More convolutional nets

Lecture 09 — CS 577 Deep Learning

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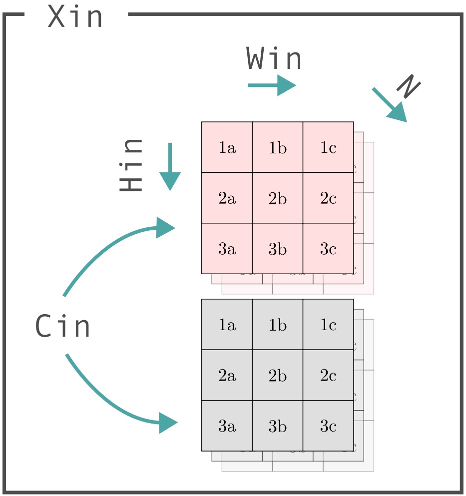
Input image tensor with shape $(N, C_{\text{in}}, H_{\text{in}}, W_{\text{in}})$



Samples

n channels_in

1
grayscale

3
RGB

height

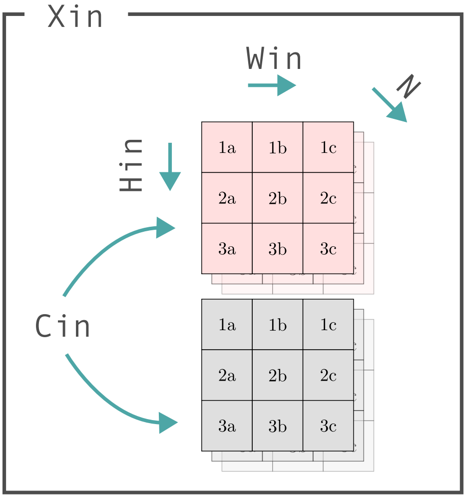
width

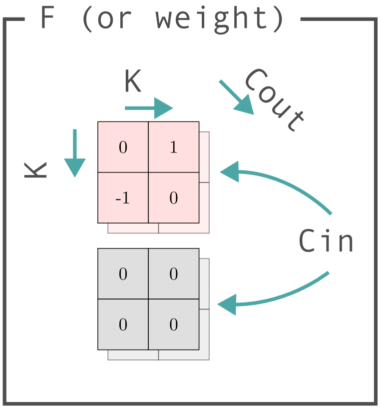
32 × 32
padded
mnist

abstract physical

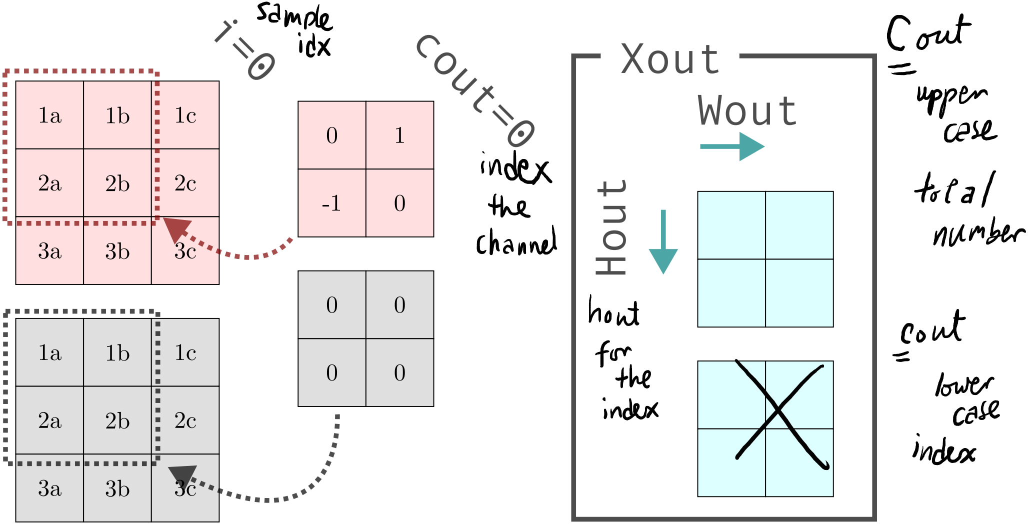
A single convolution layer $(C_{\text{out}}, C_{\text{in}}, K, K)$

nchannels_out

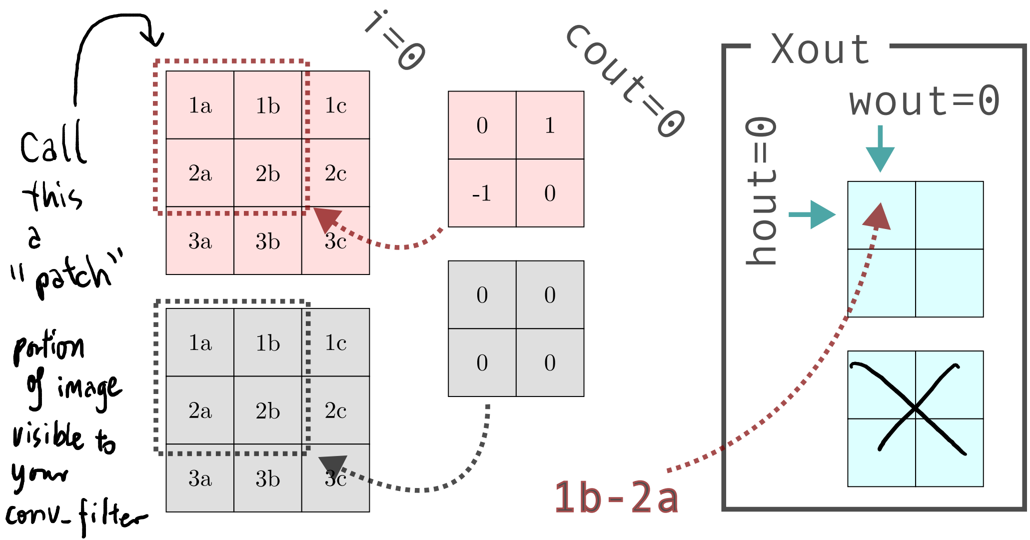




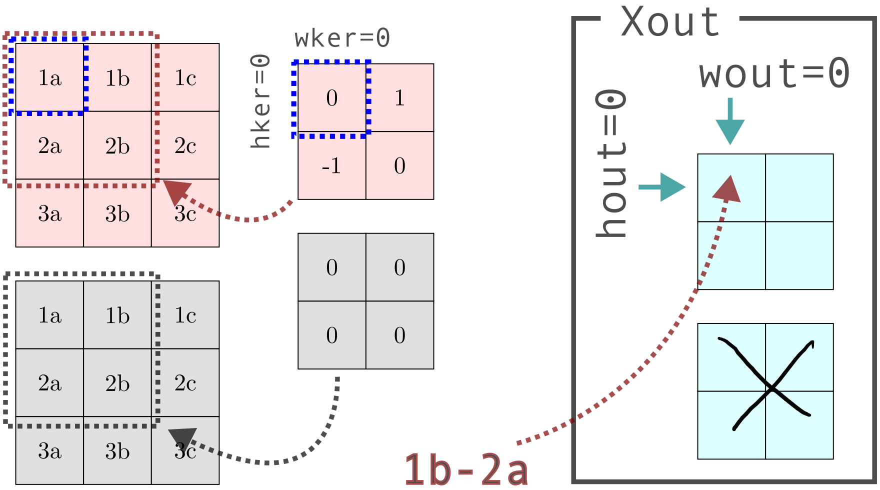
Output: `Xout` with shape $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$



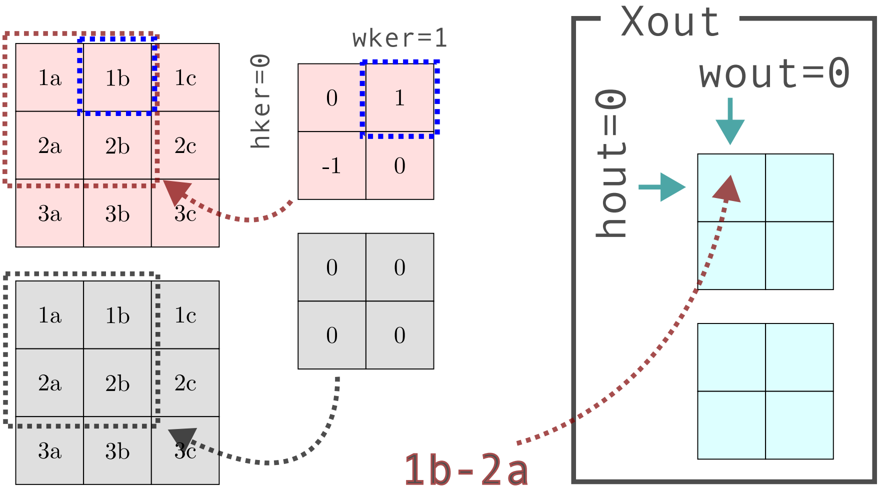
patch_idx = 0


Call this a "patch"
portion of image visible to your conv_filter
1a
1b
1c
2a
2b
2c
3a
3b
3c
i=0
cout=0
0
1
-1
0
0
0
0
0
Xout
wout=0
hout=0
1b-2a

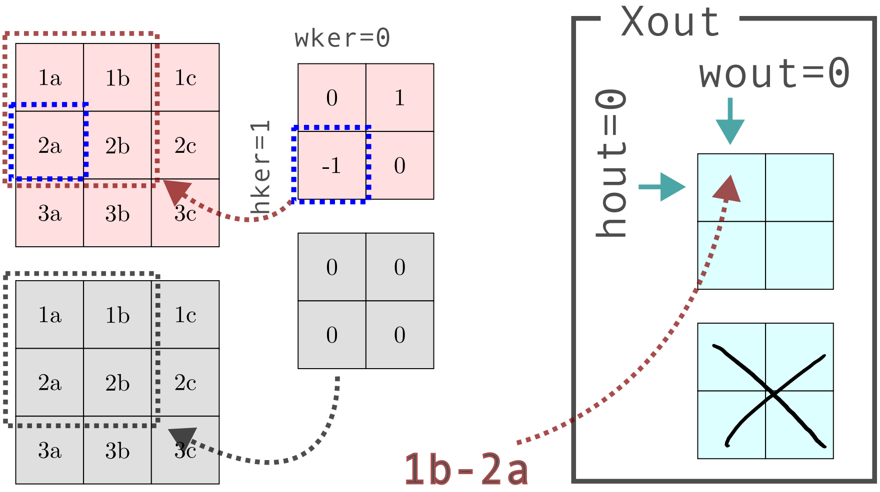
patch_idx = 0


1a
1b
1c
2a
2b
2c
3a
3b
3c
hker=0
wker=0
0
1
-1
0
0
0
0
0
1a
1b
1c
2a
2b
2c
3a
3b
3c
1b-2a
Xout
wout=0
hout=0

