Linear Programming Examples using Julia/JuMP

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February 21, 2019

Example 1: Avocado Plant

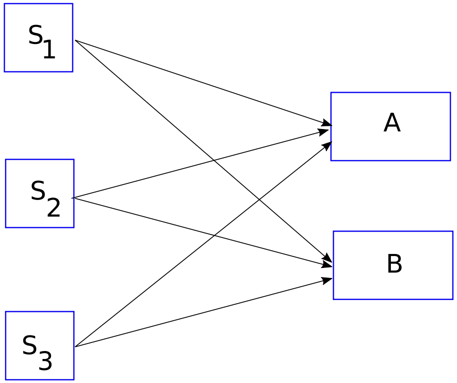
An avocado oil company owns two A and B processing plants. Plants A and B can process a maximum of 460 and 650 Ton, respectively. The labor cost turns is 26 and 21 \$/Ton for plants A and B, also respectively. The avocado oil sale price turns out to be \$50/T regardless the manufacturing plant.
To meet the avocado oil manufacturing aims, the company can purchase avocado raw material from three different sellers: S_1, S_2 and S_3. In the following table, maximum avocado supply and purchase cost from each seller are shown.

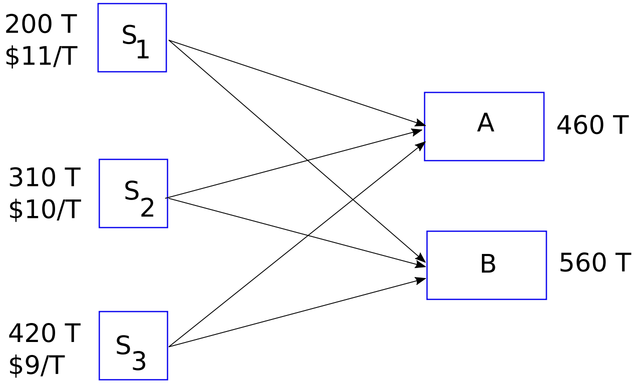
Seller	Maximum Supply [T]	Purchase cost [\$/T]
S_1	200	11
S_2	310	10
S_3	420	9

Moreover, transportation costs [\$/T] should also be taken into account to get a realistic net profit. Transportation cost are shown in next table.

Seller	Plant A	Plant B
S_1	3	3.5
S_2	2	2.5
S_3	6	4

Problem: We would like to compute the avocado amount to be purchase from each seller and the amount to be sent to each manufacturing plant such that net profit is maximized.


S_1
S_2
S_3
A
B


200 T
$11/T
S_1
310 T
$10/T
S_2
420 T
$9/T
S_3
A
460 T
B
560 T

Results

- Amount [Ton] of avocado shipped to each plant

Seller	Plant A	Plant B
S_1	60	140
S_2	310	0
S_3	0	120

- Amount of avocado oil [Ton] manufactured at each plant

Plant	Amount [Ton]
A	370
B	560

- Economic Analysis

Gross Profit	46500
Labor Cost	21380
Raw Material Cost	9080
Transport Cost	2970.0
Net Profit	13070

- JuMP code

```
using JuMP, Clp
avocado_model = Model(solver=ClpSolver())
@variable(avocado_model, s1a >= 0, start = 100)
@variable(avocado_model, s2a >= 0, start = 150)
@variable(avocado_model, s3a >= 0, start = 200)
@variable(avocado_model, s1b >= 0, start = 100)
@variable(avocado_model, s2b >= 0, start = 150)
@variable(avocado_model, s3b >= 0, start = 200)
@variable(avocado_model, pa >= 0, start = 150)
@variable(avocado_model, pb >= 0, start = 200)
@objective(avocado_model, Max, 50pa+50pb-26pa-21pb-11(s1a+s1b)-10(s2a+s2b)
-9(s3a+s3b)-3s1a-3.5s1b- 2s2a -2.5s2b - 6s3a - 4s3b)
@constraint(avocado_model, c1, s1a+ s1b <= 200)
@constraint(avocado_model, c2, s2a+ s2b <= 310)
@constraint(avocado_model, c3, s3a+ s3b <= 420)
@constraint(avocado_model, c4, s1a + s2a + s3a <= 460)
@constraint(avocado_model, c5, s1b + s2b + s3b <= 560)
@constraint(avocado_model, c6, s1a + s2a + s3a == pa)
@constraint(avocado_model, c7, s1b + s2b + s3b == pb)
print(avocado_model)
status_avocado_model = solve(avocado_model)
println("Status of solution:", status_avocado_model)
println("s1a =", getvalue(s1a))
println("s2a =", getvalue(s2a))
println("s3a =", getvalue(s3a))
println("s1b =", getvalue(s1b))
println("s2b =", getvalue(s2b))
println("s3b =", getvalue(s3b))
println("pa =", getvalue(pa))
println("pb =", getvalue(pb))
```

