Simplifying ω-Automata through the Alternating Cycle Decomposition

Antonio Casares · University of Warsaw & Corto Mascle · LaBRI, University of Bordeaux

Automata over infinite words (ω-automata)

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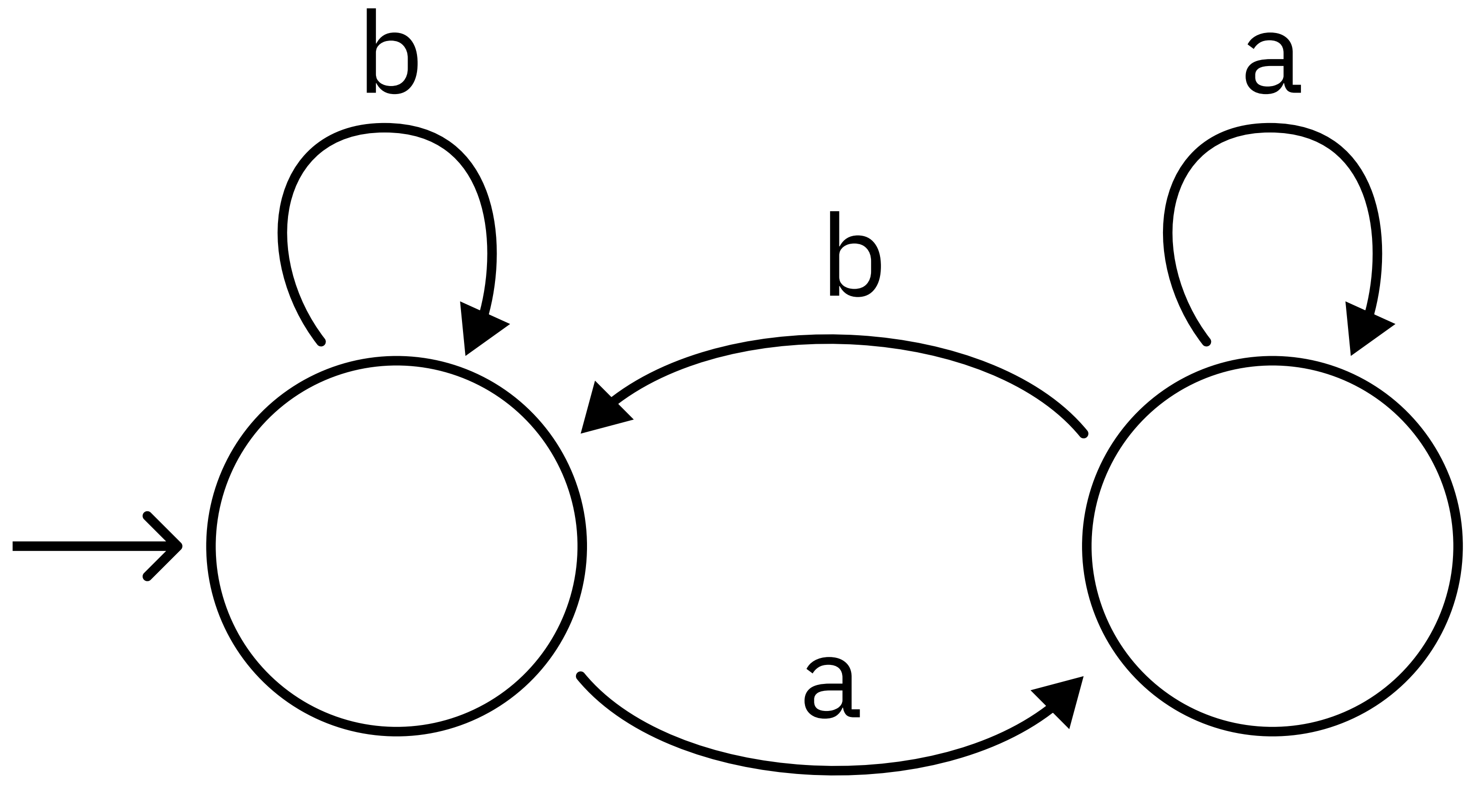
$\mathcal{A}$



Automata over infinite words (ω-automata)

Automata are deterministic
in this talk

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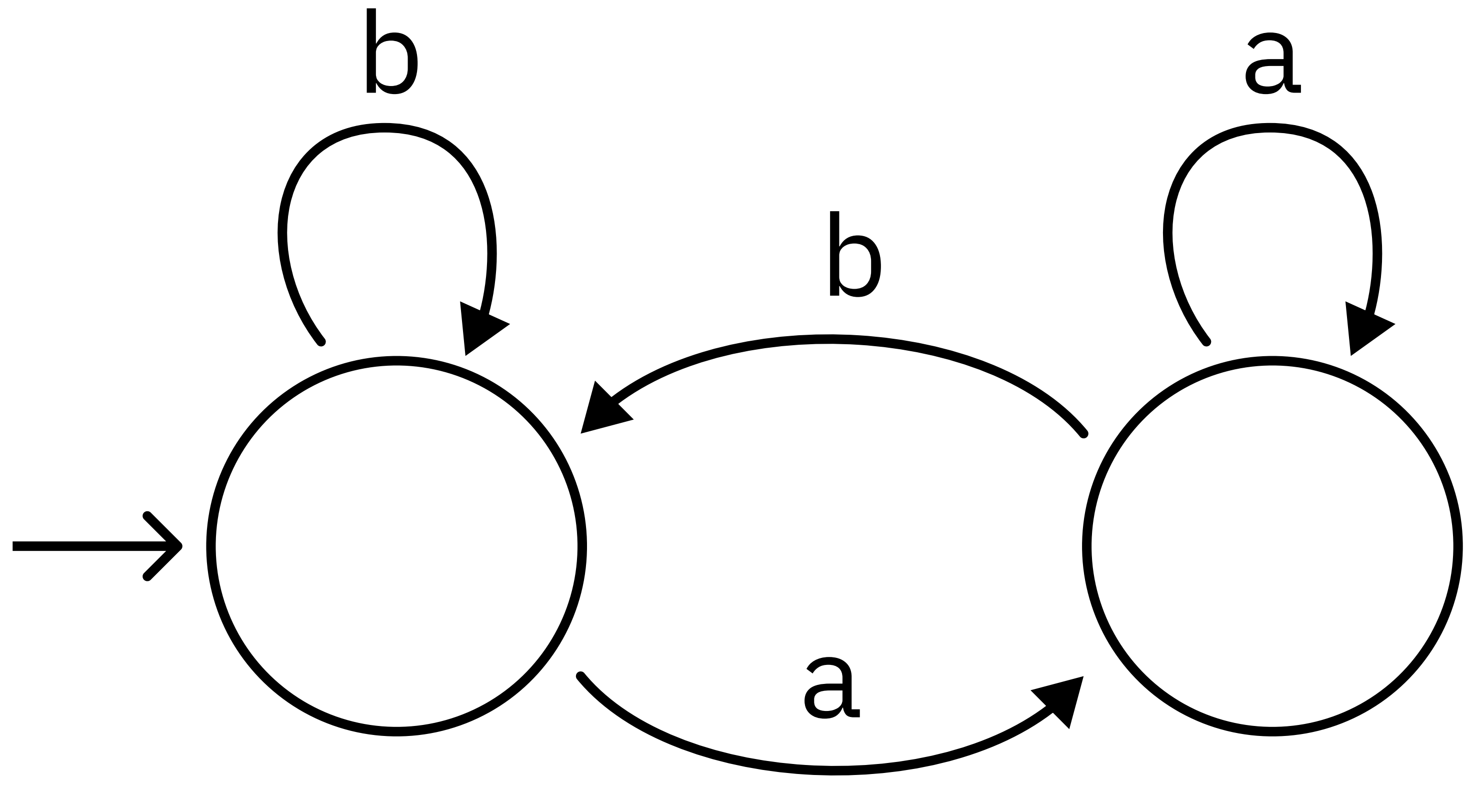

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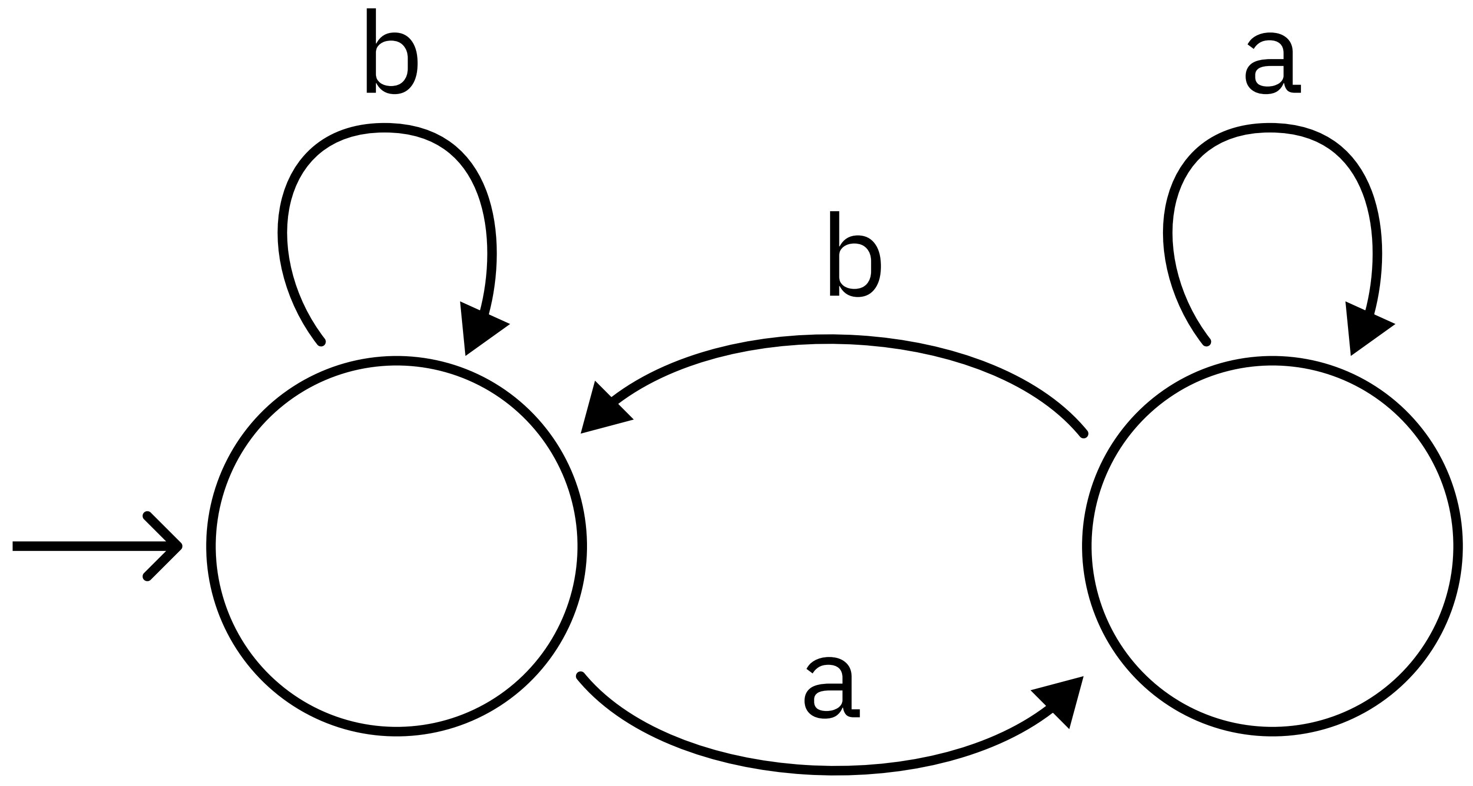

Input $w \in \Sigma^\omega$ $\longrightarrow$ Infinite path ρ

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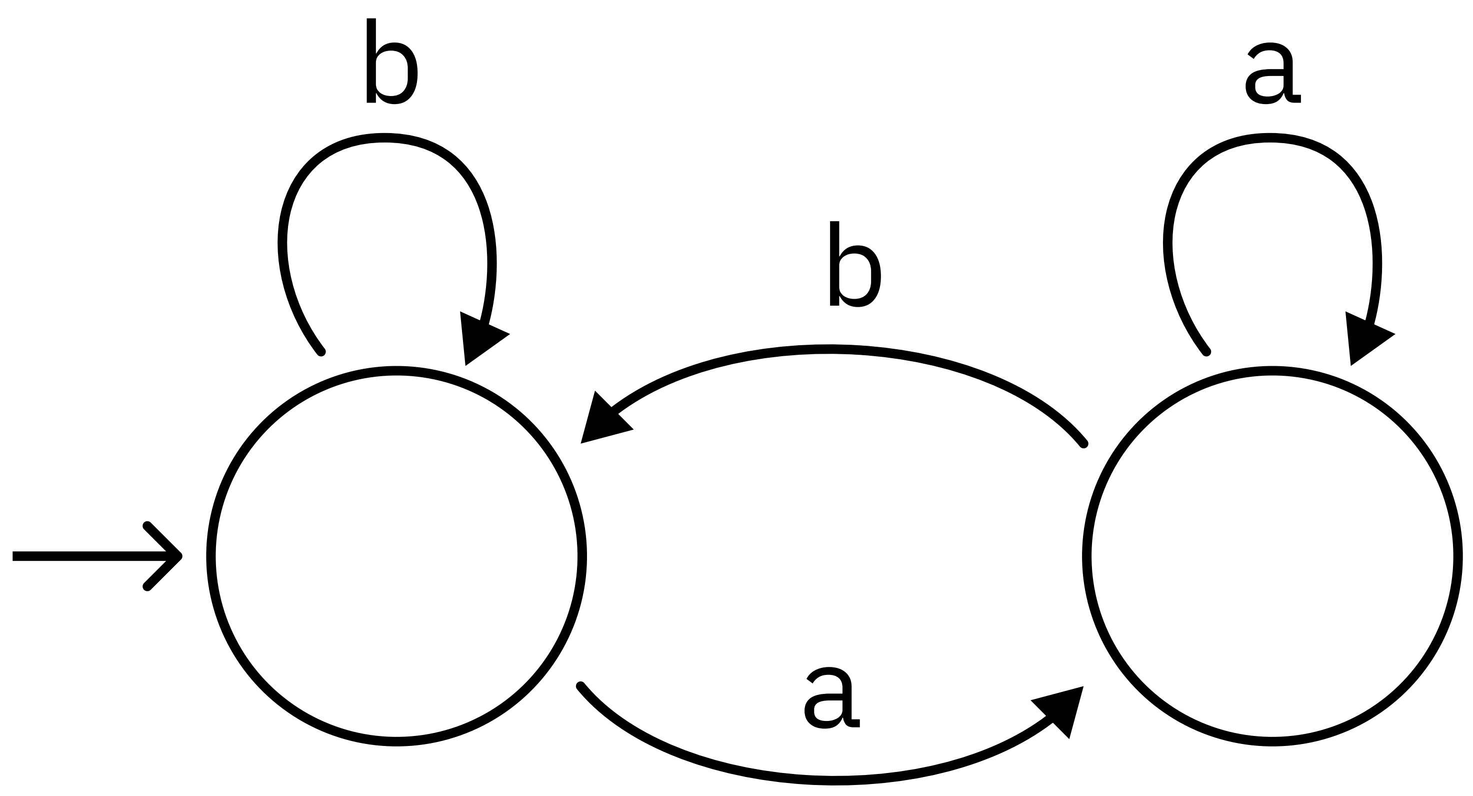

Acceptance Condition?

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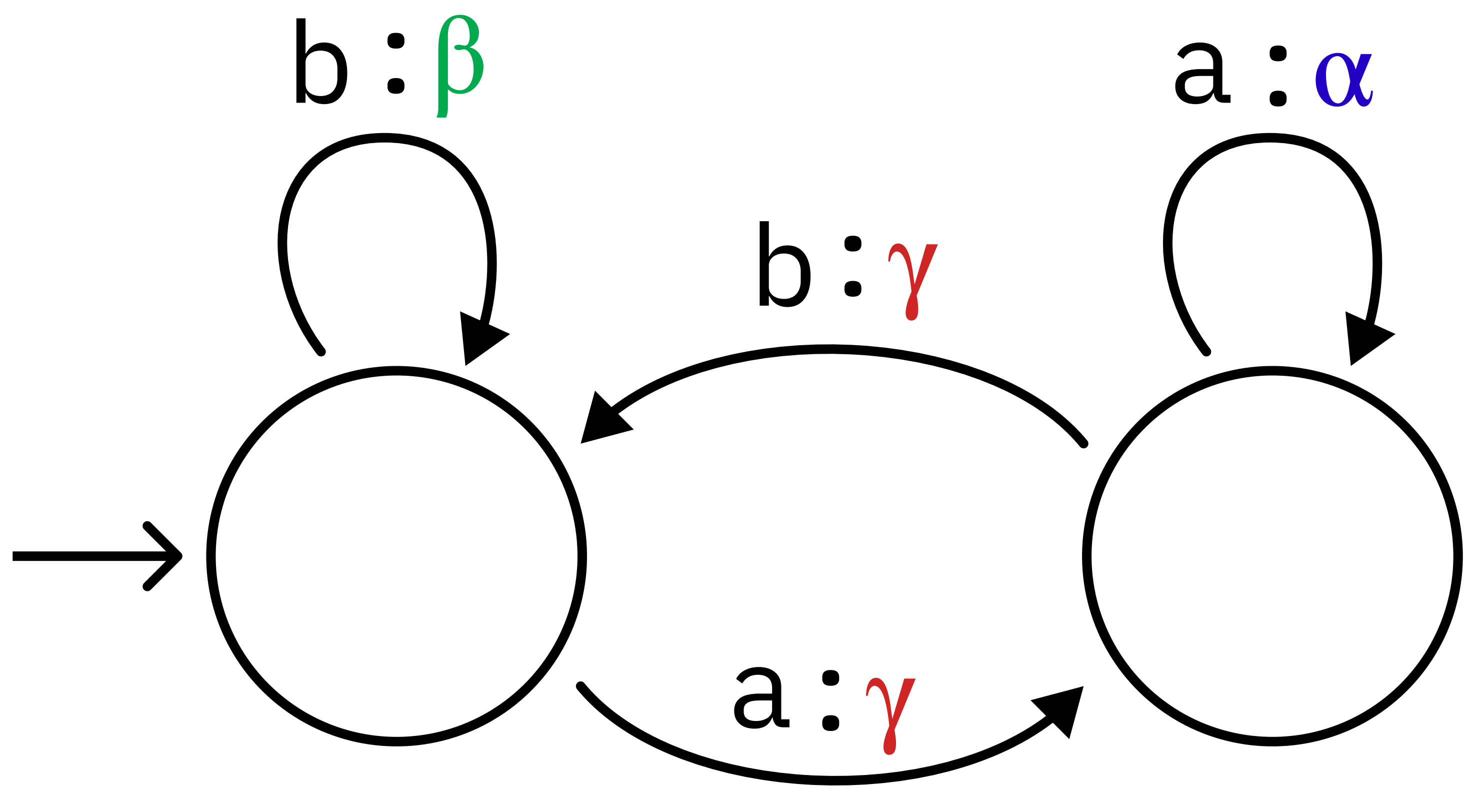

Acceptance Condition?

$C = \{\alpha, \beta, \gamma\}$

output colours

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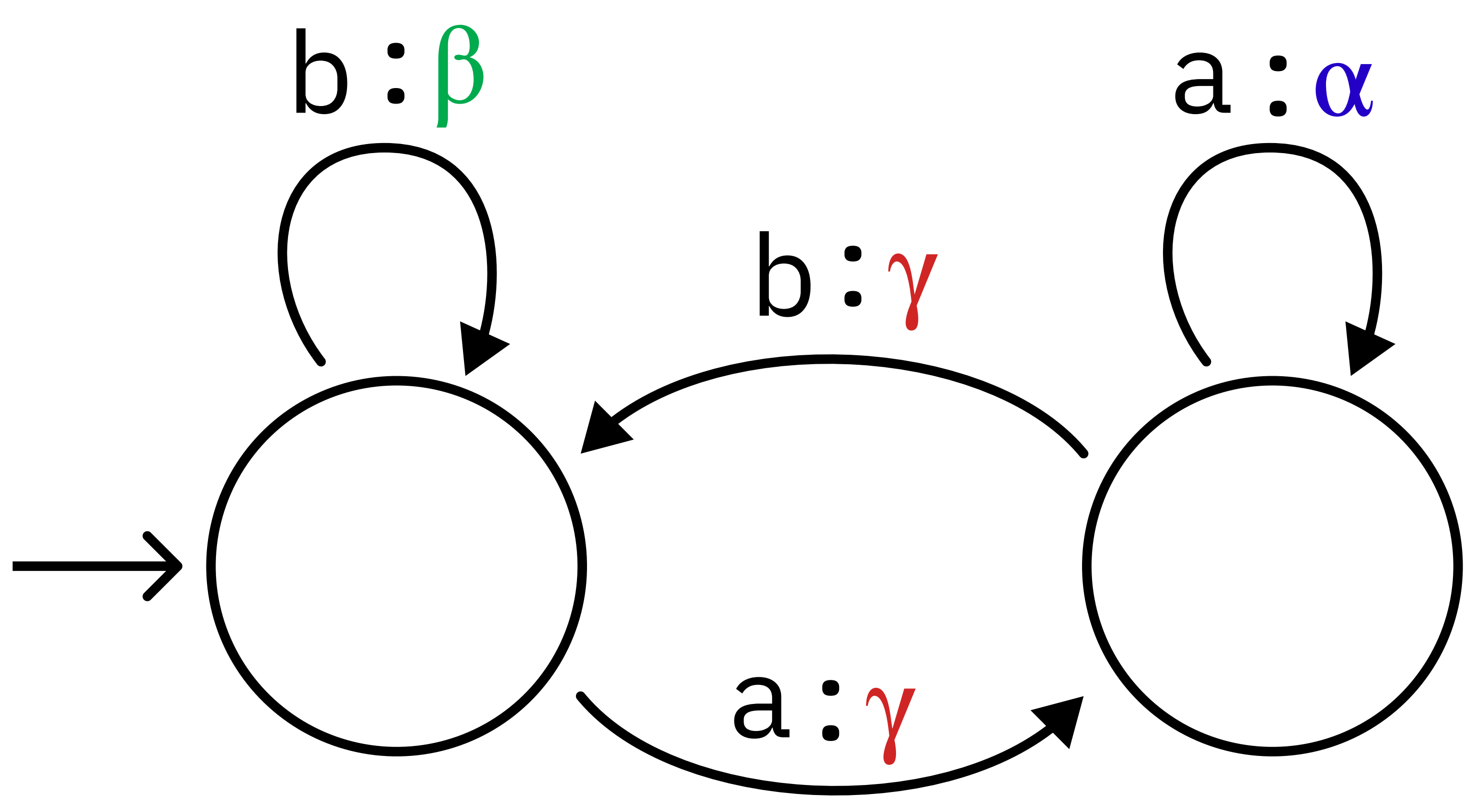

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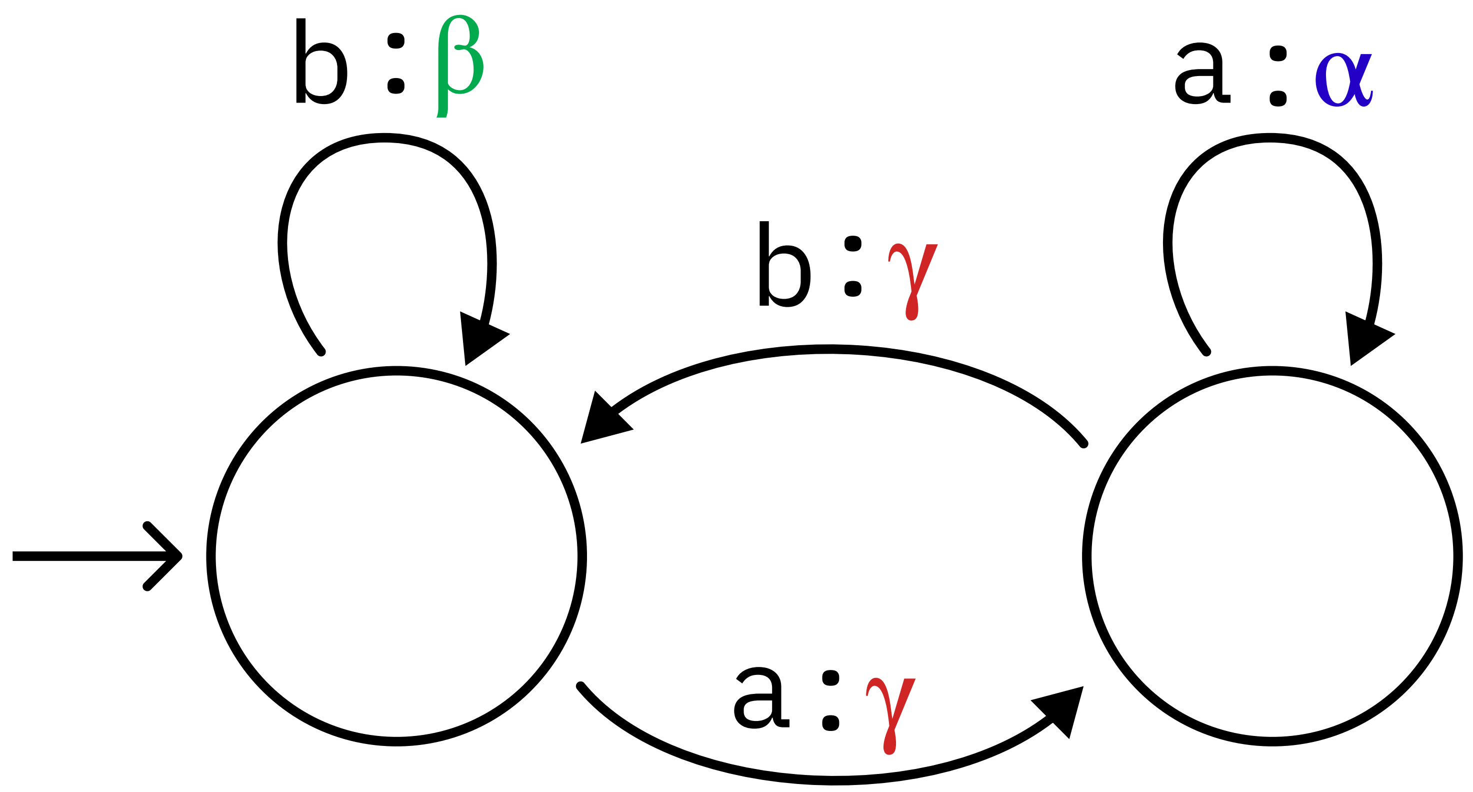