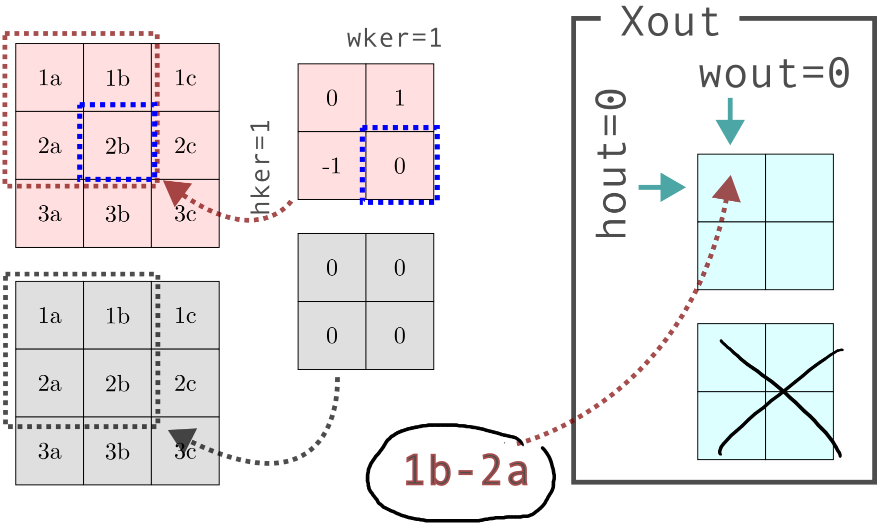
patch_idx = 0


1a
1b
1c
2a
2b
2c
3a
3b
3c
hker=0
wker=1
0
1
-1
0
1a
1b
1c
2a
2b
2c
3a
3b
3c
0
0
0
0
1b-2a
Xout
wout=0
hout=0

patch_idx = 0

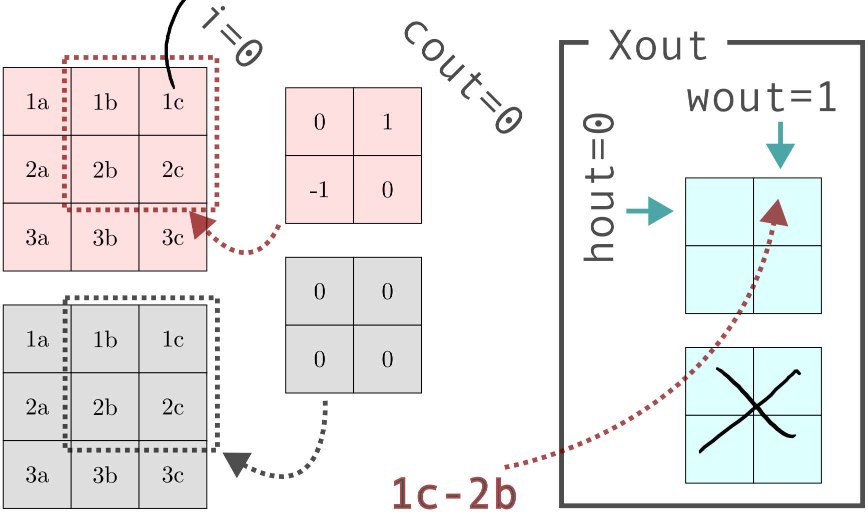

1a
1b
1c
2a
2b
2c
3a
3b
3c
wker=0
0
1
-1
0
hker=1
1a
1b
1c
2a
2b
2c
3a
3b
3c
0
0
0
0
Xout
wout=0
hout=0
1b-2a

patch_idx = 0

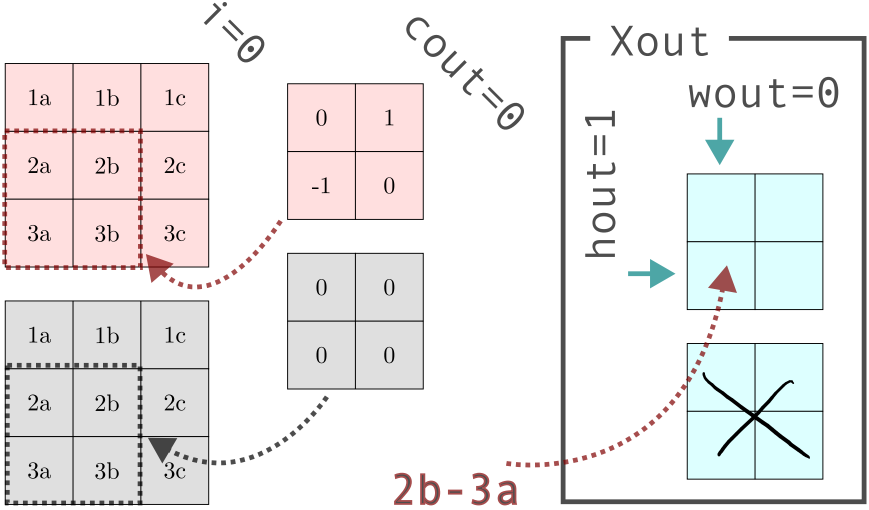

1a
1b
1c
2a
2b
2c
3a
3b
3c
hker=1
wker=1
0
1
-1
0
0
0
0
0
1a
1b
1c
2a
2b
2c
3a
3b
3c
1b-2a
Xout
wout=0
hout=0

patch_idx = 1

"1c" entire symbol substitute for actual number

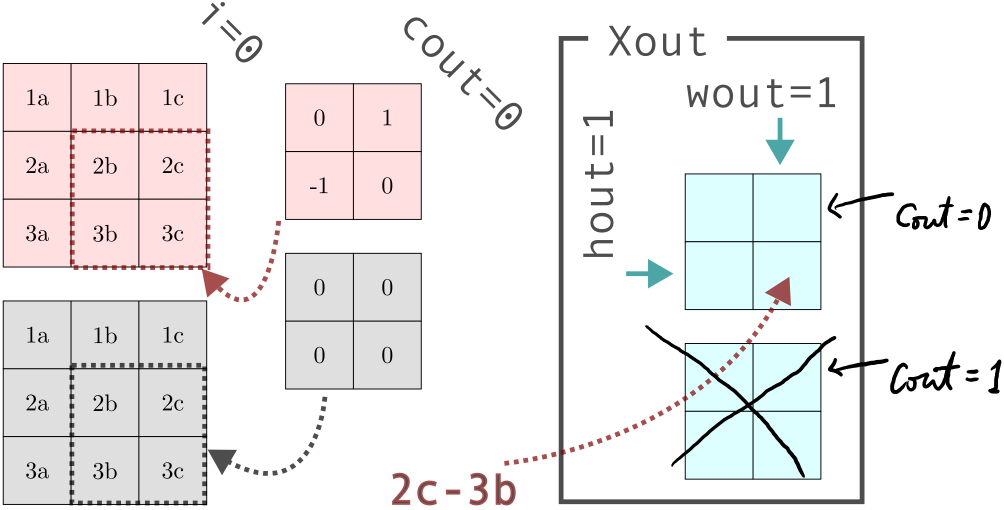


patch_idx = 2

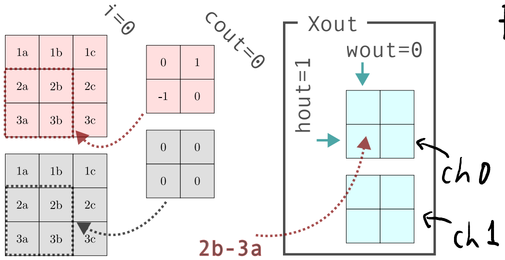


i=0
cout=0
Xout
wout=0
hout=1
1a
1b
1c
2a
2b
2c
3a
3b
3c
0
1
-1
0
1a
1b
1c
2a
2b
2c
3a
3b
3c
0
0
0
0
2b-3a

patch_idx = 3


i=0
cout=0
Xout
wout=1
hout=1
1a
1b
1c
2a
2b
2c
3a
3b
3c
0
1
-1
0
0
0
0
0
1a
1b
1c
2a
2b
2c
3a
3b
3c
Cout=0
Cout=1
2c-3b

Sliding window (simple but slow)



Full conv has 7 nested for-loops

```
# Suppose i, c_out, h_out, w_out are already defined, and
# X_out is initialized to all zeros
for cin in range(Cin):   # Input channels
    for hker in range(K):   # Kernel height
        for wker in range(K):   # Kernel width
            Xout[i, cout, hout, wout] += (
                F[cout, cin, hker, wker] *
                Xin[i, cin, hout + hker, wout + wker])
```

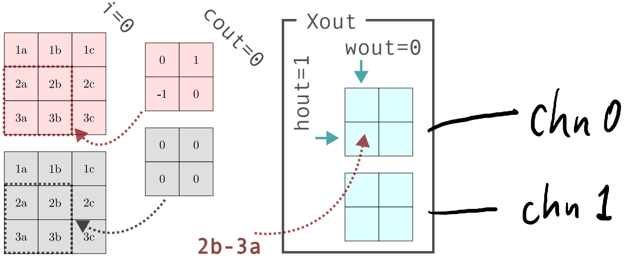
patch_idx

runs

runs

There's gotta be a better way! "im2col" (Next)

