Quinta Lista de Exercícios - OTIMIZACAO NAO LINEAR

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Chapter 7

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import warnings
from scipy import optimize
from scipy.optimize import Bounds
warnings.filterwarnings('ignore')
```

Problem 7.1

7.1 Find the solution of the problem:

$$f(X) = x_1^2 + 2x_2^2 - 2x_1x_2 - 14x_1 - 14x_2 + 10$$

subject to

$$4x_1^2 + x_2^2 - 25 \leq 0$$

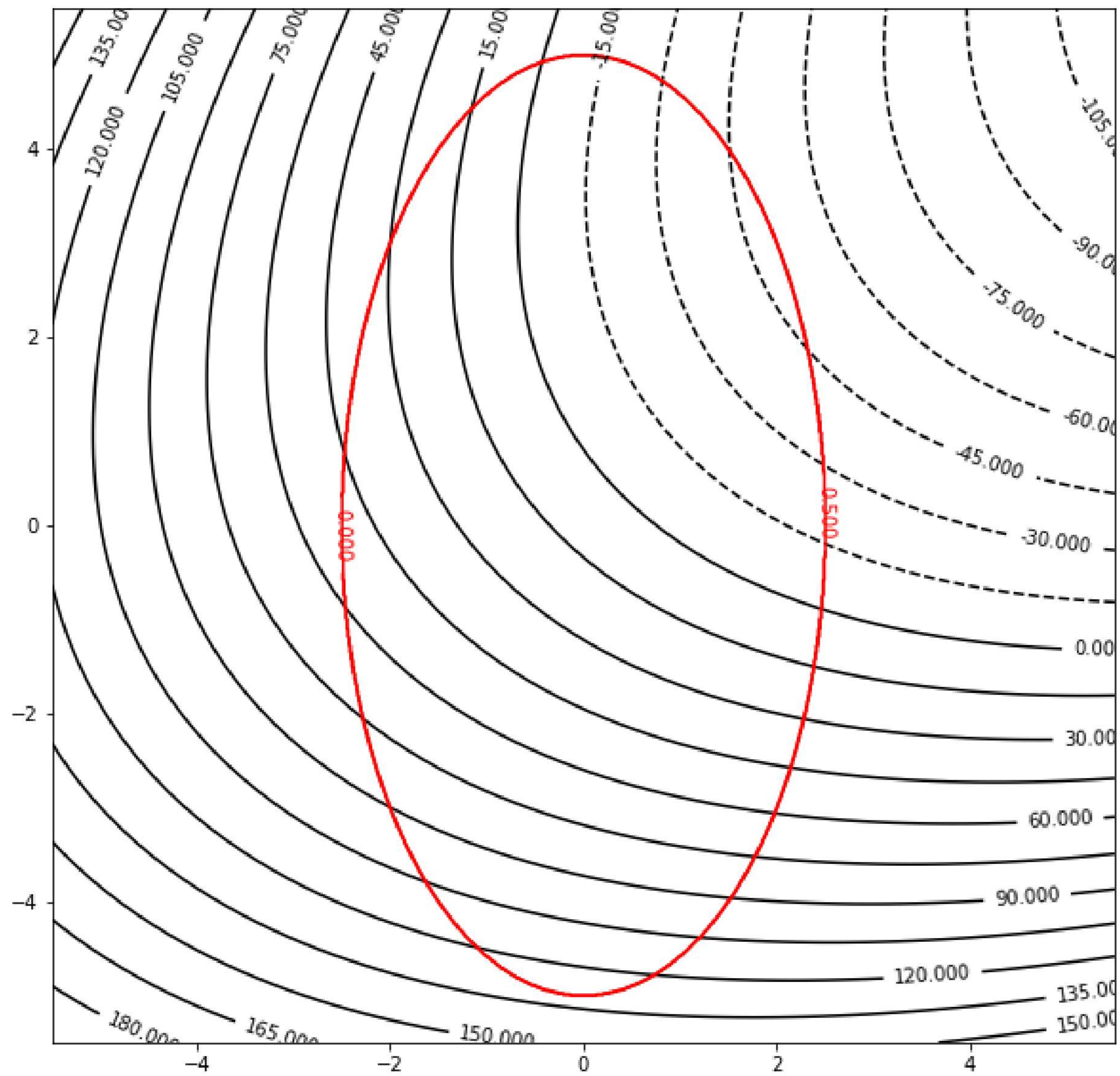
using a graphical procedure.

In [2]:
```
def F710(x1, x2):
    f = x1**2+2*x2**2-2*x1*x2-14*(x1+x2)+10
    return f

