Theoretical Neuroscience I
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Exercise02: Single-compartment model

Introduction

The components of a neuron behave like electronic circuit components. With the phospholipid bilayer, the hydrophobic core acts as a barrier to prevent free exchange of ions between the extracellular matrix (ECM) and the cytosol. On the other hand, the channel proteins on the membrane allows specific ions flowing across the membrane. These components provided the necessary bases for the electrophysiology of the neuron.

In single-compartment model (SCM), a neuron is simplified as a compartment with a lipid membrane surrounded. The membrane is modelled as an electronic circuit for mimicking the architecture and the electrophysiological property of a neuron membrane (Fig. 1). Namely, the electrolytes in ECM and cytosol act as the short circuit end, while the flow of ions as current. Most importantly, the lipid bilayer acts as a capacitor while the channels act as a resistor, and they are connected in parallel.

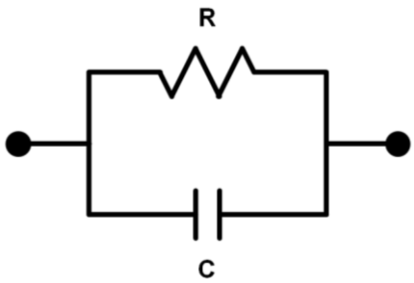

Fig. 1 Electrical Circuit used in Single-Compartment Model In SCM, the lipid bilayer is metaphorized as a capacitor (annotated as C), whereas the channels on the membrane are symbolized by a resistor (as R). The two components are connected in parallel.

In this exercise, based on the above construction, the change of membrane voltage (V_{mem}) across the time was computed under different current injected into the cell (I_e) using Matlab. In addition, the trend of current going through the capacitor (I_c) and the resistor (I_r), plus the equilibrium membrane voltage (V_{infty}) were also investigated.

Method

In this exercise, different vectors were generated for illustrating SCM. Since the focus is to observe the changes of current and voltage across the time, a time vector (*t* in the code) from 0ms to 250ms with step-size (*dt*) of 0.05ms was generated. Next, vectors for I_e (*Ie1*, *Ie2*, *Ie3* and *Ie4* in the code) were generated for four different situations: a constant current, a sinusoidal oscillating current with low frequency (10Hz), a sinusoidal oscillating current with high frequency (100Hz) and a random oscillating current with high frequency. The amplitude of the current (I_0) was set at 25nAmm^{-2}. The formula for the two sinusoidal currents were as followed:

$$I_e(t) = I_0 \cdot sin(2\pi f t)\,,\ \ f = frequency$$

For the voltage, the initial voltage (*V_0*) was set as 0mV. Based on the following equation:

$$\frac{dV_m(t)}{dt} = \frac{1}{\tau_m}[r_m \cdot i_e(t) - V_m(t)]$$

the four vectors for V_{mem} (*Vm1*, *Vm2*, *Vm3* and *Vm4* in the code) were generated using for-loop function in Matlab by:

$$V_m(i+1) = Vm(i) \cdot \left(1 - \frac{dt}{\tau_m}\right) + r_m \cdot i_e(t)\frac{dt}{\tau_m}$$

In the above, the resistance (r_m) and capacitance (c_m) of the membrane were defined as 0.9MΩmm^2 and 12nFmm^{-2}. The time-constant of membrane (τ_m) was therefore 10.8ms based on the following relationship:

$$\tau_m = r_m \cdot c_m$$

In addition, three vectors for I_c, I_r and V_{infty} were calculated using the following equations. The former two were plotted together with I_e and the latter one with V_{mem} in the respective cases of input current.

$$i_c(t) = c_m \cdot \frac{r_m \cdot i_e(t) - V_m(t)}{\tau_m}$$

$$i_r(t) = \frac{V_m(t)}{\tau_m}$$

$$V_{infty} = r_m \cdot i_e(t)$$

Results & Discussion

Membrane voltage does not respond immediately to the current input in SCM

In the first situation, the membrane voltage did not reach the equilibrium instantly when the constant current input injected (Fig. 2). According to the Ohm's Law, the relationship between current and voltage should be proportional. However, as in Fig. 2, the V_{mem} increased gradually as a logarithmical curve until it reached the equilibrium V_{infty}. This also confirmed that the membrane does not act simply as a resistor. Another component should be in the circuit.