- JuMP code

```
gross_profit = 50*getvalue(pa)+50*getvalue(pb)
labor = 26getvalue(pa)+21getvalue(pb)
raw_material = 11(getvalue(s1a)+getvalue(s1b))+10(getvalue(s2a)+getvalue(s2b))
+9(getvalue(s3a)+getvalue(s3b))
transport = 3getvalue(s1a)+3.5getvalue(s1b)+ 2getvalue(s2a) +2.5getvalue(s2b)
+ 6getvalue(s3a) + 4getvalue(s3b)
println("Gross Profit = ", gross_profit)
println("Labor cost = ", labor)
println("Raw Mat. cost = ", raw_material)
println("Transport cost = ", transport)
```

Example 2: Polymer Plant

A chemical company runs a polymer plant where three different grades A, B, C are manufactured. There are available 4 different batch reactors (denoted as R_1, R_2, R_3 and R_4) for polymers manufacture. Although the manufactured batch size for each grade is the same regardless of the type of reactor, each reactor requires different processing time [min/batch] for polymer production as shown in the next table.

Grade	R_1	R_2	R_3	R_4
A	5	7	4	10
B	6	12	8	15
C	13	14	9	17

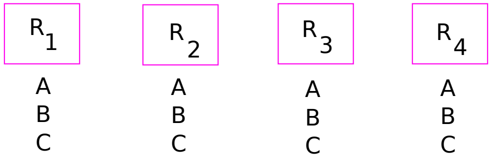
The sale price [$/batch] of each polymer is different depending on the reactor where it was manufactured as shown in the next table:

Grade	R_1	R_2	R_3	R_4
A	10	8	6	9
B	18	20	15	17
C	15	6	13	7

Polymer manufacture should meet the following grades demand:

	Demand
Grade	[number of batches]
A	100
B	150
C	100

How many batches of each grade should be manufactured such that polymer demands are met, assuming a 40 h/week working load, such that process profit is maximized?


R_1
A
B
C
R_2
A
B
C
R_3
A
B
C
R_4
A
B
C

Results

We have solved the problem under the following 2 operating scenarios:

- No Inventory is allowed

Profit = $5500

Grade	R_1	R_2	R_3	R_4	$\sum$
A	100	0	0	0	100
B	0	150	0	0	150
C	100	0	0	0	100

- Some kind of inventory would be available

Profit = $17882.5

Grade	R_1	R_2	R_3	R_4	$\sum$
A	0	0	100	0	100
B	400	200	137.5	160	897.5
C	0	0	100	0	100

