3.3 Odometry-based motion model

November 12, 2020

1 3.3. Odometry-based motion model

Odometry can be defined as the sum of wheel encoder pulses (see Fig. 1) to compute the robot pose. In this way, most robot bases/platforms provide some form of *odometry information*, a measurement of how much the robot has moved in reality.

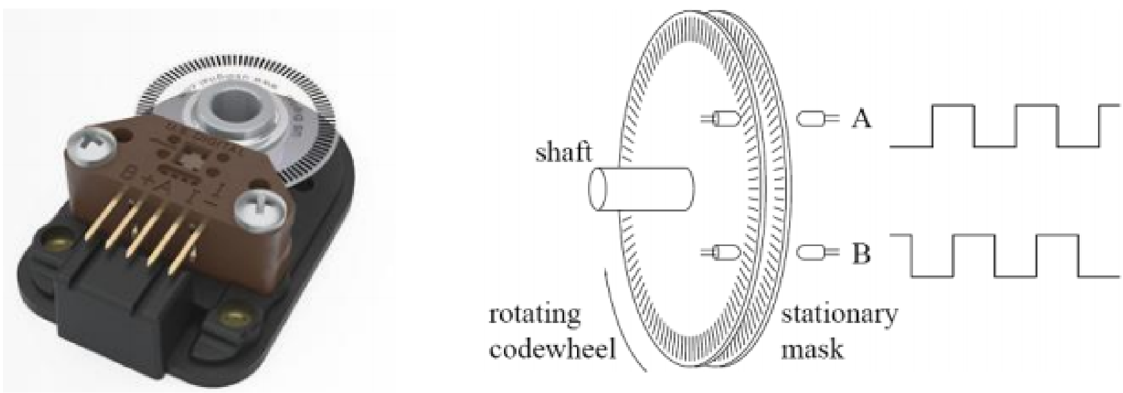


Fig. 1: Example of a wheel encoder used to sum pulses and compute the robot pose.

Such information is yielded by the firmware of the robotic base, which computes it at very high rate (*e.g.* at 100Hz) considering constant linear v_t and angular w_t velocities, and makes it available to the robot at lower rate (*e.g.* 10Hz) using a tool that we already know: the composition of poses.

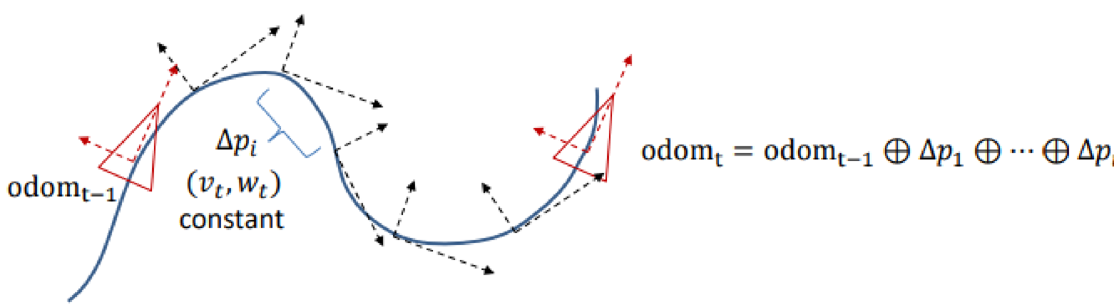


Fig. 2: Example of composition of poses based on odometry.

The **odometry motion model** consists of the utilization of such information that, although technically being a measurement rather than a control, will be treated as a control command to simplify the modeling. Thus, the odometry commands take the form of:

$$u_t = f(odom_t, odom_{t-1}) = \begin{bmatrix}\Delta x\\ \Delta y\\ \Delta\theta\end{bmatrix}$$

being $odom_t$ and $odom_{t-1}$ measurements taken as control and computed from the odometry at time instants t and $t-1$.

We will implement this motion model in two different forms: - Analytical form: $u_t = [\Delta x_t, \Delta y_t, \Delta\theta_t]^T$ - Sample form: $u_t = [\theta_1, d, \theta_2]^T$

In this way, the utilization of the odometry motion model is more suitable to keep track and estimate the robot pose in contrast to the *velocity model*.

