Simplex Method Using the Dictionaries Formulation Approach

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February 17, 2020

Solve by Simplex Method:

$$\begin{array}{rrcl} \max\limits_{x_1,x_2,x_3} & \Omega = 5x_1 + 4x_2 + 3x_3 & & \\ \text{s.t.} & 2x_1 + 3x_2 + x_3 & \leq & 5 \\ & 4x_1 + x_2 + 2x_3 & \leq & 11 \\ & 3x_1 + 4x_2 + 2x_3 & \leq & 8 \\ & x_1, x_2, x_3 & \geq & 0 \end{array}$$

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$$\begin{aligned} \max_{x_1,x_2,x_3} \quad & \Omega = 5x_1 + 4x_2 + 3x_3 \\ \text{s.t.} \quad & 2x_1 + 3x_2 + x_3 + x_4 \quad = \quad 5 \end{aligned}$$

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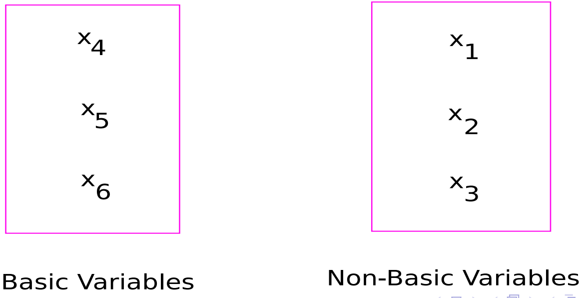

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x_4

x_5

x_6

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after setting $x_2 = 0$ and $x_3 = 0$:

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$$\begin{array}{rclcr} 2x_1 & \leq & 5 \rightarrow x_1 & = & 2.5 \\ 4x_1 & \leq & 11 \rightarrow x_1 & = & 2.75 \end{array}$$

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hence,

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$$2(2.5)=5 \quad \leq \quad 5$$

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$$\begin{array}{ccc} 2(2.66)=5.32 & \nleq & 5 \\ 4(2.66)=10.64 & \leq & 11 \end{array}$$

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$$\begin{array}{ccc} 2(2.66)=5.32 & \nleq & 5 \\ 4(2.66)=10.64 & \leq & 11 \\ 3(2.66)=7.98 & \leq & 8 \end{array}$$

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- Then we pick x_1=2.5 (to meet constraints)
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$$\begin{array}{lcccccl} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5 \end{array}$$

- In summary, from the first iteration:

$$x_1 = 2.5$$
$$x_2 = 0$$
$$x_3 = 0$$

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{ccccccc} x_4 & = & 5-2x_1-3x_2-x_3 & = & 5 - 2(2.5) - 3(0) - (0) & = & 0 \\ x_5 & = & 11-4x_1-x_2-2x_3 & = & 11 - 4(2.5) - (0) - 2(0) & = & 1 \\ x_6 & = & 8-3x_1-4x_2-2x_3 & = & 8 - 3(2.5) - 4(0) - 2(0) & = & 0.5 \end{array}$$

- In summary, from the first iteration:

$$\begin{aligned} x_1 &= 2.5 \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \end{aligned}$$

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{lclclcl}
x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\
x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\
x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5
\end{array}$$

- In summary, from the first iteration:

$$\begin{aligned}
x_1 &= 2.5 \\
x_2 &= 0 \\
x_3 &= 0 \\
x_4 &= 0 \\
x_5 &= 1
\end{aligned}$$

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{lclclcl} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5 \end{array}$$

- In summary, from the first iteration:

$$\begin{aligned} x_1 &= 2.5 \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0.5 \end{aligned}$$

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$$\begin{aligned} x_1 &= 2.5 \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0.5 \end{aligned}$$

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- Then we pick x_1=2.5 (to meet constraints)
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- In summary, from the first iteration:

$$\begin{aligned} x_1 &= 2.5 \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0.5 \end{aligned}$$

and,

$$\Omega(x) = 12.5$$

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{lclclcl} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5 \end{array}$$

- In summary, from the first iteration:

$$\begin{aligned} x_1 &= 2.5 \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0.5 \end{aligned}$$

and,

$$\Omega(x) = 12.5$$

Things relevant to recall:

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{lcccc} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & \text{5 - 2(2.5) - 3(0) - (0)} & = & 0 \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & \text{11 - 4(2.5) - (0) - 2(0)} & = & 1 \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & \text{8 - 3(2.5) - 4(0) - 2(0)} & = & 0.5 \end{array}$$

