Simplex Method Using the Dictionaries Formulation Approach

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August 19, 2019

Solve by Simplex Method:

$$\begin{array}{rrcl}
\max\limits_{x_1,x_2,x_3} & \Omega = 5x_1 + 4x_2 + 3x_3 & & \\
\text{s.t.} & 2x_1 + 3x_2 + x_3 & \leq & 5 \\
 & 4x_1 + x_2 + 2x_3 & \leq & 11 \\
 & 3x_1 + 4x_2 + 2x_3 & \leq & 8 \\
 & x_1, x_2, x_3 & \geq & 0
\end{array}$$

- Transform the set of inequalities constrains into a set of Equality constraints introducing slack variables

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\begin{aligned}
\max_{x_1,x_2,x_3} \quad & \Omega = 5x_1 + 4x_2 + 3x_3 \\
\text{s.t.} \quad & 2x_1 + 3x_2 + x_3 + x_4 = 5
\end{aligned}
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& 4x_1 + x_2 + 2x_3 + x_5 = 11
\end{aligned}
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\begin{array}{rl}
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- Note that the slacks variables also become decision variables and are bounded by a non-negativity bound

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x_1, x_2, x_3, x_4, x_5, x_6 & \geq & 0
\end{array}
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\begin{array}{|lcr|}
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- Variables in the LHS (x_4, x_5, x_6) are called Basic variables

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$$\boxed{\begin{array}{c} x_4 \\ \\ x_5 \\ \\ x_6 \end{array}} \qquad \boxed{\begin{array}{c} x_1 \\ \\ x_2 \\ \\ x_3 \end{array}}$$

Basic Variables   Non-Basic Variables

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x_4 & = & 5 - 2x_1 - 3x_2 - x_3 \\
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Basic Variables	Non-Basic Variables
x_4	x_1
x_5	x_2
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$$\begin{array}{rclc} & & & \text{Basic Variable} \\ 2x_1 + 3x_2 + x_3 & \leq & 5 & \rightarrow x_4 \end{array}$$

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after setting $x_2 = 0$ and x_3 =0 :

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$$2x_1 \quad \leq \quad 5 \rightarrow x_1 \quad = \quad 2.5$$

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$$\begin{array}{lclcr} 2x_1 & \leq & 5 \rightarrow x_1 & = & 2.5 \\ 4x_1 & \leq & 11 \rightarrow x_1 & = & 2.75 \end{array}$$

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$$\begin{array}{rcrcr} 2x_1 & \leq & 5 \rightarrow x_1 & = & 2.5 \\ 4x_1 & \leq & 11 \rightarrow x_1 & = & 2.75 \\ 3x_1 & \leq & 8 \rightarrow x_1 & = & 2.66 \end{array}$$

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hence,

- for $x_1 = 2.5$

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$$\begin{array}{|ccc|} \hline 2(2.5)=5 & \leq & 5 \\ & & \end{array}$$

hence,

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$$
\left|
\begin{array}{ccc}
2(2.5)=5 & \leq & 5 \\
4(2.5)=10 & \leq & 11
\end{array}
\right.
$$

hence,

- for $x_1 = 2.5$

$$\boxed{\begin{array}{ccc} 2(2.5)=5 & \leq & 5 \\ 4(2.5)=10 & \leq & 11 \\ 3(2.5)=7.5 & \leq & 8 \end{array}}$$

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$$2(2.75)=5.5 \quad \nleq \quad 5$$

hence,

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- for $x_1 = 2.75$

$$\begin{array}{ccc} 2(2.75){=}5.5 & \not\leq & 5 \\ 4(2.75){=}11 & \leq & 11 \end{array}$$

hence,

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- for $x_1 = 2.75$

$$\begin{array}{ccc} 2(2.75)=5.5 & \not\leq & 5 \\ 4(2.75)=11 & \leq & 11 \\ 3(2.75)=8.25 & \not\leq & 8 \end{array}$$

