Navigation Systems: Alignment with SVD

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Rigid Registration

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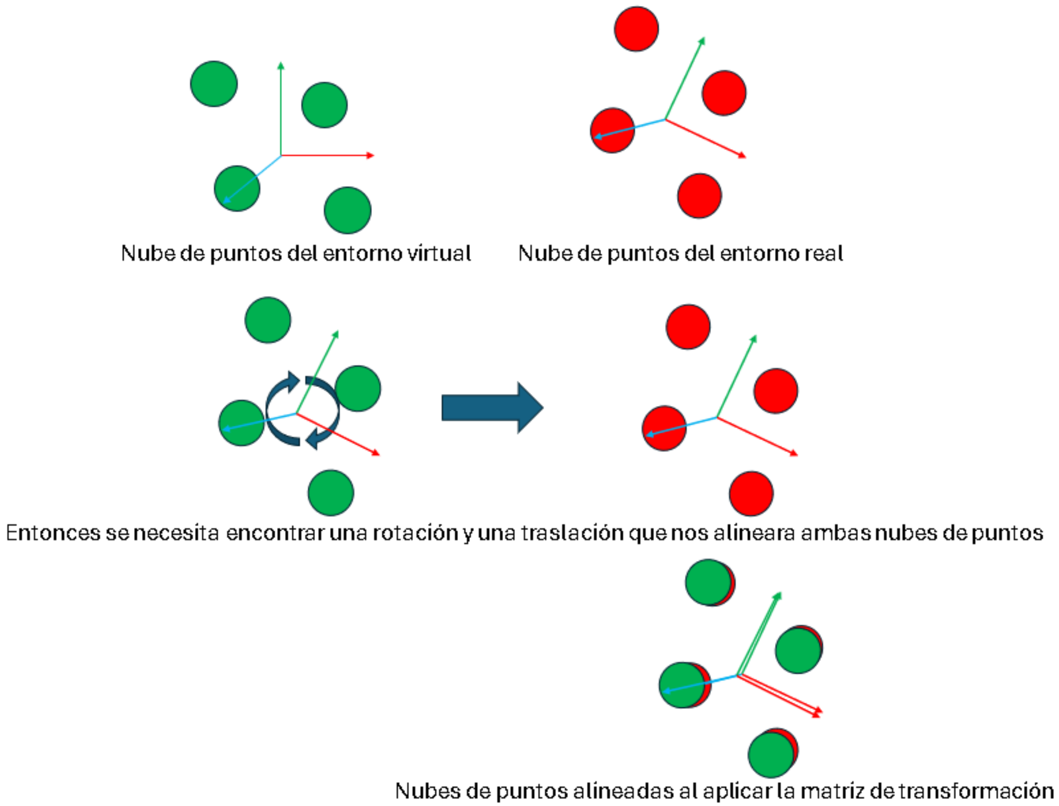
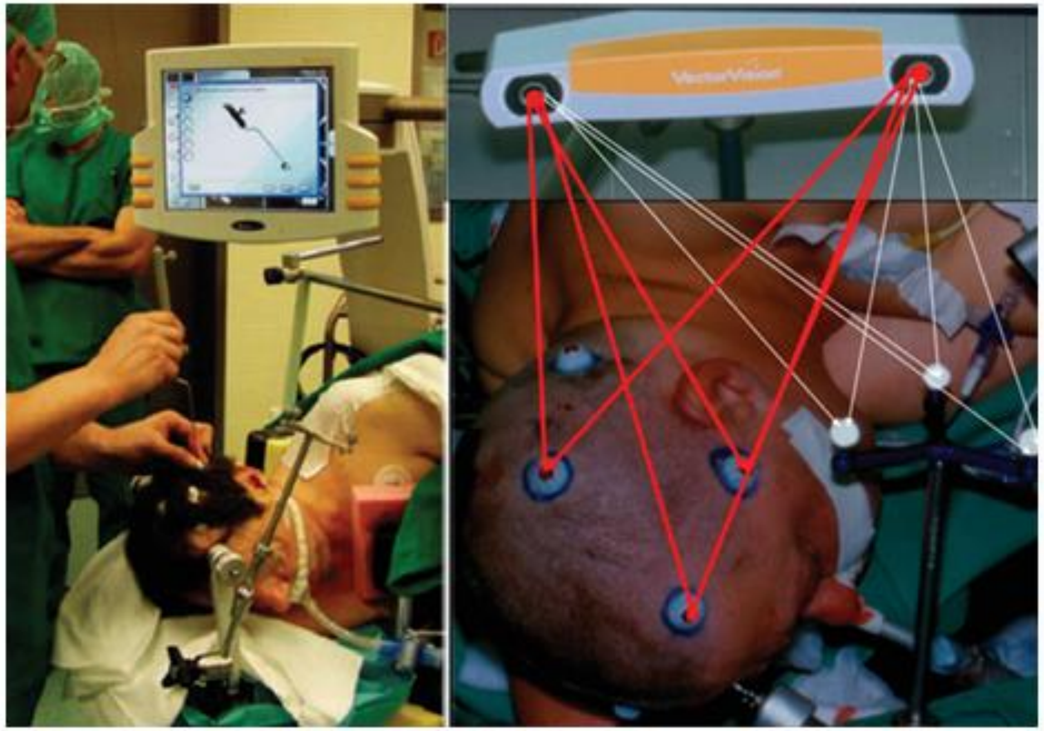


- Fiducial point-based registration methods are achieved by localizing points (fiducials) in each coordinate system:
  - namely that of physical markers placed on the patient
  - and that of the virtual environment or preoperative images of the patient.
- Often, both localizations are performed manually, first by choosing a fiducial point on the image and then by using a hand-tracked pointer to physically touch the responding fiducial on the patient.
- These manual procedures introduce user-dependent localization errors that can significantly decrease registration accuracy.
- Therefore, there is a need for a registration method that is tolerant of inaccurate fiducial localization in both the preoperative and intraoperative phases.