Sliding window (simple but slow)




```
# Suppose i, c_out, h_out, w_out are already defined, and
# X_out is initialized to all zeros
for cin in range(Cin):  # Input channels
    for hker in range(K):  # Kernel height
        for wker in range(K):  # Kernel width
            Xout[i, cout, hout, wout] += (
                F[cout, cin, hker, wker] *
                Xin[i, cin, hout + hker, wout + wker])
```

im2col

There's gotta be a better way! "im2col" (Next)

“im2col”

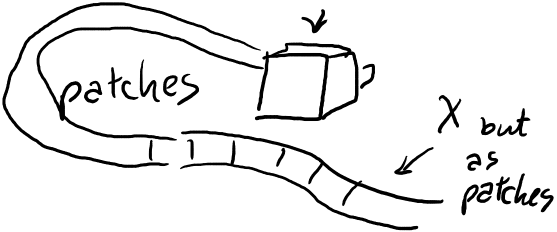
Conv_Filter

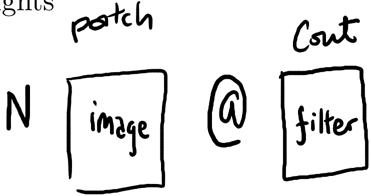
Main idea (in theory)

- Keep the window fixed in one spot
- Move the image tensor

Main idea (in ~~theory~~) practice

- Create patches of the data
- Each patch has the same shape as the convolution filter
- Do matrix multiplication between the (flattened) patches and the (flattened) convolutional filter weights





"lower the convolution to matmul"

“im2col”

Main idea (in theory)

- Keep the window fixed in one spot
- Move the image tensor

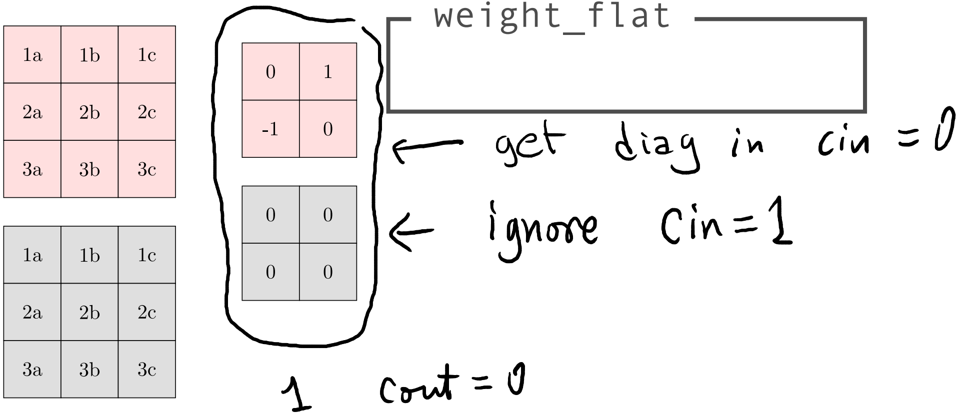
Main idea (in theory)

- Create patches of the data
- Each patch has the same shape as the convolution filter
- Do matrix multiplication between the (flattened) patches and the (flattened) convolutional filter weights

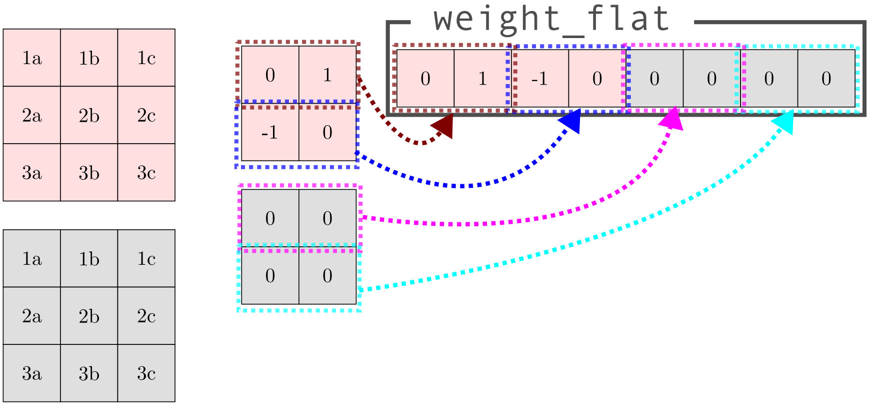
im2col physical interpretation

many way to compute

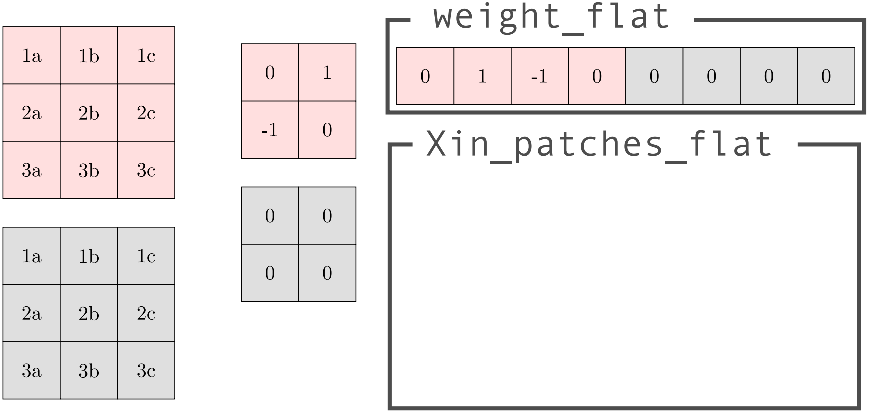
Flatten the weights



Flatten the weights

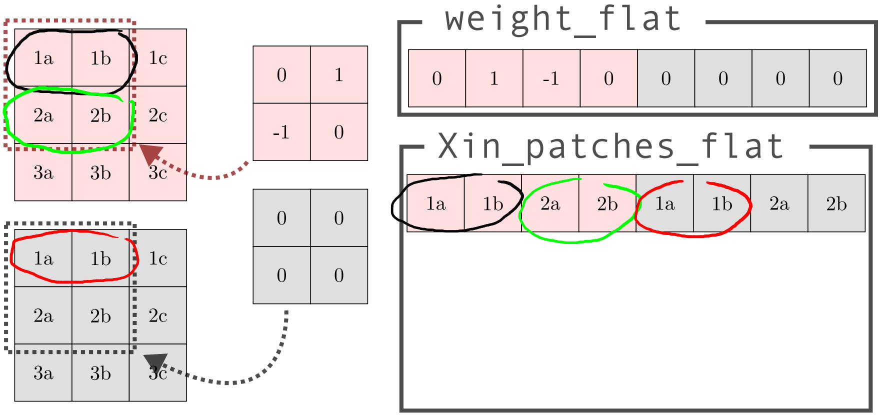


Flatten the weights

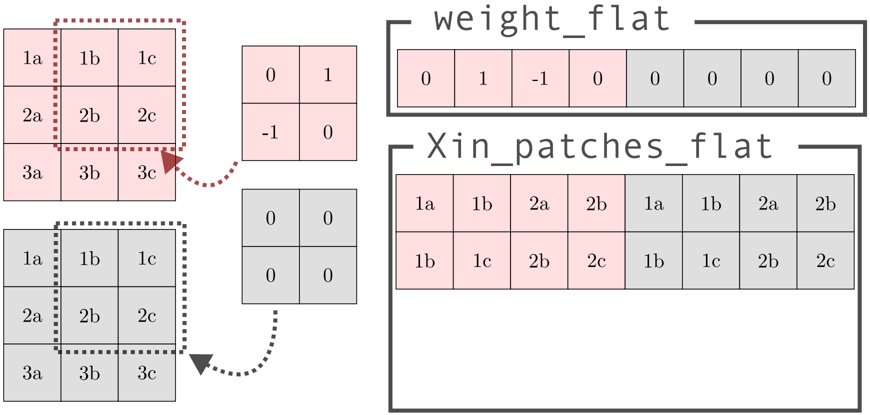


Flatten the weights

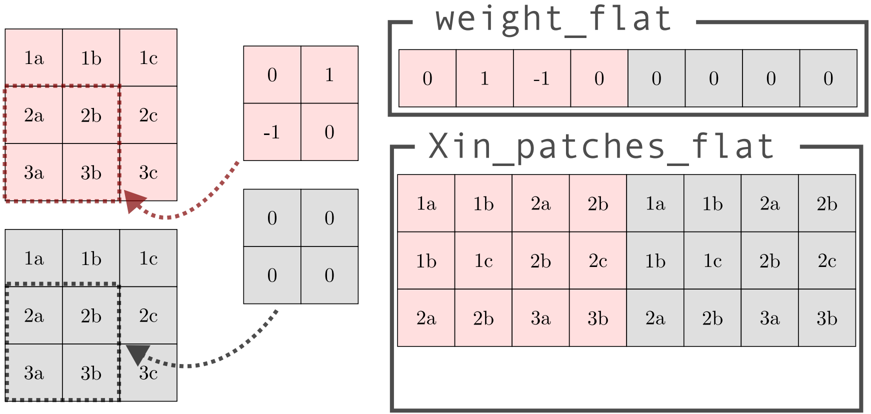
mat-vec multiply



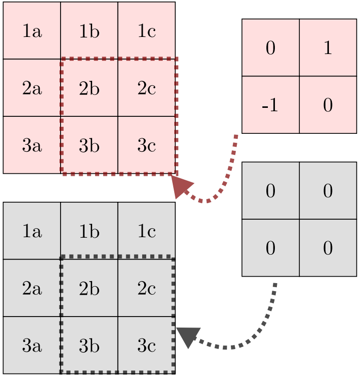
Flatten the weights



Flatten the weights



Flatten the weights



`weight_flat`

0	1	-1	0	0	0	0	0

`Xin_patches_flat`

1a	1b	2a	2b	1a	1b	2a	2b
1b	1c	2b	2c	1b	1c	2b	2c
2a	2b	3a	3b	2a	2b	3a	3b
2b	2c	3b	3c	2b	2c	3b	3c

