Layered Automata

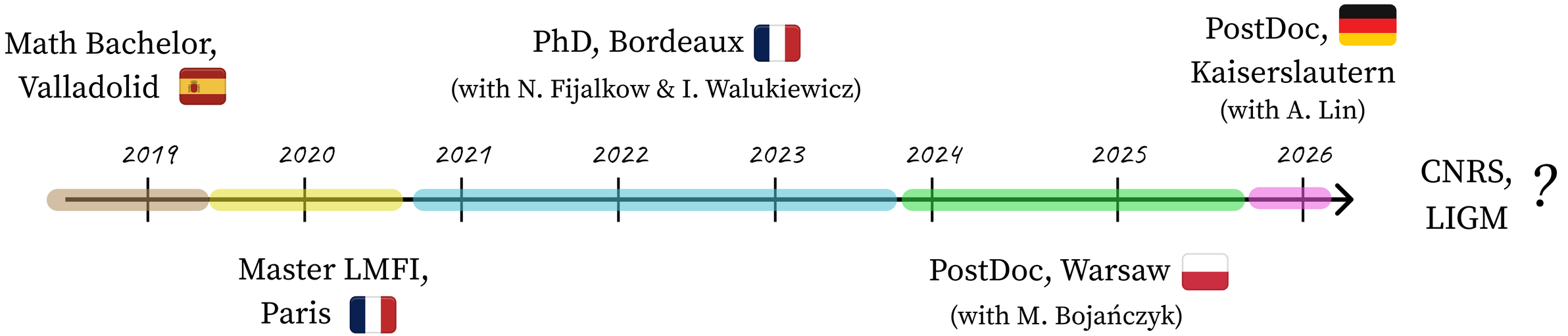
A canonical model for ω-regular languages

Antonio Casares · RPTU Kaiserslautern

Joint work with: Christof Löding and Igor Walukiewicz

Séminaire BAAM, 4 November 2025

Who am I?


Math Bachelor,
Valladolid
PhD, Bordeaux
(with N. Fijalkow & I. Walukiewicz)
PostDoc,
Kaiserslautern
(with A. Lin)
2019
2020
2021
2022
2023
2024
2025
2026
CNRS,
LIGM
?
Master LMFI,
Paris
PostDoc, Warsaw
(with M. Bojańczyk)

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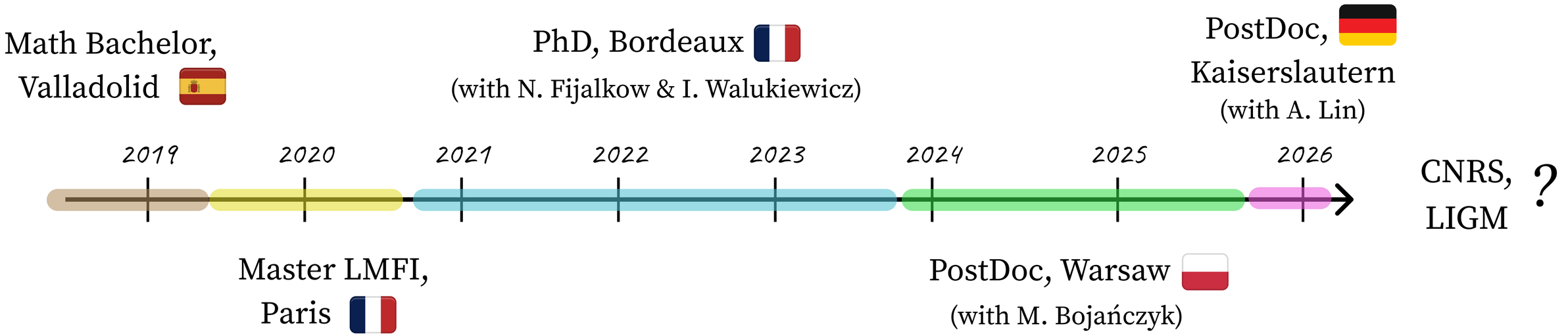

Courses in
- Model theory
- Descriptive complexity

ω-automata

Games on graphs

Logic

Transducers

Finite model theory

Graph databases

Arithmetic theories (?)

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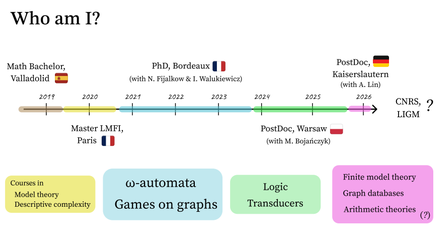

Séminaire BAAM, 4 November 2025

Everybody loves automata

✦ Finite automata $\longleftrightarrow$ MSO-logic *(Büchi, Elgot, Trakhtenbrot '60)*

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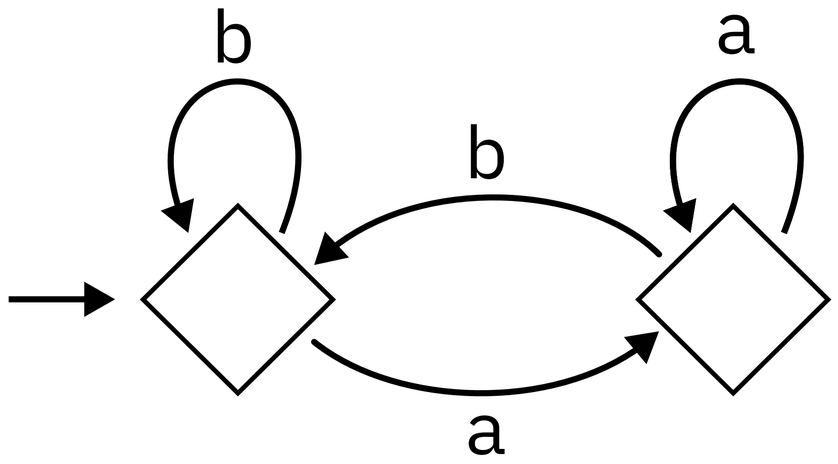

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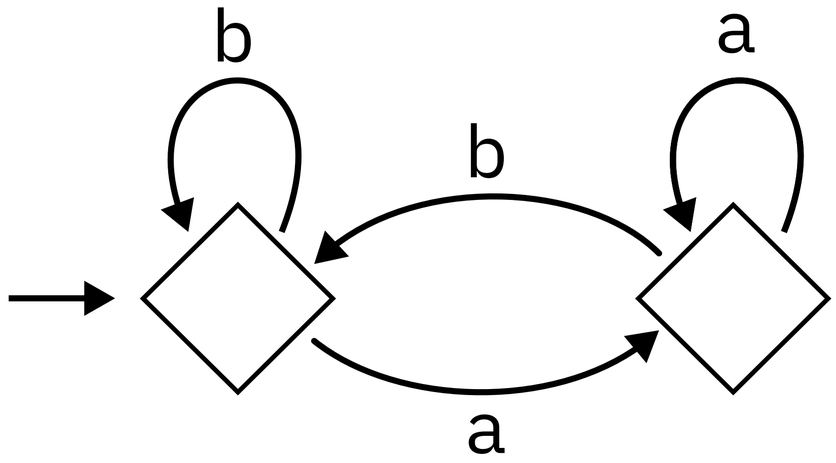
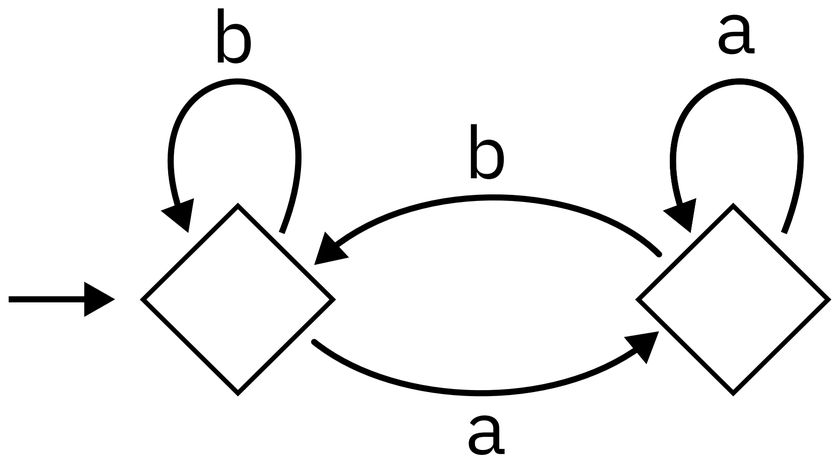


Input: abaababbaa... $\in \Sigma^{\omega}$

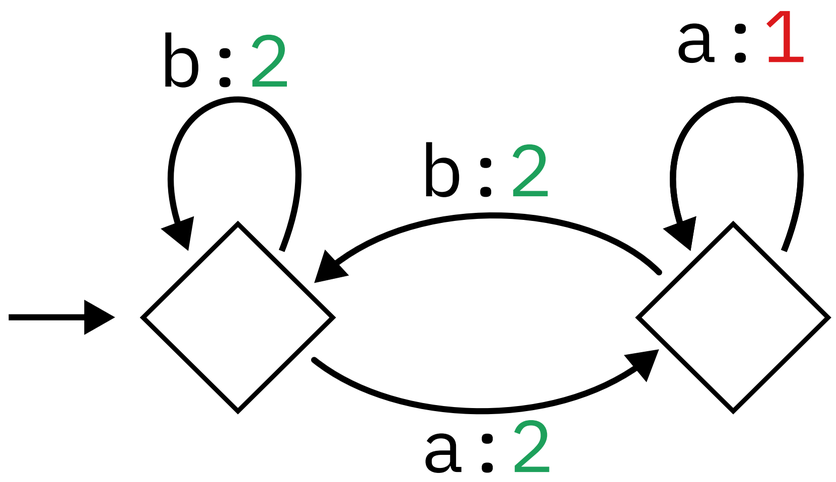
ω-automata



Acceptance?

Input: abaababbbaa... $\in \Sigma^{\omega}$

Parity automata



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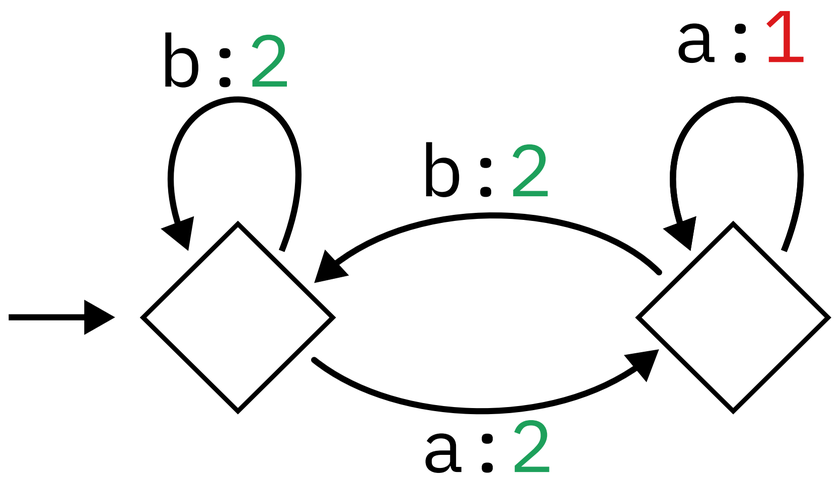

Run accepting if

min {*priorities seen infinitely often*}
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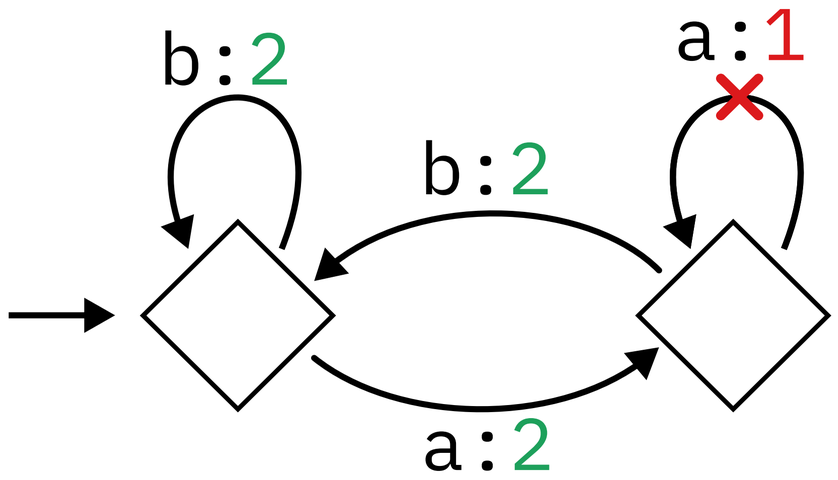

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$L(A)$ = `Finitely Often` `` `aa` ``

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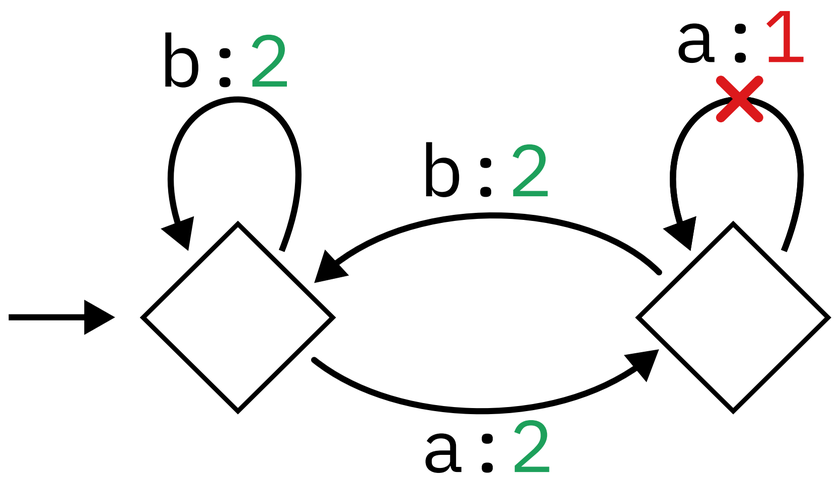
Acceptance on transitions!