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$$2(2.66)=5.32 \quad \nleq \quad 5$$

hence,

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- for $x_1 = 2.66$

$$\begin{array}{rcl} 2(2.66)=5.32 & \not\leq & 5 \\ 4(2.66)=10.64 & \leq & 11 \end{array}$$

hence,

- for $x_1 = 2.5$

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- for $x_1 = 2.66$

$$\begin{array}{ccc} 2(2.66)=5.32 & \nleq & 5 \\ 4(2.66)=10.64 & \leq & 11 \\ 3(2.66)=7.98 & \leq & 8 \end{array}$$

hence,

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$$x_4 = 5 - 2x_1 - 3x_2 - x_3 = 5 - 2(2.5) - 3(0) - (0) = 0$$

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$$
\begin{array}{ccccccc}
x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & 5 \text{ - } 2(2.5) \text{ - } 3(0) \text{ - } (0) & = & 0 \\
x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & 11 \text{ - } 4(2.5) \text{ - } (0) \text{ - } 2(0) & = & 1
\end{array}
$$

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{lllllll}
x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\
x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\
x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5
\end{array}$$

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$$
\begin{array}{lllllll}
x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\
x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\
x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5
\end{array}
$$

- In summary, from the first iteration:

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{ccccccc} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5 \end{array}$$

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\begin{array}{lllllll}
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x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\
x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5
\end{array}
$$

- In summary, from the first iteration:

$$x_1 = 2.5$$
$$x_2 = 0$$

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{lllllll} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5 \end{array}$$

- In summary, from the first iteration:

$$x_1 = 2.5$$
$$x_2 = 0$$
$$x_3 = 0$$

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{ccccccc} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5 \end{array}$$

- In summary, from the first iteration:

$$x_1 = 2.5$$
$$x_2 = 0$$
$$x_3 = 0$$
$$x_4 = 0$$

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

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- In summary, from the first iteration:

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$$x_2 = 0$$
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$$
\begin{array}{lllllll}
x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & 5 \text{ - } 2(2.5) \text{ - } 3(0) \text{ - } (0) & = & 0 \\
x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & 11 \text{ - } 4(2.5) \text{ - } (0) \text{ - } 2(0) & = & 1 \\
x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & 8 \text{ - } 3(2.5) \text{ - } 4(0) \text{ - } 2(0) & = & 0.5
\end{array}
$$

- In summary, from the first iteration:

$$
\begin{aligned}
x_1 &= 2.5 \\
x_2 &= 0 \\
x_3 &= 0 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0.5
\end{aligned}
$$

- Then we pick x_1=2.5 (to meet constraints)
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x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & 5 \text{ - } 2(2.5) \text{ - } 3(0) \text{ - } (0) & = & 0 \\
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\begin{aligned}
x_1 &= 2.5 \\
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- In summary, from the first iteration:

$$\begin{aligned} x_1 &= 2.5 \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0.5 \end{aligned}$$

and,

$$\Omega(x) = 12.5$$

- Then we pick x_1=2.5 (to meet constraints)
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$$
\begin{array}{ccccccc}
x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\
x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\
x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5
\end{array}
$$

- In summary, from the first iteration:

$$
\begin{aligned}
x_1 &= 2.5 \\
x_2 &= 0 \\
x_3 &= 0 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0.5
\end{aligned}
$$

and,

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Things relevant to recall:

- Then we pick x_1=2.5 (to meet constraints)
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$$\begin{array}{ccccccc} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5 \end{array}$$

- In summary, from the first iteration:

$$\begin{aligned} x_1 &= 2.5 \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0.5 \end{aligned}$$

and,

$$\Omega(x) = 12.5$$

Things relevant to recall:

- x_1 was the increasing variable

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$
\begin{array}{lllllll}
x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & 5 \text{ - } 2(2.5) \text{ - } 3(0) \text{ - } (0) & = & 0 \\
x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & 11 \text{ - } 4(2.5) \text{ - } (0) \text{ - } 2(0) & = & 1 \\
x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & 8 \text{ - } 3(2.5) \text{ - } 4(0) \text{ - } 2(0) & = & 0.5
\end{array}
$$

- In summary, from the first iteration:

$$
\begin{aligned}
x_1 &= 2.5 \\
x_2 &= 0 \\
x_3 &= 0 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0.5
\end{aligned}
$$

and,

$$\Omega(x) = 12.5$$

Things relevant to recall:

- x_1 was the increasing variable
- x_4 was the basic variable related to the equation from which maximum increase in x_1, without constraints violation, was achieved

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{ccccccc} x_4 & = & 5-2x_1-3x_2-x_3 & = & 5 - 2(2.5) - 3(0) - (0) & = & 0 \\ x_5 & = & 11-4x_1-x_2-2x_3 & = & 11 - 4(2.5) - (0) - 2(0) & = & 1 \\ x_6 & = & 8-3x_1-4x_2-2x_3 & = & 8 - 3(2.5) - 4(0) - 2(0) & = & 0.5 \end{array}$$

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Things relevant to recall:

- x_1 was the increasing variable
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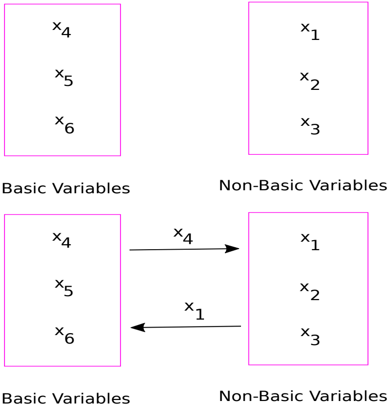
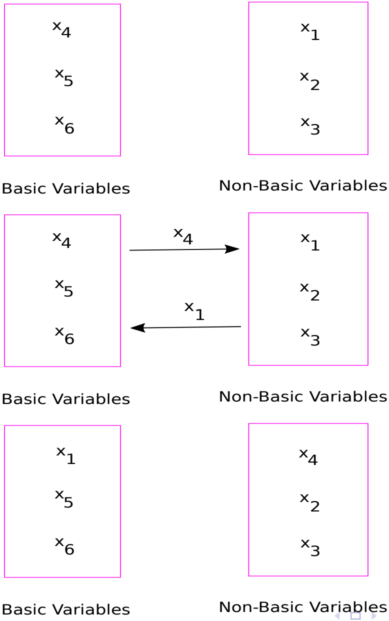
Second Iteration

Basic Variables	Non-Basic Variables
x_4	x_1
x_5	x_2
x_6	x_3

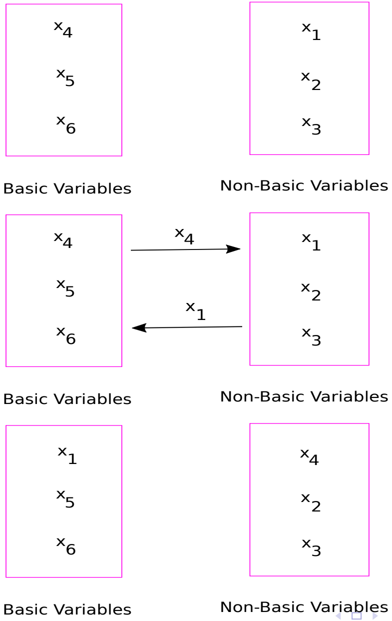
Second Iteration



Second Iteration



Second Iteration



- How to turn a non-basic into a basic variable?

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- Pick from the Dictionary formulation that equation in which x_4 is the basic variable:

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$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

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$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

and solve for x_1:

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation that equation in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

and solve for x_1:

$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4$$

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation that equation in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

and solve for x_1:

$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4$$

and replace this equation into the two remaining dictionary equations:

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$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

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$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4$$

and replace this equation into the two remaining dictionary equations:

$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \end{aligned}$$

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- Pick from the Dictionary formulation that equation in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

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For x_5:

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For x_5:

$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ &= 11 - 4\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - x_2 - 2x_3 \end{aligned}$$

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$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

and solve for x_1:

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and replace this equation into the two remaining dictionary equations:

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For x_5:

$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ &= 11 - 4\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - x_2 - 2x_3 \\ &= 1 + 5x_2 + 2x_4 \end{aligned}$$

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- Pick from the Dictionary formulation that equation in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

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For x_6:

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$$x_6 = 8 - 3x_1 - 4x_2 - 2x_3$$

For x_6:

$$\begin{aligned} x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\ &= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \end{aligned}$$