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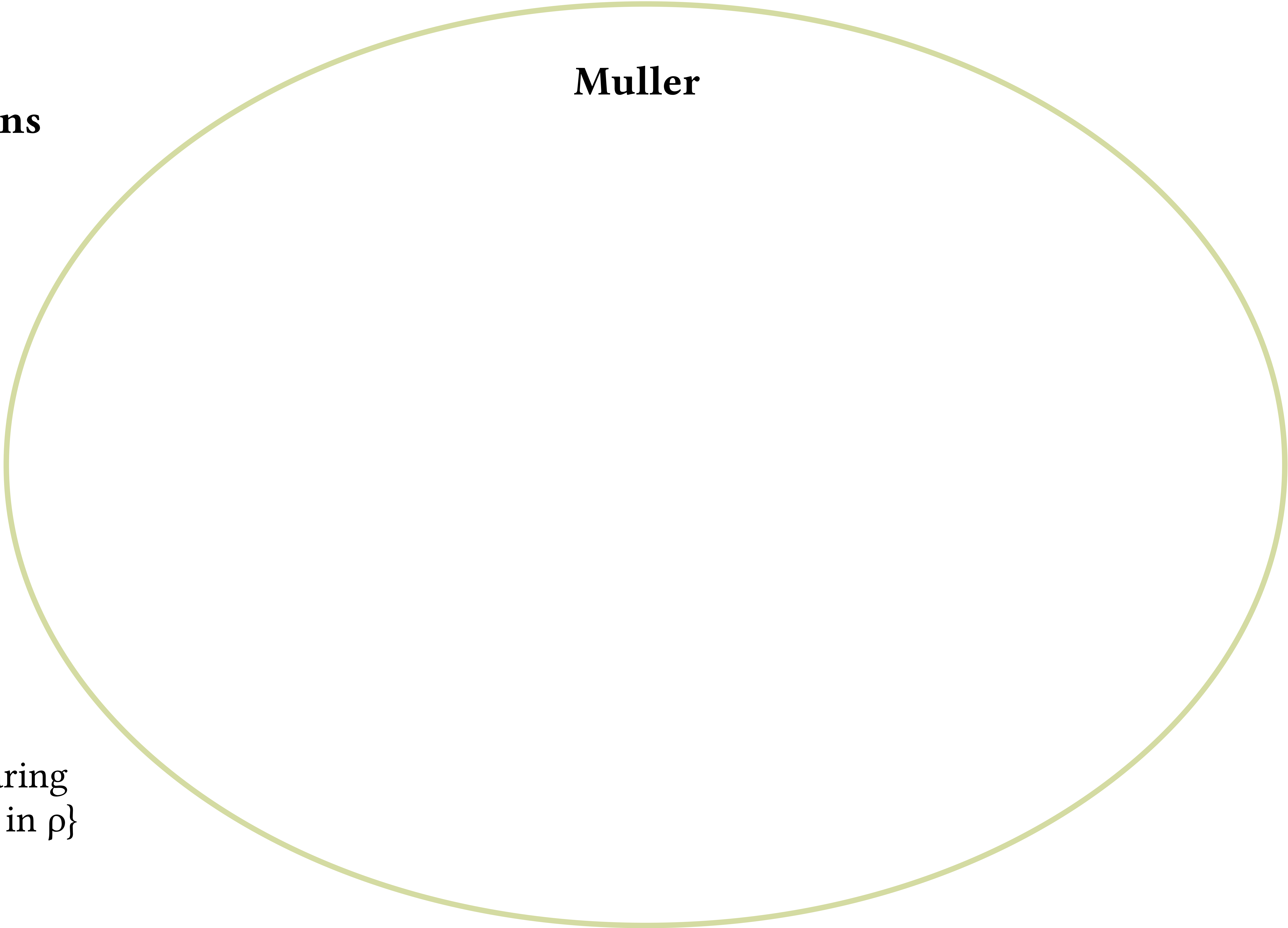

$C = \{ {\color{blue}\alpha}, {\color{green}\beta}, {\color{red}\gamma} \}$

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Muller

$\mathcal{F} \subseteq 2^{C}$ family of subsets

A run ρ is accepting $\Leftrightarrow$ $\mathrm{Inf}(\rho) \in \mathcal{F}$

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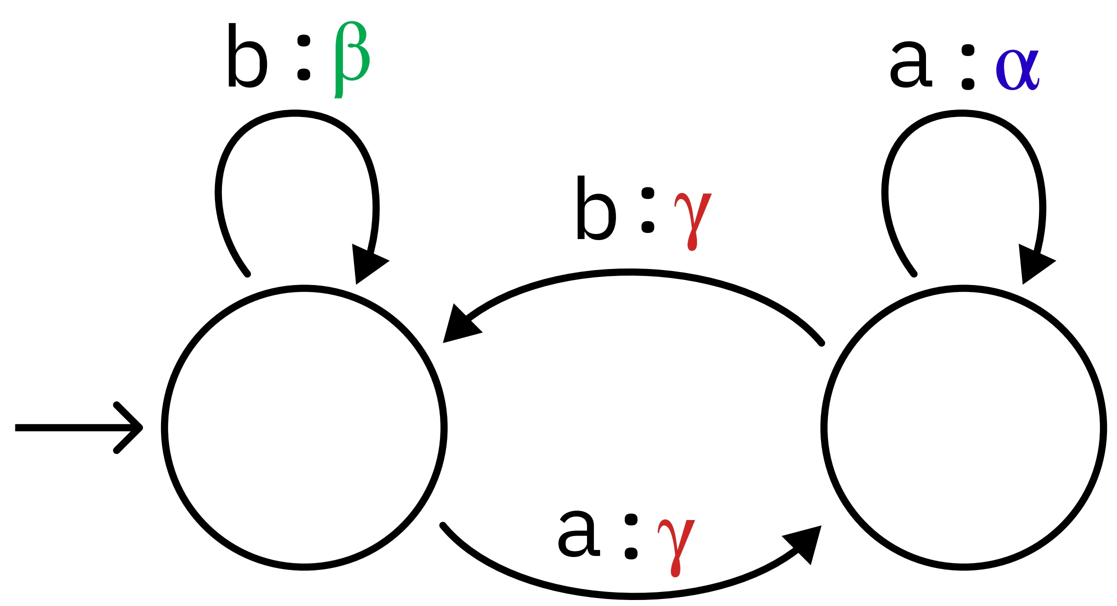

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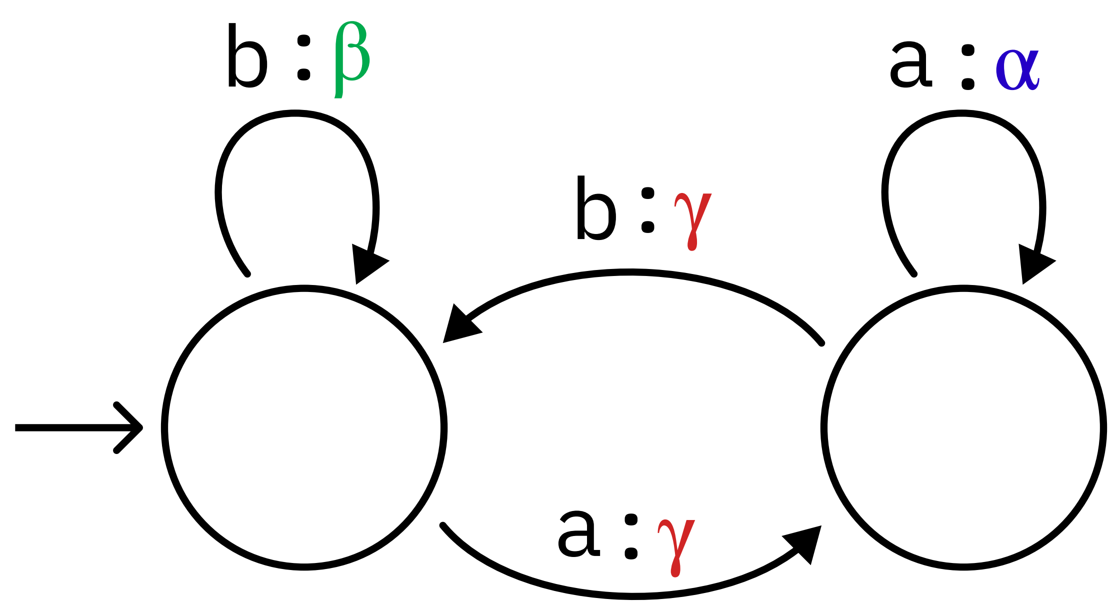

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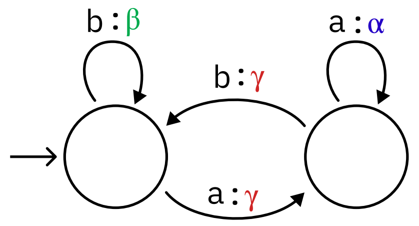

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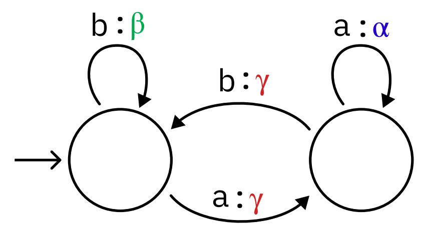

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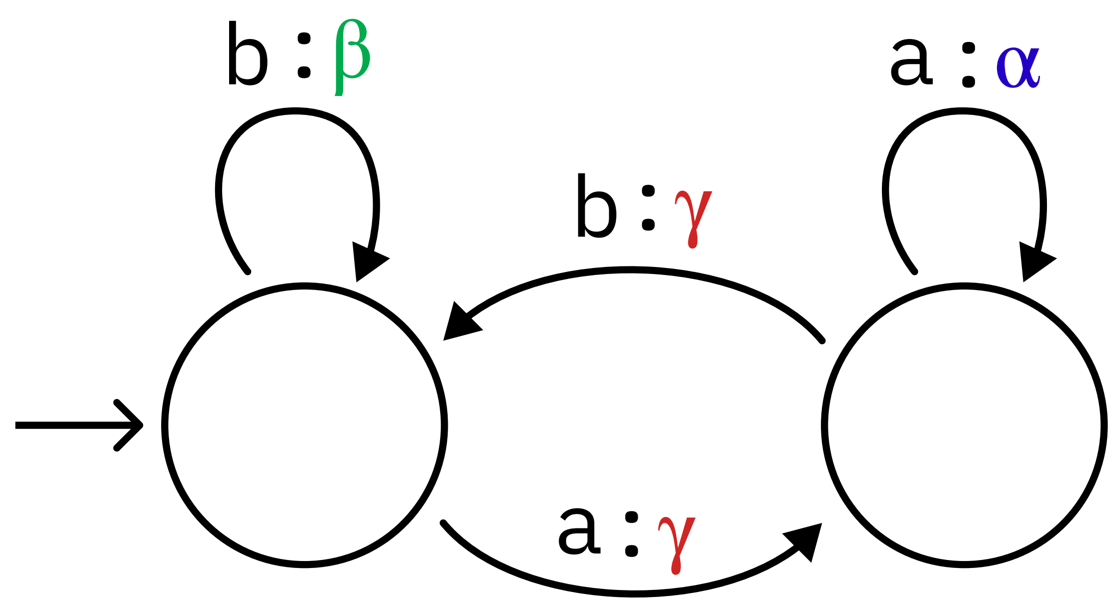

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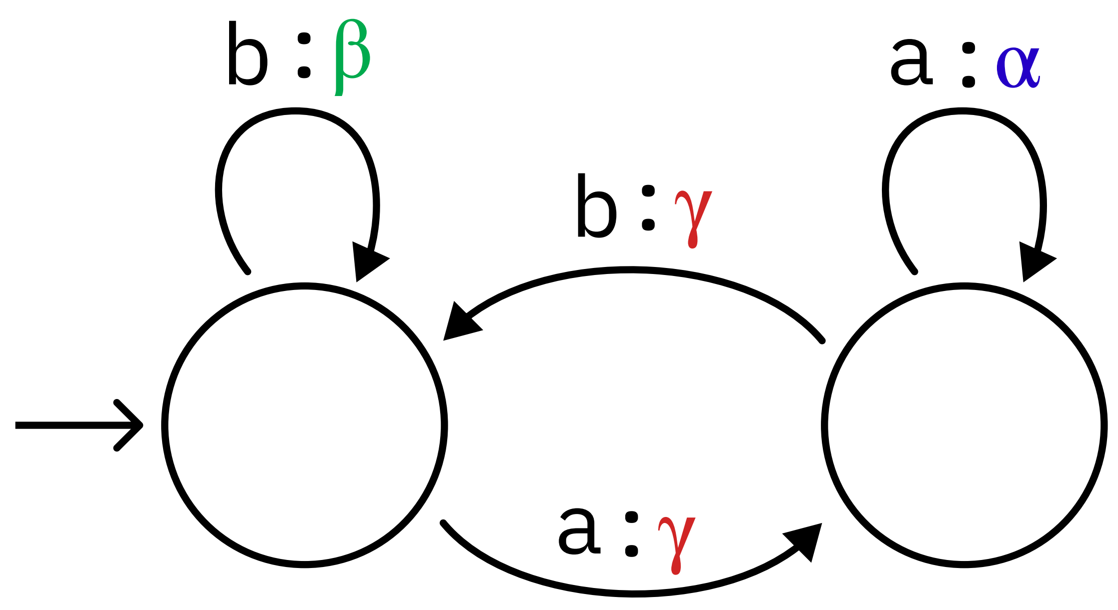

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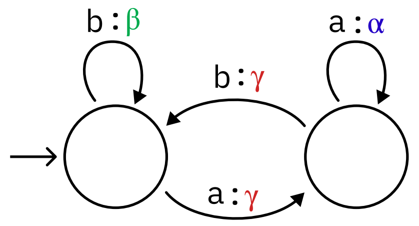

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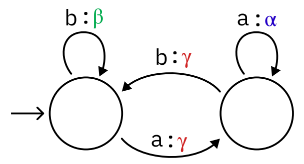

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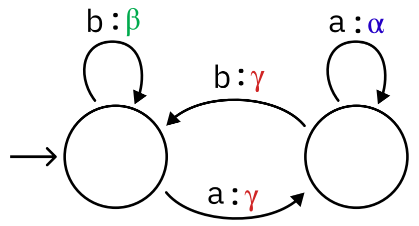

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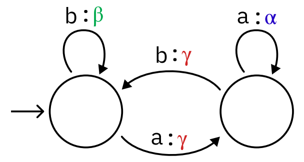

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Streett

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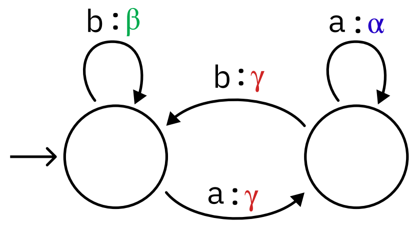

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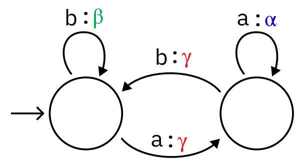

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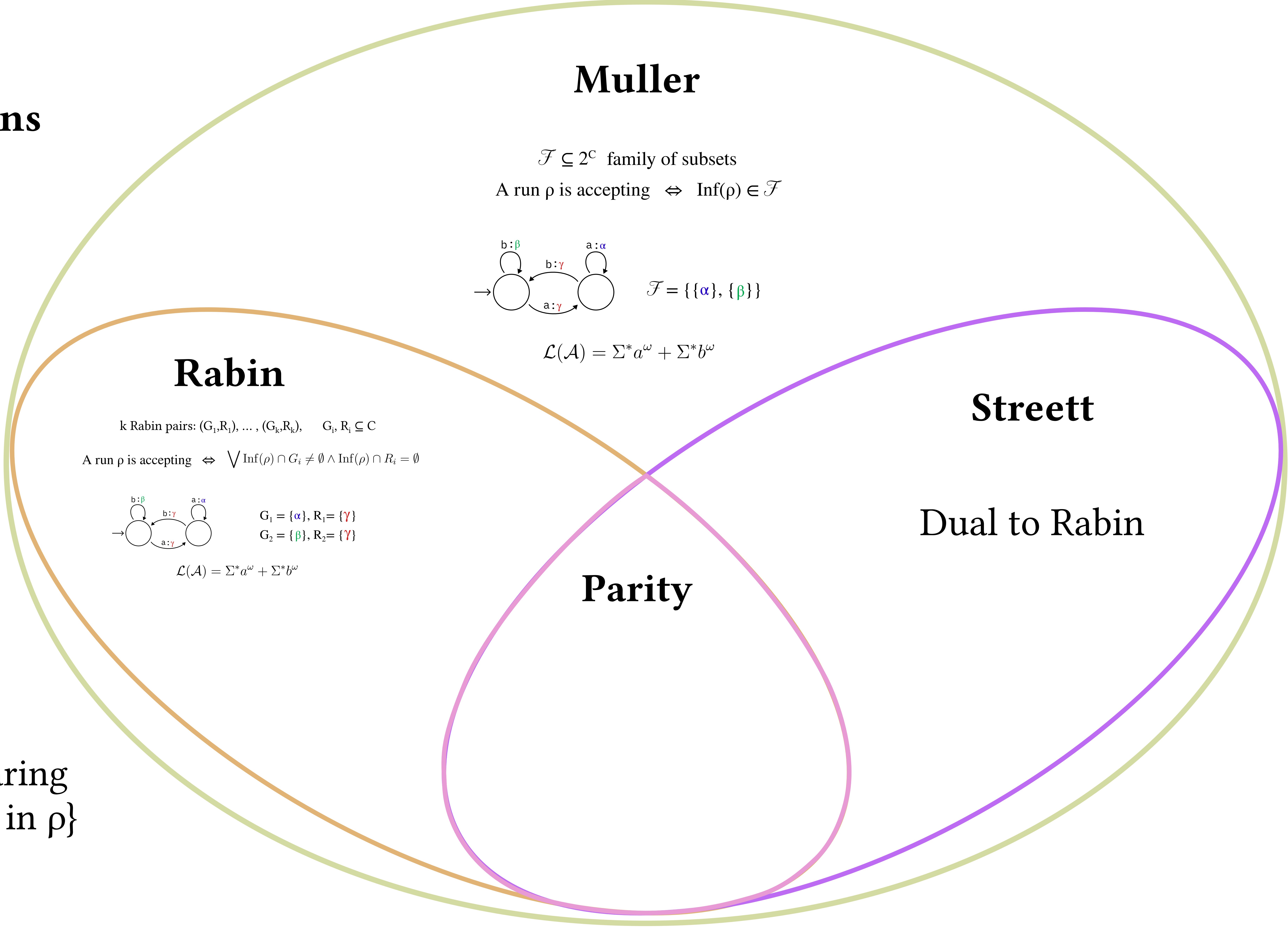

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We assume $C \subseteq \mathbb{N}$

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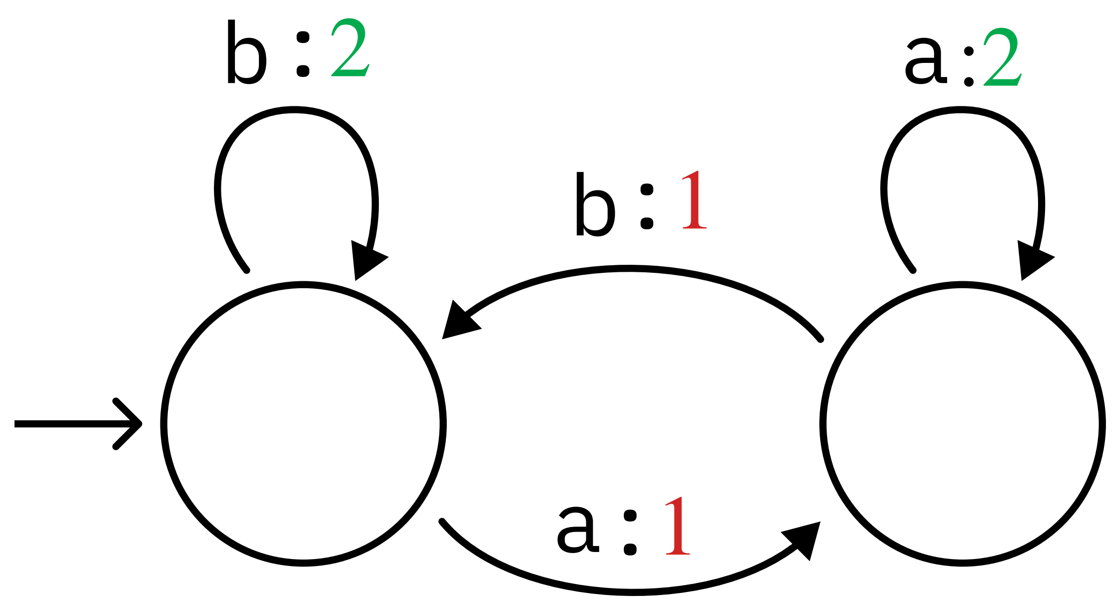

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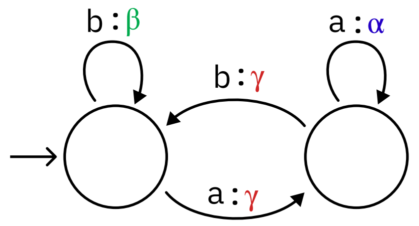

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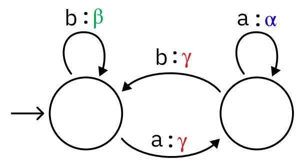

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Streett

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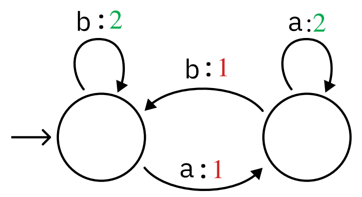

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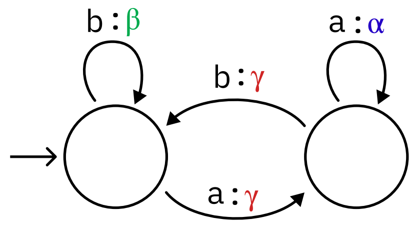

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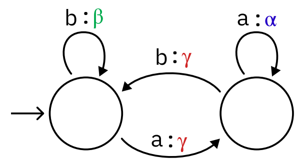

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WHY SO MANY?

Streett

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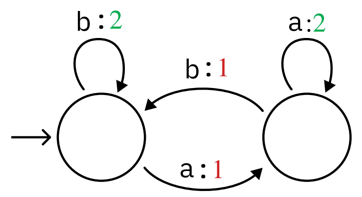

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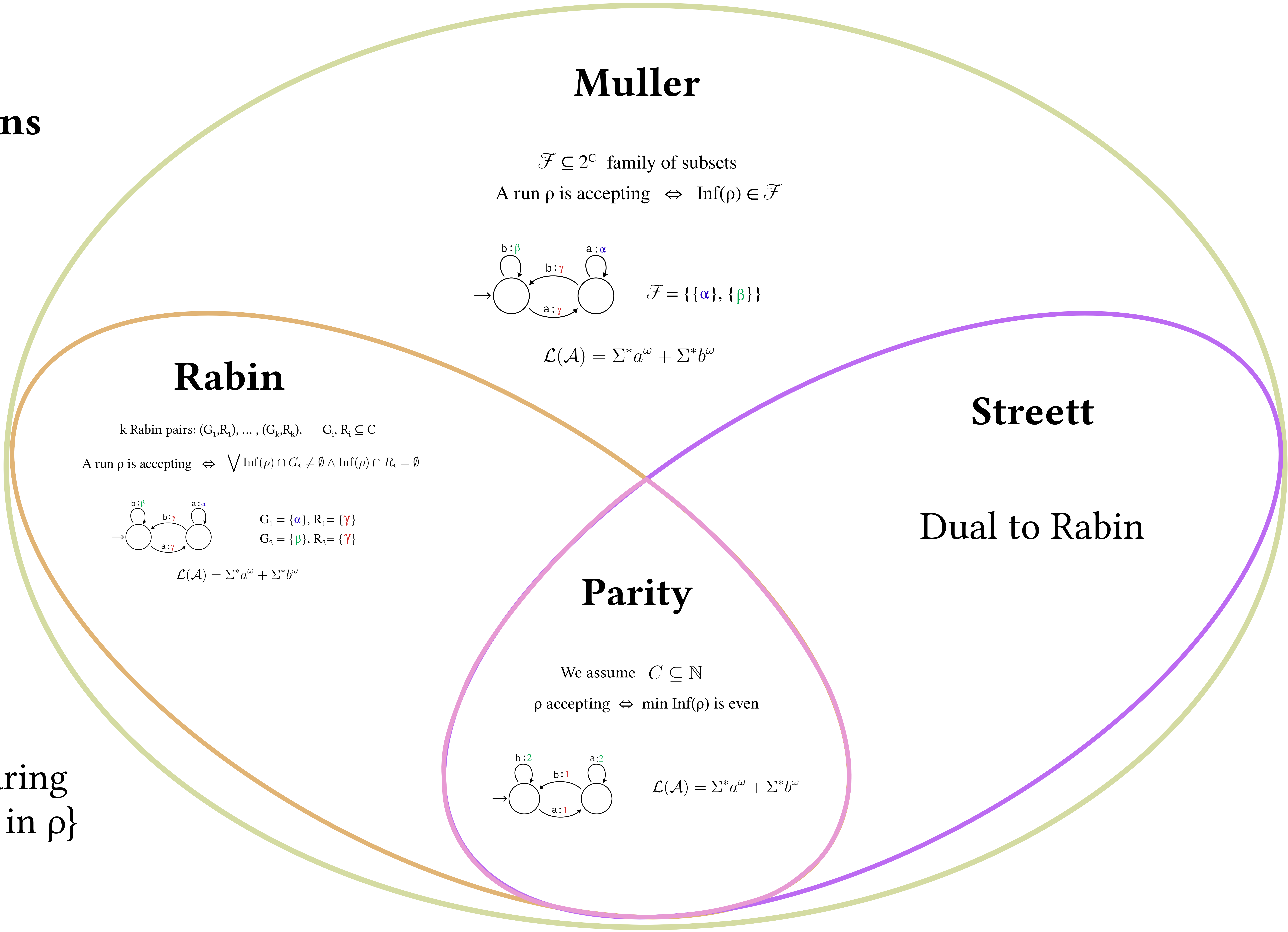

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Add states to the automaton to maximize simplification

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(Löding '99)

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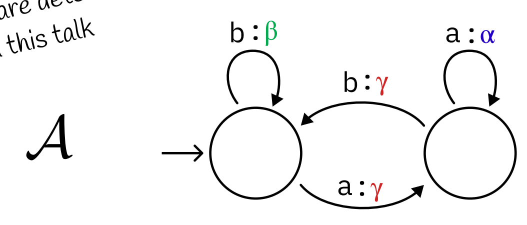