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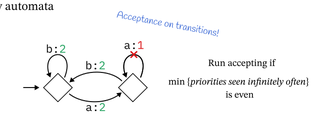

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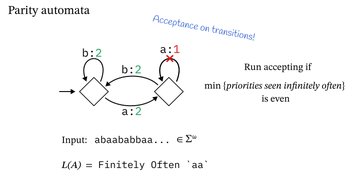

No unique minimal deterministic automata

Myhill-Nerode equivalence relation:

$$u \sim v \quad \text{if} \quad \text{for all } w \in \Sigma^{\omega}, \quad uw \in L \iff vw \in L$$

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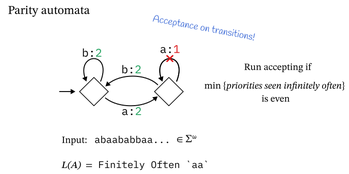

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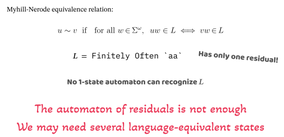

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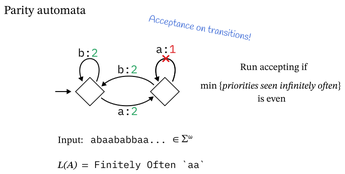

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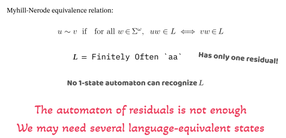

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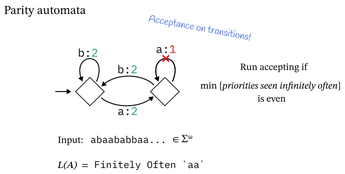

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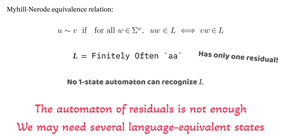

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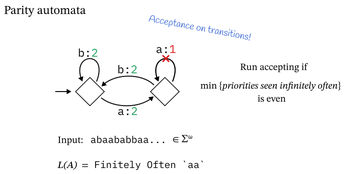

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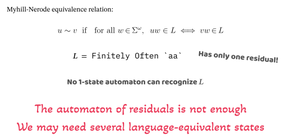

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Minimal HD coBüchi automata

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THEOREM *(Abu Radi-Kupferman '19)*

History-deterministic (HD) coBüchi automata admit minimal, canonical automata, constructible in PTIME.

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Using priorities [1,2]

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Adam gives letters:

Eve takes transitions:

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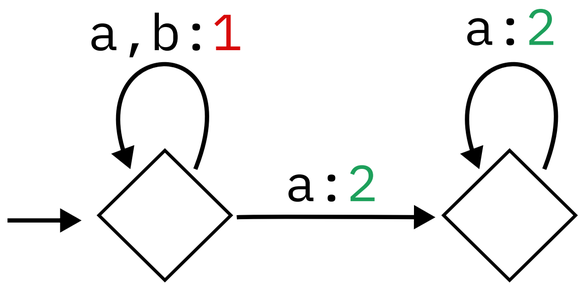
Eve wins if $w \in L(\mathcal{A}) \implies \rho$ is accepting

Examples



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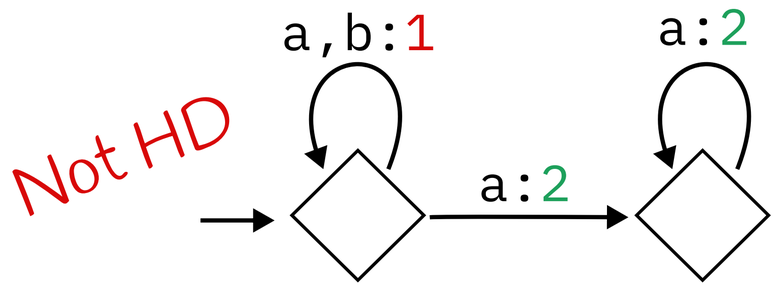

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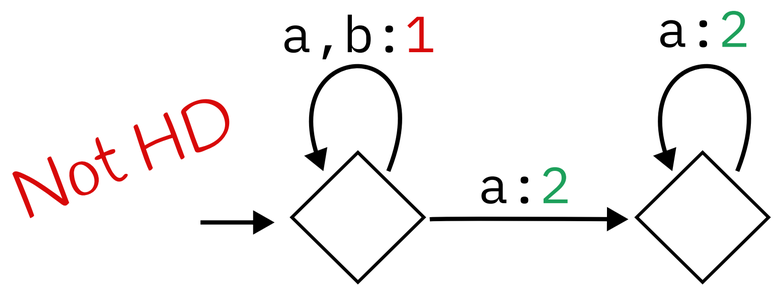

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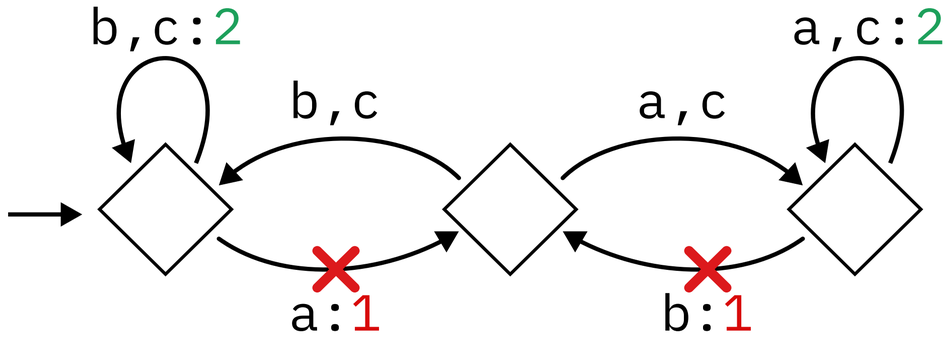

$L(A)$ = Fin(a) or Fin(b)

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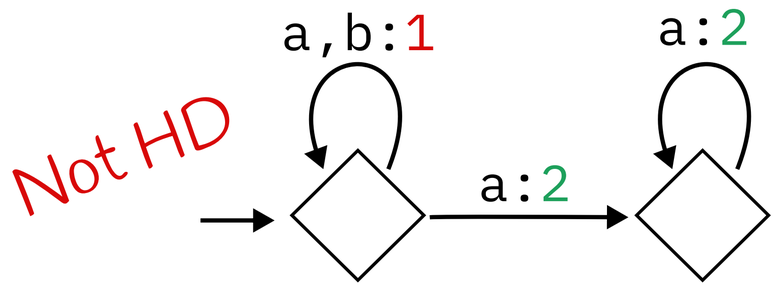

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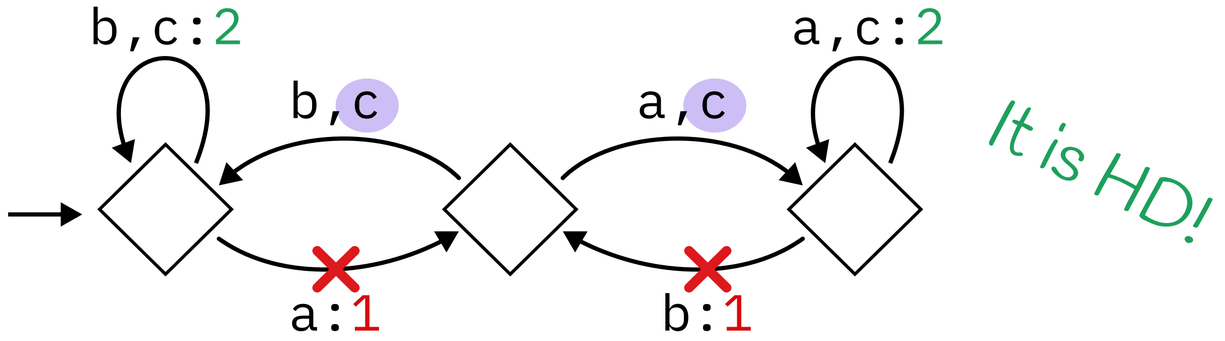

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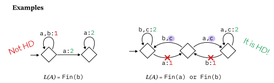

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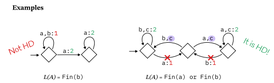

✦ History-determinism is the key property necessary in applications to verification

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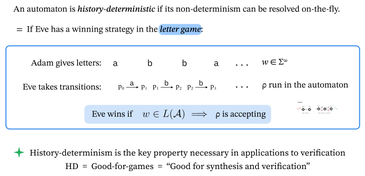
Using priorities [1,2]

History-deterministic *(HD)* *coBüchi* automata admit minimal, canonical automata, constructible in PTIME.



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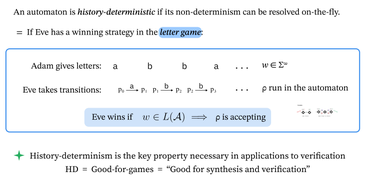

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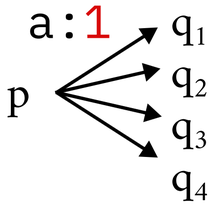
B. If $p \xrightarrow{a:1} q$ (with a crossed-out arrow), then also 

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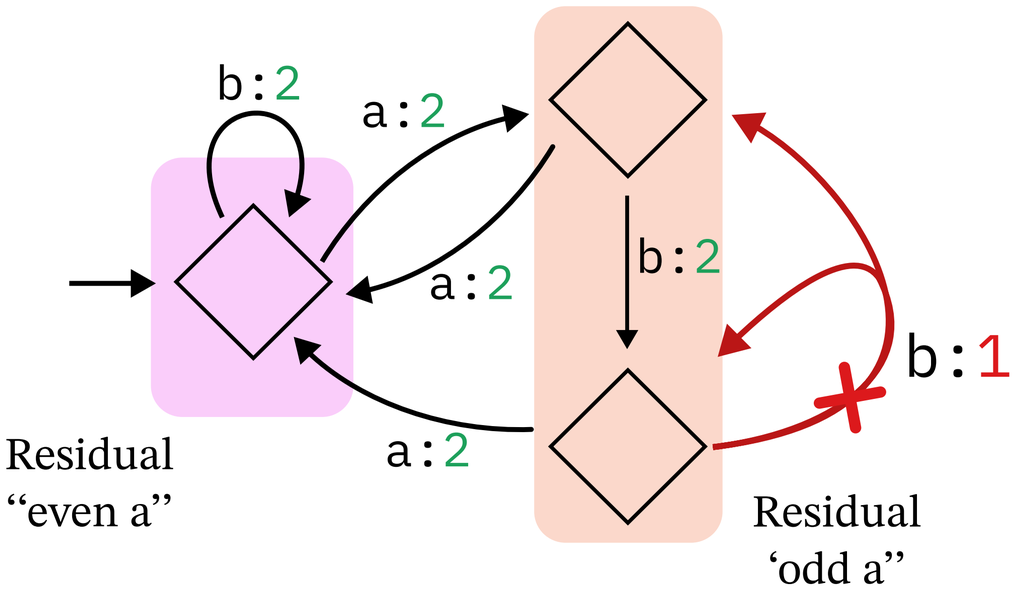

