1 Recursion

1.1 (Adapted from Fall 2013) Fill in the blanks in the implementation of `paths`, which takes as input two positive integers x and y. It returns a list of paths, where each path is a list containing steps to reach y from x by repeated incrementing or doubling For instance, we can reach 9 from 3 by incrementing to 4, doubling to 8, then incrementing again to 9, so one path is [3, 4, 8, 9]

```
def paths(x, y):
    """Return a list of ways to reach y from x by repeated
    incrementing or doubling.
    >>> paths(3, 5)
    [[3, 4, 5]]
    >>> sorted(paths(3, 6))
    [[3, 4, 5, 6], [3, 6]]
    >>> sorted(paths(3, 9))
    [[3, 4, 5, 6, 7, 8, 9], [3, 4, 8, 9], [3, 6, 7, 8, 9]]
    >>> paths(3, 3) # No calls is a valid path
    [[3]]
    """
    if _____:

        return _____

    elif _____:

        return _____

    else:

        a = _____

        b = _____

        return _____
```

1.2   We will now write one of the faster sorting algorithms commonly used, known as *merge sort.* Merge sort works like this:

1. If there is only one (or zero) item(s) in the sequence, it is already sorted!

2. If there are more than one item, then we can split the sequence in half, sort each half recursively, then merge the results, using the `merge` procedure from earlier in the notes. The result will be a sorted sequence.

Using the algorithm described, write a function `mergesort(seq)` that takes an unsorted sequence and sorts it.

```
def mergesort(seq):
```

2 Trees

2.1 Implement `long_paths`, which returns a list of all *paths* in a tree with length at least n. A path in a tree is a linked list of node values that starts with the root and ends at a leaf. Each subsequent element must be from a child of the previous value's node. The *length* of a path is the number of edges in the path (i.e. one less than the number of nodes in the path). Paths are listed in order from left to right. See the doctests for some examples.

```
def long_paths(tree, n):
    """Return a list of all paths in tree with length at least n.

    >>> t = Tree(3, [Tree(4), Tree(4), Tree(5)])
    >>> left = Tree(1, [Tree(2), t])
    >>> mid = Tree(6, [Tree(7, [Tree(8)]), Tree(9)])
    >>> right = Tree(11, [Tree(12, [Tree(13, [Tree(14)])])])
    >>> whole = Tree(0, [left, Tree(13), mid, right])
    >>> for path in long_paths(whole, 2):
    ...     print(path)
    ...
    <0 1 2>
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 6 9>
    <0 11 12 13 14>
    >>> for path in long_paths(whole, 3):
    ...     print(path)
    ...
    <0 1 3 4>
    <0 1 3 4>
    <0 1 3 5>
    <0 6 7 8>
    <0 11 12 13 14>
    >>> long_paths(whole, 4)
    [Link(0, Link(11, Link(12, Link(13, Link(14)))))]
    """
```

3   Mutation

3.1   For each row below, fill in the blanks in the output displayed by the interactive Python interpreter when the expression is evaluated. Expressions are evaluated in order, and expressions may affect later expressions.

```
>>> cats = [1, 2]
>>> dogs = [cats, cats.append(23), list(cats)]
>>> cats
[1, 2, 23]

>>> dogs[1] = list(dogs)
>>> dogs[1]
[[1, 2, 23], None, [1, 2, 23]]

>>> dogs[0].append(2)
>>> cats
[1, 2, 23, 2]

>>> cats[1::2]
[2, 2]

>>> cats[:3]
[1, 2, 23]

>>> dogs[2].extend([list(cats).pop(0), 3])
>>> dogs[3]
Error

>>> dogs
[[1, 2, 23, 2], [[1, 2, 23, 2], None, [1, 2, 23, 1, 3]], [1, 2, 23, 1, 3]]
```

4 Mutable Linked Lists and Trees

4.1 Write a recursive function `flip_two` that takes as input a linked list `lnk` and mutates `lnk` so that every pair is flipped.

```
def flip_two(lnk):
    """
    >>> one_lnk = Link(1)
    >>> flip_two(one_lnk)
    >>> one_lnk
    Link(1)
    >>> lnk = Link(1, Link(2, Link(3, Link(4, Link(5)))))
    >>> flip_two(lnk)
    >>> lnk
    Link(2, Link(1, Link(4, Link(3, Link(5)))))
    """
```

