TÉCNICO LISBOA

Circuit Theory and Electronics Fundamentals

[DEEC]

Department of Eletrical and Computing Engineering

LABORATORY 1

LINEAR CIRCUIT ANALYSIS

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1 Introduction

The aim of this laboratory assignment is to study the behaviour of a circuit containing linear components, in this case, resistors. Moreover, it also presents Voltage and Current Sources, either Independent or Dependent. In Figure 1 the stated circuit is presented. Despite being a simple circuit, it is perfect to experiment some important Circuit Theory's analysing tools, presented in Section 2, so that sotware simulation and theoretical analysis can be stacked up against each other more easily.

In Section 2, a theoretical analysis of the circuit is presented, either by the Node Analysis (2.1) and Mesh Analysis (2.2) methods, giving us some insights on different, although equivalent forms of exploring a linear circuit. In Section 5, the circuit is analysed by computacional simulation tools, via *ngspice*, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 6.

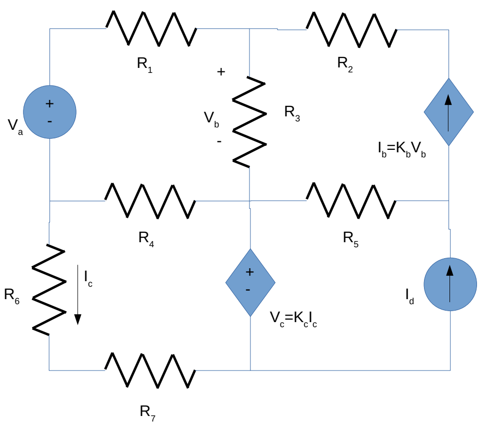

Figure 1: Linear Circuit with Resistors, Dependent and Independent Sources.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically using methods: Node and Mesh Analysis.

2.1 Node Analysis

Before applying the method, it is important to discuss how it works.
In general, it consists of discovering the voltages associated with each node of the circuit - they are the unknown variables in our equations.
In order to obtain these relations, one has to apply *Kirchhoff*'s Current Law (KCL) to the nodes, always having in mind that nodes on the ends of branches containing Independent Voltage Sources cannot be analysed in this way.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & K_c * G_6 & -1 \\ G_1 & -(G_1+G_2+G_3) & G_2 & G_3 & 0 & 0 & 0 \\ 0 & K_b+G_2 & -G_2 & -K_b & 0 & 0 & 0 \\ 0 & G_2 & -G_2 & G_5 & -G_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6+G_7 & G_7 \\ 0 & G_3 & 0 & -(G_3+G_4+G_5) & G_5 & G_7 & -G_7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ 0 \\ -I_d \\ 0 \\ I_d \end{bmatrix}$$

2.2 Mesh Analysis

In this case Usually, we create *fictious* currents in each mesh and we then apply *Kirchhoff*'s Voltage Law (KVL) to either meshes or loops, depending on if it

$$\begin{bmatrix} R_1+R_3+R_4 & R_3 & R_4 \\ -K_bR_3 & 1-K_bR_3 & 0 \\ R_4 & 0 & (R_4+R_6+R_7-K_c) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

3 Time response

The circuit consists of a single V-R-C loop where a current $i(t)$ circulates. The voltage source $v_I(t)$ drives its input, and the output voltage $v_O(t)$ is taken from the capacitor terminals. Applying the Kirchhoff Voltage Law (KVL), a single equation for the single loop in the circuit can be written as

$$Ri(t) + v_O(t) = v_I(t). \tag{1}$$

Because v_O is the voltage between capacitor C's plates, it is related to the current i by

$$i(t) = C\frac{dv_O}{dt}. \tag{2}$$

Hence, Equation (1) can be rewritten as

$$RC\frac{dv_O}{dt} + v_O(t) = v_I. \tag{3}$$

Equation (3) is a linear differencial equation whose solution is a superposition of a natural solution v_{On} and a forced solution v_{Of}:

$$v_O(t) = v_{On}(t) + v_{Of}(t). \tag{4}$$

As learned in the theory classes the natural solution is of the form

$$v_{On}(t) = Ae^{-\frac{t}{RC}}, \tag{5}$$

where A is an integration constant.

The forced solution is of the form given in Equation (6) and is illustrated in Figure **??**.

$$V_{Of}(t) = |\bar{V}_{Of}|cos(\omega t + \angle\bar{V}_{Of}), \tag{6}$$

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4 Frequency response

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5 Simulation Analysis

5.1 Operating Point Analysis

Table 1 shows the simulated operating point results for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: describe and explain the differences.

Name	**Value [A or V]**
@gb[i]	-2.38996e-04
@id[current]	1.047911e-03
@r1[i]	2.282360e-04
@r2[i]	-2.38996e-04
@r3[i]	-1.07597e-05
@r4[i]	1.224163e-03
@r5[i]	-1.28691e-03
@r6[i]	9.959274e-04
@r7[i]	9.959274e-04
v(1)	5.225520e+00
v(2)	4.989286e+00
v(3)	4.508524e+00
v(4)	5.021948e+00
v(5)	9.009942e+00
v(6)	-2.00134e+00
v(7)	-2.00134e+00
v(8)	-3.03868e+00

Table 1: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

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5.2 Transient Analysis

Figure **??** shows the simulated transient analysis results for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: describe and explain the differences.

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5.3 Frequency Analysis

5.3.1 Magnitude Response

Figure **??** shows the magnitude of the frequency response for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: describe and explain the differences.

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5.3.2 Phase Response

Figure **??** shows the magnitude of the frequency response for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: describe and explain the differences.

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5.3.3 Input Impedance

Figure **??** shows the magnitude of the frequency response for the circuit under analysis. Compared to the theoretical analysis results, one notices the following differences: describe and explain the differences.

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6 Conclusion

In this laboratory assignment the objective of analysing an RC circuit has been achieved. Static, time and frequency analyses have been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ but this is not the case in this work.

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