5.10 What is a dichotomous search method?

5.11 Define the golden mean.

5.12 What is the difference between quadratic and cubic interpolation methods?

5.13 Why is refitting necessary in interpolation methods?

5.14 What is a direct root method?

5.15 What is the basis of the interval halving method?

5.16 What is the difference between Newton and quasi-Newton methods?

5.17 What is the secant method?

5.18 Answer true or false:

(a) A unimodal function cannot be discontinuous.
(b) All elimination methods assume the function to be unimodal.
(c) The golden section method is more accurate than the Fibonacci method.
(d) Nearly 50% of the interval of uncertainty is eliminated with each pair of experiments in the dichotomous search method.
(e) The number of experiments to be conducted is to be specified beforehand in both the Fibonacci and golden section methods.

PROBLEMS

5.1 Find the minimum of the function

$$f(x) = 0.65 - \frac{0.75}{1+x^2} - 0.65x \tan^{-1}\frac{1}{x}$$

using the following methods:

(a) Unrestricted search with a fixed step size of 0.1 from the starting point 0.0
(b) Unrestricted search with an accelerated step size using an initial step size of 0.1 and starting point of 0.0
(c) Exhaustive search method in the interval (0, 3) to achieve an accuracy of within 5% of the exact value
(d) Dichotomous search method in the interval (0, 3) to achieve an accuracy of within 5% of the exact value using a value of $\delta = 0.0001$
(e) Interval halving method in the interval (0, 3) to achieve an accuracy of within 5% of the exact value

5.2 Find the minimum of the function given in Problem 5.1 using the quadratic interpolation method with an initial step size of 0.1.

5.3 Find the minimum of the function given in Problem 5.1 using the cubic interpolation method with an initial step size of $t_0 = 0.1$.

5.4 Plot the graph of the function $f(x)$ given in Problem 5.1 in the range (0, 3) and identify its minimum.

5.5 The shear stress induced along the z-axis when two cylinders are in contact with each other is given by

$$\frac{\tau_{zy}}{p_{\max}} = -\frac{1}{2}\left[-\frac{1}{\sqrt{1+\left(\frac{z}{b}\right)^2}} + \left\{ 2 - \frac{1}{1+\left(\frac{z}{b}\right)^2} \right\} \times \sqrt{1+\left(\frac{z}{b}\right)^2} - 2\left(\frac{z}{b}\right) \right] \tag{1}$$

where $2b$ is the width of the contact area and $p_{\max}$ is the maximum pressure developed at the center of the contact area (Fig. 5.21):

$$b = \left(\frac{2F}{\pi l} \frac{\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}}{\frac{1}{d_1} + \frac{1}{d_2}} \right)^{1/2} \tag{2}$$

$$p_{\max} = \frac{2F}{\pi b l} \tag{3}$$

F is the contact force; E_1 and E_2 are Young's moduli of the two cylinders; ν_1 and ν_2 are Poisson's ratios of the two cylinders; d_1 and d_2 the diameters of the two cylinders, and l the axial length of contact (length of the shorter cylinder). In many practical applications,

