Theoretical Neuroscience I
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Exercise 5: Operating Point of a Spiking Neuron

Introduction

In previous leaky integrate-and-fire model, the input current for simulating the generation of action potential was an artificial current. However, in reality the input of a neuron comes from thousands of other neurons, with some of them excitatory and some inhibitory. In general, a synaptic activity can be considered as a function of the conductances of different channels. Upon the release of neurotransmitter, channels open which causes the rise in conductance. Subsequently, the neurotransmitter will be removed, causing the closure of channels and thus the decrease in conductance. Therefore the current contributed by this synapse can be formulated as follow, where $\bar{g}_s$ is the maximum conductance, $P_s(t)$ is the time-course of fractional synaptic conductance and E_s is the reversal potential.

$$I_s(t) = \bar{g}_s P_s(t)(V_{rest} - E_s)$$

Assuming the responses for same kind of synapses are the same (i.e. no difference amongst all excitatory or inhibitory synapses), the dynamic equation for the membrane potential with multiple synapses can be formulated as:

$$\tau_m \frac{dV(t)}{dt} = -[V(t) - E_L] + r_m g_s [V(t) - E_s] \sum P_s(t)$$

For the synaptic time course $P_s(t)$, different functions can be used to approximate the dynamics. Amongst these functions, alpha function is one of the most simplified one that is suitable for us to understand the basic dynamics of a spiking neurons. Below is the form and the example figure of an alpha function with the time constant τ_s :

$$P_s(t) = \frac{t}{\tau_s} e^{(1-\frac{t}{\tau_s})}$$

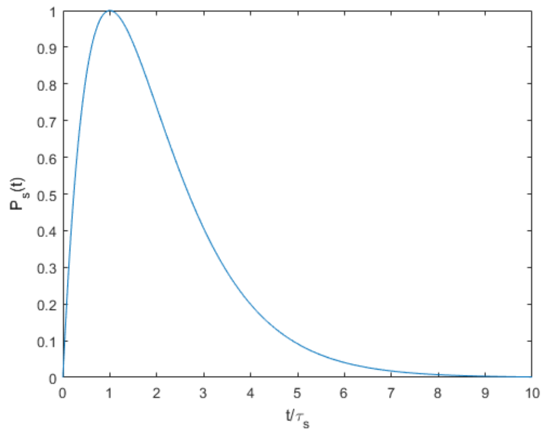

Fig. 1 Alpha function for $P_s(t)$ against normalised time Alpha function is one of the simplified functions that can be used to approximate the fractional changes in synaptic conductance. Notably, the maximum of 1 is reached when $t = \tau_s$.

However, in the case of multiple synapses, their alpha functions overlap each other depending on the spike train. Assuming the spiking events of different neurons are independent of each other, the resulting spike train approximates Poisson events. Below is an example of $P_s(t)$ with 5 synapses spiking at an average rate of 12*Hz* with time constant of 2*ms* across *t* = 0 to 100*ms*:

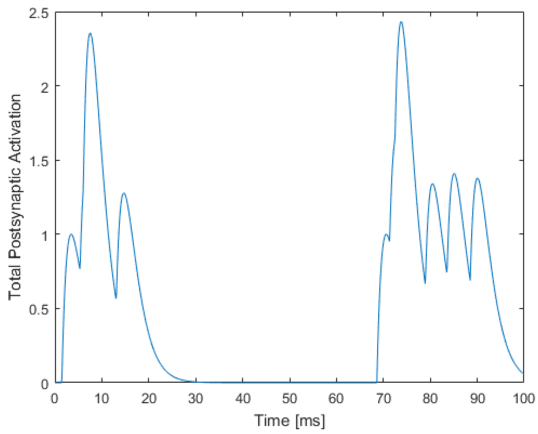

Fig. 1 Postsynaptic activation against time Noted that the spikes summated above the maximum of an individual spike (i.e. 1).

Considering a neuron as an information processor, one of its most important works is to give out information. As a result, if a neuron can spike with a higher degree of variance, it can provide a higher variety of information to its next unit. To measure this, one can bombard the neuron with random inputs and observe the variance of the inter-spike interval (t_{isi}). Coefficient of variation is one of the statistical measurements for irregularity and below is the calculation:

$$c_v = \frac{\sigma_{t_{isi}}}{\overline{t_{isi}}}$$

The variance of a neuron depends on three different factors, namely the number of excitatory synapses connected, the number of inhibitory synapses connected and the threshold of membrane potential for triggering action potential. When the maximum of variance is reached, it is defined as the operating point of a spiking neuron.

