Layered Automata

A canonical model for ω-regular languages

Antonio Casares · RPTU Kaiserslautern

Joint work with: Christof Löding and Igor Walukiewicz

GT Vérif, 7 Novembre 2025

Everybody loves automata

✦ Finite automata $\longleftrightarrow$ MSO-logic *(Büchi, Elgot, Trakhtenbrot '60)*

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for each regular language

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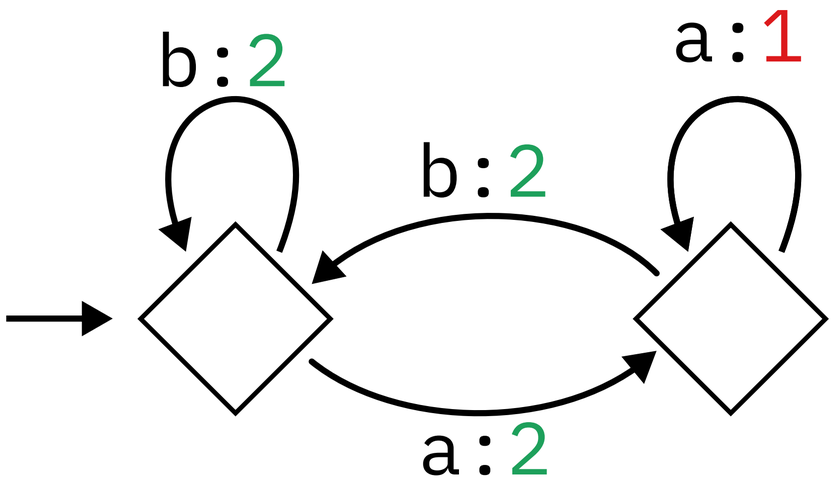
- Applications in verification and reactive synthesis

Parity automata



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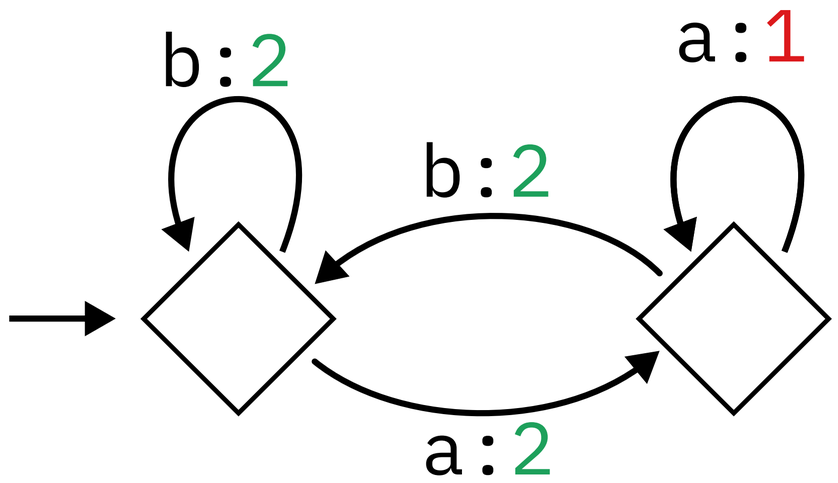

Input: $abaababbaa\ldots \in \Sigma^{\omega}$

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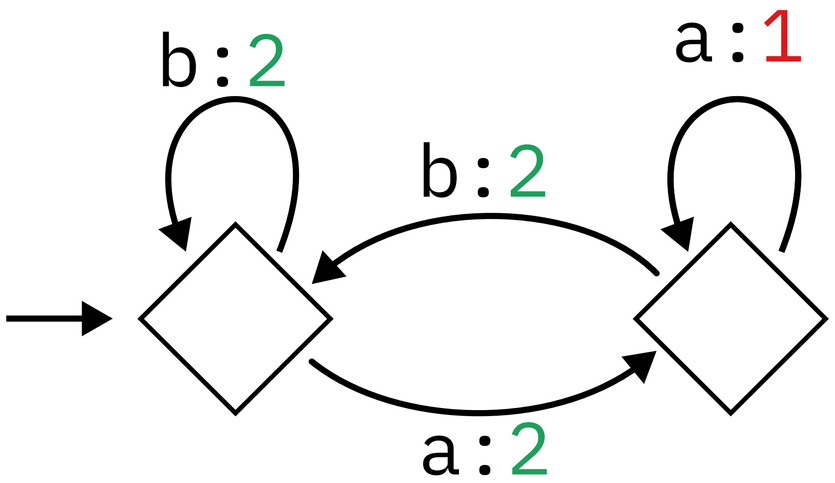

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min {*priorities seen infinitely often*}
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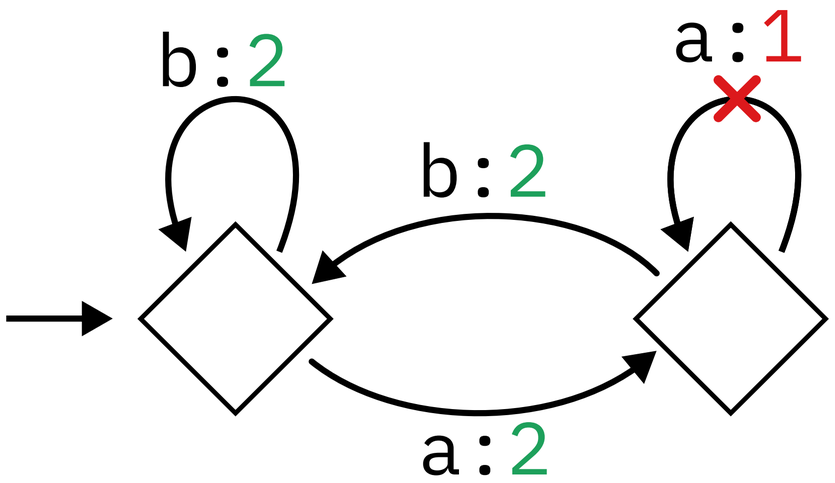

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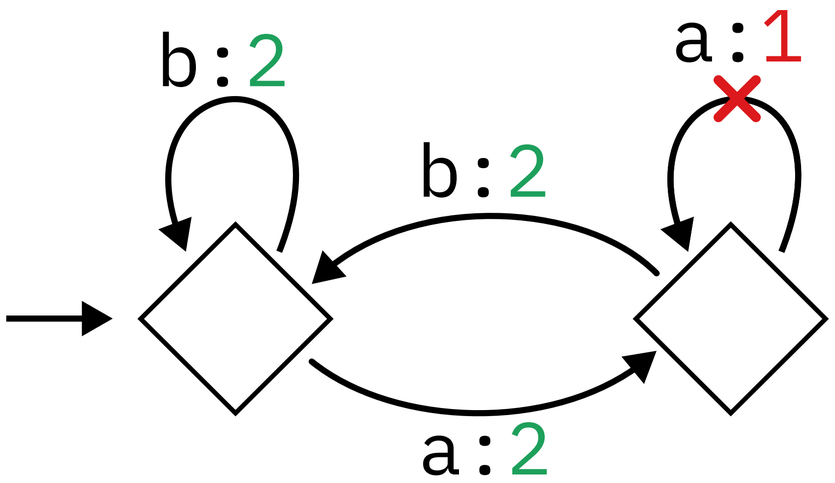

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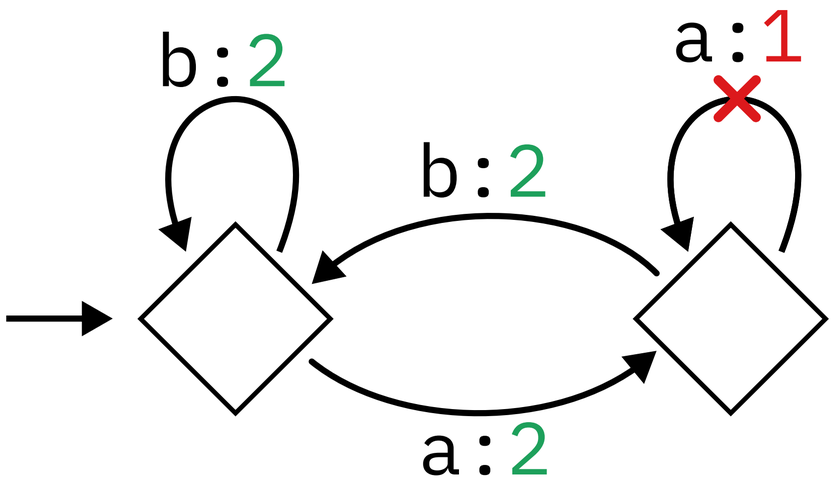
$L(A)$ = `Finitely Often` `` `aa` ``

Parity automata

Acceptance on transitions!



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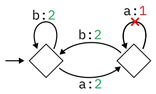

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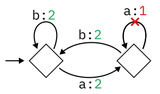

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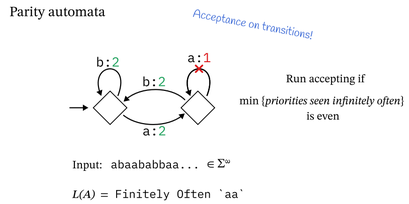

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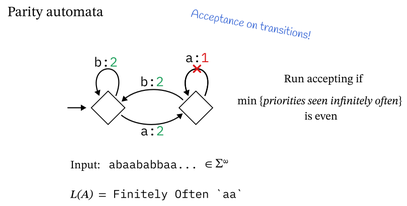

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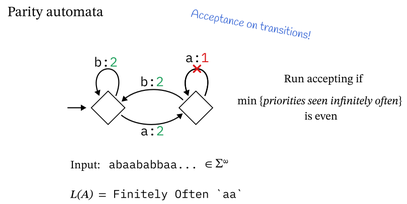

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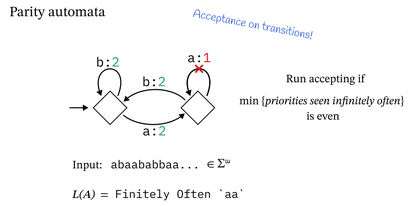

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Instead, use ***Layered Automata***

Layered Automata

Formalism to represent a subclass of alternating ω-automata

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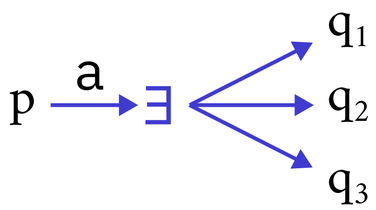
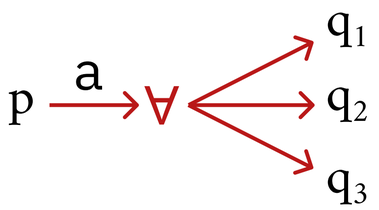
Alternating automata:



existential choices



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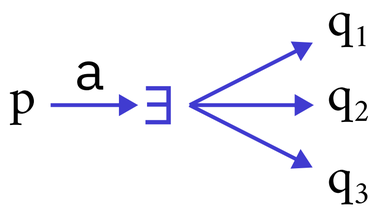

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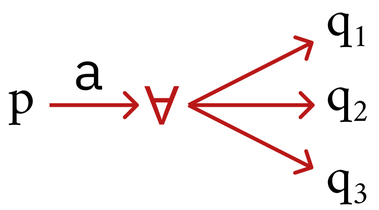

universal choices

A word w induces a parity game. $w \in L(\mathcal{A})$ if the existential player wins this game.