- In summary, from the first iteration:

$$\begin{aligned} x_1 &= 2.5 \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0.5 \end{aligned}$$

and,

$$\Omega(x) = 12.5$$

Things relevant to recall:

  - x_1 was the increasing variable

- Then we pick x_1=2.5 (to meet constraints)
- and because x_2=0 and x_3=0 from:

$$\begin{array}{lclclcl} x_4 & = & 5 - 2x_1 - 3x_2 - x_3 & = & 5\text{ - }2(2.5)\text{ - }3(0)\text{ - }(0) & = & 0 \\ x_5 & = & 11 - 4x_1 - x_2 - 2x_3 & = & 11\text{ - }4(2.5)\text{ - }(0)\text{ - }2(0) & = & 1 \\ x_6 & = & 8 - 3x_1 - 4x_2 - 2x_3 & = & 8\text{ - }3(2.5)\text{ - }4(0)\text{ - }2(0) & = & 0.5 \end{array}$$

- In summary, from the first iteration:

$$\begin{aligned} x_1 &= 2.5 \\ x_2 &= 0 \\ x_3 &= 0 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0.5 \end{aligned}$$

and,

$$\Omega(x) = 12.5$$

Things relevant to recall:

  - x_1 was the increasing variable
  - The maximum amount by which x_1 was changed was obtained from: $x_4 = 5 - 2x_1 - 3x_2 - x_3$, which is called the *Pivot Equation*. In this Equation, x_4 is the Basic variable.

Second Iteration

x_4

x_5

x_6

Basic Variables

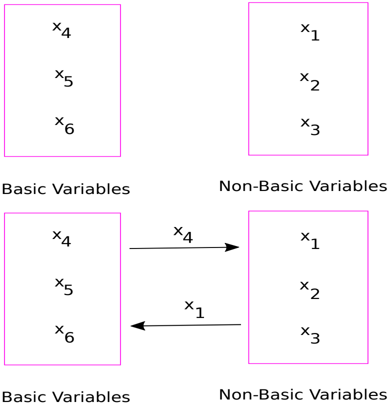
x_1

x_2

x_3

Non-Basic Variables

Second Iteration


x_4

x_5

x_6

Basic Variables

x_1

x_2

x_3

Non-Basic Variables

x_4

x_5

x_6

x_4

x_1

x_1

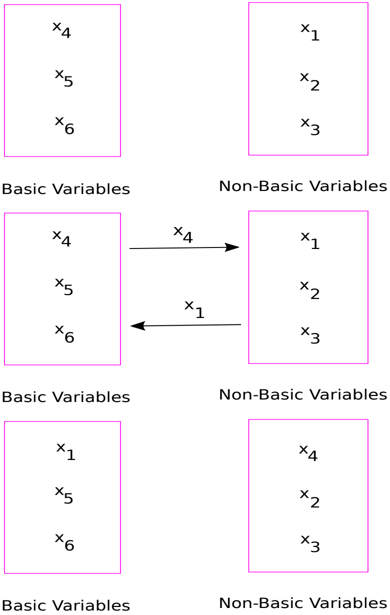
x_2

x_3

Basic Variables

Non-Basic Variables

Second Iteration



x_4

x_5

x_6

Basic Variables

x_1

x_2

x_3

Non-Basic Variables

x_4

x_5

x_6

x_4

x_1

x_1

x_2

x_3

Basic Variables

Non-Basic Variables

x_1

x_5

x_6

Basic Variables

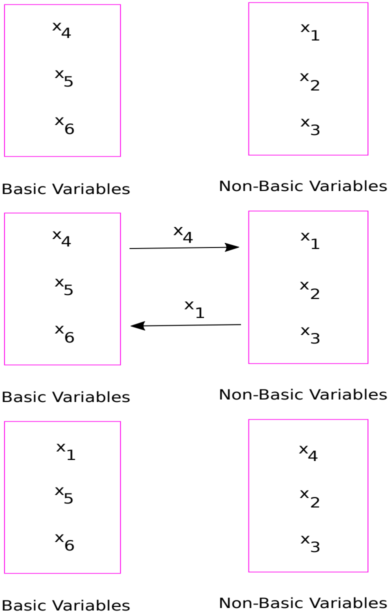
x_4

x_2

x_3

Non-Basic Variables

Second Iteration



x_4 x_5 x_6

Basic Variables

x_1 x_2 x_3

Non-Basic Variables

x_4 x_5 x_6

x_4

x_1

x_1 x_2 x_3

Basic Variables

Non-Basic Variables

x_1 x_5 x_6

Basic Variables

x_4 x_2 x_3

Non-Basic Variables

- How to turn a non-basic into a basic variable?