[1]:
```
%matplotlib notebook

# IMPORTS
import numpy as np
from numpy import random
import matplotlib.pyplot as plt
from scipy import stats

import sys
sys.path.append("..")
from utils.DrawRobot import DrawRobot
from utils.PlotEllipse import PlotEllipse
from utils.pause import pause
from utils.Jacobians import J1, J2
from utils.tcomp import tcomp
```

1.1 3.3.1 Analytic form

Just as we did in chapter 3.1, the analytic form of the odometry motion model uses the composition of poses to model the robot's movement, providing only a notion of how much the pose has changed, not how did it get there.

As with the *velocity model*, the odometry one uses a gaussian distribution to represent the **robot pose**, so $x_t \sim (\overline{x}_t, \Sigma_{x_t})$, being its mean and covariance computed as:

- **Mean:**
$$\overline{x}_t = g(\overline{x}_{t-1}, \overline{u}_t) = \overline{x}_{t-1} \oplus \overline{u}_t$$

  where $u_t = [\Delta x_t, \Delta y_t, \Delta\theta_t]^T$, so:

$$g(\overline{x}_{t-1}, \overline{u}_t) = \begin{bmatrix} x_1 + \Delta x\cos\theta_1 - \Delta y\sin\theta_1 \\ y_1 + \Delta x\sin\theta_1 - \Delta y\cos\theta_1 \\ \theta_1 + \Delta\theta \end{bmatrix}$$

- **Covariance:** \$\$$\Sigma\partial g_{\overline{\partial x_{k-1}=}}$

$$\begin{bmatrix} 1 & 0 & -\Delta x_k \sin\theta_{k-1} - \Delta y_k \cos\theta_{k-1} \\ 0 & 1 & \Delta x_k \cos\theta_{k-1} - \Delta y_k \sin\theta_{k-1} \\ 0 & 0 & 1 \end{bmatrix}$$

$, , , , , , , , , , , , \partial g_{\overline{\partial u_k =}}$

$$\begin{bmatrix} \cos\theta_{k-1} & -\sin\theta_{k-1} & 0 \\ \sin\theta_{k-1} & \cos\theta_{k-1} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

\[10pt]

$andthecovariancematrixofthismovement($

Σ_{u_t}) is defined as seen below. Typically, it is constant during robot motion:

$\Sigma\_\{u\_t\} =$

$$\begin{bmatrix} \sigma^2_{\Delta x} & 0 & 0 \\ 0 & \sigma^2_{\Delta y} & 0 \\ 0 & 0 & \sigma^2_{\Delta\theta} \end{bmatrix}$$

$$

1.1.1 ASSIGNMENT 1: The model in action

Similarly to the assignment 3.1, we'll move a robot along a 8-by-8 square (in meters), in increments of 2m. In this case you have to complete:

- The `step()` method to compute:
  - the new expected pose (`self.pose`),
  - the new true pose x_t (ground-truth `self.true_pose`) after adding some noise using `stats.multivariate_normal.rvs()` to the movement command u according to `Q` (which represents Σ_{u_t}),
  - and to update the uncertainty about the robot position in `self.P` (covariance matrix Σ_{x_t}). Note that the methods `J1()` and `J2()` already implement $\partial g / \partial x_{t-1}$ and $\partial g / \partial u_t$ for you, you just have to call them with the right input parameters.
- The `draw()` method to plot:
  - the uncertainty of the pose as an ellipse centered at the expected pose, and
  - the true position (ground-truth).

