS | 400 | | | | | |
| 5 | Annual SG&A | 300 | | | **NPV analysis** | | |
| 6 | Annual depreciation | 100 | | | Year | Cash flow | |
| 7 | | | | | 0 | –800 | <-- =–B2 |
| 8 | Tax rate | 40% | | | 1 | 220 | <-- =$B$23 |
| 9 | Discount rate | 15% | | | 2 | 220 | |
| 10 | | | | | 3 | 220 | |
| 11 | **Annual profit and loss (P&L)** | | | | 4 | 220 | |
| 12 | Sales | 1,000 | | | 5 | 220 | |
| 13 | Minus COGS | –400 | | | 6 | 220 | |
| 14 | Minus SG&A | –300 | | | 7 | 530 | <-- =$B$23+B30 |
| 15 | Minus depreciation | –100 | | | | | |
| 16 | Profit before taxes | 200 | <-- =SUM(B12:B15) | | | | |
| 17 | Subtract taxes | –80 | <-- =–B8*B16 | | **NPV** | 232 | <-- =F7+NPV(B9,F8:F15) |
| 18 | Profit after taxes | 120 | <-- =B16+B17 | | | | |
| 19 | | | | | | | |
| 20 | **Calculating the annual cash flow** | | | | | | |
| 21 | Profit after taxes | 120 | | | | | |
| 22 | Add back depreciation | 100 | | | | | |
| 23 | Cash flow | 220 | | | | | |
| 24 | | | | | | | |
| 25 | **Calculating the cash flow from salvage value** | | | | | | |
| 26 | Machine market value, year 7 | 450 | | | | | |
| 27 | Book value, year 7 | 100 | | | | | |
| 28 | Taxable gain | 350 | <-- =B26-B27 | | | | |
| 29 | Taxes paid on gain | 140 | <-- =B8*B28 | | | | |
| 30 | Cash flow from salvage value | 310 | <-- =B26–B29 | | | | |

Note the subtle changes from the previous example:

- The *cash flow from salvage value* is

$$Salvage\ value - Tax * \underbrace{\left(Salvage\ value - Book\ value\right)}_{\substack{\uparrow \\ \text{Taxable gain at time} \\ \text{of machine sale}}}$$

In our example, this is \$310 (cell B30).

- Another way to write the cash flow from the salvage value is

$$\underbrace{Salvage\ value * (1 - Tax)}_{\substack{\uparrow \\ \text{After-tax proceeds from machine} \\ \text{sale } \textit{if} \text{ the whole salvage value is} \\ \text{taxed}}} + \underbrace{Tax * Book\ value}_{\substack{\uparrow \\ \text{Tax shield on book} \\ \text{value at time of machine} \\ \text{sale}}}$$

Using this example, you can see the role taxes play even if we sell the machine at a loss. Suppose, for example, that the machine is sold in year 7 for $50, which is less than the book value:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **BUYING A MACHINE—NPV ANALYSIS With salvage value of machine sold at end of year 7** | | | | | | |
| 2 | Cost of the machine | 800 | | | | | |
| 3 | Annual anticipated sales | 1,000 | | | | | |
| 4 | Annual COGS | 400 | | | | | |
| 5 | Annual SG&A | 300 | | | **NPV analysis** | | |
| 6 | Annual depreciation | 100 | | | Year | Cash flow | |
| 7 | | | | | 0 | –800 | <-- =–B2 |
| 8 | Tax rate | 40% | | | 1 | 220 | <-- =$B$23 |
| 9 | Discount rate | 15% | | | 2 | 220 | |
| 10 | | | | | 3 | 220 | |
| 11 | **Annual profit and loss (P&L)** | | | | 4 | 220 | |
| 12 | Sales | 1,000 | | | 5 | 220 | |
| 13 | Minus COGS | –400 | | | 6 | 220 | |
| 14 | Minus SG&A | –300 | | | 7 | 290 | <-- =$B$23+B30 |
| 15 | Minus depreciation | –100 | | | | | |
| 16 | Profit before taxes | 200 | <-- =SUM(B12:B15) | | **NPV** | 142 | <-- =F7+NPV(B9,F8:F15) |
| 17 | Subtract taxes | –80 | <-- =–B8*B16 | | | | |
| 18 | Profit after taxes | 120 | <-- =B16+B17 | | | | |
| 19 | | | | | | | |
| 20 | **Calculating the annual cash flow** | | | | | | |
| 21 | Profit after taxes | 120 | | | | | |
| 22 | Add back depreciation | 100 | | | | | |
| 23 | Cash flow | 220 | | | | | |
| 24 | | | | | | | |
| 25 | **Calculating the cash flow from salvage value** | | | | | | |
| 26 | Machine market value, year 7 | 50 | | | | | |
| 27 | Book value, year 7 | 100 | | | | | |
| 28 | Taxable gain | –50 | <-- =B26–B27 | | | | |
| 29 | Taxes paid on gain | –20 | <-- =B8*B28 | | | | |
| 30 | Cash flow from salvage value | 70 | <-- =B26–B29 | | | | |

In this case, the negative taxable gain (cell B28, the jargon often heard is “loss over book”) produces a “tax shield”—the negative taxes of –$20 in cell B29. This tax shield is really a reduction in your total taxes attributable to the $50 loss in cell B28. It is added to the market value to produce a salvage value cash flow of $70 (cell B30). Thus even selling an asset at a loss can produce a positive cash flow.

# 6.5 Accelerated Depreciation

As you know by now, the *salvage value* for an asset is its value at the end of its life; another term sometimes used is *terminal value*. Here's a capital budgeting example that illustrates the importance of accelerated depreciation in computing the cash:

- Your company is considering buying a machine for $10,000.
- If bought, the machine will produce annual cost savings of $3,000 for the next 5 years; these cash flows will be taxed at the company's tax rate of 40%.
- The machine will be depreciated over the 5-year period using the accelerated depreciation percentages allowable in the United States. At the end of the 5th year, the machine will be sold; your estimate of its terminal value at this point is $4,000, even though for accounting purposes its book value is $576 (cell B19 below).

You have to decide what the NPV of the project is, using a discount rate of 12%. Here are the relevant calculations:

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **CAPITAL BUDGETING WITH ACCELERATED DEPRECIATION** | | | | | | |
| 2 | Machine cost | 10,000 | | | | | |
| 3 | Annual materials savings, before tax | 3,000 | | | | | |
| 4 | Salvage value, end of year 5 | 4,000 | | | | | |
| 5 | Tax rate | 40% | | | | | |
| 6 | Discount rate | 12% | | | | | |
| 7 | | | | | | | |
| 8 | **Accelerated depreciation schedule (ACRS)** | | | | | | |
| 9 | **Year** | ACRS depreciation percentage | Actual depreciation | Depreciation tax shield | | | |
| 10 | 1 | 20.00% | 2,000 | 800 | <-- =$B$5*C10 | | |
| 11 | 2 | 32.00% | 3,200 | 1,280 | <-- =$B$5*C11 | | |
| 12 | 3 | 19.20% | 1,920 | 768 | <-- =$B$5*C12 | | |
| 13 | 4 | 11.52% | 1,152 | 461 | <-- =$B$5*C13 | | |
| 14 | 5 | 11.52% | 1,152 | 461 | | | |
| 15 | 6 | 5.76% | 576 | 230 | | | |
| 16 | | | | | | | |
| 17 | **Terminal value, end of year 5** | | | | | | |
| 18 | End-year 5 sale price, estimated | 4,000 | <-- =B4 | | | | |
| 19 | End-year 5 book value | 576 | <-- =B2–SUM(C10:C14) | | | | |
| 20 | Taxable gain | 3,424 | <-- =B18–B19 | | | | |
| 21 | Taxes | 1,370 | <-- =B5*B20 | | | | |
| 22 | Net cash flow from terminal value | 2,630 | <-- =B18–B21 | | | | |
| 23 | | | | | | | |
| 24 | | | | | | | |
| 25 | **Net present value calculation** | | | | | | |
| 26 | Year | Cost | After-tax cost savings | Depreciation tax shield | Terminal value | Total cash flow | |
| 27 | 5 | –10,000 | | | | –10,000 | |
| 28 | 4 | | 1,800 | 800 | | 2,600 | <-- =SUM(B28:E28) |
| 29 | 3 | | 1,800 | 1,280 | | 3,080 | |
| 30 | 2 | | 1,800 | 768 | | 2,568 | |
| 31 | 1 | | 1,800 | 461 | | 2,261 | |
| 32 | 0 | | 1,800 | 461 | 2,630 | 4,891 | |
| 33 | | | | | | | |
| 34 | Net present value | 817 | <-- =F27+NPV(B6,F28:F32) | | | | |
| 35 | **IRR** | 15.01% | <-- =IRR(F27:F32) | | | | |

The book value at the end of year 5 is the initial cost of the machine ($10,000) *minus* the sum of all the depreciation taken on the machine through year 5 ($9,424).

The net cash flow from the terminal value equals the end-year 5 sale price minus applicable taxes.

