TÉCNICO LISBOA

CIRCUIT THEORY AND ELECTRONICS FUNDAMENTALS

[MEAER]

INTEGRATED MASTER'S DEGREE IN AEROSPACE ENGINEERING

LABORATORY 5

PASS-BAND FILTER

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1 Introduction

In this laboratory, we analysed in a theoretical approach as well as using software simulation, a Pass-band Filter using an Operational Amplifier (OPAMP). It was composed of three different stages: a High Pass Stage (with the objective of, as it name expresses, passing signals with high frequencies and cutting the ones with low frequencies, as we'll discuss in greater detail), an Amplification Stage (whose objective is, using the OPAMP, to amplify the signal) and, finally, a Low Pass Stage (that given the amplified signal from the previous stages, allows the low frequencies to pass and cuts off the high ones). In this report, a software simulation and theoretical analysis will be stacked up against each other. This assignment allowed us to deal with important concepts such as the **OPAMPs** - its input and output impedances, as well as gain and operation. In Figure 1 the stated circuit is presented.

Also, regarding the OPAMP it is important to notice that the model μA741 from Texas Instruments was the chosen one.

After the theoretical analysis, the circuit is analysed by computational simulation tools, via *Ngspice*, and the results are compared to the theoretical results obtained, in Section 2. The conclusions of this study are outlined in Section 6.

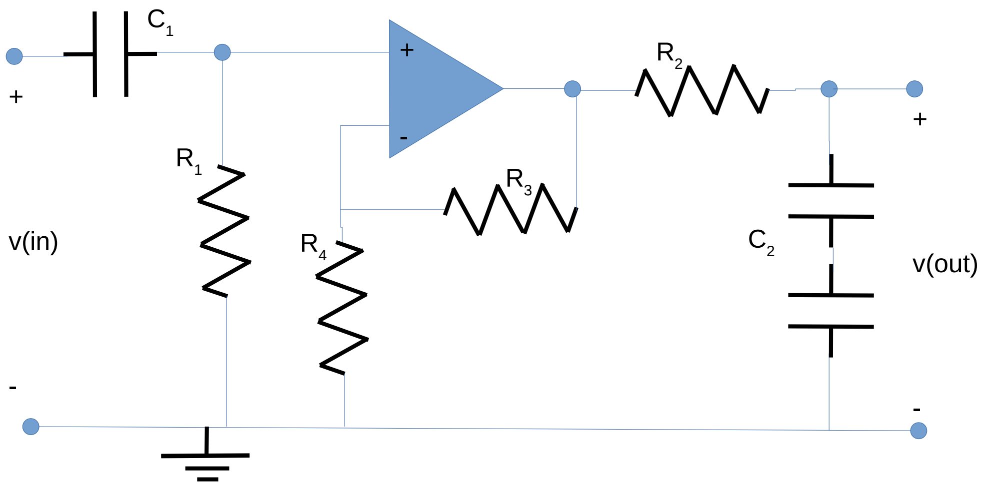

Figure 1: Active Passband Filter

2 Theoretical and Simulation Analysis

In order to compare *side by side*, we'll discuss the theoretical and simulation analysis at the same time. However, one has to present and analyse the three stages in order to fully understand the simulation. The constants values used are expressed in the table below.

Name	Value	Units
R_1	1000	Ω [Ohms]
R_2	1000	Ω [Ohms]
R_3	100000	Ω [Ohms]
R_4	1000	Ω [Ohms]
C_1	220	ηF [ηFarads]
C_2	110	ηF [ηFarads]

Table 1: Constants Values [1]

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2.1 High Pass Stage

Firstly, we must discuss the first third of the circuit that was used. It is easy to understand that it works as voltage divider,

$$v_{HP} = v_{R_1} = \frac{R_1}{R1 + \frac{1}{j\omega C_1}} v_i \tag{1}$$

which can be written as:

$$A_{HP} = \frac{v_{R_1}}{v_{in}} = \frac{jR_1\omega C_1}{jR_1\omega C_1 + 1} \tag{2}$$

For high frequencies ($\omega >> 1$), the voltage at the resistor tends to v_i and for low frequencies ($\omega << 1$), it tends to 0. In reality, this is a simple RC Series circuit, acting as a high pass filter.

A scheme of this circuit is presented below.