In SCM, the channel acts as the resistor while the membrane as a capacitor connected in parallel. This is the reason why the membrane voltage did not respond immediately, as not all current injected went to the resistor. Instead, initially part of the current went to other components. Similar results were also observed in the sinusoidal currents (Fig. 3 & Fig. 4). A phrase delay was observed for V_{mem} than V_{infty}, in which the V_{infty} had the same phase of I_e.

Current passing through the resistor and the capacitor shows reversal change across the time

Current going through the resistor increased across the time, whereas that passing through the capacitor decreased (Fig. 2). Like the V_{mem}, I_r showed a logarithmical increase until it reached I_e. On the other hand, I_c decreased exponentially towards 0mV as if a mirror image of I_r. This is as expected. In SCM, when a current injected, the current will pass through the capacitor, as it has significantly low resistance compared with the resistor. However, when current keeps going to the capacitor, it will be gradually charged, and the charging force will counteract the flow of current. Therefore, more current will go through the resistor when the capacitor is keep charging. At the end, the charging force built up at capacitor will completely prevent the current passing through it. All the current injected will then pass through the resistor.

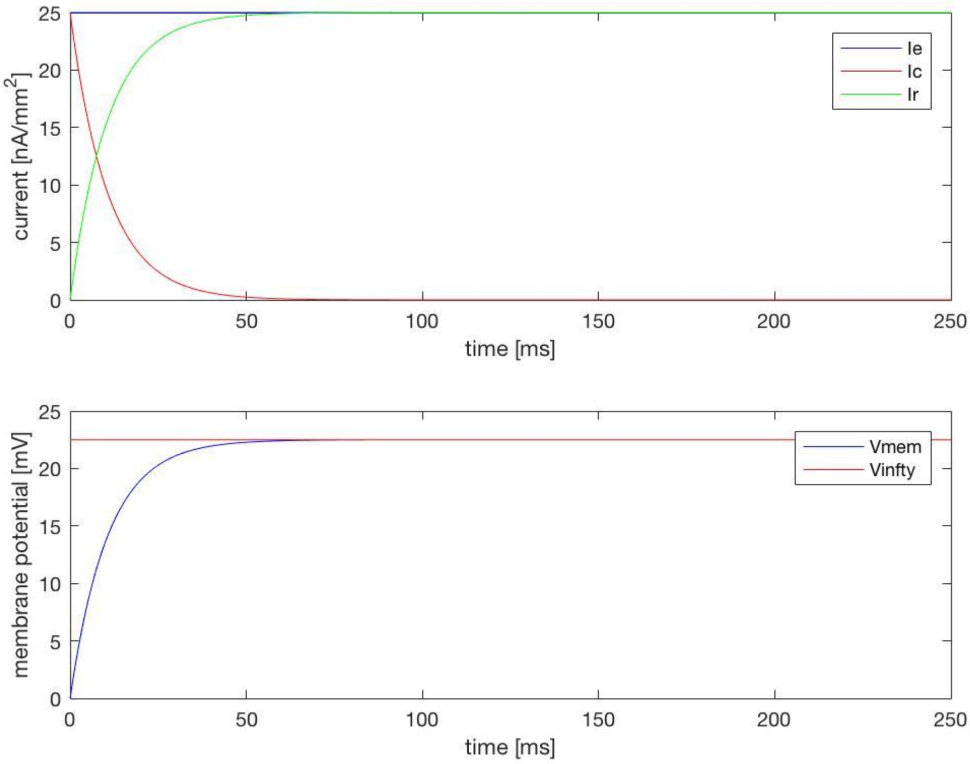

Fig. 2 Change of Current and Voltage across Time with a Constant Current Injected

Membrane acts as a low-pass filter to the current injected

For oscillating current, the amplitude of I_r was small when the frequency of oscillation was high; whereas it reached almost as I_e with a low frequency oscillation (Fig. 3, Fig. 4 & Fig. 5). In the 10Hz-situation, the oscillation of current injected in slow. For this reason, the de-charging of capacitor is also relatively slow, which allows the counteracting force to build up. As a result, current will be forced to go through the resistor, i.e. signals could pass through.