The annual after-tax cost saving is $1,800 = (1 – 40%)*$3,000. The depreciation tax shields are determined by the accelerated depreciation schedule (rows 10–15). When the asset is sold at the end of year 5, its book value is $576. This leads to a taxable gain of $3,424 (cell B20) and to taxes of $1,370 (cell B21). The net cash flow (cell B22) from selling the asset at the end of year 5 is its sale price of $4,000 minus the taxes (cell B21). The NPV of the asset is $817, and the IRR is 15.01% (cells B34 and B35).

## 6.6 Capital Budgeting Principle 4: Don't Forget the Cost of Foregone Opportunities

Undertaking a project can often involve abandoning the ability to do other projects. Principle 4 says that, where possible, you should take the cash flows from the foregone opportunities into account. **Here's an example:** You've been offered the project below, which involves buying a widget-making machine for $300 to make a new product. The cash flows in years 1–5 have been calculated by your financial analysts:

| | A | B | C |
|---|---|---|---|
| 1 | **DON'T FORGET THE COST OF FOREGONE OPPORTUNITIES** | | |
| 2 | Discount rate | 12% | |
| 3 | | | |
| 4 | **Year** | **Cash flow** | |
| 5 | 0 | –300 | |
| 6 | 1 | 185 | |
| 7 | 2 | 249 | |
| 8 | 3 | 155 | |
| 9 | 4 | 135 | |
| 10 | 5 | 420 | |
| 11 | | | |
| 12 | **NPV** | 498.12 | <– =NPV(B2,B6:B10)+B5 |
| 13 | **IRR** | 62.67% | <– =IRR(B5:B10) |

Looks like a fine project! But now someone remembers that the widget process makes use of some already existing but underused equipment. Should the value of this equipment be somehow taken into account?

The answer to this question has to do with whether the equipment has an alternative use. For example, suppose that, if you don't buy the widget machine, you can sell the equipment for $200. Then the true year 0 cost for the project is $500, and the project has a lower NPV:

| | A | B | C |
|---|---|---|---|
| 16 | Discount rate | 12% | |
| 17 | | | |
| 18 | **Year** | **Cash flow** | |
| 19 | 0 | –500 | The $300 direct cost + $200 <-- value of the existing machines |
| 20 | 1 | 185 | |
| 21 | 2 | 249 | |
| 22 | 3 | 155 | |
| 23 | 4 | 135 | |
| 24 | 5 | 420 | |
| 25 | | | |
| 26 | **NPV** | 298.12 | |
| 27 | **IRR** | 31.97% | |

While the logic here is clear, the implementation can be murky: What if the machine is to occupy space in a building that is currently unused? Should the cost of this space be taken into account? It all depends on whether there are alternative uses, now or in the future.[6]

## In-House Copying or Outsourcing? A Mini Case Illustrating Foregone Opportunity Costs

Your company is trying to decide whether to outsource its photocopying or continue to do it in-house. The current photocopier won't do anymore—it has to either be sold or be thoroughly fixed up. Here are some details about the two alternatives:

- The company's tax rate is 40%.
- Doing the copying in-house requires an investment of $17,000 to fix up the existing photocopy machine. Your accountant estimates that this $17,000 can be immediately booked as an expense, so that its after-tax cost is $(1-40\%)*17,000=10,200$. Given this investment, the copier will be good for another 5 years. Annual copying costs are estimated to be $25,000 on a before-tax basis; after tax, this is $(1-40\%)*25,000=15,000$.
- The photocopy machine is on your books for $15,000, but its market value is in fact much less—it could only be sold today for $5,000. This means that the sale of the copier will generate a loss for tax purposes of $10,000; at your tax rate of 40%, this loss gives a tax shield of $4,000. Thus the sale of the copier will generate a cash flow of $9,000.
- If you decide to keep doing the photocopying in-house, the remaining book value of the copier will be depreciated over 5 years at $3,000 per year. Since your tax rate is 40%, this will produce a tax shield of 40%*$3,000 = $1,200 per year.

[6] There's a fine Harvard case on this topic: "The Super Project," Harvard Business School case 9-112-034.

- Outsourcing the copying will be $33,000 per year—$8,000 more expensive than doing it in-house on the rehabilitated copier. Of course this $33,000 is an expense for tax purposes, so the net savings from doing the copying in-house is

$$(1 - Tax\,rate) * Outsourcing\ costs = (1 - 40\%) * \$33,000 = \$19,800$$

- The relevant discount rate is 12%.

We will show you two ways to analyze this decision. The first method evaluates each of the alternatives separately. The second method looks only at the differential cash flows; while this produces a somewhat "cleaner" set of cash flows that take explicit account of foregone opportunity costs, we recommend the first method—it's simpler and leads to fewer mistakes.

## Method 1: Write Down the Cash Flows of Each Alternative

This is often the simplest way to do things; if you do it correctly, this method takes care of all the foregone opportunity costs without your thinking about them. Below we write down the cash flows for each alternative:

| | In-house | Outsourcing |
|---|---|---|
| **Year 0** | $-(1 - Tax\,rate) * Machine\ rehab\ cost$ <br> $= -(1 - 40\%) * 17,000 = -\$10,200$ | $Sale\ price\ of\ machine$ <br> $+\ Tax\ rate * loss\ over\ book\ value$ <br> $= \$5,000 + 40\% * (\$15,000 - 5,000)$ <br> $= \$9,000$ |
| **Years 1–5 annual cash flow** | $-(1 - Tax\,rate) * In\text{-}house\ costs$ <br> $+ Tax\ rate * Depreciation$ <br> $= -(1 - 40\%) * \$25,000$ <br> $+ 40\% * \$3,000 = -\$13,800$ | $-(1 - Tax\ rate) * Outsourcing\ costs$ <br> $= -(1 - 40\%) * \$33,000$ <br> $= -\$19,800$ |

Putting these data in a spreadsheet and discounting at the discount rate of 12% shows that it is cheaper to do the in-house copying. The NPV of the in-house cash flows is –$59,946, whereas the NPV of the outsourcing cash flows is –$62,375. Note that both NPVs are negative, but the in-house alternative is less negative (i.e., more positive) than the outsourcing alternative. Therefore the in-house is preferred:

| | A | B | C |
|---|---|---|---|
| 1 | **SELL THE PHOTOCOPIER OR FIX IT UP?** | | |
| 2 | Annual cost savings (before tax) after fixing up the machine | 8,000 | |
| 3 | Book value of machine | 15,000 | |
| 4 | Market value of machine | 5,000 | |
| 5 | Rehab cost of machine | 17,000 | |
| 6 | Tax rate | 40% | |
| 7 | Annual depreciation if machine is retained | 3,000 | |
| 8 | Annual copying costs | | |
| 9 | In-house | 25,000 | |
| 10 | Outsourcing | 33,000 | |
| 11 | Discount rate | 12% | |
| 12 | | | |
| 13 | **Alternative 1: Fix up machine and do copying in-house** | | |
| 14 | Year | Cash flow | |
| 15 | 0 | –10,200 | <-- =–B5*(1–B6) |
| 16 | 1 | –13,800 | <-- =–$B$9*(1–$B$6)+$B$6*$B$7 |
| 17 | 2 | –13,800 | |
| 18 | 3 | –13,800 | |
| 19 | 4 | –13,800 | |
| 20 | 5 | –13,800 | |
| 21 | **NPV** of fixing up machine and in-house copying | –59,946 | <-- =B15+NPV(B11,B16:B20) |
| 22 | | | |
| 23 | **Alternative 2: Sell machine and outsource copying** | | |
| 24 | Year | Cash flow | |
| 25 | 0 | 9,000 | <-- =B4+B6*(B3–B4) |
| 26 | 1 | –19,800 | <-- =–(1–$B$6)*$B$10 |
| 27 | 2 | –19,800 | |
| 28 | 3 | –19,800 | |
| 29 | 4 | –19,800 | |
| 30 | 5 | –19,800 | |
| 31 | **NPV** of selling machine and outsourcing | –62,375 | <-- =B25+NPV(B11,B26:B30) |

## Method 2: Discounting the Differential Cash Flows

In this method, we subtract the cash flows of Alternative 2 from those of Alternative 1:

| | A | B | C |
|---|---|---|---|
| 34 | **Subtract Alternative 2 CFs from Alternative 1 CFs** | | |
| 35 | Year | Cash flow | |
| 36 | 0 | –19,200 | <-- =B15–B25 |
| 37 | 1 | 6,000 | <-- =B16–B26 |
| 38 | 2 | 6,000 | |
| 39 | 3 | 6,000 | |
| 40 | 4 | 6,000 | |
| 41 | 5 | 6,000 | |
| 42 | NPV(Alternative 1–Alternative 2) | 2,429 | <-- =B36+NPV(B11,B37:B41) |

The NPV of the differential cash flows is positive. This means that Alternative 1 (in-house) is better than Alternative 2 (outsourcing):

$$NPV\left(In\text{-}house - Outsourcing\right) = NPV\left(In\text{-}house\right) - NPV(Outsourcing) > 0$$

This means that

$$NPV\left(In\text{-}house\right) > NPV(Outsourcing)$$

If you look carefully at the differential cash flows, you'll see that they take into account the cost of the foregone opportunities:

| | Differential cash flow | Explanation |
|---|---|---|
| Year 0 | –$19,200 | This is the after-tax cost of rehabilitating the old copier (–$10,200) and the foregone opportunity cost of selling the copier (–$9,000). In other words: This is the cost in year 0 of deciding to do the copying in-house. |
| Years 1–5 | $6,000 | This is the after-tax saving of doing the copying in-house: If you do it in-house, you save $8,000 pre-tax (= $4,800 after tax) and you get to take depreciation on the existing copier (= tax shield of $1,200). Relative to in-house copying, the outsourcing alternative has a foregone opportunity cost of the loss of the depreciation tax shield. |

If you examine the convoluted prose in the table above ("the outsourcing alternative has a foregone opportunity cost of the loss of the depreciation tax shield"), you'll agree that it may just be simpler to list each alternative's cash flows separately.

