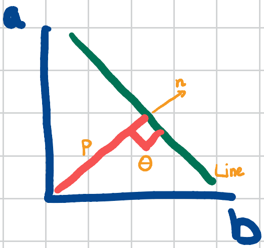
1.1 Explain the problem of using the slope and y-intercept as line parameters when using the Hough transform.

Hough transform uses polar coordinate representation of a line:

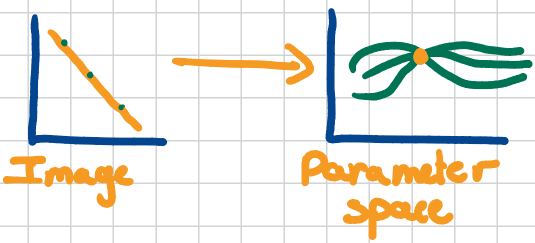


$$\rho = x\cos\theta + y\cos\theta$$

ρ: Perpendicular distance from origin to line

θ: Angle of the perpendicular line with the positive x-axis

- For each point of the image, we got a line in parameter space



- The parameter space is finite

  x-axis: (θ) From 0 to 180

  y-axis: (ρ) From 0 to $\sqrt{m^2+n^2}$

- The sinusoidal curve represent all possible lines passing through the point (x,y).
- When multiple points share a line, their sinousidal curves share a common inteception in the paramter space

1.2 When using the polar representation of lines, what does the vote of each point in the image look like in the parameter plane?

- Each point of the images contributes a sinusoidal curve in the parameter space.
- Their curves are sinusoidal combines cos and sin which are periodic trigonometric functions

1.3 Given the point $(1, 1)$, find the vote in parameter space it will cast for angle $\theta = 0$.

- Vote is a pair (ρ, θ) which represents the line

$$\rho = x \cos\theta + y \cos\theta$$

$$\rho = \boxed{1}$$

- Therefore, it cast a vote at localization (0,1) in Hough space

1.4 Explain how lines are detected by checking the parameter plane.

- Interceptions in Hough space are true lines in the picture

1.5 Explain the trade-off regarding bin size in the parameter plane.

- Bin size: Is the resolution of the hough space
  - → Small bin: (higher resolution) → More votes, → Slower, → Higher accuracy
  - → Large bin: (lower resolution) → Fewer votes, → Faster, → Lower accuracy

1.6 Describe how voting in the parameter plane can be improved if the normal at each voting point is known.

- The normal is the gradient direction
- When the normal is known, we can reduce (narrow down) the range of θ (normally from 0 to 180) to

$$\theta \in [\theta_n - \nabla\theta, \theta_n + \nabla\theta]$$

θ_n → Normal direction

1.7 When using the Hough transform for circles, explain what should be the number of dimensions of the parameter space.

- It should be three dimensions
  - A center point (a, b)
  - A radius (r)

1.8 Detecting "Nearly Horizontal" Lines

- Assume an $m \times n$ image.
- Parameters of a detected line are (θ, d) where $\theta \in [45°, 135°]$.
- Derive the equation to compute pixel coordinates (x, y) by scanning $x \in [0, n]$ and computing:

$$y = -\frac{\cos(\theta)}{\sin(\theta)} x + \frac{d}{\sin(\theta)}$$

- Explain why x is scanned, and y is computed.

$d = \cos\theta x + \sin\theta y$

$-\cos\theta x + d = \sin\theta y$

$\underbrace{-\frac{\cos\theta}{\sin\theta}}_{\text{Slope}} x + \underbrace{\frac{d}{\sin\theta}}_{y \text{ intercept}} = y$

- y is scanned to avoid zero in the denominador ($\cos\theta = 90$)

2.1 Explain the disadvantage of using the equation $y = a \cdot x + b$ for line fitting. What kind of lines cannot be fitted accurately using this equation?

- Vertical lines or near verticals because the slope is close to infinite.

- A line with a slope of 45° passes at a distance of 10 from the origin.
- Write the coefficients a, b, c in the explicit line equation $a \cdot x + b \cdot y + c = 0$.
- Verify the answer by drawing the line and checking points on it.

$$ax + by + c = 0$$

$$y = \frac{a}{b} x + \frac{c}{b}$$

$\frac{a}{b} \rightarrow 1$ "Slope"

$\frac{c}{b} \rightarrow 10$ "Distance"

If $b = 0.5$

$$\frac{a}{0.5} = 1 \Rightarrow a = 0'5$$

$$\frac{c}{0.5} = 10 \Rightarrow c = 5$$

$$0.5x + 0.5y + 5 = 0$$

2.3 Line Defined by Two Points

- Given points $(10, 10)$ and $(20, 20)$, write the implicit line equation they define.
- Write the normalized normal to this line.