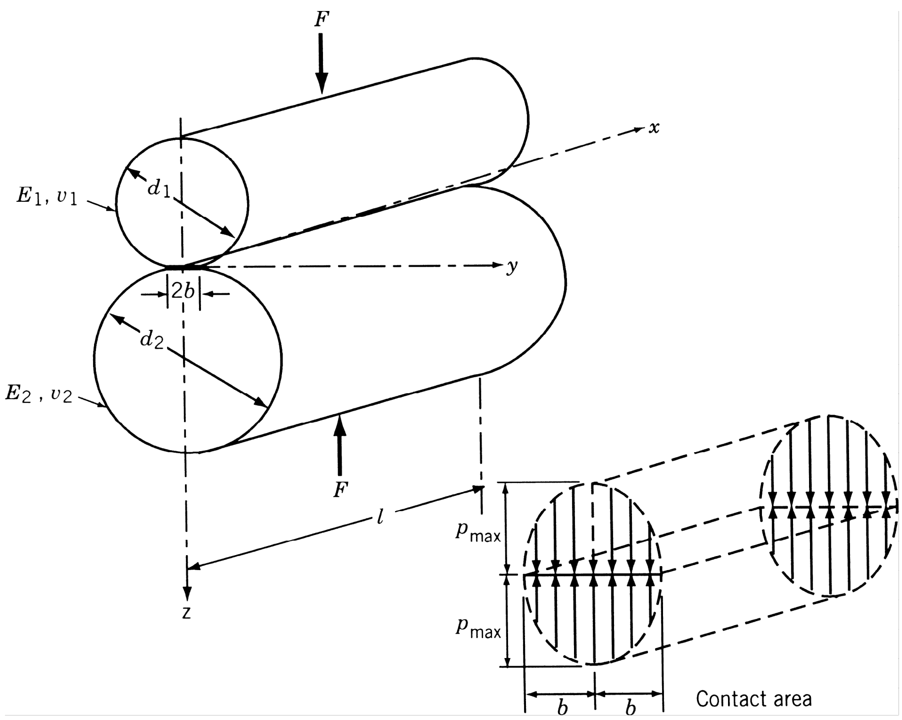

Figure 5.21 Contact stress between two cylinders.

such as roller bearings, when the contact load (F) is large, a crack originates at the point of maximum shear stress and propagates to the surface leading to a fatigue failure. To locate the origin of a crack, it is necessary to find the point at which the shear stress attains its maximum value. Show that the problem of finding the location of the maximum shear stress for $\nu_1 = \nu_2 = 0.3$ reduces to maximizing the function

$$f(\lambda) = \frac{0.5}{\sqrt{1+\lambda^2}} - \sqrt{1+\lambda^2}\left(1 - \frac{0.5}{1+\lambda^2}\right) + \lambda \tag{4}$$

where $f = \tau_{zy}/p_{\max}$ and $\lambda = z/b$.

5.6 Plot the graph of the function $f(\lambda)$ given by Eq. (4) in Problem 5.5 in the range $(0, 3)$ and identify its maximum.

5.7 Find the maximum of the function given by Eq. (4) in Problem 5.5 using the following methods:

(a) Unrestricted search with a fixed step size of 0.1 from the starting point 0.0
(b) Unrestricted search with an accelerated step size using an initial step length of 0.1 and a starting point of 0.0
(c) Exhaustive search method in the interval (0, 3) to achieve an accuracy of within 5% of the exact value
(d) Dichotomous search method in the interval $(0, 3)$ to achieve an accuracy of within 5% of the exact value using a value of $\delta = 0.0001$
(e) Interval halving method in the interval (0, 3) to achieve an accuracy of within 5% of the exact value

5.8 Find the maximum of the function given by Eq. (4) in Problem 5.5 using the following methods:

(a) Fibonacci method with $n = 8$
(b) Golden section method with $n = 8$

5.9 Find the maximum of the function given by Eq. (4) in Problem 5.5 using the quadratic interpolation method with an initial step length of 0.1.

5.10 Find the maximum of the function given by Eq. (4) in Problem 5.5 using the cubic interpolation method with an initial step length of $t_0 = 0.1$.

5.11 Find the maximum of the function $f(\lambda)$ given by Eq. (4) in Problem 5.5 using the following methods:

(a) Newton method with the starting point 0.6
(b) Quasi-Newton method with the starting point 0.6 and a finite difference step size of 0.001
(c) Secant method with the starting point $\lambda_1 = 0.0$ and $t_0 = 0.1$

5.12 Prove that a convex function is unimodal.

5.13 Compare the ratios of intervals of uncertainty (L_n/L_0) obtainable in the following methods for $n = 2, 3, \ldots, 10$:

(a) Exhaustive search
(b) Dichotomous search with $\delta = 10^{-4}$

(c) Interval halving method
(d) Fibonacci method
(e) Golden section method

5.14 Find the number of experiments to be conducted in the following methods to obtain a value of $L_n/L_0 = 0.001$:

(a) Exhaustive search
(b) Dichotomous search with $\delta = 10^{-4}$
(c) Interval halving method
(d) Fibonacci method
(e) Golden section method

5.15 Find the value of x in the interval $(0, 1)$ which minimizes the function $f = x(x - 1.5)$ to within ± 0.05 by (**a**) the golden section method and (**b**) the Fibonacci method.