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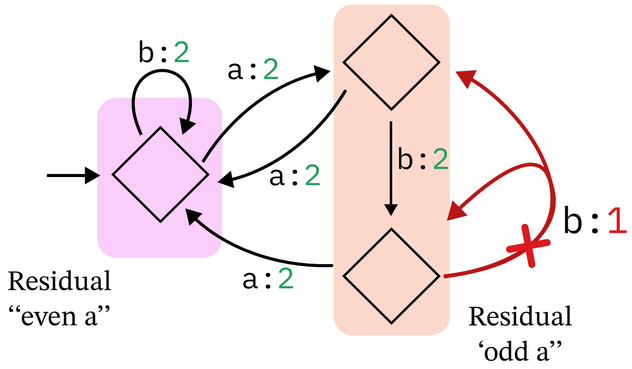

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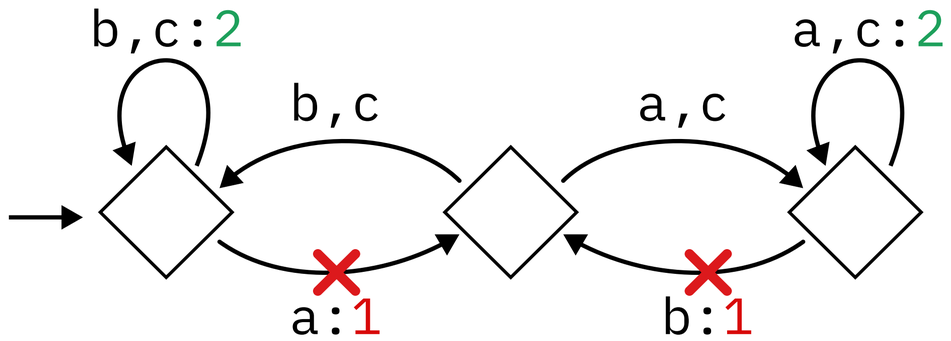

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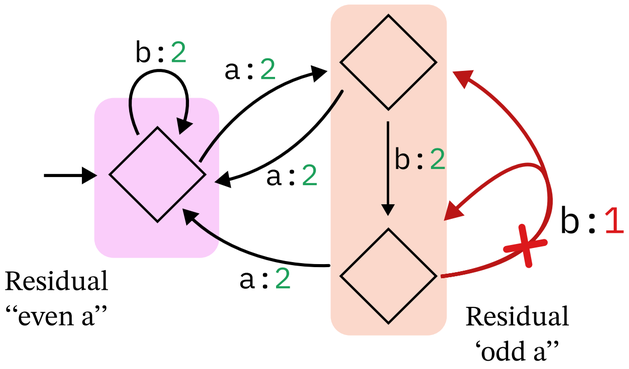

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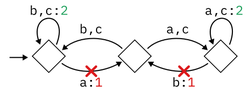

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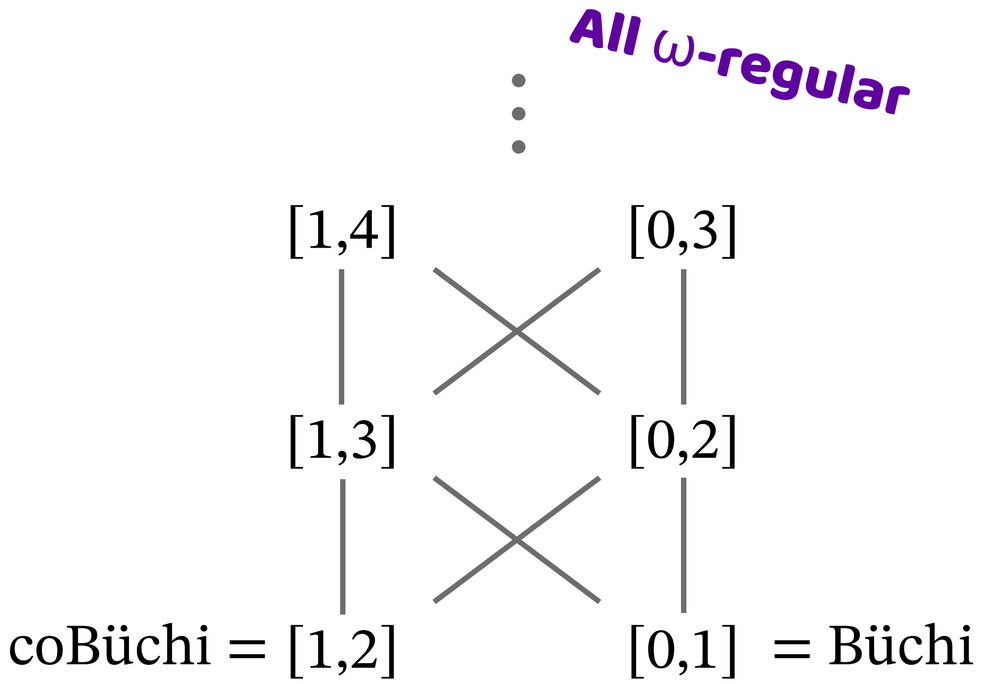

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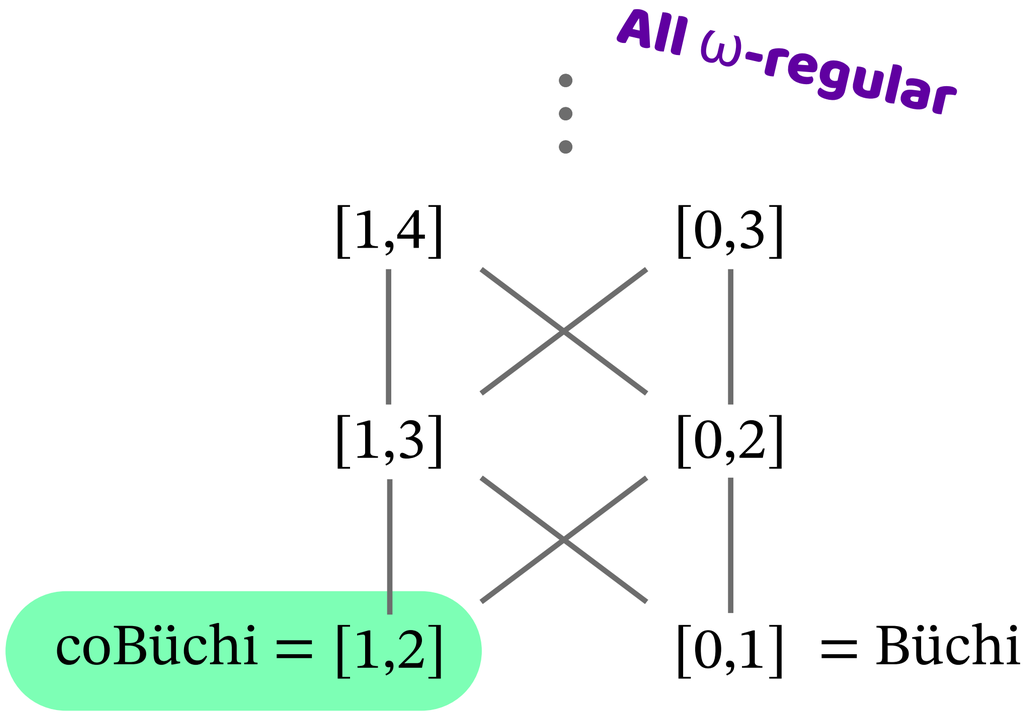

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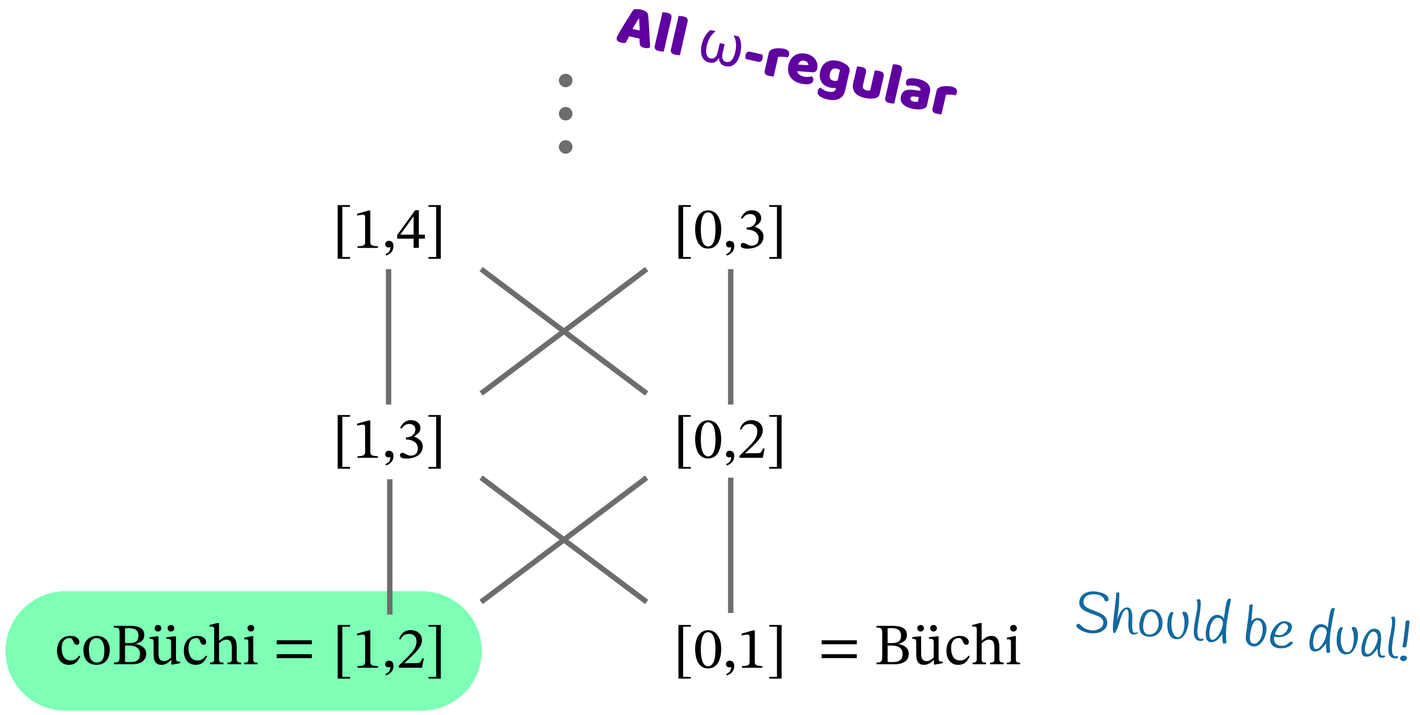

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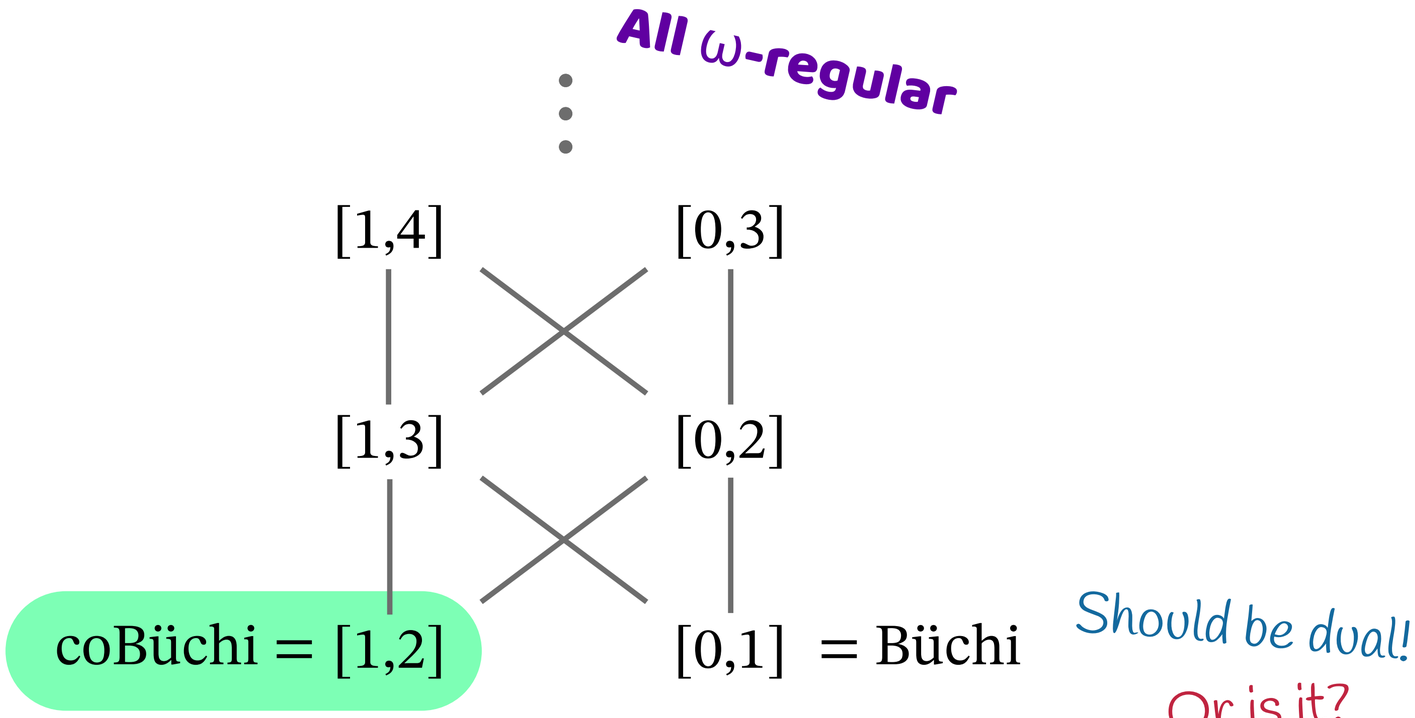