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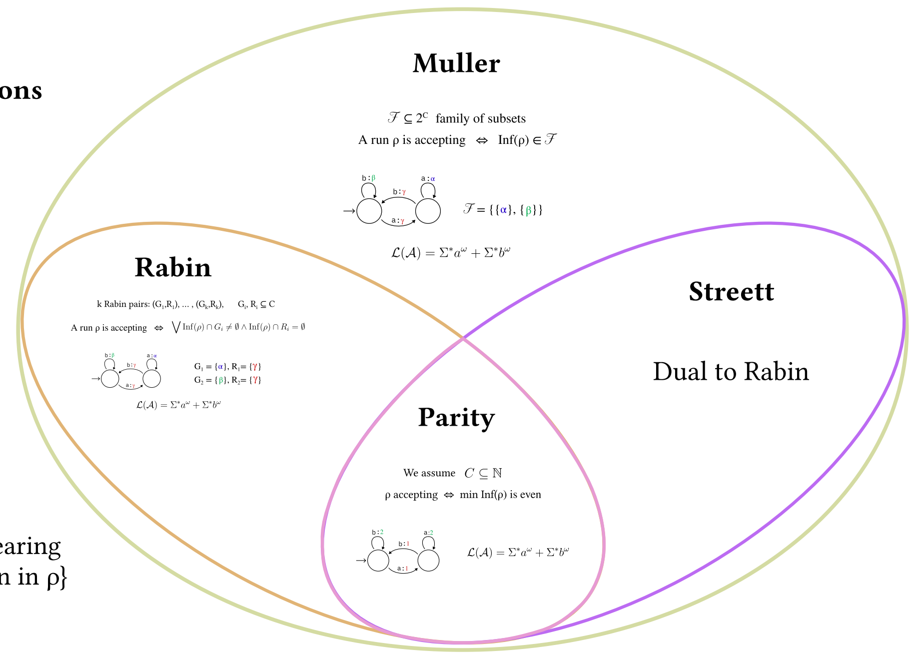

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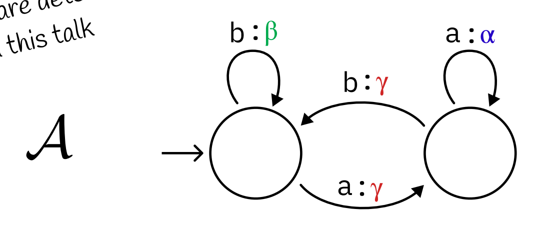
Simplifying ω-Automata through the Alternating Cycle Decomposition

Antonio Casares · University of Warsaw & Corto Mascle · LaBRI, University of Bordeaux

Automata over infinite words (ω-automata)

Automata are deterministic in this talk

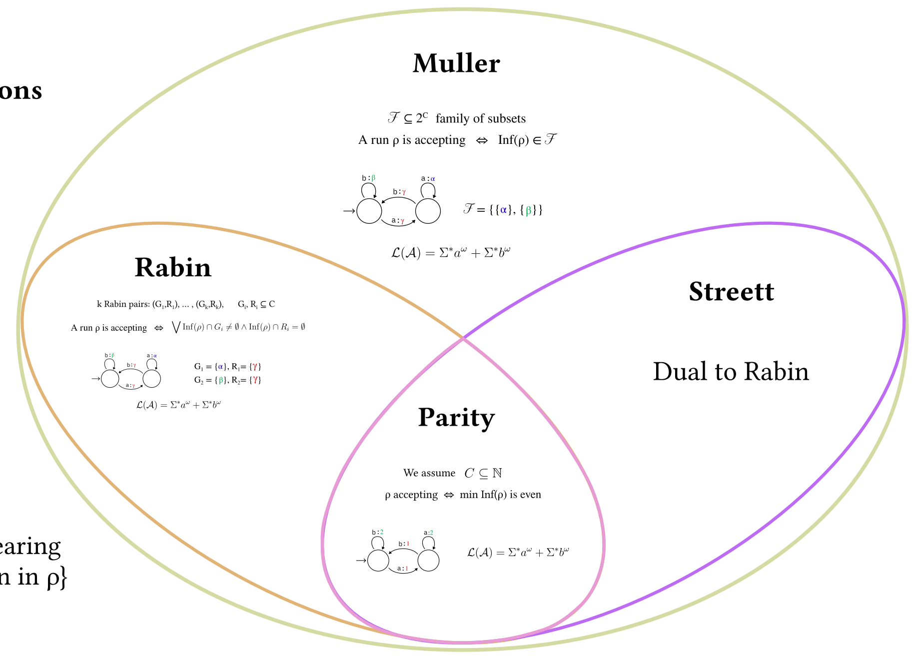


Input $w \in \Sigma^\omega$ ⟶ Infinite path ρ

Different Acceptance Conditions

$C = \{\alpha, \beta, \gamma\}$
output colours

Inf(ρ) = {Colours appearing infinitely often in ρ}



More general conditions → Automata are simpler to produce

- LTL → Det. Muller automata
- ND Büchi → Det. Rabin automata

Simpler conditions → More efficient algorithms

Complexity of solving 2-player games. n = Size of input graph

Muller	PSPACE-complete	$\mathcal{O}(k^{5k}n^5)$	k = #Colours
Rabin	NP-complete	$\mathcal{O}(n^{k+3}kk!)$	k = #Rabin pair
Parity	NP∩coNP. In P?	$\mathcal{O}(n^{\log(k)})$	k = #Colours

Goal: Simplify the acceptance condition of automata

a) Simplify the type (Muller → Rabin → Parity)

b) Minimise #Colours / #Rabin pairs

Approach 1

Add states to the automaton to maximize simplification

√ Well understood

✦ Optimal transformation to parity automata *(C. Colcombet, Fijalkow '21)*

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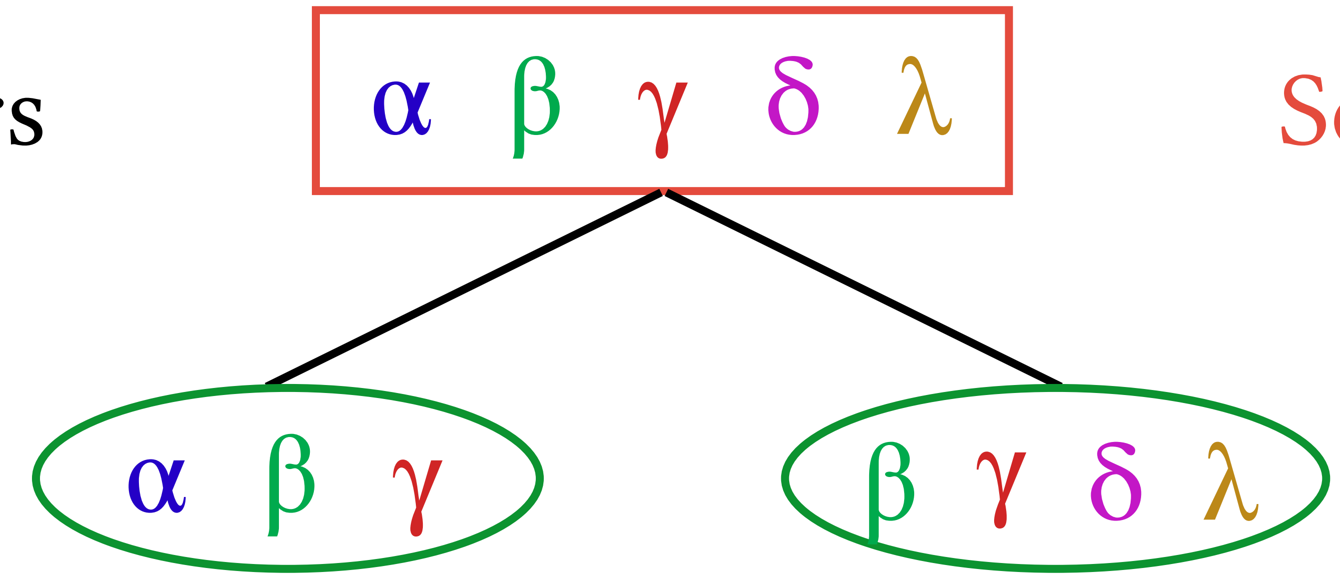

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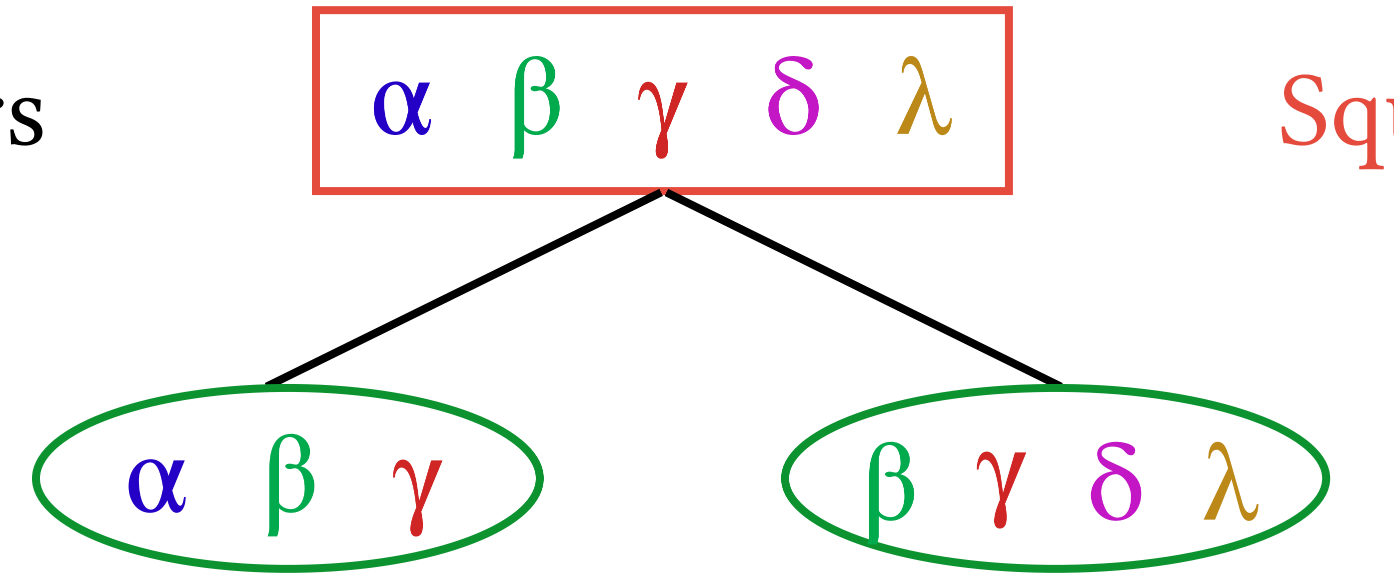

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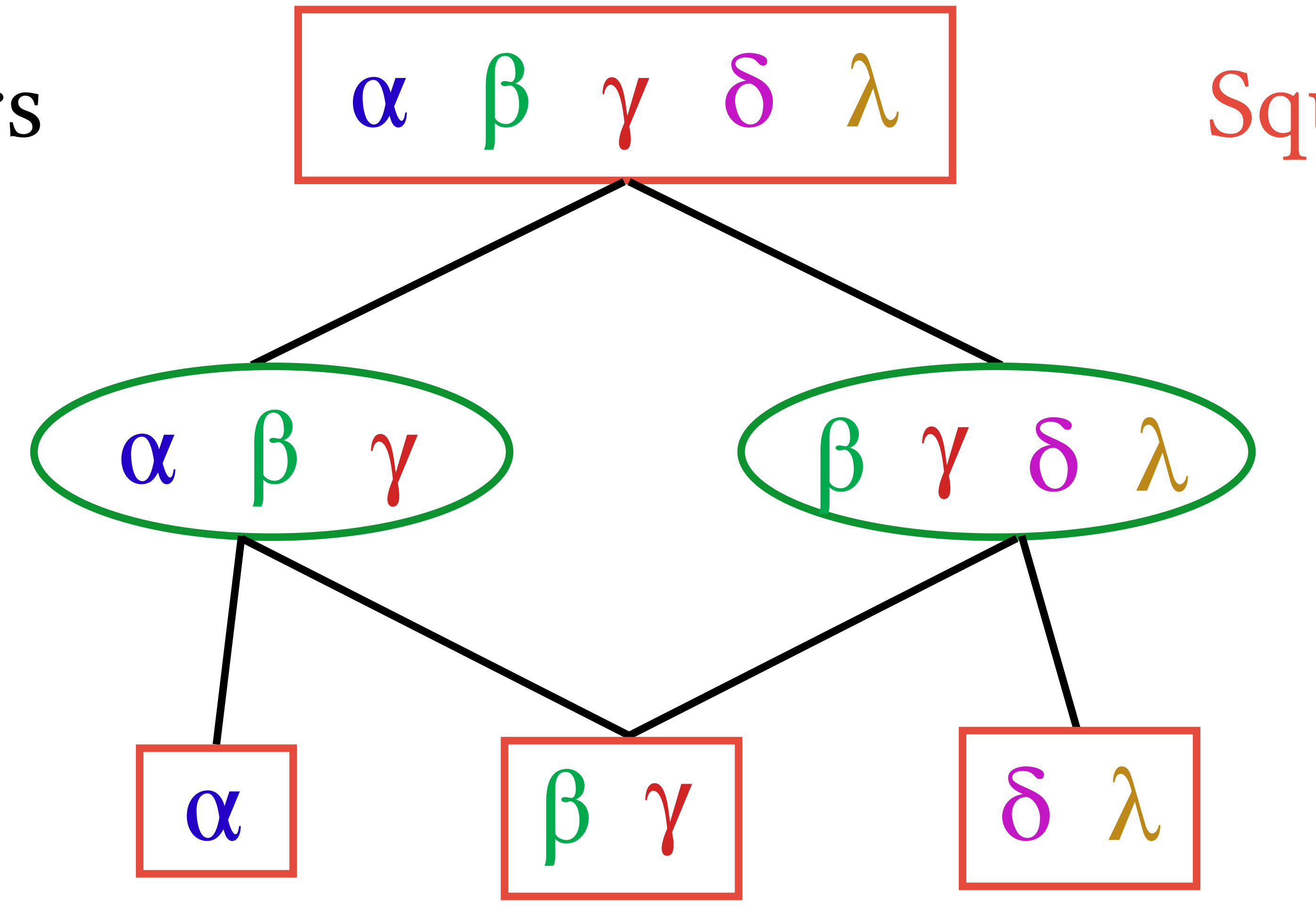

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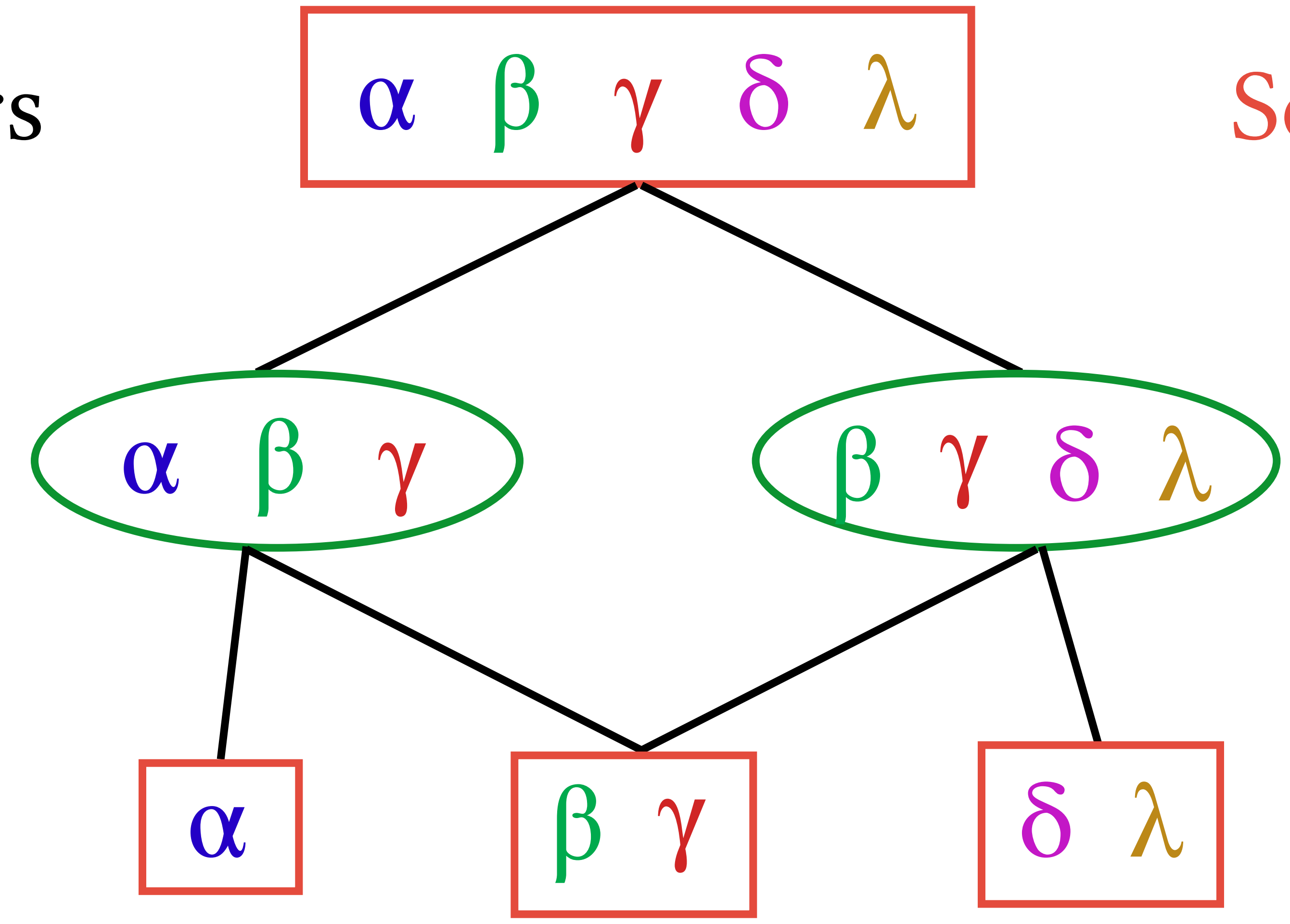

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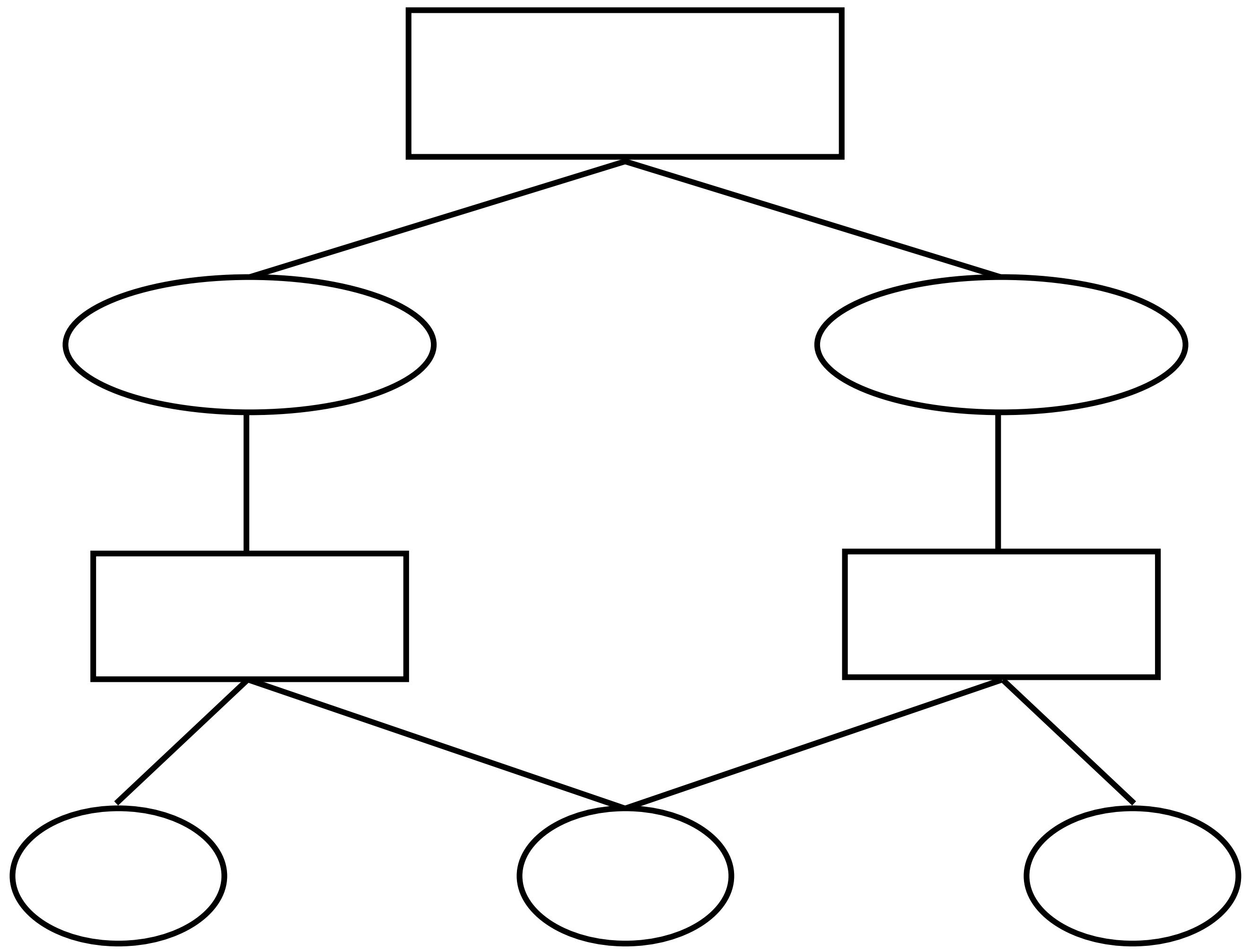

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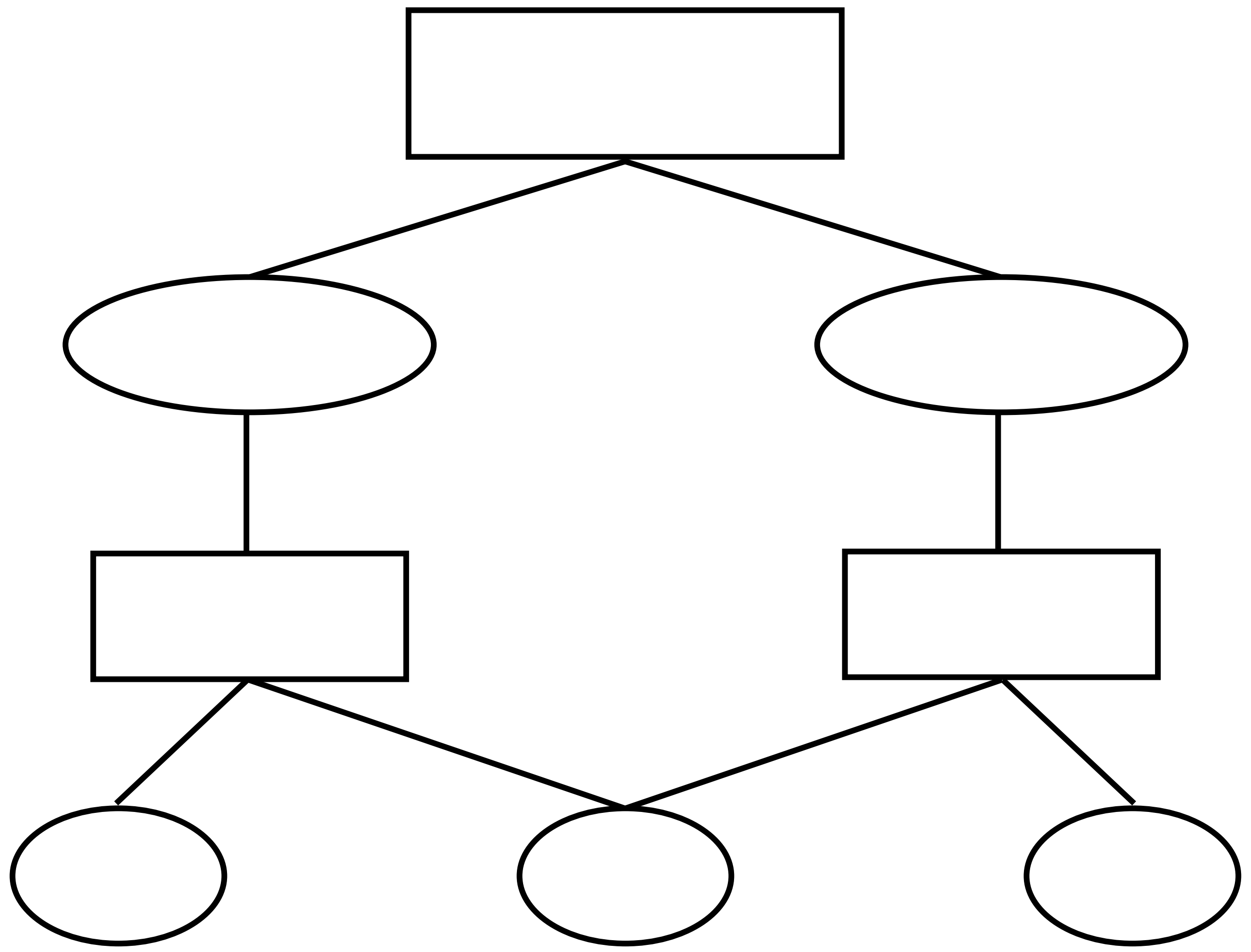

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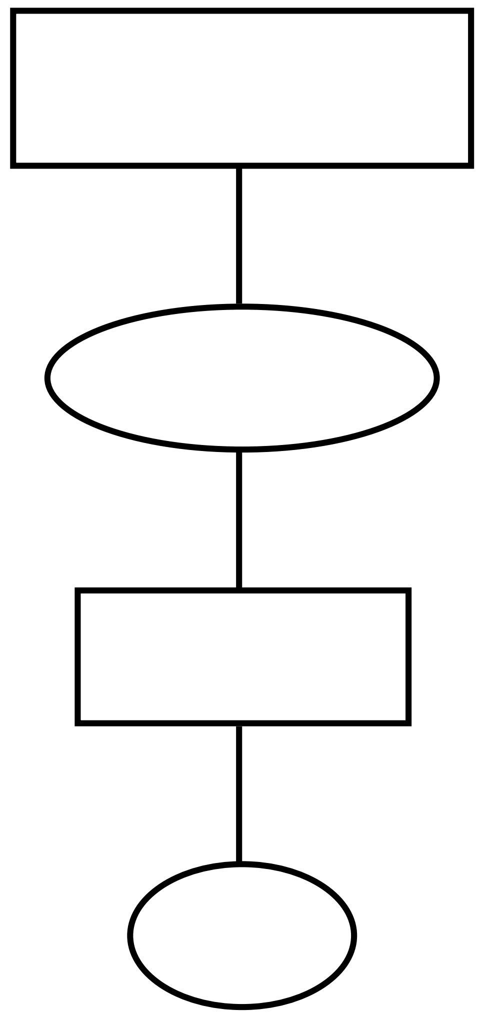