Fiducial point-based registration methods



- Example of a typical preoperative registration using skin markers (so-called fiducials) and an optical camera system (**BrainLab Vector Vision 2**) that correlates presurgical imaging data sets with a three-arm star, allowing intraoperative orientation of tracking devices at the surgical site.

- The accuracy of the registration is correlated with the number of fiducial points used.
- The **internal square root (RMS)** of the different neuronavigation systems is between **2 millimeters and 3.2 millimeters**.
- **The difference between the pointer tip and the anatomical reference points is 1.7 to 2.2 millimeters**.
- **This difference must be added to the known difference of 5 to 10 millimeters between the functional maps and the site of neuronal activity, determined by direct electrical stimulation**.

Singular Value Decomposition (SVD)

- Singular value decomposition (SVD) is a matrix factorization technique widely used in computing.
- SVD allows us to find the optimal transformation between two sets of points, that is, the rotation and translation.
- SVD is achieved by decomposing the covariance matrix between the point clouds into orthogonal matrices, extracting the rotations and minimizing the squared error between corresponding points.
- SVD is essential in applications such as surgical navigation, where accuracy in superimposing virtual and physical models is crucial.

SVD Method for Rigid Registration

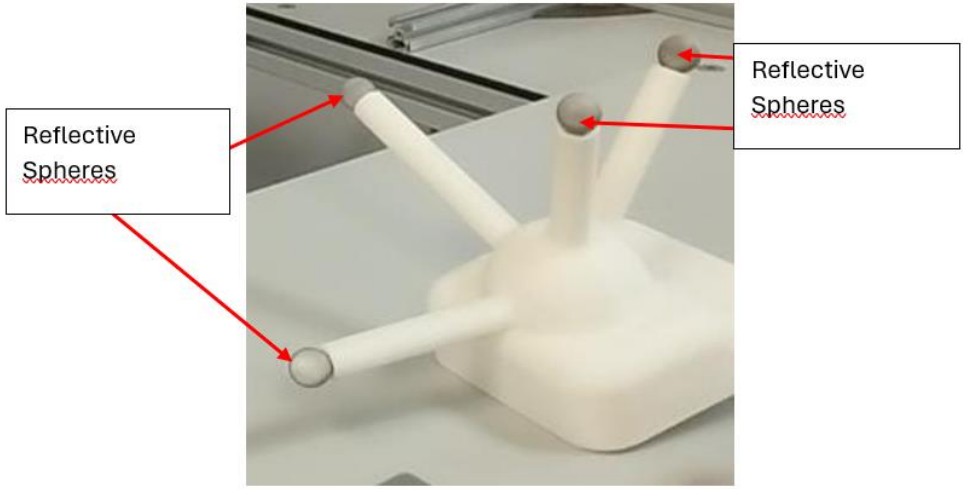
- It is a non-iterative method to obtain the transformation matrix by a relatively fast estimation of the **orientation and translation between point clouds**.
- The main purpose of **point cloud registration (PCR)** is to establish the **three-dimensional correspondence between different models**, facilitating their alignment in a common spatial system.
- The **PCR** is **solving the homogeneous matrix equation**:

$$\mathbf{AX = XB}$$

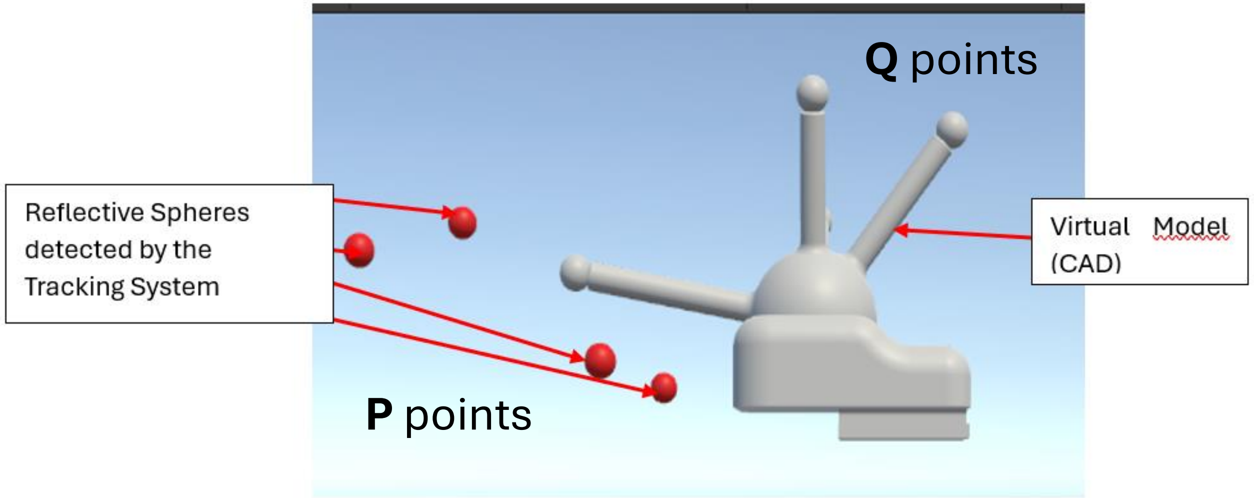
Where X is the transformation matrix composed of the R rotation matrix and the T translation vector that solves the problem of establishing the positions (poses) at A, against the positions (poses) determined at B.

SVD Method for Rigid Registration

A set of points **P** corresponds to the positions recorded by the optical tracking device, while the other set **Q** matches the positions of each marker given by the CAD model of the object.



P points



$$\mathbf{PX = XQ}$$

SVD Method for Rigid Registration

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- First, the centroids c_p and c_q of both point clouds **P** and **Q** are calculated:

$$c_p = \frac{1}{N}\sum_{i=1}^{N} p_i \qquad\qquad c_q = \frac{1}{N}\sum_{i=1}^{N} q_i$$

- Subsequently, $\forall p_i \in \mathbf{P}, \forall q_i \in \mathbf{Q}$ both point clouds are centered at the origin, by subtracting the centroids c_p and c_q from their respective point clouds to center both point clouds at the origin:

$$r_i = p_i - c_p \qquad\qquad s_i = q_i - c_q$$

- The next step is to calculate the correlation matrix **H**:

$$\mathbf{H} = \sum_{i=1}^{N} s_i * r_i^T$$

- Where the correlation matrix **H** is of size m x n

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SVD Method for Rigid Registration

- Then the matrix **H** is then decomposed into its singular values (SVD) such that:

$$\mathbf{H} = \mathbf{U} * \mathbf{D} * \mathbf{V}^{\mathbf{T}}$$

- Where **U** is an m x n matrix, **D** is an n x n diagonal matrix and **V** is an n x n matrix.