Layered Automata

Formalism to represent a subclass of alternating ω-automata

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Alternating automata: $p \xrightarrow{a} \exists \{q_1, q_2, q_3\}$ $\qquad$ $p \xrightarrow{a} \forall \{q_1, q_2, q_3\}$

existential choices $\qquad$ *universal choices*

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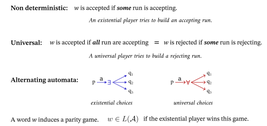
Alternating automata are too complex to use in practice 😢



Layered Automata

well-behaved

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Some subclasses are usable in verification!! 🥳

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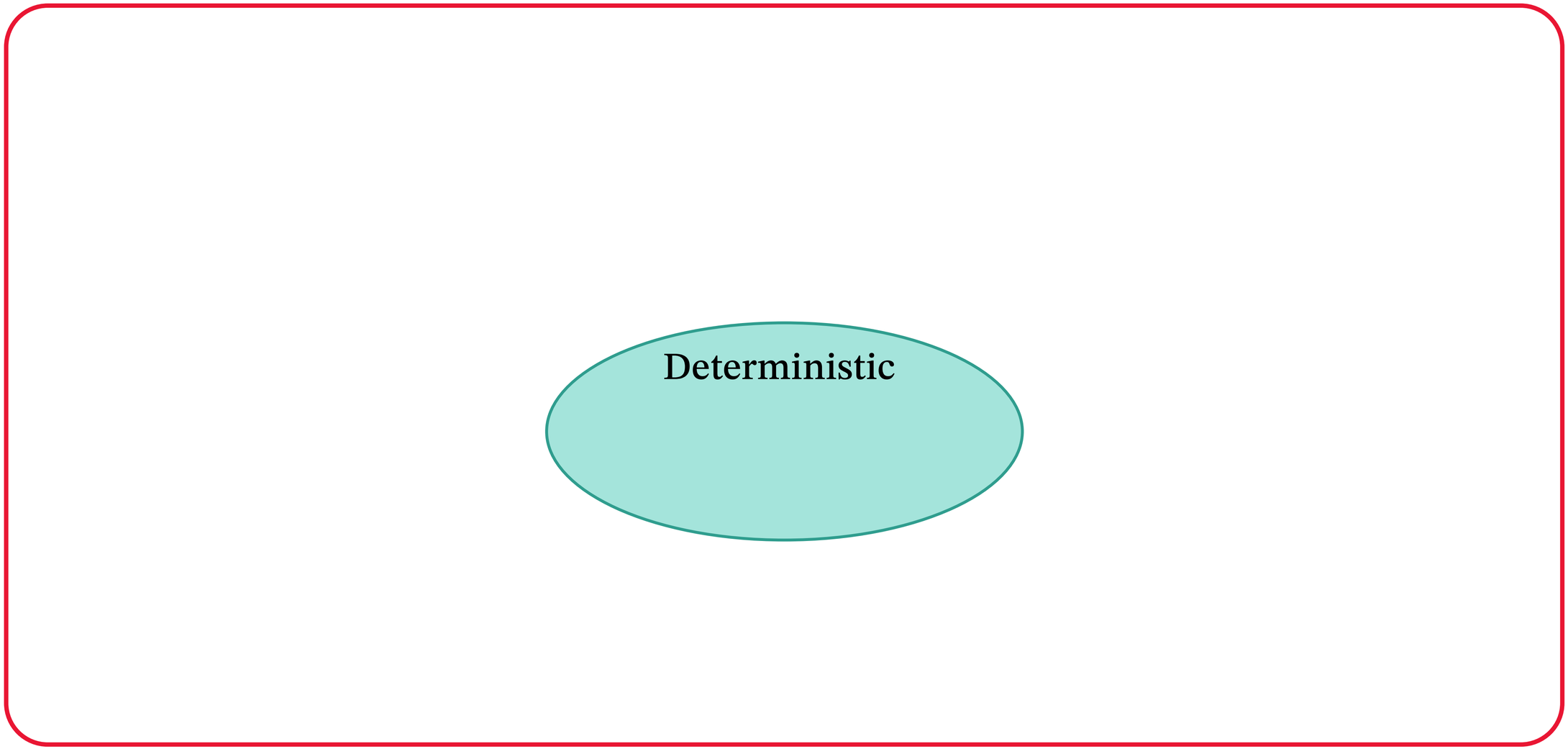

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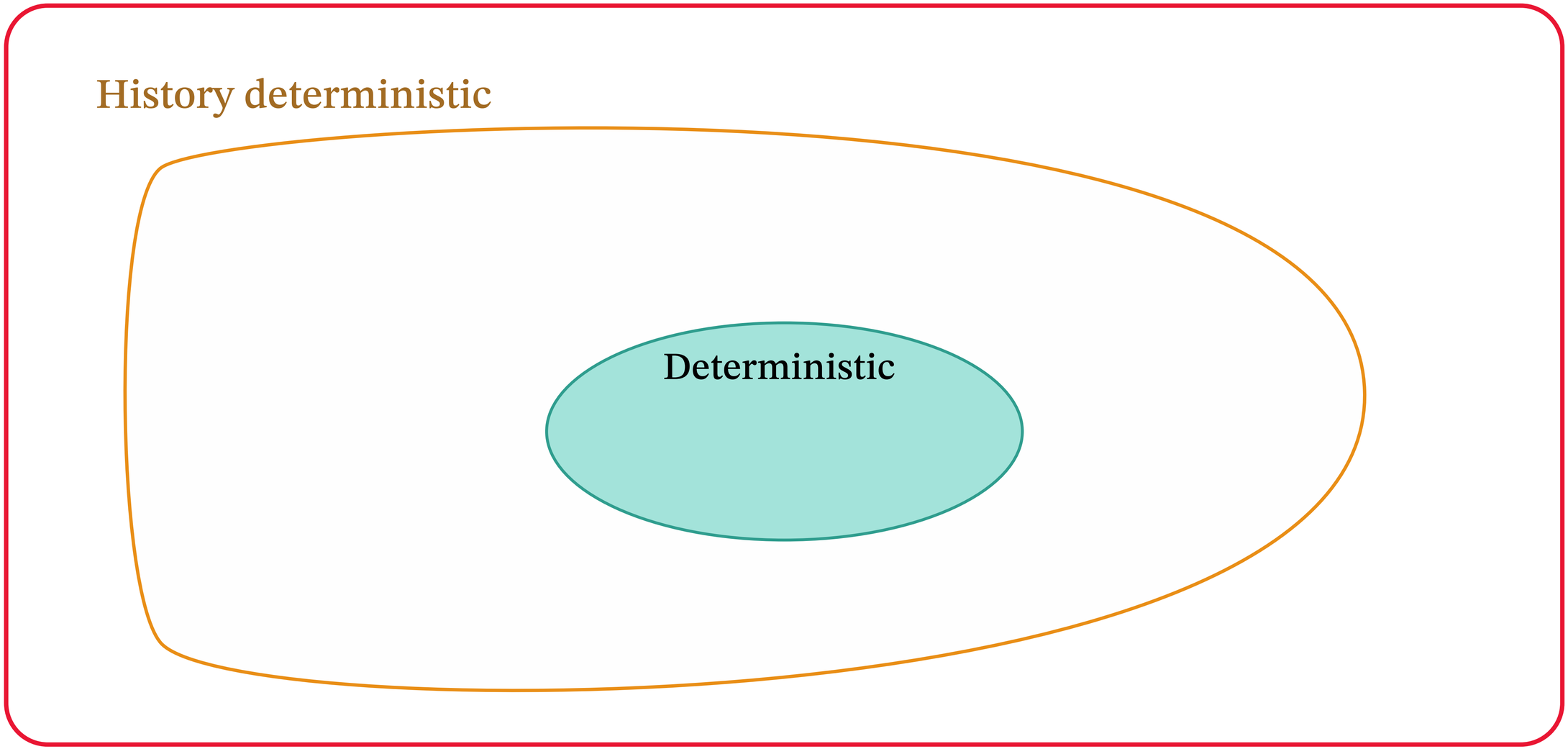

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Adam gives letters:

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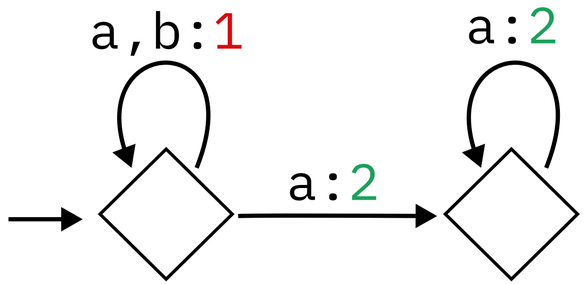

$L(A) = \texttt{Fin(b)}$

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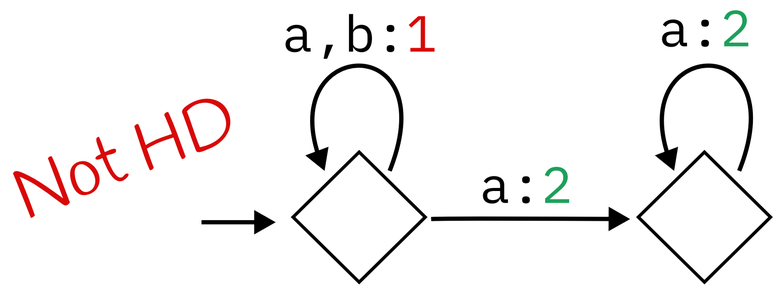

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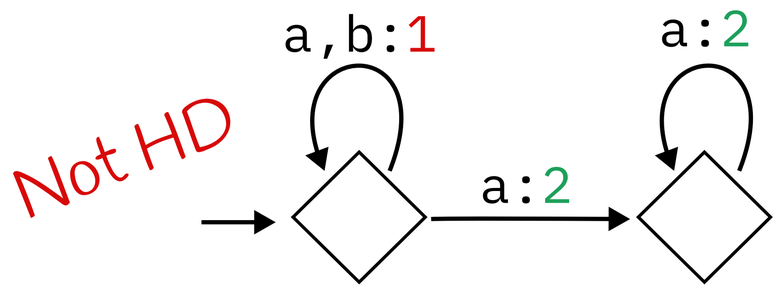

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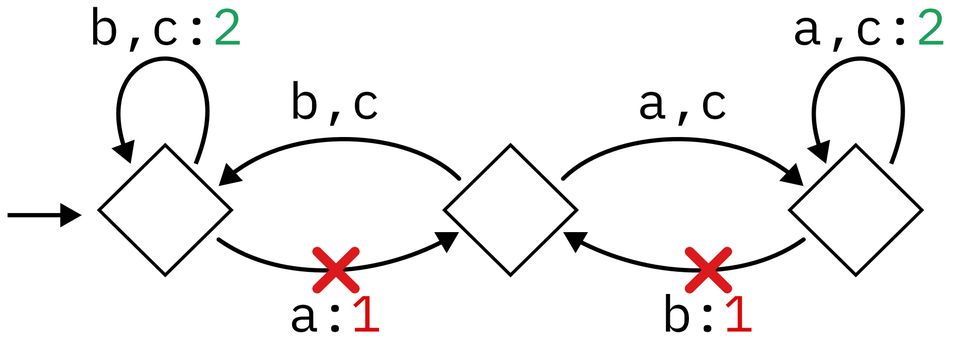