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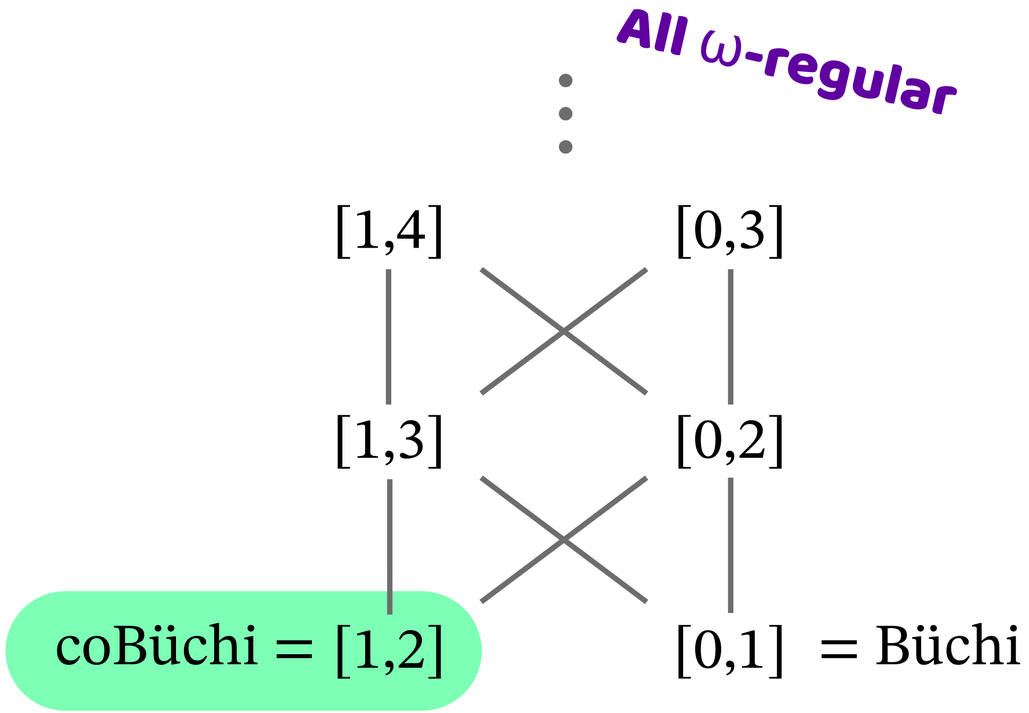

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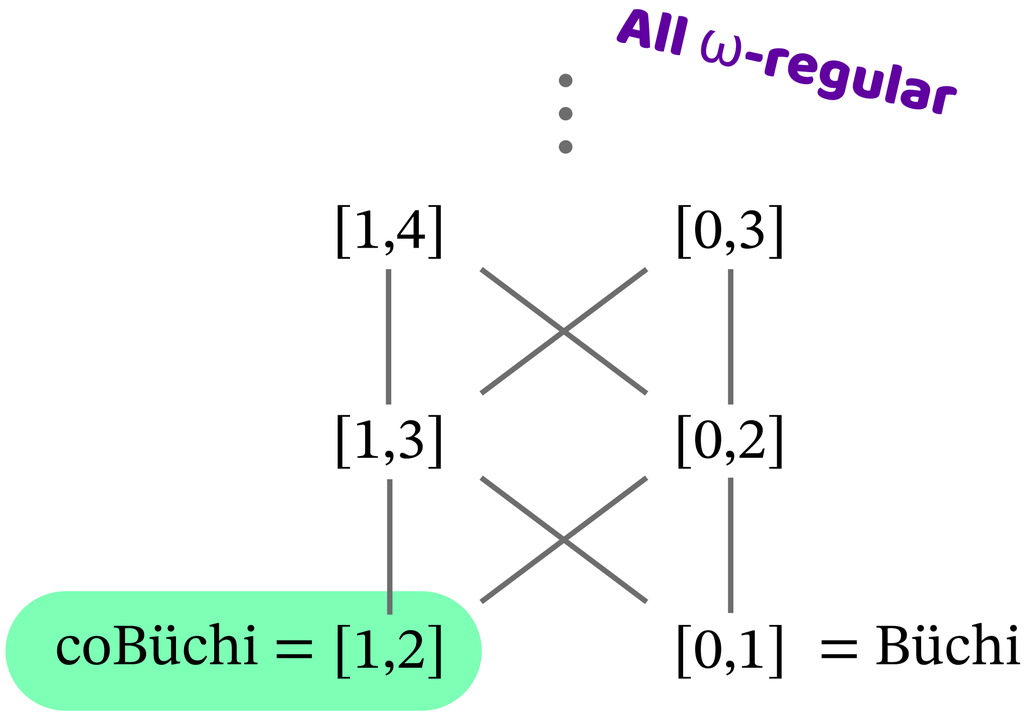

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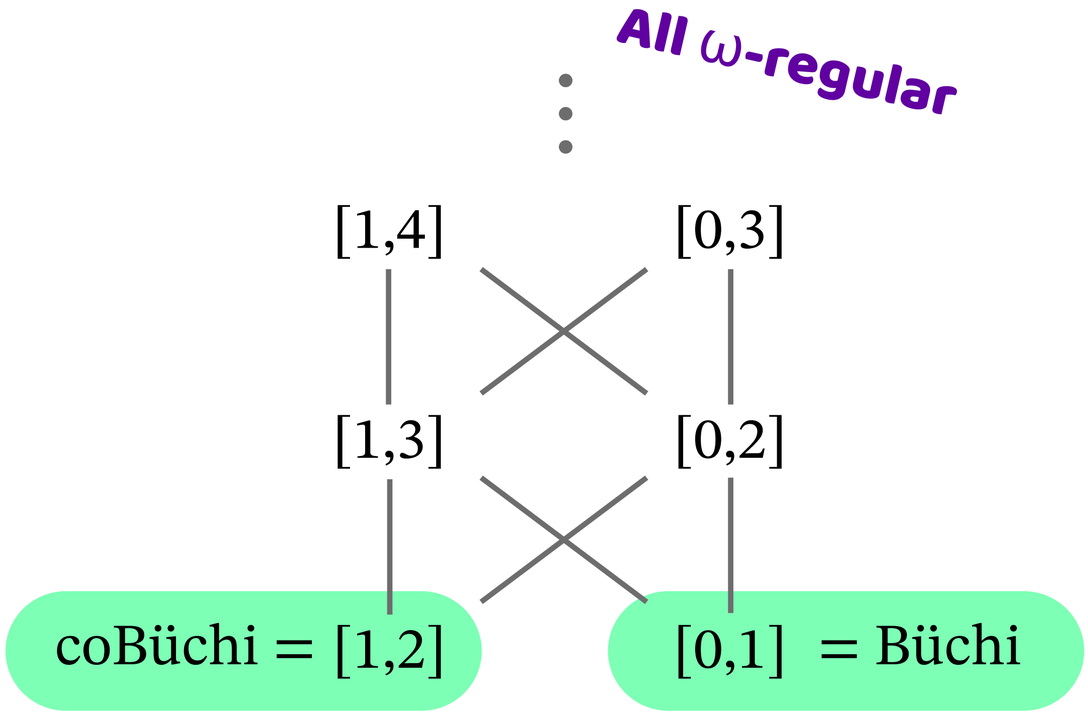

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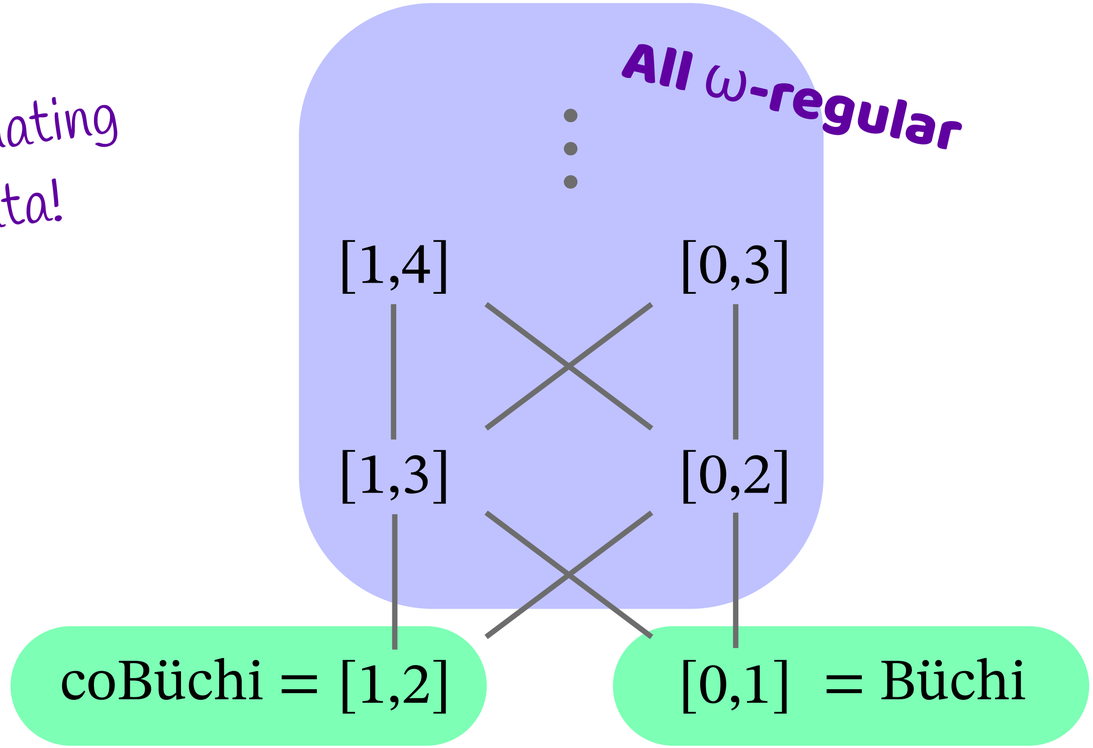

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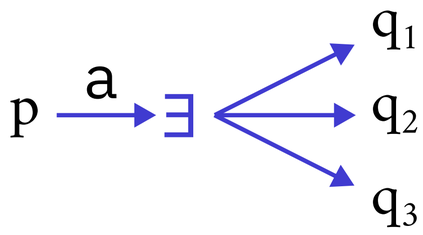

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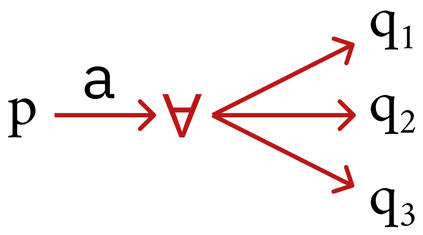

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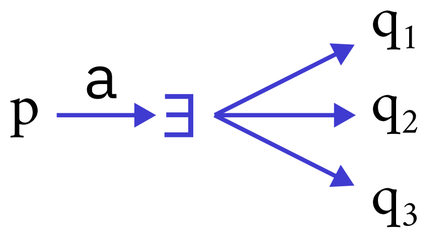

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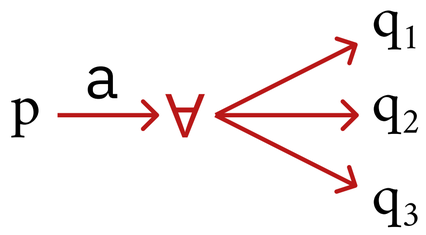

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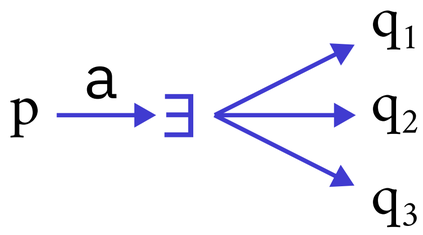