$$ax + by + c = 0$$

$$\text{Slope} = \frac{20-10}{10} = \boxed{1}$$

$$\begin{cases} 10a + 10b = 0 \\ 20a + 20b = 0 \end{cases}$$

$a = -b \quad b = 1$

$$y = 1x + c$$

$10 = 10 + c$ | $20 = 20 + c$

$\boxed{0 = c}$ | $\boxed{0 = c}$

$-x + y = 0 \quad n = \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

$x - y = 0 \quad n = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$

2.4 Vector Representation of a Line

- Given a line with normal $(1, 2)$ and a distance of 2 from the origin, write the vector l representing the line in the implicit equation $l^T x = 0$.

$n = (1, 2)$, $1 \to a$, $2 \to b$

$$d = \frac{|c|}{\sqrt{a^2 + b^2}} \Rightarrow 2 = \frac{c}{\sqrt{1 + 2^2}} =$$

$$\Rightarrow 2 = \frac{c}{\sqrt{5}} \Rightarrow 2\sqrt{5} = c \longrightarrow x + 2y + 2\sqrt{5} = 0$$

$$\left[\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \cancel{\frac{2\sqrt{5}}{\sqrt{5}}}\right]^T$$

2.5 Find y-Coordinate on a Line

- Given line coefficients $(1, 2, 3)$, find the y-coordinate where $x = 2$.

$$x + 2y + 3 = 0 \to 2y = -x - 3$$

$$y = \frac{-2-3}{2} \Rightarrow \boxed{y = \frac{-5}{2}}$$

2.6 Line Fitting with Implicit Equation

- Explain how to fit a line using the implicit equation.
- Write the equation to solve for unknown line parameters.

2.8 Algebraic vs. Geometric Distance

- Explain the difference between algebraic and geometric distance.

- Algebraic distance is the own point $|f(p)|$
- Geometric distance is $\dfrac{|f(p)|}{|\nabla f(x^*)|}$

Closed point to p of the function

2.9 Geometric Distance Approximation

- Explain how the geometric distance of a point p from an implicit curve $f(p) = 0$ is measured exactly and approximated.
- Discuss why the approximation is used.

- Sometimes is not possible (computationality) to calculate the close point, for that reason we use the own point

$$\frac{|f(p)|}{|\nabla f(x^*)|} \longrightarrow \frac{|f(p)|}{|\nabla f(p)|}$$

1. Convolution Layers

1.1 Let I be a 4×4 RGB image where:

- The R channel is all 1's.
- The G channel is all 2's.
- The B channel has values:
  - 1 in the first row,
  - 2 in the second row,
  - 3 in the third row,
  - 4 in the fourth row. Compute the convolution of this image with a 3×3 filter having all ones without zero padding.

Remember: Filter 3x3 all 1s

$$R = \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{matrix} = \begin{matrix} 9 & 9 \\ 9 & 9 \end{matrix}$$

$$G = \begin{matrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{matrix} = \begin{matrix} 18 & 18 \\ 18 & 18 \end{matrix}$$

$$B = \begin{matrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \end{matrix} = \begin{matrix} 18 & 18 \\ 27 & 27 \end{matrix}$$

$$\begin{pmatrix} 45 & 45 \\ 54 & 54 \end{pmatrix}$$

$9+18+18$

$2+2+2+$
$3+3+3+$
$4+4+4=27$

1.2 Repeat the previous question with zero padding.

$$R = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} = \begin{matrix} 4 & 6 & 6 & 4 \\ 6 & 9 & 9 & 9 \\ 6 & 9 & 9 & 9 \\ 4 & 6 & 6 & 4 \end{matrix}$$

$$G = \begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} = \begin{matrix} 8 & 12 & 12 & 8 \\ 12 & 18 & 18 & 12 \\ 12 & 18 & 18 & 12 \\ 8 & 12 & 12 & 8 \end{matrix}$$

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 2 & 2 & 2 & 0 \\ 0 & 3 & 3 & 3 & 3 & 0 \\ 0 & 4 & 4 & 4 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \qquad \begin{matrix} 6 & 9 & 9 & 6 \\ 12 & 18 & 18 & 12 \\ 18 & 27 & 27 & 18 \\ 14 & 21 & 21 & 14 \end{matrix}$$

$$\begin{matrix} 18 & 27 & 27 & 18 \\ 30 & 45 & 45 & 30 \\ 36 & 54 & 54 & 36 \\ 26 & 39 & 39 & 26 \end{matrix}$$