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation the Pivot Equation, in which x_4 is the basic variable:

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$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation the Pivot Equation, in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

and solve for x_1:

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation the Pivot Equation, in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

and solve for x_1:

$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4$$

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation the Pivot Equation, in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

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and replace this equation into the two remaining dictionary equations:

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$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4$$

and replace this equation into the two remaining dictionary equations:

$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \end{aligned}$$

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation the Pivot Equation, in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

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$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \end{aligned}$$

For x_5:

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation the Pivot Equation, in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

and solve for x_1:

$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4$$

and replace this equation into the two remaining dictionary equations:

$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \end{aligned}$$

For x_5:

$$x_5 = 11 - 4x_1 - x_2 - 2x_3$$

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation the Pivot Equation, in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

and solve for x_1:

$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4$$

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$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \end{aligned}$$

For x_5:

$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ &= 11 - 4\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - x_2 - 2x_3 \end{aligned}$$

- How to turn a non-basic into a basic variable?
- Pick from the Dictionary formulation the Pivot Equation, in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

and solve for x_1:

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$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \end{aligned}$$

For x_5:

$$\begin{aligned} x_5 &= 11 - 4x_1 - x_2 - 2x_3 \\ &= 11 - 4\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - x_2 - 2x_3 \\ &= 1 + 5x_2 + 2x_4 \end{aligned}$$

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- Pick from the Dictionary formulation the Pivot Equation, in which x_4 is the basic variable:

$$x_4 = 5 - 2x_1 - 3x_2 - x_3$$

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For x_6:

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$$x_6 = 8 - 3x_1 - 4x_2 - 2x_3$$

For x_6:

$$\begin{aligned} x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\ &= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \end{aligned}$$

For x_6:

$$\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
\end{aligned}$$

For x_6:

$$\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
\end{aligned}$$

Also replace into the Objective Function:

For x_6:

$$\begin{aligned} x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\ &= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\ &= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \end{aligned}$$

Also replace into the Objective Function:

$$\Omega = 5x_1 + 4x_2 + 3x_3$$

For x_6:

$$\begin{aligned} x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\ &= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\ &= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \end{aligned}$$

Also replace into the Objective Function:

$$\begin{aligned} \Omega &= 5x_1 + 4x_2 + 3x_3 \\ &= 5\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) + 4x_2 + 3x_3 \end{aligned}$$

For x_6:

$$\begin{aligned} x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\ &= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\ &= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \end{aligned}$$

Also replace into the Objective Function:

$$\begin{aligned} \Omega &= 5x_1 + 4x_2 + 3x_3 \\ &= 5\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) + 4x_2 + 3x_3 \\ &= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4 \end{aligned}$$

For x_6:

$$\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
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&= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{aligned}$$

In summary, the new dictionary reads:

For x_6:

$$\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
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&= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{aligned}$$

In summary, the new dictionary reads:

$$x_1 = \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4$$

For x_6:

$$\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
\end{aligned}$$

Also replace into the Objective Function:

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\Omega &= 5x_1 + 4x_2 + 3x_3 \\
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&= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
\end{aligned}$$

In summary, the new dictionary reads:

$$\begin{array}{|lcl|}
\hline
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \\
x_5 & = & 1 + 5x_2 + 2x_4 \\
 & &
\end{array}$$

For x_6:

$$\begin{aligned}
x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\
&= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\
&= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4
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Also replace into the Objective Function:

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&= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4
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In summary, the new dictionary reads:

$$\begin{array}{|lcl|}
\hline
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \\
x_5 & = & 1 + 5x_2 + 2x_4 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \\
\hline
\end{array}$$

For x_6:

$$\begin{aligned} x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\ &= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\ &= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \end{aligned}$$

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$$\begin{aligned} \Omega &= 5x_1 + 4x_2 + 3x_3 \\ &= 5\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) + 4x_2 + 3x_3 \\ &= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4 \end{aligned}$$

In summary, the new dictionary reads:

$$\boxed{\begin{array}{lcl} x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \\ x_5 & = & 1 + 5x_2 + 2x_4 \\ x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \\ \hline \\ \Omega & = & \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4 \end{array}}$$

