Spiking Network
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Exercise 5: Autocorrelation and Power

Introduction

In a renewal process with linearly increasing hazard$\rho(t) = \frac{t}{a^2}$, the survivor function, interval-distribution and average interval will be as followed:

$$S(t) = e^{-\int_0^t \rho(t)dt} = e^{-\frac{t^2}{2a^2}}$$
$$f(t) = -\frac{dS(t)}{dt} = \frac{t}{a^2} e^{-\frac{t^2}{2a^2}}$$
$$E(t) = \sqrt{\frac{\pi}{2}}a$$

In this assignment, the model described above was studied using MatLab. In particular, the power spectral density and the auto-covariance of the process were compared with reference to the Wiener-Khinchin theorem.

Method

In this exercise, the main script *SN_Exercise_5_LeePoShing.m* was divided into two parts. In the first part, a realization of the process was computed. a (also a in the code) was firstly calculated based on the average interval with the average spiking rate of $100Hz$, i.e. $E(t) = 0.01s$. After that, an array of uniformly distributed numbers $\in [0, 1]$ (S_t in the code) were drawn in order to generate inter-spike interval t_{isi} (t_isi in the code) using the inverse formula of survivor function, namely

$$t_{isi} = \sqrt{-2a^2 \ln S(t)}$$

With that, the spiking time t_i (t_i in the code) was calculated. Next, an array of time t (t in the code) was generated with the total length of the process t_{end} (t_end in the code) defined as the last t_{isi} and an interval Δt (dt in the code) of $0.0001s$. By comparing t_i and t_{isi}, an array of point process x_i (x_i in the code) was calculated with those appear in t_i assigned as $1 - \nu\Delta t$ while those not as $-\nu\Delta t$. By that, the process would be zero-mean on average. The realization was calculated and plotted. Lastly, the Fourier transform of the point process $\tilde{x}(\omega)$ (x_hat in the code) was calculated and thus the power spectral density $S(\omega)$ by the equation $S(\omega) = |\tilde{x}(\omega)|^2/T$. Since only the positive frequencies were concerned, the density was adjusted by multiplying 2 and plotted against the frequency ω (omega in the code). The realization $X(t)$ (X_t in the code) also plotted against time as a reference.

In the second part, the autocorrelation was studied and related to the power spectral density. With an array of time τ (tau in the code), the interval density $f(\tau)$ (f_tau in the code) was calculated by the formula

$$f(\tau) = \frac{\tau}{a^2} e^{\frac{-\tau^2}{sa^2}}$$

Then the Fourier transform of the density $\tilde{f}(\omega)$ (f_hat in the code) was calculated. With the equation

$$\tilde{C}(\omega) = \nu \, Re(\frac{1 + \tilde{f}(\omega)}{1 - \tilde{f}(\omega)})$$

the Fourier transform of the auto-covariance $\tilde{C}(\omega)$ (C_hat in the code) was calculated and plotted against the corresponding frequencies (C_omega in the code). The interval density was also plotted against time as reference.

Results & Discussion

Power spectral density of the point process shares the same trend as the Fourier transform of auto-covariance

In *Fig. 1*, the empirical power spectral density calculated using a randomly generated realization and the Fourier transform of the auto-covariance of spike calculated based on the Fourier transform of the theoretical inter-spike interval density were plotted separately in (a) and (b). Despite the noisy nature of the former graph, its shape resembled the latter one as a sigmoidal curve with an increasing value in higher frequencies. This agrees with the Wiener-Khinchin theorem that the power spectral density of a process equal to the auto-correlation of the process. The difference between the two graphs lies mainly on the value, which can be accounted as the square of the expected value of the spiking rate.

(a)

(b)

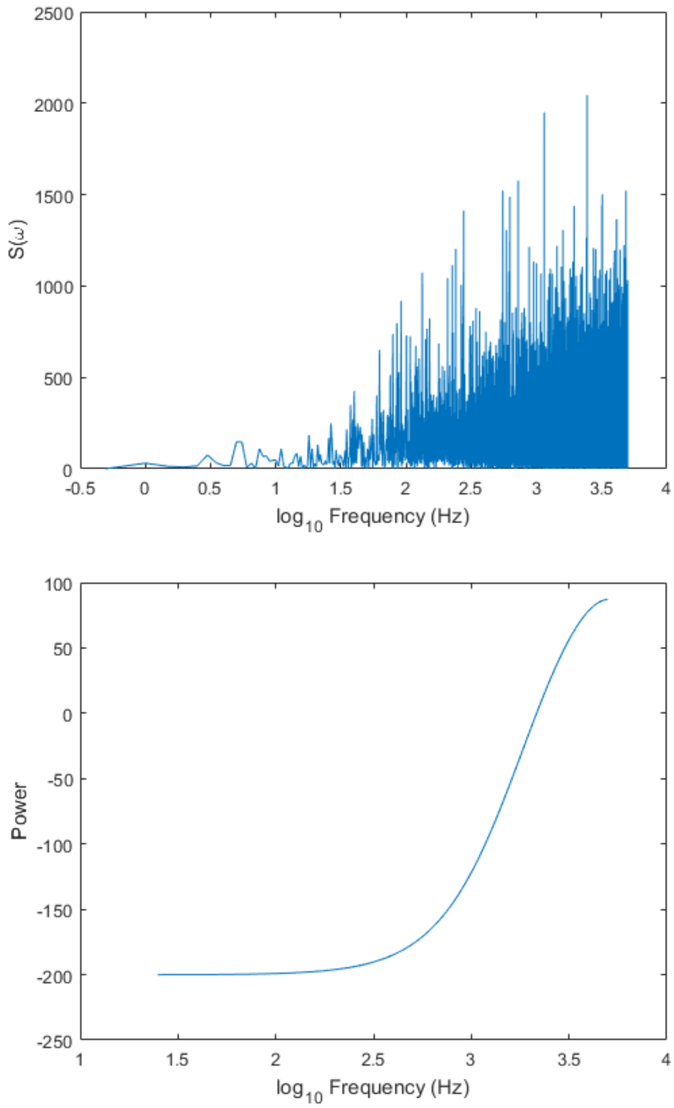

Fig. 1 Power spectral density mimicked the trend of Fourier transform of the auto-covariance of the point process. In (a), the power spectral density showed a sigmoidal trend with increasing value in the higher frequencies. The shape is rather noisy due to the randomness of the hazard. In (b), the Fourier transform of the auto-covariance also showed a sigmoidal sharp. Interestingly, the starting value is negative, which reflects the square of the expected spiking rate.