## 6.7 Capital Budgeting Principle 5: Think About Mid-Year Discounting

We could have called this section "Think About the Timing of Cash Flows," but "Mid-Year Discounting" is catchier. To show what we mean, we present two examples. In our first example, a company is thinking about spending $10,000 in order to produce an annual cash flow of $3,000 per year for the next 5 years. If the discount rate is 15% and the cash flows occur at year end, then the NPV of the project is $56.47:

| | A | B | C |
|---|---|---|---|
| 1 | **NPV, CASH FLOWS OCCUR AT YEAR END** | | |
| 2 | Initial cost | 10,000.00 | |
| 3 | Annual cash flow | 3,000.00 | |
| 4 | Discount rate | 15% | |
| 5 | | | |
| 6 | **Year** | **Cash flow** | |
| 7 | 0 | –10,000.00 | |
| 8 | 1 | 3,000 | |
| 9 | 2 | 3,000 | |
| 10 | 3 | 3,000 | |
| 11 | 4 | 3,000 | |
| 12 | 5 | 3,000 | |
| 13 | | | |
| 14 | **NPV** of year-end cash flows | 56.47 | <-- =B7+NPV(B4,B8:B12) |

The NPV of $56.47 assumes that the cash flow for each year occurs at the end of the year:

$$NPV = -10{,}000 + \frac{3{,}000}{(1.15)} + \frac{3{,}000}{(1.15)^2} + \frac{3{,}000}{(1.15)^3} + \frac{3{,}000}{(1.15)^4} + \frac{3{,}000}{(1.15)^5} = 56.47$$

For many capital budgeting situations, this end-year cash flow assumption is not realistic. Think of a company buying a machine and getting cash flows by selling the machine's products—in this case, the cash flows are likely to occur as a stream throughout the year rather than a single, end-year cash flow. Since it's always better to get cash earlier, the NPV of the project will be higher than $56.47.

To get some feeling for whether this is important, suppose that the $3,000 annual cash flow is actually received as $750 at the end of each quarter. Then, as the spreadsheet below shows, the NPV would increase significantly:

| | A | B | C |
|---|---|---|---|
| 1 | **NPV, CASH FLOWS OCCUR EACH QUARTER** | | |
| 2 | Initial cost | 10,000.00 | |
| 3 | Annual cash flow | 3,000.00 | |
| 4 | Discount rate | 15% | |
| 5 | Quarterly discount rate | 3.56% | <-- =(1+B4)^(1/4)–1 |
| 6 | | | |
| 7 | **Quarter** | **Quarterly cash flow** | |
| 8 | 0 | –10,000.00 | |
| 9 | 1 | 750.00 | |
| 10 | 2 | 750.00 | |
| 11 | 3 | 750.00 | |
| 12 | 4 | 750.00 | |
| 13 | 5 | 750.00 | |
| 14 | 6 | 750.00 | |
| 15 | 7 | 750.00 | |
| 16 | 8 | 750.00 | |
| 17 | 9 | 750.00 | |
| 18 | 10 | 750.00 | |
| 19 | 11 | 750.00 | |
| 20 | 12 | 750.00 | |
| 21 | 13 | 750.00 | |
| 22 | 14 | 750.00 | |
| 23 | 15 | 750.00 | |
| 24 | 16 | 750.00 | |
| 25 | 17 | 750.00 | |
| 26 | 18 | 750.00 | |
| 27 | 19 | 750.00 | |
| 28 | 20 | 750.00 | |
| 29 | | | |
| 30 | **NPV**, quarterly cash flows | 605.68 | <-- =B8+NPV(B5,B9:B28) |

Notice that in calculating the NPV of the quarterly cash flows (cell E29), we've used the *quarterly discount rate* which is equivalent to the annual discount rate of 15% (3.56%, cell E4). This quarterly discount rate is calculated by

$$(1+Quarterly\ discount\ rate)=(1+Annual\ discount\ rate)^{1/4}$$

So far, the message of this section is clear and uncontroversial: When you discount, you should take the timing of the cash flows into account. The problem is that for many capital budgeting problems we project annual cash flows, even though the actual flows occur throughout the year.[7] In many cases, it is difficult to project the precise timing of the cash flows throughout the year, even though our example shows that this timing is very important.

## Mid-Year Discounting—an Elegant Compromise

On the one hand, the timing of cash flows is important, but on the other hand, it's difficult to deviate from end-year cash flow projections and project the precise timing of each cash flow. An elegant compromise is to project annual cash flow numbers but to assume that they occur in mid-year. Here's how this looks in Excel:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **MID-YEAR DISCOUNTING** | | | |
| 2 | Initial cost | 10,000.00 | | |
| 3 | Annual cash flow | 3,000.00 | | |
| 4 | Discount rate | 15% | | |
| 5 | | | | |
| 6 | **Year** | **Cash flow** | **Discounted value** | |
| 7 | 0 | –10,000.00 | –10,000.00 | <-- =B7 |
| 8 | 1 | 3,000 | 2,797.51 | <-- =B8/(1+$B$4)^(A8–0.5) |
| 9 | 2 | 3,000 | 2,432.62 | <-- =B9/(1+$B$4)^(A9–0.5) |
| 10 | 3 | 3,000 | 2,115.32 | |
| 11 | 4 | 3,000 | 1,839.41 | |
| 12 | 5 | 3,000 | 1,599.49 | |
| 13 | | | | |
| 14 | **NPV**, mid-year | 784.36 | <-- =SUM(C7:C12) | |
| 15 | | 784.36 | <-- =B7+NPV(B4,B8:B12)*(1+B4)^0.5 | |

[7] This has a lot to do with most firms' accounting cycles, which are annual. (There we go again, blaming the accountants!)

The spreadsheet shows two ways to do the calculation:

- In cells B8:B12, each cash flow has been discounted by a factor $(1+r)^{year-0.5}$. This is equivalent to calculating the following NPV:

$$NPV = -10{,}000 + \frac{3{,}000}{(1.15)^{0.5}} + \frac{3{,}000}{(1.15)^{1.5}} + \frac{3{,}000}{(1.15)^{2.5}} + \frac{3{,}000}{(1.15)^{3.5}} + \frac{3{,}000}{(1.15)^{4.5}} = \underbrace{784.36}_{\text{Cell C43}}$$

- In cell B15, we show a simple Excel formula which produces the same result: Simply take the Excel **NPV** formula and multiply by $(1+r)^{0.5}$.

## Using the XNPV Function

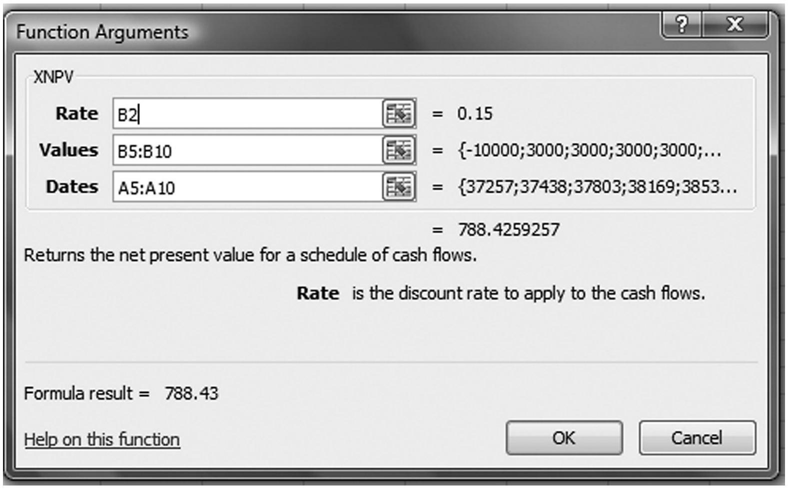
We can also do the mid-year NPV calculation by using Excel's **XNPV** function.[8] To use **XNPV,** you have to indicate the dates on which the cash flows will be received. The spreadsheet below shows an implementation of the function to our problem.

| | A | B | C |
|---|---|---|---|
| 1 | **CALCULATING THE MID-YEAR NPV WITH EXCEL'S XNPV FUNCTION** | | |
| 2 | Annual discount rate | 15% | |
| 3 | | | |
| 4 | **Date** | **Cash flow** | |
| 5 | 1-Jan-02 | –10,000 | |
| 6 | 1-Jul-02 | 3,000 | |
| 7 | 1-Jul-03 | 3,000 | |
| 8 | 1-Jul-04 | 3,000 | |
| 9 | 1-Jul-05 | 3,000 | |
| 10 | 1-Jul-06 | 3,000 | |
| 11 | | | |
| 12 | **NPV** | 788.43 | <-- =XNPV(B2,B5:B10,A5:A10) |

[8] If this function does not appear in your list of Excel functions, go to **File|Options|Add-ins** on the Excel menu, and check **Analysis Toolpak**.