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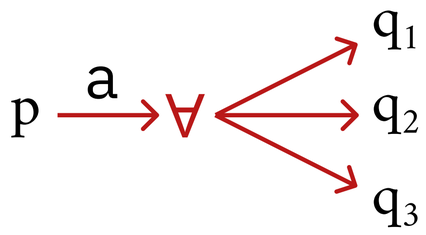

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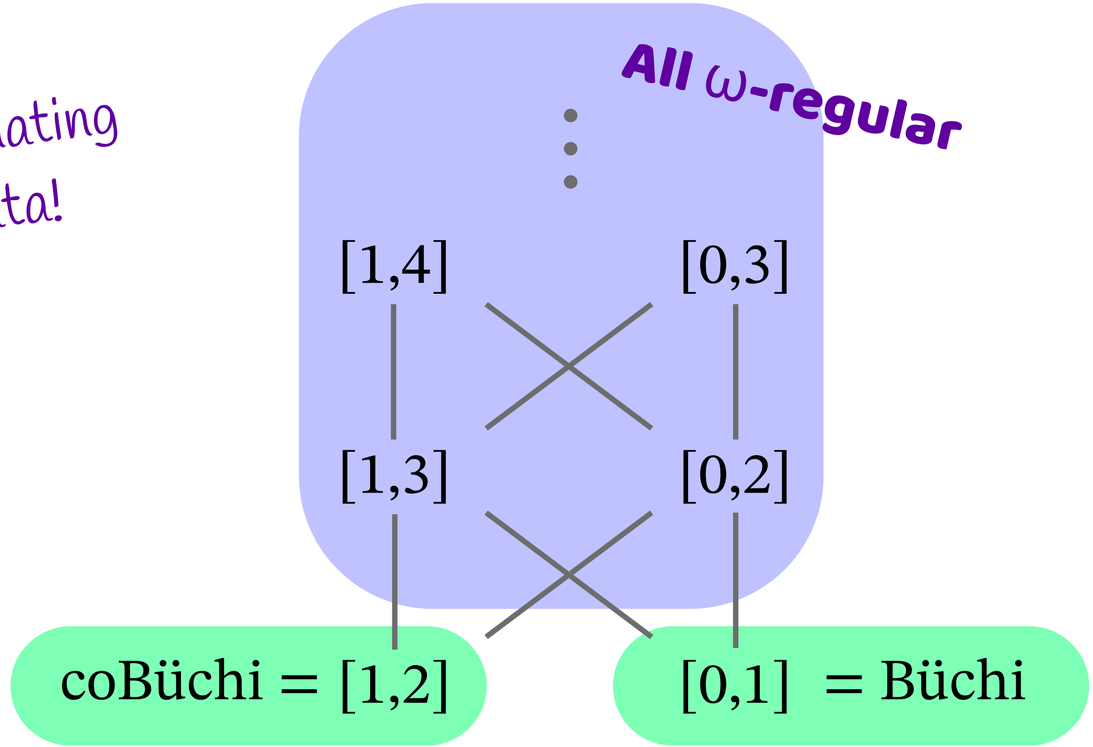

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Everybody loves automata

- Finite automata ⟷ MSO-logic *(Büchi, Elgot, Trakhtenbrot '60)*

Beautiful mathematical theory!

A minimal, canonical DFA for each regular language

Efficiency in applications!

- Minimisation in $O(n\log n)$ *(Hopcroft '71)*
- Automata learning in PTIME *(Gold '67, Angluin 87')*
- Simple characterization of FO-definability

⋮

Let's generalize to infinite words!

- ω-automata ⟷ MSO-logic *(Büchi '62)*
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No unique minimal deterministic automata

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- Minimisation is NP-complete *(Schewe '10, AbuRadi-Ehlers '25)*
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Minimal HD coBüchi automata

Instead, use Layered Automata *but before that...*

THEOREM *(Abu Radi-Kupferman '19)*

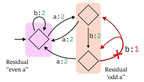
Using priorities {1,2}

History-deterministic (HD) coBüchi automata admit minimal, canonical automata, constructible in PTIME.

Properties of the minimal HD coBüchi automaton

A. If $p \xrightarrow{a:2} q$, then this is the only a-transition from p *(2-deterministic)*

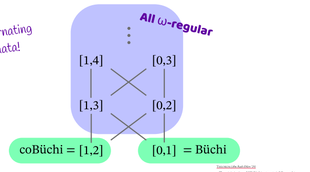
B. If $p \not\xrightarrow{a:2} q$, then also $p \xrightarrow{a:\{1,2\}} \{q_1, q_2, q_3, q_4\}$ to all states that are languages-equivalent to q *(1-saturated)*



Minimal HD coBüchi are great!

But they only recognize a small part of all ω-regular languages

Use alternating automata!



Everybody loves automata

- Finite automata ⟷ MSO-logic *(Büchi, Elgot, Trakhtenbrot '60)*

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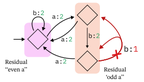

Residual "even a"

Residual "odd a"

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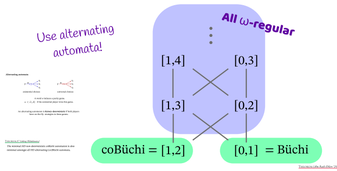

Instead, use ✨ ***Layered Automata*** ✨

Layered Automata

Formalism to represent a subclass of alternating parity automata

Layered Automata

Formalism to represent a subclass of alternating parity automata

Definition

Layered automaton: A sequence of deterministic transition systems with morphisms

$$\mathcal{T}_1 \xleftarrow{\mu_1} \mathcal{T}_2 \xleftarrow{\mu_2} \quad \dots \quad \mathcal{T}_{d-1} \xleftarrow{\mu_{d-1}} \mathcal{T}_d$$

with morphisms

$$\mu : V \to V'$$

$$\mathrm{p} \xrightarrow{\mathtt{a}} \mathrm{q} \quad \text{in } \mathcal{T} \qquad \Longrightarrow \qquad \mu(\mathrm{p}) \xrightarrow{\mathtt{a}} \mu(\mathrm{q}) \quad \text{in } \mathcal{T}'$$

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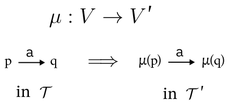

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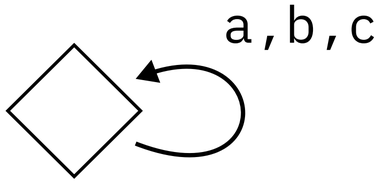

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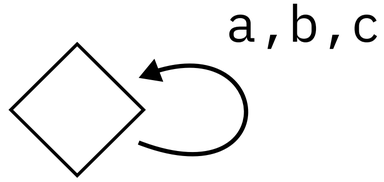

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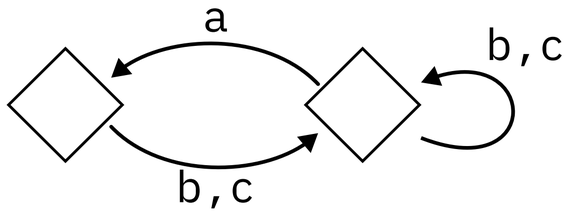

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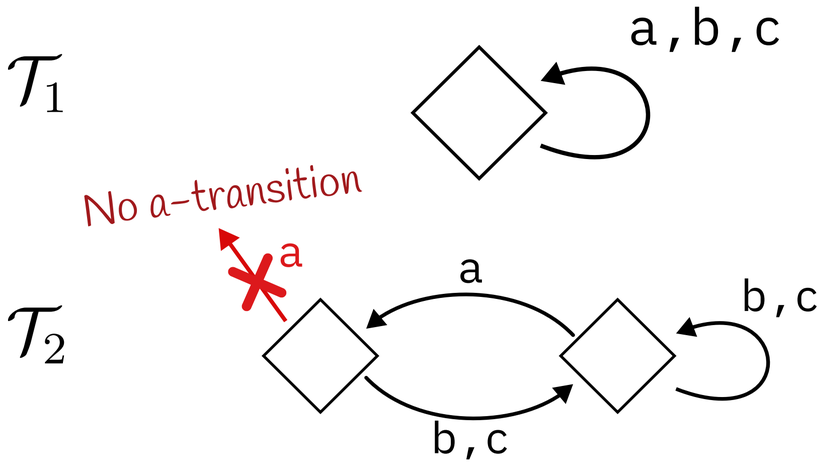

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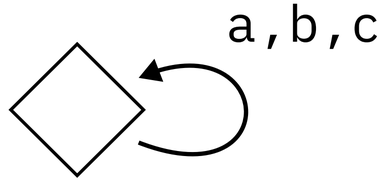

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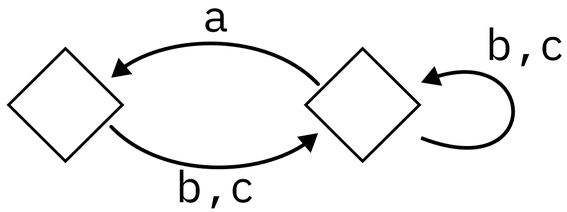

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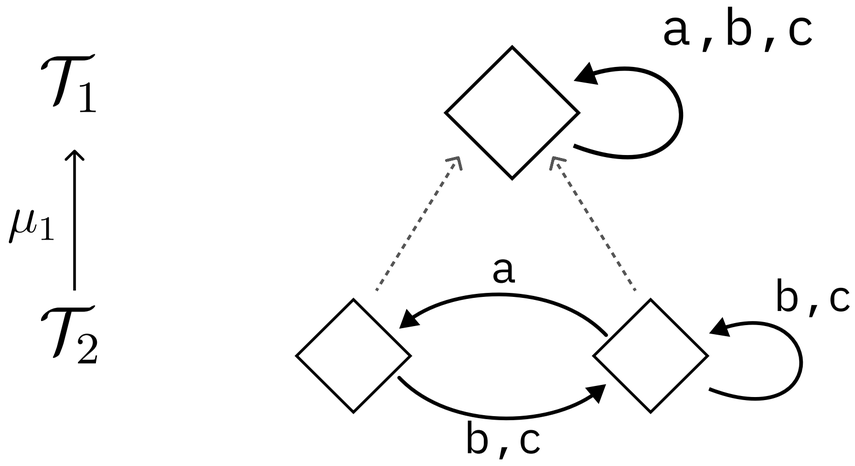

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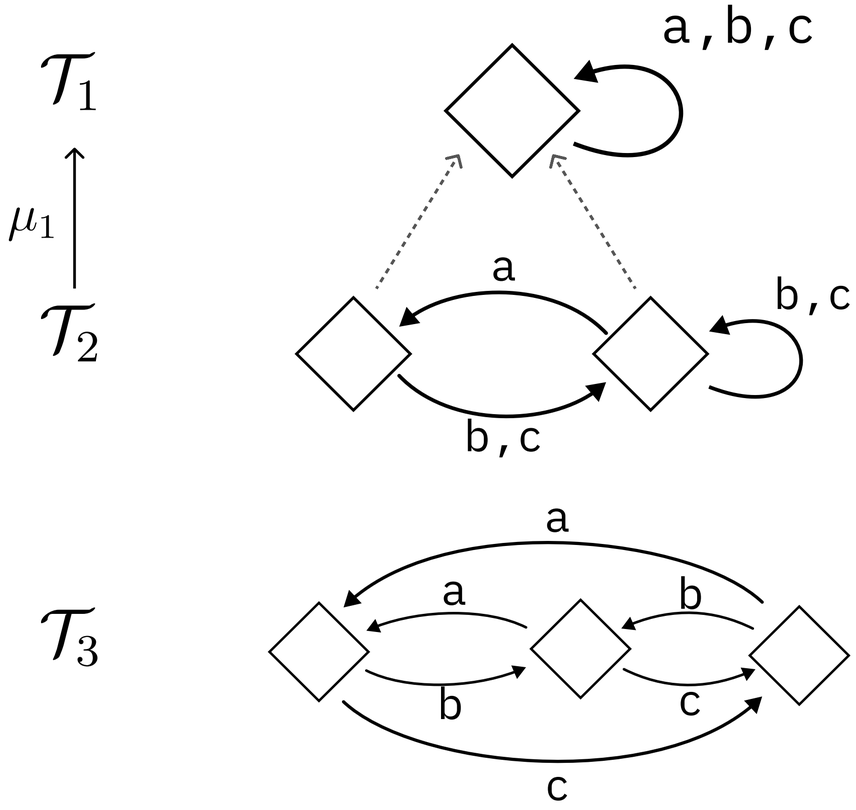