In this exercise, based on the LIF, the changes of membrane voltage with the spiking mechansim (V) and without it (V_{ns}) across the time were computed under different numbers of excitatory and inhibitory inputs. Corresponding v_{spk} and c_v of V in different cases were also analysed using Matlab.

Method

In this exercise, a function file named *spike.m* was created for the spiking pattern with inputs of time (*t* in the code), number of excitatory (*ex*) and inhibitory (*in*) (*N* in the code) and the threshold of membrane potential (V_{th}, *V_th* in the code). In the very beginning of the function, different basic parameters were input, including membrane capacity (c_m, also c_*m* in the code), maximal conductances of leaky conductance (*L*), *ex* and *in* ($\bar{g}$, also *g* in the code), corresponding reversal potentials (*E* in the code), time constant of *ex* and *in* (τ, also tau in the code), spiking rate of *ex* and *in* (v), and reset value of membrane potential (*V_reset* in the code).The values of these seven parameters are as below:

c_m = 10nF/mm^2; $\bar{g}_L$ = 1.2$\mu S/mm^2$; $\bar{g}_{ex}$ = 0.1$\mu S/mm^2$; $\bar{g}_K$ = 0.5$\mu S/mm^2$; v_{ex} = 20Hz; v_{in} = 20Hz

E_L = -65mV; E_{ex} = 0mV; E_{in} = -80mV; τ_{ex}= 1ms; τ_{in}= 2ms; V_{reset} = -70mV.

Since the aim of this exercise is to observe the changes of variables across the time, a time vector (*t* in the code) from 0 to 20000*ms* with step-size (*dt* in the code) of 0.1*ms* was generated. Only *dt* was included in the function as the size of time vector could then be flexibly changed to a smaller one of 0 to 2000*ms* during preliminary stage. Next, the vectors for fractional synaptic conductances (also known as synaptic time course, *P*) of *ex* and *in* were generated based on the v, τ, *N* (from input of function in main script) and *t* (also from input) using the function file *synaptic_activation.m*. In particular, in this file, *P* firstly generated random spike trains with average frequency of $v*N$. Then assuming the time course of a spike changed as an alpha function from t/τ = 0 to 10, the value from different spikes were summed at each time value. The equation of the alpha function is as followed

$$P_s(t) = \frac{t}{\tau_s} e^{(1-\frac{t}{\tau_s})}$$

Afterwards, a for-loop was used for the generation of dynamic variables, including the membrane voltage with a spiking mechanism (*V*) and that without the mechanism (V_{ns}, *V_ns* in the code). In each round of the for-loop, the effective time constant (τ_{eff}, also *tau_eff* in the code) and effective equilibrium state of membrane voltage (V_∞^{eff}, also *V_infty* in the code) were calculated based on the following equations:

$$\tau_{eff}(t) = \frac{c_m}{\bar{g}_L + \bar{g}_{ex}P_{ex}(t) + \bar{g}_{in}P_{in}(t)} \qquad V_\infty^{eff}(t) = \frac{\bar{g}_L E_L + \bar{g}_{ex}P_{ex}(t)E_{ex} + \bar{g}_{in}P_{in}(t)E_{in}}{\bar{g}_L + \bar{g}_{ex}P_{ex}(t) + \bar{g}_{in}P_{in}(t)}$$

Then, the new *V* and V_{ns} were calculated with the value of τ_{eff} and V_∞^{eff} as below:

$$V(i+1) = V_\infty^{eff}(t) + [V(i) - V_\infty^{eff}(t)]e^{\frac{-dt}{\tau_{eff}(t)}}$$

In addition, an if-statement was added after calculation of *V(i+1)* in the for-loop for the spiking mechanism. If the calculated value was larger than V_{th} (from input of function) the value would be reset to V_{reset}, i.e.

$$V(i+1) = V_{reset} \quad if\ V(i+1) \geq V_{th}$$

Correspondingly, the time *t(i+1)* was saved in a vector *t_i* in the code. This will be used to calculate the inter-spike time interval (t_{isi}, *t_isi* in the code), the corresponding coefficient of variance (c_v, *c_v* in the code), the average spiking frequency (v_{spk}, *v_spk* in the code), and the average V_{ns} at the end of the function.