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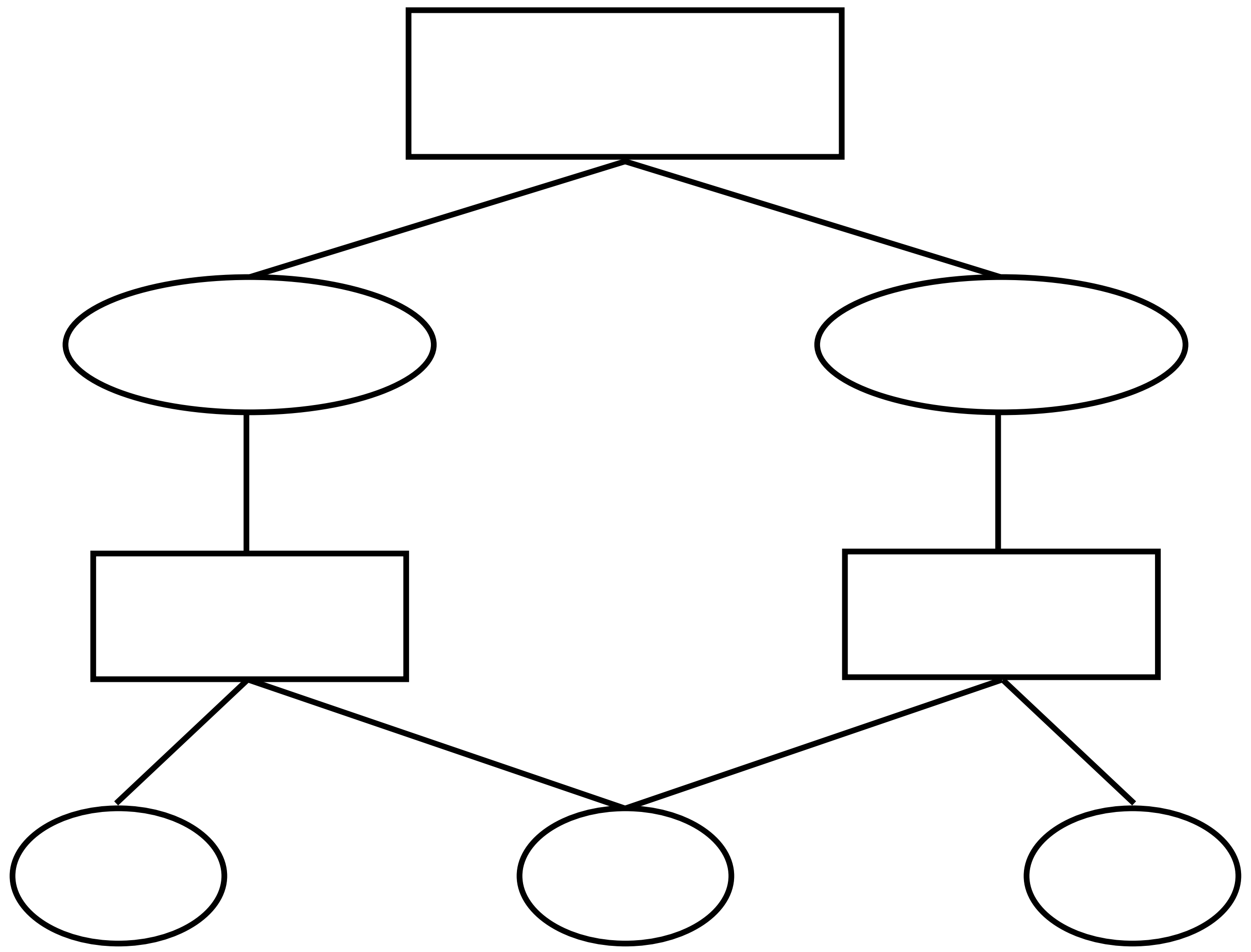

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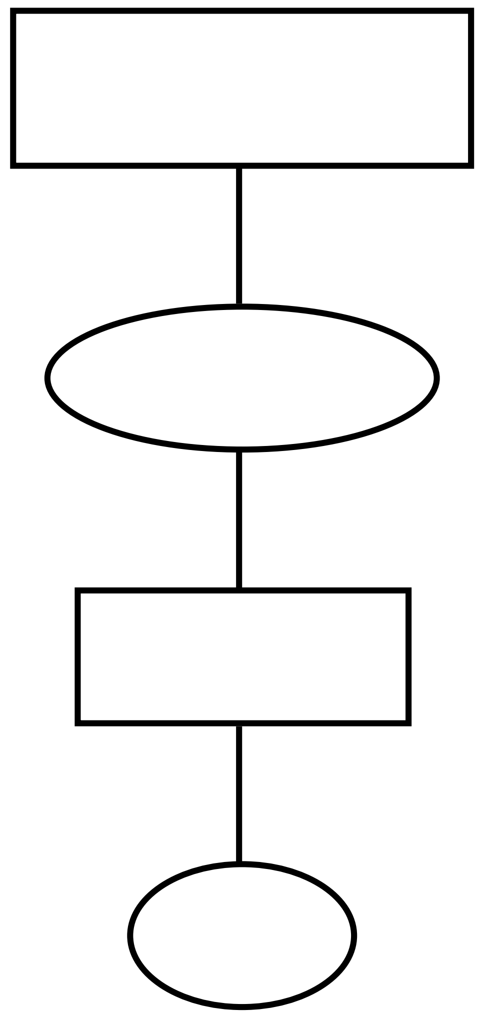

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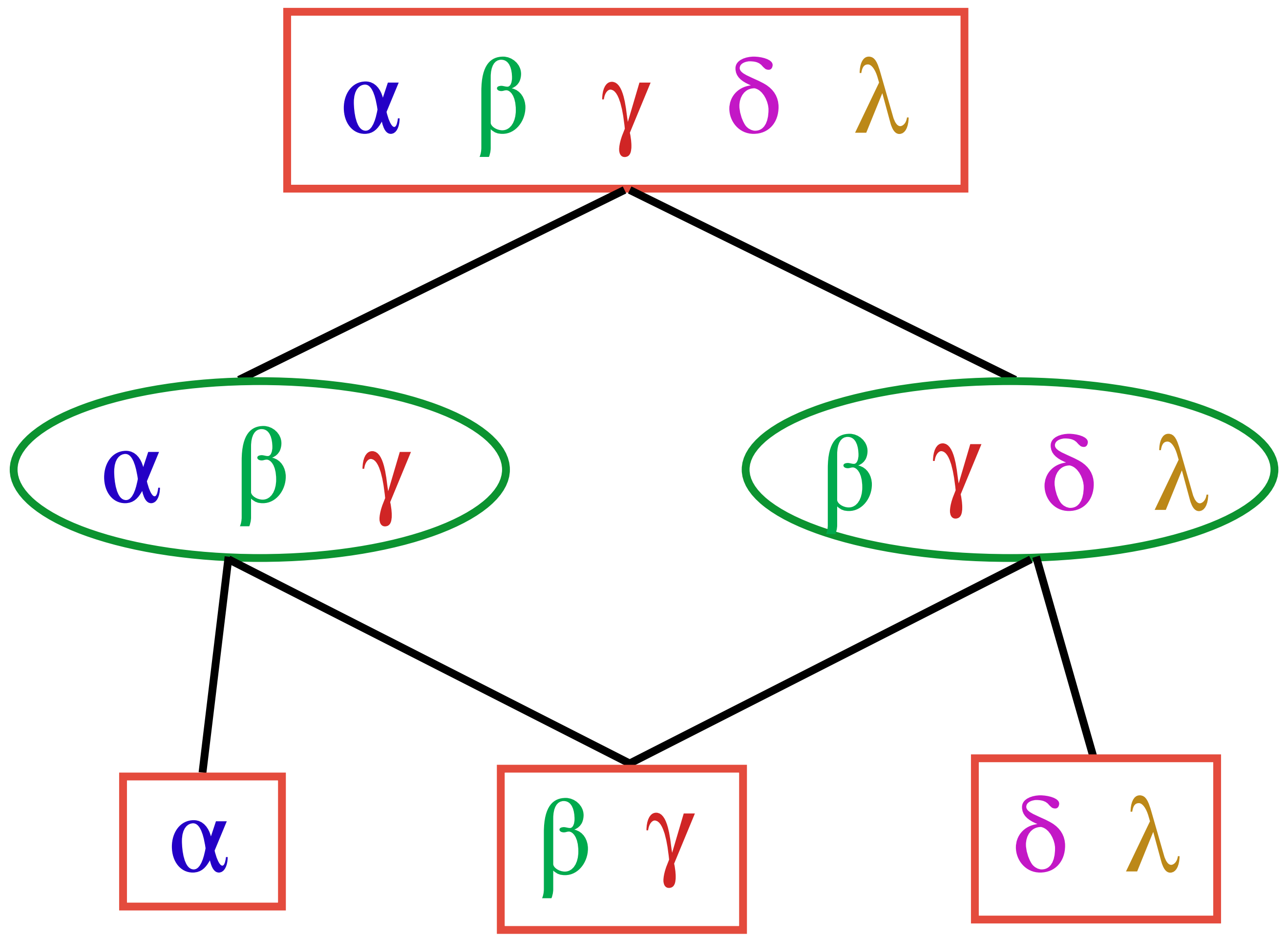

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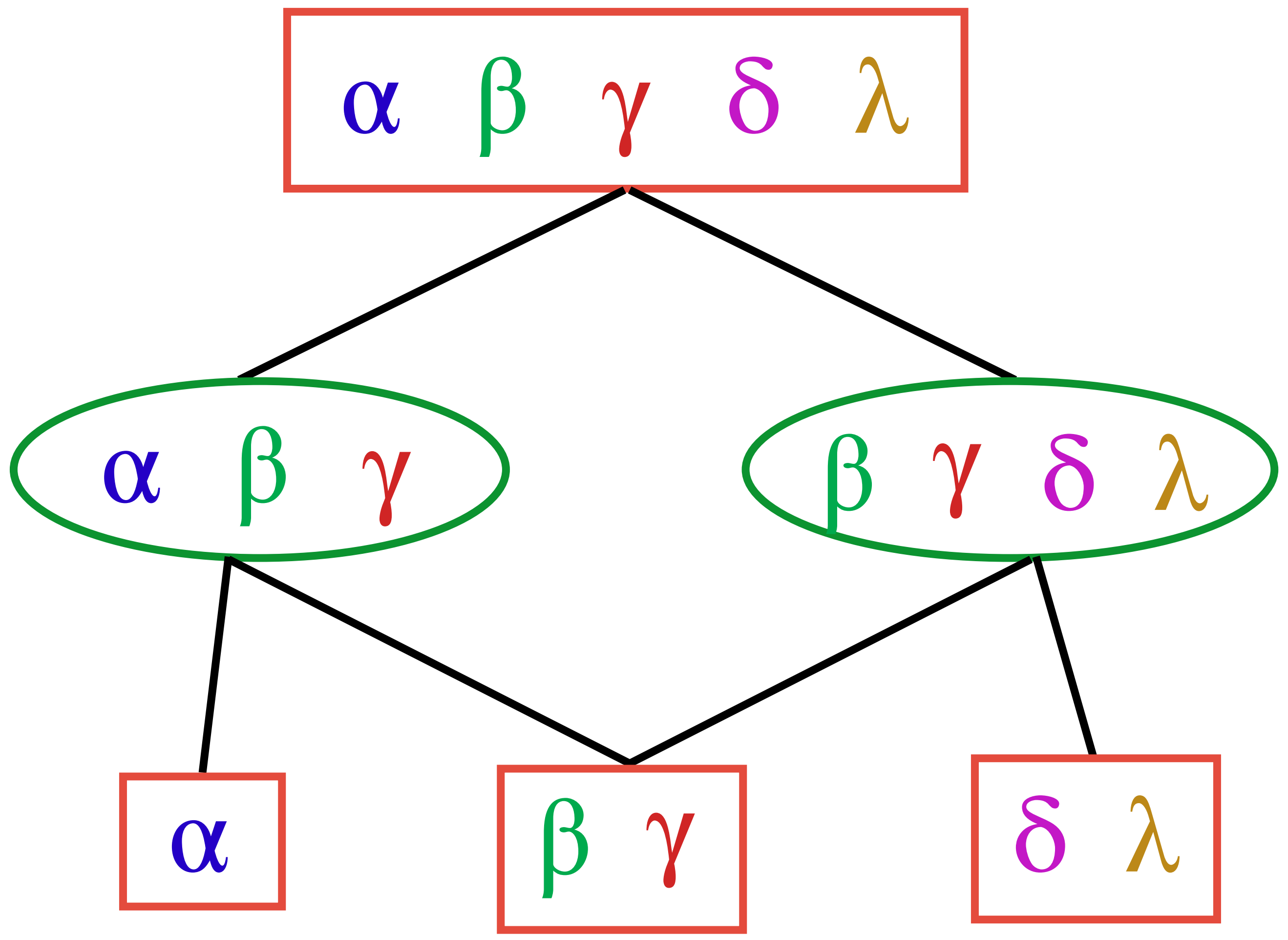

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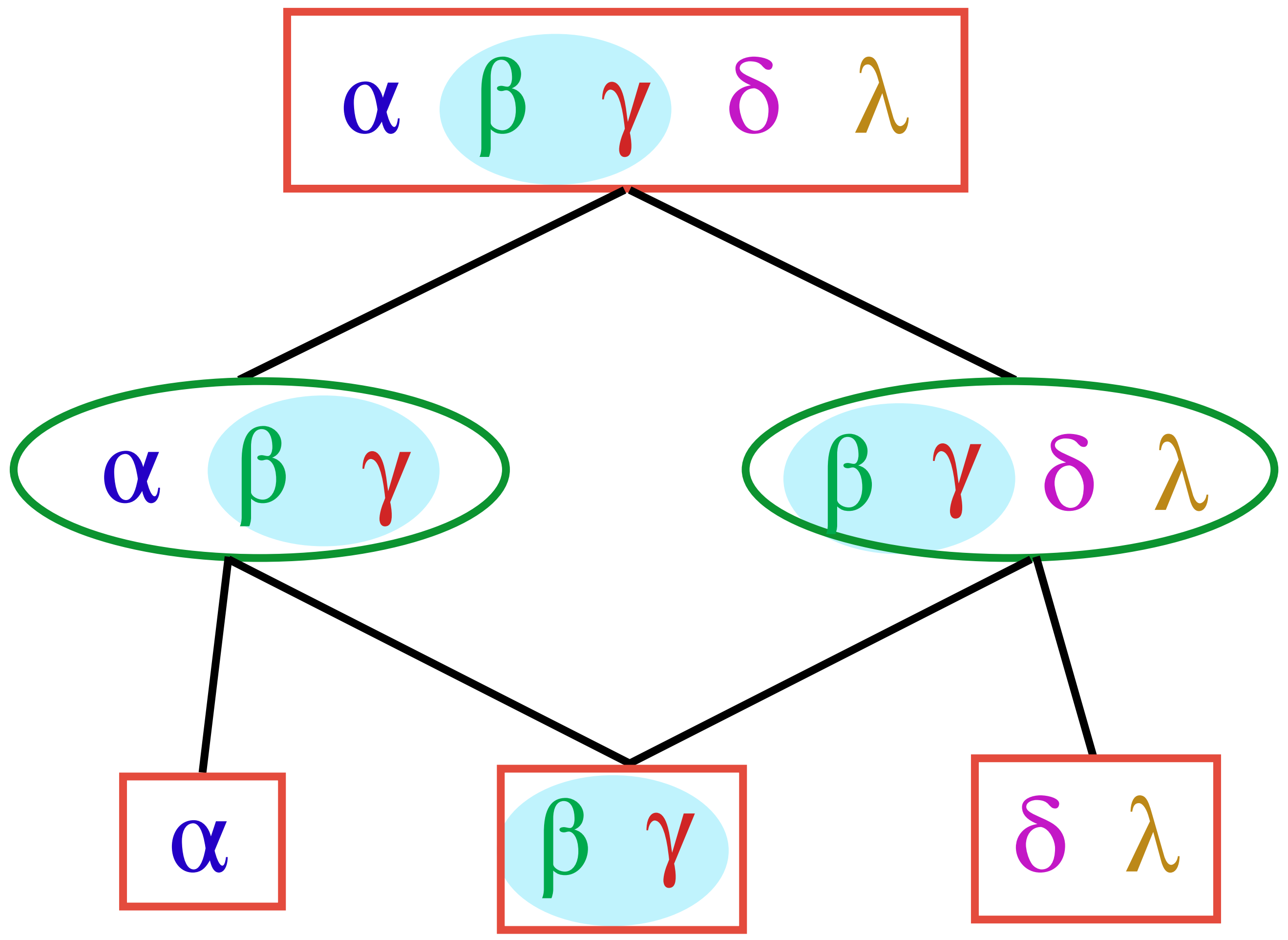

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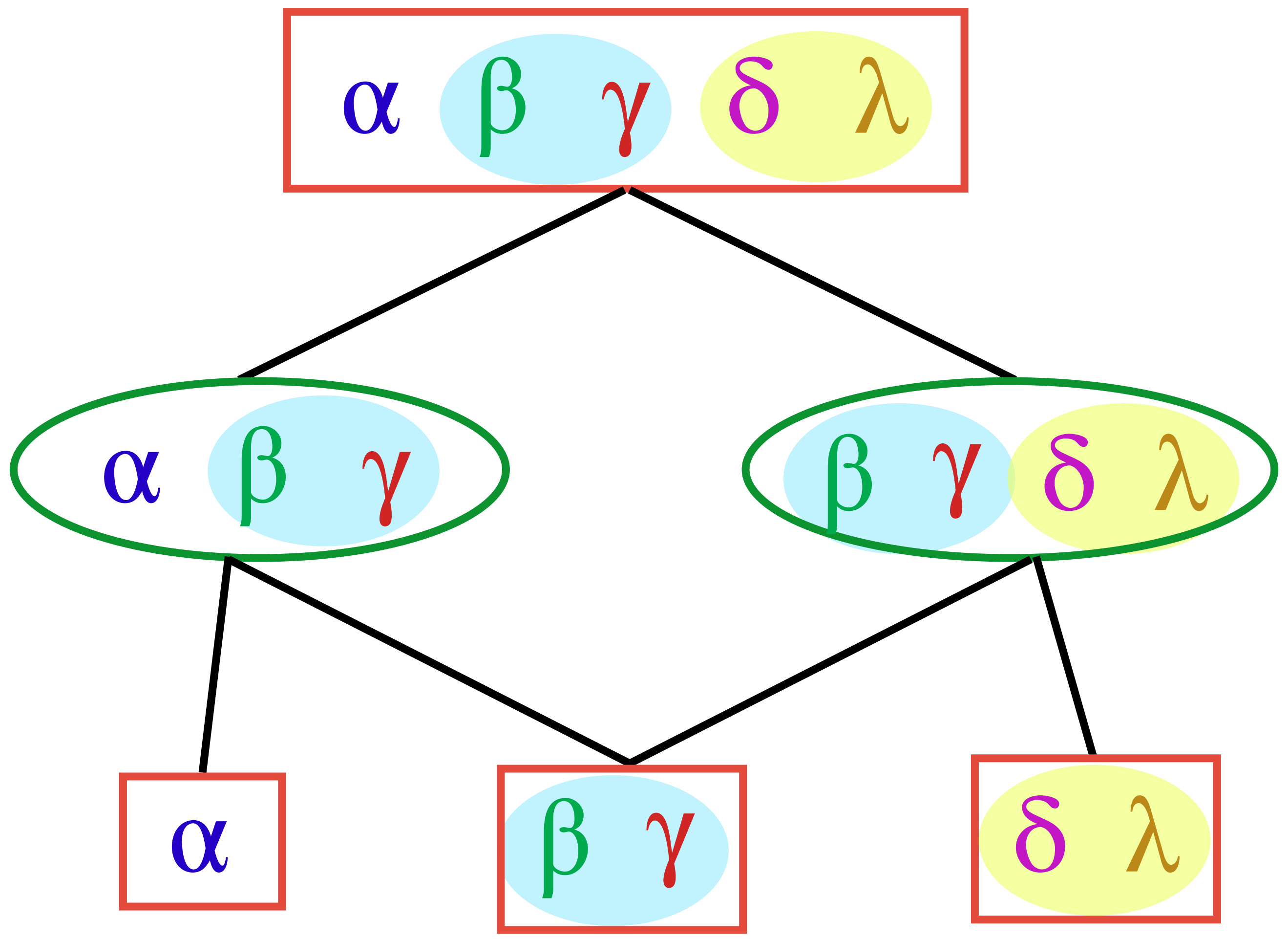

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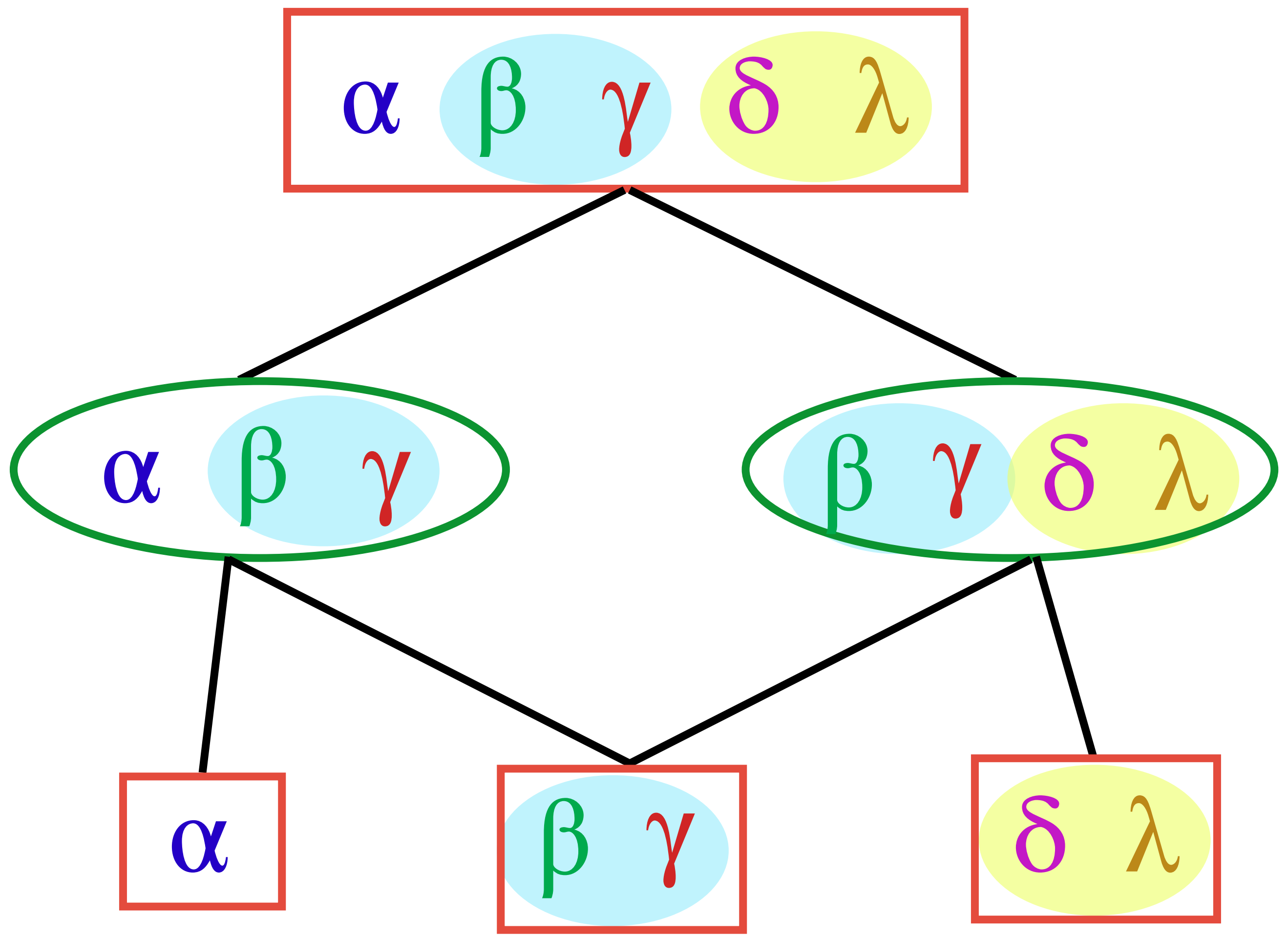

