221 Spiking Networks
Exercise 6: Wiener process

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1 Langevin equation of Wiener process

A Wiener process is specified in terms of its Langevin equation

$$\tau \dot{x} = \mu + \sigma \sqrt{\tau}\, \xi(t)$$

where $\tau = 5\,ms$, $x_0 = -70\,mV$, $\mu = 20\,mV$, and $\sigma = 10\,mV$. White noise $\xi(t)$ assumes independent random values with

$$\langle \xi(t) \rangle = 0, \qquad \langle \xi(t)\, \xi(t') \rangle = \delta(t - t')$$

2 Assignments

1. Derive iterative updating rule from the Langevin equation, replacing $\xi(t)$ with $\frac{n(t)}{\sqrt{\Delta t}}$ where $n(t)$ are independent samples from a normal distribution ($n \in N(0,1)$). Simulate numerically several 100 realisations to an upper bound of $x_{max} = 150\,mV$.

2. Establish numerically and plot the *cumulative density* $P[x'(t) \leq x]$ at time $t = 5\,\tau$. Using results from the lecture, compute the density analytically and compare in the same plot!

3. For zero drift ($\mu = 0$), compute numerically the Fourier transform of your process (average over many realisations) and plot the spectral density! Using results from the lecture, compute the spectral density analytically and compare in the same plot!