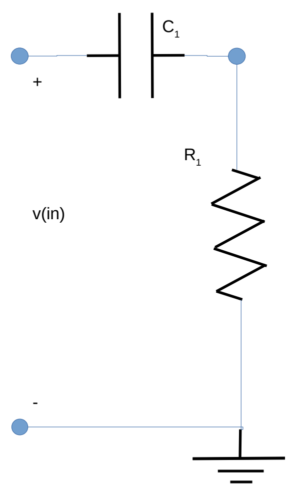


Figure 2: High Pass Stage

[1] Not that the value of C_1 reflects two 220 ηF capacitors in series, as seen in Figure 1

2.2 Amplification Stage

In this second stage we have the amplification of the signal. It is where the gain is maximum. This configuration uses a Non-Inverting Amplifier (the + part of the OPAMP is connected to the input *i.e.*, the output of the first stage and the - is connected to the feedback resistors). It is worth noticing that, depending on the values of R_3 and R_4 we can achieve higher or lower gains, hence the gain is given by,

$$AV_{AMP} = 1 + \frac{R_3}{R_4} \tag{3}$$

A scheme of this stage is presented below,

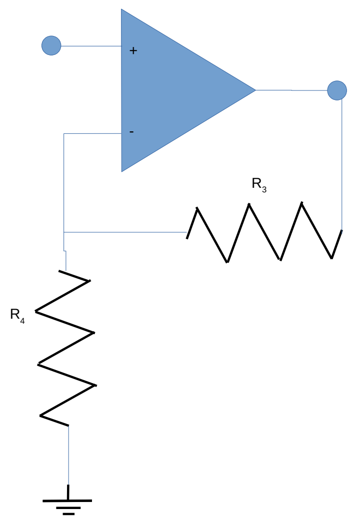


Figure 3: Amplification Stage

2.3 Low Pass Stage

In this last stage, we have an analogous circuit to the the High Pass one, except that at this time we are interested in the voltage at the capacitor, instead of the one on the resistor, in order to cut high frequencies.

A scheme of this stage is presented below, in Figure 4

By applying the voltage divider law, this time we get:

$$v_{LP} = \frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_2} + R_2} v_{amp} \tag{4}$$

which can be simplified to:

$$A_{LP} = \frac{v_{LP}}{v_{amp}} = \frac{1}{1 + j\omega C_2 R_2} \tag{5}$$

For high frequencies ($\omega >> 1$), the voltage at the capacitor tends to 0 and for low frequencies ($\omega << 1$), it tends to v_{amp}.

When combining the three stages, we end up with our Active Pass-band filter as seen in Figure 1.

Finally, it is worth noticing that we chose, among other possible options, this circuit's architecture due to its understandable behaviour: a first part blocks low frequencies, a second part amplifies the signal and a last part cuts high frequencies.

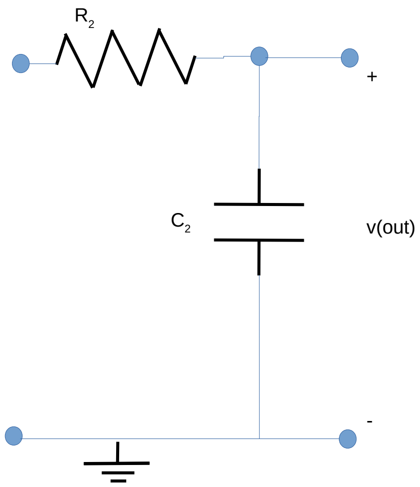


Figure 4: Low Pass Stage

2.4 Theoretical Analysis

On the Theoretical side, we will evaluate the central frequency (f_o), the two impedances: input and output, respectively (Z_I and Z_O), as well as the three gains associated with each stage (AV_{HP},AV_{amp} and AV_{LP}).

2.4.1 Central Frequency

The Central Frequency (f_o) is the frequency at the center of the passband of the filter. In this laboratory we want $f_o = \omega_o/2\pi$ to be as close to 1kHz as possible.

As for the mathematical expressions, the Lower Cut-off Frequency (ω_L), the Higher Cut-off Frequency (ω_H) and the Central Frequency are given by,

$$\omega_L = \frac{1}{R_1C_1} \qquad \omega_H = \frac{1}{R_2C_2} \qquad \omega_o = \sqrt{w_L * w_H} = \sqrt{\frac{1}{R_1C_1R_2C_2}} \tag{6}$$

As a remark, if both frequencies, f_L and f_H, are equal, f_o is going to be also equal and the passband width is going to be 0 - in the plot, we can see a peak instead of a highland.

The theoretical values of the lower cut-off frequency, the high cut-off frequency, and the central frequency are presented in the table below and will be better analysed and compared in Section 2.6.