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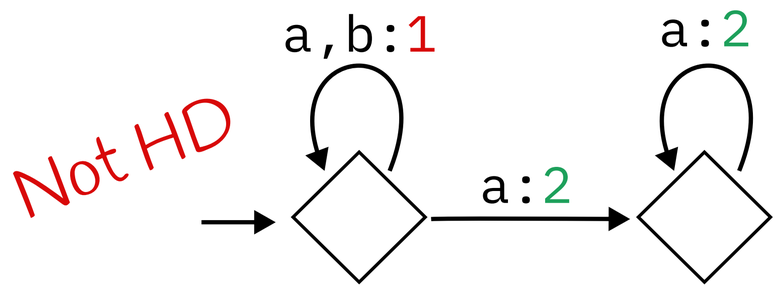

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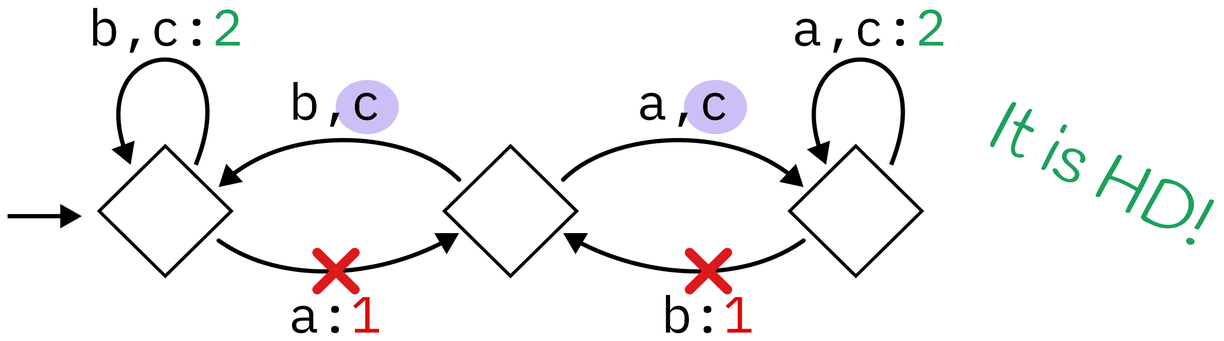

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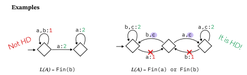


Non-deterministic case (*For alternating automata → natural generalization*)

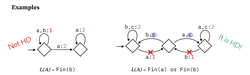
An automaton is ***history-deterministic*** if its non-determinism can be resolved on-the-fly.

= If Eve has a winning strategy in the ***letter game***:

Adam gives letters: a b b a . . . $w \in \Sigma^\omega$

Eve takes transitions: $p_0 \xrightarrow{a} p_1$ $p_1 \xrightarrow{b} p_2$ $p_2 \xrightarrow{b} p_3$ . . . ρ run in the automaton

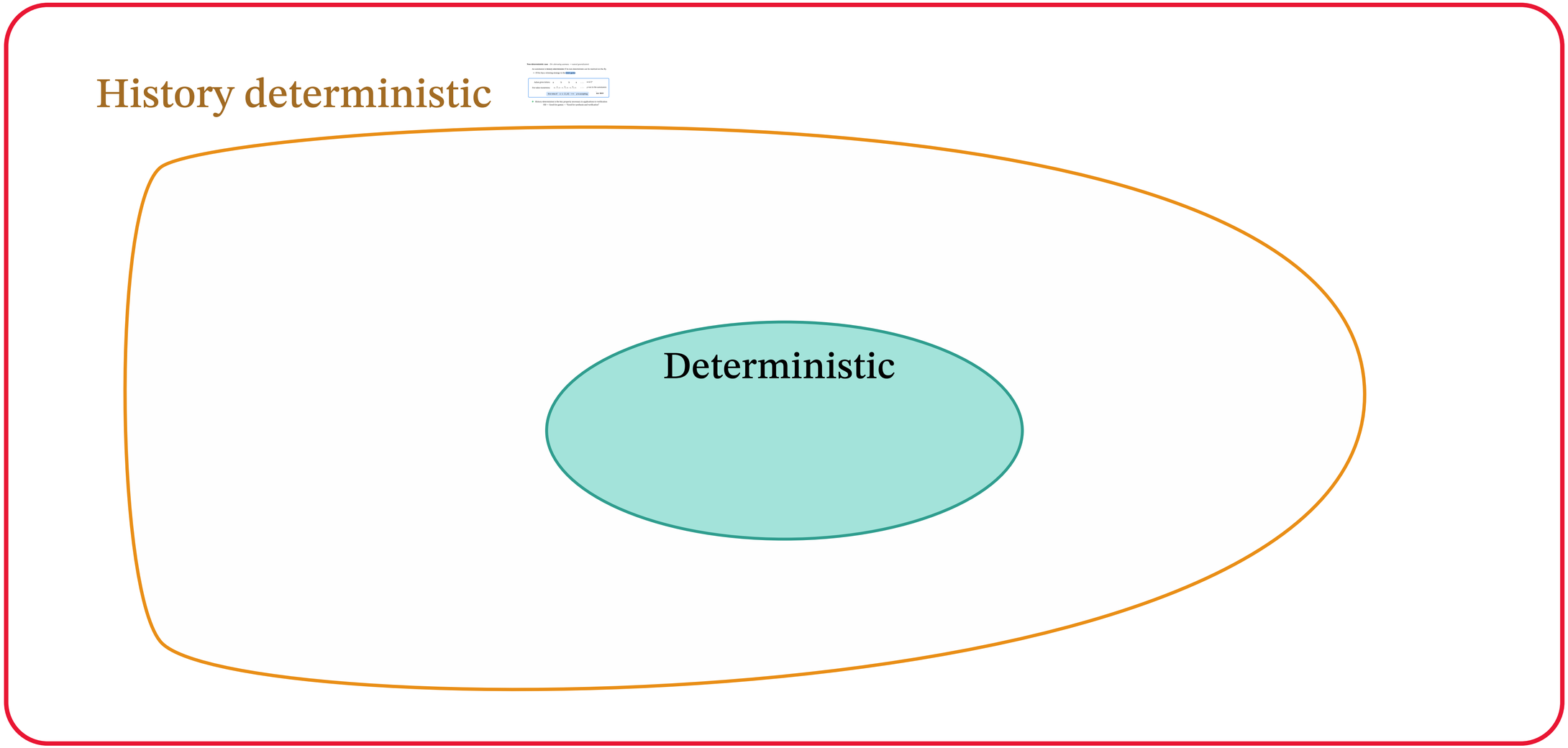
Eve wins if $w \in L(\mathcal{A}) \implies$ ρ is accepting



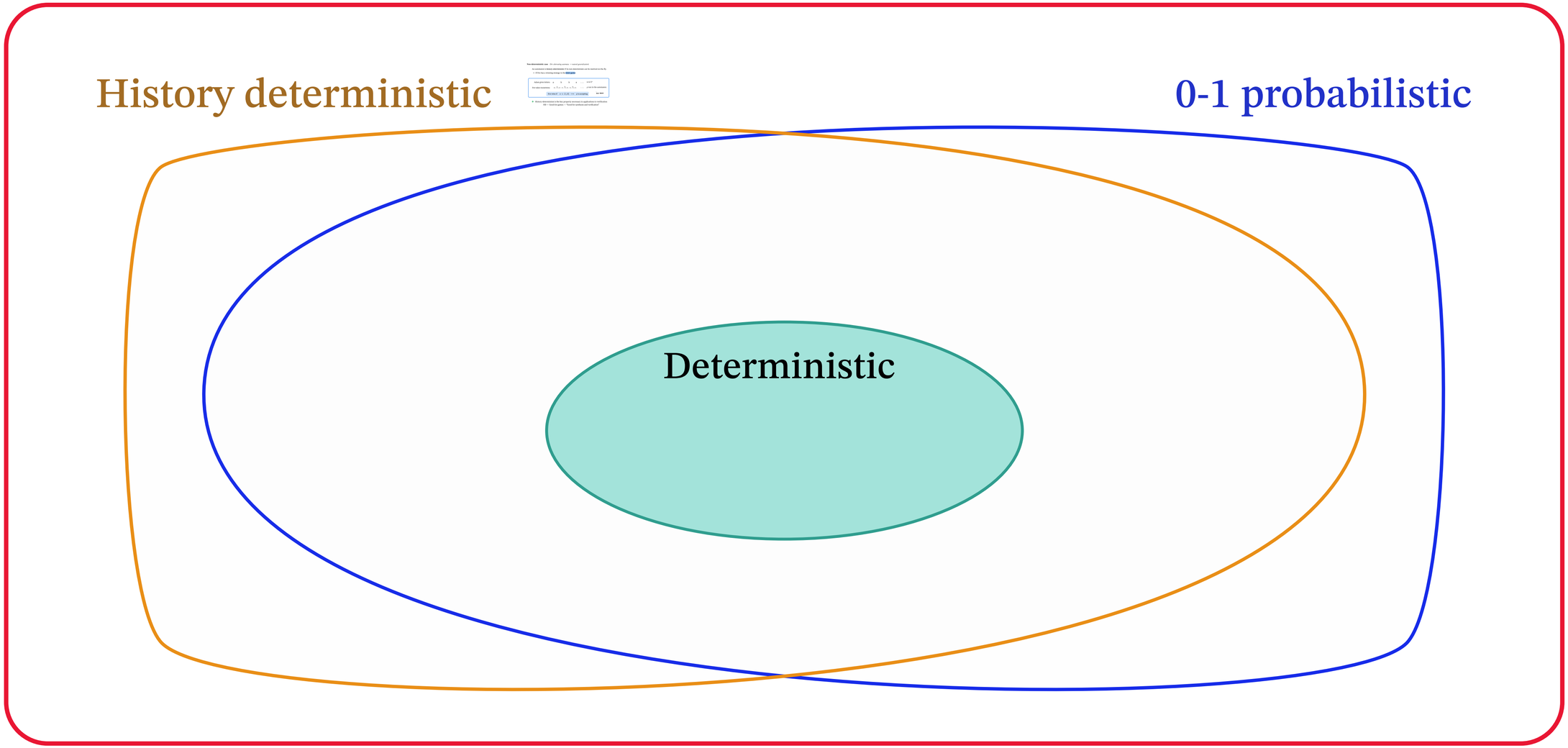
✦ History-determinism is the key property necessary in applications to verification

HD = Good-for-games = “Good for synthesis and verification”

Alternating parity automata



Alternating parity automata



An automaton is ***0-1 probabilistic*** if for every *w,* and a random run ρ over *w:*

- $Pr(\rho \text{ is accepting}) = 1 \iff w \in L(\mathcal{A})$
- $Pr(\rho \text{ is accepting}) = 0 \iff w \notin L(\mathcal{A})$

see the automaton as a graph

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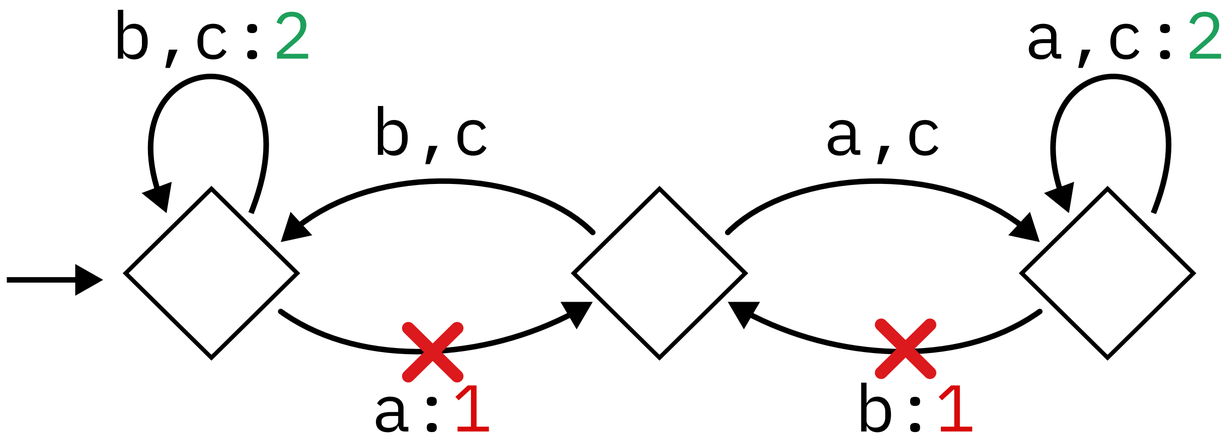
The players do not need to think!