$i=0 \rightarrow 2\times 3\times 3 \rightarrow$ $\underset{\text{num\_patches}}{4} \times 8 \sim \underset{C}{2}\cdot\underbrace{\underset{H}{2}\cdot\underset{W}{2}}_{\text{kernel}}$ size

Sliding window via matmul

Xin_patches_flat

1a	1b	2a	2b	1a	1b	2a	2b
1b	1c	2b	2c	1b	1c	2b	2c
2a	2b	3a	3b	2a	2b	3a	3b
2b	2c	3b	3c	2b	2c	3b	3c

@

weight_flat

0
1
-1
0
0
0
0
0

matmul

Sliding window via matmul

in ch0

diag finder

Xin_patches_flat

1a	1b	2a	2b	1a	1b	2a	2b
1b	1c	2b	2c	1b	1c	2b	2c
2a	2b	3a	3b	2a	2b	3a	3b
2b	2c	3b	3c	2b	2c	3b	3c

@

matmul

weight_flat

0	0
1	0
-1	0
0	0
0	1
0	0
0	-1
0	0

Cout

horizontal edge finder in Ch 1

output channel

output

(N, P, Cout)

Hout*Wout

Sliding window via matmul

A filter can have Cin Cout both be larger than 1

Many-to-many

Xin_patches_flat

1a	1b	2a	2b	1a	1b	2a	2b
1b	1c	2b	2c	1b	1c	2b	2c
2a	2b	3a	3b	2a	2b	3a	3b
2b	2c	3b	3c	2b	2c	3b	3c

@

weight_flat

0	0
1	0
-1	0
0	0
0	1
0	0
0	-1
0	0

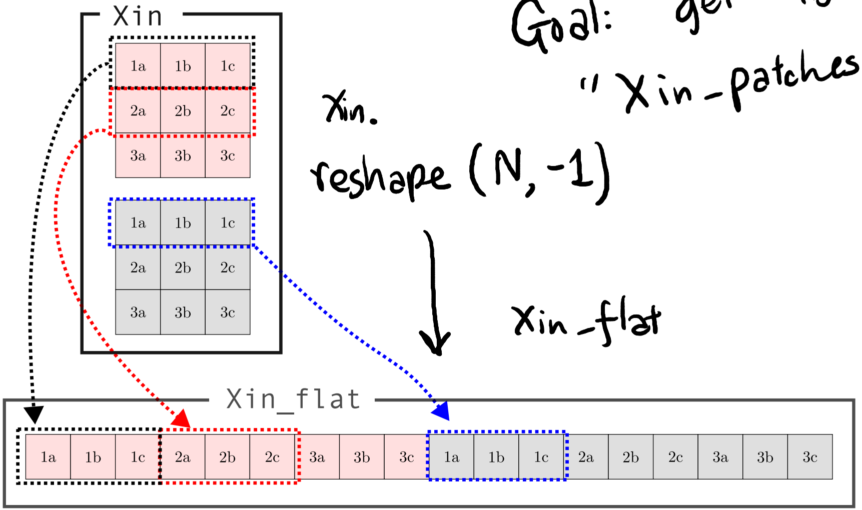
Cout

how to get this?

matmul

Flattening `Xin` into `Xin_flat`

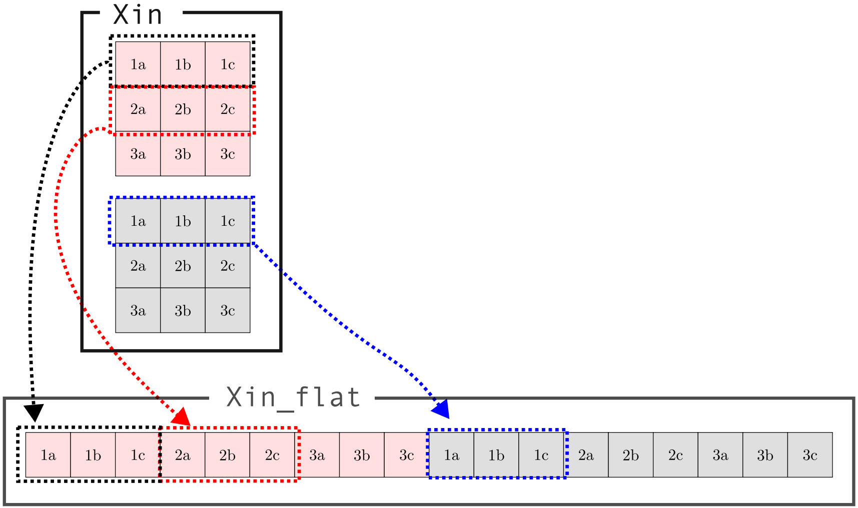
Goal: get to "Xin_patches"



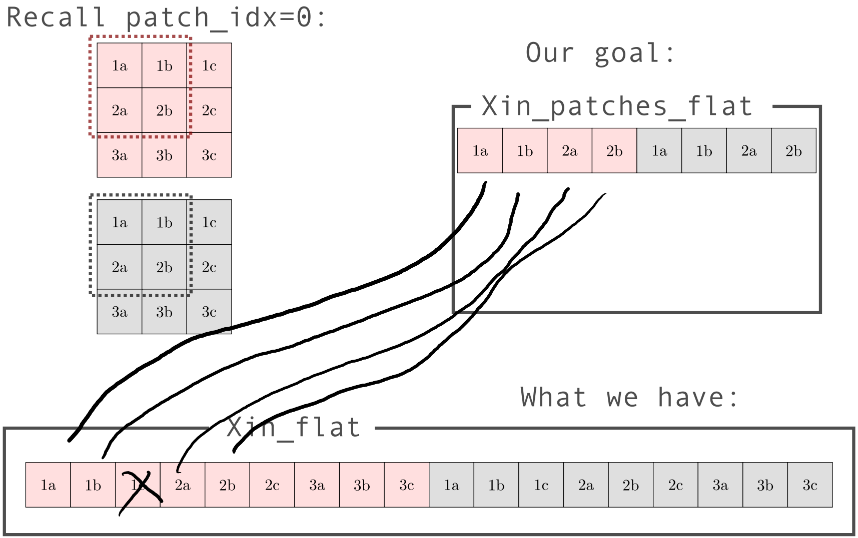
Xin.
reshape (N, -1)

Xin_flat

Flattening `Xin` into `Xin_flat`



Flattening `Xin` into `Xin_flat`

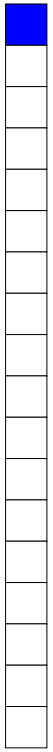


The "im2col" matrix

blue = 1

white = 0

Xin_flat @



Our goal:

Xin_patches_flat

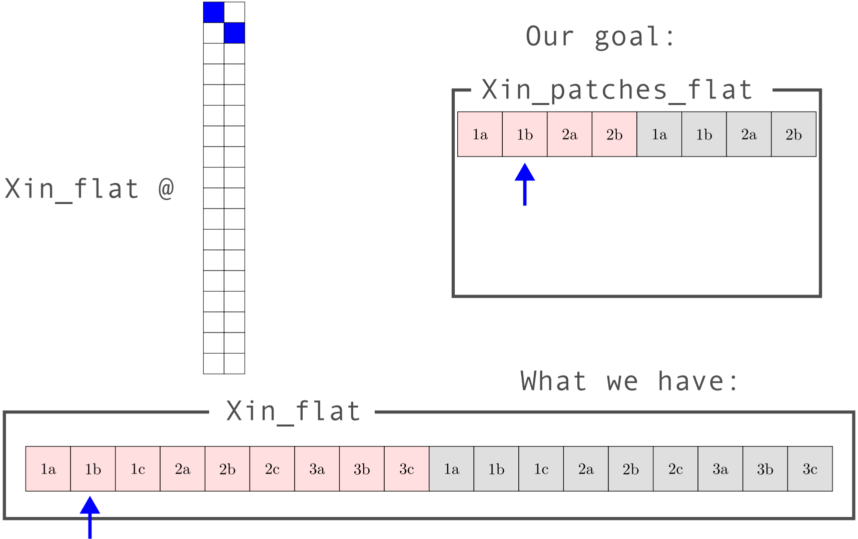
1a	1b	2a	2b	1a	1b	2a	2b

What we have:

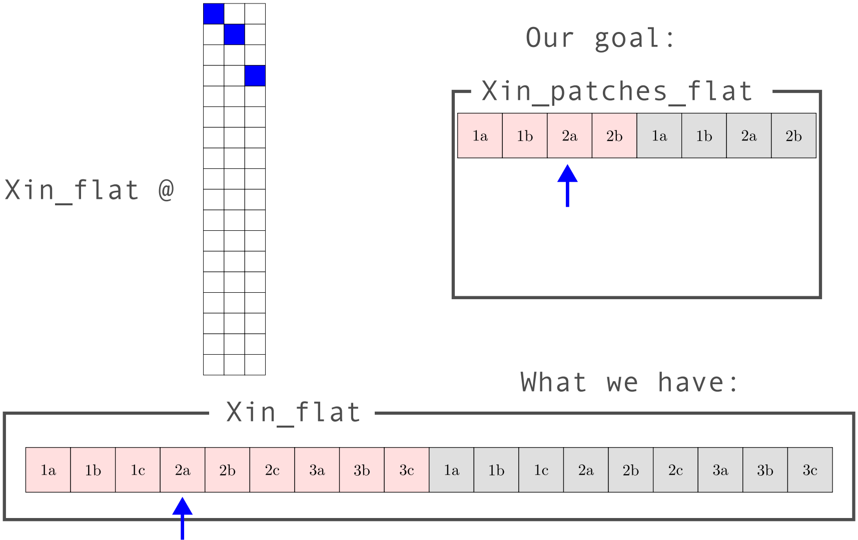
Xin_flat

1a	1b	1c	2a	2b	2c	3a	3b	3c	1a	1b	1c	2a	2b	2c	3a	3b	3c

The “im2col” matrix

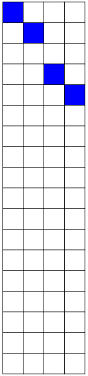


The “im2col” matrix



The "im2col" matrix

Xin_flat @



Our goal:

Xin_patches_flat

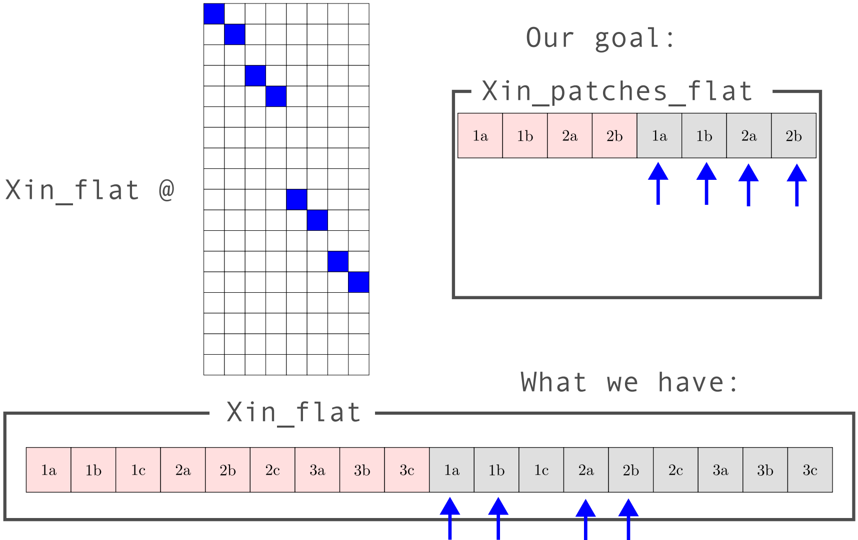
1a	1b	2a	2b	1a	1b	2a	2b

What we have:

Xin_flat

1a	1b	1c	2a	2b	2c	3a	3b	3c	1a	1b	1c	2a	2b	2c	3a	3b	3c

The “im2col” matrix



The “im2col” matrix

Recall patch_idx=1:

1a	1b	1c
2a	2b	2c
3a	3b	3c

1a	1b	1c
2a	2b	2c
3a	3b	3c

Our goal:

Xin_patches_flat

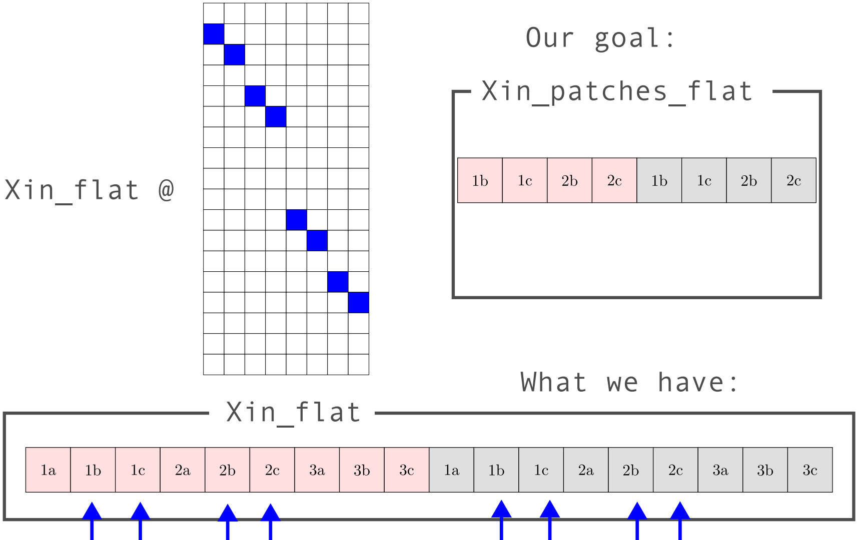
1b	1c	2b	2c	1b	1c	2b	2c

What we have:

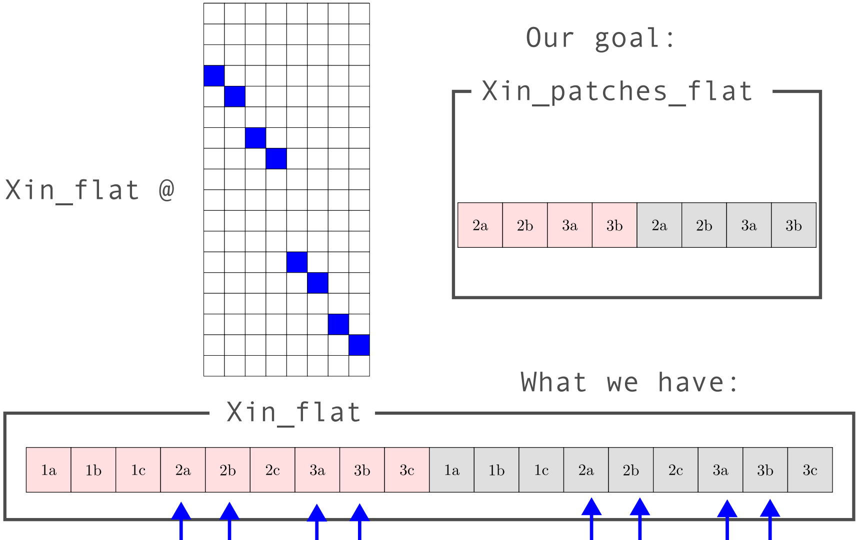
Xin_flat

1a	1b	1c	~~2a~~	2b	2c	3a	3b	3c	1a	1b	1c	2a	2b	2c	3a	3b	3c

The “im2col” matrix

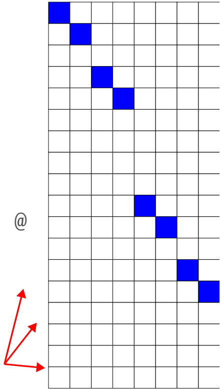


The “im2col” matrix



The “im2col” matrix

Xin_flat @



im2col

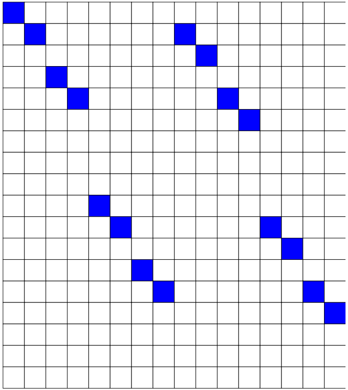
=

Xin_im2col

1a	1b	2a	2b	1a	1b	2a	2b

The “im2col” matrix

Xin_flat @



im2col

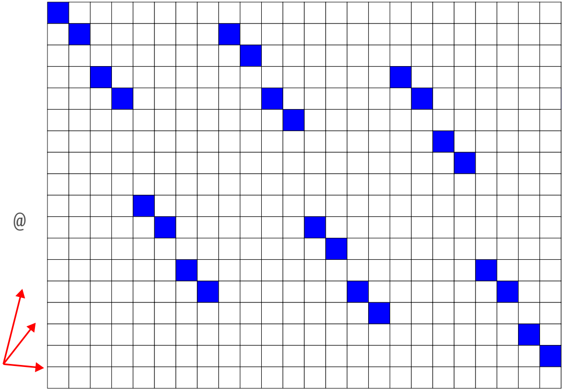
=

Xin_im2col

1a	1b	2a	2b	1a	1b	2a	2b	1b	1c	2b	2c	1b	1c	2b	2c

The “im2col” matrix

Xin_flat @



im2col

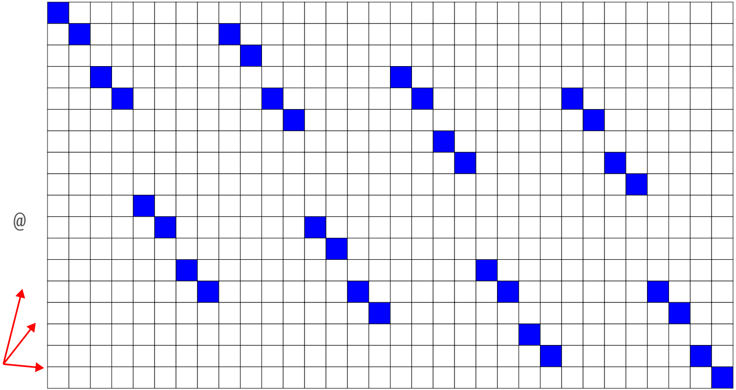
=

Xin_im2col

1a	1b	2a	2b	1a	1b	2a	2b	1b	1c	2b	2c	1b	1c	2b	2c	2a	2b	3a	3b	2a	2b	3a	3b

The "im2col" matrix

Xin_flat @



im2col

=

pid=0 1 2 3

Xin_im2col

1a	1b	2a	2b	1a	1b	2a	2b	1b	1c	2b	2c	1b	1c	2b	2c	2a	2b	3a	3b	2a	2b	3a	3b	2b	2c	3b	3c	2b	2c	3b	3c