5.16 Find the minimum of the function $f = \lambda^5 - 5\lambda^3 - 20\lambda + 5$ by the following methods:

(a) Unrestricted search with a fixed step size of 0.1 starting from $\lambda = 0.0$
(b) Unrestricted search with accelerated step size from the initial point 0.0 with a starting step length of 0.1
(c) Exhaustive search in the interval $(0, 5)$
(d) Dichotomous search in the interval $(0, 5)$ with $\delta = 0.0001$
(e) Interval halving method in the interval $(0, 5)$
(f) Fibonacci search in the interval $(0, 5)$
(g) Golden section method in the interval $(0, 5)$

5.17 Find the minimum of the function $f = (\lambda/\log \lambda)$ by the following methods (take the initial trial step length as 0.1):

(a) Quadratic interpolation method
(b) Cubic interpolation method

5.18 Find the minimum of the function $f = \lambda/\log \lambda$ using the following methods:

(a) Newton method
(b) Quasi-Newton method
(c) Secant method

5.19 Consider the function

$$f = \frac{2x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3}{x_1^2 + x_2^2 + 2x_3^2}$$

Substitute $\mathbf{X} = \mathbf{X}_1 + \lambda\mathbf{S}$ into this function and derive an exact formula for the minimizing step length λ^*.

5.20 Minimize the function $f = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ starting from the point $\mathbf{X}_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ along the direction $\mathbf{S} = \begin{Bmatrix} -1 \\ 0 \end{Bmatrix}$ using the quadratic interpolation method with an initial step length of 0.1.

5.21 Consider the problem

$$\text{Minimize } f(\mathbf{X}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

and the starting point, $\mathbf{X}_1 = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$. Find the minimum of $f(\mathbf{X})$ along the direction, $\mathbf{S}_1 = \begin{Bmatrix} 4 \\ 0 \end{Bmatrix}$ using quadratic interpolation method. Use a maximum of two refits.

5.22 Solve Problem 5.21 using the cubic interpolation method. Use a maximum of two refits.

5.23 Solve Problem 5.21 using the direct root method. Use a maximum of two refits.

5.24 Solve Problem 5.21 using the Newton method. Use a maximum of two refits.

5.25 Solve Problem 5.21 using the Fibonacci method with $L_0 = (0, 0.1)$.

5.26 Write a computer program, in the form of a subroutine, to implement the Fibonacci method.

5.27 Write a computer program, in the form of a subroutine, to implement the golden section method.

5.28 Write a computer program, in the form of a subroutine, to implement the quadratic interpolation method.

5.29 Write a computer program, in the form of a subroutine, to implement the cubic interpolation method.

5.30 Write a computer program, in the form of a subroutine, to implement the secant method.

5.31 Find the maximum of the function given by Eq. (4) in Problem 5.5 using MATLAB. Assume the bounds on λ as 0 and 3.

5.32 Find the minimum of the function f(λ) given in Problem 5.16, in the range 0 and 5, using MATLAB.

5.33 Find the minimum of $f(x) = x(x - 1.5)$ in the interval (0, 1) using MATLAB.

5.34 Find the minimum of the function $f(x) = \frac{x^3}{16} - \frac{27x}{4}$ in the range (0, 10) using MATLAB.

5.35 Find the minimum of the function $f(x) = x^3 + x^2 - x - 2$ in the interval -4 and 4 using MATLAB.

5.36 Find the minimum of the function $f(x) = -\frac{1.5}{x} + \frac{6(10^{-6})}{x^9}$ in the interval -4 and 4 using MATLAB.