We are going to consider the following motion covariance matrix (it is already coded for you):

$$\Sigma_{u_t} = \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

Example

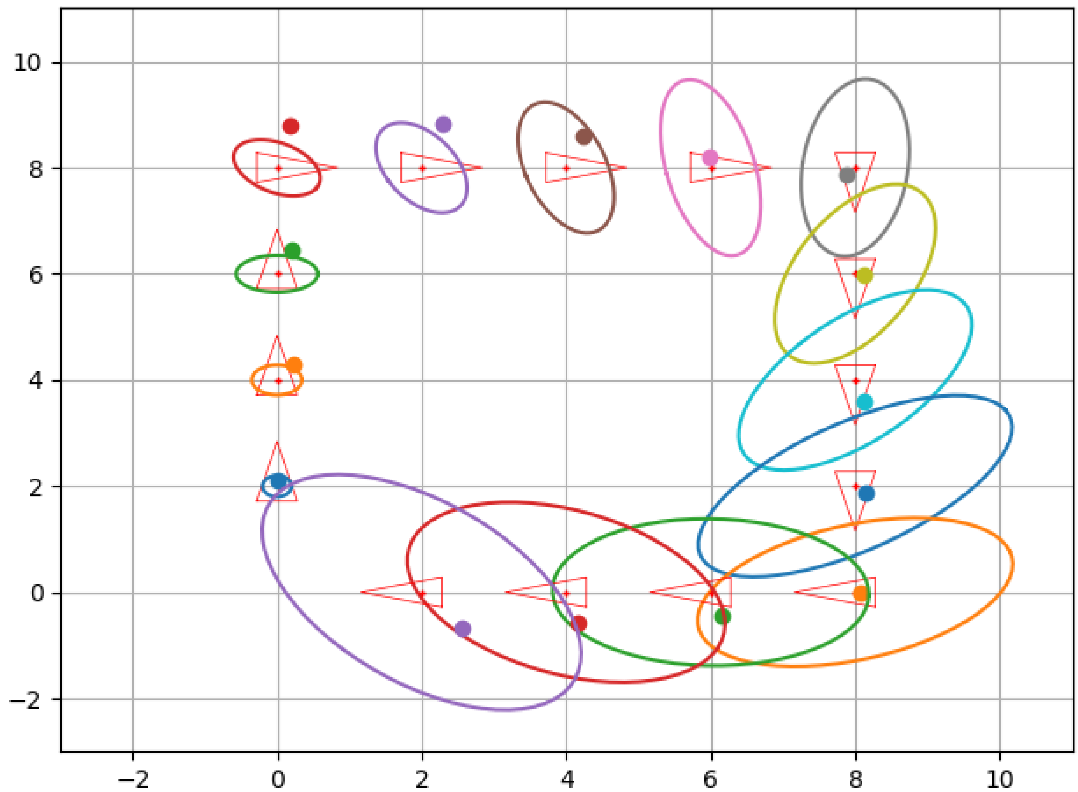


Fig. 2: Movement of a robot using odometry commands. Representing the expected pose (in red), the true pose (as dots) and the confidence ellipse.

[18]:
```
class Robot():
    """ Simulation of a robot base

        Attrs:
            pose: Expected pose of the robot
            P: Covariance of the current pose
            true_pose: Real pose of the robot(affected by noise)
            Q: Covariance of the movement
    """
    def __init__(self, x, P, Q):
        self.pose = x
        self.P = P
        self.true_pose = self.pose
        self.Q = Q

    def step(self, u):
        # TODO Update expected pose
        prev_pose = self.pose
        self.pose = tcomp(prev_pose, u)
```

```
        # TODO Generate true pose
        noisy_u = np.vstack(stats.multivariate_normal.rvs(mean=u.flatten(),␣
↪cov=self.Q))
        self.true_pose = tcomp(self.true_pose, noisy_u)

        ## Preguntar ##
        # TODO Update covariance
        JacF_x = J1(prev_pose, u)
        JacF_u = J2(prev_pose, u)

        self.P = (
            (JacF_x @ self.P @ JacF_x.T)
            + (JacF_u @ self.Q @ JacF_u.T)
        )


    def draw(self, fig, ax):
        DrawRobot(fig, ax, self.pose)
        el = PlotEllipse(fig, ax, self.pose, self.P)
        ax.plot(self.true_pose[0,0], self.true_pose[1,0], 'o', color=el[0].
↪get_color())
```