For x_6:

$$
\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
\end{aligned}
$$

For x_6:

$$
\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
\end{aligned}
$$

Also replace into the Objective Function:

For x_6:

$$\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
\end{aligned}$$

Also replace into the Objective Function:

$$\Omega = 5x_1 + 4x_2 + 3x_3$$

For x_6:

$$\begin{aligned} x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\ &= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\ &= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \end{aligned}$$

Also replace into the Objective Function:

$$\begin{aligned} \Omega &= 5x_1 + 4x_2 + 3x_3 \\ &= 5\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) + 4x_2 + 3x_3 \end{aligned}$$

For x_6:

$$
\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
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\end{aligned}
$$

Also replace into the Objective Function:

$$
\begin{aligned}
\Omega &= 5x_1 + 4x_2 + 3x_3 \\
&= 5\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) + 4x_2 + 3x_3 \\
&= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{aligned}
$$

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&= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{aligned}
$$

In summary, the new dictionary reads:

For x_6:

$$\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
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\end{aligned}$$

In summary, the new dictionary reads:

$$x_1 = \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4$$

For x_6:

$$
\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
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$$

In summary, the new dictionary reads:

$$
\begin{array}{|lcl|}
\hline
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \\
x_5 & = & 1 + 5x_2 + 2x_4 \\
\end{array}
$$

For x_6:

$$
\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
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$$

In summary, the new dictionary reads:

$$
\boxed{
\begin{aligned}
x_1 &= \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 \\
x_5 &= 1 + 5x_2 + 2x_4 \\
x_6 &= \tfrac{1}{2} + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4
\end{aligned}
}
$$

For x_6:

$$
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x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
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\begin{aligned}
\Omega &= 5x_1 + 4x_2 + 3x_3 \\
&= 5\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) + 4x_2 + 3x_3 \\
&= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{aligned}
$$

In summary, the new dictionary reads:

$$
\boxed{
\begin{array}{lcl}
x_1 &=& \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \\
x_5 &=& 1 + 5x_2 + 2x_4 \\
x_6 &=& \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \\
\hline
\Omega &=& \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{array}
}
$$

For x_6:

$$
\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
\end{aligned}
$$

Also replace into the Objective Function:

$$
\begin{aligned}
\Omega &= 5x_1 + 4x_2 + 3x_3 \\
&= 5\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) + 4x_2 + 3x_3 \\
&= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{aligned}
$$

In summary, the new dictionary reads:

$$
\boxed{
\begin{array}{lcl}
x_1 &=& \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \\
x_5 &=& 1 + 5x_2 + 2x_4 \\
x_6 &=& \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \\
\hline
\Omega &=& \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{array}
}
$$

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$

[1] when solving this equation set the basic variable x_1 to zero

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- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

[1] when solving this equation set the basic variable x_1 to zero

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- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$x_1 \quad = \quad \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \quad = \quad \frac{5}{2} - \frac{1}{2}x_3 \quad \rightarrow x_3{=}5$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables ($x_2 = 0, x_4 = 0$)
- From the present dictionary:

$$x_1 \quad = \quad \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \quad = \quad \frac{5}{2} - \frac{1}{2}x_3 \quad \rightarrow x_3{=}5 \text{ [1]}$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{array}{lclcll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 \ ^1 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1
\end{array}$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{array}{lllll} x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 \; ^1 \\ x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\ \color{red}{x_6} & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow \color{red}{x_3{=}1} \end{array}$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables ($x_2 = 0, x_4 = 0$)
- From the present dictionary:

$$
\begin{array}{lclclcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow & x_3{=}5 \; ^{1} \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow & x_5{=}1 \\
\color{red}{x_6} & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow & \color{red}{x_3{=}1} \; ^{2}
\end{array}
$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables ($x_2 = 0, x_4 = 0$)
- From the present dictionary:

$$\begin{array}{lclcll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5\ ^1 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\
{\color{red}x_6} & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow {\color{red}x_3{=}1}\ ^2
\end{array}$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{array}{lllll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3 = 5 \; {}^{1} \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5 = 1 \\
\color{red}{x_6} & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow \color{red}{x_3 = 1} \; {}^{2}
\end{array}$$