Example:



$L(A) = \texttt{Fin(a) or Fin(b)}$

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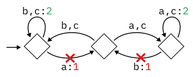

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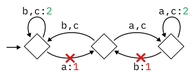
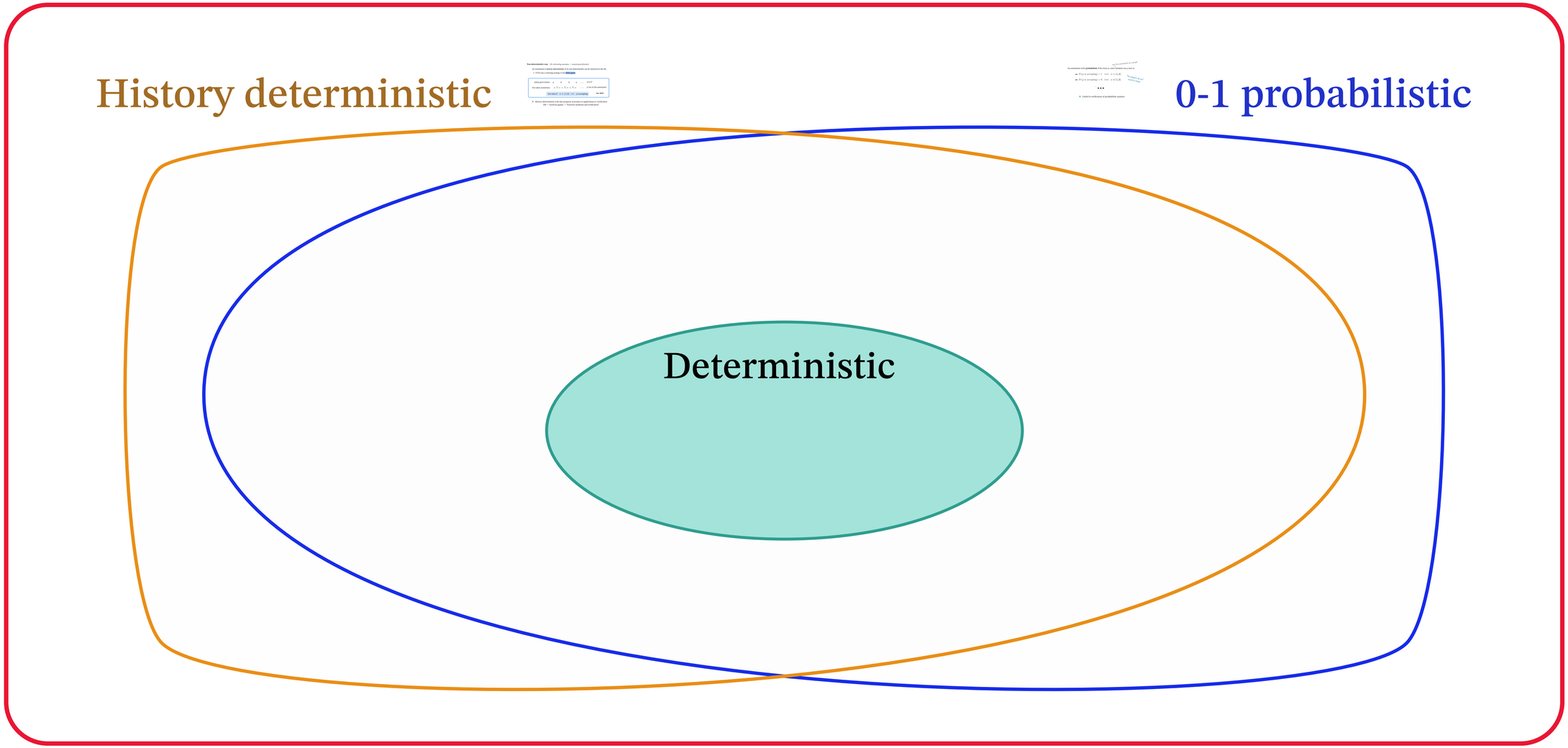


$L(A) =$ Fin(a) or Fin(b)

✦ Useful in verification of probabilistic systems

Alternating parity automata



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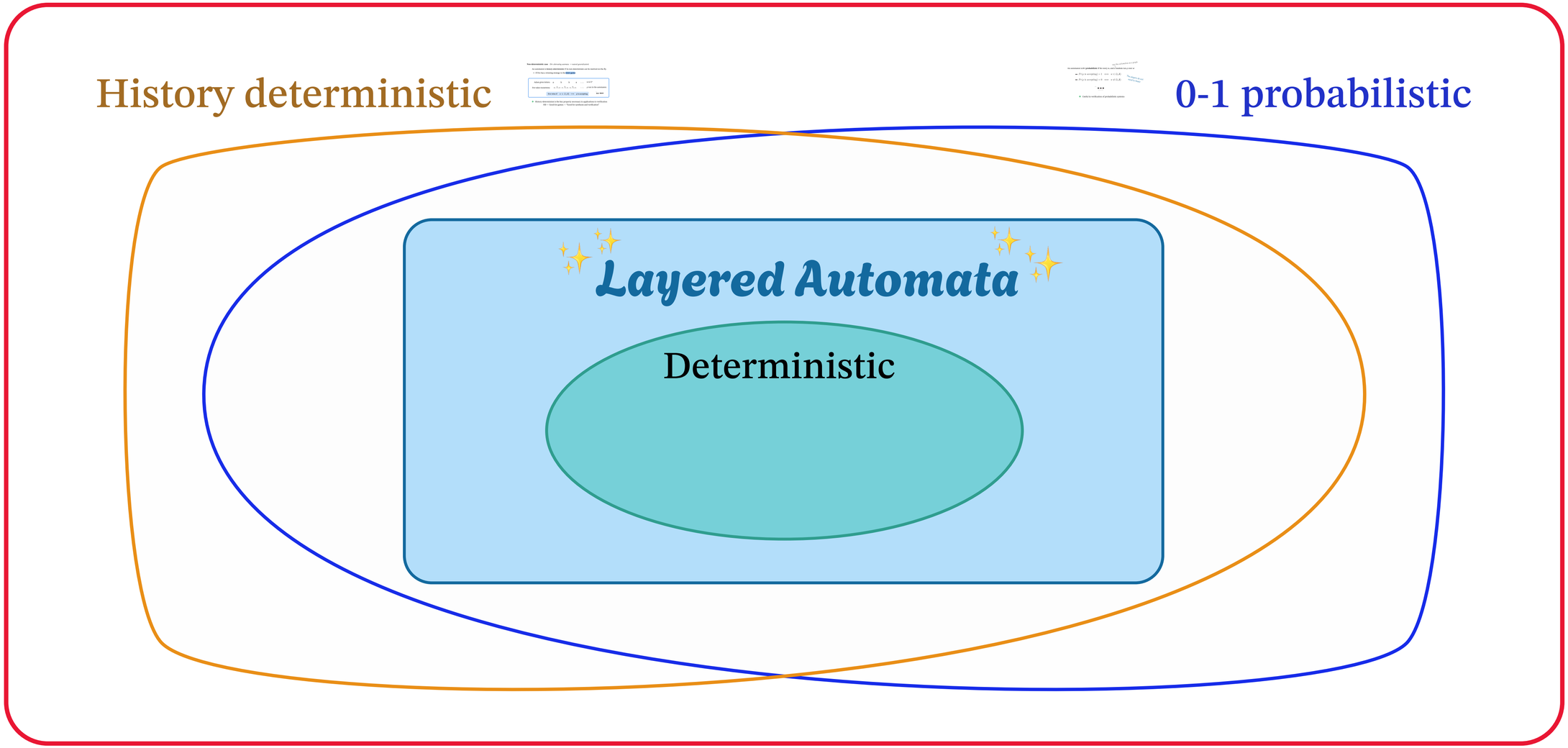
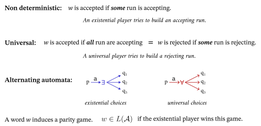


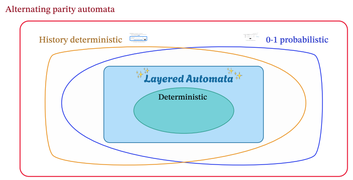
Layered Automata

well-behaved

Formalism to represent a subclass of alternating ω-automata

Some subclasses are usable in verification!!