- By obtaining the orthogonal matrices **V** and **U** from the correlation matrix, the rotation and translation required to align the point cloud **P** with the point cloud **Q** can be determined:

$$\mathbf{X} = \mathbf{V}\mathbf{U}^{\mathbf{t}}$$

SVD Method for Rigid Registration

- If det(**X**) = 1 we can say that X is the rotation matrix (**R = X**).

- If det(**X**) = -1, it would mean that **X** is a reflection, so the rotation will be given by:

$$\mathbf{X}' = \mathbf{V}'\mathbf{U}^{\mathbf{t}}$$

- Finally, the translation is given by:

$$\mathbf{T} = c_p - \mathbf{R} * c_q$$

$H=U*D*V^t$

- Computing the SVD of **H** consists of finding the eigenvalues and eigenvectors of $\mathbf{HH^t}$ and $\mathbf{H^t H}$.

- The eigenvectors of $\mathbf{H^t H}$ form the columns of **V**, the eigenvectors of $\mathbf{HH^t}$ form the columns of **U**.

- The singular values in **D** are square roots of the eigenvalues of $\mathbf{HH^t}$ or $\mathbf{H^t H}$. The singular values are the diagonal entries of the matrix **D** and are arranged in descending order. The singular values are always real numbers.

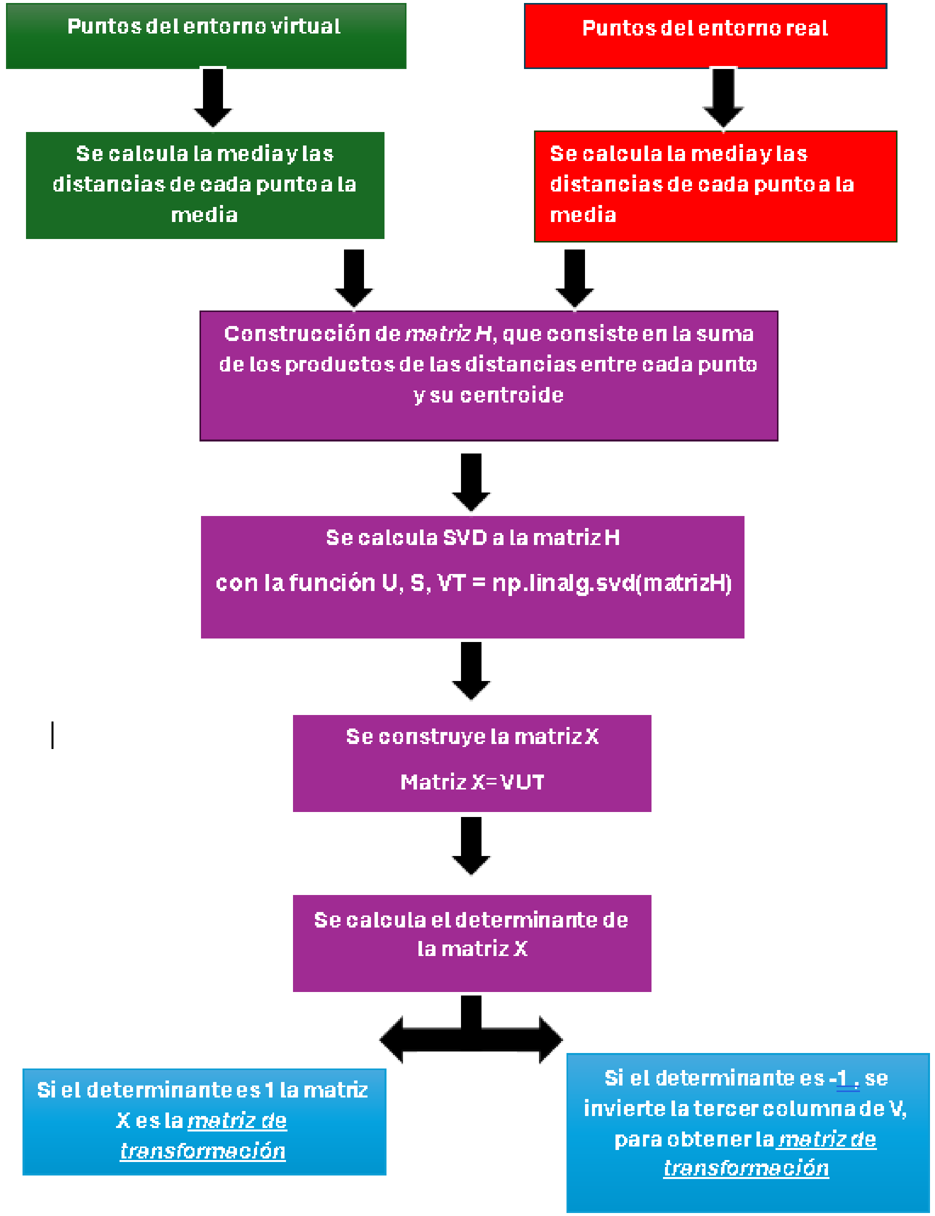
- Since the matrix **H** is a real matrix, then **U** and **V** are also real.