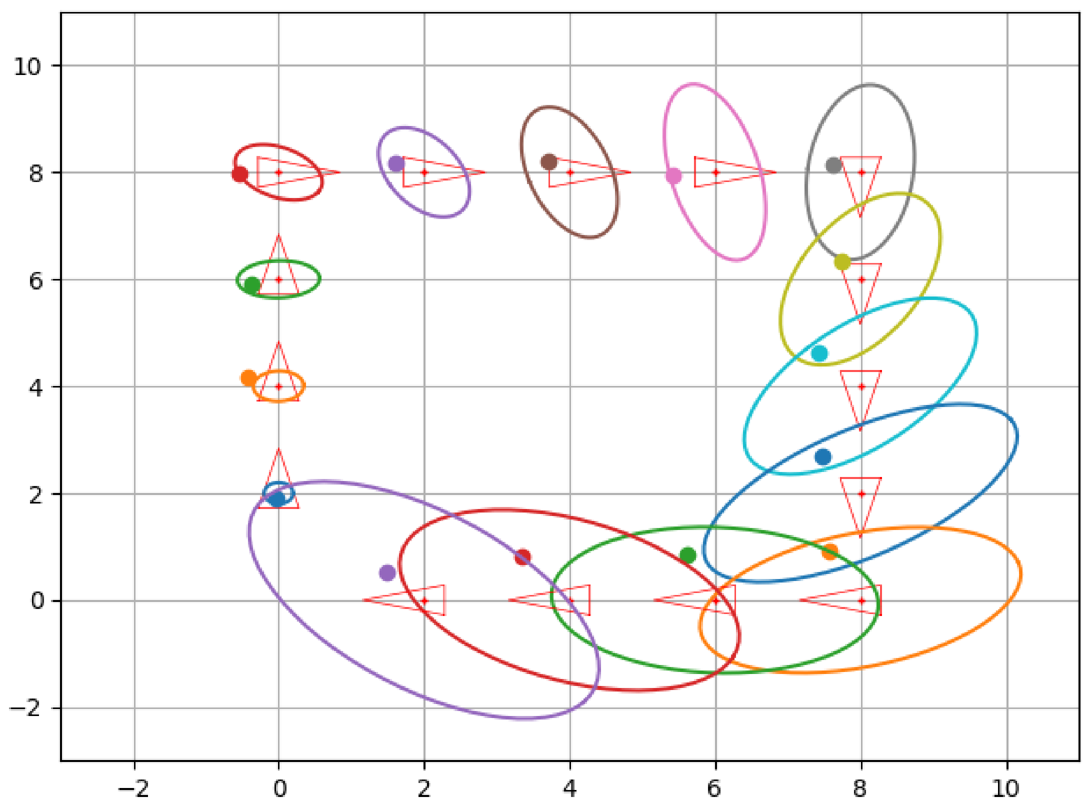
You can use the following demo to **try your new** `Robot()` **class**.

[3]:
```
def demo_odometry_commands_analytical(robot):
    # MATPLOTLIB
    fig, ax = plt.subplots()
    ax.set_xlim([-3, 11])
    ax.set_ylim([-3, 11])
    plt.ion()
    plt.grid()
    plt.tight_layout()
    fig.canvas.draw()

    # MOVEMENT PARAMETERS
    nSteps = 15
    ang = -np.pi/2 # angle to turn in corners
    u = np.vstack((2., 0., 0.))

    # MAIN LOOP
    for i in range(nSteps):
        # change angle on corners
        if i % 4 == 3:
            u[2, 0] = ang

        #Update positions
```

```
        robot.step(u)

        # Restore angle iff changed
        if i % 4 == 3:
            u[2, 0] = 0

        # Draw every loop
        robot.draw(fig, ax)
        fig.canvas.draw()
        plt.pause(0.3)
```

[19]:

```
## PREGUNTAR ##
x = np.vstack([0., 0., np.pi/2]) # pose inicial

# Probabilistic parameters
P = np.diag([0., 0., 0.])
Q = np.diag([0.04, 0.04, 0.01])

robot = Robot(x, P, Q)
demo_odometry_commands_analytical(robot)
```



1.1.2 Thinking about it (1)

Once you have completed this assignment regarding the analytical form of the odometry model, **answer the following questions**:

- Which is the difference between the $g(\cdot)$ function used here, and the one in the velocity model?