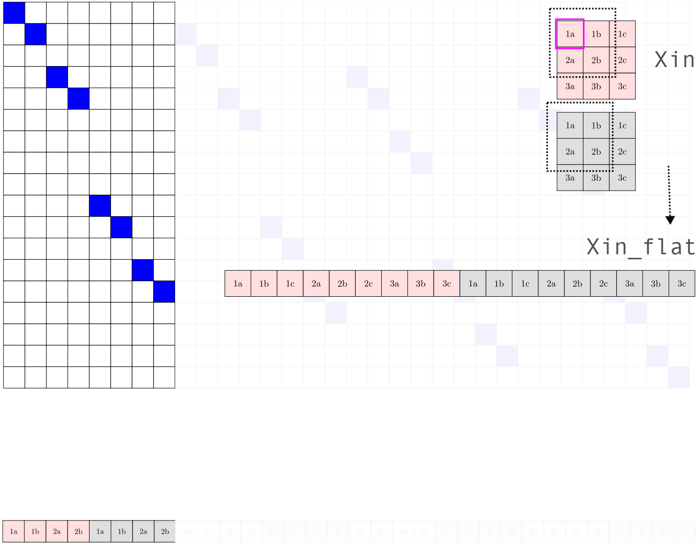
The “im2col” matrix

Xin_flat @

im2col

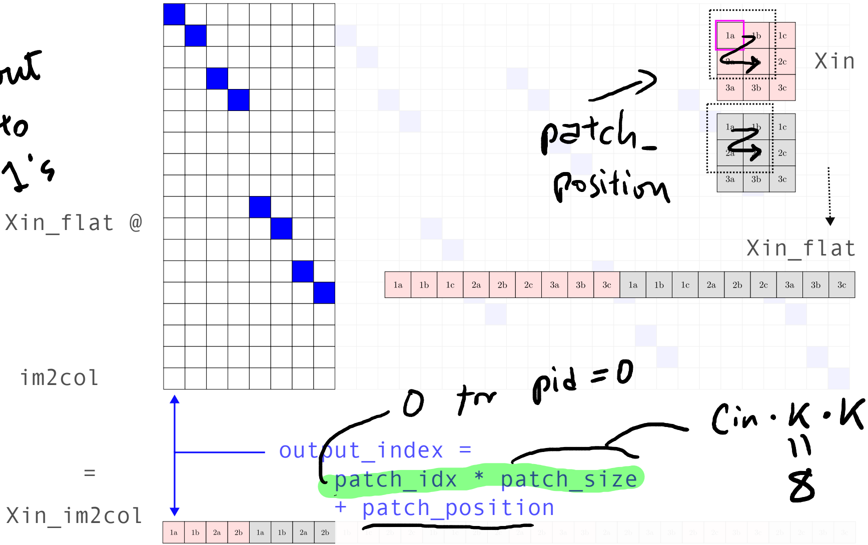
=

Xin_im2col

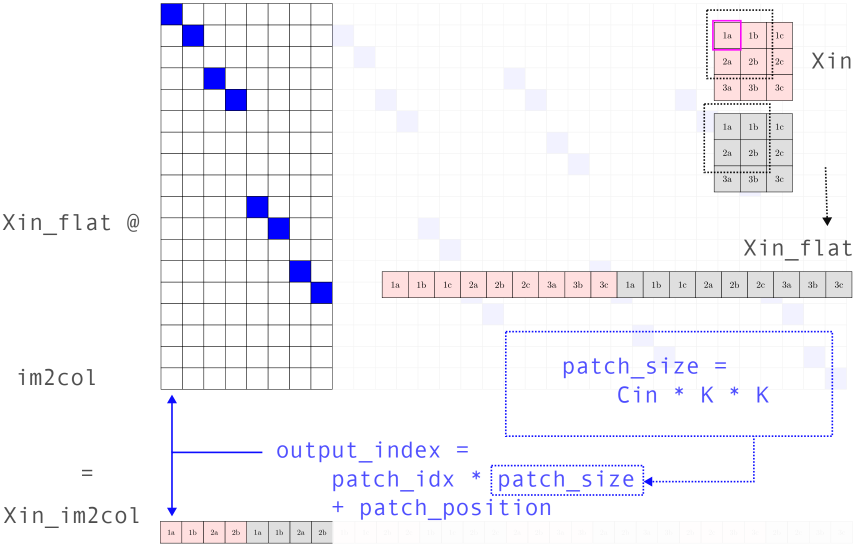


The "im2col" matrix

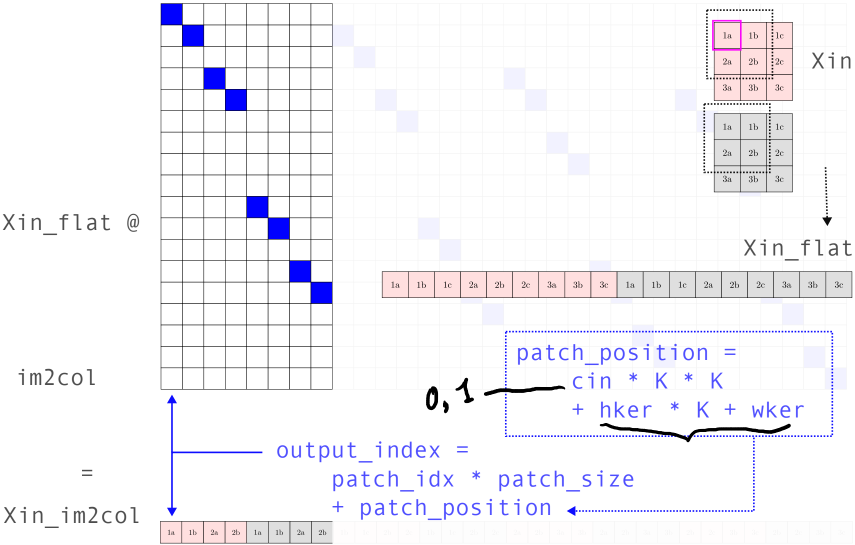
Need to figure out where to put 1's



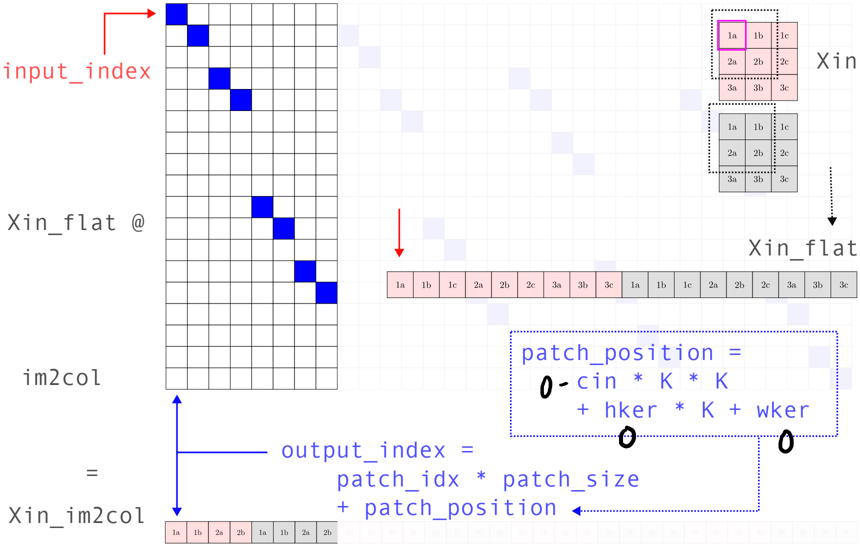
The "im2col" matrix



The “im2col” matrix

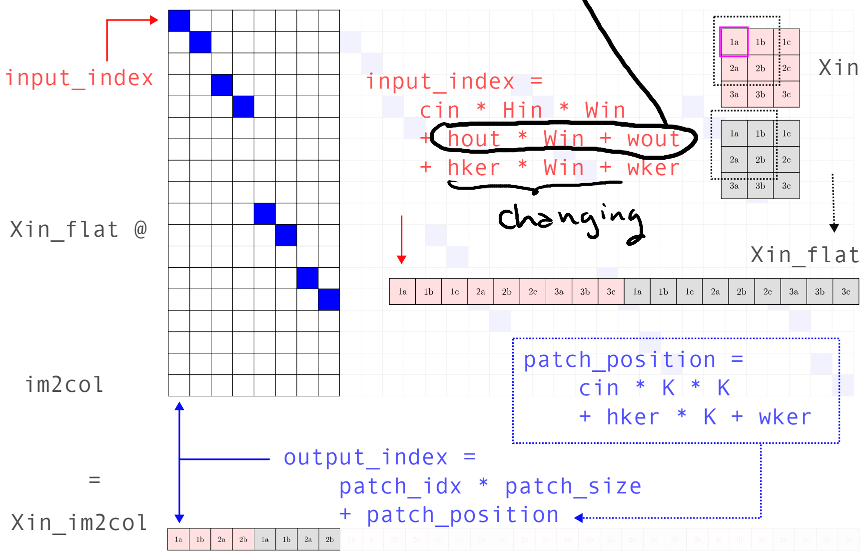


The "im2col" matrix

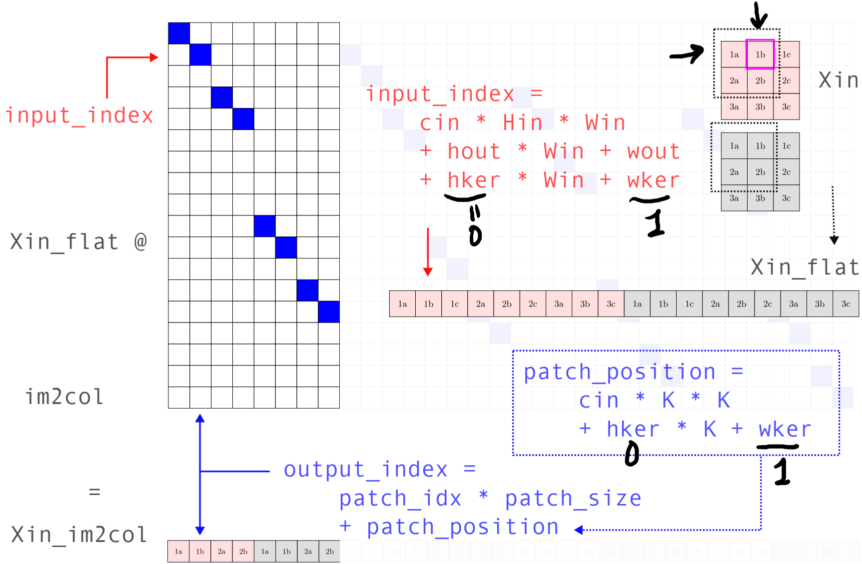


The “im2col” matrix

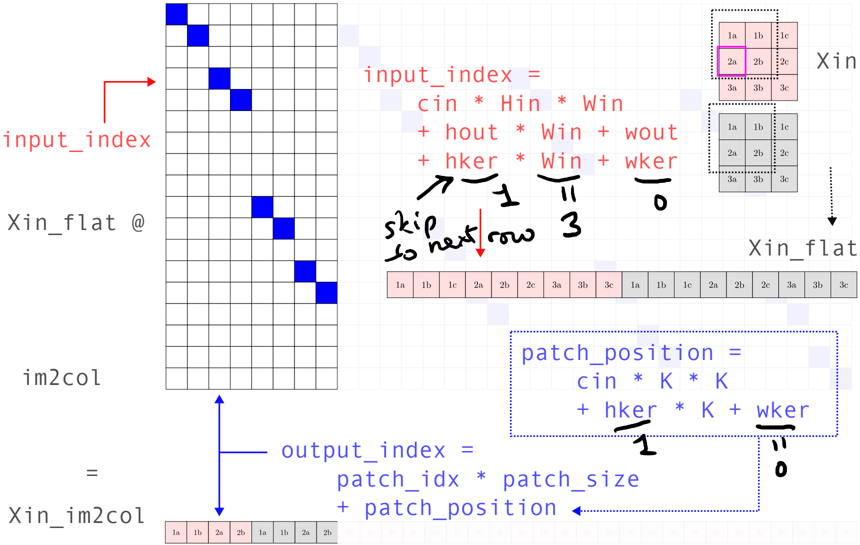
patch_idx = 0



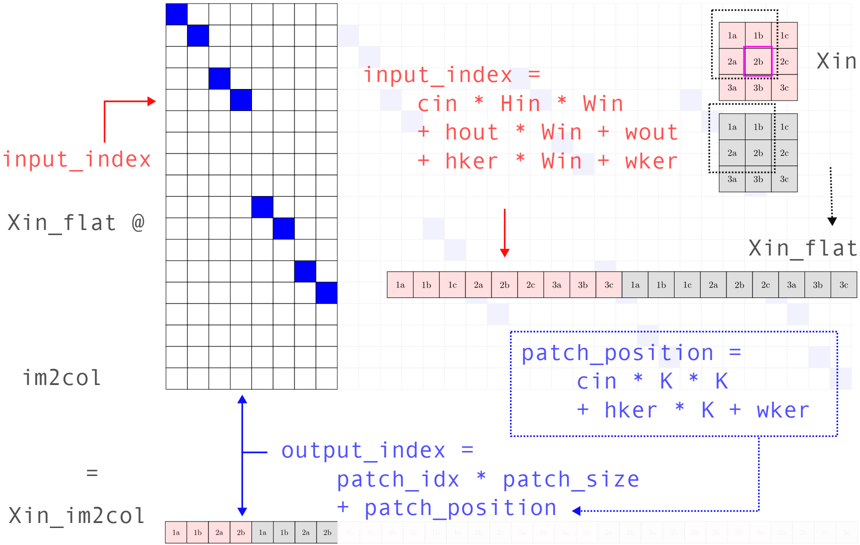
The "im2col" matrix



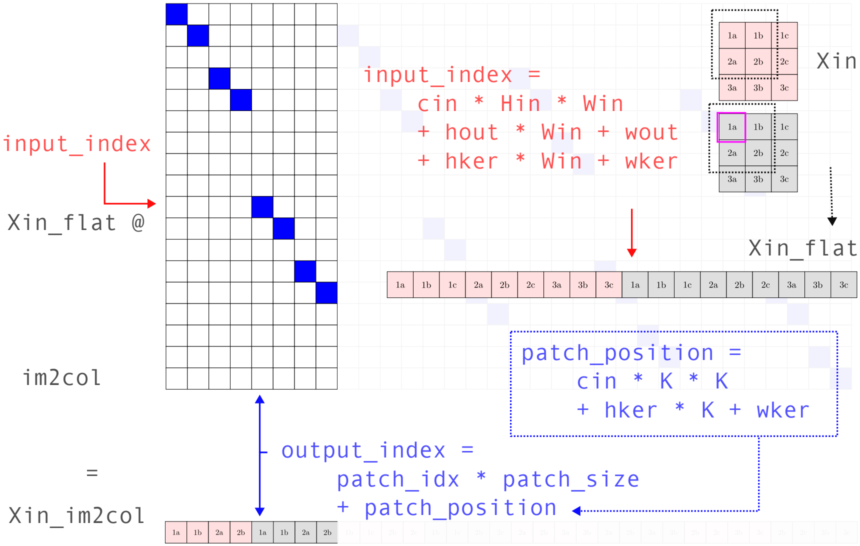
The “im2col” matrix



The “im2col” matrix

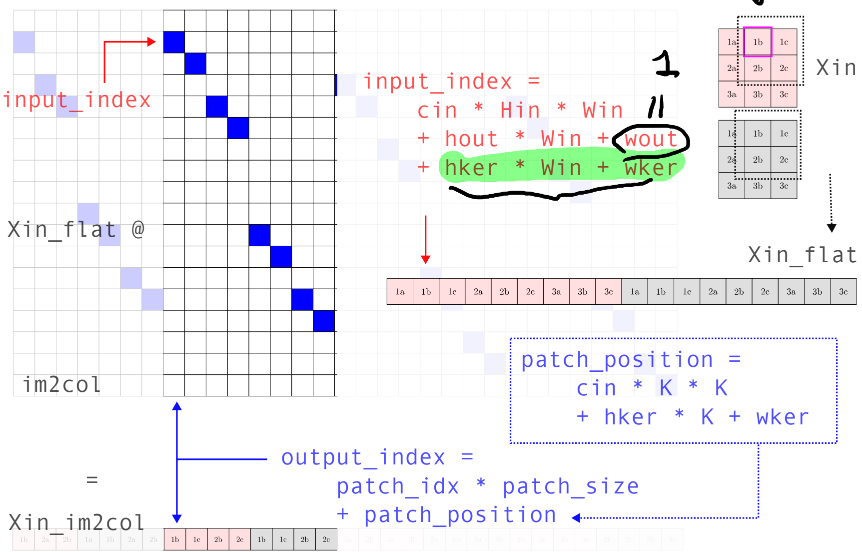


The “im2col” matrix

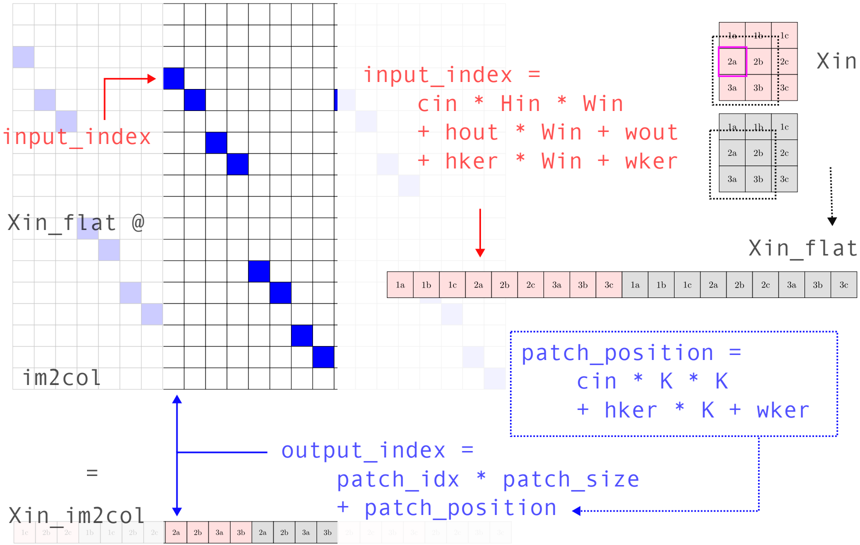


The “im2col” matrix

wout = 1



The “im2col” matrix



Putting it all together

stride

create the im2col

```
def im2col_matrix_dense(Xin, K, S=1):
    N, Cin, Hin, Win = Xin.shape
    Hout, Wout = Hin - K + 1, Win - K + 1
    P = Hout * Wout  # Total number of patches per image
    patch_size = Cin * K * K # Size of each flattened patch
    im2col_mat_dense = np.zeros((Cin * Hin * Win, P * patch_size))