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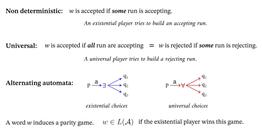

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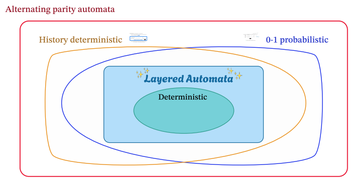

Definition

Canonicity properties

Layered automaton: A sequence of deterministic transition systems with morphisms

$$\mathcal{T}_1 \xleftarrow{\mu_1} \mathcal{T}_2 \xleftarrow{\mu_2} \quad \dots \quad \mathcal{T}_{d-1} \xleftarrow{\mu_{d-1}} \mathcal{T}_d$$

ms with morphisms

$$\mu : V \to V'$$

$$\mathrm{p} \xrightarrow{\mathtt{a}} \mathrm{q} \quad \text{in } \mathcal{T} \qquad \Longrightarrow \qquad \mu(\mathrm{p}) \xrightarrow{\mathtt{a}} \mu(\mathrm{q}) \quad \text{in } \mathcal{T}'$$

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incomplete

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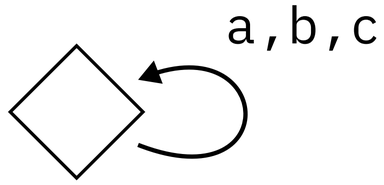
$\mathcal{T}_1$



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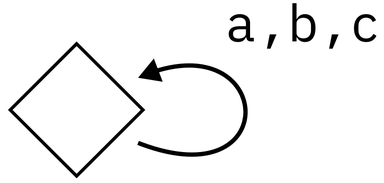
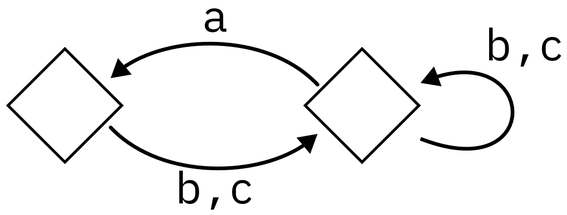


$\mathcal{T}_2$



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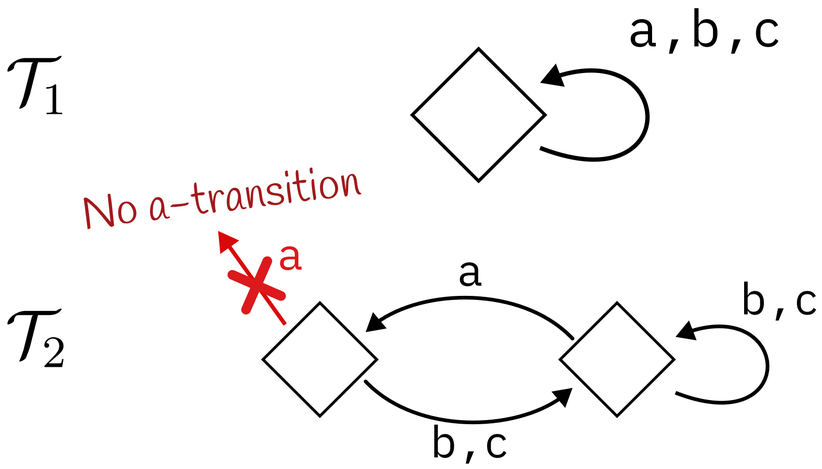

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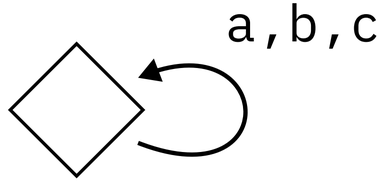

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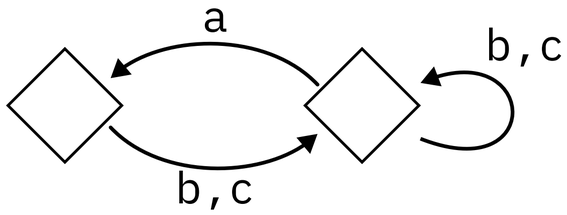

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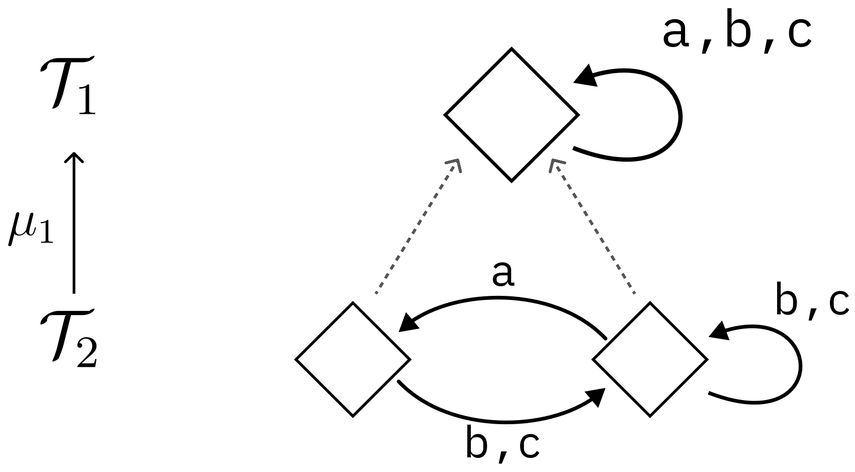

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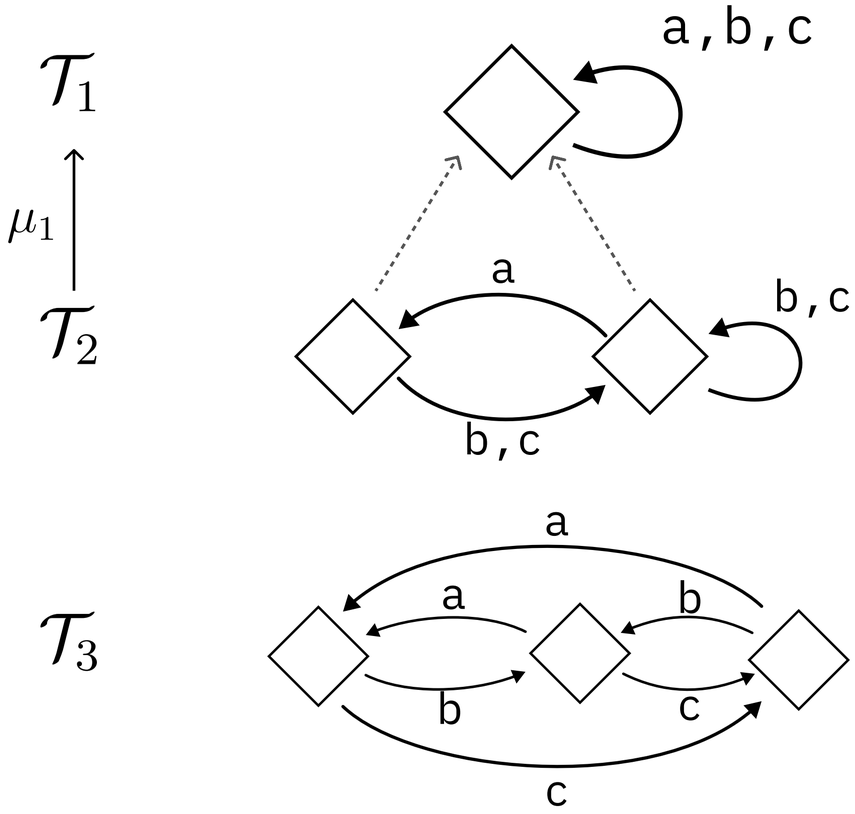

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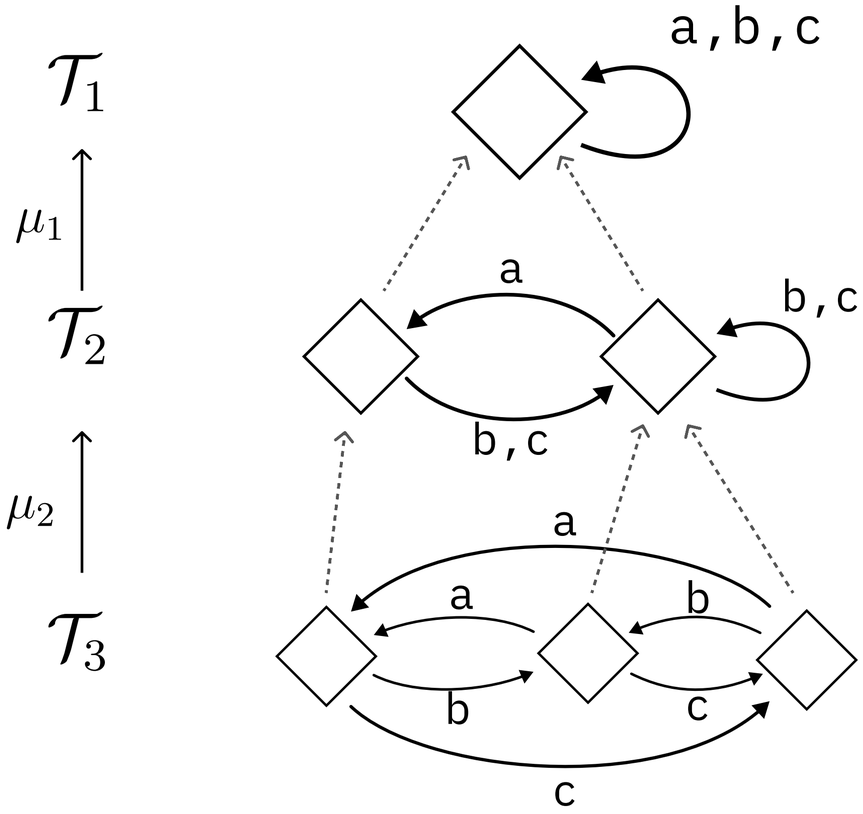
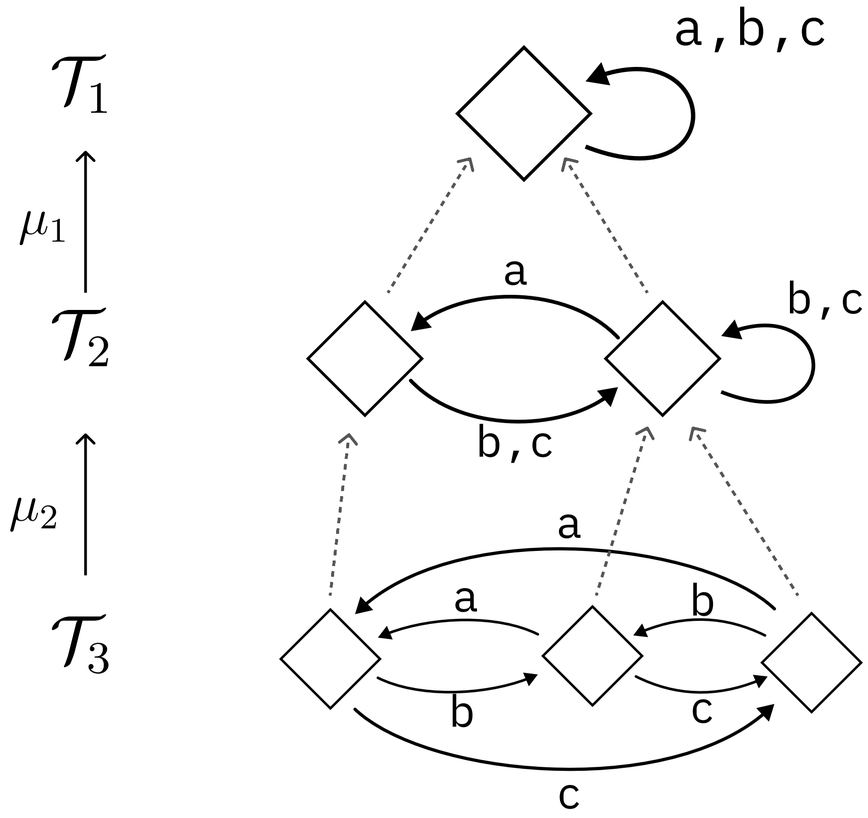
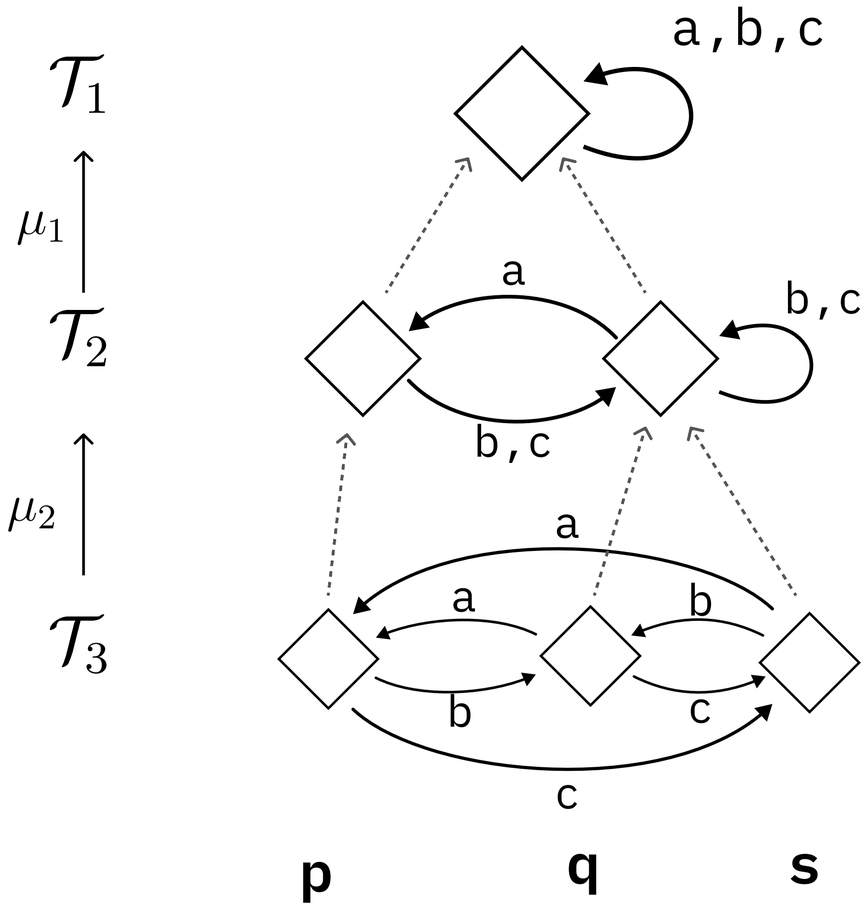


