Theoretical Neuroscience II
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Exercise 1: Linear Recurrent Network

Introduction

In a recurrent network, it can be considered as a group of neurons as Fig. 1, each of which receives input **h** (**u** in the figure) from somewhere and giving output to each other **v**. The synaptic strength for the interconnection between these neurons is denoted as **M**. Mathematically, it can be understood as the following equation:

$$\frac{d\boldsymbol{v}}{dt} = -\boldsymbol{v} + \boldsymbol{h} + \boldsymbol{M} \cdot \boldsymbol{v}$$

In this exercise, based on different equations, the properties of a recurrent network were studied with different constant input. In particular, the projection of different inputs (which was also the initial state), the corresponding steady-state activity on the vector space formed by the eigenvectors of the connection matrix.

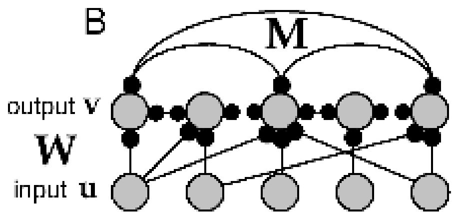

Fig. 1 Representation of an example of a recurrent network.

Method

In this exercise, the properties of a recurrent network were studied with a constant input. As required, the input vector (*h*, *h* also in the code) has to be in the form that, except two adjacent elements (1st and 20th were considered as adjacent) having the same value between 1 and 3, all others are equal to zero. To create such kind of input vectors, a function file named *inputvect.m* was created.

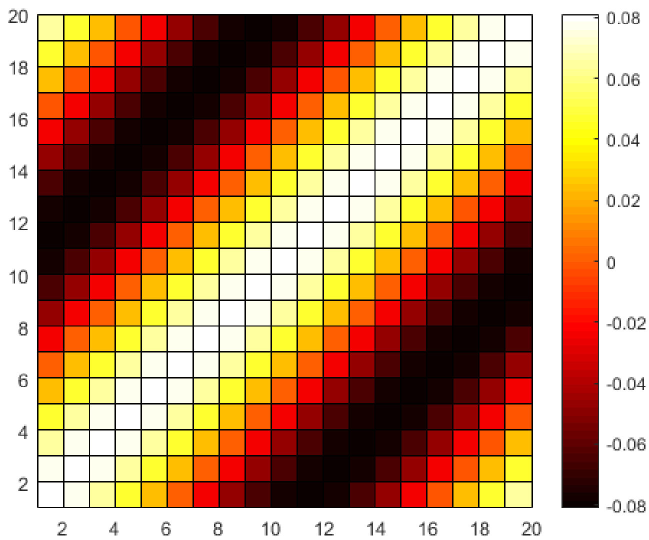

Fig. 2 False colour representation of the connection. As indicated in the graph, the synaptic weight is higher for the connection of a neuron with itself than the neighbouring cells.

In the main script, by inputting the size of population into a given function file *ConnectionMatrix.m*, the connectivity matrix (*M*, *M* also in the code; see Fig. 2), the first two principal eigenvectors (e_1 and e_2, *E_vect_1* and *E_vect_2 in the code*) and the corresponding eigenvalues (λ_1 and λ_2, *lambda_1* and *lambda_2*) were firstly extracted. Then e_1 and e_2 were plotted against the population index (*k* in the code) and also projected on the vector plane formed by e_1 and e_2 as the axis. To project a vector on this plane, the dot product of h with each eigenvector was done. It can be understood as the following equation:

$$v_0 = (h \cdot e_1)e_1 + (h \cdot e_2)e_2$$

This is also the same case as the two eigenvectors, as the dot product with each other is 0, whereas with itself is 1.