# EXCEL NOTE

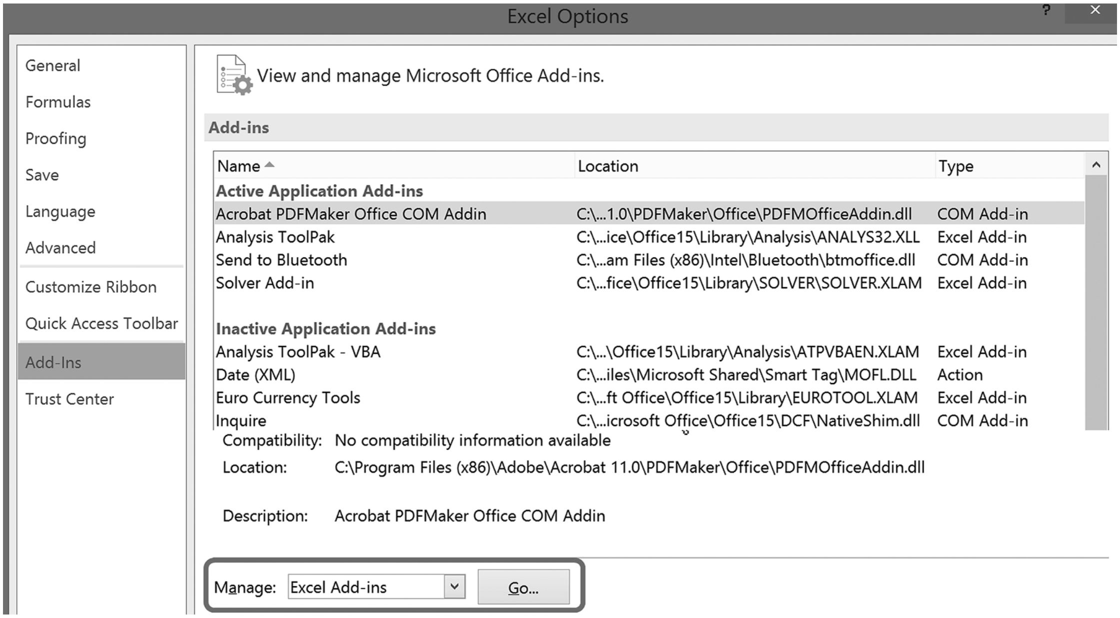
## Using the XNPV Function



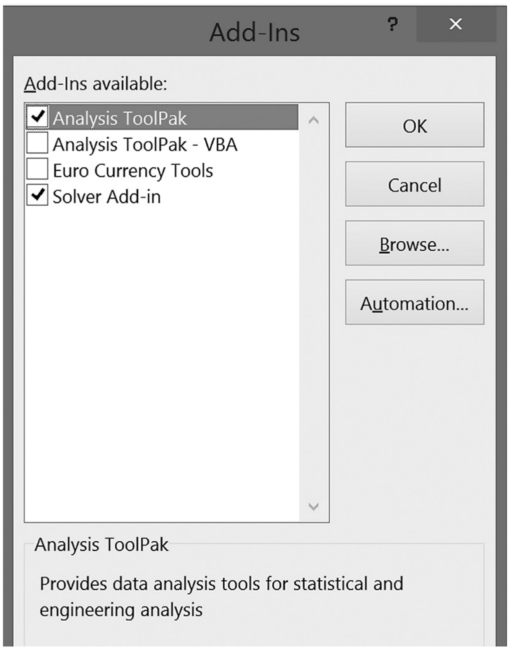

As the dialog box shows, **XNPV** requires you to input the *annual* discount rate, the values to be discounted, and the dates on which these values occur. The function then finds the net present value on the first date of the series (in our example, 1-Jan-02). The **XNPV** function differs from the **NPV** function in one very important aspect: In Chapter 3 we stressed that Excel's **NPV** calculates the present value of future cash flows; to calculate the true net present value, you have to add in the initial cash flow separately. The **XNPV** function has *all* the cash flows as inputs (including the initial cash flow) and has as output the true net present value.

The **XNPV** function (and its cousin, the **XIRR** function discussed below) are part of the standard Excel package, but sometimes they have to be separately installed as an add-in. Here's what you do:

**Step 1: Go to File|Options|Add-Ins|Manage Excel Add-Ins**




**Step 2: Click Analysis ToolPak (while you're there, also click the Solver box)**



## Calculating the Mid-Year IRR

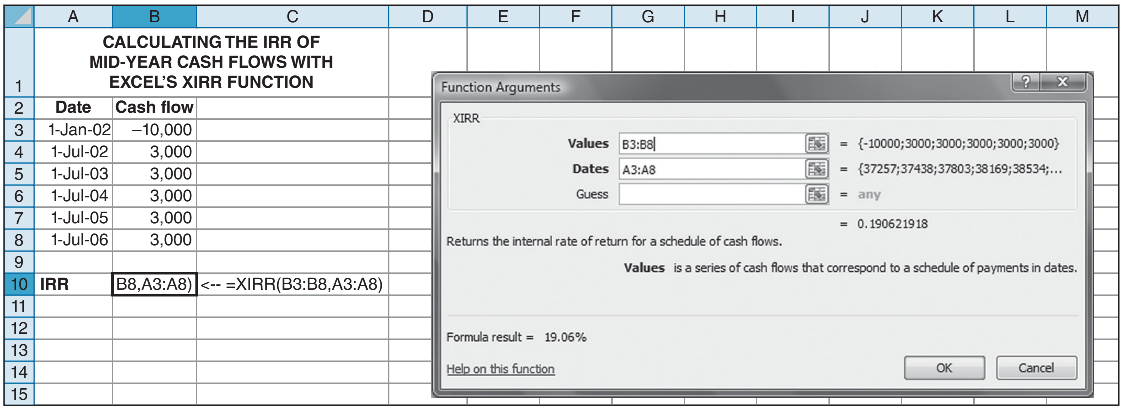
What if you want to compute the IRR of the cash flows, taking into account the fact that they occur in mid-year? The easiest way to do this is to use the Excel function **XIRR**, as shown below:

| | A | B | C |
|---|---|---|---|
| 1 | **CALCULATING THE IRR OF MID-YEAR CASH FLOWS WITH EXCEL'S XIRR FUNCTION** | | |
| 2 | **Date** | **Cash flow** | |
| 3 | 1-Jan-02 | –10,000 | |
| 4 | 1-Jul-02 | 3,000 | |
| 5 | 1-Jul-03 | 3,000 | |
| 6 | 1-Jul-04 | 3,000 | |
| 7 | 1-Jul-05 | 3,000 | |
| 8 | 1-Jul-06 | 3,000 | |
| 9 | | | |
| 10 | **IRR** | 19.06% | <-- =XIRR(B3:B8,A3:A8) |

# EXCEL NOTE

## The XIRR Function

The **XIRR** function requires you to put in a list of dates at which the cash flows occur. The syntax of the function is given in the dialogue box below:




For cash flows with multiple IRRs, the **XIRR** function allows you to use a **Guess** (like Excel's **IRR** function discussed on Chapter 3).

## Applying Mid-Year Cash Flows to The Sally and Dave Condo

In this section, we have stressed the importance of cash flow timing in determining the NPV of a project. We have also suggested that—rather than try to determine the precise timing of each cash flow—it may be roughly equivalent to assume that the cash flows occur in mid-period.

The implementation of this simple idea can be complicated. Take Sally and Dave's condo, for example, which was discussed in Section 6.3. Recall that Sally and Dave's annual cash flow of $18,050 from the condo rental was computed as follows:

- The annual rent of $24,000 is taxable income, and the annual property taxes ($1,500) and maintenance ($1,000) are expenses for tax purposes. Since Sally and Dave's tax rate is 30%, these three items produce $\left(1-30\%\right)*\left(\$24,000-1,000-1,500\right)=\$15,050$ of after-tax income per year.
- The condo's annual depreciation of $10,000 produces a tax shield of $30\%*\$10,000=\$3,000$. Adding this tax shield to the $15,050 gives Sally and Dave's annual cash flow of $18,050 in years 1-10.
- Sally and Dave plan to sell the condo for $100,000 after 10 years. At this point the condo will be fully depreciated, so that all the money they receive from the sale will be income. Thus the after-tax terminal value of the condo is $\left(1-30\%\right)*\$100,000=\$70,000$. Adding this to the condo's year 10 cash flow produces a total year 10 cash flow of $88,050.