Fiducial Registration Error (FRE)

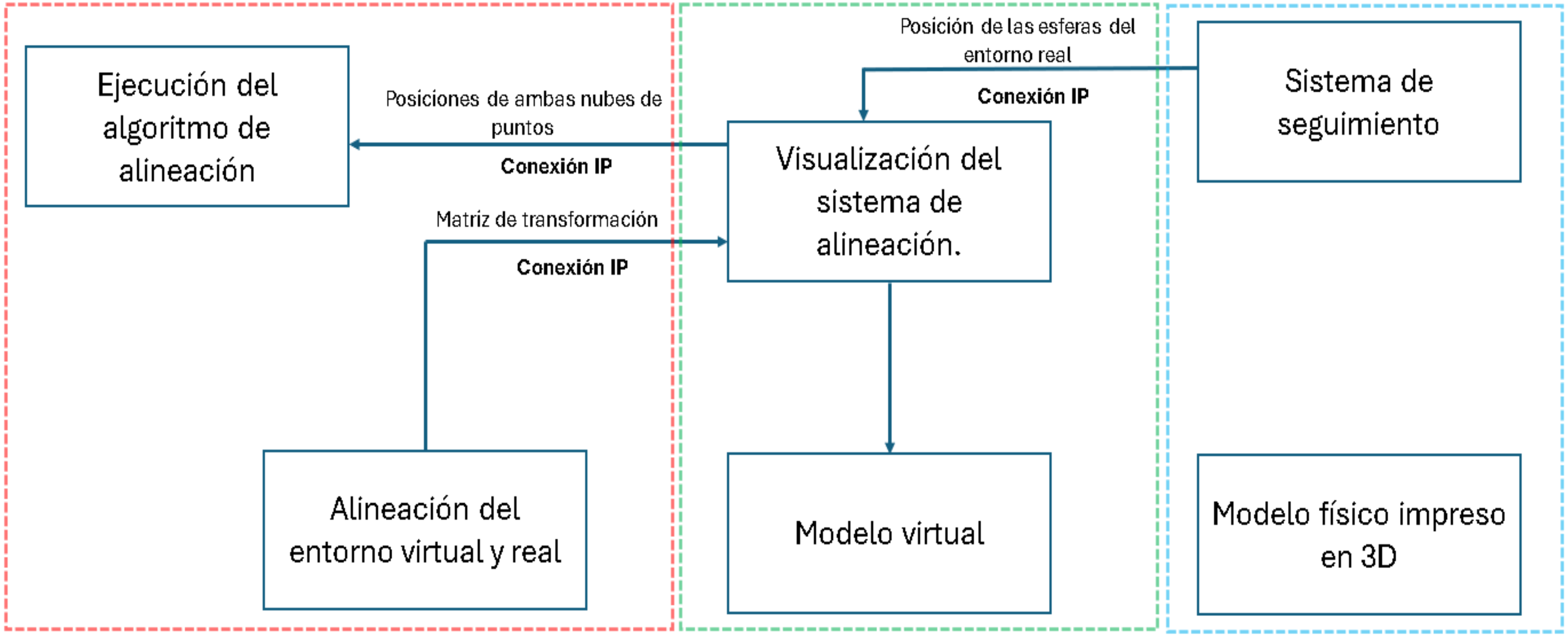
- This metric determines the error that exists after performing the alignment process between the fiducial points.

$$FRE^2 \equiv \frac{1}{\mathrm{N}}\sum_{i=1}^{N}|(Rp_i + t) - q_i|^2$$

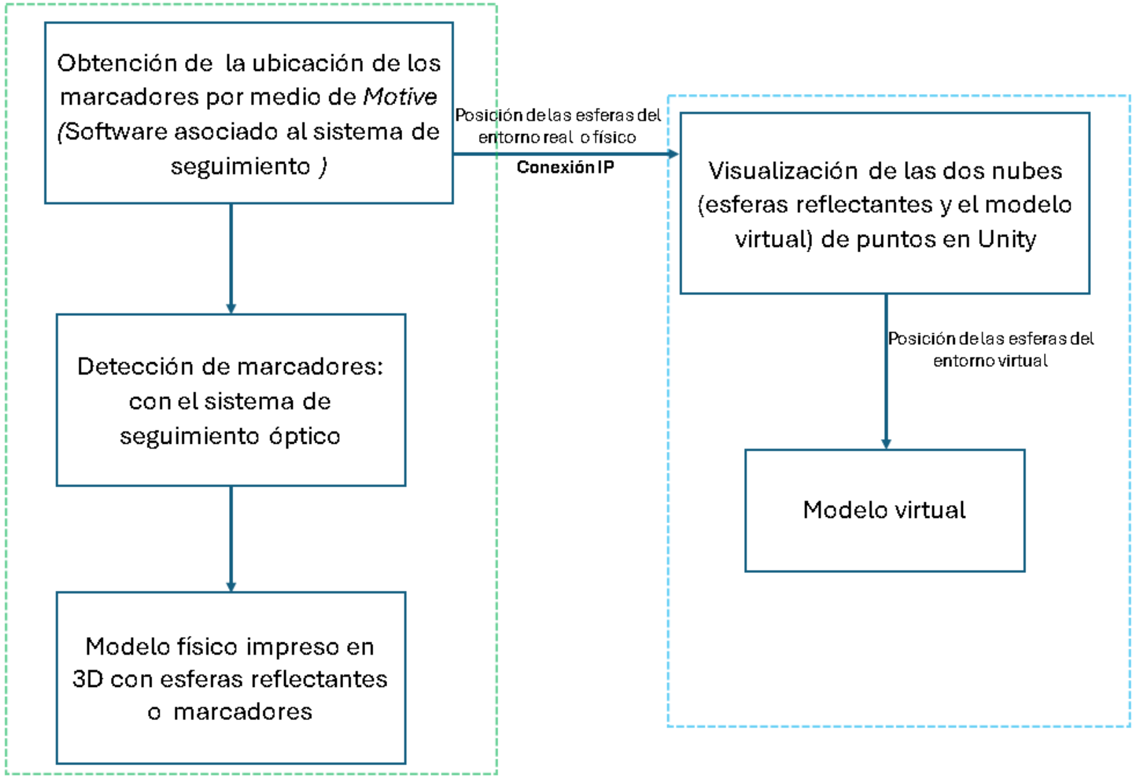
- Typical **FRM** for neuronavigation systems is between **2 millimeters and 3.2 millimeters**.


