Spiking Networks
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# Exercise 1: Random Variables

**Binomial Distribution**

According to the formula $f(n) = \binom{N}{n} p^n (1-p)^{N-n}$ with $N = 3$ and $p = 0.5$, the values of $f(n)$ and $F(n)$ with $n \in \{0, 1,2,3\}$ is as followed:

| $n$ | $f(n)$ | $F(n)$ |
|---|---|---|
| 0 | 0.125 | 0.125 |
| 1 | 0.375 | 0.5 |
| 2 | 0.375 | 0.875 |
| 3 | 0.125 | 1 |

When 1000 realizations were generated to compute the above, the result was similar to the theoretical value calculated above (*Fig. 1*).

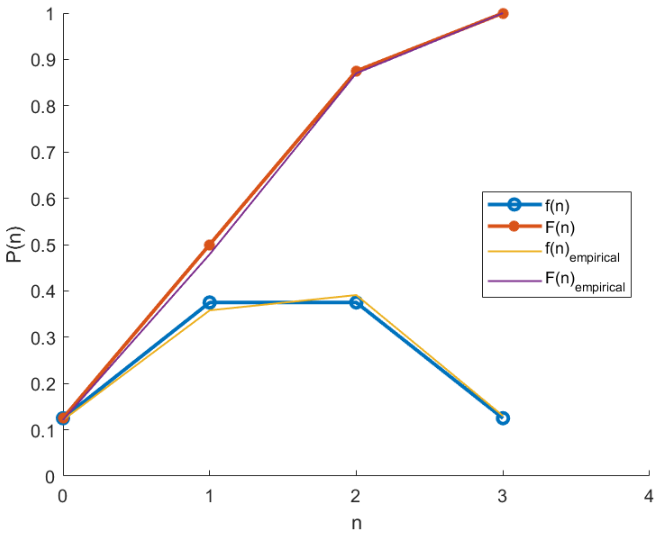


*Fig. 1 Experimental and theoretical values of probability and cumulative probability of binomial distribution with $N = 3$ and $p = 0.5$.* The lines with markers showed the theoretical values of either the probability distribution f(n) or the cumulative value F(n); whereas the ones without showed the experimental values. The differences between experimental and theoretical values were not pronounced.

**Moments of Discrete Poisson Distribution**

With $f(n) = \frac{\lambda^n}{n!} e^{-\lambda}$ for discrete Poisson distribution and the Taylor expansion $e^{\lambda} = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$, the first moment and the second moment will be

$$E(n) = \sum_{k=0}^{\infty} k f(k)$$

$$E(n) = \sum_{k=1}^{\infty} k \frac{\lambda^k}{k!} e^{-\lambda}$$

$$E(n) = e^{-\lambda} \sum_{k=1}^{\infty} k \frac{\lambda^k}{k!}$$

$$E(n) = e^{-\lambda} \lambda \sum_{k=1}^{\infty} \frac{\lambda^{(k-1)}}{(k-1)!}$$

$$E(n^2) = \sum_{k=0}^{\infty} k^2 f(k)$$

$$E(n^2) = \sum_{k=1}^{\infty} k^2 \frac{\lambda^k}{k!} e^{-\lambda}$$

$$E(n^2) = e^{-\lambda} \sum_{k=1}^{\infty} k^2 \frac{\lambda^k}{k!}$$

$$E(n^2) = e^{-\lambda} \lambda (\sum_{k=2}^{\infty} (k-1) \frac{\lambda^{(k-1)}}{(k-1)!} + \sum_{k=1}^{\infty} \frac{\lambda^{(k-1)}}{(k-1)!})$$

$$E(n) = e^{-\lambda}\lambda\sum_{k=0}^{\infty}\frac{\lambda^k}{k!}$$

$$E(n) = e^{-\lambda}\lambda e^{\lambda}$$

$$E(n) = \lambda$$

$$E(n^2) = e^{-\lambda}\lambda(\lambda\sum_{k=2}^{\infty}\frac{\lambda^{(k-2)}}{(k-2)!} + \sum_{k=0}^{\infty}\frac{\lambda^k}{k!})$$

$$E(n^2) = e^{-\lambda}\lambda(\lambda\sum_{k=0}^{\infty}\frac{\lambda^k}{k!} + e^{\lambda})$$

$$E(n^2) = e^{-\lambda}\lambda(\lambda e^{\lambda} + e^{\lambda})$$

$$E(n^2) = \lambda^2 + \lambda$$

For the variance, it will therefore be$E(n^2) - \left(E(n)\right)^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$.