def G710(x1, x2):
    g = 4*x1**2+x2**2 -25
    return g
```

In [3]:
```
a1 = np.linspace(-5.5, 5.5, 1000)
a2 = np.linspace(-5.5, 5.5, 1000)
A1, A2 = np.meshgrid(a1, a2)
Z1 = F710(A1, A2)
G2 = G710(A1, A2)
```

In [4]:
```
plt.figure(figsize=(10,10))
contours1 = plt.contour(A1, A2, Z1, 25, colors='black')
contours2 = plt.contour(A1, A2, G2 <= 0, 1 , colors='red')
plt.clabel(contours1, inline=True, fontsize=10)
plt.clabel(contours2, inline=True, fontsize=10)
plt.show()
```



A solução são os valores contindo nas curvas de niveis dentra da região vermelha.

Problem 7.24:

Construct the φk function according to the exterior penalty function approach and complete the minimization of φk for the following problem.

$$Minimize f(x) = (x-1)^2$$

subject to

$$g_1(x) = 2 - x \leq 0$$

$$g_2(x) = x - 4 \leq 0$$

Solution:

$$\phi_k = (x-1)^2 + r_k([max(0, 2-x)]^2 + [max(0, x-4)]^2)$$

$$d\phi_k/dx = 2(x-1) + 2r_k([-max(0, 2-x)] + [max(0, x-4)]) = 0$$

$$x * (2 + 2r_k) = 2 + 4r_k$$

$$x* = \frac{2+4r_k}{2+2r_k}$$

$$x* = \lim_{r_k \to \infty} \frac{\frac{2}{r_k} + 4}{\frac{2}{r_k} + 2}$$

$$x* = 2$$

$$Min = f(2-1)^2 = 1$$

Problem 7.31:

Find the solution of the following problem using an exterior penalty function method with classical method of unconstrained minimization:

$$Minimize f(x_1, x_2) = (2x_1 - x_2)^2 + (x_2 + 1)^2$$

subject to

$$x_1 - x_2 = 10$$

Consider the limiting case as rk → ∞ analytically.

Solution:

$$\phi = (2x_1 - x_2)^2 + (x_2 + 1)^2 + r(x_1 + x_2 - 10)^2$$

$$\nabla \phi = [0, 0]$$

$$\nabla \phi = [4(2x_1 - x_2) + 2r(x_1 + x_2 - 10), -2(2x_1 + 1) + 2r(x_1 + x_2 - 10)] = [20r, 20r - 2]$$

$$\begin{cases} (4+r)x_1 - (2-r)x_2 = 10r \\ -(2-r)x_1 + (2+r)x_2 = 10r - 1 \end{cases}$$

solven to r → ∞

$$x_1* = 4.1$$

$$x_2* = 5.9$$

$$f_{min} = 53.53$$

As proximas questões foram resolvidas usando aproximação quadratica, do scipy:

Problem 7.52:

Find the solution of the following problem (known as Rosen–Suzuki problem) using the MATLAB function fmincon with the starting point X1 = {0, 0, 0, 0}T:

Minimize

$$f(X) = x_1^2 + x_2^2 + 2x_3^2 - x_4^2 - 5x_1 - 5x_2 - 21x_3 + 7x_4 + 100$$

subject to

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_1 - x_2 + x_3 - x_4 - 100 \leq 0$$

$$x_1^2 + 2x_2^2 + x_3^2 + 2x_4^2 - x_1 - x_4 - 10 \leq 0$$

$$2x_1^2+x_2^2+x_3^2+2x_1-x_2-x_4-5\le 0$$

$$-100\le x_i\le 100, i=1,2,3,4$$

In [5]:

```
def F752(x):
    f = x[0]**2+x[1]**2+2*x[2]**2-x[3]**2-5*(x[0]+x[1])-21*x[2]+7*x[3]+100
    return f

def gradF752(x):
    gradf = np.array([[2*x[0]-5], [2*x[1]-5], [4*x[2]-21], [-2*x[3]+7]])
    return gradf

def G752_1(x):
    g = x[0]**2+x[1]**2+x[2]**2+x[3]**2+x[0]-x[1]+x[2]-x[3]-100
    return g

def G752_2(x):
    g = x[0]**2+2*x[1]**2+x[2]**2+2*x[3]**2-x[0]-x[3]-10
    return g

def G752_3(x):
    g = 2*x[0]**2+x[1]**2+x[2]**2-x[1]-x[3]-5
    return g
```

In [6]:

```
con1 = {'type':'ineq','fun':G752_1}
con2 = {'type':'ineq','fun':G752_2}
con3 = {'type':'ineq','fun':G752_3}
cons = (con1,con2,con3)

x0 = np.array([0, 0, 0, 0])
bounds = Bounds([-100, -100, -100, -100], [100, 100, 100, 100])
```

In [7]:

```
result = optimize.minimize(F752, x0, method='SLSQP', jac=gradF752,
               constraints=cons, options={'ftol': 1e-9, 'disp': True},
               bounds=bounds)
```

```
Optimization terminated successfully    (Exit mode 0)
            Current function value: -10667.624999995127
            Iterations: 18
            Function evaluations: 17
            Gradient evaluations: 14
```

In [8]:

```
print("Resultados: X* = [", format(result.x[0], '.2f'), ", ",
      format(result.x[0], '.2f'), ", ",
      format(result.x[1], '.2f'), ", ",
      format(result.x[2], '.2f'), ", ",
      format(result.x[3], '.2f'),"]")
```

```
Resultados: X* = [ 2.50 ,  2.50 ,  2.50 ,  5.25 ,  -100.00 ]
```

Problem 7.53:

Find the solution of the following problem using the MATLAB function fmincon with the starting point X1 = {0.5, 1.0}T:

Minimize

$$f(X)=x_1^2+x_2^2-4x_1-6x_2$$

subject to

$$x_1 + x_2 \leq 2$$

$$2x_1 + 3x_2 \leq 12$$

$$x_i \geq 0, i = 1, 2$$

In [9]:

```
def F753(x):
    f = x[0]**2+x[1]**2-4*x[0]-6*x[1]
    return f

def gradF753(x):
    f = np.array([ [2*x[0]-4], [2*x[1]-6] ])
    return f

def G753_1(x):
    g = x[0]+x[1] - 2
    return g

def G753_2(x):
    g = 2*x[0]+3*x[1]-12
    return g
```

In [10]:

```
bounds = Bounds([0,0], [10**10, 10**10])
con1 = {'type':'ineq','fun':G753_1}
con2 = {'type':'ineq','fun':G753_2}
cons = (con1,con2)
```

In [11]:

```
x0 = np.array([0.5, 1])
bounds = Bounds([0, 0], [10**10, 10**10])
```

In [12]:

```
result = optimize.minimize(F753, x0, method='SLSQP', jac=gradF753,
               constraints=cons, options={'ftol': 1e-9, 'disp': True},
               bounds=bounds)
```

```
Optimization terminated successfully    (Exit mode 0)
            Current function value: -13.0
            Iterations: 2
            Function evaluations: 3
            Gradient evaluations: 2
```

In [13]:

```
print("Resultados: X* = [", format(result.x[0], '.2f'), ", ",
      format(result.x[1], '.2f'),"]")
```

```
Resultados: X* = [ 2.00 ,  3.00 ]
```

References:

1. Engineering Optimization Theory and Practice, 4th, Singiresu S. Rao

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Link:

https://github.com/antonioanunciacao/Otimizacao-Nao-Linear