    # [main loop on next slide...]

    return im2col_mat_dense
```

Putting it all together

```
    # [continued from previous slide...]
    patch_idx = 0
    for hout in range(Hout):
        for wout in range(Wout):
            for cin in range(Cin):
                for hker in range(K):
                    for wker in range(K):
                        input_index = cin * Hin * Win + hout * Win + wout + hker * Win + wker
                        patch_position = cin * K * K + hker * K + wker
                        output_index = patch_idx * patch_size + patch_position
                        im2col_mat_dense[input_index, output_index] = 1
            patch_idx += 1
```

which row to place a 1

which column to place the 1

Stride = 2

1a	1b	1c	1d	1e
2a	2b	2c	2d	2e
3a	3b	3c	3d	3e
4a	4b	4c	4d	4e
5a	5b	5c	5d	5e

1a	1b	1c	1d	1e
2a	2b	2c	2d	2e

Stride = 2

1a	1b	1c	1d	1e
2a	2b	2c	2d	2e
3a	3b	3c	3d	3e
4a	4b	4c	4d	4e
5a	5b	5c	5d	5e

1a	1b	1c	1d	1e
2a	2b	2c	2d	2e

Stride = 2

1a	1b	1c	1d	1e
2a	2b	2c	2d	2e
3a	3b	3c	3d	3e
4a	4b	4c	4d	4e
5a	5b	5c	5d	5e

1a	1b	1c	1d	1e
2a	2b	2c	2d	2e

Implement `im2col_matrix_dense` with stride

See `lec09-in-class-ex1-im2col.ipynb`
End of part 1.