On the other hand, in the 100Hz-situation, the amplitude of current passing through the resistor was very small (Fig. 4). This is because the capacitor keeps charging and de-charging quickly in an alternating manner, which does not provide a prolong counteract effect against the current. Hence, most of the current injected went to the capacitor instead of the resistor, i.e. signals were filtered.

Similar phenomenon was also observed in the situation of random oscillation with high frequency (Fig. 5). The fluctuation of I_r was within a very narrow range, i.e. the signals were filtered.

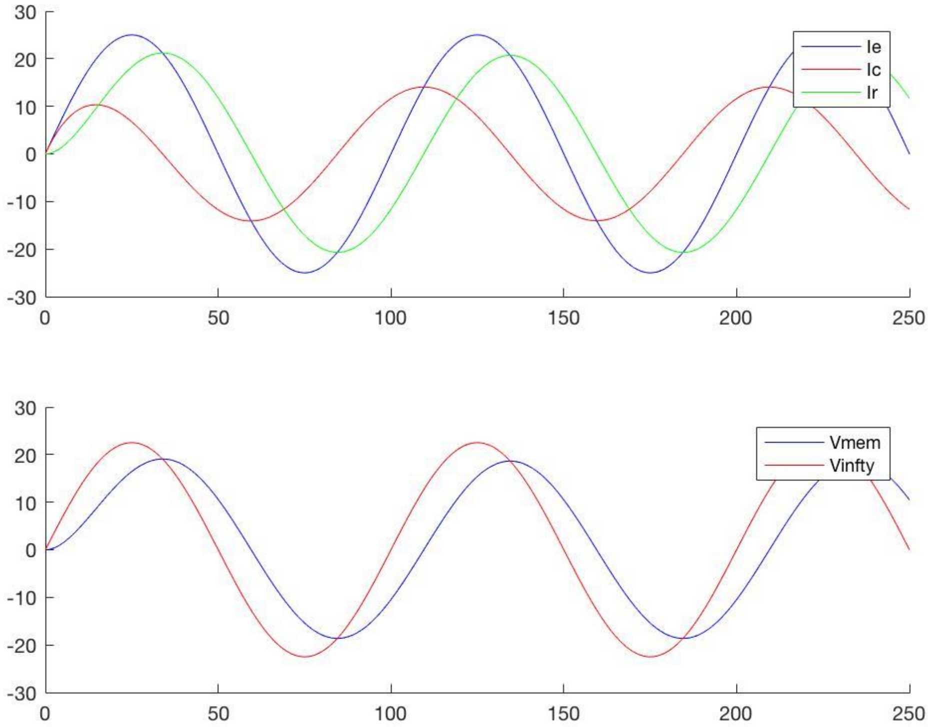

Fig. 3 Change of Current and Voltage across Time with a Sinusoidal Oscillation with Low Frequency 10Hz