Puntos del entorno virtual
Puntos del entorno real
Se calcula la media y las distancias de cada punto a la media
Se calcula la media y las distancias de cada punto a la media
Construcción de *matriz H*, que consiste en la suma de los productos de las distancias entre cada punto y su centroide
Se calcula SVD a la matriz H
con la función U, S, VT = np.linalg.svd(matrizH)
Se construye la matriz X
Matriz X=VUT
Se calcula el determinante de la matriz X
Si el determinante es 1 la matriz X es la *matriz de transformación*
Si el determinante es -1 , se invierte la tercer columna de V, para obtener la *matriz de transformación*

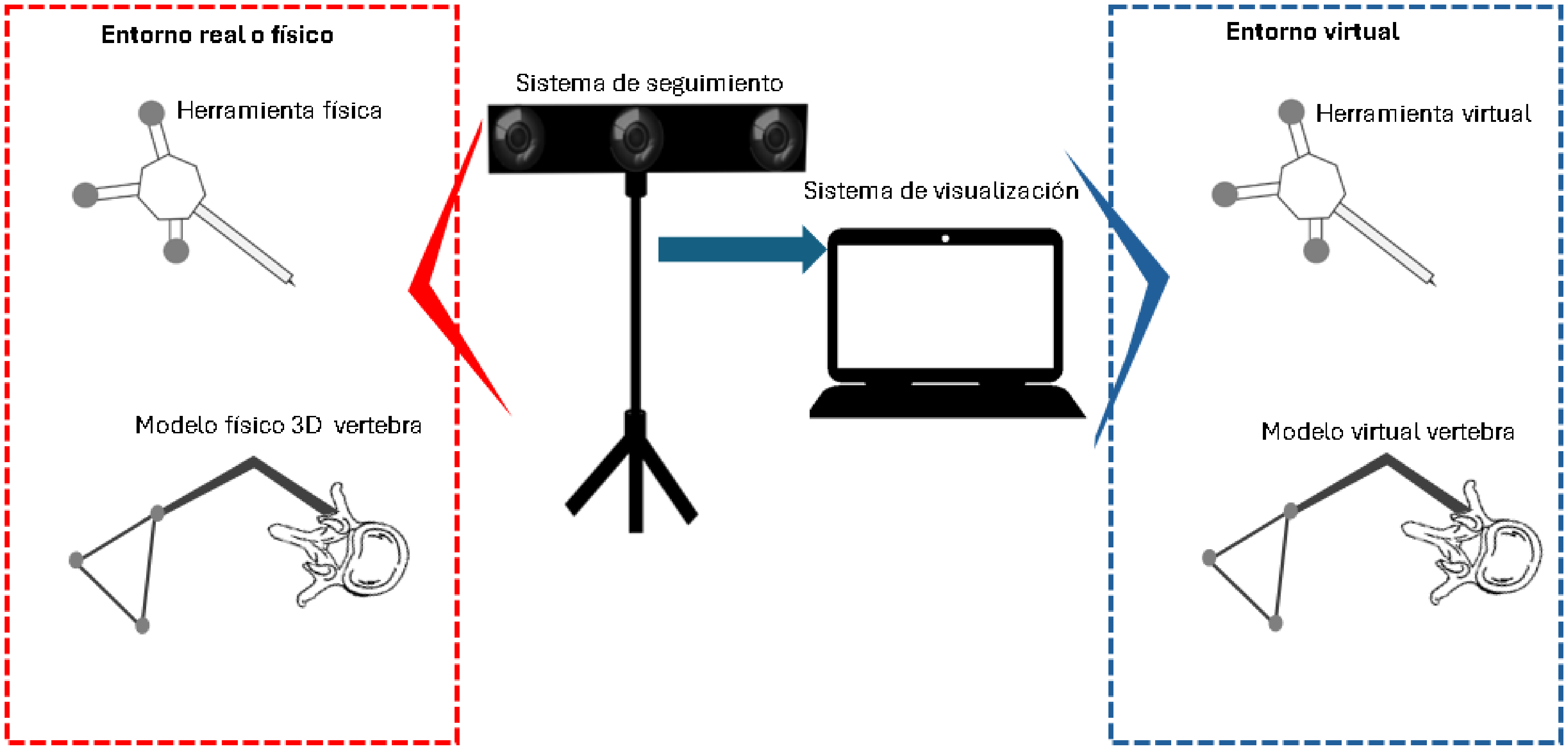
PCR for alignement with SVD



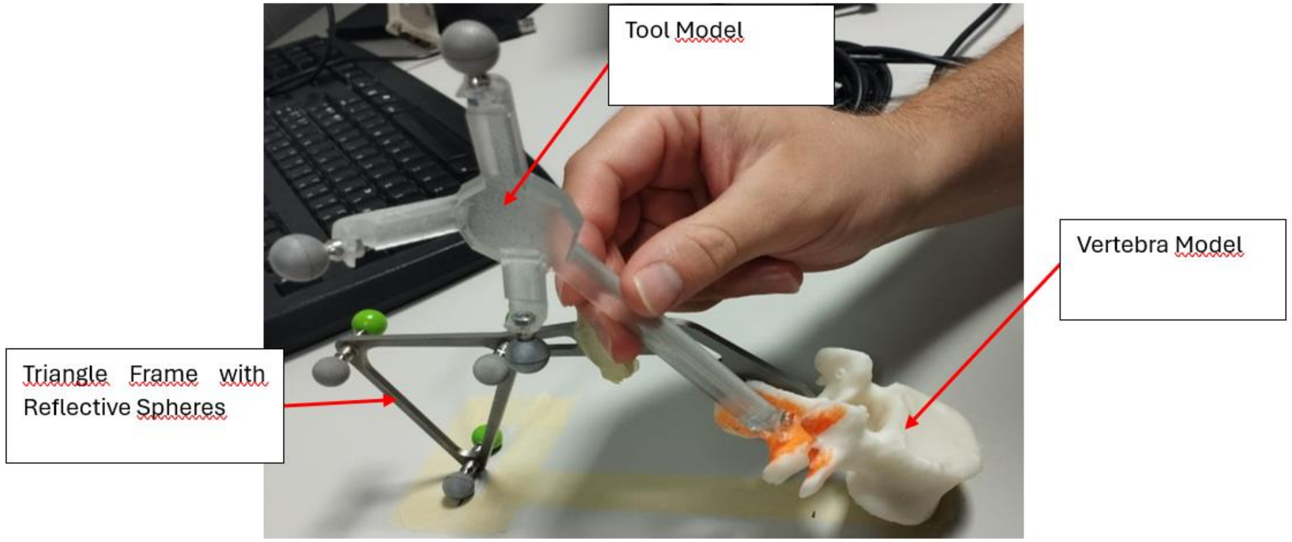
Real-time tracking with SVD

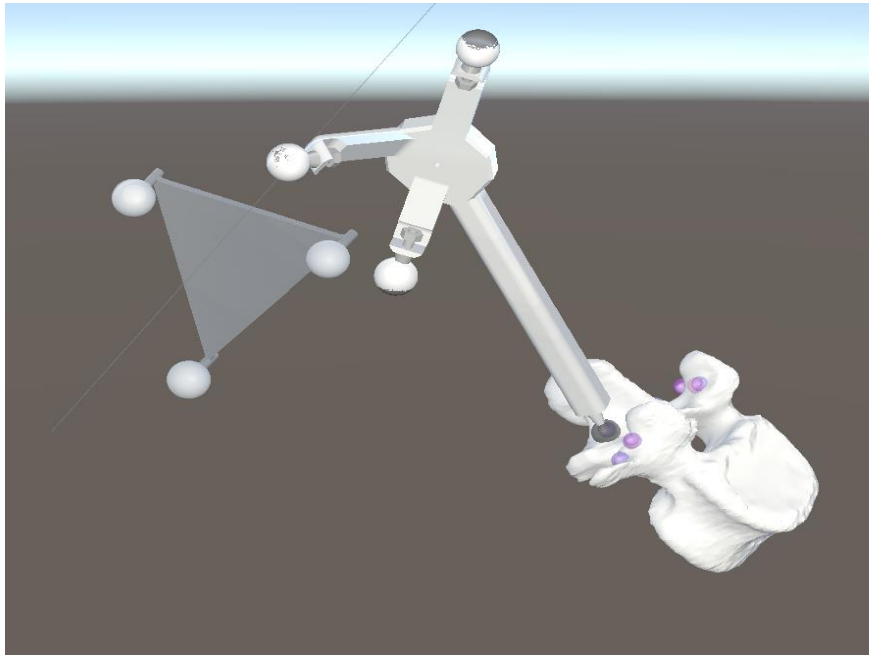


Real-time Navigation with SVD



Real-time Navigation with starting fast alignment with SVD