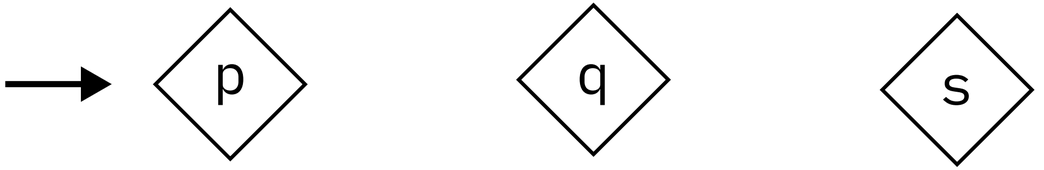
Alternating Parity Automaton

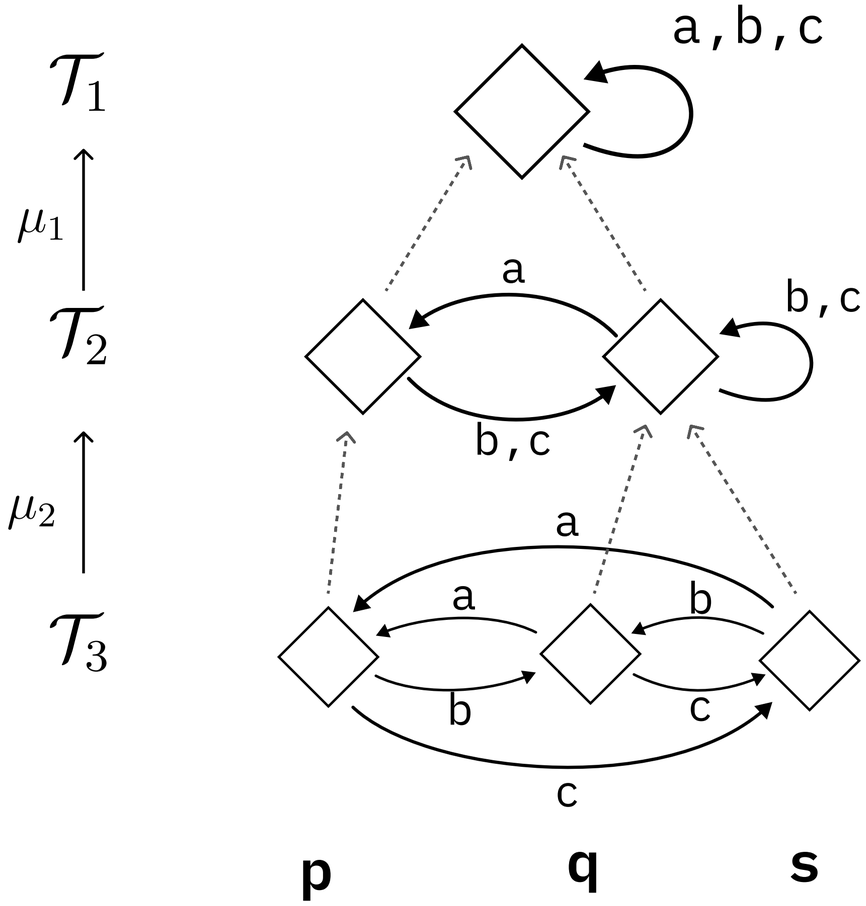




Alternating Parity Automaton

States: leaves

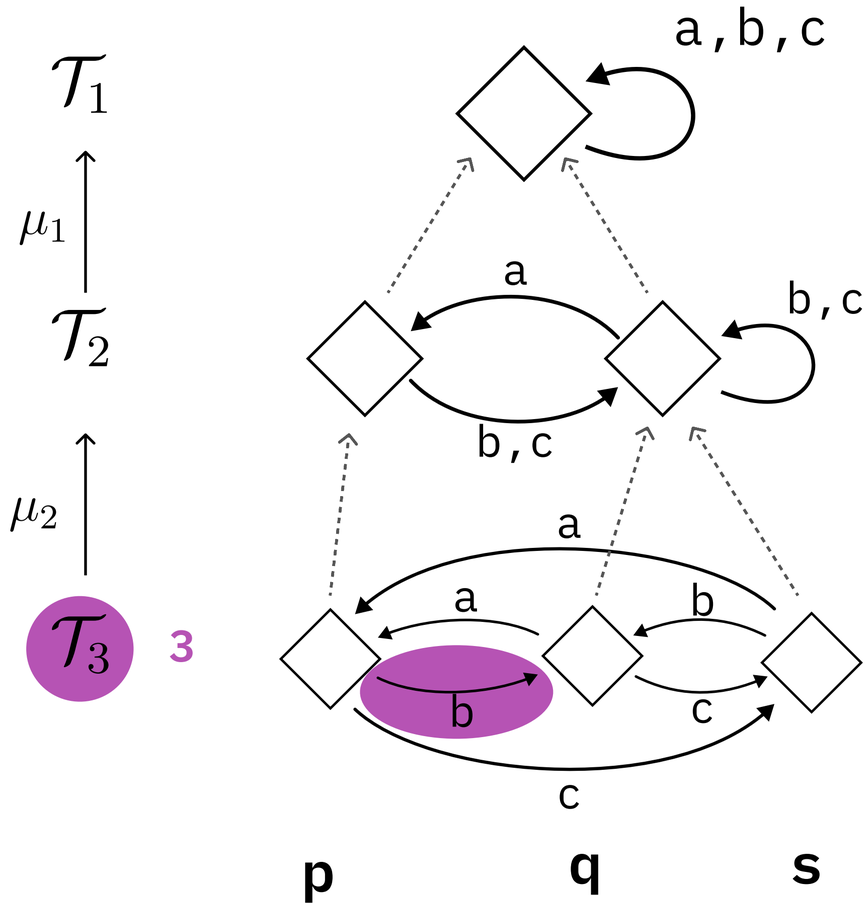




Alternating Parity Automaton

States: leaves

Transitions: given by the deepest $\mathcal{T}_i$ where the transition exists



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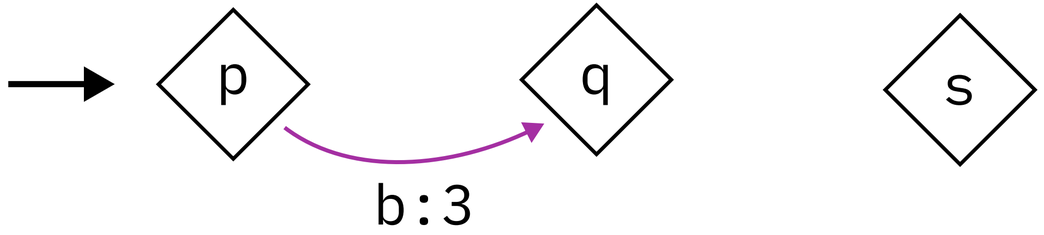

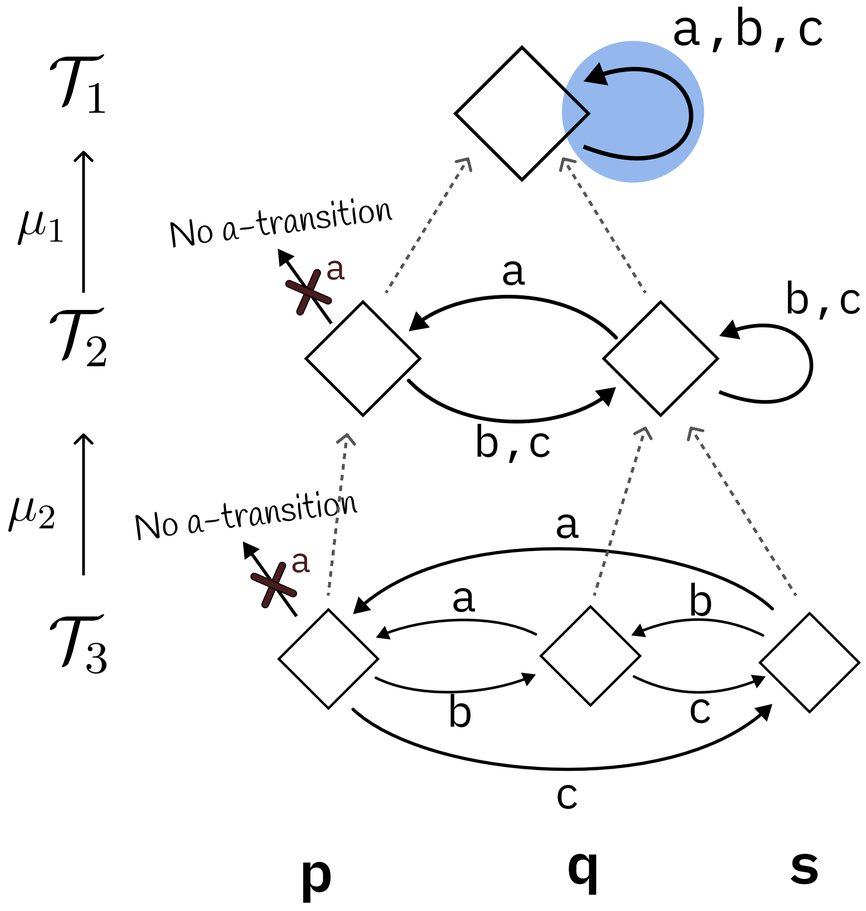

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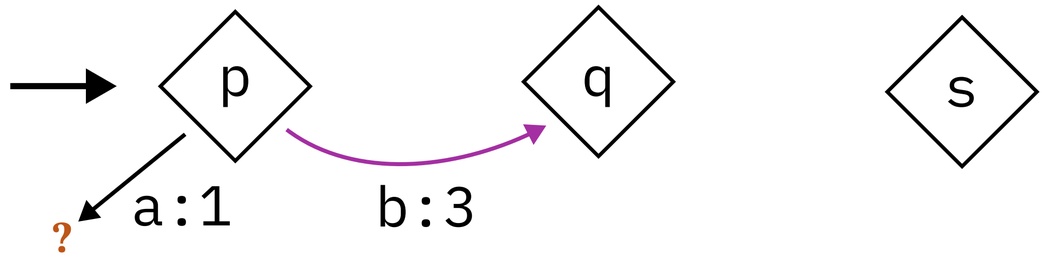