Name	Value [Hz]
f_L	723.431560
f_H	1446.863119
f_O	1023.086723

Table 2: Frequencies (Lower, Higher and Central)

2.4.2 Impedances

As for the impedances, we have the input and output impedances to be,

$$Z_I = R_1 + Z_{C_1} = R_1 + \frac{1}{j\omega C_1} \qquad Z_O = Z_{C2}||R_2 = \frac{1}{j\omega C_2}||R_2 \tag{7}$$

Below, we present the values of these impedances, for the asked value of 1kHz,

Name	Value [Hz]
Z_{in}	1000.000000-723.431560i
Z_{out}	676.732451-467.723894i

Table 3: Impedances

When it comes to the functionality of the circuit, one has to remember that it is important to have a high input impedance (as high as possible) - in order to have a minimum degradation of the input signal. On the other hand, the output impedance needs to be as low as possible, especially when connecting a load to the output of this circuit. Of course, our results regarding the output impedance of the circuit are not satisfactory, however, because of the limitations of the components it was very tough to reduce this. One solution would be to replace R_2 with a parallel of several resistances, so that the equivalent resistance would be much less. Another solution would be to add more capacitors in series with C_2, decreasing the equivalent reactance seen from a load. However this would be a hard bill to pay because the overall gain would decrease too much below the 40dB wanted, which will be explained in greater detail in the next Section. Adding to this, we were not giving any specific impedance value for the load, but we only can assure good compatibility with very high impedance loads with this specific circuit.

2.4.3 Gain

In order to calculate the total gain (A_V) we performed a simple multiplication, so that we have $A_{V_{HP}} A_{V_{amp}} A_{V_{LP}}$ (when working with dBs, this multiplication is, in reality, a sum).

The Transfer Function is given by,

$$T(s) = \frac{R_1 C_1 s}{1 + R_1 C_1 s} \left(1 + \frac{R_3}{R_4}\right) \frac{1}{1 + R_2 C_2 s} \tag{8}$$

where $s = j\omega + \sigma$ and $\sigma = 0$ (for sinusoidal waves).

We can identify $T(s)$ to be the multiplication of $T_i(s) \equiv$ transfer function of the $i - th$ stage.

When substituting the frequency, $f = \frac{\omega}{2\pi} = 1000Hz$, we have the results below,

Name	Value [dB]
AV_{HP}	-1.828006
AV_{Amp}	40.086427
AV_{LP}	-1.695830
AV	36.562591

Table 4: Theoretical Gains

2.5 Simulation Analysis

When it comes to the simulation analysis, using *Ngspice*, we can get the values of Z_I, Z_O, f_O and also A_v.

Below, we present some tables with this data,

Name	**Value [Ω]**
Input Imp	999.991 + -723.534 j

Table 5: Experimental Input Impedance

Name	**Value [Ω]**
Output Imp	680.05 + -466.901 j

Table 6: Experimental Output Impedance

Name	**Value**	**Units**
AV	36.5323	dB
fO	1013.91	Hz

Table 7: Experimental Gain[2]and Central Frequency

In the next section we'll discuss these results, comparing them to the theoretical ones.

2.6 Comparison

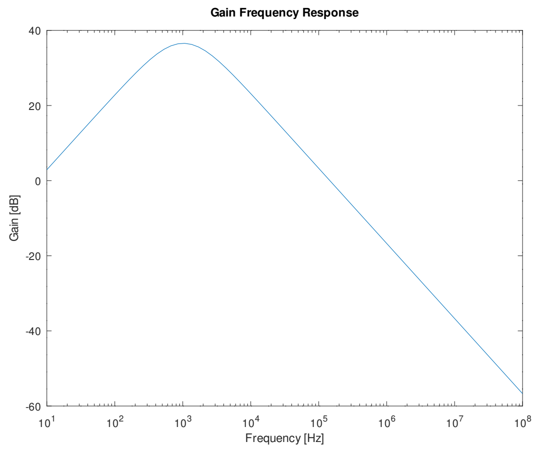
We are now ready to make a global comparison of the two approaches, with the chosen values for the constants. In the following page, we present both the theoretical and simulation graphs of the frequency response of the gain and the phase.

Overall, we have pretty satisfactory results. When one performs an initial comparison of the gain response, using the shape of the graph, it is easy to realise that they have a similar behaviour. In the initial section, there is a constant slope of +20dB/dec; after that, there exists a small passband (its width is not aim of study in this laboratory), also the region where we achieve the central frequency, and a last part with a slope of -20dB/dec.