Different input vectors were then generated using *inputvect.m* and analysed. As all the analysis for every *h* was the same, a for-loop was used. Firstly, the input was projected on the e_1-e_2 plane, then using the equation

$$v_{ss} = \frac{h * e_1}{1 - \lambda_1} e_1 + \frac{h * e_2}{1 - \lambda_2} e_2$$

The steady-state activity (v_{ss}, *v_s_s* in the code) was derived and projected again on the e_1-e_2 plane. After that, the time evolution of the activity of the population (v_i, *v_t* in the code) was observed by using another for-loop and plotted on the e_1-e_2 plane. For each time point, the new v^{ss}_i was calculated as followed:

$$v_i^{ss} = M \cdot v_i + h$$

And the new activity would be

$$v_{i+1} = v_i^{ss} + (v_i - v_i^{ss})\exp(-\Delta t)$$

The time evolution was performed from t = 0 to 20 with intervals of 0.25. Lastly, with the last v_i calculated, it was plotted against the population and time to visually illustrate the change.

Results & Discussion

Two principal eigenvectors of the connection matrix showed sinusoidal patterns when plotted against population index

From the connectivity matrix ***M*** provided, two principal eigenvectors (e_1 and e_2) were determined, which provides information about the properties of the matrix. Both the eigenvectors had equal eigenvalues of 0.81057, which was quite close to 1, indicating that they both exerted equally significant amplification on the projection of *h* on the e_1 and e_2. The other eigenvectors were neglected because their eigenvalues were close to zero (Data not shown), and thus had less impact. e_1 *and* e_2, along with various input vectors were plotted as a function of the population index (Fig. 3). Both showed a sinusoidal pattern with e_1 having the peak at the 15th neuron while e_2 the 20th neuron.

Steady-state activity showed the same direction but with amplification as the input when projected on the e_1-e_2 plane

The input vectors *h* (also as the initial activity v_0), the steady-state response v_{ss} together with the activity vectors v_i across time were projected onto the vector plane formed by e_1 *and* e_2 (Fig.4). According to the two following equations,

$$v_0 = (h \cdot e_1)e_1 + (h \cdot e_2)e_2 \qquad\qquad v_{ss} = (\frac{h * e_1}{1-\lambda_1})e_1 + (\frac{h * e_2}{1-\lambda_2})e_2$$

the direction of the vectors are based on ratio between the coefficients before e_1 and before e_2 (i.e. the calculations in the bracket. Significantly, v_0 and v_{ss} had same direction in the eigenvector space, which means the projection of them on e_1 and on e_2 has the same ratio. This is because the λ_1 and λ_2 are equal. In addition, the factor $\frac{1}{1-\lambda}$ determines how much the input projection on eigenvector is amplified by recurrent dynamics. The closer the eigenvalue is to one, the larger is the amplification. In our case, the amplification was around 5.

The activity relaxed from the initial state to the steady state

As in Fig. 4, the cyan dots represented the time-progression of the activity for each input. The distance between the dots got smaller and smaller when the time moved from the initial to the steady state. This is because, as the following equation, the change followed a logarithmic pattern. Therefore the activity was relaxed towards the steady state.