In main script, different sets of *N* and V_{th} were entered by trial and error to find out the values that led to the following situations: (i) v_{spk} = 10, 30 or 70*Hz* with N_{in} = 0; (ii) v_{spk} = 10, 30 or 70*Hz* with N_{ex} = 100; and (iii) c_v is maximised with N_{ex} = 100. Each set of values were tested for five times to get the average results (±S.D.). The relationship between c_v and average V_{ns} in each situations and that and V_{th} in the last situation were also assessed.

Results & Discussion

Spikes generation with varying number of excitatory but without inhibitory synapses

As one can expect, v_{spk} increases when N_{ex} increases, since excitatory input serves as an inward current to the cell (Fig. 3). As in Table 1, with N_{ex} = 30, 40 or 56, v_{spk} = 9.56±0.44, 30.26±0.60 or 69.23±1.22*Hz*. In fact, the relationship between the two values is roughly linear, as one may simplify the excitatory input as a constant input current when the time frame is long enough. This was also shown in previous exercise on LIF. Representative figures for above N_{ex} with a shorter time frame of 2000*ms* were shown in Fig. 4.

In addition, with an increasing N_{ex}, c_v decreases while the average V_{ns} increases (Fig. 3). As in Table 1, with N_{ex} = 30, 40 or 56, c_v = 0.83±0.05, 0.66±0.02 or 0.50±0.01 whereas average V_{ns} = -57.30±0.06, -55.09±0.05 or -51.96±0.09*mV*. Again, the relationships between c_v and N_{ex}, and average V_{ns} and N_{ex} was in general linear.

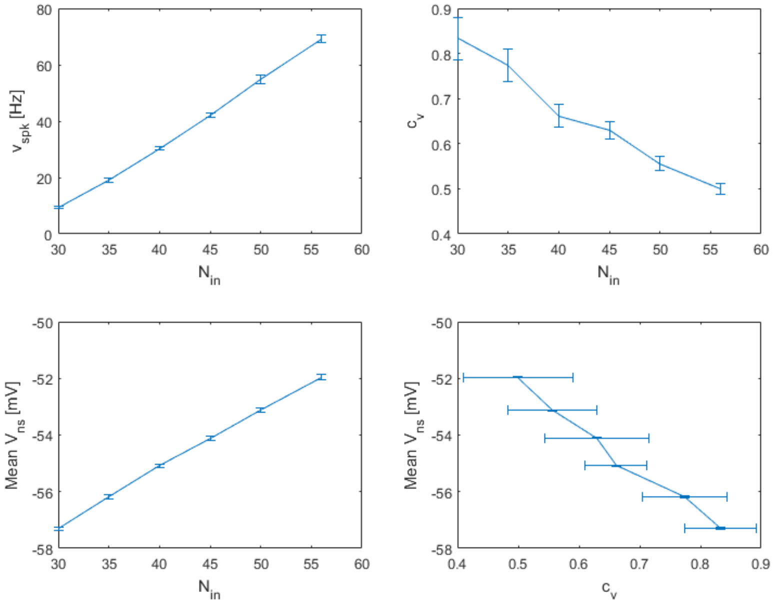

Fig. 2 Relationship amongst v_{spk}, c_v, average V_{ns} and the number of excitatory synapses without inhibitory input On the left panels, both v_{spk} and *average* V_{ns} increased linearly with increasing number of N_{ex}. On the upper-right panel, c_v decreased with increasing number of N_{ex}. On the lower-right panel, mean V_{ns} decreased when c_v increased.

N_{ex}	v_{spk} *(Hz)*	c_v	*Average* V_{ns} *(mV)*
30	9.56±0.44	0.83±0.05	-57.30±0.06
35	19.16±0.73	0.77±0.04	-56.18±0.07
40	30.26±0.60	0.66±0.02	-55.09±0.05
45	42.18±0.70	0.63±0.02	-54.11±0.09
50	54.63±1.54	0.56±0.02	-53.13±0.07
56	69.23±1.22	0.50±0.01	-51.96±0.09

Table 1 v_{spk}, c_v and average V_{ns} with varying number of excitatory but no inhibitory synapses With an increasing N_{ex}, v_{spk} *and average* V_{ns} increased, whereas c_v decreased.