- JuMP code

```
using JuMP, Clp
polymer_plant = Model(solver=ClpSolver())
@variable(polymer_plant, bar1 >= 0, start = 50)
@variable(polymer_plant, bar2 >= 0, start = 50)
@variable(polymer_plant, bar3 >= 0, start = 50)
@variable(polymer_plant, bar4 >= 0, start = 50)
@variable(polymer_plant, bbr1 >= 0, start = 50)
@variable(polymer_plant, bbr2 >= 0, start = 50)
@variable(polymer_plant, bbr3 >= 0, start = 50)
@variable(polymer_plant, bbr4 >= 0, start = 50)
@variable(polymer_plant, bcr1 >= 0, start = 50)
@variable(polymer_plant, bcr2 >= 0, start = 50)
@variable(polymer_plant, bcr3 >= 0, start = 50)
@variable(polymer_plant, bcr4 >= 0, start = 50)
@objective(polymer_plant, Max, 10bar1+8bar2+6bar3+9bar4+18bbr1+20bbr2+15bbr3
+17bbr4+15bcr1+6bcr2+13bcr3+7bcr4)
@constraint(polymer_plant, c1, bar1 + bar2 + bar3 + bar4 >= 100)
@constraint(polymer_plant, c2, bbr1 + bbr2 + bbr3 + bbr4 >= 150)
@constraint(polymer_plant, c3, bcr1 + bcr2 + bcr3 + bcr4 >= 100)
@constraint(polymer_plant, c4, 5bar1 + 6bbr1 + 13bcr1 <= 2400)
@constraint(polymer_plant, c5, 7bar2 + 12bbr2 + 14bcr2 <= 2400)
@constraint(polymer_plant, c6, 4bar3 + 8bbr3 + 9bcr3 <= 2400)
@constraint(polymer_plant, c7, 10bar4 + 15bbr4 + 17bcr4 <= 2400)
print(polymer_plant)
status_polymer_model = solve(polymer_plant)
```

- JuMP code

```
println("Objective function value = ", getobjectivevalue(polymer_plant))
println("bar1 =", getvalue(bar1))
println("bar2 =", getvalue(bar2))
println("bar3 =", getvalue(bar3))
println("bar4 =", getvalue(bar4))
println("bbr1 =", getvalue(bbr1))
println("bbr2 =", getvalue(bbr2))
println("bbr3 =", getvalue(bbr3))
println("bbr4 =", getvalue(bbr4))
println("bcr1 =", getvalue(bcr1))
println("bcr2 =", getvalue(bcr2))
println("bcr3 =", getvalue(bcr3))
println("bcr4 =", getvalue(bcr4))
```

Optimal Power Contract Capacity

Commonly, power generating enterprises have different tariffs depending upon the type of consumers. Hence, consumers are divided as follows:

- Residential
- Commercial
- Industrial

Moreover, during a typical day power supply is split in fixed power periods:

- Peak
- Medium
- Off-Peak

Among other factors, power tariff depends upon the above power supply periods.

Optimal Power Contract Capacity

Due to its economic impact , we will only address the optimal negotiating of a one year Industrial power contract.
The tariff of an Industrial power contract is split into two parts:

$$\text{Cost} = \text{Energy Charge} + \text{Capacity Charge}$$

- Energy Charge: This charge is fixed and depends only on power consumption and power supply period
- Capacity Charge: Commonly, this is the term that we should negotiate in an optimal manner. Due to its importance we we will address it in a separate way.

Optimal Power Contract Capacity

Some Capacity Charge Considerations...

- Determined by user consumption [kW/month] based on maximum demand during time of use
- If power demand in Peak period does not exceed the nominal contract capacity then a fixed capacity charge is levied
- If power demand in Peak period exceeds nominal contract capacity then a penalty charge (i.e. 2-3 times power basic cost) applies
- Hence, choosing a low power capacity contract likely will enforce high capacity charge
- On the other hand, choosing a high power contract capacity may result in unnecessary capacity charge

Optimal Power Contract Capacity

Constraints

- Energy charge: Power cost during a given consumption period
- Capacity charge: (a) If power consumer demand exceeds the nominal capacity contract within 10% of the nominal contract capacity then a penalty is charged at twice the rate of nominal contract capacity and (b) Excess power demand over 10% of the nominal contract capacity is charged three times that rate
- Power factor adjustment: (a) Monthly bill will be reduced by 0.15% for every 1% of the average monthly power factor above 80%, (b) It will be increased by 0.3% for every 1% below 80%

Optimal Power Contract Capacity

Constraints

- Expanding construction fee: Power consumers may request a change in their peak contract capacity each month. However, an expanding construction fee will be charged. The expanding construction rate is 1759 [$/kW] for summer months and 1320 for non-summer months.
- Disallowed decreased in contract capacities: At end of each month customers can ask for a decrease in the power contract capacity. However, if they ask for an increase in power capacity within two yers they are forced to pay a maintenance bill that will cost more than staying with the original power contract capacity. Therefore, avoid this scenario.