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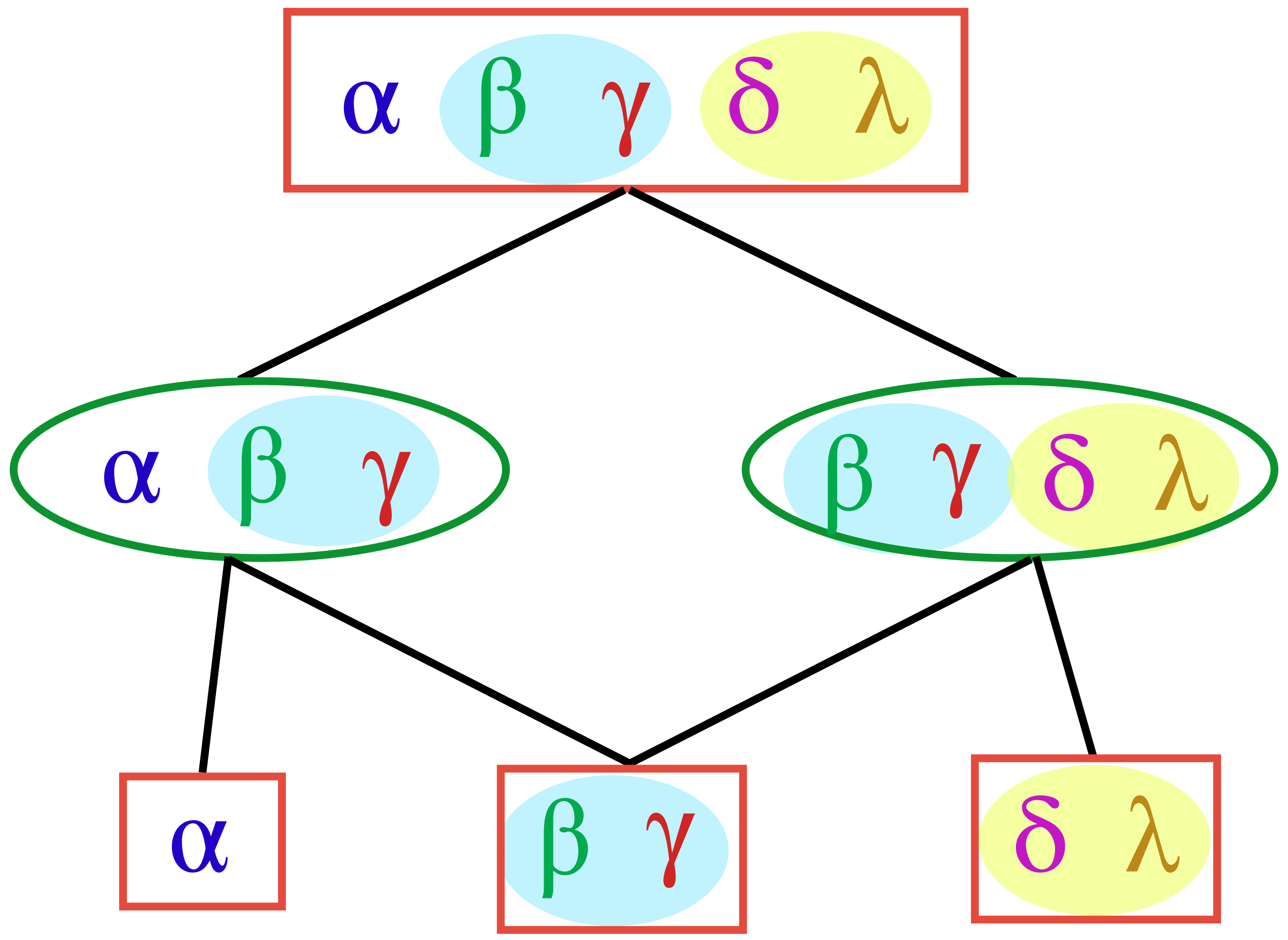

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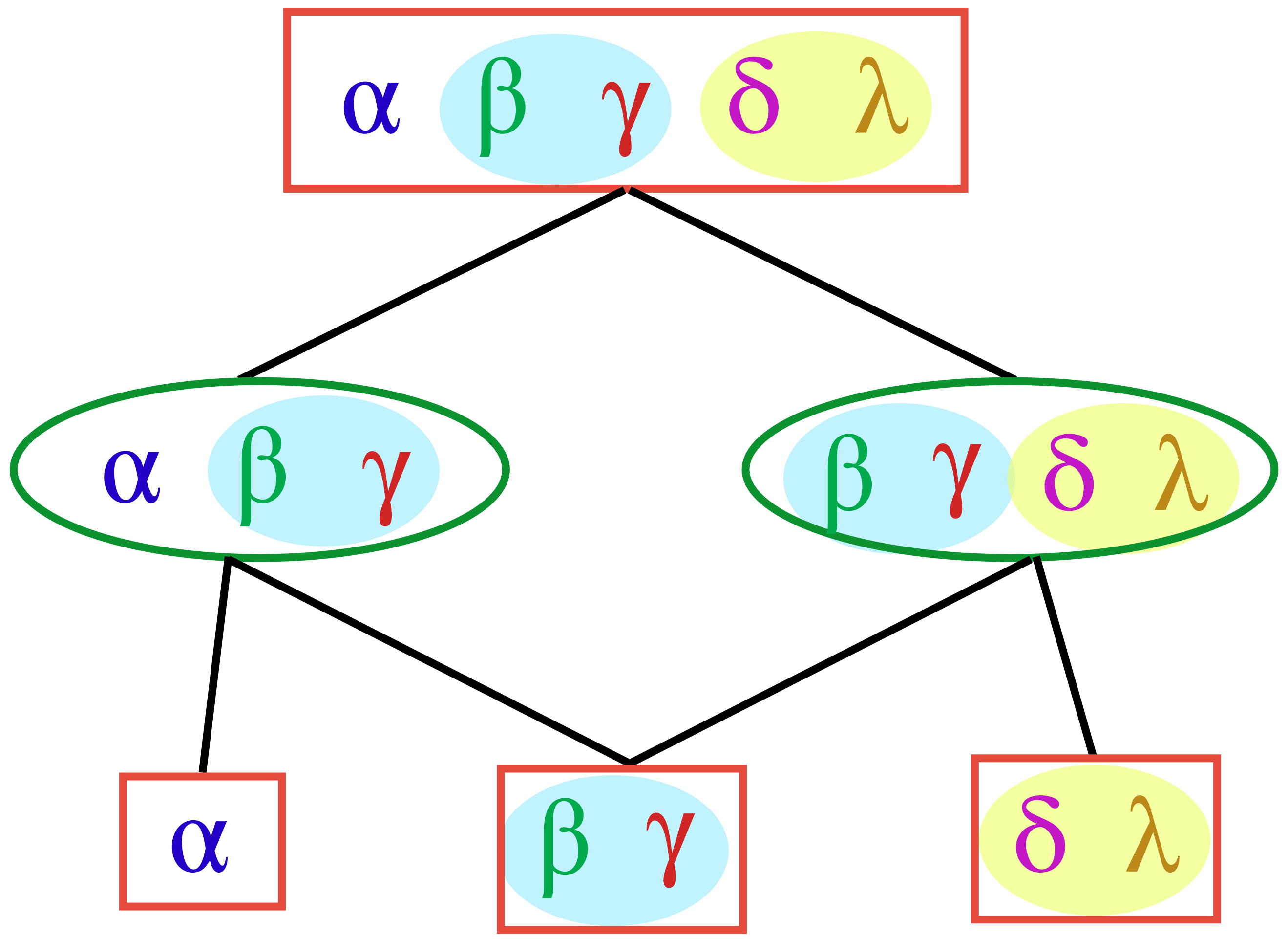

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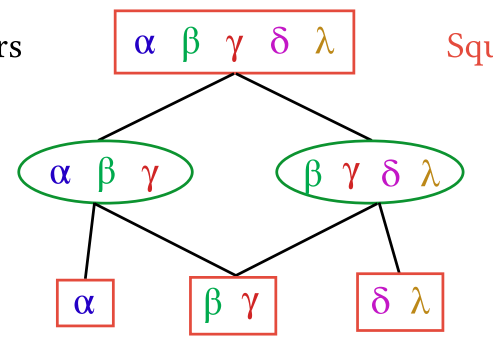

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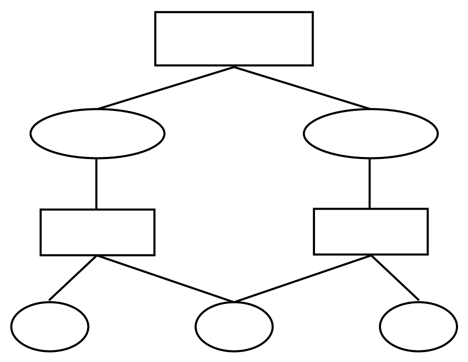

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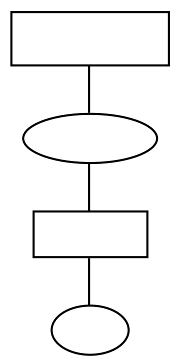

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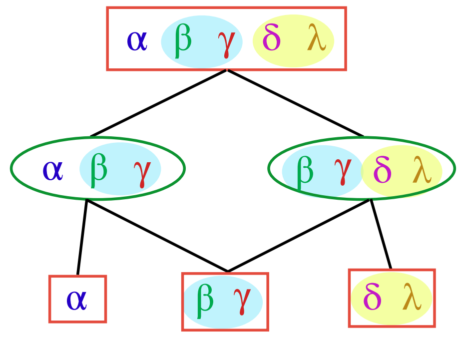

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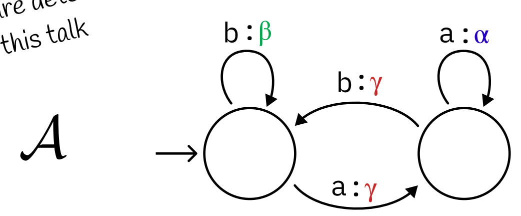

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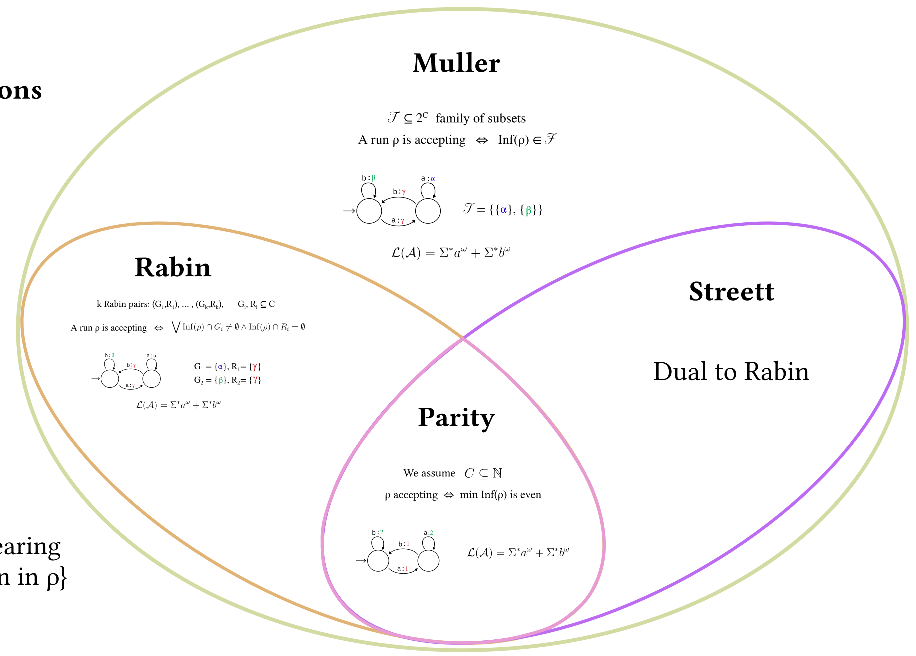

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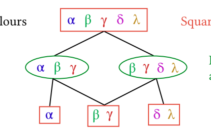

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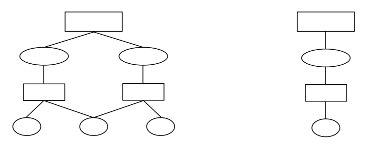

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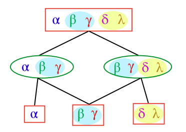

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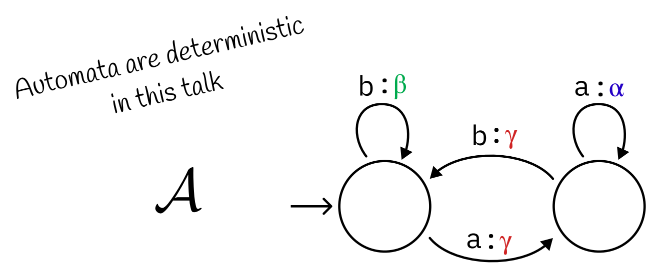

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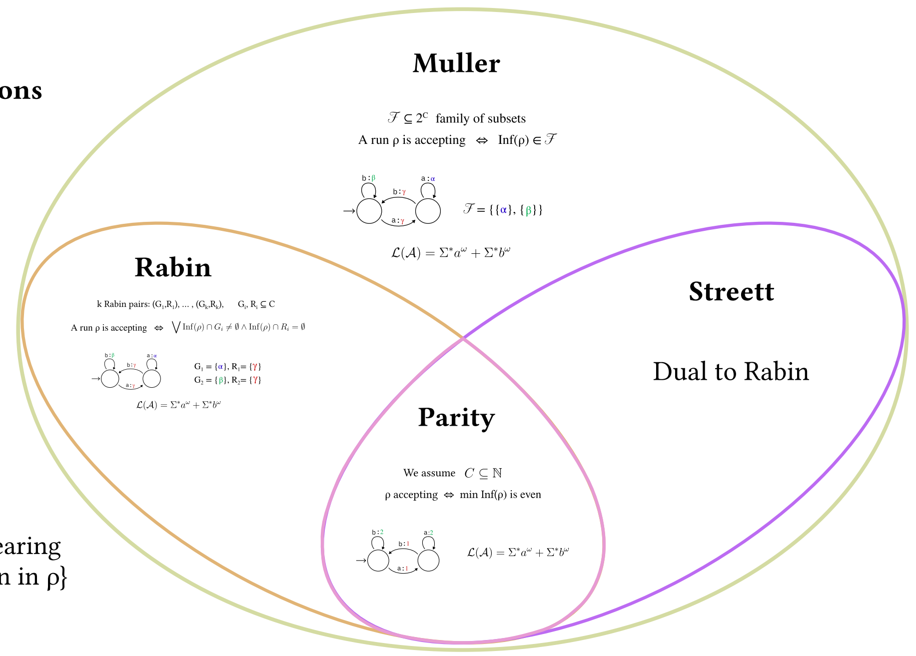

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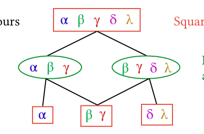

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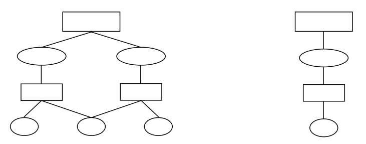

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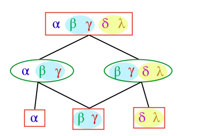

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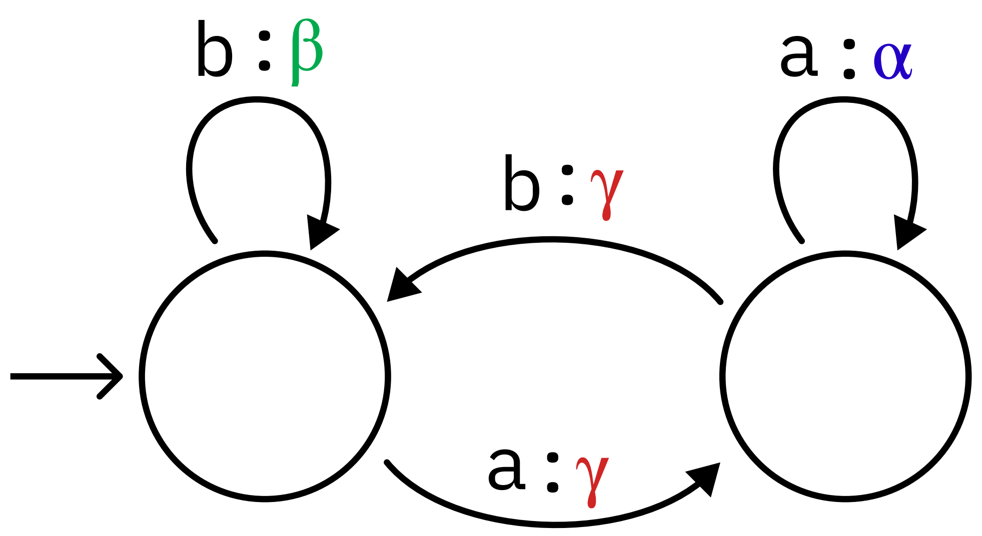

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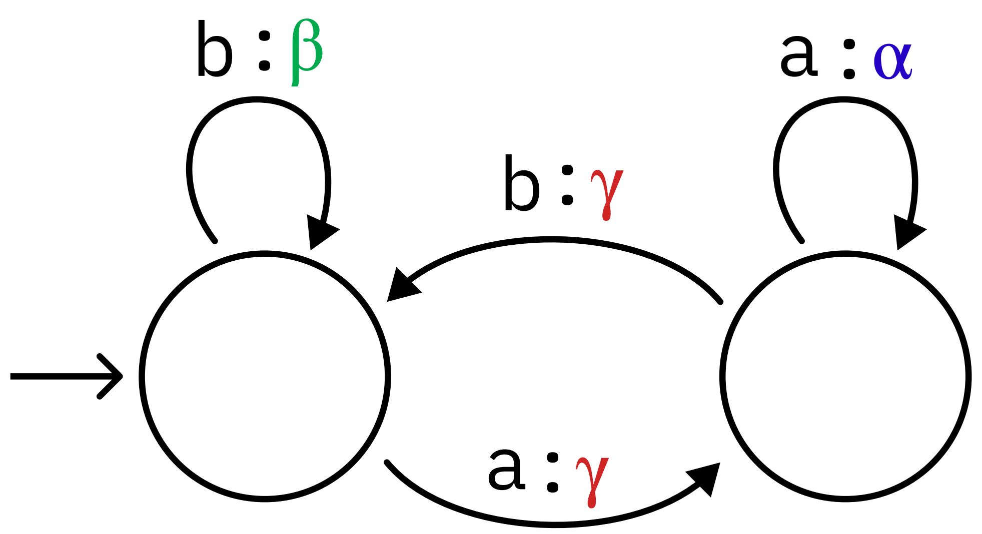

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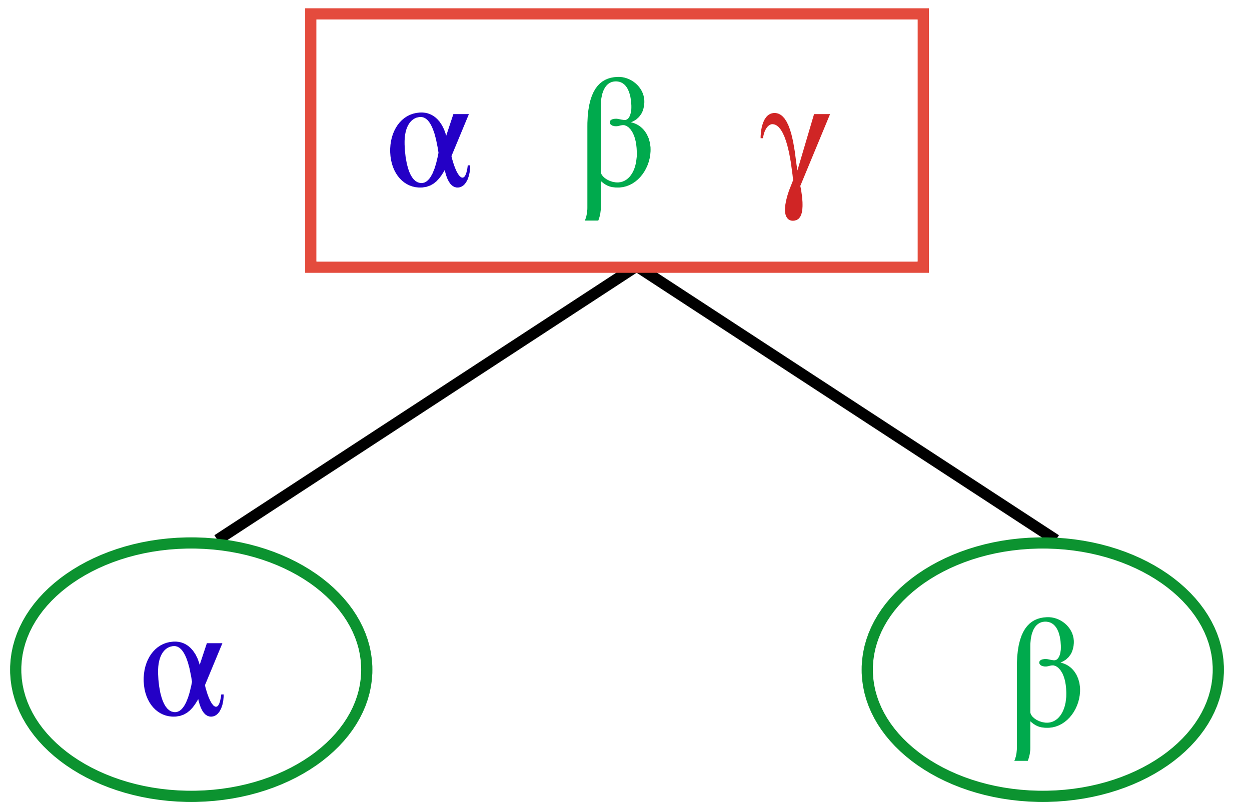

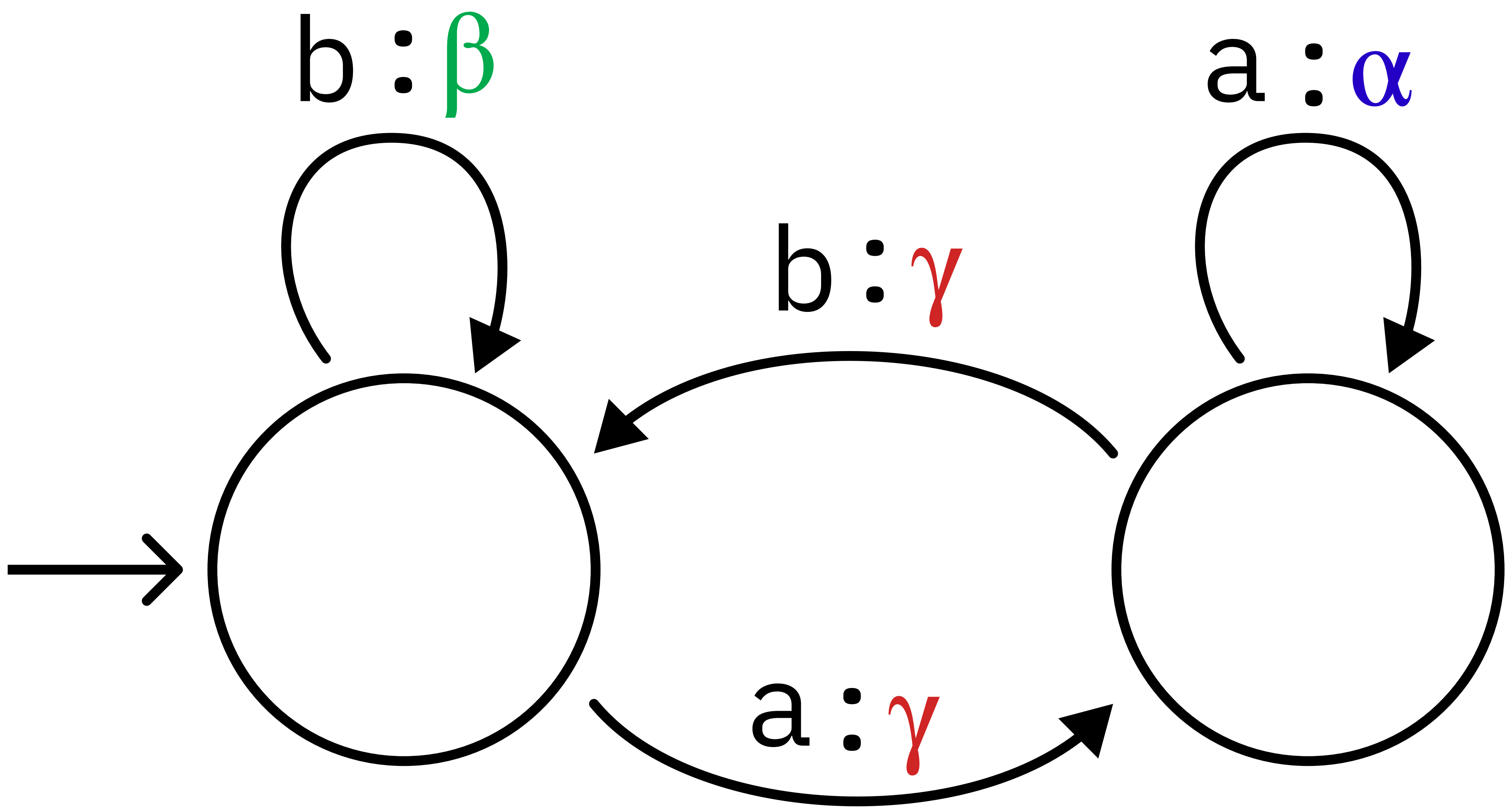

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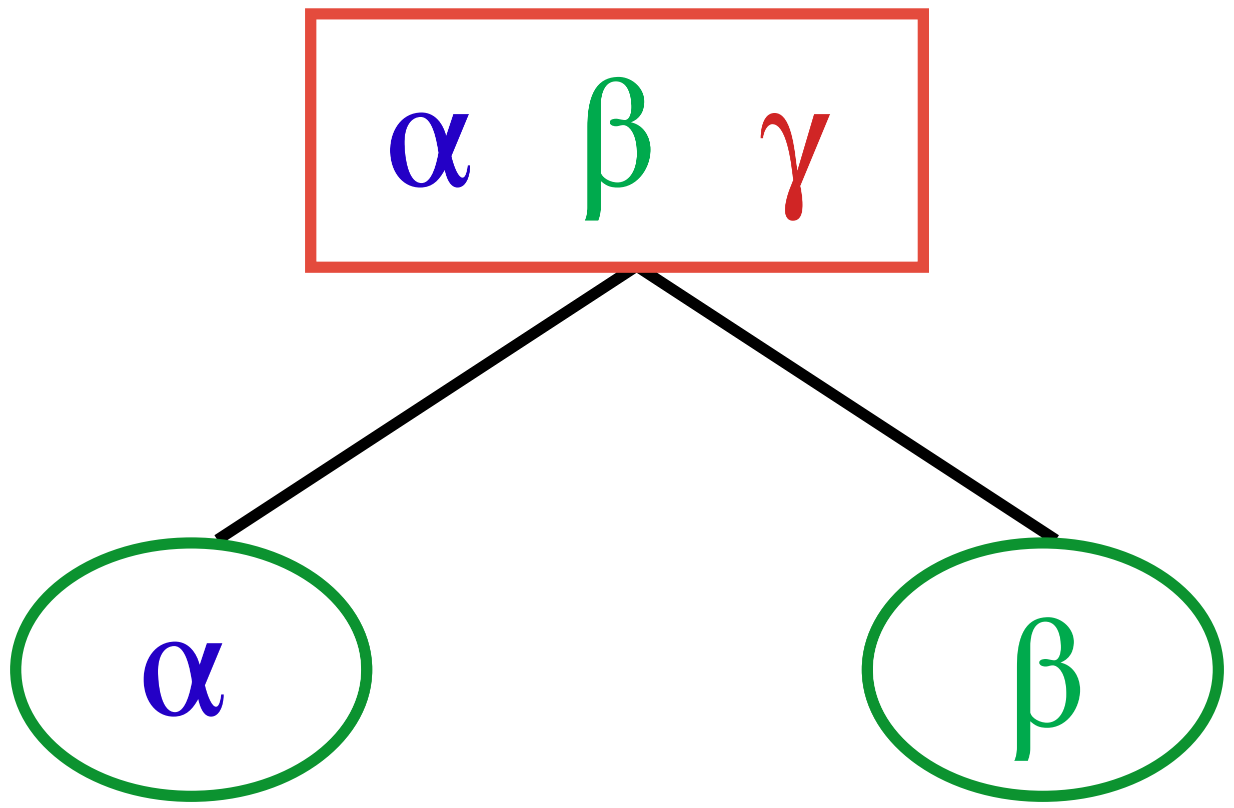