Our initial calculation gave us an IRR of 16.69% on Sally and Dave's investment (cell B37 below):

| | A | B | C |
|---|---|---|---|
| 1 | **SALLY & DAVE'S CONDO**<br>**Example from Section 6.3** | | |
| 2 | **Cost of condo** | 100,000 | |
| 3 | **Sally & Dave's tax rate** | 30% | |
| 4 | | | |
| 5 | **Annual reportable income calculation** | | |
| 6 | Rent | 24,000 | |
| 7 | Expenses | | |
| 8 | Property taxes | –1,500 | |
| 9 | Miscellaneous expenses | –1,000 | |
| 10 | Depreciation | –10,000 | |
| 11 | Reportable income | 11,500 | <-- =SUM(B6:B10) |
| 12 | Taxes (rate = 30%) | –3,450 | <-- =–B3*B11 |
| 13 | Net income | 8,050 | <-- =B11+B12 |
| 14 | | | |
| 15 | Annual cash flow | 18,050 | <-- =B13–B10 |
| 16 | | | |
| 17 | **Terminal value** | | |
| 18 | Estimated resale value, year 10 | 100,000 | |
| 19 | Book value | 0 | |
| 20 | Taxable gain | 100,000 | <-- =B18–B19 |
| 21 | Taxes | 30,000 | <-- =0.3*B20 |
| 22 | Net after tax—cash flow from terminal value | 70,000 | <-- =B20–B21 |
| 23 | | | |
| 24 | **Year** | **Cash flow** | |
| 25 | 0 | –100,000 | |
| 26 | 1 | 18,050 | <-- =$B$15 |
| 27 | 2 | 18,050 | |
| 28 | 3 | 18,050 | |
| 29 | 4 | 18,050 | |
| 30 | 5 | 18,050 | |
| 31 | 6 | 18,050 | |
| 32 | 7 | 18,050 | |
| 33 | 8 | 18,050 | |
| 34 | 9 | 18,050 | |
| 35 | 10 | 88,050 | <-- =$B$15+B22 |
| 36 | | | |
| 37 | IRR | 16.69% | |

## Incorporating the Timing of Cash Flows

Now suppose we try to incorporate the timing of the cash flows into our analysis of the condo IRR. We make the following assumptions:

- The annual rent of $24,000 occurs at mid-year. This is an approximation to the fact that the renters pay their rent monthly.
- Miscellaneous expenses of $1,000 also occur at mid-year.

- Property taxes and income taxes occur at the end of each year.
- The resale of the property (which produces a cash flow of \$70,000) occurs at the end of year 10.

These assumptions lead to the cash flows given in cells E4:E44 below. The IRR of these cash flows (9.59%, cell E46) is the *semi-annual IRR* (remember that our cash flows are now semi-annual). The *annualized IRR* is $(1+9.59\%)^2 - 1 = 20.10\%$, which is significantly higher than the 16.69% we calculated above assuming that all cash flows occur at year-end. Since the IRR gets higher when positive cash flows occur earlier, this is not surprising.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | **SALLY & DAVE'S CONDO—INCORPORATING MID-YEAR CASH FLOWS** | | | | | |
| 2 | Tax rate | 30% | | **Return on the condo—Taking into account mid-year cash flows** | | |
| 3 | | | | **Year** | **Cash flow** | |
| 4 | **Mid-year cash flows** | | | 0 | –100,000 | |
| 5 | Rent | 24,000 | | 0.5 | 23,000 | <-- Rent + miscellaneous expenses |
| 6 | Miscellaneous expenses | –1,000 | | 1 | –4,950 | <-- Property and income taxes |
| 7 | Sum of mid-year cash flows | 23,000 | <-- =SUM(B5:B6) | 1.5 | 23,000 | |
| 8 | | | | 2 | –4,950 | |
| 9 | **End-year cash flows** | | | 2.5 | 23,000 | |
| 10 | Depreciation | 10,000 | | 3 | –4,950 | |
| 11 | Property taxes | –1,500 | | 3.5 | 23,000 | |
| 12 | Reported income | 11,500 | | 4 | –4,950 | |
| 13 | Income taxes | –3,450 | <-- =–B2*B12 | 4.5 | 23,000 | |
| 14 | Sum of end-year cash flows | –4,950 | <-- =B11+B13 | 5 | –4,950 | |
| 15 | | | | 5.5 | 23,000 | |
| 16 | Annual cash flow | 18,050 | <-- =B7+B14 | 6 | –4,950 | |
| 17 | | | | 6.5 | 23,000 | |
| 18 | | | | 7 | –4,950 | |
| 19 | | | | 7.5 | 23,000 | |
| 33 | | | | 14.5 | 23,000 | |
| 34 | | | | 15 | –4,950 | |
| 35 | | | | 15.5 | 23,000 | |
| 36 | | | | 16 | –4,950 | |
| 37 | | | | 16.5 | 23,000 | |
| 38 | | | | 17 | –4,950 | |
| 39 | | | | 17.5 | 23,000 | |
| 40 | | | | 18 | –4,950 | |
| 41 | | | | 18.5 | 23,000 | |
| 42 | | | | 19 | –4,950 | |
| 43 | | | | 19.5 | 23,000 | |
| 44 | | | | 20 | 65,050 | <-- Sale of condo + taxes |
| 45 | | | | | | |
| 46 | | | | **IRR** (semi-annual) | 9.59% | <-- =IRR(E4:E44) |
| 47 | | | | **IRR** (annualized) | 20.10% | <-- =(1+E46)^2–1 |

Note: Some rows have been hidden. For full version, see the book's website.

## Two "Reality" Notes

**Note 1:** The Sally–Dave condo example shows that getting the dates of the cash flows right is important, but it also shows that this can be cumbersome. As a compromise, perhaps we should have gone back to the mid-year IRR. In the spreadsheet below, we use the **XIRR** function in cell B37 to compute the IRR on the assumption that all the condo cash flows occur in mid-year:

| | A | B | C |
|---|---|---|---|
| 1 | **SALLY & DAVE'S CONDO—MID-YEAR CASH FLOWS** | | |
| 2 | **Cost of condo** | 100,000 | |
| 3 | **Sally & Dave's tax rate** | 30% | |
| 4 | | | |
| 5 | **Annual reportable income calculation** | | |
| 6 | Rent | 24,000 | |
| 7 | Expenses | | |
| 8 | Property taxes | –1,500 | |
| 9 | Miscellaneous expenses | –1,000 | |
| 10 | Depreciation | –10,000 | |
| 11 | Reportable income | 11,500 | <-- =SUM(B6:B10) |
| 12 | Taxes (rate = 30%) | –3,450 | <-- =–B3*B11 |
| 13 | Net income | 8,050 | <-- =B11+B12 |
| 14 | | | |
| 15 | Annual cash flow | 18,050 | <-- =B13–B10 |
| 16 | | | |
| 17 | **Terminal value** | | |
| 18 | Estimated resale value, year 10 | 100,000 | |
| 19 | Book value | 0 | |
| 20 | Taxable gain | 100,000 | <-- =B18–B19 |
| 21 | Taxes | 30,000 | <-- =0.3*B20 |
| 22 | Net after tax—cash flow from terminal value | 70,000 | <-- =B20–B21 |
| 23 | | | |
| 24 | **Date** | **Cash flow** | |
| 25 | 1-Jan-02 | –100,000 | |
| 26 | 1-Jul-02 | 18,050 | <-- =$B$15 |
| 27 | 1-Jul-03 | 18,050 | |
| 28 | 1-Jul-04 | 18,050 | |
| 29 | 1-Jul-05 | 18,050 | |
| 30 | 1-Jul-06 | 18,050 | |
| 31 | 1-Jul-07 | 18,050 | |
| 32 | 1-Jul-08 | 18,050 | |
| 33 | 1-Jul-09 | 18,050 | |
| 34 | 1-Jul-10 | 18,050 | |
| 35 | 1-Jul-11 | 88,050 | <-- =$B$15+B22 |
| 36 | | | |
| 37 | **IRR** | 18.69% | <-- =XIRR(B25:B35,A25:A35) |

**Note 2:** In this book, we often ignore mid-year discounting—not because we don't believe it's important, but because it's cumbersome to explain this, along with all the other myriad capital budgeting problems. In this case, our advice to you is: "Do as we say, don't do as we do."

# 6.8 Capital Budgeting Principle 6: Don't Ignore Inflation

You're considering an investment in a new widget machine. The machine will cost \$9,500 today; cells B9:B14 give the widget sales forecasts for years 1–6. Widgets today sell for \$15 each (cell B3), and the widget price in the future is expected to rise at the inflation rate of 4% (cell B2). Your nominal discount rate is 12% (cell B4).