For x_6:

$$\begin{aligned} x_6 &= 8 - 3x_1 - 4x_2 - 2x_3 \\ &= 8 - 3\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) - 4x_2 - 2x_3 \\ &= \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \end{aligned}$$

Also replace into the Objective Function:

$$\begin{aligned} \Omega &= 5x_1 + 4x_2 + 3x_3 \\ &= 5\left(\frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4\right) + 4x_2 + 3x_3 \\ &= \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4 \end{aligned}$$

In summary, the new dictionary reads:

$$\boxed{\begin{array}{lcl} x_1 &=& \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \\ x_5 &=& 1 + 5x_2 + 2x_4 \\ x_6 &=& \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 \\ \hline \\ \Omega &=& \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4 \end{array}}$$

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$

---

[1] when solving this equation set the basic variable x_1 to zero

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- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

---

[1] when solving this equation set the basic variable x_1 to zero

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- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$x_1 \quad = \quad \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 \quad = \quad \frac{5}{2} - \frac{1}{2}x_3 \quad \rightarrow x_3{=}5$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 = \frac{5}{2} - \frac{1}{2}x_3 \rightarrow x_3{=}5$$ [1]

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{array}{lclcll} x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 \\ x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \end{array}$$ [1]

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{array}{lclclcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow & x_3{=}5\ ^1 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow & x_5{=}1 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow & x_3{=}1
\end{array}$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{array}{lclclr}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 \text{ [1]} \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\
\color{red}{x_6} & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow \color{red}{x_3{=}1} \text{ [2]}
\end{array}$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables ($x_2 = 0, x_4 = 0$)
- From the present dictionary:

$$\begin{array}{lcllcll} x_1 &=& \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 &=& \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 & \text{[1]} \\ x_5 &=& 1 + 5x_2 + 2x_4 &=& 1 & \rightarrow x_5{=}1 & \\ x_6 &=& \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 &=& \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}1 & \text{[2]} \end{array}$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables ($x_2 = 0, x_4 = 0$)
- From the present dictionary:

$$\begin{array}{lclcll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 \text{ [1]} \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}1 \text{ [2]}
\end{array}$$

- Hence, setting $x_3 = 1$ (recall that in the Pivot Equation, x_6 is the Basic variable):

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{array}{lllllll} x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 & \text{[1]} \\ x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 & \\ x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}1 & \text{[2]} \end{array}$$

- Hence, setting $x_3 = 1$ (recall that in the Pivot Equation, x_6 is the Basic variable):

$$x_1 = \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 = \frac{5}{2} - \frac{3}{2}(0) - \frac{1}{2}(1) - \frac{1}{2}(0) = 2$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{array}{lclcll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}1
\end{array}$$

[1] [2]

- Hence, setting $x_3 = 1$ (recall that in the Pivot Equation, x_6 is the Basic variable):

$$\begin{array}{lclclcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{3}{2}(0) - \frac{1}{2}(1) - \frac{1}{2}(0) & = & 2 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 + 5(0) + 2(0) & = & 1
\end{array}$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{aligned}
x_1 &= \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 &&= \tfrac{5}{2} - \tfrac{1}{2}x_3 &&\rightarrow x_3{=}5 \text{ [1]}\\
x_5 &= 1 + 5x_2 + 2x_4 &&= 1 &&\rightarrow x_5{=}1\\
x_6 &= \tfrac{1}{2} + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4 &&= \tfrac{1}{2} - \tfrac{1}{2}x_3 &&\rightarrow x_3{=}1 \text{ [2]}
\end{aligned}$$

- Hence, setting $x_3 = 1$ (recall that in the Pivot Equation, x_6 is the Basic variable):

$$\begin{aligned}
x_1 &= \tfrac{5}{2} - \tfrac{3}{2}x_2 - \tfrac{1}{2}x_3 - \tfrac{1}{2}x_4 &&= \tfrac{5}{2} - \tfrac{3}{2}(0) - \tfrac{1}{2}(1) - \tfrac{1}{2}(0) &&= 2\\
x_5 &= 1 + 5x_2 + 2x_4 &&= 1 + 5(0) + 2(0) &&= 1\\
x_6 &= \tfrac{1}{2} + \tfrac{1}{2}x_2 - \tfrac{1}{2}x_3 + \tfrac{3}{2}x_4 &&= \tfrac{1}{2} + \tfrac{1}{2}(0) - \tfrac{1}{2}(1) + \tfrac{3}{2}(0) &&= 0
\end{aligned}$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables $(x_2 = 0, x_4 = 0)$
- From the present dictionary:

$$\begin{array}{lclcll}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}5\ ^{[1]} \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5{=}1 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow x_3{=}1\ ^{[2]}
\end{array}$$

