221 Spiking Networks
Exercise 5: Autocorrelation and power

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1 Renewal process with increasing hazard

We return to the renewal process of Exercise 2 with linearly increasing hazard

$$\rho(t) = \frac{t}{a^2}$$

survivor function

$$s(t) = \exp\left[-\int_0^t \rho(t)\,dt\right] = \exp\left(-\frac{t^2}{2a^2}\right)$$

interval-distribution

$$f(t) = -\frac{ds(t)}{dt} = \frac{t}{a^2}\,\exp\left(-\frac{t^2}{2a^2}\right).$$

and average interval

$$\langle t \rangle = \sqrt{\frac{\pi}{2}}\,a$$

2 Numerical computation assignements

1. Choose a value of a such as to bring the spike rate to 100 Hz! Use the inverse survivor function to generate intervals t_{si} from uniformly distributed random numbers u:

$$t_{isi} = s^{-1}(u) = \sqrt{-2\,a^2\,\ln(u)}$$

   Obtain lists of spike times t_i from the cumulative sum of intervals t_{isi}!

2. Use the t_i to generate realisations of a zero-mean process $X(t)$ sampled at discrete intervals $\Delta t = 0.1\,ms$. To ensure $\langle X \rangle = 0$, assign to samples *with* spike values $x_1 = 1 - \alpha$ and to samples *without* spike values $x_0 = -\alpha$, where $\alpha = \nu\,\Delta t$. This serves to guarantee

$$\langle X \rangle = \nu\,\Delta t\,(1-\alpha) + (1 - \nu\,\Delta t)\,(-\alpha) = 0$$

3. Use Matlab function ‘fft’ to compute the Fourier transform $\tilde{X}(\omega)$. $\tilde{X}(\omega = 0)$ **should be zero!** Compute and plot the power spectral density

$$S(\omega) = \frac{1}{T}\,\tilde{X}(\omega)\,\tilde{X}^{*}(\omega)$$

4. Numerically compute the Fourier transform $\tilde{f}(\omega)$ of the analytical interval distribution $f(t)$.

$$f(t) = \Theta(t)\,\frac{t}{a^2}\,\exp\left(-\frac{t^2}{2\,a^2}\right), \qquad\qquad \tilde{f}(\omega) = ?$$

5. Numerically compute the Fourier transform $\tilde{C}_x(\omega)$ of the *auto-covariance* from the Wiener-Khinchin theorem

$$\tilde{C}_x(\omega) = \nu\, Re\left(\frac{1+\tilde{f}(\omega)}{1-\tilde{f}(\omega)}\right)$$

and compare with the empirical power spectral density computed above!