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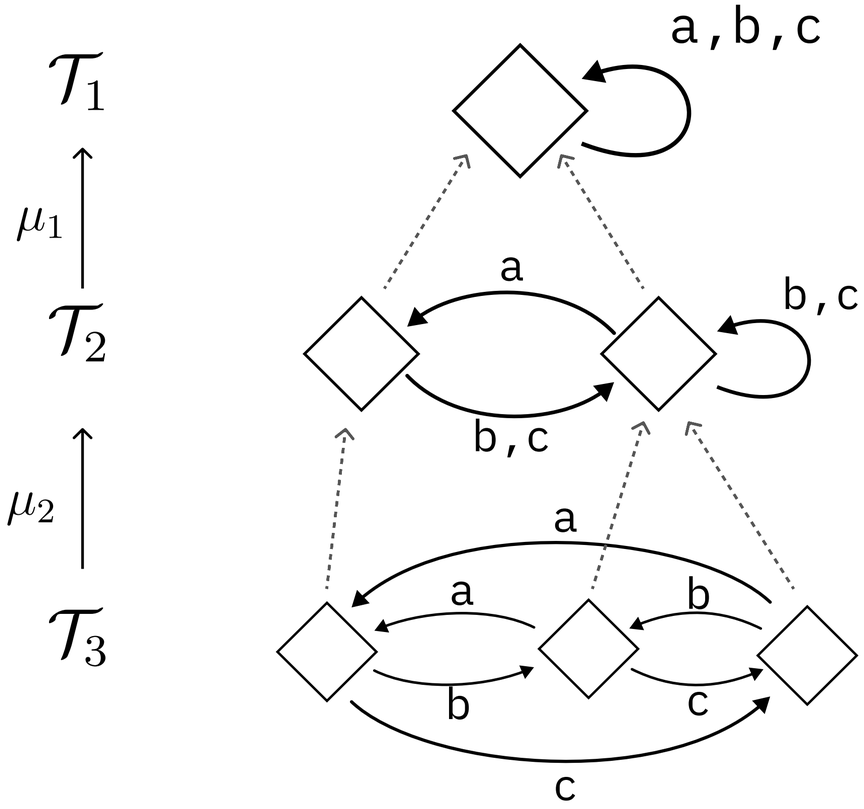
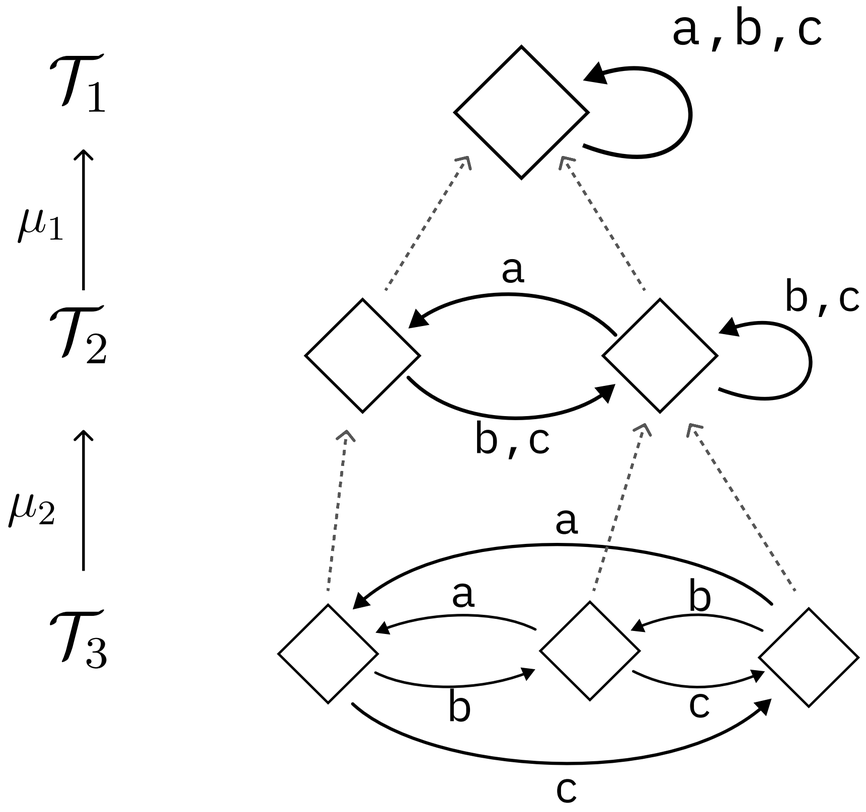
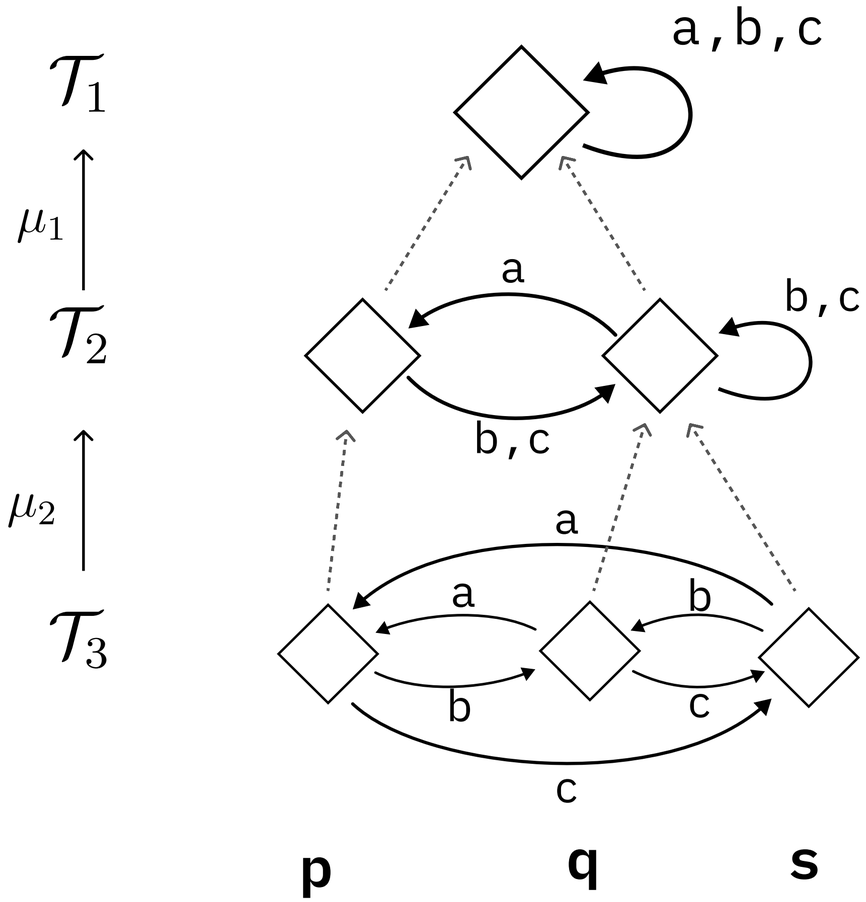


Alternating Parity Automaton





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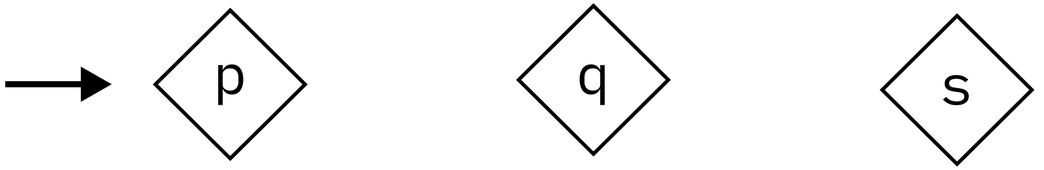
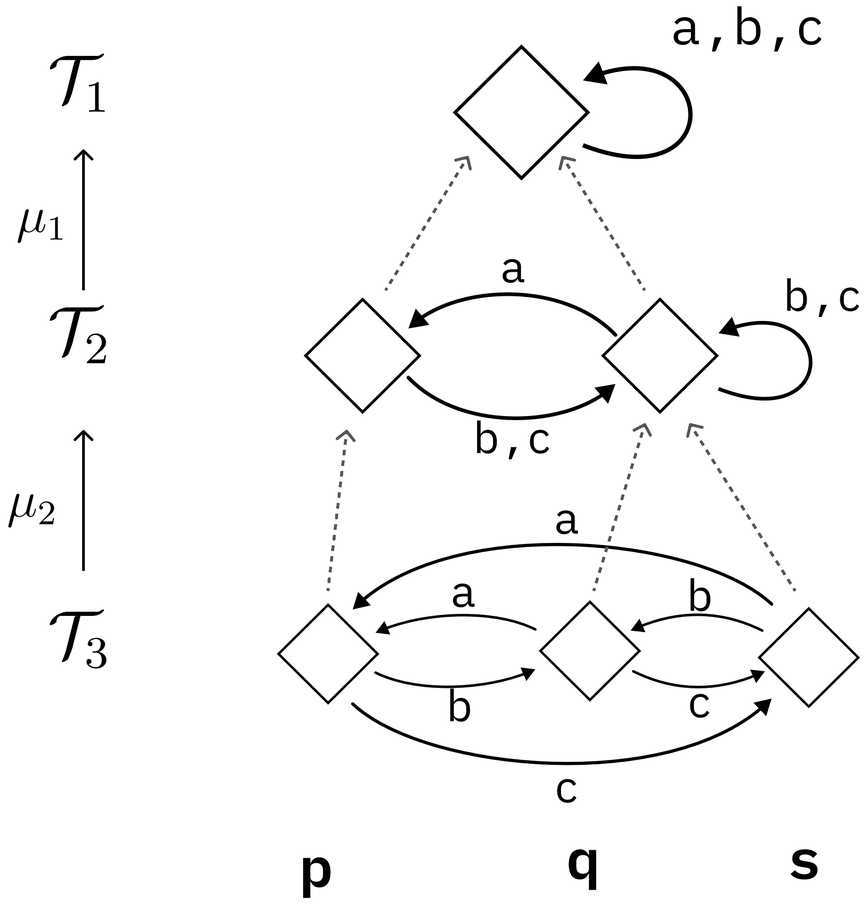
States: leaves





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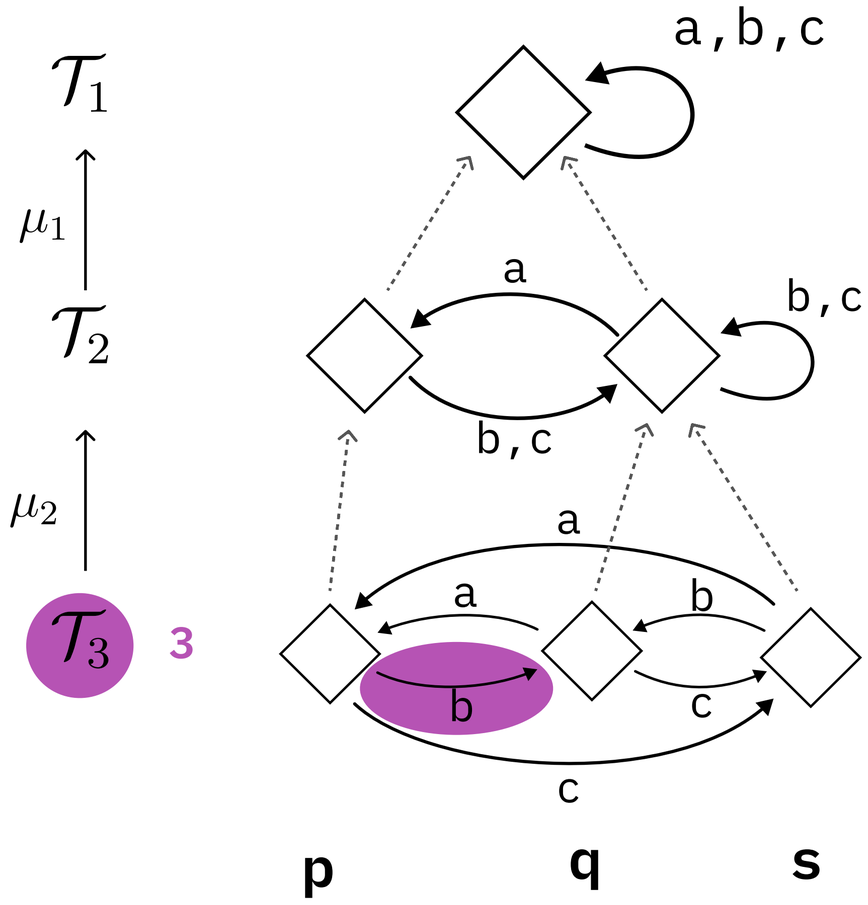