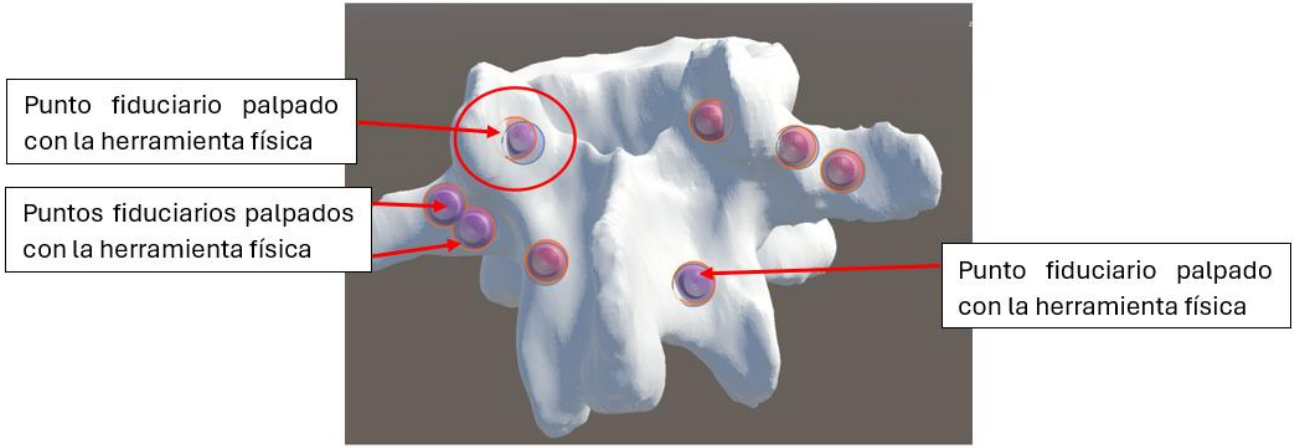


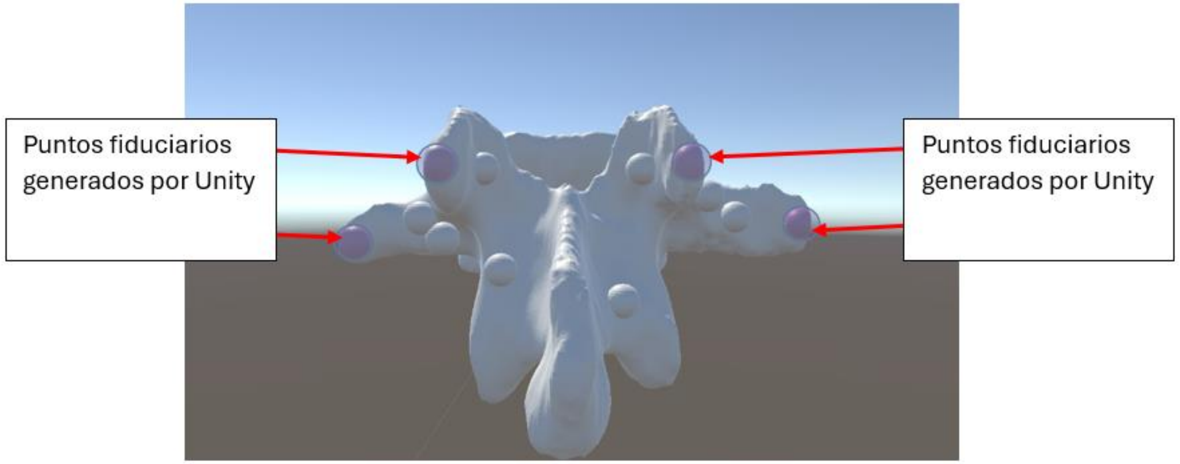


Experimental validation

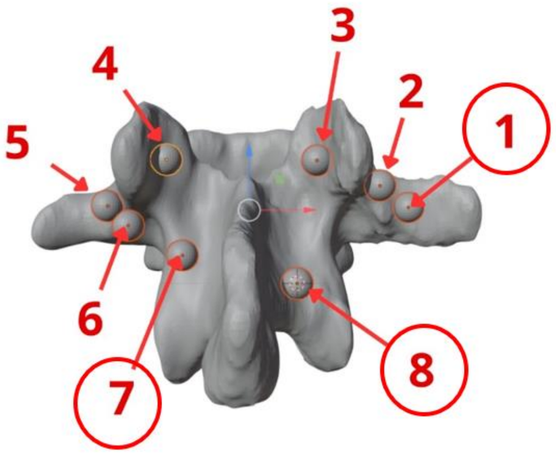
1. Se eligen tres puntos fiduciarios de los ocho puntos de la vertebra.
2. Se realiza la alineación de la vertebra
3. Con la herramienta física se palpa en el modelo físico de la vertebra los cinco puntos fiduciarios restantes.
4. Por cada punto palpado en el entorno real , se coloca una esfera virtual en el modelo virtual de la vertebra.
5. De manera virtual tenemos los cinco puntos fiduciarios palpados representados con una esfera virtual y también tenemos los puntos fiduciarios del modelo virtual de la vertebra
6. Por cada punto palpado en el entorno real , se coloca una esfera virtual en el entorno virtual .
7. Se calculó la distancia euclidiana de cada punto palpado y cada punto del modelo virtual, obteniendo así veinticinco puntos por cada prueba.