- Hence, setting $x_3 = 1$:

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$
\begin{array}{lclcll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}1
\end{array}
$$

[1] [2]

- Hence, setting $x_3 = 1$:

$$
x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 = \frac{5}{2} - \frac{3}{2}(0) - \frac{1}{2}(1) - \frac{1}{2}(0) = 2
$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$
\begin{array}{lcllcll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 \; ^1 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\
\color{red}{x_6} & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow \color{red}{x_3{=}1} \; ^2
\end{array}
$$

- Hence, setting $x_3 = 1$:

$$
\begin{array}{lcllclcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{3}{2}(0) - \frac{1}{2}(1) - \frac{1}{2}(0) & = & 2 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 + 5(0) + 2(0) & = & 1
\end{array}
$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$
\begin{array}{lcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5\ ^{1} \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\
{\color{red}x_6} & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow {\color{red}x_3{=}1}\ ^{2}
\end{array}
$$

- Hence, setting $x_3 = 1$:

$$
\begin{array}{lcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{3}{2}(0) - \frac{1}{2}(1) - \frac{1}{2}(0) & = & 2 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 + 5(0) + 2(0) & = & 1 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} + \frac{1}{2}(0) - \frac{1}{2}(1) + \frac{3}{2}(0) & = & 0
\end{array}
$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$
\begin{array}{lclcll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 \; ^1 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\
\color{red}{x_6} & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow \color{red}{x_3{=}1} \; ^2
\end{array}
$$

- Hence, setting $x_3 = 1$:

$$
\begin{array}{lclclcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{3}{2}(0) - \frac{1}{2}(1) - \frac{1}{2}(0) & = & 2 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 + 5(0) + 2(0) & = & 1 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} + \frac{1}{2}(0) - \frac{1}{2}(1) + \frac{3}{2}(0) & = & 0 \\
\Omega & = & \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4 & = & \frac{25}{2} - \frac{7}{2}(0) + \frac{1}{2}(1) - \frac{5}{2}(0) & = & 13
\end{array}
$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$
\begin{array}{lclcll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3 = 5 \; ^{1} \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5 = 1 \\
\color{red}{x_6} & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow \color{red}{x_3 = 1} \; ^{2}
\end{array}
$$

- Hence, setting $x_3 = 1$:

$$
\begin{array}{lclclcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{3}{2}(0) - \frac{1}{2}(1) - \frac{1}{2}(0) & = & 2 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 + 5(0) + 2(0) & = & 1 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} + \frac{1}{2}(0) - \frac{1}{2}(1) + \frac{3}{2}(0) & = & 0 \\
\Omega & = & \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4 & = & \frac{25}{2} - \frac{7}{2}(0) + \frac{1}{2}(1) - \frac{5}{2}(0) & = & 13
\end{array}
$$

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- In summary, from the second iteration:

- In summary, from the second iteration:

$$x_1 = 2$$

- In summary, from the second iteration:

$$x_1 = 2$$
$$x_2 = 0$$

- In summary, from the second iteration:

$$x_1 = 2$$
$$x_2 = 0$$
$$x_3 = 1$$

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
x_3 &= 1 \\
x_4 &= 0
\end{aligned}
$$

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
x_3 &= 1 \\
x_4 &= 0 \\
x_5 &= 1
\end{aligned}
$$

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
x_3 &= 1 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0
\end{aligned}
$$

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
x_3 &= 1 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0
\end{aligned}
$$

and,

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
x_3 &= 1 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0
\end{aligned}
$$

and,

$$\Omega(x) = 13$$

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
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x_6 &= 0
\end{aligned}
$$

and,

$$\Omega(x) = 13$$

Things relevant to recall:

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
x_3 &= 1 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0
\end{aligned}
$$

  and,

$$\Omega(x) = 13$$

Things relevant to recall:

- x_3 was the increasing variable

- In summary, from the second iteration:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0 \end{aligned}$$

and,

$$\Omega(x) = 13$$

Things relevant to recall:

- x_3 was the increasing variable
- x_6 was the basic variable related to the equation from which maximum increase in x_3, without constraints violation, was achieved