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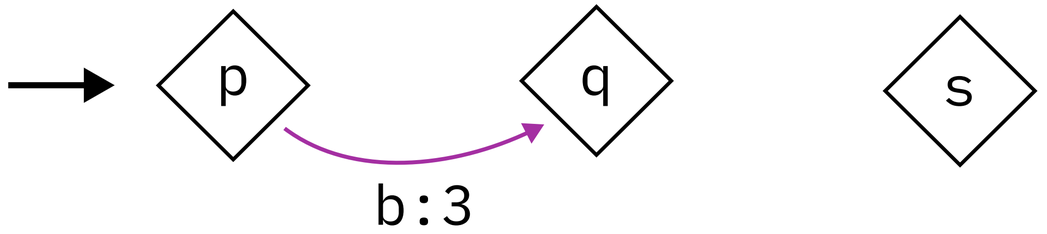

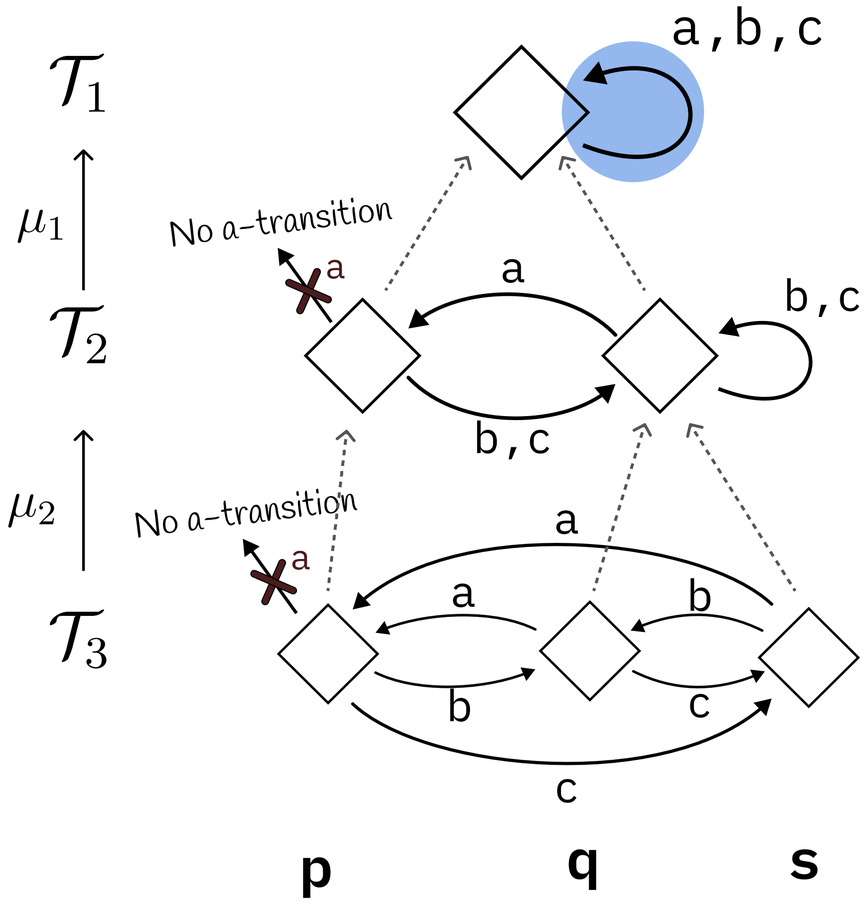

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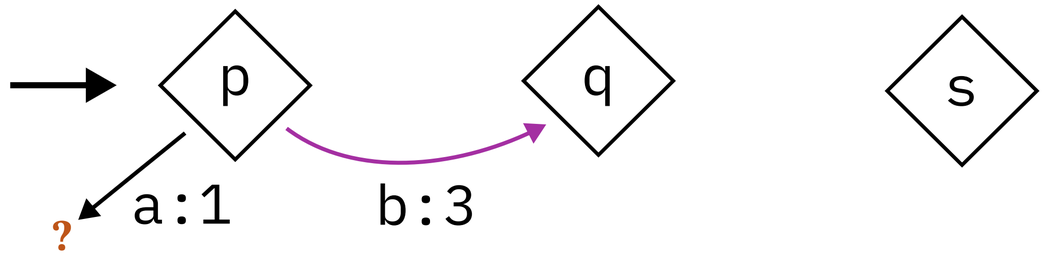

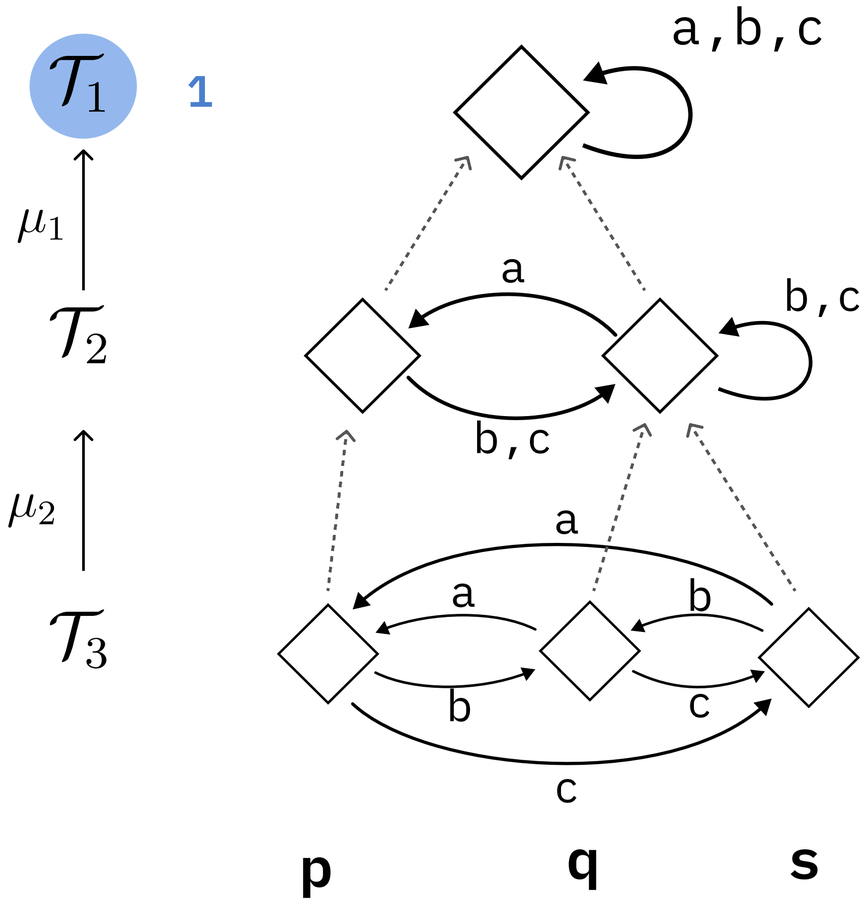

Odd → existential choice

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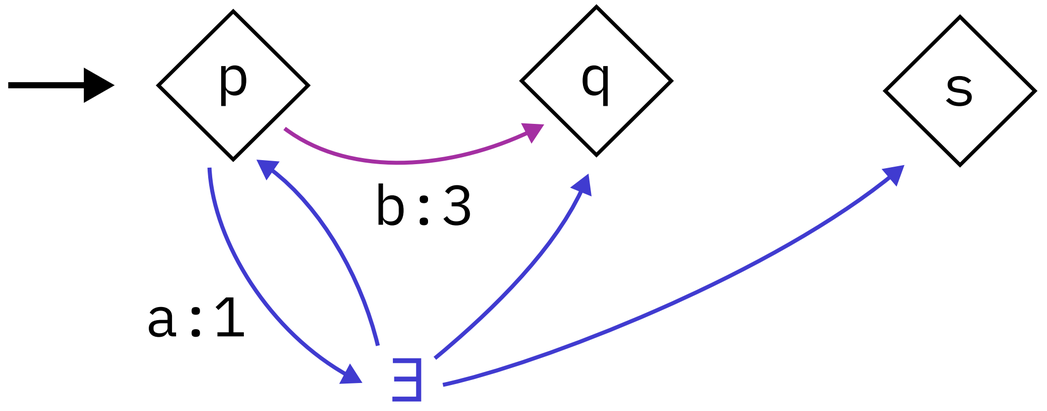

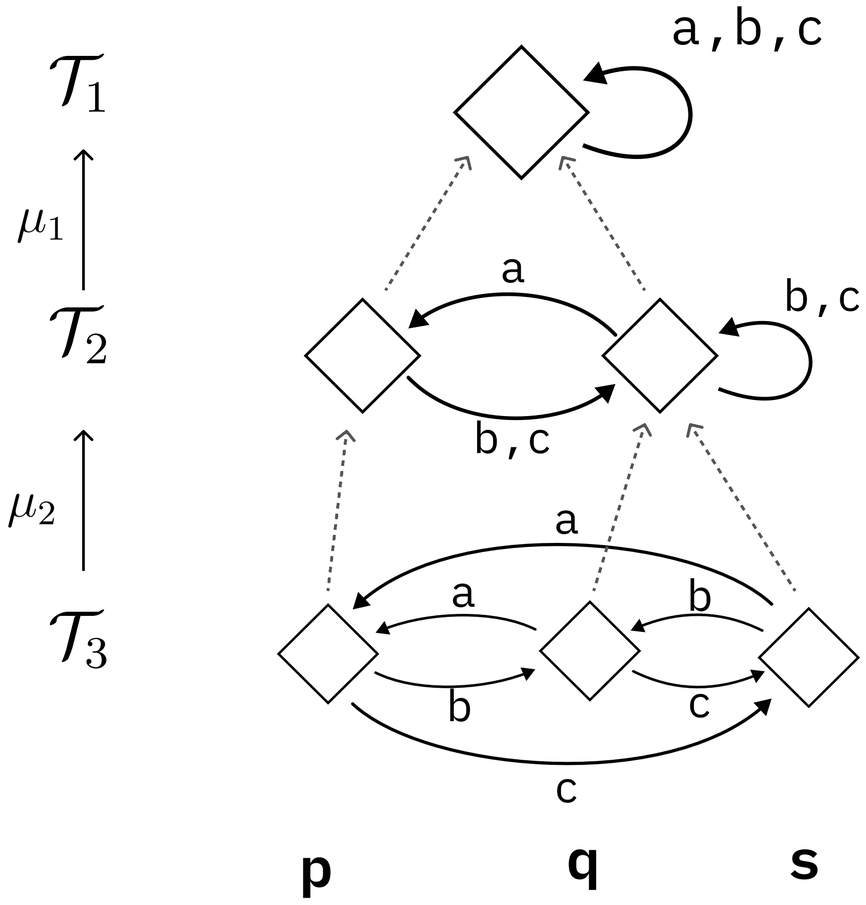

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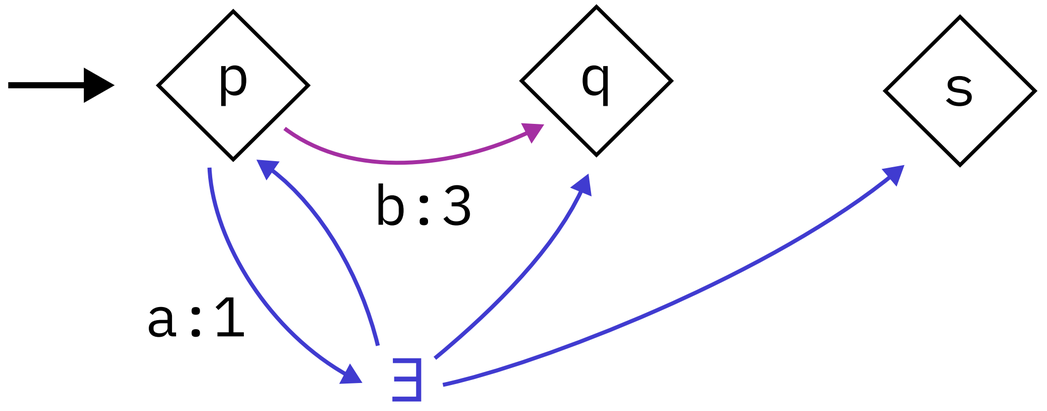