- Hence, setting $x_3 = 1$ (recall that in the Pivot Equation, x_6 is the Basic variable):

$$\begin{array}{lclclcl}
x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{3}{2}(0) - \frac{1}{2}(1) - \frac{1}{2}(0) & = & 2 \\
x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 + 5(0) + 2(0) & = & 1 \\
x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} + \frac{1}{2}(0) - \frac{1}{2}(1) + \frac{3}{2}(0) & = & 0 \\
\Omega & = & \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4 & = & \frac{25}{2} - \frac{7}{2}(0) + \frac{1}{2}(1) - \frac{5}{2}(0) & = & 13
\end{array}$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- Now, pick the non-basic variable featuring the largest positive coefficient in Ω: x_3
- Check the maximum amount by which x_3 can be increased without constraints violation and set to zero the remaining non-basic variables ($x_2 = 0, x_4 = 0$)
- From the present dictionary:

$$\begin{array}{lclcll} x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{1}{2}x_3 & \rightarrow x_3 = 5 \\ x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 & \rightarrow x_5 = 1 \\ x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} - \frac{1}{2}x_3 & \rightarrow x_3 = 1 \end{array}$$

[1] [2]

- Hence, setting $x_3 = 1$ (recall that in the Pivot Equation, x_6 is the Basic variable):

$$\begin{array}{lclclcl} x_1 & = & \frac{5}{2} - \frac{3}{2}x_2 - \frac{1}{2}x_3 - \frac{1}{2}x_4 & = & \frac{5}{2} - \frac{3}{2}(0) - \frac{1}{2}(1) - \frac{1}{2}(0) & = & 2 \\ x_5 & = & 1 + 5x_2 + 2x_4 & = & 1 + 5(0) + 2(0) & = & 1 \\ x_6 & = & \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4 & = & \frac{1}{2} + \frac{1}{2}(0) - \frac{1}{2}(1) + \frac{3}{2}(0) & = & 0 \\ \Omega & = & \frac{25}{2} - \frac{7}{2}x_2 + \frac{1}{2}x_3 - \frac{5}{2}x_4 & = & \frac{25}{2} - \frac{7}{2}(0) + \frac{1}{2}(1) - \frac{5}{2}(0) & = & 13 \end{array}$$

---

[1] when solving this equation set the basic variable x_1 to zero

[2] when solving the basic variable this equation set x_6 to zero

- In summary, from the second iteration:

- In summary, from the second iteration:

$$x_1 = 2$$

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0
\end{aligned}
$$

- In summary, from the second iteration:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \end{aligned}$$

- In summary, from the second iteration:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \end{aligned}$$

- In summary, from the second iteration:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \\ x_5 &= 1 \end{aligned}$$

- In summary, from the second iteration:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0 \end{aligned}$$

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
x_3 &= 1 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0
\end{aligned}
$$

and,

- In summary, from the second iteration:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0 \end{aligned}$$

and,

$$\Omega(x) = 13$$

- In summary, from the second iteration:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0 \end{aligned}$$

and,

$$\Omega(x) = 13$$

Things relevant to recall:

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
x_3 &= 1 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0
\end{aligned}
$$

  and,

$$\Omega(x) = 13$$

Things relevant to recall:

- x_3 was the increasing variable

- In summary, from the second iteration:

$$\begin{aligned} x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0 \end{aligned}$$

  and,

$$\Omega(x) = 13$$

Things relevant to recall:

- x_3 was the increasing variable
- x_6 was the basic variable related to the equation from which maximum increase in x_3, without constraints violation, was achieved

- In summary, from the second iteration:

$$
\begin{aligned}
x_1 &= 2 \\
x_2 &= 0 \\
x_3 &= 1 \\
x_4 &= 0 \\
x_5 &= 1 \\
x_6 &= 0
\end{aligned}
$$

  and,

$$\Omega(x) = 13$$

Things relevant to recall:

- x_3 was the increasing variable
- x_6 was the basic variable related to the equation from which maximum increase in x_3, without constraints violation, was achieved