  In the Velocity Based motion model, we use the old pose and the velocity of the robot meanwhile in the Odometry Based motion model, we use the old pose and the movement we want to do.

- How many parameters compound the motion command u_t in this model?

  Three. X and Y coords and the angle.

- Which is the role of the Jacobians $\partial g / \partial x_{t-1}$ and $\partial g / \partial u_t$?

  Jacobians are used to obtain the new value of the covariance. (Noise propagated)

- What happens if you modify the covariance matrix Σ_{u_t} modeling the uncertainty in the motion command u_t? Try different values and discuss the results.

  If we increase the values of the covariance matrix, the real_pose will probably be more dispersed from the mean. If we decrease them, the real_pose will probably be near the mean.

1.2 3.3.2 Sample form

The analytical form used above, although useful for the probabilistic algorithms we will cover in this course, does not work well for sampling algorithms such as particle filters.

The reason being, if we generate random samples from the gaussian distributions as in the previous exercise, we will find some poses that are not feasible to the non-holonomic movement of a robot, i.e. they do not correspond to a velocity command (v, w) with noise.

The following *sample form* is a more realistic way to generate samples of the robot pose. In this case, the movement of the robot is modeled as a sequence of actions (see Fig 3):

1. **Turn** (θ_1): to face the destination point.
2. **Advance** (d): to arrive at the destination.
3. **Turn** (θ_2): to get to the desired angle.

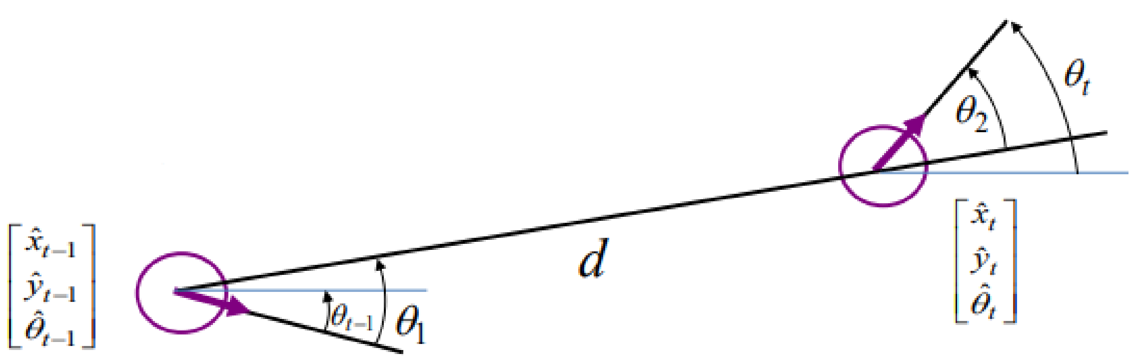


Fig. 3: Movement of a robot using odometry commands in sampling form.

So this type of order is expressed as:

$$u_t = \begin{bmatrix} \theta_1 \\ d \\ \theta_2 \end{bmatrix}$$

It can easily be generated from odometry poses $[\hat{x}_t, \hat{y}_t, \hat{\theta}_t]^T$ and $[\hat{x}_{t-1}, \hat{y}_{t-1}, \hat{\theta}_{t-1}]^T$ given the following equations:

$$\theta_1 = atan2(\hat{y}_t - \hat{y}_{t-1}, \hat{x}_t - \hat{x}_{t-1}) - \hat{\theta}_{t-1} d = \sqrt{(\hat{y}_t - \hat{y}_{t-1})^2 + (\hat{x}_t - \hat{x}_{t-1})^2} \theta_2 = \hat{\theta}_t - \hat{\theta}_{t-1} - \theta_1 \quad (1)$$