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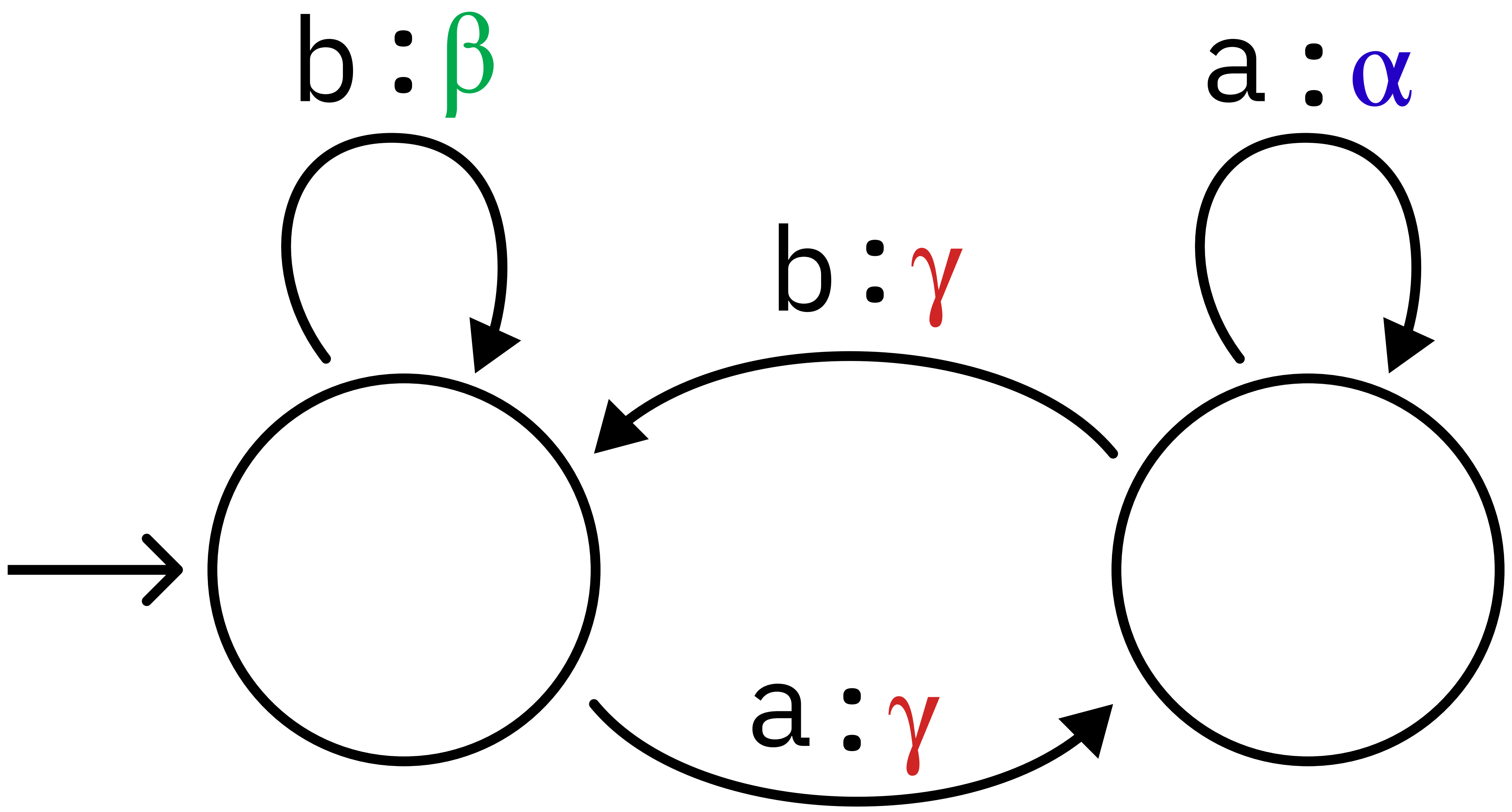

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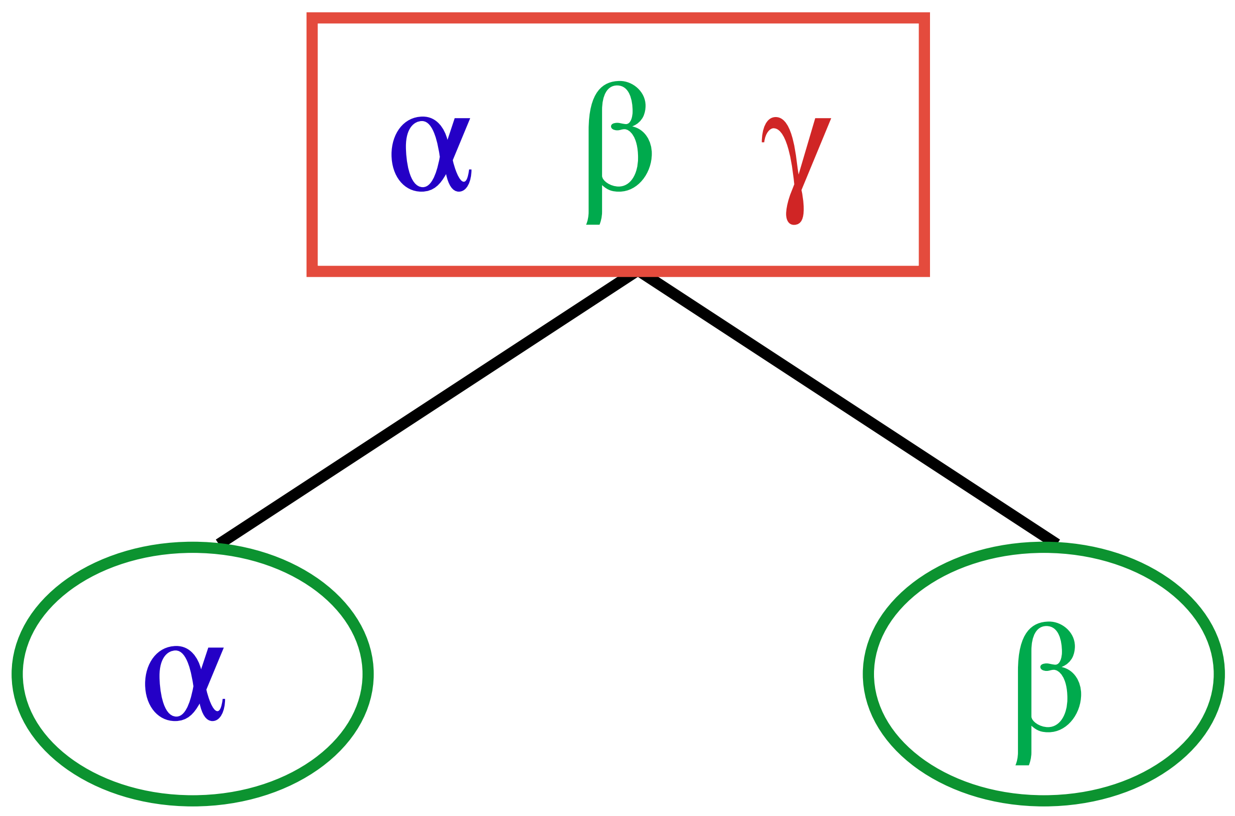

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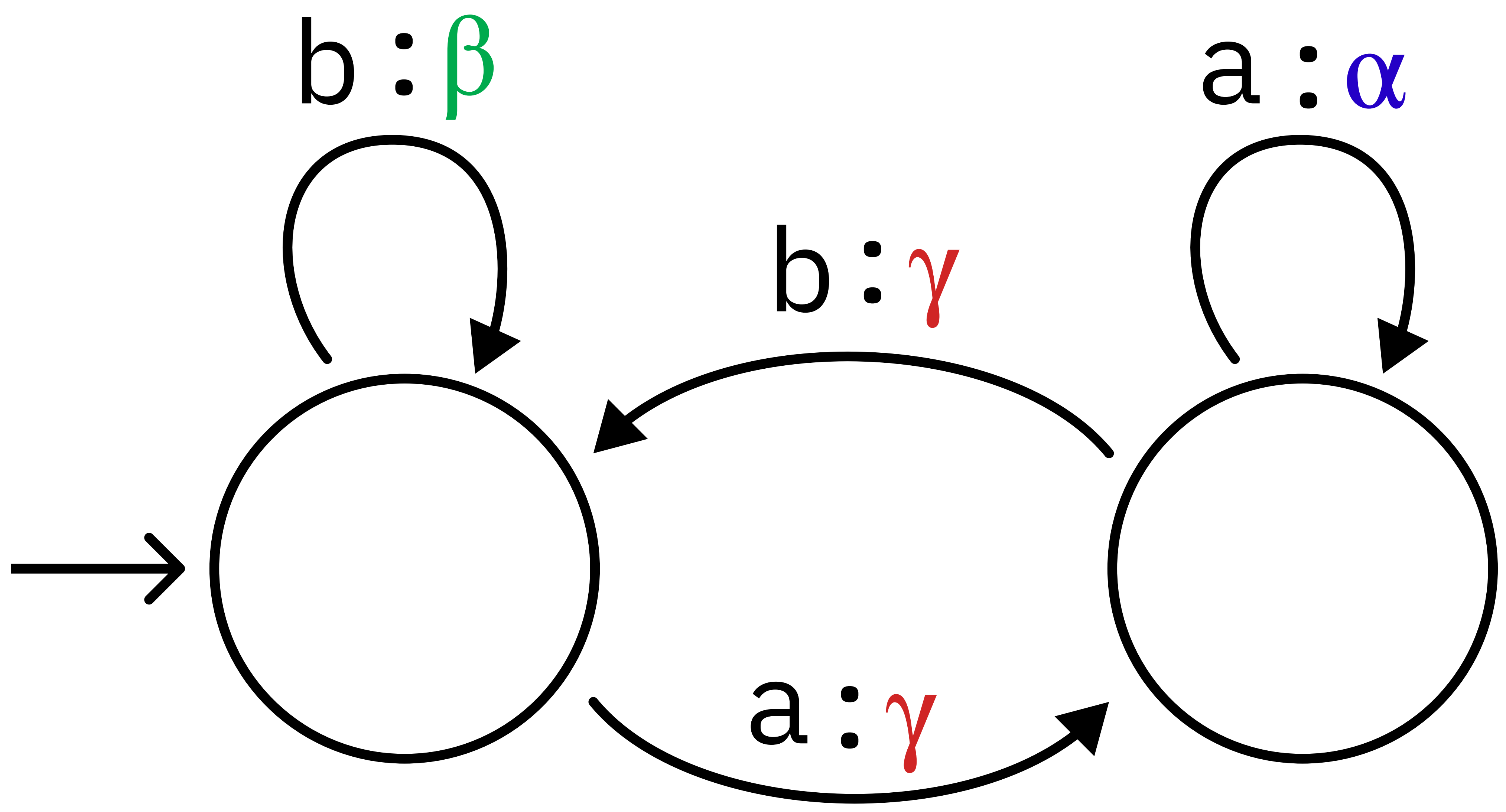

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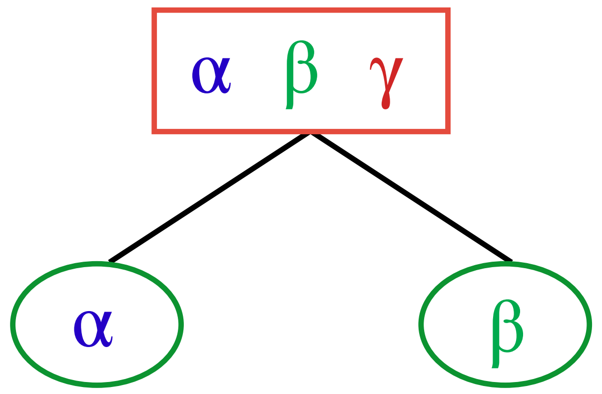

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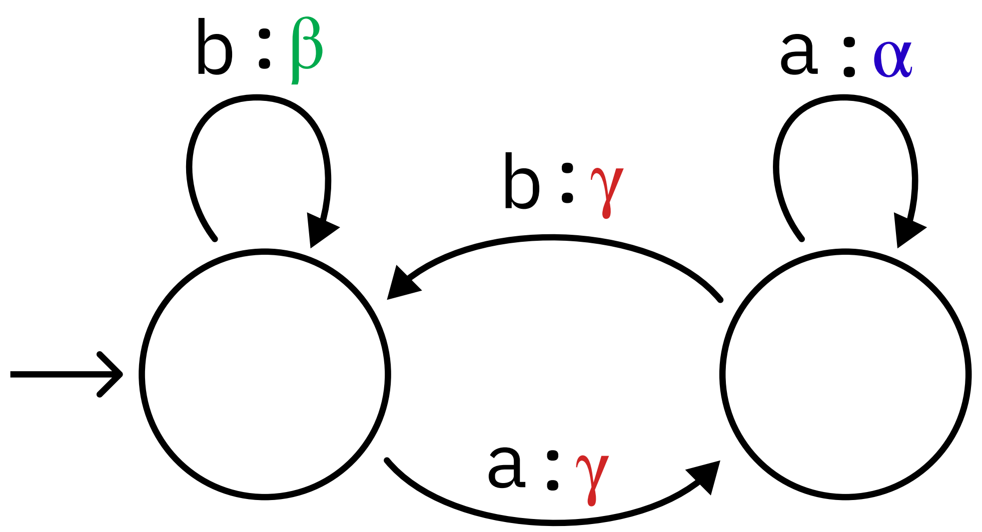

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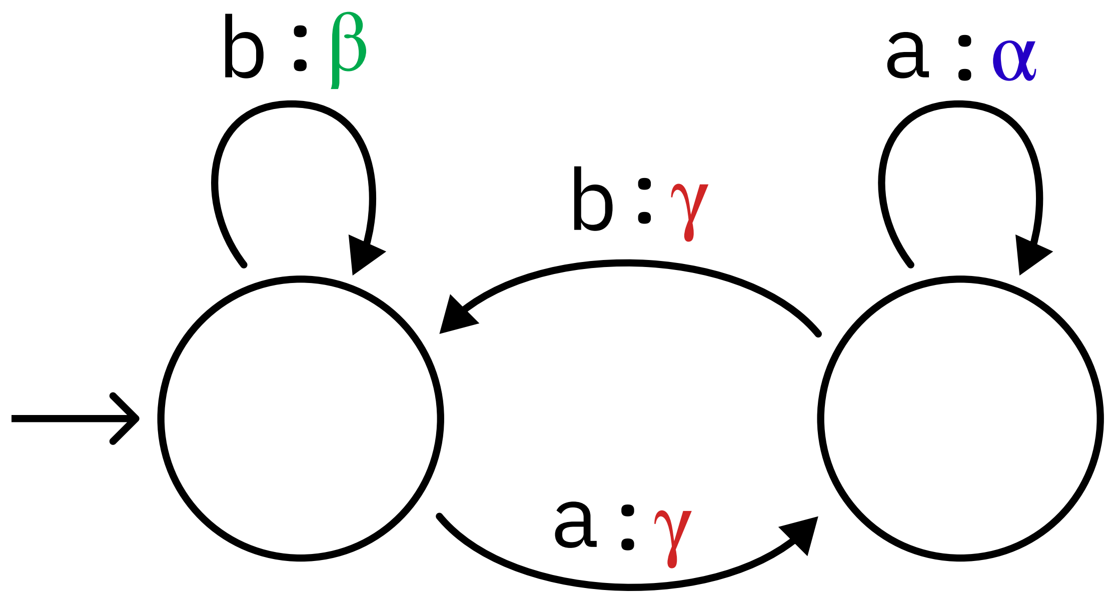

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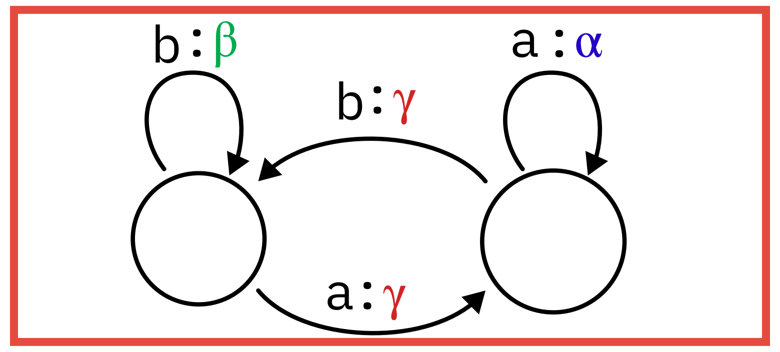

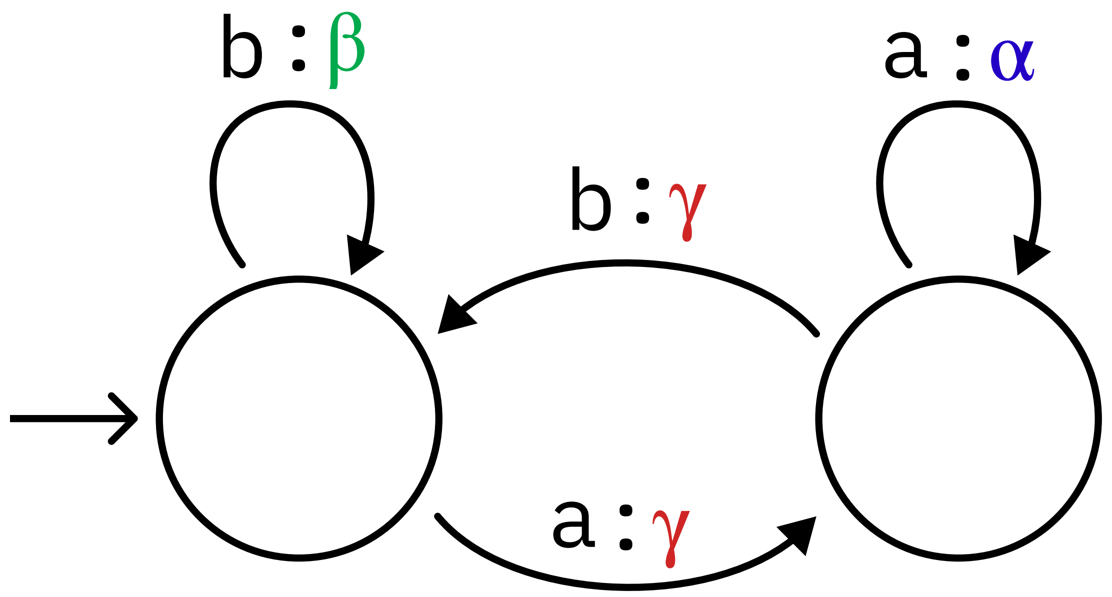

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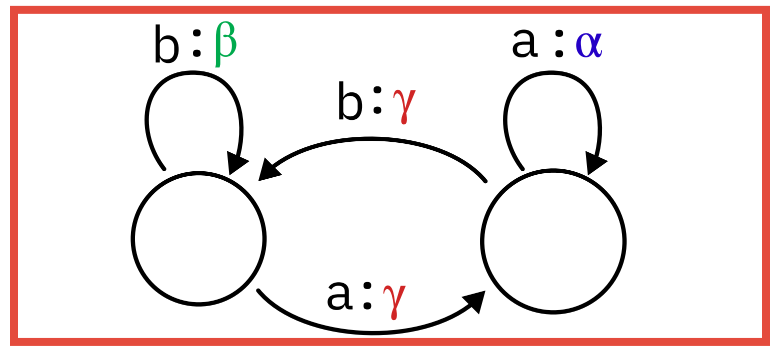

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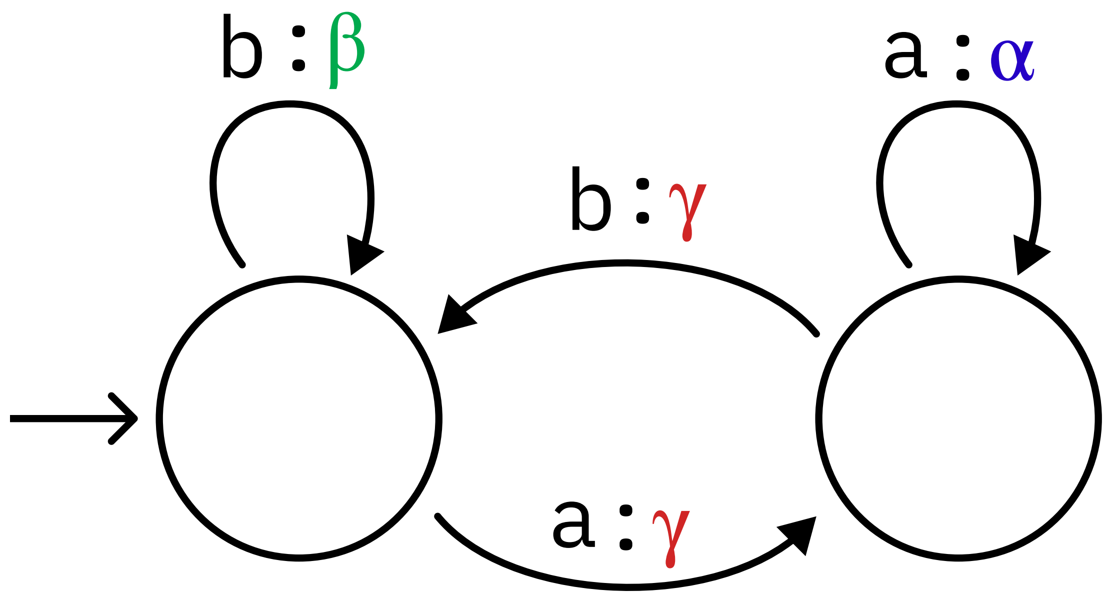

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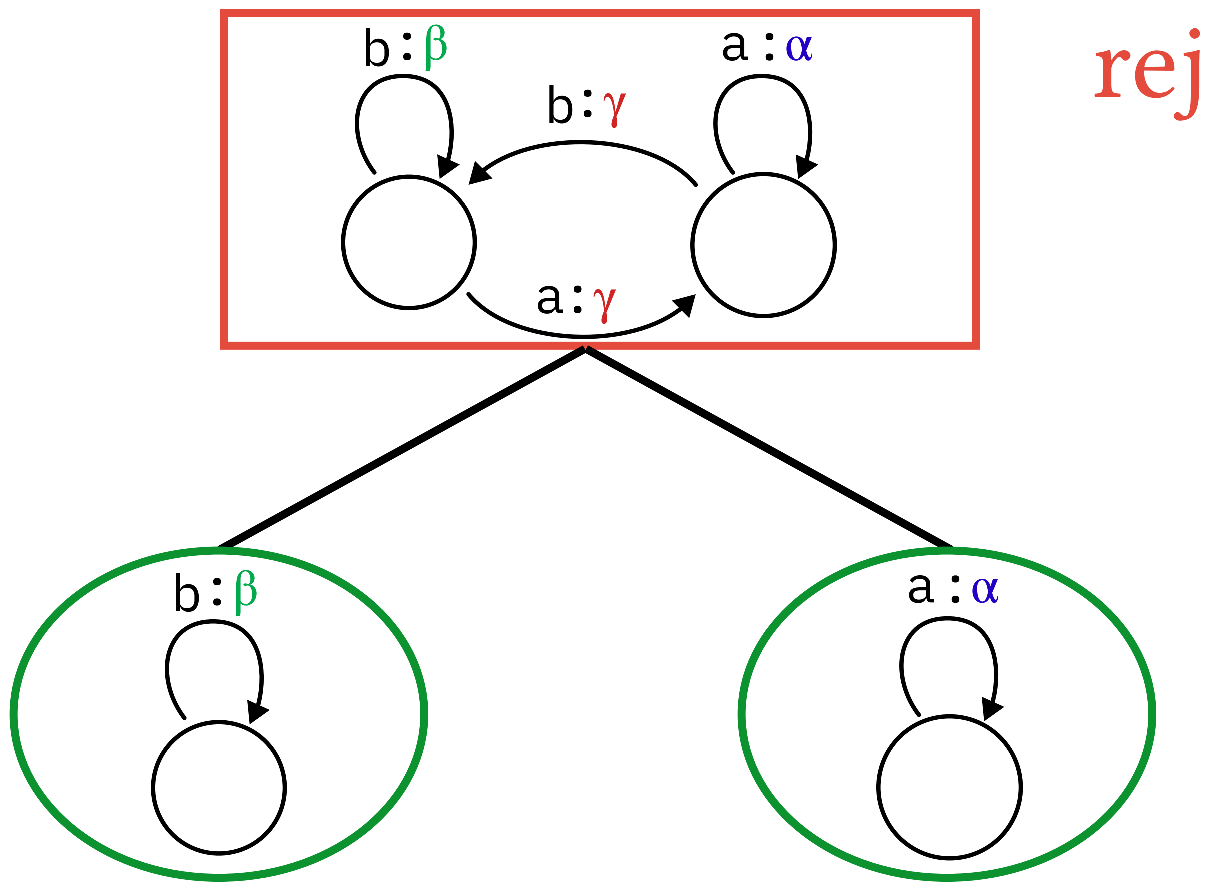