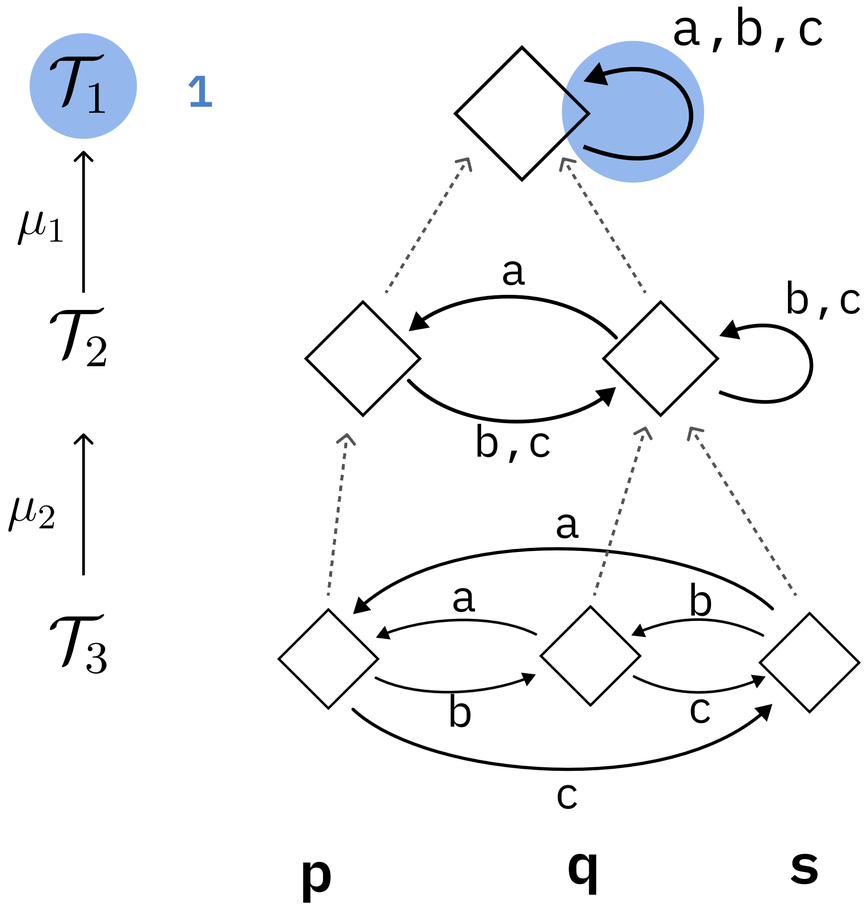

Odd → existential choice

Alternating Parity Automaton

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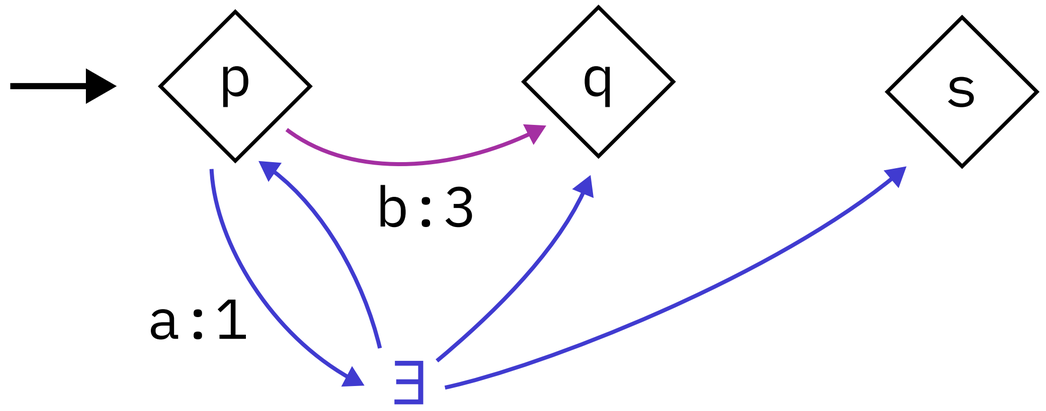

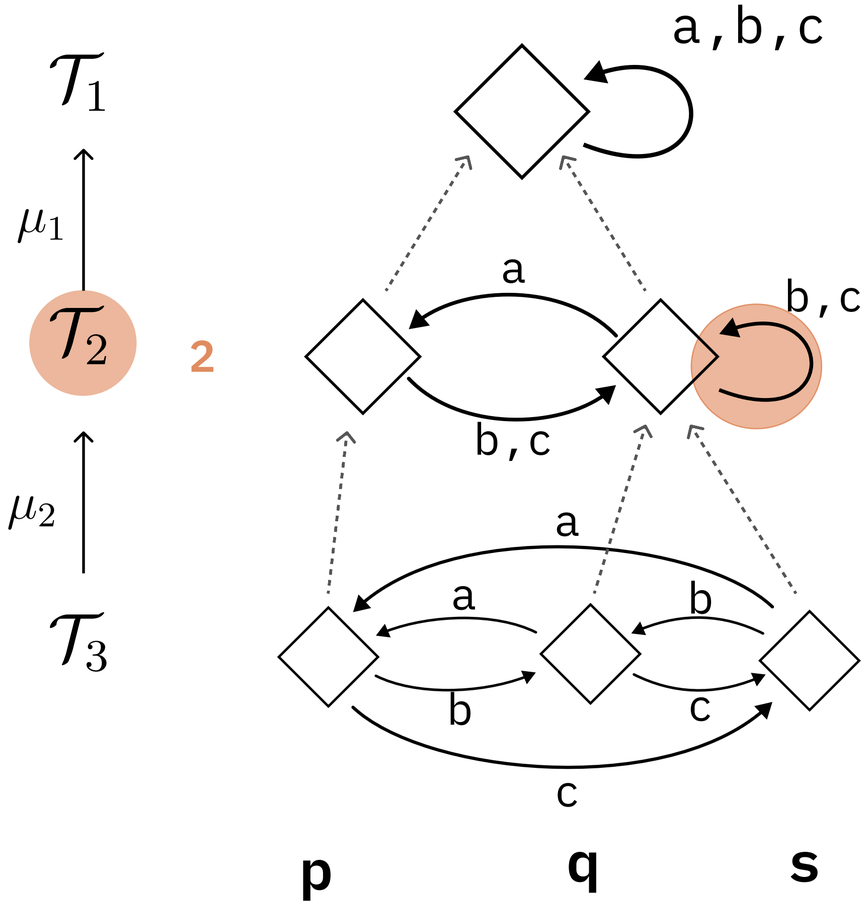

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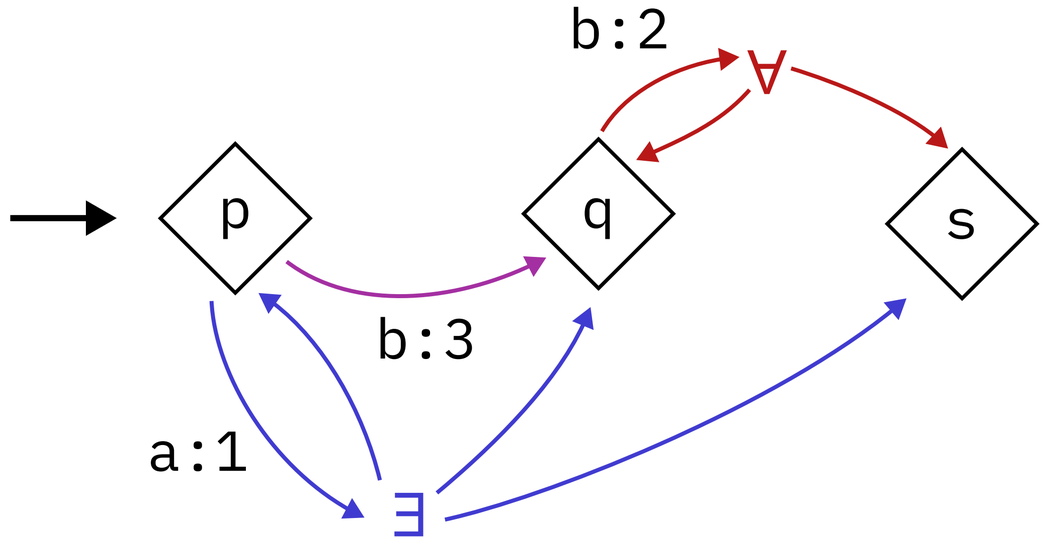
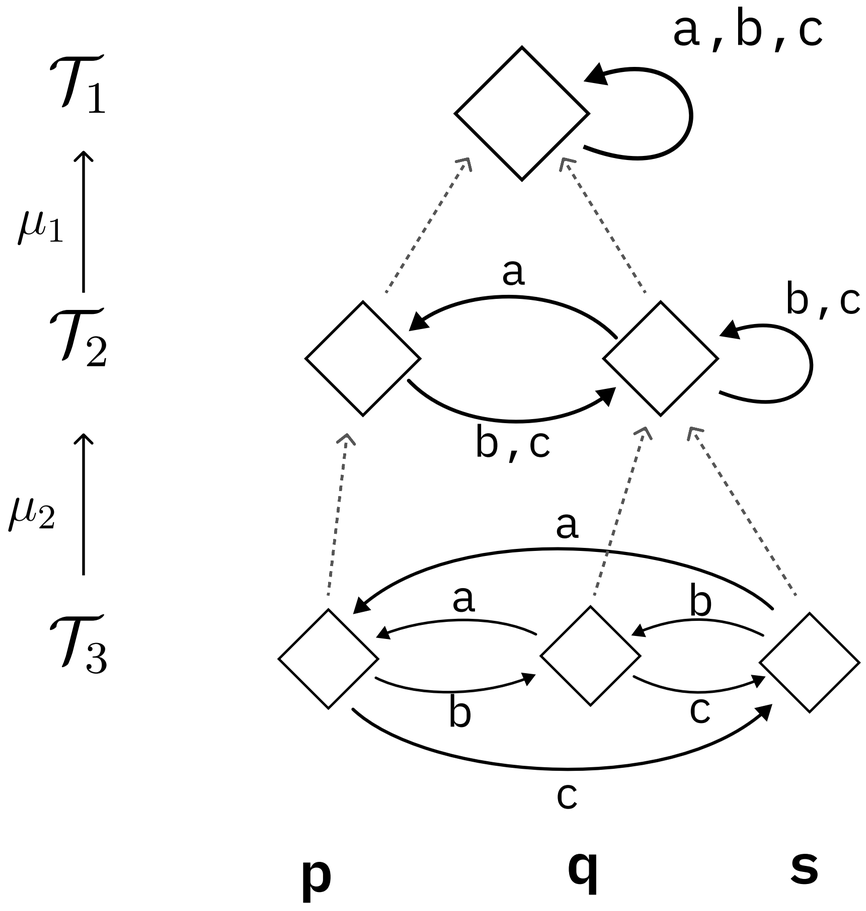