Spikes generation with fixed number of excitatory and varying inhibitory synapses

As one can expect, v_{spk} decreases when N_{in} increases, since incitatory input serves as an outward current to the cell (Fig. 5). As in Table 2, with N_{in} = 13, 21 or 31, v_{spk} = 70.58±2.40, 32.39±1.67 or 10.10±0.55*Hz*. In fact, the relationship between the two values is roughly linear, as one may simplify the excitatory input as a constant input current when the time frame is long enough. This was also shown in previous exercise on LIF. Representative figures for above N_{in} with a shorter time frame of 2000*ms* were shown in Fig. 6.

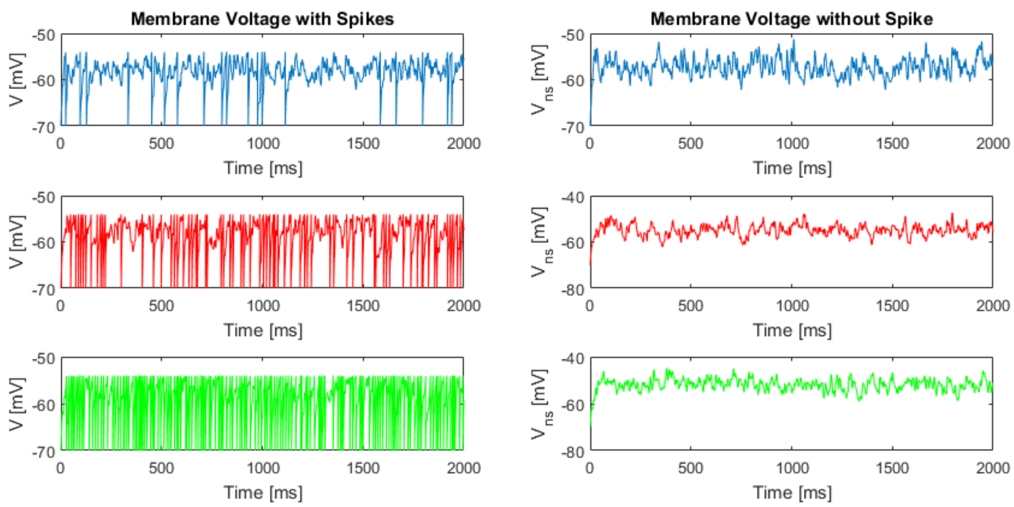

Fig. 3 Representative figures of membrane voltage with (left panels) and without (right panels) spikes across time with varying number of excitatory but without inhibitory synapses From up to down, different rods showed the corresponding V and V_{ns} of a neuron with 30, 40 and 56 excitatory synaptic inputs. With increasing number of N_{ex}, the spiking pattern became denser. The figures shown were from simulation with a time length of 2000ms.

In addition, with an increasing N_{in}, the average V_{ns} increases while c_v in general decreases (Fig. 5). As in Table 2, with N_{ex} = 13, 21 or 31, c_v = 0.84±0.03, 0.96±0.03 or 1.10±0.06 whereas average V_{ns} = -54.47±0.10, -58.13±0.09 or -61.57±0.06. But with by comparing more input options of N_{in}, c_v roughly showed a bell-shaped relationship with N_{in}. From Fig. 5, c_v was at maximum with 31 inhibitory inputs. Further analysis showed that it maximized with 34 inhibitory inputs (Fig. 7). The corresponding average V_{ns} was -62.45±0.08, which is around -8.5mV than the threshold. As defined, this is the operation point of this spiking neuron.

N_{in}	v_{spk} *(Hz)*	c_v	*Average* V_{ns} *(mV)*
13	70.58±2.40	0.84±0.03	-54.47±0.10
17	47.93±0.70	0.92±0.02	-56.67±0.07
21	32.39±1.67	0.96±0.03	-58.13±0.09
26	17.82±1.11	1.05±0.07	-59.98±0.09
31	10.10±0.55	1.10±0.06	-61.57±0.06
36	5.31±0.38	1.01±0.06	-62.85±0.06

Table 2 v_{spk}*,* c_v *and average* V_{ns} *with constant number of 100 excitatory varying inhibitory synapses* With an increasing N_{in}, v_{spk} *and average* V_{ns} decreased, whereas c_v increased, peaked at N_{in} = 31 and decreased.

