Ray-Cylinder Intersection

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We have a ray defined by his direction and his origin as such:

$$x(t) = x_0 + u_0 t$$
$$y(t) = y_0 + u_1 t$$
$$z(t) = z_0 + u_2 t$$

And the equation of a cylinder of radius r:

$$y^2 + z^2 = r^2$$

So we can just replace y and z in the cylinder equation by the ray equation:

$$(y_0 + u_1 t)^2 + (z_0 + u_2 t)^2 = r^2$$

And we get:

$$u_1^2 t^2 + 2u_1 y_0 t + y_0^2 + u_2^2 t^2 + 2u_2 z_0 t + z_0^2 - r^2 = 0$$

By grouping the terms we get:

$$(u_1^2 + u_2^2)t^2 + (2u_1 y_0 + 2u_2 z_0)t + (y_0^2 + z_0^2 - r^2) = 0$$

from here we can derive 3 parameters a, b and c:

$$a = u_1^2 + u_2^2$$
$$b = 2u_1 y_0 + 2u_2 z_0$$
$$c = y_0^2 + z_0^2 - r^2$$

So that we have the quadratic equation:

$$at^2 + bt + c = 0$$

That we can easily solve to get the intersection points:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

They are then 3 differents cases:

1. $b^2 - 4ac < 0$: no intersection
2. $b^2 - 4ac = 0$: one intersection
3. $b^2 - 4ac > 0$: two intersections

If we have two intersections, we can choose the one that is the closest to the origin of the ray.

One last remark, here we just check for a cylinder of infinite height, but we can easily add a check for the height of the cylinder after computing the intersection points and just keep the ones that are actually in the cylinder.