Odd → existential choice

Even → universal choice



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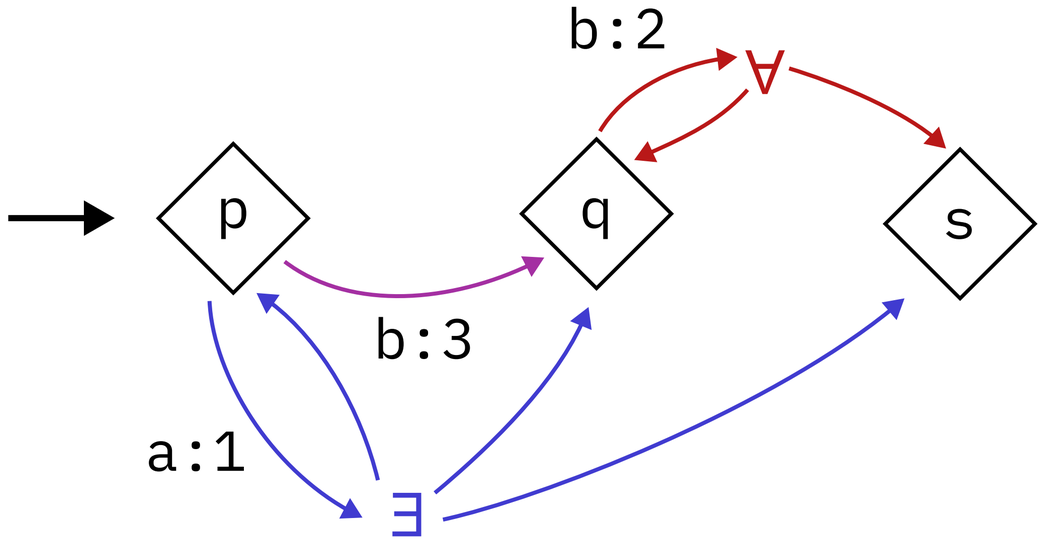

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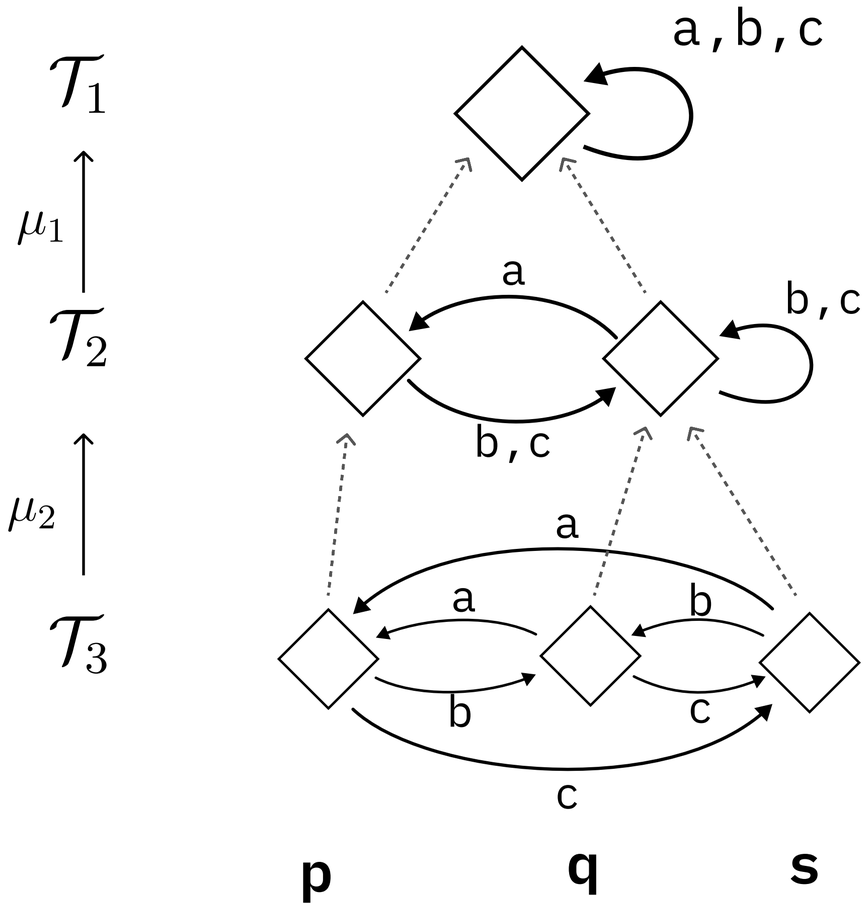

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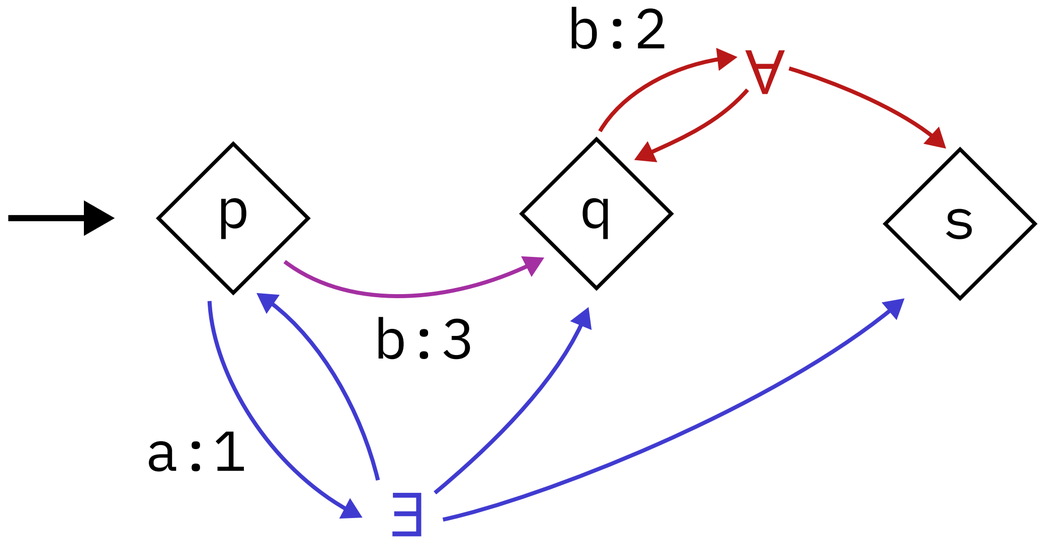
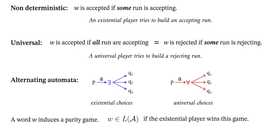


+ $\mathcal{T}_1$ must correspond to residuals

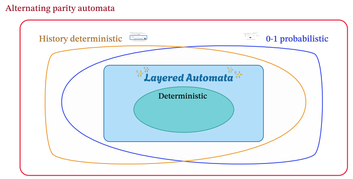
Layered Automata

well-behaved

Formalism to represent a subclass of alternating ω-automata



Some subclasses are usable in verification!!



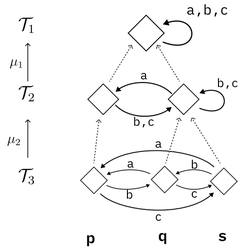
Definition

incomplete

Layered automaton: A sequence of deterministic transition systems with morphisms

$$\mathcal{T}_1 \xleftarrow{\mu_1} \mathcal{T}_2 \xleftarrow{\mu_2} \cdots \mathcal{T}_{d-1} \xleftarrow{\mu_{d-1}} \mathcal{T}_d$$

$$\mu : V \to V' \quad \text{s.t.} \quad s \xrightarrow{a} s \implies \mu s \xrightarrow{a} \mu s \text{ in } \mathcal{T}'$$



Odd → existential choice

Even → universal choice

Alternating Parity Automaton

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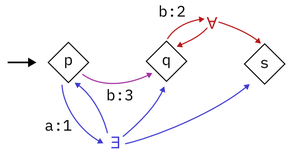

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Canonicity properties

✦ THEOREM: All layered automata are history deterministic and 0-1 probabilistic.

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Each ω-regular language admits a canonical minimal layered automaton.

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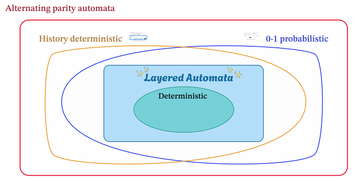
Non deterministic: w is accepted if ***some*** run is accepting.
An existential player tries to build an accepting run.

Universal: w is accepted if ***all*** run are accepting $=$ w is rejected if ***some*** run is rejecting.
A universal player tries to build a rejecting run.

Alternating automata: $p \xrightarrow{a} \{q_1, q_2, q_3\}$ $p \xrightarrow{a} \{q_1, q_2, q_3\}$
existential choices *universal choices*

A word w induces a parity game. $w \in L(\mathcal{A})$ if the existential player wins this game.

Some subclasses are usable in verification!!



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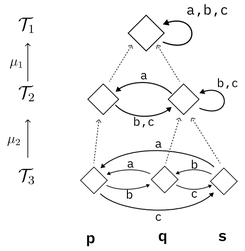

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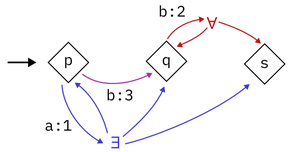

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✦ Congruence-based characterisation of the minimal layered automaton.

Everybody loves automata

- Finite automata ⟷ MSO-logic *(Büchi, Elgot, Trakhtenbrot '60)*

Beautiful mathematical theory!

A minimal, canonical DFA for each regular language

Efficiency in applications!

- Minimisation in $O(n \log n)$ *(Hopcroft '71)*
- Automata learning in PTIME *(Gold '67, Angluin 87')*
- Simple characterization of FO-definability

⋮

Let's generalize to infinite words!

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- Applications in verification and reactive synthesis

No unique minimal deterministic automata

Use of ω-automata very costly in applications!

- × Minimisation is NP-complete *(Schewe '10, AbuRadi-Ehlers '25)*
- × Costly determinization and boolean operations
- × No efficient learning of ω-automata

The theory of deterministic parity automata is not satisfactory...

Instead, use ✨ *Layered Automata* ✨

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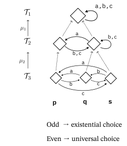

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Future work

Conjecture

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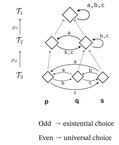

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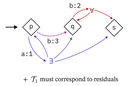

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Layered Automata

A canonical model for ω-regular languages

Antonio Casares · RPTU Kaiserslautern

Joint work with: Christof Löding and Igor Walukiewicz

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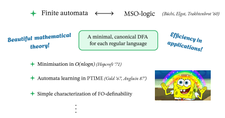

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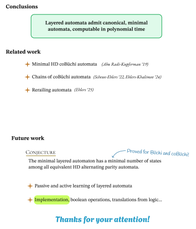

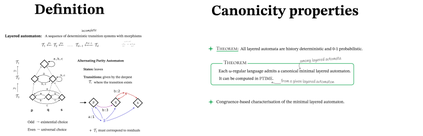

GT Vérif, 7 Novembre 2025