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | CAPITAL BUDGETING FOR THE WIDGET MACHINE | | | | | | |
| 2 | Inflation rate | 4.00% | | | | | |
| 3 | Widget price today | 15.00 | | | | | |
| 4 | Nominal discount rate | 12.00% | | | | | |
| 5 | Equivalent real discount rate | 7.69% | <-- =(1+B4)/(1+B2)–1 | | | | |
| 6 | | | | | | | |
| 7 | **Year** | **Widgets sold** | **Anticipated nominal widget price** | **Anticipated nominal cash flow** | | **Anticipated real cash flow in year 0 dollars** | |
| 8 | 0 | | | –9,500.00 | | –9,500.00 | |
| 9 | 1 | 100 | 15.60 | 1,560.00 | <–=C9*B9 | 1,500.00 | <-- =D9/(1+\$B\$2)^A9 |
| 10 | 2 | 125 | 16.22 | 2,028.00 | <– =C10*B10 | 1,875.00 | <-- =D10/(1+\$B\$2)^A10 |
| 11 | 3 | 150 | 16.87 | 2,530.94 | | 2,250.00 | |
| 12 | 4 | 160 | 17.55 | 2,807.66 | | 2,400.00 | |
| 13 | 5 | 170 | 18.25 | 3,102.46 | | 2,550.00 | |
| 14 | 6 | 200 | 18.98 | 3,795.96 | | 3,000.00 | |
| 15 | | | | | | | |
| 16 | **NPV calculations** | | | =\$B\$3*(1+\$B\$2)^A9 | | | |
| 17 | Discounting nominal cash flows at nominal discount rates | 778.93 | <-- =NPV(B4,D9:D14)+D8 | | | | |
| 18 | Discounting real cash flows at real discount rates | 778.93 | <-- =NPV(B5,F9:F14)+F8 | | | | |
| 19 | | | | | | | |
| 20 | **IRR calculations** | | | | | | |
| 21 | Nominal IRR | 14.47% | <-- =IRR(D8:D14) | | | | |
| 22 | Real IRR | 10.06% | <-- =IRR(F8:F14) | | | | |
| 23 | (1+nominal IRR)/(1+inflation)–1 | 10.06% | <-- =(1+B21)/(1+B2)–1 | | | | |

- Assuming a nominal discount rate of 12%, an equivalent real discount rate is given in cell B5. This rate is computed by the formula

$$Real\ discount\ rate = \frac{1+Nominal\ rate}{1+Inflation\ rate} - 1 = \frac{1+12\%}{1+4\%} - 1 = 7.69\%$$

- Column C of the spreadsheet shows the anticipated widget price in each of years 1–6. We've computed this price by computing the *nominal widget price* in each of years 1–6 using the formula

$$Nominal\ time\text{-}t\ \ price = Price\ today * (1 + Inflation)^t$$

- Column D shows the nominal cash flow.[9] Discounting these cash flows at 12% gives the NPV of $778.93 in cell B17. Because the NPV is positive, you should invest in the new machine.
- Assuming that the increase in widget prices and the increase in general prices are the same, we can compute the real cash flows as in column F:

$$Real\ cash\ flow,\ year\ t = (Widgets\ sold) * Widget\ price\ today$$

Column F actually shows a different formula, which gives the same result:

$$Real\ cash\ flow,\ year\ t = \left(\frac{Nominal\ value\ of\ widgets\ sold}{(1+Inflation\ rate)^t}\right)$$

- The NPV of these real cash flows, computed in cell B18, is the same as the $778.93 computed in B17.

Discounting nominal cash flows at a nominal discount rate and discounting real cash flows at a real discount rate gives the same net present value in year 0 dollars.

## Computing the Real and Nominal IRR

We can compute the real and nominal IRR for the widget machine as follows:

- Taking the IRR of the nominal cash flows (cell B22) gives a nominal IRR of 14.47%. Because the nominal IRR is greater than the nominal discount rate of 12%, the widget machine is a good investment.
- Computing the IRR of the real cash flows (cell B23) gives a real IRR of 10.06%. The investment decision given by the real IRR is the same as the investment decision given by the nominal IRR: Because the real IRR is greater than the real discount rate of 7.69%, the machine is a good investment. Note that we've computed the real discount rate in cell B5 using the formula

$$Real\ discount\ rate = \frac{1+Nominal\ discount\ rate}{1+Anticipated\ inflation\ rate} = \frac{1+12\%}{1+4\%} - 1 = 7.69\%$$

- The real IRR can also be computed by the formula

$$Real\ IRR = \frac{1+Nominal\ IRR}{1+Inflation\ rate} - 1 = \frac{1+14.47\%}{1+4\%} - 1 = 10.06\%$$

[9] We've assumed that it won't cost you anything to produce the widgets once you buy the machine. (Alternatively, you can assume that the widget price is net of production costs.)

## Two Ways of Computing the Real IRR

The real IRR can be computed either by:

- Taking the IRR of the projected real cash flows (direct calculation of the real IRR)
- Taking the IRR of the nominal cash flows, dividing by (1 + inflation rate), and subtracting 1.

To see that these two are the same, notice that the nominal NPV is computed by:

$$\begin{aligned}
Nominal\ NPV = CF_0 &+ \frac{CF_1(real)*(1+Inflation\ rate)}{(1+Real\ interest\ rate)*(1+Inflation\ rate)} \\
&+ \frac{CF_2(real)*(1+Inflation\ rate)^2}{\left[(1+Real\ interest\ rate)*(1+Inflation\ rate)\right]^2} \\
&+ \frac{CF_2(real)*(1+Inflation\ rate)^3}{\left[(1+Real\ interest\ rate)*(1+Inflation\ rate)\right]^3} \\
&+ \cdots
\end{aligned}$$

Throughout this formula, the (1 + inflation rate) term cancels out, so that

$$\begin{aligned}
Nominal\ NPV = CF_0 &+ \frac{CF_1(real)}{(1+Real\ interest\ rate)} \\
&+ \frac{CF_2(real)}{(1+Real\ interest\ rate)^2} + \frac{CF_3(real)}{(1+Real\ interest\ rate)^3} + \cdots \\
= Real\ NPV &
\end{aligned}$$

### Widget Prices Have a Different Inflation Rate Than the General Inflation Rate

In the previous problem, the anticipated increase in widget prices was the same as the inflation rate. Suppose this isn't true—in the spreadsheet below, we assume that inflation (understood as the increase in the CPI) will be 4% per year, but that widget prices will increase at 8% per year. (Widget demand is expected to rise sharply, causing a big increase in prices.)

The analysis for this case is shown below. Though in principle it is not different from the analysis for the previous problem, the results are, of course, different—making widgets an even more profitable business.

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | **CAPITAL BUDGETING FOR THE WIDGET MACHINE**<br>**Widget prices increase at a different rate than the inflation rate** | | | | | | |
| 2 | Inflation rate | 4.00% | | | | | |
| 3 | Widget price today | 15.00 | | | | | |
| 4 | Annual increase in widget prices | 8.00% | | | | | |
| 5 | Nominal discount rate | 12.00% | | | | | |
| 6 | Equivalent real discount rate | 7.69% | <-- =(1+B5)/(1+B2)–1 | | | | |
| 7 | | | | | | | |
| 8 | **Year** | **Widgets sold** | **Anticipated nominal widget price** | **Anticipated nominal cash flow** | | **Anticipated real cash flow in year 0 dollars** | |
| 9 | 0 | | | –9,500.00 | | –9,500.00 | |
| 10 | 1 | 100 | 16.20 | 1,620.00 | <–=C10*B10 | 1,557.69 | <-- =D10/(1+$B$2)^A10 |
| 11 | 2 | 125 | 17.50 | 2,187.00 | <– =C11*B11 | 2,022.00 | <-- =D11/(1+$B$2)^A11 |
| 12 | 3 | 150 | 18.90 | 2,834.35 | | 2,519.73 | |
| 13 | 4 | 160 | 20.41 | 3,265.17 | | 2,791.08 | |
| 14 | 5 | 170 | 22.04 | 3,746.79 | | 3,079.59 | |
| 15 | 6 | 200 | 23.80 | 4,760.62 | | 3,762.39 | |
| 16 | | | | | | | |
| 17 | **NPV calculations** | | | =$B$3*(1+$B$4)^A10 | | | |
| 18 | Discounting nominal cash flows at nominal discount rates | 2,320.31 | <-- =NPV(B5,D10:D15)+D9 | | | | |
| 19 | Discounting real cash flows at real discount rates | 2,320.31 | <-- =NPV(B6,F10:F15)+F9 | | | | |
| 20 | | | | | | | |
| 21 | **IRR calculations** | | | | | | |
| 22 | Nominal IRR | 18.87% | <-- =IRR(D9:D15) | | | | |
| 23 | Real IRR | 14.30% | <-- =IRR(F9:F15) | | | | |
| 24 | (1+nominal IRR)/(1+inflation)–1 | 14.30% | <-- =(1+B22)/(1+B2)–1 | | | | |

Compare this spreadsheet to the calculations we did in the previous example: Since widget prices increase faster than the inflation rate, both the nominal and the real anticipated cash flows are greater in every year. Thus the project is more profitable, whether measured by real or nominal NPV or real or nominal IRR.

## Summary

In this chapter, we've discussed the basics of capital budgeting using NPV and IRR. Capital budgeting decisions can be crudely separated into "yes–no" decisions ("Should we undertake a given project?") and into "ranking" decisions ("Which of the following list of projects do we prefer?"). We've concentrated on two important areas of capital budgeting:

- In many cases, NPV and IRR give the same answer to the capital budgeting question. However, there are cases—especially when we rank projects—where NPV and IRR give different answers. Where they differ, NPV is the preferable criterion to use because the NPV is the additional wealth derived from a project.

- Every capital budgeting decision ultimately involves a set of anticipated cash flows, so when you do capital budgeting, it's important to get these cash flows right. We've illustrated the importance of sunk costs, taxes, foregone opportunities, and salvage values in determining the cash flows.

## Exercises

**Note:** The data for these exercises can be found on the Benninga, *Principles of Finance with Excel,* Third Edition companion website (www.oup.com/us/Benninga).

**1.** (Sunk costs) We return to the sunk cost example of Section 6.2. You have invested \$100,000 in a badly built house. For \$20,000 invested today, you can fix up the house and sell it 1 year from today for \$90,000. As an alternative, you can sell the house today for \$60,000.

> **a.** Should you take into account the \$100,000 cost already invested in the house?
>
> **b.** If the relevant discount rate is 9%, which alternative should you prefer?
>
> **c.** What is the discount rate that makes you indifferent between the two alternatives?