Hint 1:

$$H_{out} = \frac{H_{in} - K}{S} + 1$$

Same for

$$W_{out} = \frac{W_{in} - K}{S} + 1$$

Hint 2:

wout

hout

multiplied by something

Goal for today: implement LeNet5

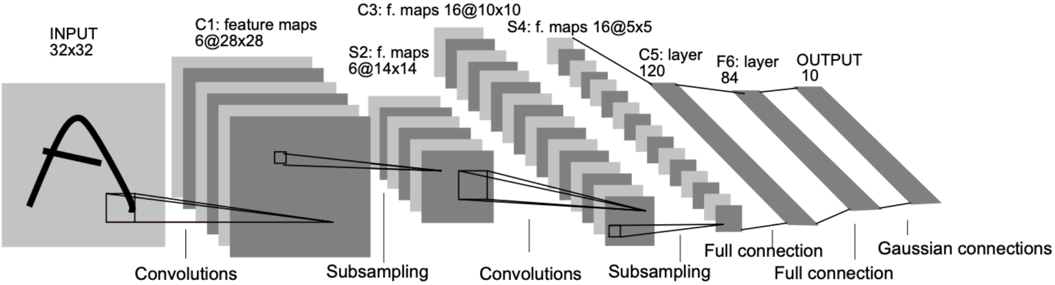


Image from LeCun et al. 1998

im2col.shape = $C_{\text{in}} \cdot H_{\text{in}} \cdot W_{\text{in}}$-by-$P \cdot \overset{C_{in}}{\cancel{C_{\text{out}}}} \cdot K^2$

dense

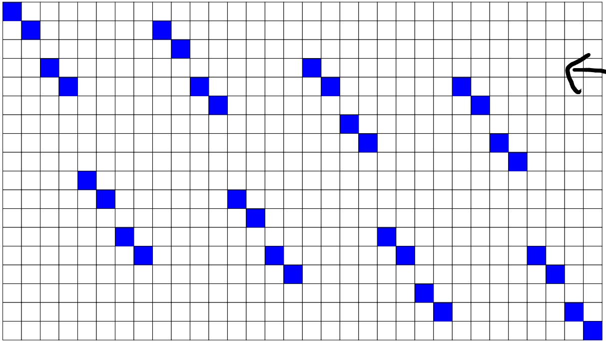
versus

sparse

Xin_flat @

dense

im2col



im2col

mostly zero

only a single 1 in every col

=

Xin_im2col

1a	1b	2a	2b	1a	1b	2a	2b	1b	1c	2b	2c	1b	1c	2b	2c	2a	2b	3a	3b	2a	2b	3a	3b	2b	2c	3b	3c	2b	2c	3b	3c

`im2col.shape` = $C_{\text{in}} \cdot H_{\text{in}} \cdot W_{\text{in}}$**-by-**$P \cdot C_{\text{in}} \cdot K^2$

For 1st convolution layer in LeNet5.

- $H_{\text{in}} = W_{\text{in}} = 32$
- $C_{\text{in}} = 1$
- $K = 5$
- $S = 1$
- $H_{\text{out}} = H_{\text{in}} - K + 1 = 28$
- $W_{\text{out}} = W_{\text{in}} - K + 1 = 28$
- $P = 28 * 28$ × 25
- Final shape of `im2col` = 1024 -by- 19600
- 20,070,400 float64's to store ≈ 160 MB

$(32-5+1) = 28 = H_{out} = W_{out}$

not very scalable

Sparse matrices

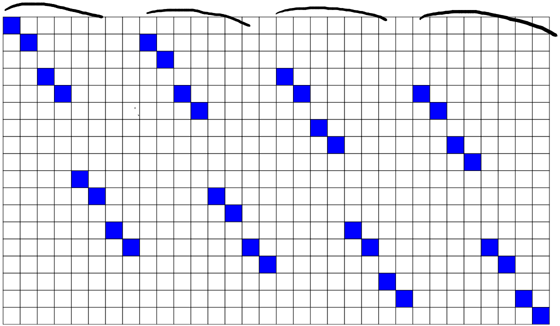
Pros everything else assume to zero

- Store just the non-zero values of the matrix (memory efficient)
- Faster than dense matmul when matrices are mostly zeros (speedier)

Cons

- Irregular memory access (less cache-optimization friendly)
- Nightmare for GPUs (as of now, to the best of my knowledge)

from scipy.sparse import csr_matrix



```
data = [1,1,1,1,1,...] # len(data) = P * patch_size
row_indices = [0,1,3,4,9,...]
col_indices = [0,1,2,3,4,...]
im2col_mat_sparse = csr_matrix((data,
                               (row_indices, col_indices)),
                              shape=(n_rows, n_cols))
```

Example

```
data = [1,2,3,1]
row_indices = [1,2,0,1]
col_indices = [0,1,2,1]
sparse_mat_example = csr_matrix((data, (row_indices, col_indices)),
    shape=(3, 3))
sparse_mat_example.toarray()
```

converts back to numpy

Output:

```
array([[0, 0, 3],
       [1, 1, 0],
       [0, 2, 0]])
```

Dense-sparse multiplication

```
X = np.arange(9).reshape(3,3)
## this won't work
# np.matmul(X, sparse_mat_example)
## this works bc method resolution order
X                    @   sparse_mat_example
```

toarray()

```
array([[0, 1, 2],         array([[0, 0, 3],         array([[ 1,  5,  0],
       [3, 4, 5],    @           [1, 1, 0],    =           [ 4, 14,  9],
       [6, 7, 8]])               [0, 2, 0]])               [ 7, 23, 18]])
```

Implement `im2col_matrix_sparse`

- `lec09-in-class-ex2-sparse.ipynb`

modify im2col_matrix_dense

$$\underbrace{\text{Xin\_flat}}_{\text{dense}} \; @ \; \text{im2col\_mat} = \text{Xin\_patches\_flat}$$

Quick update

Can add/mul constants both on left and right now.

```python
        def __radd__(self, other):
            return self + other

        def __rmul__(self, other):
            return self * other

        def __rsub__(self, other):
            return (-self) + other

        def __rtruediv__(self, other):
            return ag.Tensor(other) / self
```

Quick update

np.moveaxis(X, s , d)   if s= -1 d= -2
≡ transpose

```
def moveaxis(input, source, destination):
    output = ag.Tensor(np.moveaxis(input.value, source, destination), inputs=[input], op="moveaxis")

    def _backward():
        input.grad += np.moveaxis(output.grad, source, destination)
        return None
    output._backward = _backward
    return output
```

ag. moveaxis

Enabling autograd for dense-sparse matmul

- Problem: our current autograd-enabled matmul only supports when both inputs are numpy arrays
- Observation: we don't need to track gradients on the im2col_mat
- Solution:

does not change in training

```
# [sp]arse [c]onstant (non-AG-enabled) [mat]rix [mul]tiplication
def spcmatmul(input, sparse_mat: csr_matrix):
    output = ag.Tensor(input.value @ sparse_mat,
                       inputs = [input],
                       op="spcmatmul")

    def _backward():
        input.grad += output.grad @ sparse_mat.T
        return None

    output._backward = _backward
    return output
```

✗ Xin @ im2col

✓ spcmatmul(Xin, im2col)

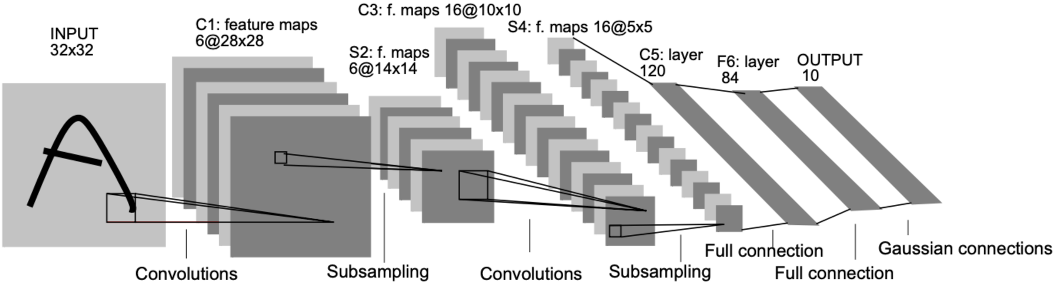
don't update sparse_mat grad

Exercise 3

lec 09_ data.csv ✓

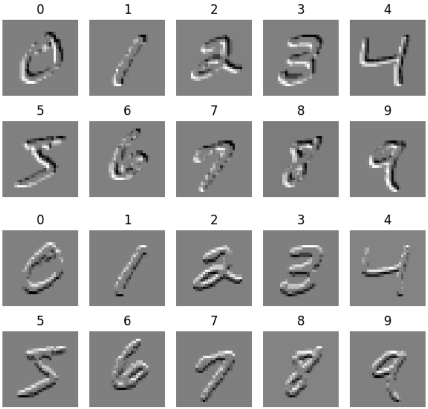
tiny_mnist

- Create the `Conv2d` layer
- Create the `AvgPool2d` layer
- Create the `LeNet5` model



- Create the `nn.CrossEntropyLoss`

Sanity check for conv2d



Channel 1

Channel 2

Hints:

moveaxis

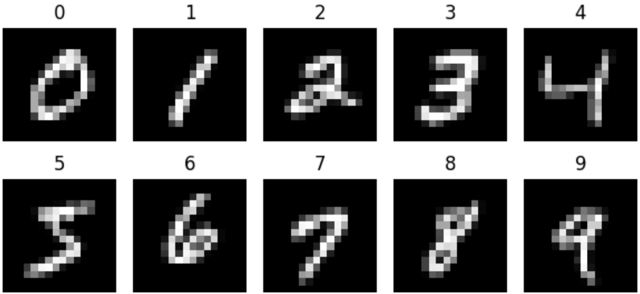
spcmatmul

from Ex1:

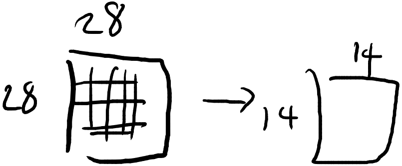
Sanity check for pool

Cout = 0 is the average of Cin = 0
and Cout = 1 Cin = 1
and so on.

$K = S$



Cin = 0
I just average the value in patch in Cin ⇝ Cout



like a conv 2d
except
$C_{in} = C_{out}$

Sanity check for training run

You should achieve 100 percentage training accuracy

References I

[LeC+98] Yann LeCun, Léon Bottou, Yoshua Bengio, and Patrick Haffner. "Gradient-based learning applied to document recognition". In: *Proceedings of the IEEE* 86.11 (1998), pp. 2278–2324.