- In summary, from the second iteration:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0 \end{aligned}$$

and,

$$\Omega(x) = 13$$

Things relevant to recall:

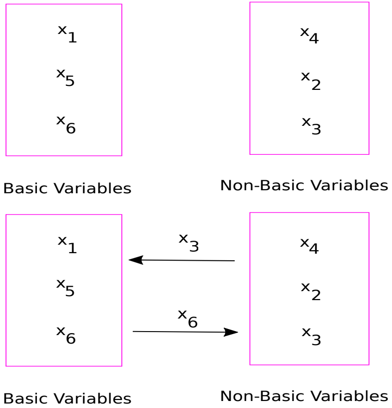
- x_3 was the increasing variable
- x_6 was the basic variable related to the equation from which maximum increase in x_3, without constraints violation, was achieved

Third Iteration

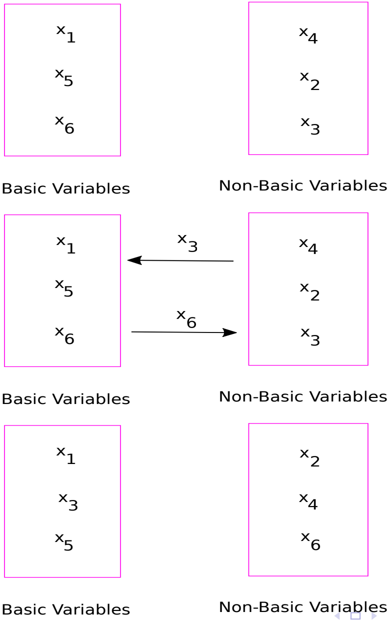
Third Iteration

Basic Variables	Non-Basic Variables
x_1	x_4
x_5	x_2
x_6	x_3

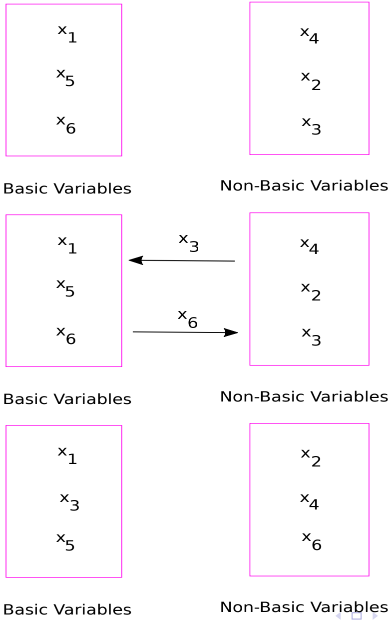
Third Iteration



Third Iteration



Third Iteration



- Once again, turn a non-basic into a basic variable (x_3)

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- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

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$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

- Once again, turn a non-basic into a basic variable (x_3)
- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

and solve for x_3:

- Once again, turn a non-basic into a basic variable (x_3)
- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

and solve for x_3:

$$x_3 = 1 + x_2 + 3x_4 - 2x_6$$

- Once again, turn a non-basic into a basic variable (x_3)
- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

and solve for x_3:

$$x_3 = 1 + x_2 + 3x_4 - 2x_6$$

and replace in the remaining dictionary equations:

- Once again, turn a non-basic into a basic variable (x_3)
- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

and solve for x_3:

$$x_3 = 1 + x_2 + 3x_4 - 2x_6$$

and replace in the remaining dictionary equations:

$$\begin{aligned} x_1 &= \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 \\ x_5 &= 1 + 5x_2 + 2x_4 \end{aligned}$$

- Once again, turn a non-basic into a basic variable (x_3)
- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

and solve for x_3:

$$x_3 = 1 + x_2 + 3x_4 - 2x_6$$

and replace in the remaining dictionary equations:

$$\begin{array}{rcl} x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \\ x_5 & = & 1 + 5x_2 + 2x_4 \end{array}$$

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$$x_1 = 2$$
$$x_2 = 0$$
$$x_3 = 1$$
$$x_4 = 0$$
$$x_5 = 1$$
$$x_6 = 0$$
$$\Omega(x) = 13$$