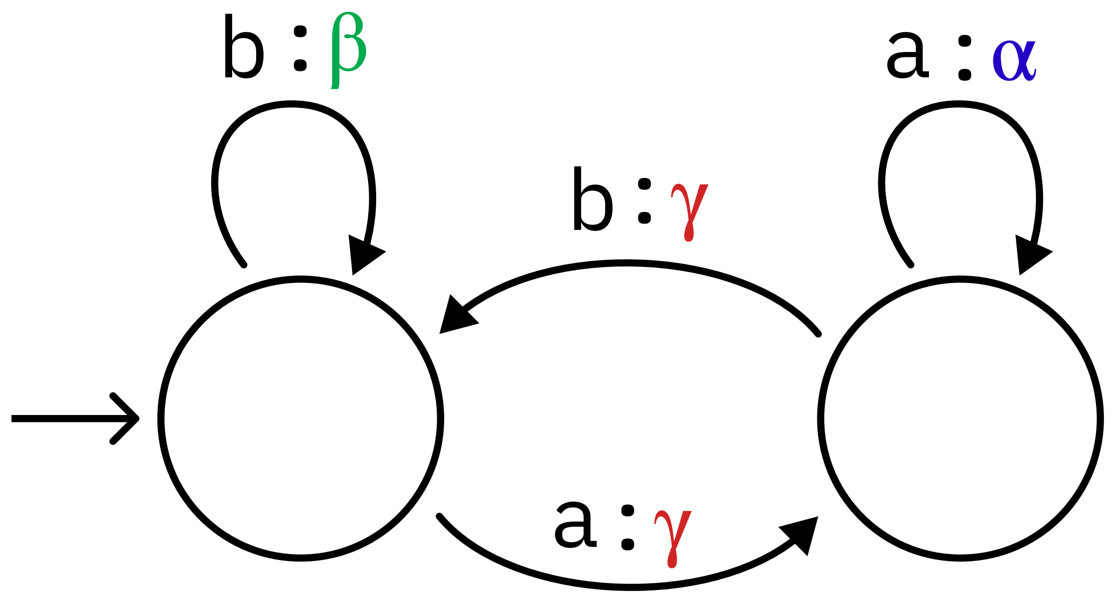

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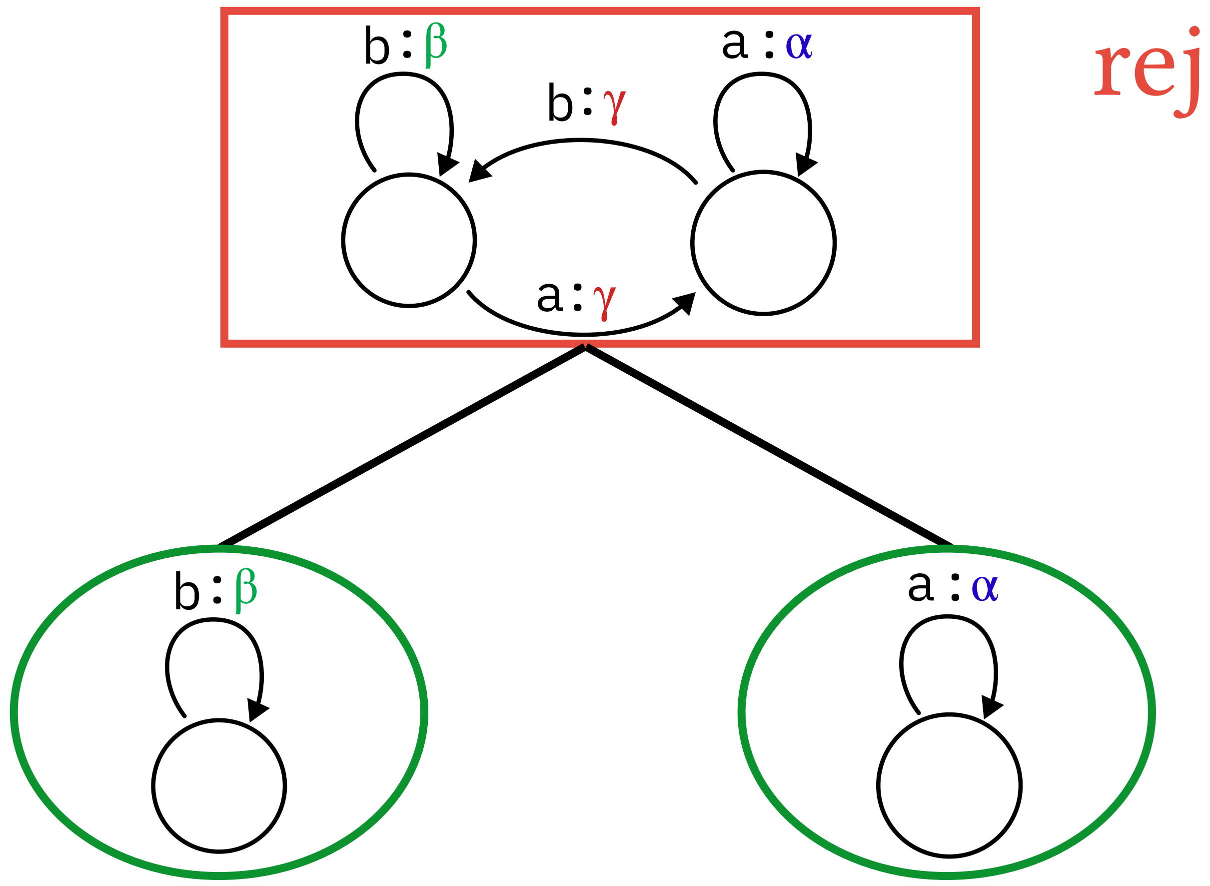

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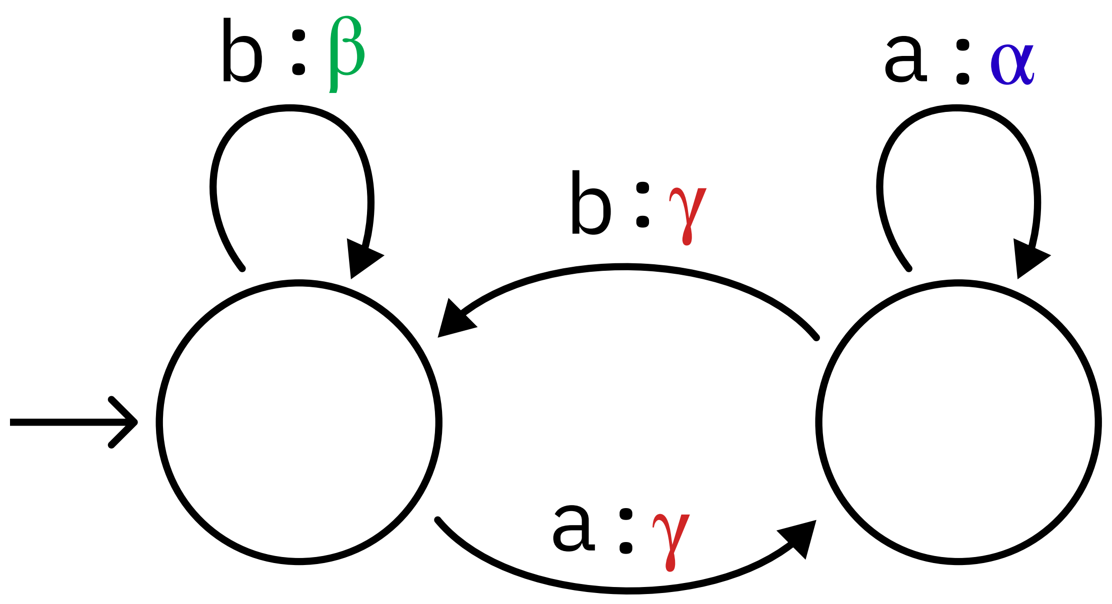

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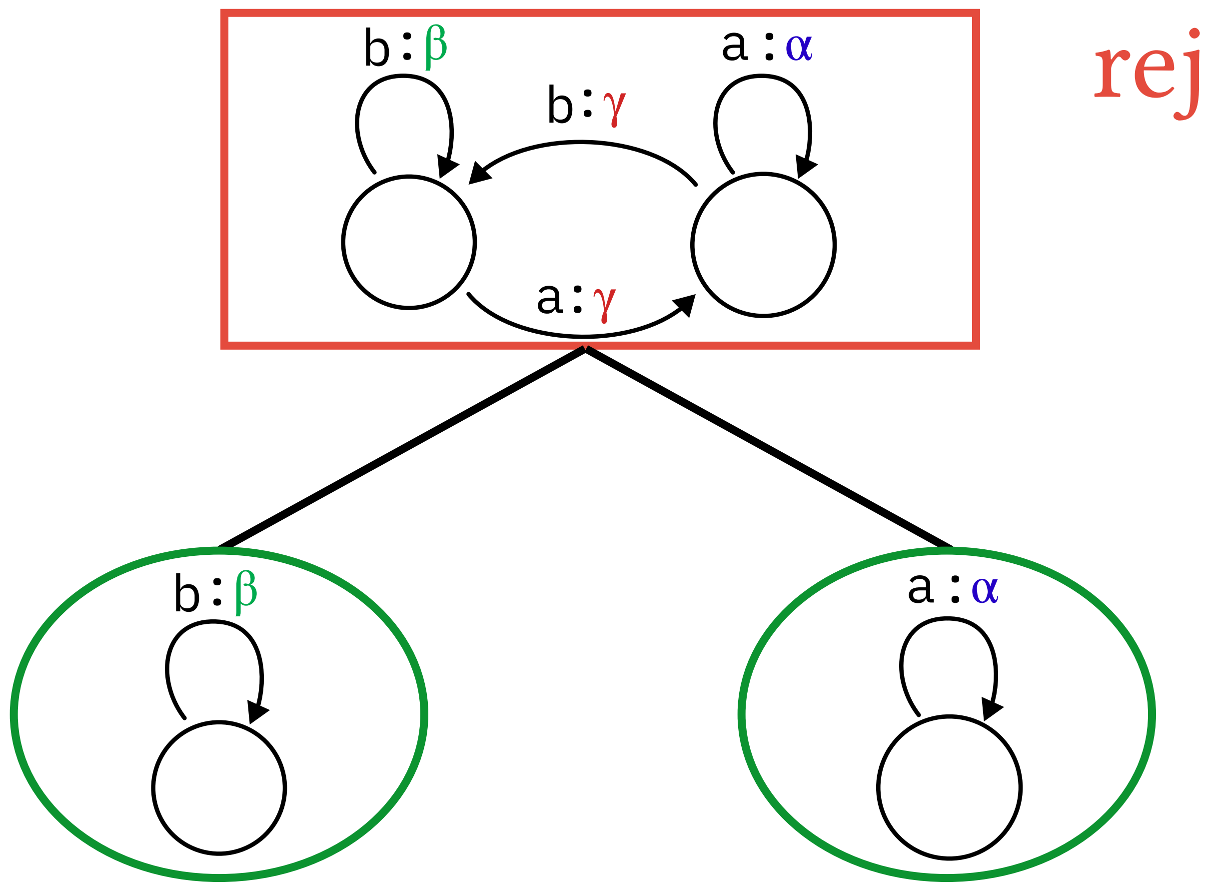

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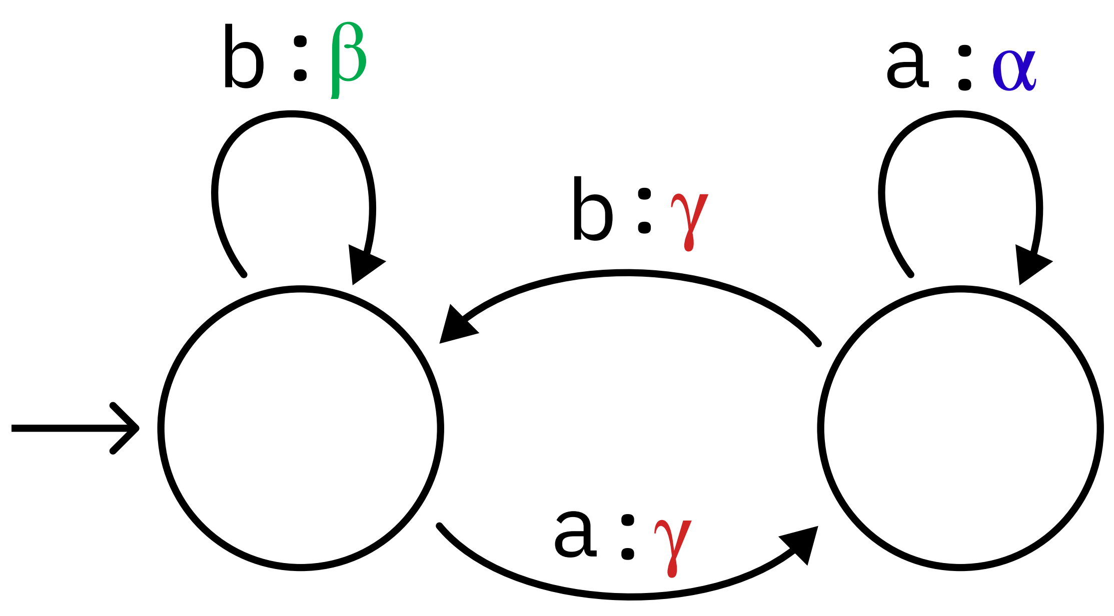

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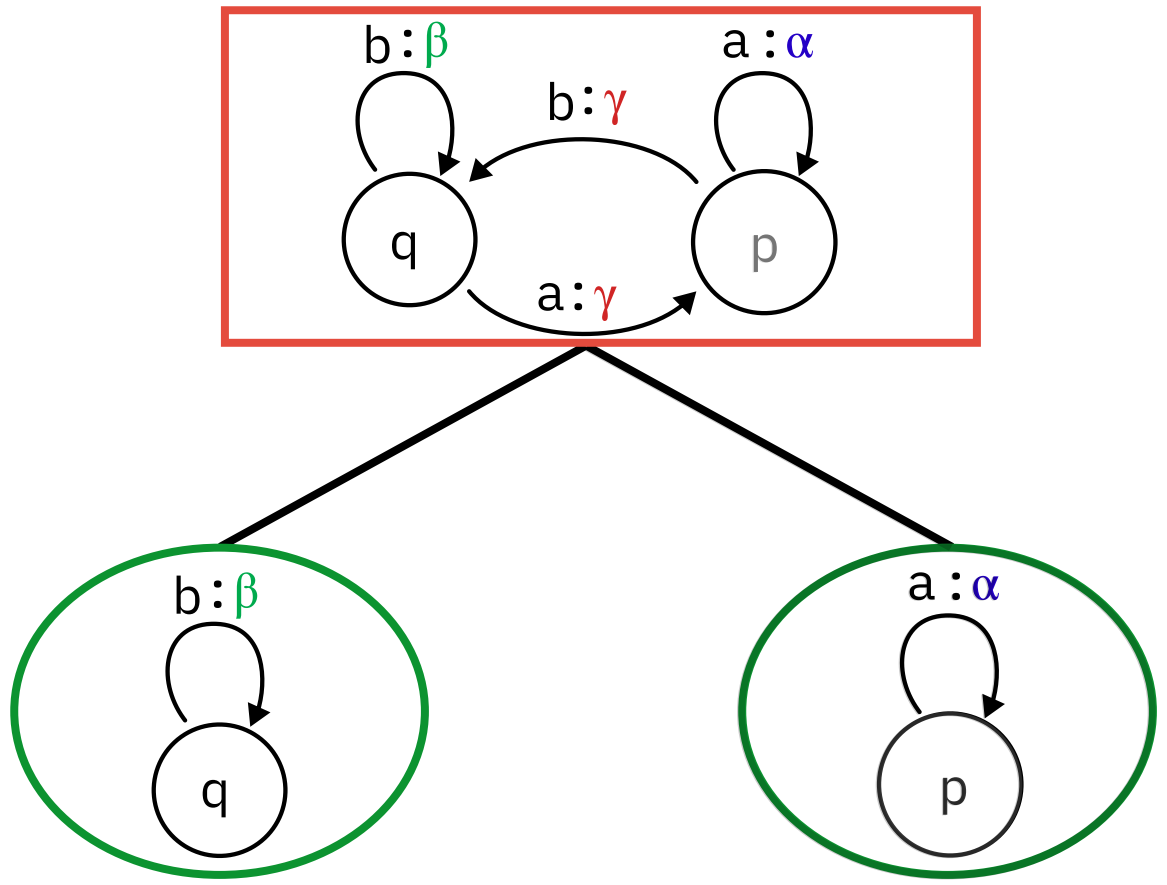

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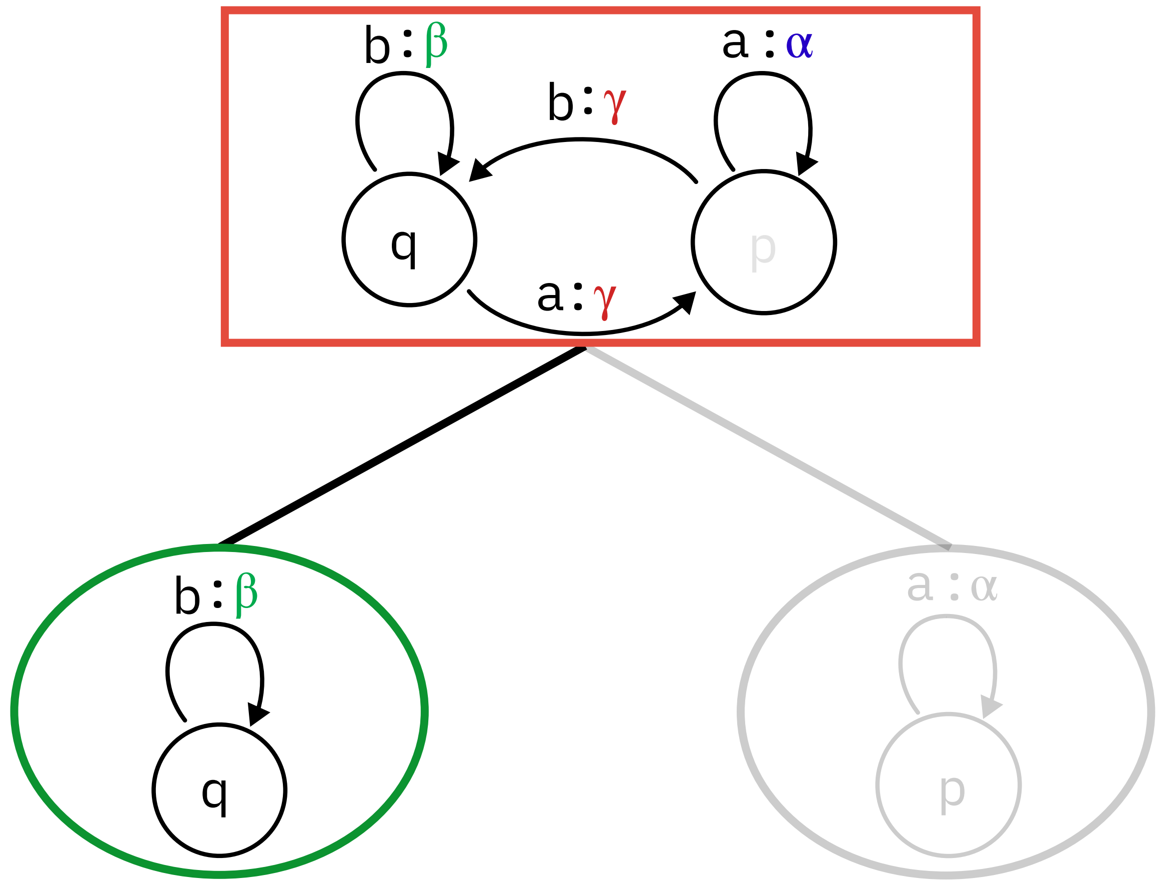

Restriction to the tree of a state q

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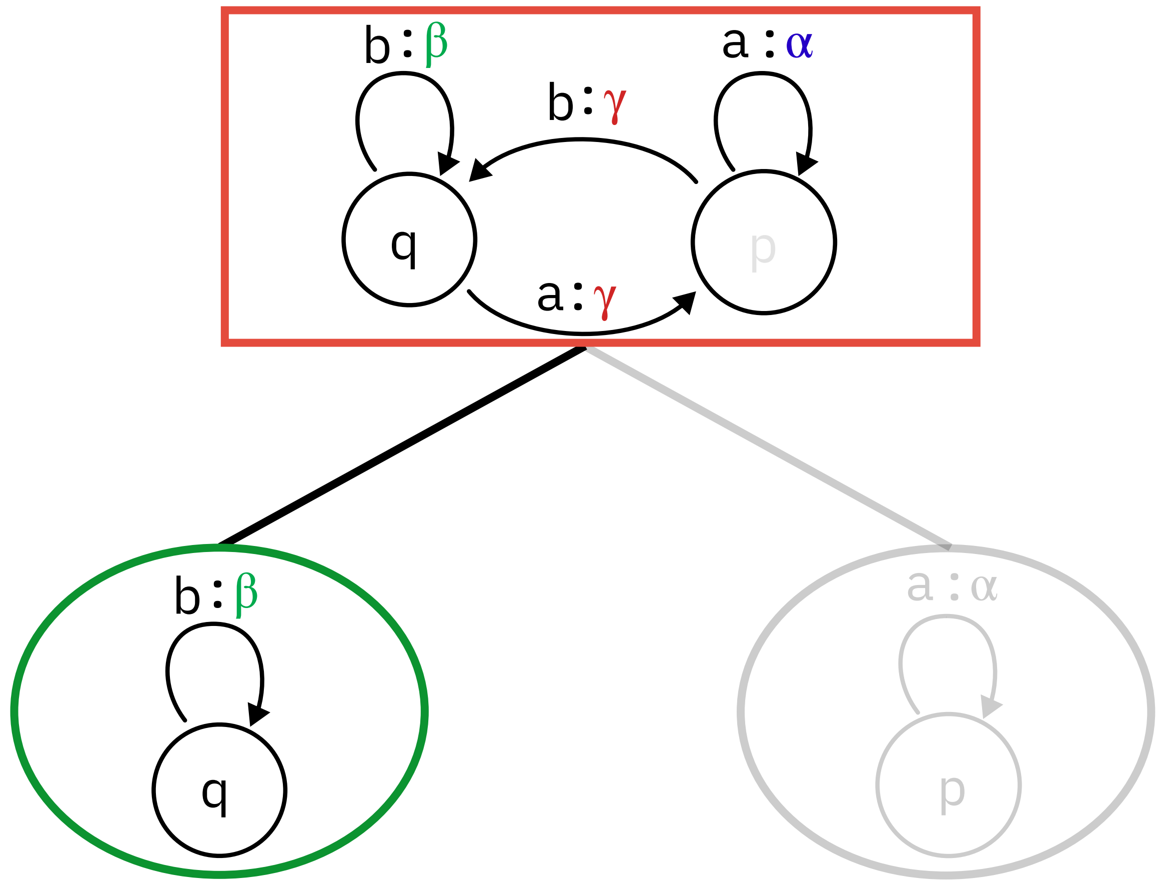

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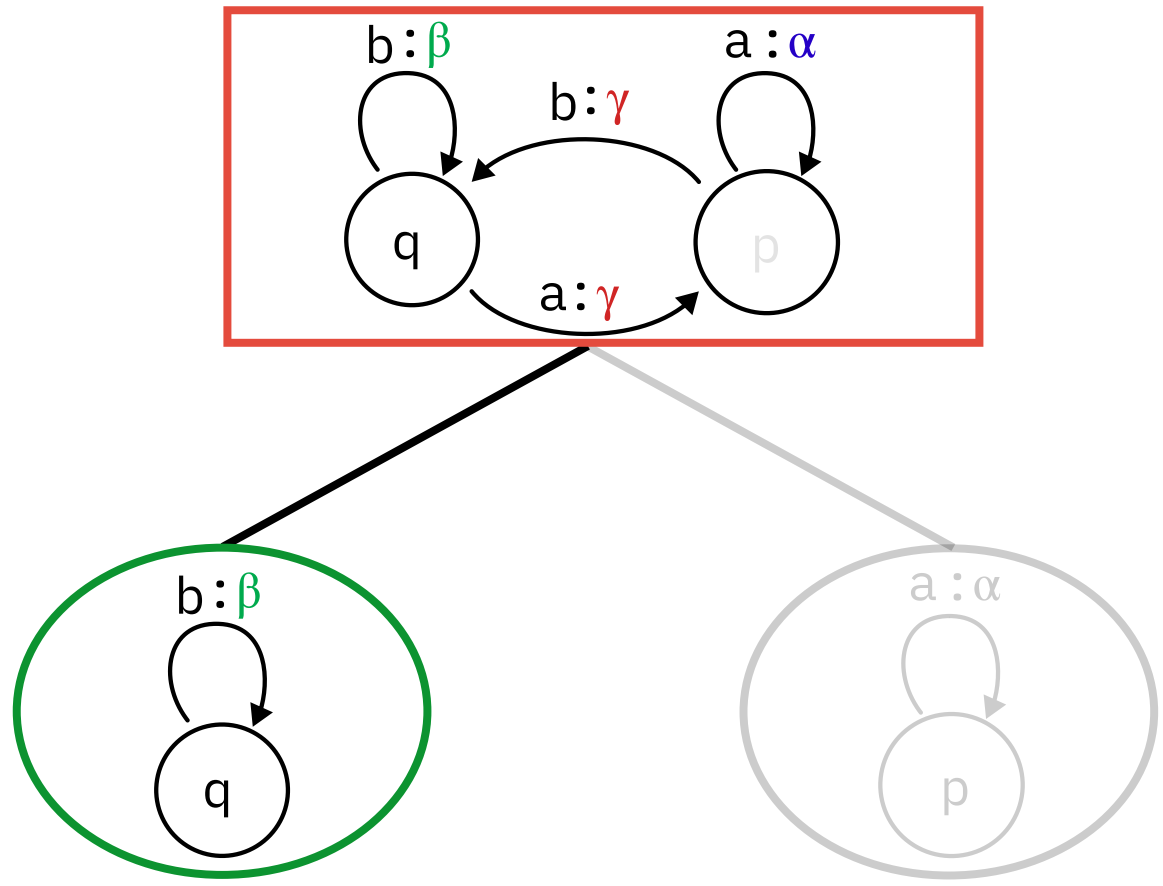

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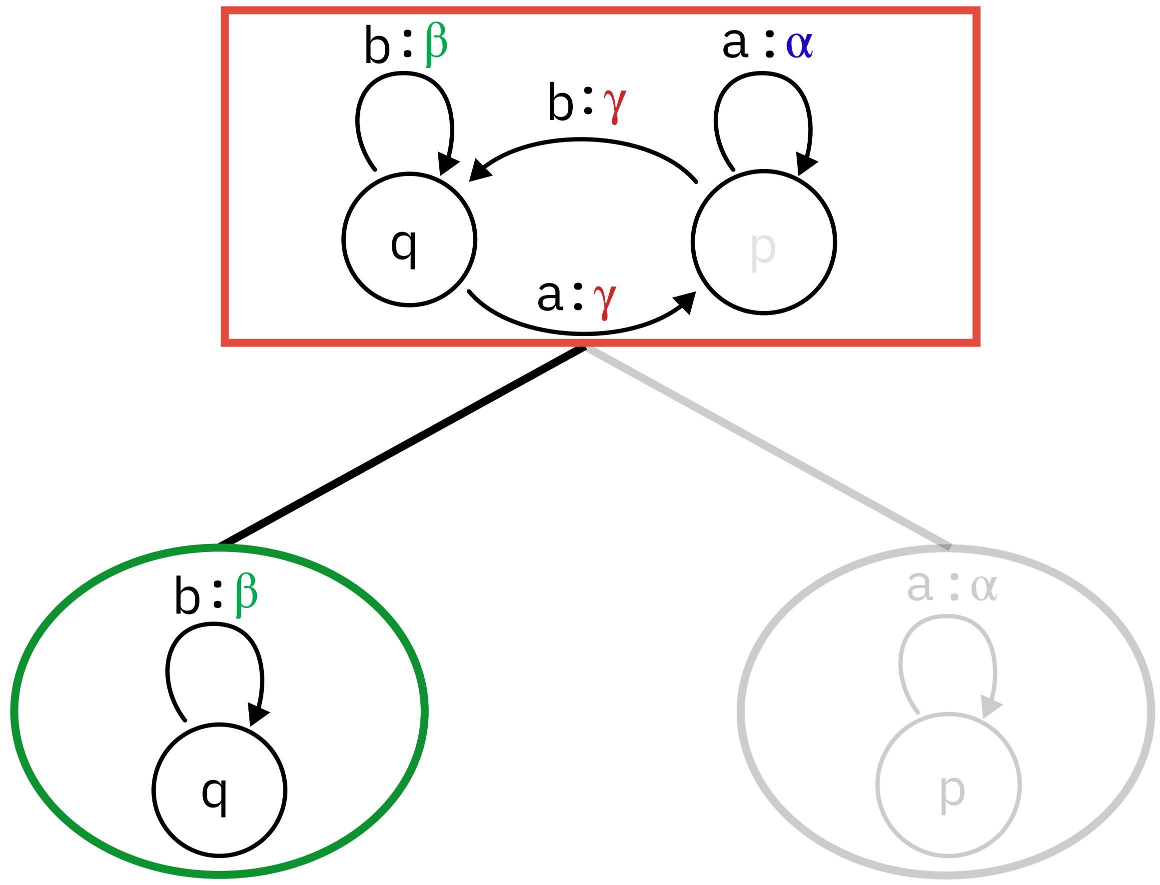