1.2.1 ASSIGNMENT 2: Implementing the sampling form

Complete the following cells to experience the motion of a robot using the sampling form of the odometry model. For that:

1. Implement a function that, given the previously mentioned $[\hat{x}_t, \hat{y}_t, \hat{\theta}_t]^T$ and $[\hat{x}_{t-1}, \hat{y}_{t-1}, \hat{\theta}_{t-1}]^T$ generates an order $u_t = [\theta_1, d, \theta_2]^T$

[5]:
```
def generate_move(pose_now, pose_old):
    diff = pose_now - pose_old
    theta1 = np.arctan2(pose_now[1]-pose_old[1], pose_now[0]-pose_old[0]) -
 ↪pose_old[2]
    d = np.sqrt((pose_now[1]-pose_old[1])**2 + (pose_now[0]-pose_old[0])**2)
    theta2 = pose_now[2] - pose_old[2] - theta1
    return np.vstack((theta1, d, theta2))
```

Try such function with the code cell below:

[6]:
```
generate_move(np.vstack([0., 0., 0.]), np.vstack([1., 1., np.pi/2]))
```

[6]:
```
array([[-3.92699082],
       [ 1.41421356],
       [ 2.35619449]])
```

Expected output for the commented example:

```
array([[-3.92699082],
    [ 1.41421356],
    [ 2.35619449]])
```

2. Using the resulting control action $u_t = [\hat{\theta}_1, \hat{d}, \hat{\theta}_2]^T$ we can model its noise in the following way:

$$\theta_1 = \hat{\theta}_1 + \text{sample}\left(\alpha_0 \hat{\theta}_1^2 + \alpha_1 \hat{d}^2\right) d = \hat{d} + \text{sample}\left(\alpha_2 \hat{d}^2 + \alpha_3 \left(\hat{\theta}_1^2 + \hat{d}^2\right)\right) \theta_2 = \hat{\theta}_2 + \text{sample}\left(\alpha_0 \hat{\theta}_2^2 + \alpha_1 \hat{d}^2\right) \quad (2)$$

Where $sample(b)$ generates a random value from a distribution $N(0, b)$. The vector $\alpha = [\alpha_0, \ldots, \alpha_3]$ (a in the code), models the robot's intrinsic noise.

The pose of the robot at the end of the movement is computed as follows:

$$x_t = x_{t-1} + d\cos(\theta_{t-1} + \theta_1)\, y_t = y_{t-1} + d\sin(\theta_{t-1} + \theta_1)\, \theta_t = \theta_{t-1} + \theta_1 + \theta_2 \tag{3}$$

Complete the `step()` and `draw()` methods to: - Update the expected robot pose (`self.pose`) and generate new samples. The number of samples is set by `n_samples`, and `self.samples` is in charge of storing such samples. Each sample can be interpreted as one possible pose reached by the robot. - Draw the true pose of the robot (without angle) as a cloud of particles (samples of possible points which the robot can be at). Play a bit with different values of `a`. To improve this visualization the robot will move in increments of 0.5 and we are going to plot the particles each 4 increments.

