Spiking Network
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Date: 5th January, 2019

Exercise 6: Wiener Process

Introduction

In a Wiener process defined by its Langevin equation $\tau\frac{dx}{dt} = \mu + \sigma\sqrt{\tau}\xi(t)$, which $\xi(t)$ denoted white noise, the property of the white noise is assumed to be $E(\xi(t)) = 0$ and $E(\xi(t)\xi(s)) = \delta(|t-s|)$.

In this assignment, the model described above was studied using MatLab. In particular, the empirical and theoretical cumulative densities, and those of power spectral densities were compared.

Method

In this exercise, the main script *SN_Exercise_6_LeePoShing.m* was divided into three parts. In the first part, realizations of the Wiener Process $X(t)$ (X_t in the code) was computed based on the above equation with parameter as followed: $\tau = 5\text{ms}$, $x_0 = -70\text{mV}$, $\mu = 20\text{mV}$, and $\sigma = 10\text{mV}$ (tau, x_0, mu, and rho in the code). To generate white noise $\xi(t)$ (xi_t in the code), random number $n(t)$ (n_t in the code) was drawn according to $n(t) \in N(0,1)$ and divided by $\sqrt{\Delta t}$ which $\Delta t = \tau/10$ (dt in the code). Then $x(t)$ (x_t in the code) was calculated as the Langevin equation and $X(t)$ as the cumulative sum of $x(t)$.

In the second part, the empirical and theoretical cumulative densities $P[x5\tau) \leq x]$ were compared. Using the *histogram* function in Matlab, the empirical density was extracted from the realization generated above. For the theoretical values, according to the following equation saying that the probability density $p(t,w)$ (p_t_w in the code) evolves as a Gaussian distribution, the cumulative value (P_t_w in the code) was calculated as the cumulative sum.

$$p(5\tau, w) = \frac{1}{\sqrt{2\pi\sigma^2(5\tau)}} e^{\frac{-(w-\mu(5\tau)-x_0)^2}{2\sigma^2(5\tau)}}$$

In the last part, the empirical and theoretical power spectral densities of processes without drift were compared. The empirical value for each process was calculated as $S(\omega) = \frac{1}{T}|\tilde{x}(\omega)|$, followed by average across process (S_omega in the code). According to the lecture, the theoretical value (S_theo in the code) should equal to $S(\omega)_{theoretical} = \frac{\sigma^2}{\tau}$.

Results & Discussion

Realizations developed with a generally increasing drift and dispersal

In *Fig. 1*, the 1500 realizations according to the parameter defined were shown. In general, all showed a general upward trend due the presence of μ. In addition, they became more dispersed along the time, as the variance increased in proportion of time.

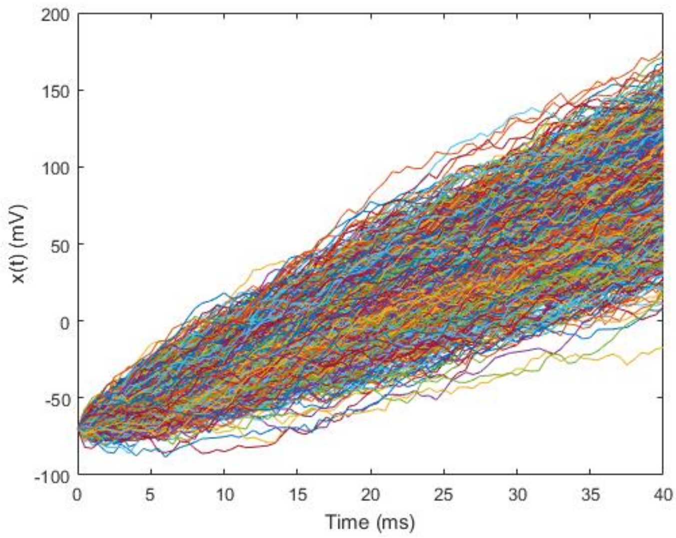

Fig. 1 Realizations developed with a generally increasing trend plus individual dispersal. In additional to the upward trend, the dispersal increased across the time.