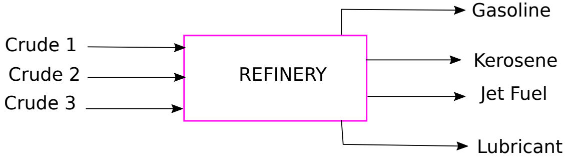
Optimal Power Contract Capacity

- Power factor = 98%
- Peak period power cost (summer months) = 224 [$/kW]
- Peak period power cost (non-summer months) = 167 [$/kW]

Month	Maximum Demand [kWh]	Peak Contract Capacity [kWh]
1	5048	4950
2	5144	4950
3	4616	4950
4	4296	4950
5	3696	4950
6	3392	4950
7	5048	4950
8	5144	4950
9	4616	4950
10	4296	4950
11	3696	4950
12	3392	4950

Optimal Crude Blending

In this problem we are interested in determining the optimal blending of 3 different crudes to meet target demands of Gasoline, Kerosene, Jet Fuel and Lubricants such that process profit is maximized.



Purchase cost information regarding the raw materials and their processing costs are shown in the next table:

Crude	Purchase Cost [$/bbl]	Processing Cost [$/bbl]
1	35	1.6
2	25	1.4
3	55	1.0

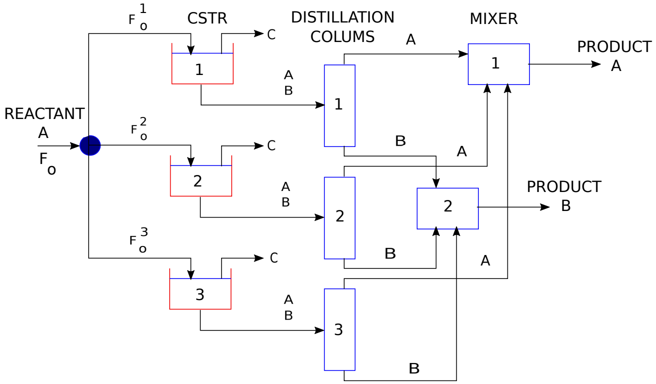
Product	Cost [$/bbl]	Demand [bbl/day]	Crude 1	Crude 2	Crude 3
Gasoline	45	35000	0.75	0.50	0.60
Kerosene	37	7000	0.1	0.15	0.18
Jet Fuel	30	10000	0.1	0.25	0.20
Lubricant	18	5000	0.05	0.1	0.12

Small Plant Optimal Operating

The following isothermal first-order irreversible chemical reaction takes place in a parallel set of continuous stirred tank reactors (CSTR):

$$A \rightarrow B + C$$

Component C is strongly volatile, such that we can safely assume that only components A and B leave any reactor in liquid phase. Afterwards, the main product B is separated from the unreacted raw material A in a set of high-purity distillation columns. Finally, compounds A and B from the reaction-separation train are collected in two separated mixers. We would like to compute the flowrate of the main feedstream (F_o), as well as the optimal split (F_o^1, F_o^2, F_o^3) of such stream, such that process profit is maximized subject to meet product demands and holding raw material availability.



Small Plant Optimal Operating

The maximum available amount of reactant A is 60000 [Kg/h], whereas the demand of the main product B is 30000 [kg/h]. Moreover, the molecular weights of components A, B and C are 27, 20 and 7 [kg/kgmol], respectively. The cost of reactant A is 10 [$/Kg] and the selling price of product B is 20 [$/Kg], respectively. You should also notice that there are bounds on the maximum amount of raw material that each CSTR can manufacture.

CSTR	K [1/h]	Residence time (θ) [h]	Maximum load [Kg]
1	1	3	21600
2	0.8	4.5	27000
3	0.5	5	16200

Column	Recovery Factors α	β
1	0.97	0.03
2	0.98	0.02
3	0.96	0.01

- For a given isothermal first-order chemical reactor i the output molar flowrate is given as follows (the superscript f stands for feed stream):

$$F_A, i = \frac{F_A^f, i}{1 + K_i \theta_i}$$

- You should note that for a given column i the recovery factors are defined as follows (both recovery factors are defined for the distillate stream):

$$\alpha_i = \frac{F_A, i}{F_A^f, i}, \quad \beta_i = \frac{F_B, i}{F_B^f, i}$$