Ray-Cylinder Intersection

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First, we need to find a parameterization for every point X on a cylinder. We assume A is normalized. Using the right triangle formed by the center C, a point X on the cylinder and the projection P of C onto the axis passing through X :

$$r^2 = \|\vec{XC} - \vec{A}(\vec{XC} \cdot \vec{A})\|^2 \tag{1}$$

where $\vec{A}(\vec{XC} \cdot \vec{A})$ is the projection of $\vec{XC}$ onto $\vec{A}$

We can now modify this equation to obtain a quadratic equation in the following steps :

$$r^2 = \|(X - C) - \vec{A}((X - C) \cdot \vec{A})\|^2 \tag{2}$$

We introduce our ray $\vec{Ray}(t) = \vec{O} + t\vec{D}$

$$\begin{aligned} r^2 &= \|(\vec{O} + t\vec{D} - C) - \vec{A}((\vec{O} + t\vec{D} - C) \cdot \vec{A})\|^2 \\ &= \|(\vec{O} + t\vec{D} - C) - \vec{A}(\vec{A} \cdot \vec{OC} + t\vec{A} \cdot \vec{D})\|^2 \\ &= \|\vec{OC} - \vec{A}(\vec{A} \cdot \vec{OC}) + t(\vec{D} - \vec{A}(\vec{A} \cdot \vec{D})\|^2 \end{aligned} \tag{3}$$

By distributing the terms, we obtain a quadratic equation with the following coefficients :

$$\begin{aligned} a &= \|\vec{D} - \vec{A}(\vec{A} \cdot \vec{D})\|^2 \\ b &= 2(\vec{OC} - \vec{A}(\vec{A} \cdot \vec{OC}))(\vec{D} - \vec{A}(\vec{A} \cdot \vec{D})) \\ c &= \|\vec{OC} - \vec{A}(\vec{A} \cdot \vec{OC})\|^2 - r^2 \end{aligned} \tag{4}$$

So that we have the quadratic equation:

$$at^2 + bt + c = 0 \tag{5}$$

That we can easily solve to get the intersection points:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{6}$$

They are then 3 differents cases:

1. $b^2 - 4ac < 0$: no intersection
2. $b^2 - 4ac = 0$: one intersection
3. $b^2 - 4ac > 0$: two intersections

If we have two intersections, we can choose the one that is the closest to the origin of the ray. One last remark, here we just check for a cylinder of infinite height, but we can easily add a check for the height of the cylinder after computing the intersection points and just keep the ones that are actually in the cylinder.