**2.** (Sunk costs) Your uncle is a proud owner of an up-market clothing store. Because business is down, he is considering replacing the languishing tie department with a new sportswear department. In order to examine the profitability of such move, he has hired a financial advisor to estimate the cash flows of the new department. After 6 months of hard work, the financial advisor came up with the following calculation:

| | A | B | C |
|---|---|---|---|
| 2 | **Investment** | **t=0** | |
| 3 | Rearranging the shop | –40,000 | |
| 4 | Loss of business during renovation | –15,000 | |
| 5 | Payment for financial advisor | –12,000 | |
| 6 | **Total** | **–67,000** | <-- =SUM(B3:B5) |

| | A | B | C | D |
|---|---|---|---|---|
| 8 | **Annual profits** | **t=1 to infinity** | | |
| 9 | Annual earnings from the sport department | 75,000 | | |
| 10 | Loss of earnings from the tie department | –20,000 | | |
| 11 | Loss of earnings from other departments* | –15,000 | | |
| 12 | Additional worker for the sport department | –18,000 | | |
| 13 | Municipal taxes | –15,000 | | |
| 14 | **Total** | **7,000** | <-- =SUM(B9:B13) | |
| 15 | * Some of your uncle's stuck-up clients will not buy in a shop that sells sportswear. | | | |

The discount rate is 12%, and there are no additional taxes. Thus the financial advisor calculated the NPV as follows: $NPV = -67{,}000 + \frac{7{,}000}{0.12} = -8{,}667$. Your surprised uncle asked you (a promising finance student) to go over the calculation. After taking a close look, you identify two mistakes: (1) Although municipal taxes do not change as a result of the replacement of the tie department, the financial advisor has attributed part of existing municipal taxes to the cost of the store change. (2) The "Payment for financial advisor" will be paid even if we do not rearrange the shop. What are the correct NPV and IRR of the project?

**3.** (Marginal vs. average costs) You are the owner of a factory that supplies chairs and tables to schools in Denver. You sell each chair for \$1.76 and each table for \$4.40 based on the following calculation:

| | A | B | C | D |
|---|---|---|---|---|
| 1 | **FURNITURES FACTORY** | | | |
| 2 | | **Chair department** | **Table department** | |
| 3 | No. of units | 100,000 | 20,000 | |
| 4 | Cost of material | 80,000 | 35,000 | |
| 5 | Cost of labor | 40,000 | 20,000 | |
| 6 | Fixed cost | 40,000 | 25,000 | |
| 7 | Total cost | 160,000 | 80,000 | <-- =SUM(C4:C6) |
| 8 | Cost per unit | 1.6 | 4 | <-- =C7/C3 |
| 9 | Normative profit | 10% | 10% | |
| 10 | **Unit cost** | **1.76** | **4.4** | <-- =C8*(1+C9) |

You have received a "take it or leave it" offer from a school in Colorado Springs to supply an additional 10,000 chairs and 2,000 tables for the price of \$1.5 and \$3.5, respectively. Your financial advisor advises you *not* to take up the offer because the price does not even cover the cost of production. You have pointed out to the advisor that the fixed costs (row 6) will not increase with additional chair production. You also claim that the existing labor force can make the additional furniture. Redo the calculations to see if you should accept the Colorado Springs offer.

**4.** (NPV and cash flows) A factory is considering the purchase of a new machine for one of its units. The machine costs \$100,000. The machine will be depreciated on a straight-line basis over its 10-year life to a salvage value of zero. The machine is expected to save the company \$50,000 annually, but in order to operate it, the factory will have to transfer an employee (with a salary of \$40,000 a year) from one of its other units. A new employee (with a salary of \$20,000 a year) will be required to replace the transferred employee. What is the NPV of the purchase of the new machine if the relevant discount rate is 8% and the corporate tax rate is 35%?

5. (Depreciation) You are considering the following investment:

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 3 | Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 4 | Earnings before depreciation and taxes | | 3,000 | 3,000 | 3,000 | 2,500 | 2,500 | 2,500 | 2,500 |
| 5 | Depreciation | | | | | | | | |
| 6 | Earnings before taxes | | | | | | | | |
| 7 | Tax (34%) | | | | | | | | |
| 8 | Net operating profit after tax | | | | | | | | |
| 9 | Capital investment (no salvage value) | –10,500 | | | | | | | 0 |
| 10 | Add back depreciation | | | | | | | | |
| 11 | Free cash flow | | | | | | | | |
| 12 | | | | | | | | | |
| 13 | Discount rate | 11% | | | | | | | |
| 14 | **NPV** | | | | | | | | |

a. Assuming that the investment can be depreciated using 7-year straight-line depreciation with no salvage value, calculate the project NPV.

b. What will be the company's gain in present value if it uses a 7-year modified accelerated depreciation (MACRS) schedule, given below:

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 17 | Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 18 | MACRS depreciation | | 14.29% | 24.49% | 17.49% | 12.49% | 8.93% | 8.93% | 8.93% | 4.45% |

6. (NPV and cash flows) A company is considering buying a new machine for one of its factories. The cost of the machine is $60,000, and its expected life span is 5 years. The machine will save the cost of a worker estimated at $22,500 annually. The net book value (salvage value) of the machine at the end of year 5 is $10,000. However, the company estimates that the market value will be only $5,000. Calculate the NPV of purchasing the machine if the discount rate is 12% and the tax rate is 30%. Assume straight-line depreciation over the 5-year life of the machine.

7. (NPV and cash flows) The ABD Company is considering buying a new machine for one of its factories. The machine cost is $100,000, and its expected life span is 8 years. The machine will be depreciated on a straight-line basis to a salvage value of zero; nevertheless, the company anticipates that it can sell the machine at the end of year 8 for $15,000. The machine is expected to reduce the production costs by $25,000 annually. If the appropriate discount rate is 15% and the corporate tax is 40%:

a. Calculate the project's NPV.

b. Calculate the project's IRR.

**8.** You are the owner of a factory located in a hot tropical climate. The monthly production of the factory is $100,000 except during June–September, when it falls to $80,000 due to the heat in the factory. In January 2017, you get an offer to install an air-conditioning system in your factory. The cost of the air-conditioning system is $150,000, and its expected life span is 10 years. If you install the air-conditioning system, the production in the summer months will equal the production in the winter months. However, the cost of operating the air-conditioning system is $9,000 per month (only in the 4 months that you operate the system). You will also need to pay a maintenance fee of $5,000 annually in October. Also assume that the depreciation costs are recognized in December of each year and that taxes are paid in December of each year. The depreciation is straight-line to salvage value zero (which is also the anticipated terminal value).

- **a.** What is the December equivalent pre-tax and pre-depreciation annual profits?
- **b.** What is the NPV of purchasing the air-conditioning system if the discount rate is 12% and corporate tax rate is 35%?

**9.** (Cash-flow analysis) The "Cold and Sweet" (C&S) company manufactures ice cream bars. The company is considering the purchase of a new machine that will top the bar with high-quality chocolate. The cost of the machine is $900,000. The machine will be depreciated over 10 years to zero salvage value. However, the company intends to use the machine for only 5 years. Management thinks that the sale price of the machine at the end of 5 years will be $100,000.

The machine can produce up to 1 million ice cream bars annually. The marketing director of C&S believes that if the company will spend $30,000 on advertising in the first year and another $10,000 in each of the following years, the company will be able to sell 600,000 bars for $1.30 each. The cost of producing each bar is $0.50, and other costs related to the new products are $40,000 annually. C&S's cost of capital is 14%, and the corporate tax rate is 30%.

- **a.** What are the capital gains/losses from selling the machine after 5 years?
- **b.** What is the NPV of the project if the marketing director's projections are correct?
- **c.** What is the minimum price that the company should charge for each bar if the project is to be profitable? Assume that the price of the bar does not affect sales.
- **d.** The C&S Marketing Vice President suggested canceling the advertising campaign. In his opinion, the company sales will not be reduced significantly due to the cancellation. What is the minimum

quantity that the company needs to sell in order to be profitable if the Vice President's suggestion is accepted?

**e.** The Marketing Vice President would like some sensitivity analysis done. He asks, "What would be the NPV of the project if annual unit sales vary from 400,000, 450,000, . . ., 900,000 and if the unit price per bar varies from $1.20, $1.30, . . ., $1.70?" Show the **Data Table** that answers his question.

**10.** (Cash-flow analysis) The "Less Is More" company manufactures swimsuits. The company is considering expanding to the bathrobe market. The proposed investment plan includes:

- **Purchase of a new machine:** The cost of the machine is $150,000, and its expected life span is 5 years. The machine will be depreciated to zero salvage value, but the chief economist of the company estimates that it can be sold for $20,000 at the end of 5 years.
- **Advertising campaign:** The head of the marketing department estimates that the campaign will cost $80,000 annually.
- **Fixed costs:** Incremental fixed costs of the new department will be $40,000 annually.
- **Variable costs:** First-year variable costs are estimated at $30 per bathrobe, but due to the expected increase in labor costs, they are expected to rise at 5% per year.
- **Price per bathrobe:** Each of the bathrobes will be sold at a price of $45 at the first year. The company estimates that it can raise the price of the bathrobes by 10% in each of the following years.