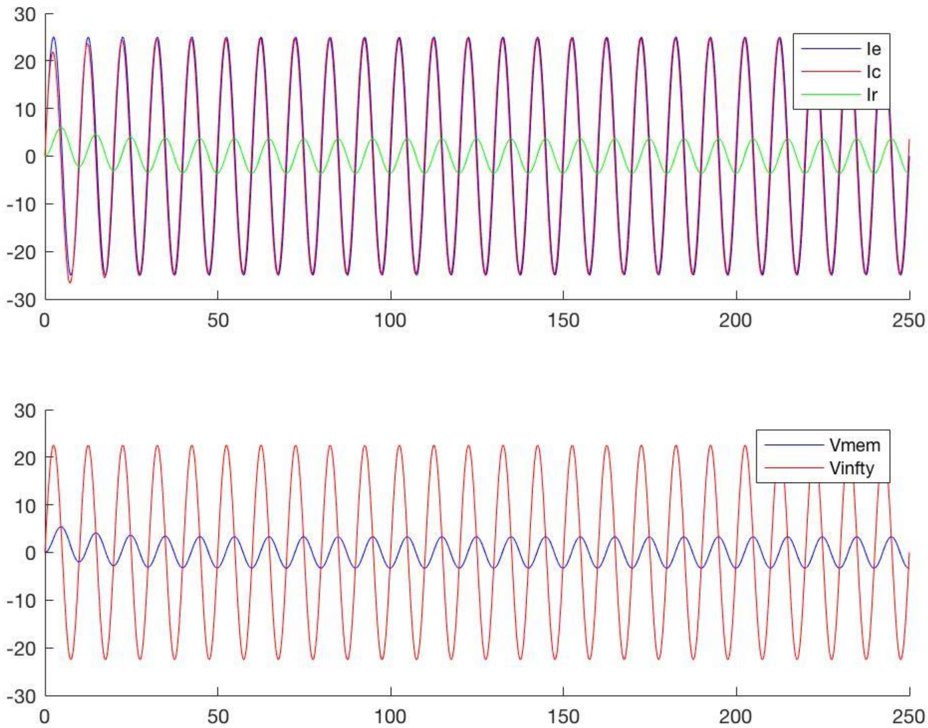

Fig. 4 Change of Current and Voltage across Time with a Sinusoidal Oscillation with High Frequency 100Hz

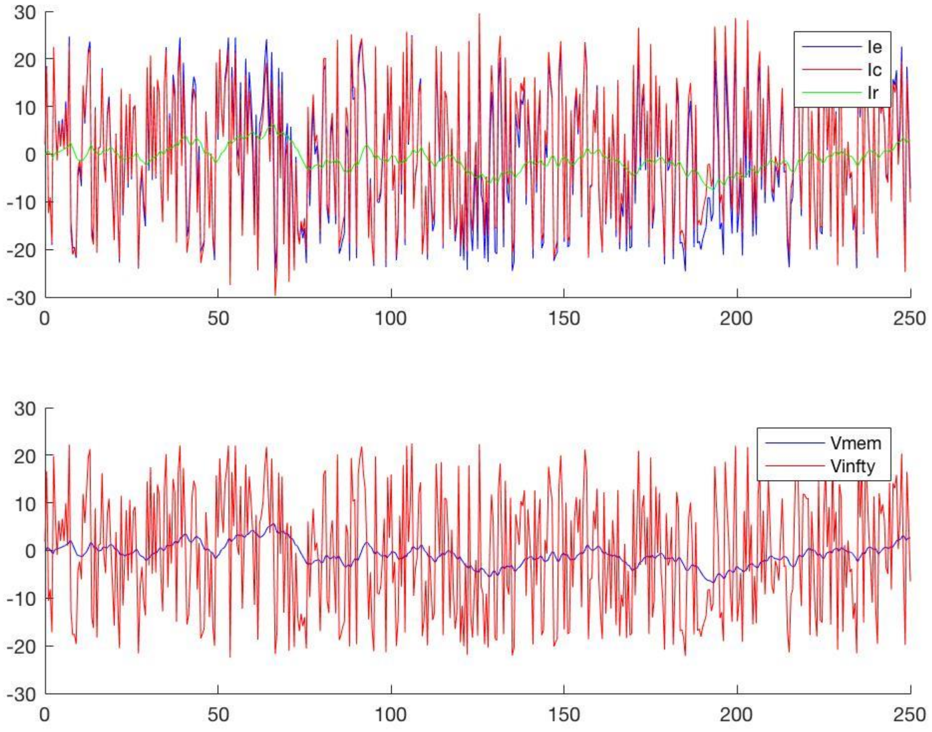

Fig. 5 Change of Current and Voltage across Time with a High Frequency Random Oscillation