$$v_{i+1} = v_i^{ss} + (v_i - v_i^{ss})\exp(-\Delta t)$$

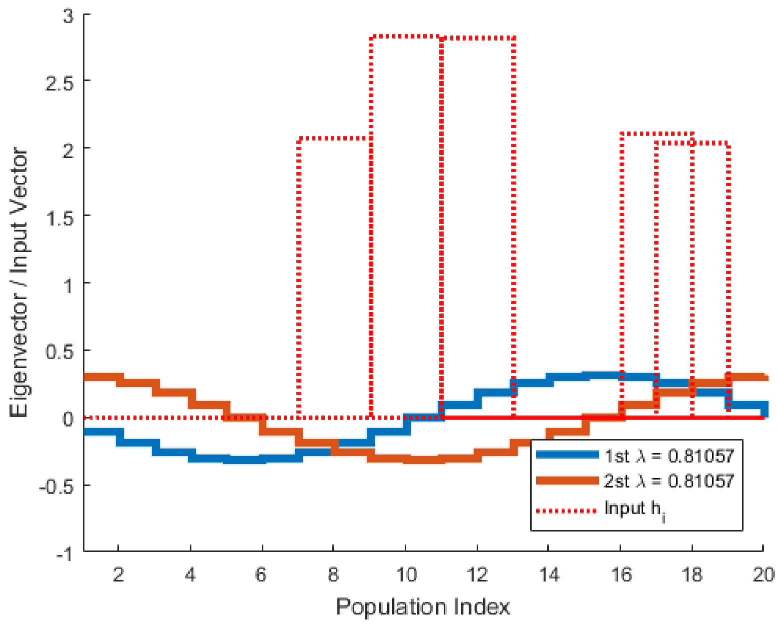

Fig. 3 The first two principal eigenvector e1 and e_2 along with five input vectors against the population index. Extracted from the connection matrix, the first two principal eigenvector (the blue and the orange curves) were plotted against the population index. Both showed a sinusoidal pattern, with the first showed the peak at the 15th neuron while the second the 20th neuron. The two eigenvectors are equally dominant. For each input *h*, the value and the position were randomly chosen.