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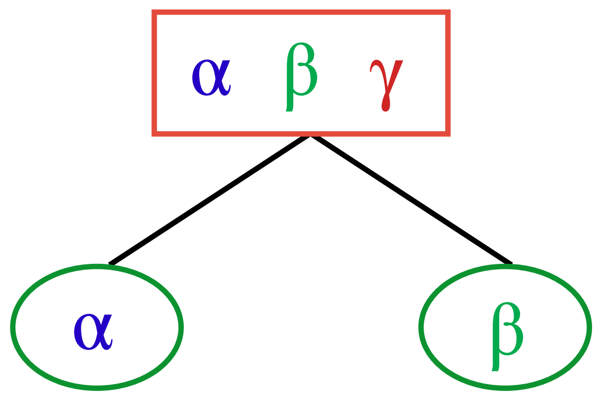

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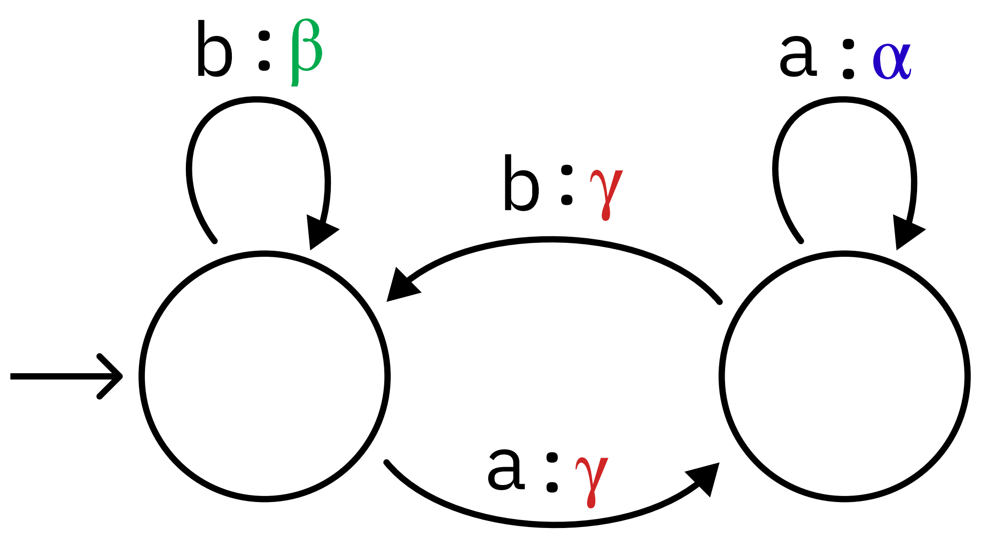

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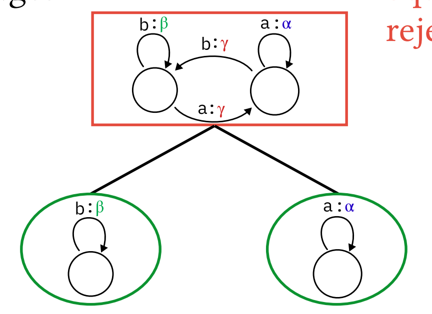

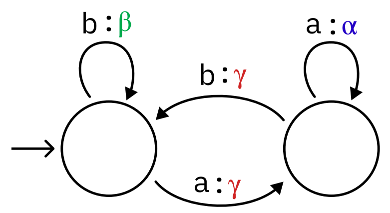

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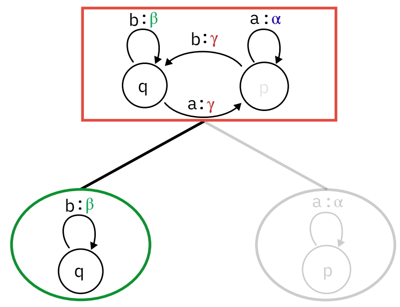

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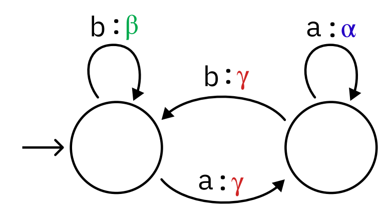
Simplifying ω-Automata through the Alternating Cycle Decomposition

Antonio Casares · University of Warsaw & Corto Mascle · LaBRI, University of Bordeaux

Automata over infinite words (ω-automata)

Automata are deterministic in this talk

$\mathcal{A}$



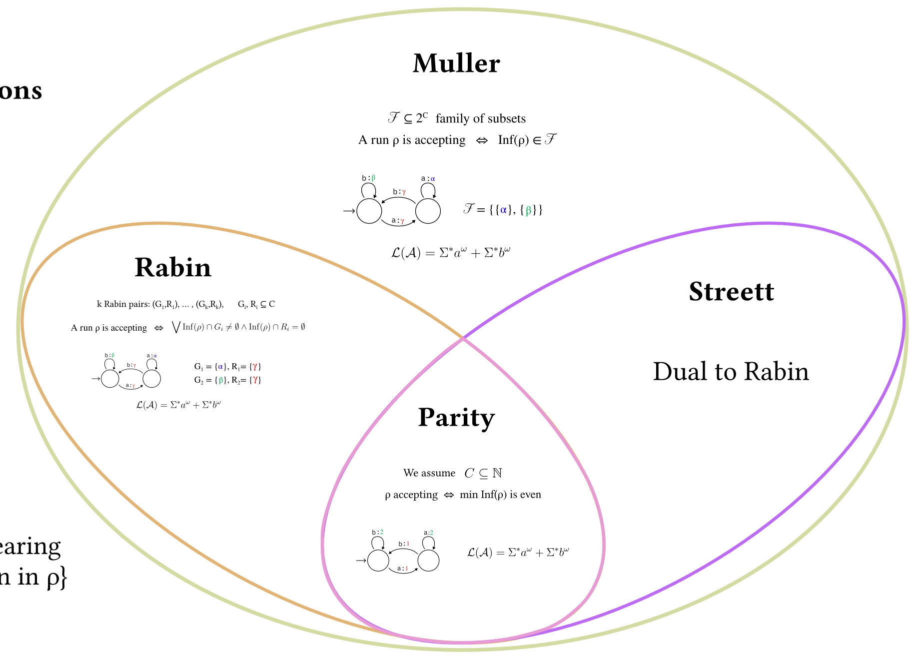
Input $w \in \Sigma^\omega$ ⟶ Infinite path ρ

Different Acceptance Conditions

$C = \{\alpha, \beta, \gamma\}$

output colours

$\text{Inf}(\rho)$ = {Colours appearing infinitely often in ρ}



More general conditions → Automata are simpler to produce

- LTL ⟶ Det. Muller automata
- ND Büchi ⟶ Det. Rabin automata

Simpler conditions → More efficient algorithms

Complexity of solving 2-player games. n = Size of input graph

Muller	PSPACE-complete	$\mathcal{O}(k^{5k}n^5)$	k = #Colours
Rabin	NP-complete	$\mathcal{O}(n^{k+3}kk!)$	k = #Rabin pair
Parity	NP⋂coNP. In P?	$\mathcal{O}(n^{\log(k)})$	k = #Colours

Goal: Simplify the acceptance condition of automata

a) Simplify the type (Muller → Rabin → Parity)

b) Minimise #Colours / #Rabin pairs

Approach 1

Add states to the automaton to maximize simplification

√ Well understood

✦ Optimal transformation to parity automata *(C. Colcombet, Fijalkow '21)*

✗ Exponential blowup in the worst case *(Löding '99)*

In this paper

Simplify the condition **without** adding further states

Pure Conditions

Reduction of conditions

A reduction of a condition $\mathcal{F} \subseteq 2^C$ to $\mathcal{G} \subseteq 2^{C'}$ is a mapping

$$\varphi : C \longrightarrow C' \quad \text{such that}$$

$$X \in \mathcal{F} \iff \varphi(X) \in \mathcal{G}, \quad \text{for all } X \subseteq C$$

Given a condition $\mathcal{F}$, can we reduce it to

- ✦ a Muller condition with ≤ k colours?
- ✦ a Rabin condition? With ≤ k Rabin pairs?
- ✦ a parity condition? With ≤ k colours?

Main tool: Zielonka DAG *(Zielonka '98, Hunter-Dawar '06)*

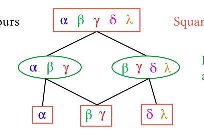
$C = \{\alpha, \beta, \gamma, \delta, \lambda\}$

$\mathcal{F} = \{\{\alpha,\beta,\gamma\}, \{\alpha,\gamma\}, \{\alpha,\beta\}, \{\beta,\gamma,\delta,\lambda\}, \{\beta,\delta,\lambda\}, \{\gamma,\delta,\lambda\}, \{\beta,\gamma,\lambda\}, \{\beta,\gamma,\delta\}, \{\beta,\delta\}, \{\beta,\lambda\}, \{\gamma,\delta\}, \{\gamma,\lambda\}\}$

Root ⟶ All colours

Children: Maximal accepting subsets

Children: Maximal rejecting subsets



Square if it is a rejecting set

Round because they are accepting sets

Proposition *(Hunter-Dawar '06)*

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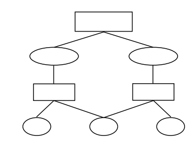

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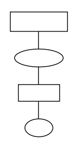

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Theorem

We can minimise the #Colours (resp. #Rabin pairs) in polynomial time.

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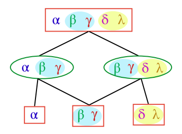

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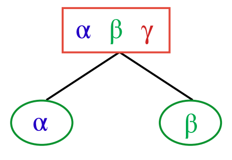

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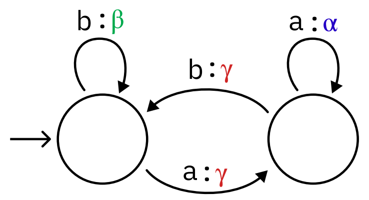

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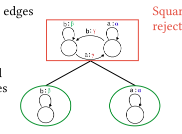

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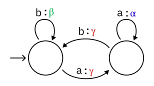

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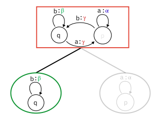

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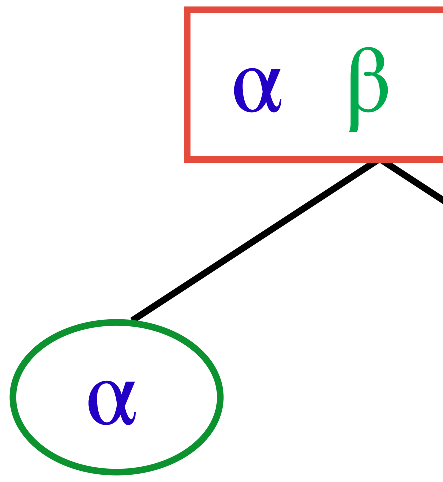
	Pure Conditions	On top of automata
Typeness		
Min. colours/ Rabin pairs		



Not parity

Requires 3 co

Main tool: Alter

Generalisation of

Root ⟶ All ed

Children: Maximal accepting subcycles

Tr

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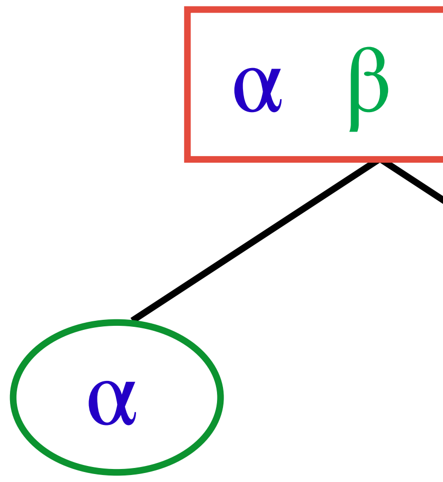
	Pure Conditions	On top of automata
Typeness	PTIME	
Min. colours/ Rabin pairs	PTIME	

ion



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Generalisation of

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Tr

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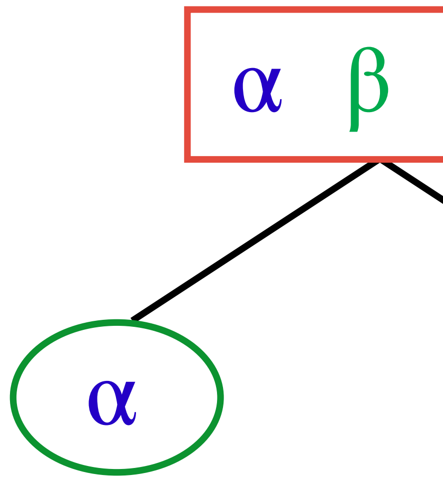
	Pure Conditions	On top of automata
Typeness	PTIME	
Min. colours/ Rabin pairs	PTIME	

Zielonka tree

ion



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Generalisation of

Root → All ed

Children: Maximal accepting subcycles

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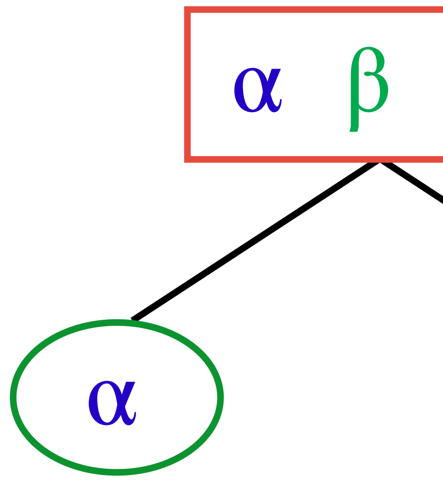
	Pure Conditions	On top of automata
Typeness	PTIME	PTIME
Min. colours/ Rabin pairs	PTIME	

Zielonka tree



Not parity

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Main tool: Alter

Generalisation of

Root ⟶ All ed

Children: Maximal accepting subcycles

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	Pure Conditions	On top of automata
Typeness	PTIME	PTIME
Min. colours/ Rabin pairs	PTIME	

Zielonka tree

Alternating Cycle Decomposition

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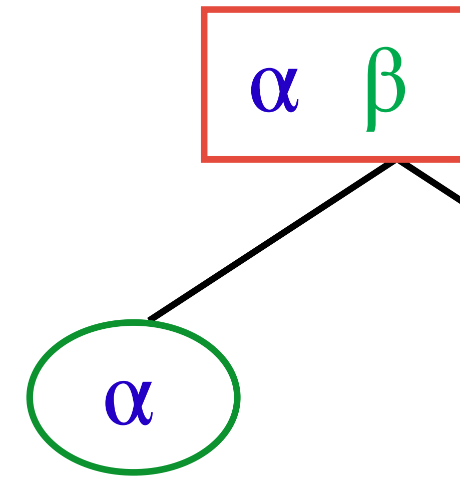

Not parity

Requires 3 co

Main tool: Alter

Generalisation of

Root ⟶ All ed

Children: Maximal accepting subcycles

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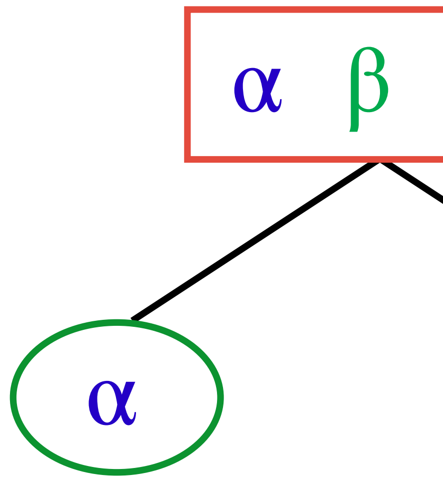
	Pure Conditions	On top of automata
Typeness	PTIME	PTIME
Min. colours/ Rabin pairs	PTIME	NP-complete

Zielonka tree

Alternating Cycle Decomposition

ion



Not parity

Requires 3 co

Main tool: Alter

Generalisation of

Root → All ed

Children: Maximal accepting subcycles

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	Pure Conditions	On top of automata
Typeness	PTIME	PTIME
Min. colours/ Rabin pairs	PTIME	NP-complete

	Zielonka tree	*Alternating Cycle Decomposition*

THANK YOU!

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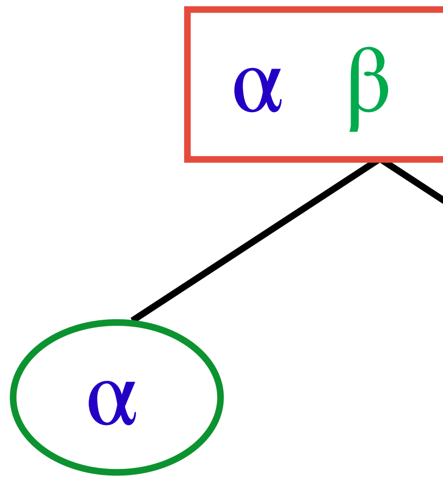

Not parity

Requires 3 co

Main tool: Alter

Generalisation of

Root → All ed

Children: Maximal accepting subcycles

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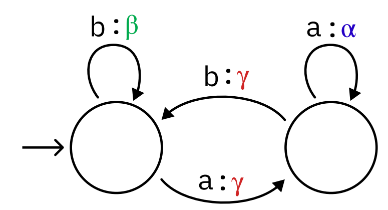
Simplifying ω-Automata through the Alternating Cycle Decomposition

Antonio Casares · University of Warsaw & Corto Mascle · LaBRI, University of Bordeaux

Automata over infinite words (ω-automata)

Automata are deterministic in this talk

$\mathcal{A}$

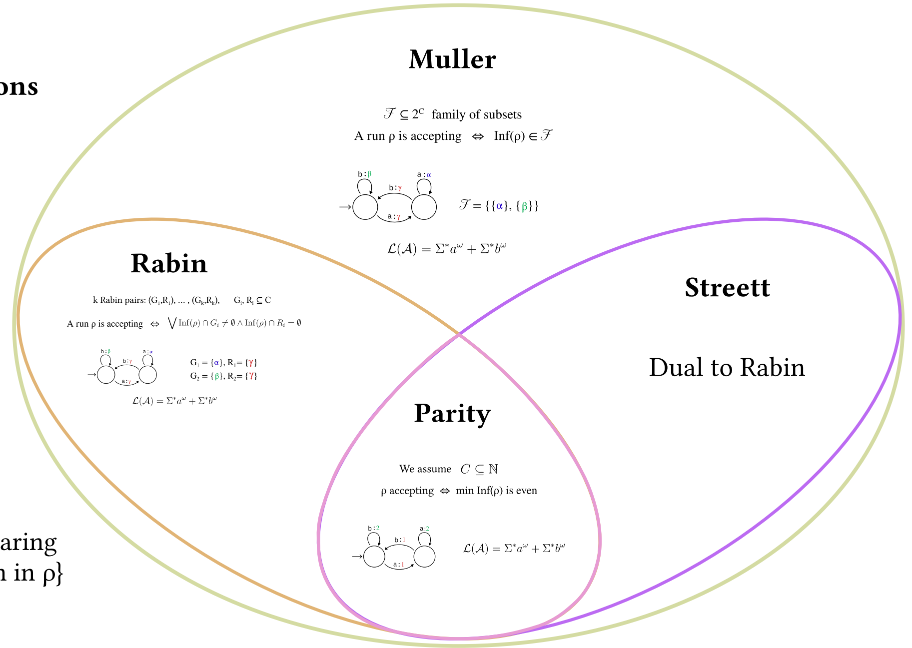


Input w ∈ Σ^ω ⟶ Infinite path ρ

Different Acceptance Conditions

C = {α, β, γ}
output colours

Inf(ρ) = {Colours appearing infinitely often in ρ}



More general conditions → Automata are simpler to produce

- LTL → Det. Muller automata
- ND Büchi → Det. Rabin automata

Simpler conditions → More efficient algorithms

Complexity of solving 2-player games. n = Size of input graph

Muller	PSPACE-complete	$\mathcal{O}(k^{5k}n^5)$	k = #Colours
Rabin	NP-complete	$\mathcal{O}(n^{k+3}kk!)$	k = #Rabin pair
Parity	NP∩coNP. In P?	$\mathcal{O}(n^{\log(k)})$	k = #Colours

Goal: Simplify the acceptance condition of automata

a) Simplify the type (Muller → Rabin → Parity)

b) Minimise #Colours / #Rabin pairs

Approach 1

Add states to the automaton to maximize simplification

✓ Well understood

✦ Optimal transformation to parity automata *(C. Colcombet, Fijalkow '21)*

✗ Exponential blowup in the worst case *(Löding '99)*

In this paper

Simplify the condition **without** adding further states

Pure Conditions

Reduction of conditions

A reduction of a condition $\mathcal{F} \subseteq 2^C$ to $\mathcal{G} \subseteq 2^{C'}$ is a mapping

$$\varphi : C \to C' \quad \text{such that}$$

$$X \in \mathcal{F} \iff \varphi(X) \in \mathcal{G}, \quad \text{for all } X \subseteq C$$

Given a condition $\mathcal{F}$, can we reduce it to

- ✦ a Muller condition with ≤ k colours?
- ✦ a Rabin condition? With ≤ k Rabin pairs?
- ✦ a parity condition? With ≤ k colours?

Main tool: Zielonka DAG *(Zielonka '98, Hunter-Dawar '06)*

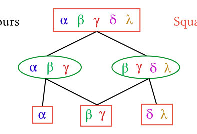
C = {α, β, γ, δ, λ}

$\mathcal{F}$ = {{α,β,γ},{α,γ},{α,β},{β,γ,δ,λ},{β,δ,λ},{γ,δ,λ},{β,γ,λ},{β,γ,δ},{β,δ},{β,λ},{γ,δ},{γ,λ}}

Root → All colours

Children: Maximal accepting subsets

Children: Maximal rejecting subsets



Square if it is a rejecting set

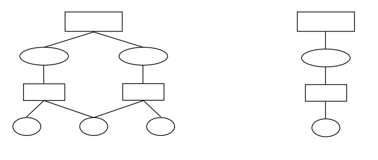
Round because they are accepting sets

Proposition *(Hunter-Dawar '06)*

The Zielonka DAG can be computed in polynomial time in $|\mathcal{F}|$.

Typeness: Can we reduce $\mathcal{F}$ to a Rabin/parity condition?

Just look to the Zielonka DAG! *(Zielonka '98)*



Rabin ⇔ Round nodes have a single child

Parity ⇔ A single branch

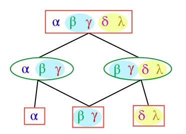
+ Minimal #Colours = Height of the branch

Minimisation of colours and Rabin pairs

Theorem

We can minimise the #Colours (resp. #Rabin pairs) in polynomial time.

Idea (for colours):



Two colours are equivalent if they appear in the same nodes

Merge equivalent colours

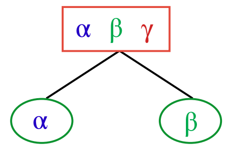
We show that the obtained condition is optimal

- Minimisation of Rabin pairs is quite involved.

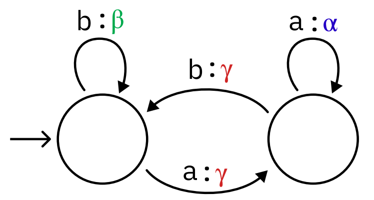
	Pure Conditions	On top of automata
Typeness	PTIME	PTIME
Min. colours/ Rabin pairs	PTIME	NP-complete
	Zielonka tree	*Alternating Cycle Decomposition*

THANK YOU!

Automata



Not parity
Requires 3 colours



$\mathcal{F} = \{\{\alpha\}, \{\beta\}\}$

But we can relabel the automaton with a parity condition with 2 colours!

Can we relabel $\mathcal{A}$ with

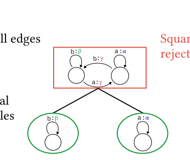
- ✦ a parity/Rabin condition?
- ✦ ≤ k colours / Rabin pairs?

Main tool: Alternating Cycle Decomposition (ACD) *(C. Colcombet Fijalkow '21)*

Generalisation of the Zielonka DAG

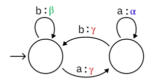
Root → All edges

Children: Maximal accepting subcycles



Square if they form a rejecting cycle

Round because they are accepting



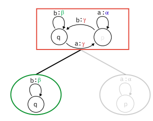
$\mathcal{F} = \{\{\alpha\}, \{\beta\}\}$

Theorem (main technical contribution)

The Alternating Cycle Decomposition can be computed in **polynomial time** in $|\mathcal{A}| + |\mathcal{F}|$.

Typeness: Can we relabel $\mathcal{A}$ with a Rabin/parity condition?

Just look to the Altenating Cycle Decompositon! *(C. Colcombet Fijalkow '21)*



Restriction to the tree of a state q

Rabin ⇔ In the restriction-tree of every state round nodes have a single child

Parity ⇔ The restriction-tree of every state consists of a single branch.

+ Minimal #Colours = Height of the ACD

Minimization of colours and Rabin pairs

First idea: Do as with the Zielonka DAG!

But... It does not work!

Theorem

Given an automaton $\mathcal{A}$ and its ACD, it is **NP-complete** to determine if we can relabel it with a condition using:

- ≤ k colours.
- ≤ k Rabin pairs.