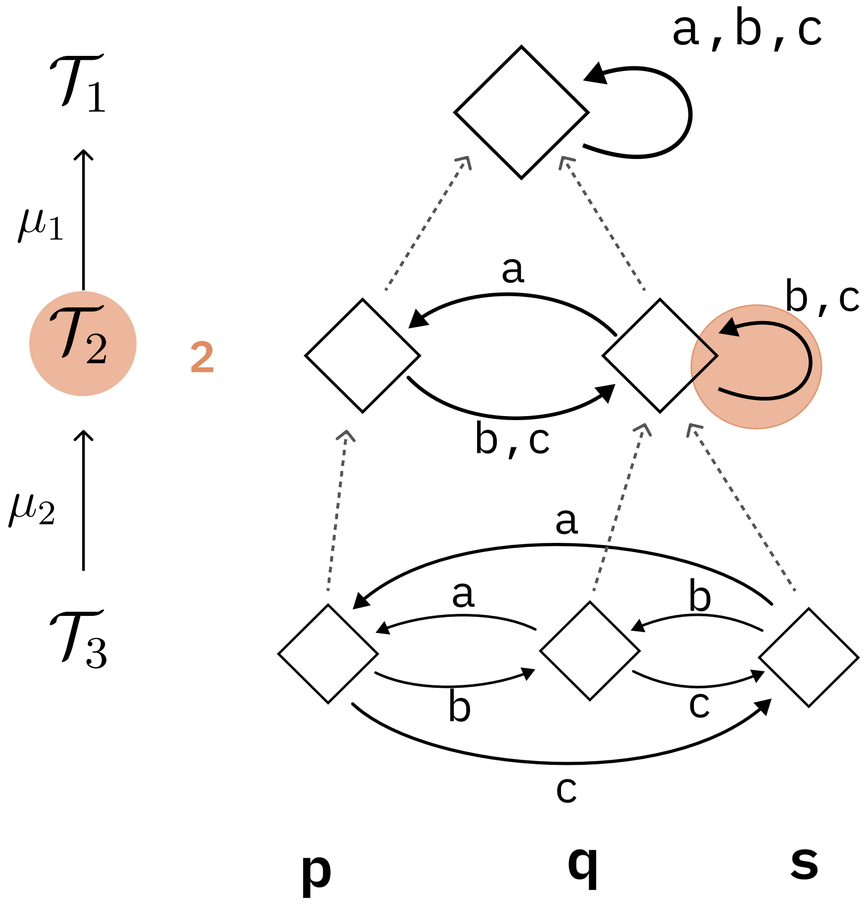

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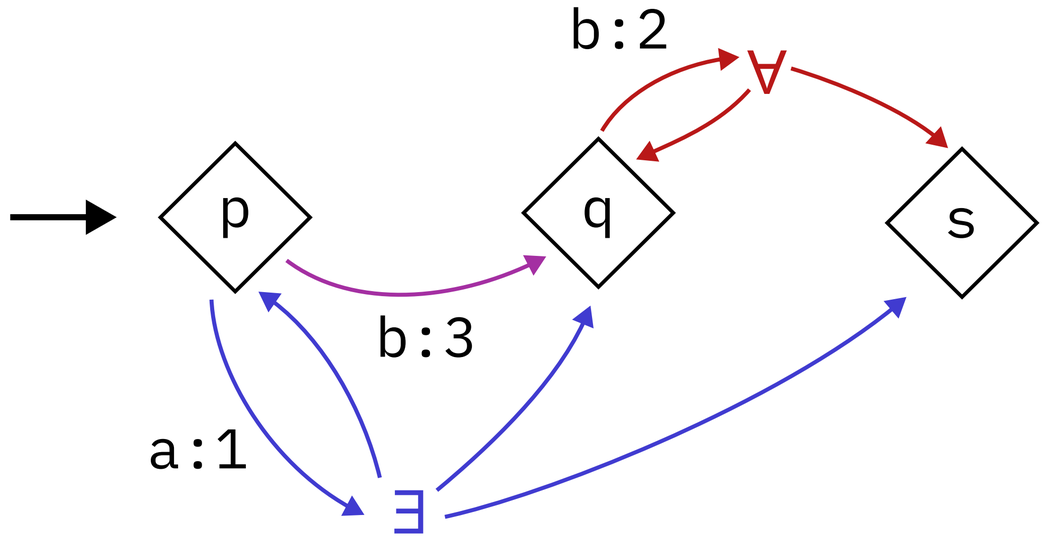

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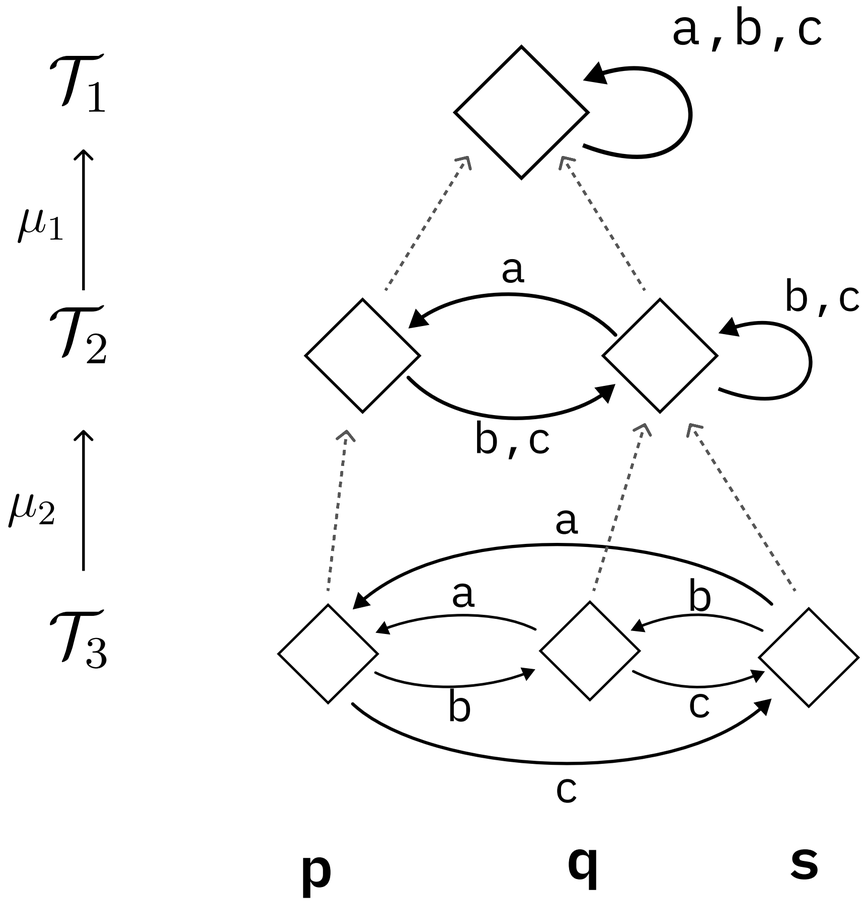
Even → universal choice



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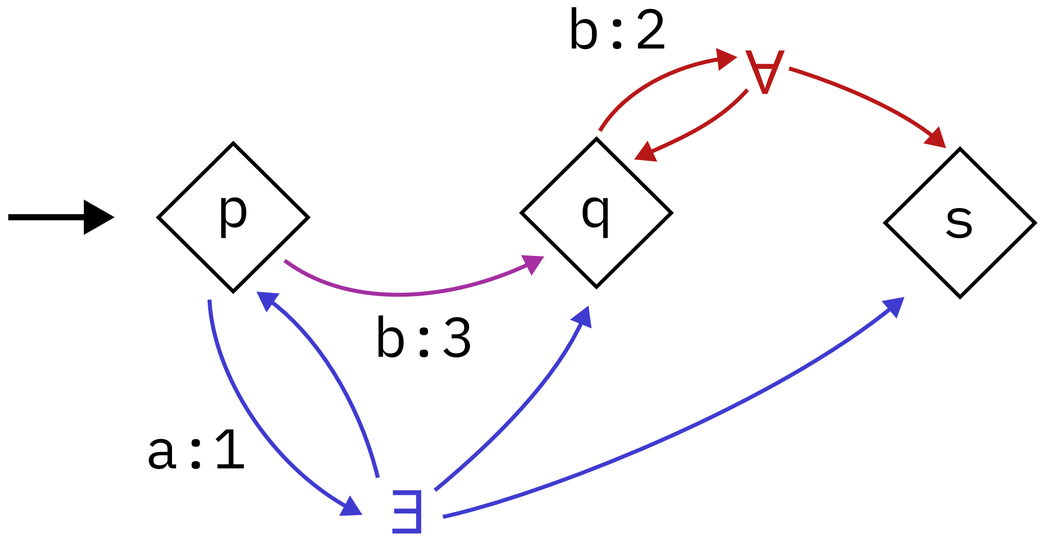

\+ $\mathcal{T}_1$ must correspond to residuals

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Formalism to represent a subclass of alternating parity automata

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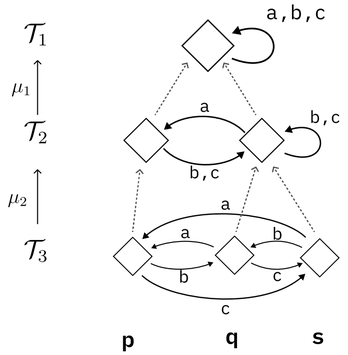

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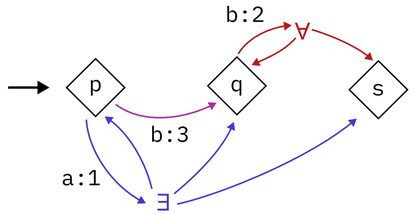

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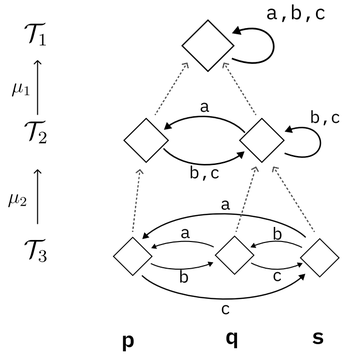

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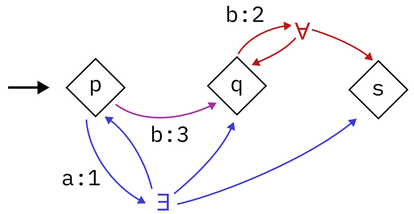

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Layered Automata

Formalism to represent a subclass of alternating parity automata

Definition

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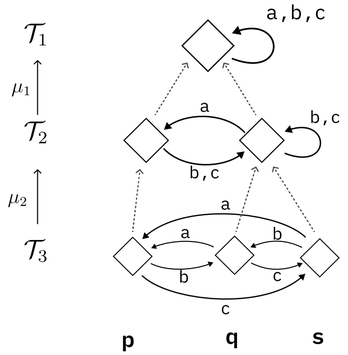
$\mu : V \to V'$

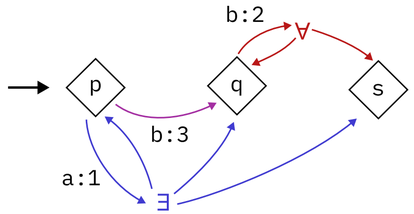
$p \xrightarrow{a}_{\text{in } \mathcal{T}} q \implies \mu(p) \xrightarrow{a}_{\text{in } \mathcal{T}'} \mu(q)$



Alternating Parity Automaton

States: leaves

Transitions: given by the deepest $\mathcal{T}_i$ where the transition exists



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Beautiful mathematical theory! — A minimal, canonical DFA for each regular language — **Efficiency in applications!**

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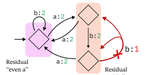

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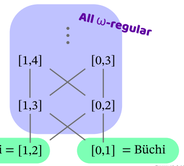

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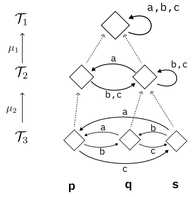

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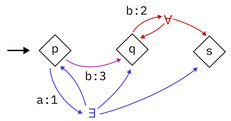

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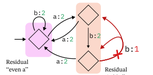

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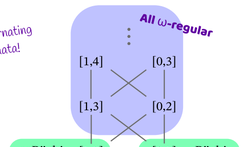

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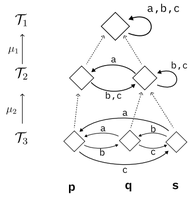

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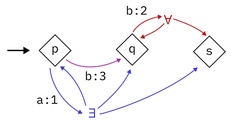

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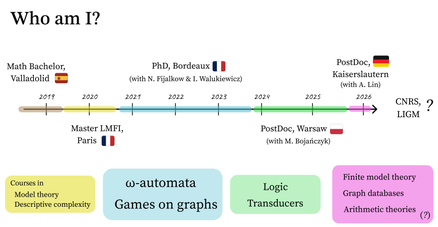
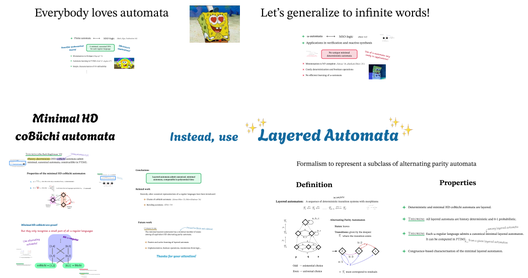
A canonical model for ω-regular languages



Antonio Casares · RPTU Kaiserslautern

Joint work with: Christof Löding and Igor Walukiewicz



Séminaire BAAM, 4 November 2025