Operation point of a spiking neuron

As mentioned in the introduction, a neuron served as a carrier of information. Therefore, if the variance of its spiking pattern is higher, the more types of information can be passed to its next unit. This is similar to the language ability of a person: the more words you know, the more information you can express to the others.

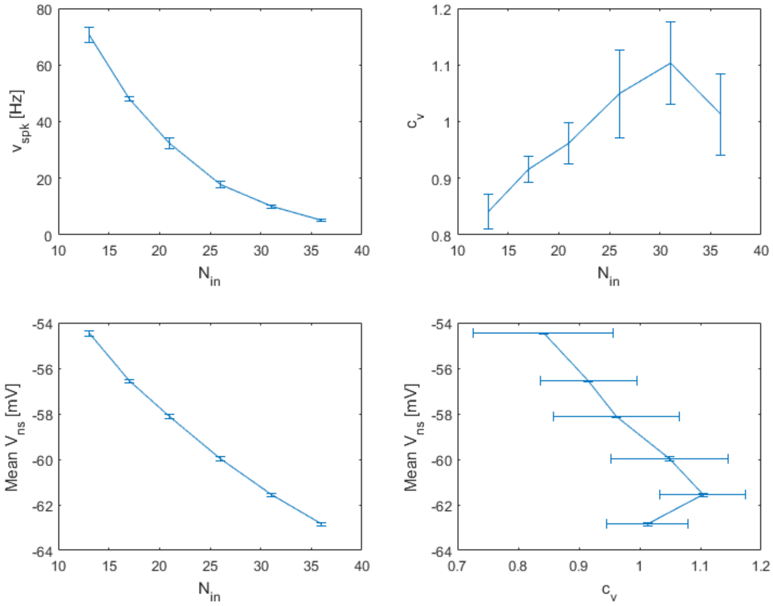

Fig. 4 Relationship amongst v_{spk}, c_v, average V_{ns} and the number of inhibitory synapses with 100 excitatory inputs On the left panels, both v_{spk} and *average* V_{ns} decreased with increasing number of N_{in}. On the upper-right panel, c_v increased, peaked at N_{in} = 31, and decreased with increasing number of N_{in}. On the lower-right panel, mean V_{ns} in general decreased when c_v increased.

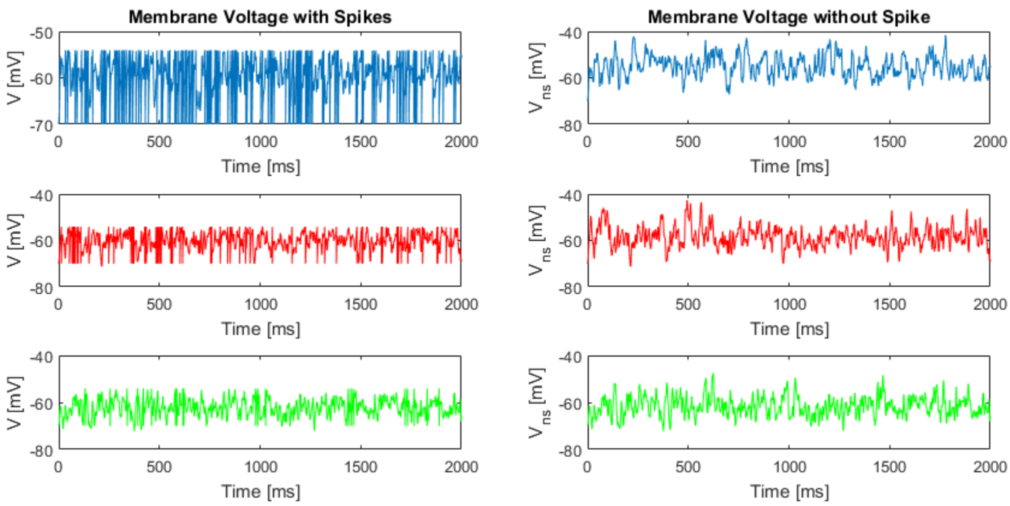

Fig. 5 Representative figures of membrane voltage with (left panels) and without (right panels) spikes across time with 100 excitatory and varying number of inhibitory synapses From up to down, different rods showed the corresponding V and V_{ns} of a neuron with 13, 21 and 31 inhibitory synaptic inputs. With increasing number of N_{in}, the spiking pattern became less dense. The figures shown were from simulation with a time length of 2000*ms*.