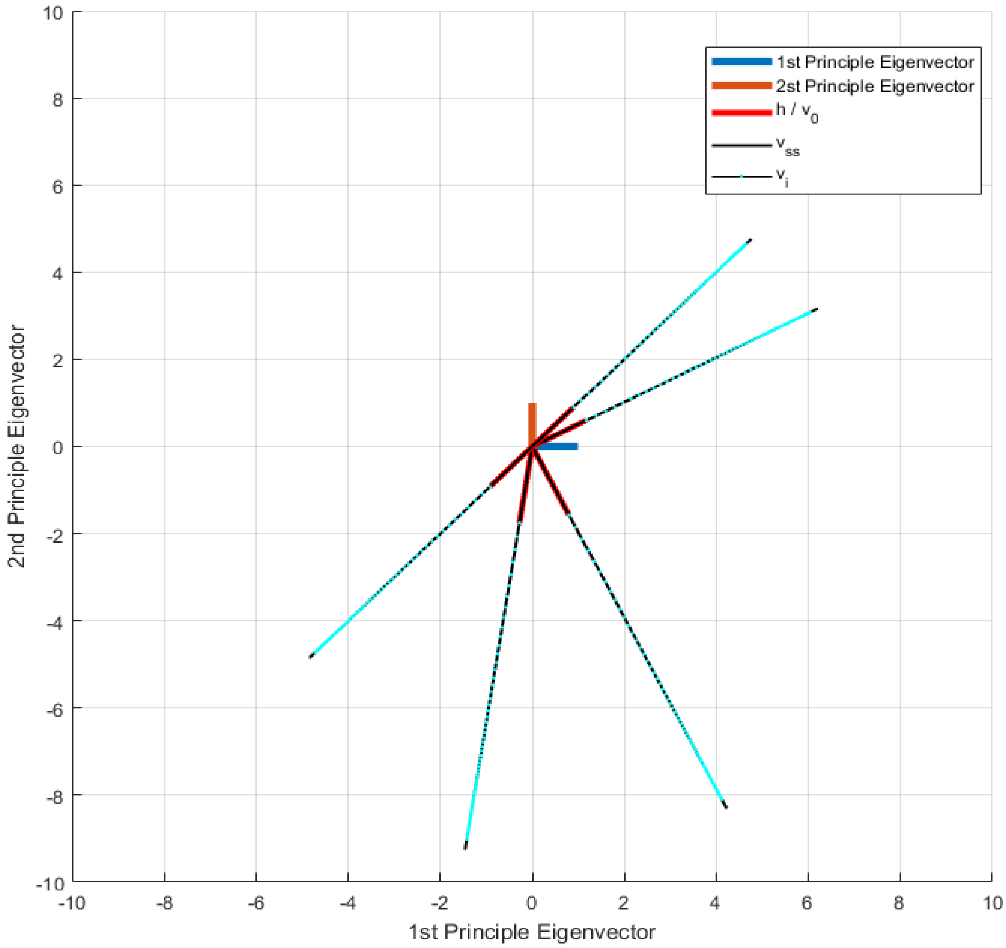

Fig. 4 Input vectors, the corresponding steady state activities and the time-development of the activity in the vector space of e_1-e_2. With e_1 and e_2 (blue and orange lines) forming a plane, the randomly generated h were projected onto it. With $h = v_0$ (red lines), v_{ss} (black lines) had the same direction as of its corresponding v_0 with an amplification of roughly 5 times. Time development of v_i displayed a relaxing manner as the cyan dots got closer and closer when it was approaching v_{ss}.

The steady state activity showed an interposed pattern of a sinusoidal curve and the input activity

When v_i was plotted against time and the population index, interestingly, v_{ss} showed a pattern similar to the summation of the input and a sinusoidal curve (Fig. 5). As in Fig. 5(a), at t = 0, the activity correctly reflected the pattern of the input, validating the programme. Interestingly, if one tried to project the vector to the e_1-e_2 plane first and calculated the v_i using the following equation from the lecture, one cannot get the initial state (Data not shown). This is because the two eigenvalues were not sufficient to provide all the information about the vector.

$$v(t) = c_1 e_1 + c_2 e_2,; \; c_i(t) = \frac{h \cdot e_i}{1 - \lambda_i}\left[1 - e^{\left(-\frac{1-\lambda_i}{\tau}t\right)}\right] + (v_0 \cdot e_i)e^{-\frac{1-\lambda_i}{\tau}t}$$

When the time progressed, a sinusoidal pattern interposed on the input, which is the same as e_1 and e_2. This is because these two eigenvectors were the first two principals and the dominating ones of the connection matrix, and therefore any input would be mainly projected on them. In other words, the input would be amplified based on that. For example, when an input of two pairs of adjacent neurons having a positive value was given, the steady-state activity was the interposed pattern of the input and the two sinusoidal waves with peaks at the neurons with input (Data not shown). This reflects one major feature of the connection matrix is to generate a steady-state pattern of sinusoidal waves with the peak at the neurons with input.

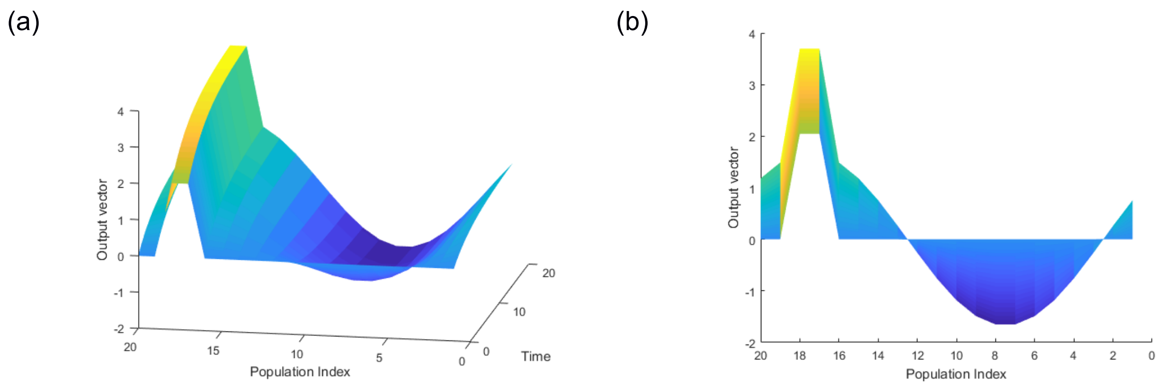

Fig. 5 Activity of the neurons along time showed a sinusoidal pattern similar to the e_1 and e_2 at steady state. When time progressed, the activity changed from the initial input pattern to a sinusoidal pattern with the peak at the position with positive input. The steady state activity resembled the eigenvectors with the peak shifted. (a) Slide view (b) Front view.