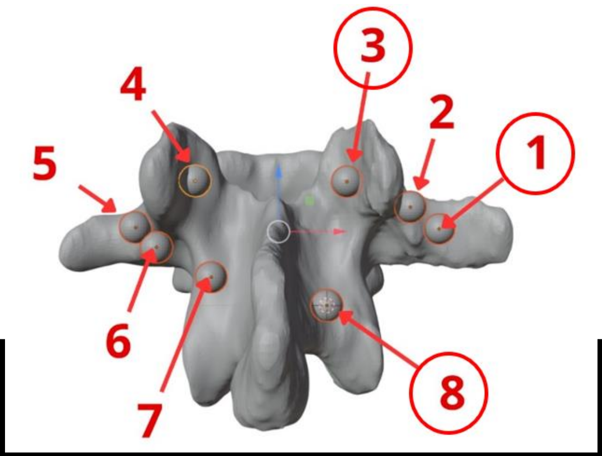
Landmarks selection





Different Configurations of landmarks





Numero de experimento	Puntos fiduciarios							
	1	2	3	4	5	6	7	8
1	*						*	*
2	*		*					*
3				*	*		*	
4		*			*		*	
5			*	*				*
6		*	*		*		*	
7	*	*			*		*	
8	*					*	*	*
9		*			*		*	*

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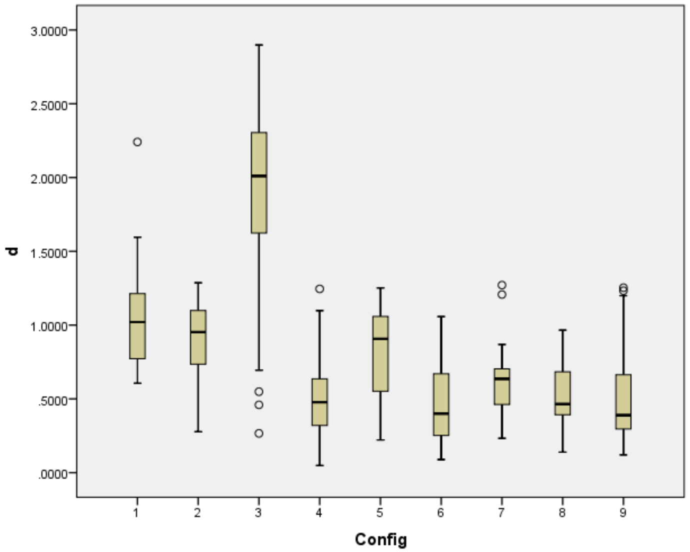
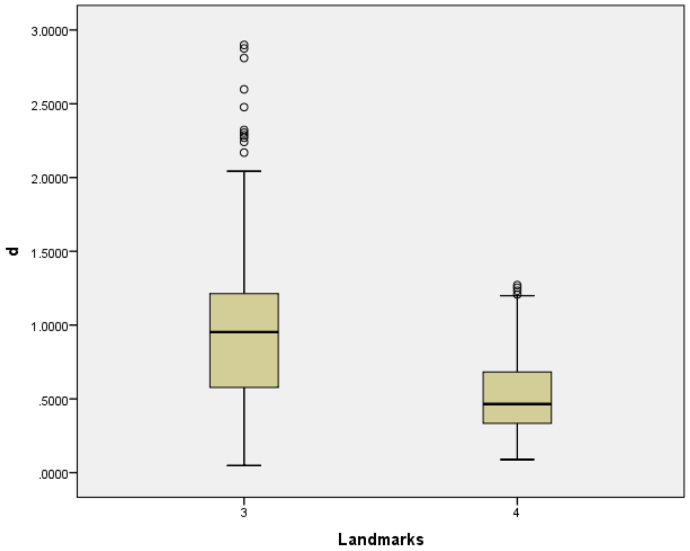
Alignement errors for different Configurations

No Experimento	Muestras	Media	Error típico	Desviación típica
1	25	1.0582	.0751	.3755
2	25	.8675	.0590	.2950
3	25	1.8391	.1528	.7640
4	25	.5255	.0626	.3132
5	25	.8235	.0611	.3059
6	20	.4655	.0618	.2763
7	20	.6308	.0590	.2639
8	20	.5120	.0462	.2068
9	20	.5329	.0822	.3674





It can be seen from the results that the values in the SVD alignment, **the largest error was 2.15mm**, followed by 1.88mm.