In order to find out the maximum amount of variance, a balance of excitatory input and inhibitory input plus a suitable threshold of membrane potential for action potential is required. This is understandable, as too much excitatory inputs will cause the neuron to spike very often; whereas too much inhibitory inputs will suppress the neuron to spike. In this exercise, with a fixed number of excitatory inputs, c_v for different number of inhibitory inputs were obtained (Fig. 4, Fig. 6, Table 2 & Table 3). It showed that, with 34 inhibitory inputs, the highest c_v can be 1.11±0.08, which is more than that of a Poisson distribution. However, since the S.D. is not small and also the graph for c_v against N_{in} is not smooth, it may require more trials to reach a more definite conclusion.

N_{in}	v_{spk} (Hz)	c_v	Average V_{ns} (mV)
26	17.78±1.27	1.02±0.08	-60.07±0.08
28	14.28±1.65	1.05±0.06	-60.67±0.09
30	10.36±0.48	1.04±0.07	-61.36±0.08
32	8.46±0.34	1.02±0.07	-61.85±0.09
34	6.03±0.29	1.11±0.08	-62.45±0.08
36	5.37±0.36	1.03±0.03	-62.85±0.07

Table 3 v_{spk}, c_v and average V_{ns} with constant number of 100 excitatory varying inhibitory synapses With a small increment of an increasing N_{in}, c_v peaked at N_{in} = 34.

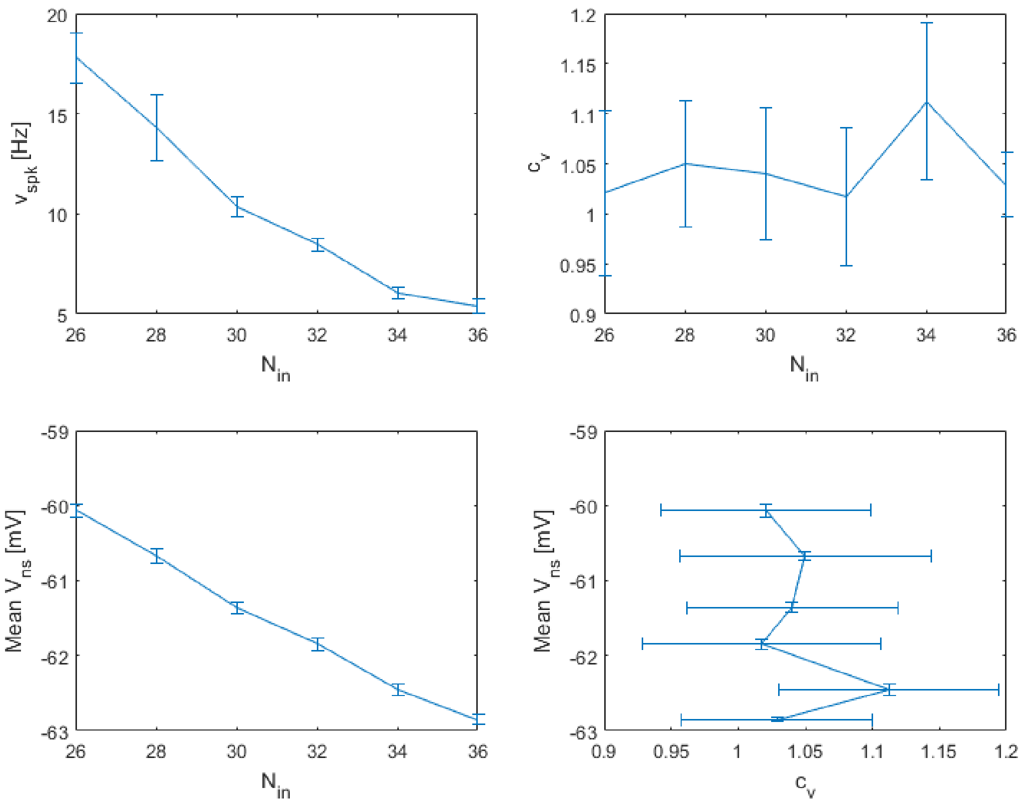

Fig. 6 Relationship amongst v_{spk}, c_v, average V_{ns} and the number of inhibitory synapses with 100 excitatory inputs Results are similar to Fig. 4. Notably, c_v maximized when N_{in} = 34.