Cumulative probability density from the realizations obeyed that of theoretical Gaussian distribution

In *Fig. 2*, the empirical and theoretical cumulative probability densities were compared. Clearly, the empirical value followed the same trend as the theoretical one based on the Gaussian distribution.

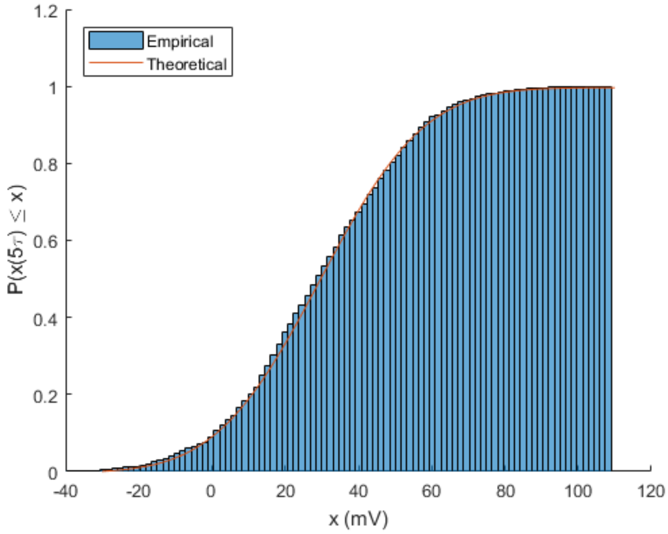

Fig. 2 Cumulative probability density from the empirical realization matched the theoretical distribution. In general, both showed a sigmoidal curve with an increasing value from x = -30 to 110mV.

Power spectral density calculated from point process of realizations had an average similar to the theoretical value

In *Fig. 3*, the empirical power spectral density calculated from the point processes, the average of it, and the theoretical value of the power spectral density were shown. As the processes generated were supposed to be white noise, the power spectrum should be a horizontal line across the frequencies as the yellow line shown in the graph, i.e. $S(\omega)$ is a constant. However, that coming from the average of different realizations showed a periodic pattern of peaks. This may due to the size of Δt. The average power value, as the red line in the graph, has a similar value to the theoretical one. Hence, the calculation should be correct. More testing is needed to see what caused the periodic pattern of the empirical pattern as increasing the number of realization did not eliminate it (Data not shown).

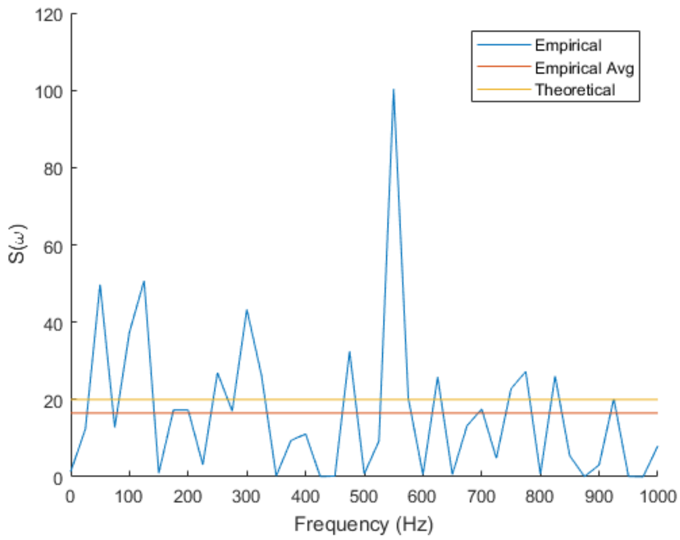

Fig. 3 Power spectral density of the empirical point process showed an average similar to the theoretical one. Despite the periodic pattern of the empirical value, the average value of the power spectral density was very close to the theoretical value.