Example

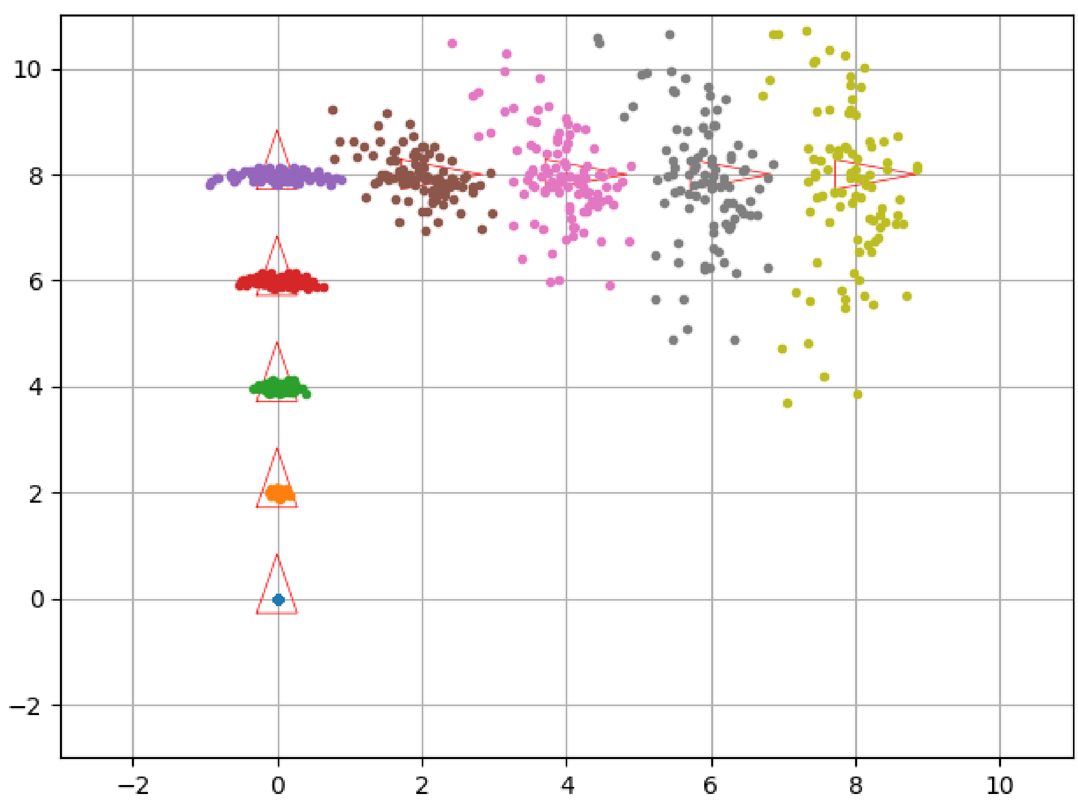


Fig. 1: Movement of a robot using odometry commands in sampling form. Representing the expected pose (in red) and the samples (as clouds of dots)

[7]:
```
class SampledRobot(object):
    def __init__(self, mean, a, n_samples):
```