The "Less Is More" discount rate is 10%, and the corporate tax rate is 36%.

**a.** What is the break-even point of the bathrobe department (what is the minimal number of units it needs to sell so the expansion is profitable)?

**b.** Plot a graph in which the NPV is the dependent variable of the annual production.

**11.** (Cash-flow analysis, replacing equipment) The "Car Clean" company operates a car wash business. The company's current cleaning machine was bought 2 years ago at the price of $60,000. The life span of this machine is 6 years, the machine has no disposal value, and the current market value of the machine is $20,000; depreciation is on a straight-line basis. The company is considering replacing the old car washing machine with a new machine. The cost of the new machine is $100,000, and its life span is 4 years. The straight-line depreciation is computed on the basis of the purchase price net of that the estimated year 4 value of $15,000. The new machine is faster than the old one; thus the company believes the annual net revenue will increase from $1 million to $1.03 million. In addition, the new machine is expected to save the company $10,000 in water and electricity costs every year.

The discount rate of "Car Clean" is 15% and the corporate tax rate is 40%. What is the NPV of replacing the old machine?

**12.** (Cash-flow analysis) A company is considering whether to buy a regular or color photocopier for the office. The cost of the regular machine is $10,000, its life span is 5 years, and the company has to pay another $1,500 annually in maintenance costs. The color photocopier's price is $30,000, its life span is also 5 years, and the annual maintenance costs are $4,500. Compared to the regular photocopier, the color photocopier is expected to increase the revenue of the office by $8,500 annually before taxes. For both machines, the depreciation is straight-line to zero salvage value, and this salvage value is also the assumed market value at the end of the machines' lives. Assuming that the company is profitable and pays 40% corporate tax, the relevant discount rate is 11%. Which photocopy machine should the firm buy?

**13.** (Cash-flow analysis, break-even points) The Coka company is a soft drink company. Until today, the company bought empty cans from an outside supplier that charges $0.20 per can. In addition, the transportation cost is $1,000 per truck that transports 10,000 cans. The Coka company is considering whether to start manufacturing cans in its plant. The cost of a can machine is $1,000,000, and its life span is 12 years. The terminal value of the machine is $160,000, but the machine will be depreciated on a straight-line basis to a salvage value of zero. Maintenance and repair costs will be $150,000 for every 3-year period and will be paid at the end of every 3 years. The additional space for the new operation will cost the company $100,000 annually. The marginal cost of producing a can in the factory is $0.17.

The cost of capital of Coka is 11% and the corporate tax rate is 40%. The machine will be straight-line depreciated over 12 years to its terminal value.

**a.** What is the minimum number of cans that the company has to sell annually in order to justify self-production of cans? Start by assuming that annual production is 3 million cans, and then use **Goal Seek** to find a break-even point.

**b.** **Advanced:** Use data tables in order to show the NPV and IRR of the project as a function of the number of cans.

**14.** (Cash-flow analysis) The ZZZ Company is considering investing in a new machine for one of its factories. The company can choose either Machine A or Machine B. The life span of each machine is 5 years, and depreciation is straight-line to zero salvage value. The widgets produced by the machines are sold for $6 each. The company has a cost of capital of 12%, and its tax rate is 35%.

| | A | B | C |
|---|---|---|---|
| 1 | **ZZZ WIDGET MACHINE** | | |
| 2 | Cost of capital | 12% | |
| 3 | Tax rate | 35% | |
| 4 | Widget sales price/unit | 6.00 | |
| 5 | | **Machine A** | **Machine B** |
| 6 | Cost | 4,000,000 | 10,000,000 |
| 7 | Annual fixed cost | 300,000 | 210,000 |
| 8 | Variable cost per widget | 1.20 | 0.80 |
| 9 | Life span (years) | 5 | 5 |
| 10 | Annual depreciation | 800,000 | 2,000,000 |

**a.** If the company manufactures 1,000,000 units per year, which machine should it buy?

**b.** Plot a graph showing the profitability of investment in each machine type depending on the annual production.

**15.** (Cash-flow analysis, replacing a machine) The Easy Sight Company manufactures sunglasses. The company has two machines, each of which produces 1,000 sunglasses per month. The book value of each of the old machines is $10,000, and their expected life span is 5 years from today. The machines are being depreciated on a straight-line basis to zero salvage value over their remaining lifetime. The company assumes it will be able to sell a machine today for $6,000. The price of a new machine is $20,000, and its expected life span is 5 years. The new machine will save the company $0.85 for every pair of sunglasses produced.

Demand for sunglasses is seasonal. During the 5 months of the summer (May–September), demand is 2,000 sunglasses per month, while during the winter months, it falls down to 1,000 per month.

Assuming that due to insurance and storage costs it is uneconomical to store sunglasses at the factory. Should Easy Sight replace its two old machines with new ones if its discount rate is 10% and its corporate tax rate is 25%? Assume that the current date is 1 January 2017. Depreciation and all other cash flows are taken at the end of December of each year.

**16.** (Cash-flow analysis) Poseidon is considering opening a shipping line from Athens to Rhodes. In order to open the shipping line, Poseidon will have to purchase two ships that cost 1,000 gold coins each. The life span of each ship is 10 years, and Poseidon estimates that each ship will earn 300 gold coins in the first year and that the earnings will increase by 5% per year. The annual costs of the shipping line are estimated at 60 gold coins annually, Poseidon's interest rate is 8%, and he is charged by Zeus a tax rate of 50%. Depreciation is not an expense for tax purposes.

**a.** Will the shipping line be profitable?

**b.** Due to Poseidon's good connections on Olympus, he can get a tax reduction. What is the maximum tax rate at which the project will be profitable?

**17.** (Cash-flow analysis) Kane Running Shoes is considering the manufacturing of a special shoe for race walking that will indicate if an athlete is running (this happens if neither of the walker's legs are touching the ground simultaneously). The chief economist of the company presented the following calculation for Smart Walking Shoes (SWS):

- **R&D:** \$200,000 annually today and in each of the next 3 years (years 0, . . ., 3).
- **Production and sales:** Can only begin in year 4, after an appropriate machine is purchased at the end of year 3.
- **Expected life span of project:** 10 years from purchase of the machine in year 4.
- **Investment in machinery:** \$250,000 (at $t = 3$); expected life span of the machine, 10 years. The machine will produce in years 4, 5, . . ., 13.
- **Expected annual sales:** 5,000 pairs of shoes at \$150 per pair.
- **Fixed costs:** \$300,000 annually.
- **Variable costs:** \$50 per pair of shoes.

Kane's discount rate is 12%, the corporate rate is 35%, and R&D expenses are tax deductible against other profits of the company. Assume that at the end of the project (i.e., after year 13), the new technology will have been superseded by other technologies and therefore have no value.

Compute the NPV and the IRR of the project.

**18.** (Cash-flow analysis) The Aphrodite Company is a manufacturer of perfume. The company is about to launch a new line of products. The marketing department has to decide whether to use an aggressive or regular campaign.

| **Aggressive campaign** | **Regular campaign** |
|---|---|
| Initial cost (production of commercial advertisement using a top model): \$400,000 | Initial cost (using a less famous model): \$150,000 |
| First month profit: \$20,000 | First month profit: \$10,000 |
| Monthly growth in profit (month 2–12): 10% | Monthly growth in profits (month 2–12): 6% |
| After 12 months the company is going to launch a new line of products and it is expected that the monthly profits from the current line would be \$20,000 forever. | Monthly profit (month 13–∞): \$20,000 |

The annual cost of capital is 7%. Calculate the NPV of each campaign, and decide which campaign the company should undertake.

**19.** (Cash-flow analysis) The Long-Life Company has a new vaccine. The company estimates that it has a 10-year monopoly for the production of the vaccine, and it is trying to estimate how many vaccines it should try to sell annually.

Machines to produce the new vaccine cost \$70 million, have a 5-year life, straight-line depreciation, and a zero salvage value. Each machine is capable of producing 75,000 doses annually. Annual fixed costs for producing the vaccine are \$50 million, and the variable costs per dose is \$1,000. The company's discount rate for this type of vaccine is 15%, and its corporate tax rate is 30%. The total market for the vaccine could be as high as 1,000,000 annually.

**a.** If the annual sales are 200,000, what will be the NPV and IRR of the product over its 10-year life?

**b.** Fill in the data table at the bottom of the template to show the relation between annual vaccine sales and the NPV.

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 31 | **Data table: sensitivity of NPV to number of vaccines sold and to discount rate** | | | | | | | | |
| 32 | ↓ Discount rate, → Vaccines sold | | | | | | | | |
| 33 | | **100,000** | **200,000** | **300,000** | **400,000** | **500,000** | **600,000** | **800,000** | **1,000,000** |
| 34 | **10%** | | | | | | | | |
| 35 | **12%** | | | | | | | | |
| 36 | **14%** | | | | | | | | |
| 37 | **16%** | | | | | | | | |
| 38 | **18%** | | | | | | | | |
| 39 | **20%** | | | | | | | | |
| 40 | **22%** | | | | | | | | |
| 41 | **24%** | | | | | | | | |
| 42 | **26%** | | | | | | | | |
| 43 | **28%** | | | | | | | | |
| 44 | **30%** | | | | | | | | |