221 Spiking Networks
Exercise 1: Random variables

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Binomial distribution (Papoulis 4-13)

A fair coin is tossed three times and the random variable $n \in \{0, 1, 2, 3\}$ equals the total number of heads.

Find $f(n)$ and $F(n)$ analytically.

Computer generate 1000 realizations to establish $f(n)$ and $F(n)$ empirically, and compare.

Moments of discrete Poisson distribution

Consider a discrete random variable $n \in \{0, 1, \ldots\}$ with the distribution

$$f(n) = \frac{\lambda^n}{n!} e^{-\lambda}$$

Starting from the Taylor expansion

$$e^{\lambda} = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$$

obtain its first and second derivatives with respect to λ and derive the first and second moments as well as the variance of n

$$E(n) \equiv \sum_{k=0}^{\infty} k\, f(k) \stackrel{!}{=} \lambda, \qquad E(n^2) \equiv \sum_{k=0}^{\infty} k^2\, f(k) \stackrel{!}{=} \lambda^2 + \lambda, \qquad E(n^2) - E^2(n) \stackrel{!}{=} \lambda$$