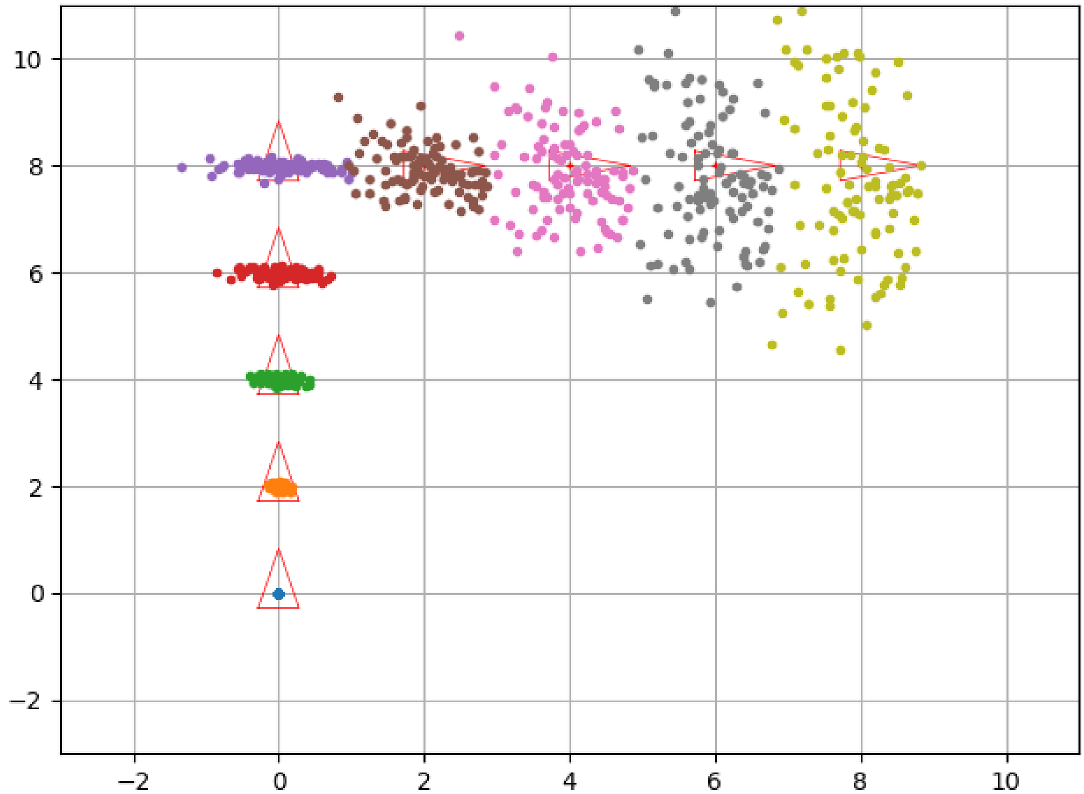
```
        self.pose = mean
        self.a = a
        self.samples = np.tile(mean, n_samples)

    def step(self, u):
        # TODO Update pose
        ang = self.pose[2, 0] + u[0, 0]
        self.pose[0, 0] += u[1, 0] * np.cos(ang)
        self.pose[1, 0] += u[1, 0] * np.sin(ang)
        self.pose[2, 0] = u[2, 0] + ang

        # TODO Generate new samples
        sample = lambda b: stats.norm(loc=0, scale=b).rvs(size=self.samples.
↪shape[1])

        u2 = u**2

        noisy_u = u + np.vstack((
            sample(self.a[0] * u2[0, 0] + self.a[1] * u2[1, 0]),
            sample(self.a[2]*u2[1, 0] + self.a[3] * (u2[0, 0] + u2[1, 0])),
            sample(self.a[0] * u2[2, 0] + self.a[1] * u2[1, 0])
        ))

        # TODO Update particles (robots) poses
        ang = self.samples[2, :] + noisy_u[0, :]

        self.samples[0, :] += noisy_u[1, :] * np.cos(ang)
        self.samples[1, :] += noisy_u[1, :] * np.sin(ang)
        self.samples[2, :] = noisy_u[2, :] + ang

    def draw(self, fig, ax):
        DrawRobot(fig, ax, self.pose)
        ax.plot(self.samples[0, :], self.samples[1, :], '.')
```

Run the following demo to **test your code**:

[8]:
```
def demo_odometry_commands_sample(robot):
    # PARAMETERS
    inc = .5
    show_each = 4
    limit_iterations = 32

    # MATPLOTLIB
    fig, ax = plt.subplots()
    ax.set_xlim([-3, 11])
    ax.set_ylim([-3, 11])
    plt.ion()
```

```
    plt.grid()
    plt.tight_layout()

    # MAIN LOOP
    robot.draw(fig, ax)
    inc_pose = np.vstack((0., inc, 0.))

    for i in range(limit_iterations):
        if i == 16:
            inc_pose[0, 0] = inc
            inc_pose[1, 0] = 0
            inc_pose[2, 0] = -np.pi/2

        u = generate_move(robot.pose+inc_pose, robot.pose)

        robot.step(u)

        if i == 16:
            inc_pose[2, 0] = 0

        if i % show_each == show_each-1:
            robot.draw(fig, ax)
            fig.canvas.draw()
            plt.pause(0.1)
```

[9]:
```
# RUN
n_particles = 100
a = np.array([.07, .07, .03, .05])
x = np.vstack((0., 0., np.pi/2))

robot = SampledRobot(x, a, n_particles)
demo_odometry_commands_sample(robot)
plt.close();
```



1.2.2 Thinking about it (2)

Now you are an expert in the sample form of the odometry motion model! **Answer the following questions**:

- Which is the effect of modifying the robot's intrinsic noise α (a in the code)?

  As more as we increase a value, more dispersed will the values be

- How many parameters compound the motion command u_t in this model?

  Three. Angle towards the destiny point, distance to destiny point and the desired angle

- After moving the robot a sufficient number of times, what shape does the distribution of samples take?

  It have a shape of a half ellipse