Significant differences were found between both with a p<0.001, with a **mean error in 4 of 0.535mm** significantly lower than **mean error in 3 of 1.023mm**.

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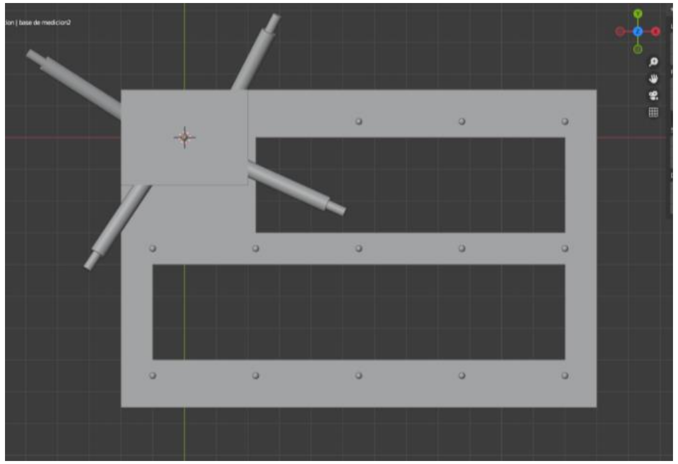
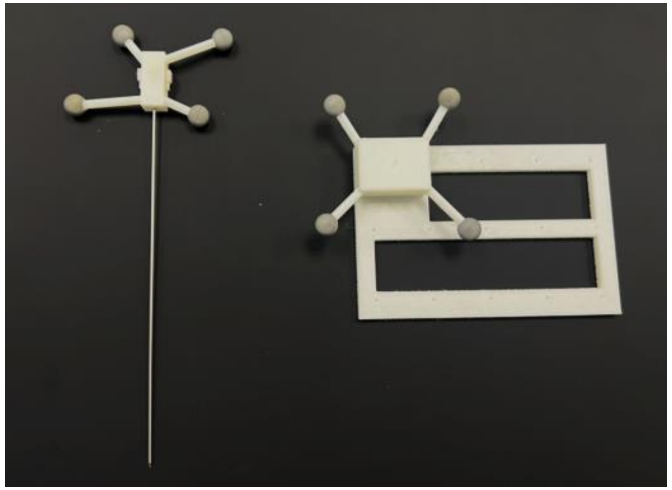
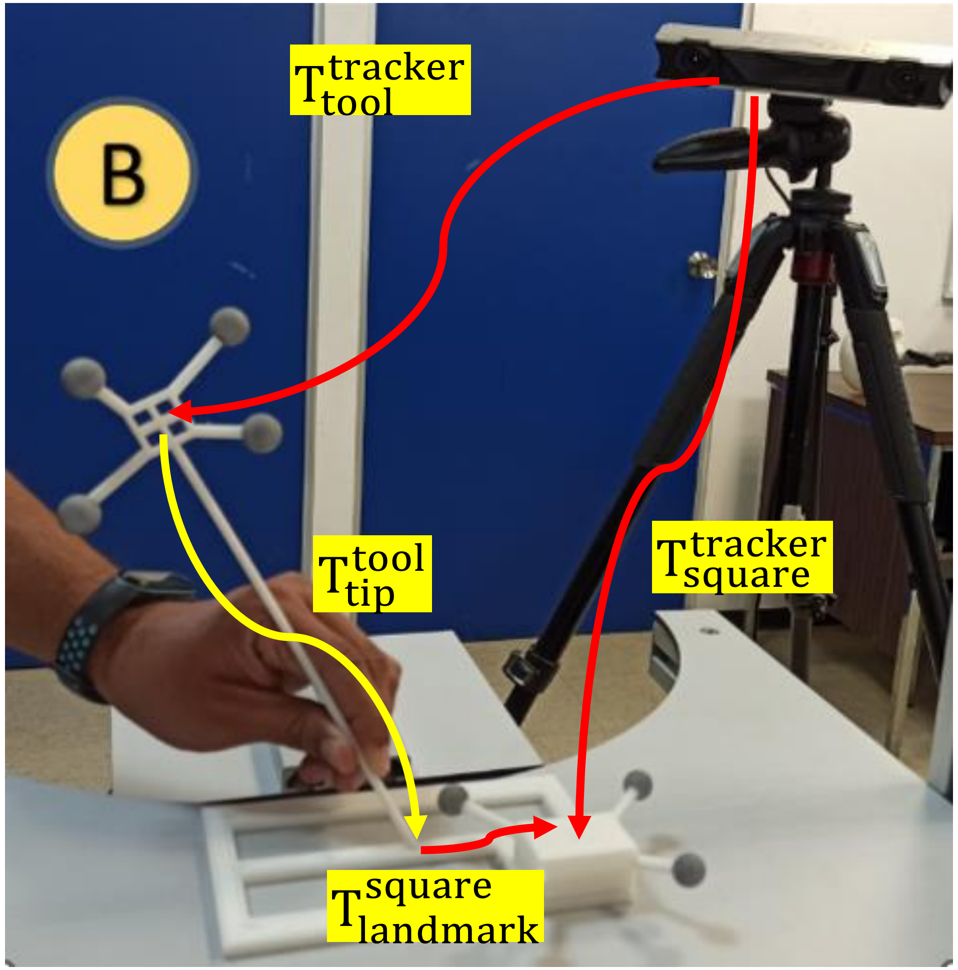
Tool Calibration with SVD







With SVD during calibration:

$$Q = T_{tool}^{tracker}$$

$$P = T_{landmark}^{square}\,T_{square}^{tracker}$$

$$T_{tip}^{tool} = c_p - R\,c_q$$

During navigation:

$$T_{tip}^{tracker} = T_{tip}^{tool}\,T_{tool}^{tracker}$$

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Thank you!

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