5   Generators

5.1   Write a generator function that yields functions that are repeated applications of a one-argument function f. The first function yielded should apply f 0 times (the identity function), the second function yielded should apply f once, etc.

```python
def repeated(f):
    """
    >>> double = lambda x: 2 * x
    >>> funcs = repeated(double)
    >>> identity = next(funcs)
    >>> double = next(funcs)
    >>> quad = next(funcs)
    >>> oct = next(funcs)
    >>> quad(1)
    4
    >>> oct(1)
    8
    >>> [g(1) for _, g in
    ...  zip(range(5), repeated(lambda x: 2 * x))]
    [1, 2, 4, 8, 16]
    """


    g = _____


    while True:


        _____



        _____
```

5.2   Ben Bitdiddle proposes the following alternate solution. Does it work?

```python
def ben_repeated(f):
    g = lambda x: x
    while True:
        yield g
        g = lambda x: f(g(x))
```

5.3 Implement `accumulate`, which takes in an `iterable` and a function `f` and yields
each accumulated value from applying `f` to the running total and the next element.

```
from operator import add, mul

def accumulate(iterable, f):
    """
    >>> list(accumulate([1, 2, 3, 4, 5], add))
    [1, 3, 6, 10, 15]
    >>> list(accumulate([1, 2, 3, 4, 5], mul))
    [1, 2, 6, 24, 120]
    """
    it = iter(iterable)
```

_____

_____

  for _____:

  _____

  _____

6   Streams

6.1   Write a function `merge` that takes 2 sorted streams `s1` and `s2`, and returns a new sorted stream which contains all the elements from `s1` and `s2`.
Assume that both `s1` and `s2` have infinite length.

```
(define (merge s1 s2)



    (if _____


        _____


        _____))
```

6.2   (Adapted from Fall 2014) Implement `cycle` which returns a stream repeating the digits 1, 3, 0, 2, and 4, forever. Write `cons-stream` only once in your solution!
Hint: (3+2) % 5 == 0.

```
(define (cycle start)


    _____)
```

7   Macros

7.1   Using macros, let's make a new special form, when, that has the following structure:

```
(when <condition>
      (<expr1> <expr2> <expr3> ...))
```

If the condition is not false (a truthy expression), all the subsequent operands are evaluated in order and the value of the last expression is returned. Otherwise, the entire when expression evaluates to okay.

```
scm> (when (= 1 0) ((/ 1 0) 'error))
okay
scm> (when (= 1 1) ((print 6) (print 1) 'a))
6
1
a
```

(a) Fill in the skeleton below to implement this without using quasiquotes.

```
(define-macro (when condition exprs)

   (list 'if_____))
```

(b) Now, implement the macro using quasiquotes.

```
(define-macro (when condition exprs)

   `(if _____))
```

7.2   Write a macro called zero-cond that takes in a list of clauses, where each clause is a two-element list containing two expressions, a predicate and a corresponding result expression. All predicates evaluate to a number. The macro should return the value of the expression corresponding to the first true predicate, *treating 0 as a false value.*

```
scm> (zero-cond
            ((0 'result1)
             ((- 1 1) 'result2)
             ((* 1 1) 'result3)
             (2 'result4)))
result3

(define-macro (zero-cond clauses)
    (cons 'cond
        (map      _____


                  _____


                  _____)))
```

8   SQL

Questions

Our tables:

dogs: **Name   Age   Phrase, DEFAULT="woof"**

8.1   What would SQL display? **Keep track of the contents of the table after every statement below.** Write Error if you think a statement would cause an error.

```
sqlite> SELECT * FROM dogs;
Fido|1|woof
Sparky|2|woof
Lassie|2|I'll save you!
Floofy|3|Much doge

sqlite> INSERT INTO dogs(age, name) VALUES ("Rover", 3);
sqlite> SELECT * FROM dogs;
```

```
sqlite> UPDATE dogs SET name=age, age=name WHERE name=3;
sqlite> SELECT * FROM dogs;
```

```
sqlite> UPDATE dogs SET phrase="Hi there!" WHERE name LIKE "F%";
sqlite> SELECT * FROM dogs;
```

```
sqlite> DELETE FROM dogs WHERE age < 3;
sqlite> SELECT * FROM dogs;
```

```
sqlite> INSERT INTO dogs VALUES ("Spot", 2), ("Buster", 4);
```

```
sqlite> INSERT INTO dogs(name, phrase) VALUES ("Spot", "bark"), ("Buster", "barkbark");
sqlite> SELECT * FROM dogs;
```

```
sqlite> INSERT INTO dogs(name, age) SELECT name, phrase from dogs where age = 3;
sqlite> DELETE FROM dogs WHERE phrase != "woof";
sqlite> SELECT * FROM dogs;
```