Third Iteration

Third Iteration

x_1

x_5

x_6

Basic Variables

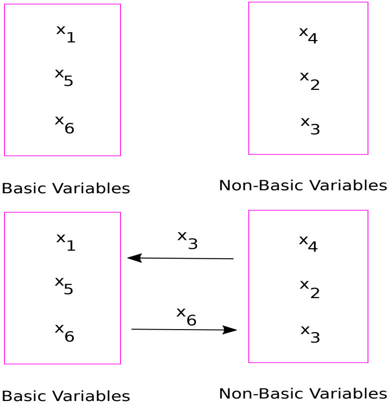
x_4

x_2

x_3

Non-Basic Variables

Third Iteration



x_1
x_5
x_6

Basic Variables

x_4
x_2
x_3

Non-Basic Variables

x_1
x_5
x_6

Basic Variables

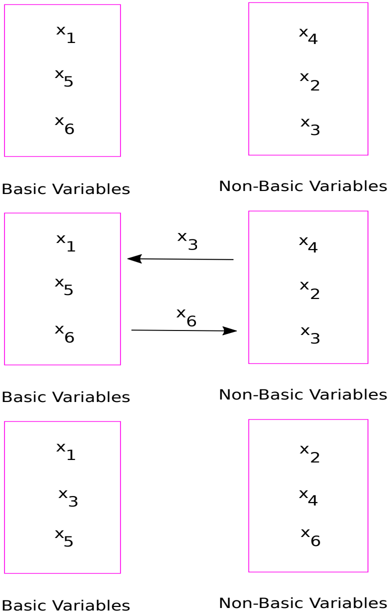
x_3

x_6

x_4
x_2
x_3

Non-Basic Variables

Third Iteration



x_1

x_5

x_6

Basic Variables

x_4

x_2

x_3

Non-Basic Variables

x_1

x_5

x_6

Basic Variables

x_3

x_6

x_4

x_2

x_3

Non-Basic Variables

x_1

x_3

x_5

Basic Variables

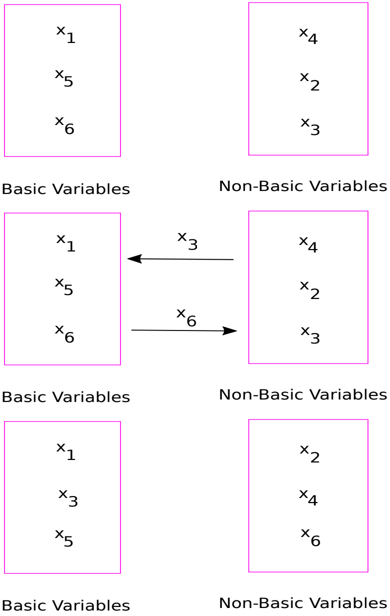
x_2

x_4

x_6

Non-Basic Variables

Third Iteration



x_1

x_5

x_6

Basic Variables

x_4

x_2

x_3

Non-Basic Variables

x_1

x_5

x_6

Basic Variables

x_3

x_6

x_4

x_2

x_3

Non-Basic Variables

x_1

x_3

x_5

Basic Variables

x_2

x_4

x_6

Non-Basic Variables

- Once again, turn a non-basic into a basic variable (x_3)

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- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

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- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

- Once again, turn a non-basic into a basic variable (x_3)
- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

and solve for x_3:

- Once again, turn a non-basic into a basic variable (x_3)
- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

and solve for x_3:

$$x_3 = 1 + x_2 + 3x_4 - 2x_6$$

- Once again, turn a non-basic into a basic variable (x_3)
- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

and solve for x_3:

$$x_3 = 1 + x_2 + 3x_4 - 2x_6$$

and replace in the remaining dictionary equations:

- Once again, turn a non-basic into a basic variable (x_3)
- Similarly to the previous iteration, pick from the past Dictionary formulation that equation in which x_6 is the basic variable:

$$x_6 = \frac{1}{2} + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{3}{2}x_4$$

and solve for x_3:

$$x_3 = 1 + x_2 + 3x_4 - 2x_6$$

and replace in the remaining dictionary equations:

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$$\begin{aligned}x_1 &= 2 \\ x_2 &= 0 \\ x_3 &= 1 \\ x_4 &= 0 \\ x_5 &= 1 \\ x_6 &= 0 \\ \Omega(x) &= 13\end{aligned}$$