221 Spiking Networks
Exercise 4: State space of deterministic system

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1 A model for short-term memory

We model the interaction between an excitatory population E_1 and an inhibitory population E_2 with a deterministic (Wilson-Cowan) model.

When suitably configured, this system will retain an externally imposed activity state even after the external stimulus has ceased, by means of recurrent interactions. If activity starts high, it will remain high. If it starts low, it will remain low. The reason is that the system exhibits two distinct steady-states (attractor states).

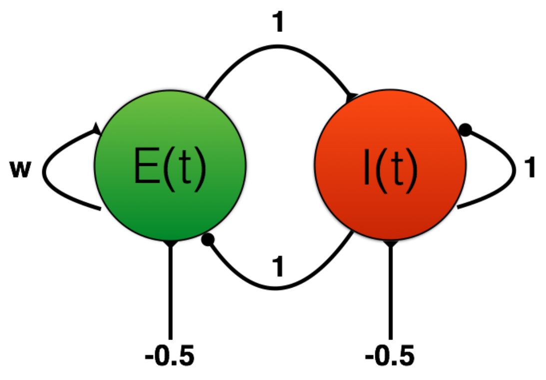
Wilson ("Spikes, decisions, actions", page 77) proposed this system as a model for short-term memory.

2 Deterministic system

The dynamic equations are

$$
\begin{aligned}
\tau \frac{dE_1}{dt} &= -E_1 + \Phi\left[w\, E_1 - E_2 - 0.5\right] \\
\tau \frac{dE_2}{dt} &= -E_2 + \Phi\left[E_1 - E_2 - 0.5\right] \\
\Phi(x) &= \frac{x^2}{\kappa^2 + x^2}
\end{aligned}
$$

where $\tau = 10$ and $\kappa = 0.75$. Note that this leaves w as a free parameter. Our aim is to understand how the dynamics changes with different values of $w \in [1, 4]$.



3 Simulation assignment

Simulate the time-evolution of the system by iterative integration. Visualize the time-evolution in the state space (E_1, E_2), starting from random initial points in the range $-0.2 \le E_{1,2} \le 1.4$. Assess the number and position of steady-states as a function of free parameter w. **Select two values of** w that provide one and two steady-states, respectively.

4 Analysis assignment

- Determine the isoclines $\frac{dE_1}{dt} = 0$ and $\frac{dE_2}{dt} = 0$ by solving

$$E_1 = \Phi\left[wE_1 - E_2 - 0.5\right]$$

  for E_2, given different values of E_1 in the range $0 < E_1 < 1$ and, similarly,

$$E_2 = \Phi\left[E_1 - E_2 - 0.5\right]$$

  for E_1, given different values of E_2 in the range $0 < E_2 < 1$.

- Determine the fixed points E_{ss} as the intersections of your isoclines.
- Plot isoclines and fixed-points in the state space (E_1, E_2), within the range $-0.2 \leq E_{1,2} \leq 1.4$.
- Repeat the analysis for different values of w. Plot steady-states E_{ss} as a function of w! This is called a 'bifurcation' diagram!

5 Simulation assignment

- Define auxiliary variables F_1 and F_2 and compute them as 2D functions of (E_1, E_2) for your two chosen values of w.

$$F_1 \equiv \tau \frac{dE_1}{dt}, \qquad F_2 \equiv \tau \frac{dE_2}{dt}$$

- Draw a dynamic "flow field" over the state space. To this end, compute the dynamic vectors (F_1, F_2) at selected positions (E_1, E_2). Give the list of positions and vectors to Matlab function **quiver** in order to plot. Add the isoclines and steady-states to your plot.
- Simulate the time-evolution of the system, starting in the vicinity of steady states (intersections of isoclines). Add trajectories from several initial conditions to the above plot.
- Do the trajectories evolve consistently with flow-field, isoclines, and steady-states?

6 Optional

- For each steady-state, compute the *Jacobian*

$$A = \begin{pmatrix} \frac{\delta F_1}{\delta E_1} & \frac{\delta F_1}{\delta E_2} \\ \\ \frac{\delta F_2}{\delta E_1} & \frac{\delta F_2}{\delta E_1} \end{pmatrix}$$

- Use the provided Matlab script **LinearOrder2.m** to compute the eigenvalues and eigenvectors of the Jacobian for each steady state and to determine the stability of each state.
- Compare with your simulation results. Does the formal analysis correctly predict whether or not a steady state is stable?