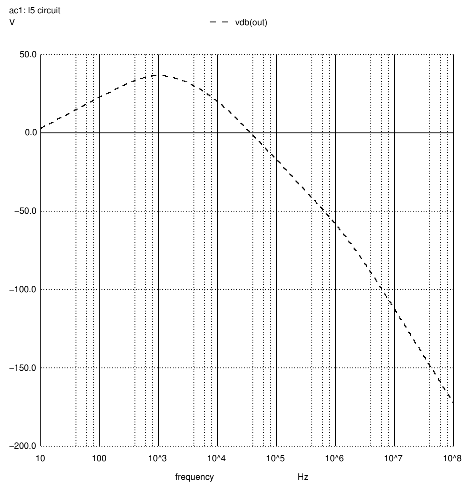
When it comes to the phase response, both plots differ. This is because our theoretical model does not predict the existence of capacitors in the OPAMP, as it idealises its gain to be purely real (no shift in phase). In reality, because the OPAMP has two capacitors, it is expected that in the phase frequency response plot each would introduce a shift of -90º, making the overall phase to go down to -270º = +90º.

Remembering the tables presented above, we have small relative errors for the values of interest (the impedances, the total gain and the central frequency). This somehow proves that, eventhough the OPAMPs are made of dozens of components, some of them non-linear, we can predict with a fairly simple model its behaviour.

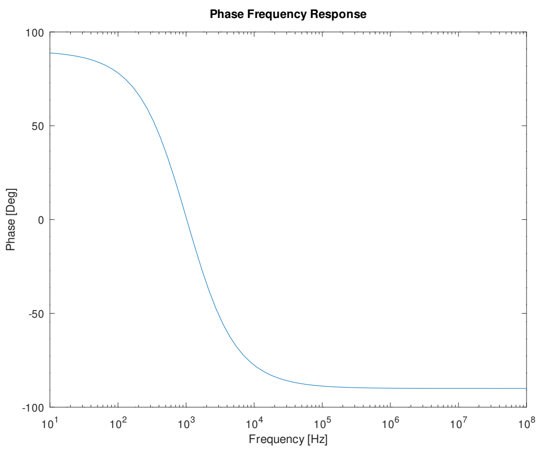
[2]Measured at 1000Hz



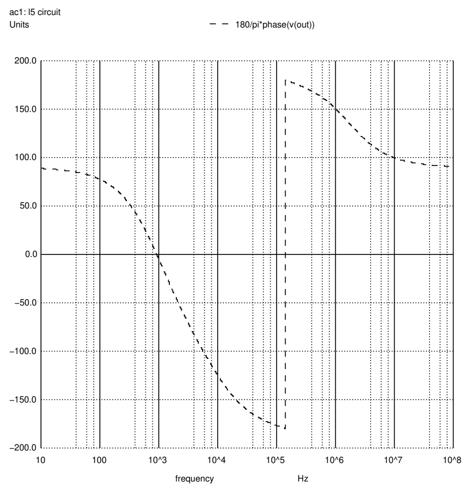
(a) Theoretical Gain Response



(b) Simulation Gain Response



(c) Theoretical Phase Response



(d) Simulation Phase Response

Figure 5: Frequency Response

3 Non-Improved Merit Results

From the results obtained through the *Ngspice* simulation (see Section 2.6) and considering we used the data shown in Section 1, we can compute the price and the merit using the *formulae* given in the lab assignment:

Name	Value	Units
AV dev	32.9164	
Freq dev	13.911	Hz
Cost	13426.8	MU
Merit	1.59048E-06	

Table 8: Merit Results

4 Circuit Improvements

Because some specifications of our circuit are a bit lacklustre, some additional improvements can be made. The main approach to improve the overall results was to increase the gain of the OPAMP stage. This was done by adding a bypass capacitor in series with resistor R_4. The expression of the gain is:

$$A_v = 1 + \frac{Z_3}{Z_4} \tag{9}$$

By adding the bypass capacitor, the impedance Z_4 decreases for high frequencies, and so the gain increases.

In order do adjust the low and high cut-off frequencies, some modifications were made to perfect these values, since the goal is that:

$$f_o = \frac{\sqrt{w_H w_L}}{2\pi} = 1000Hz \tag{10}$$

These modifications were mainly the addition of resistors in series/parallel to increase/decrease the equivalent resistance seen and add, in the same way, capacitors in series/parallel in order to decrease/increase the overall capacity. This perfected both the gain at $1000Hz$ as well as the low and high cut-off frequencies.

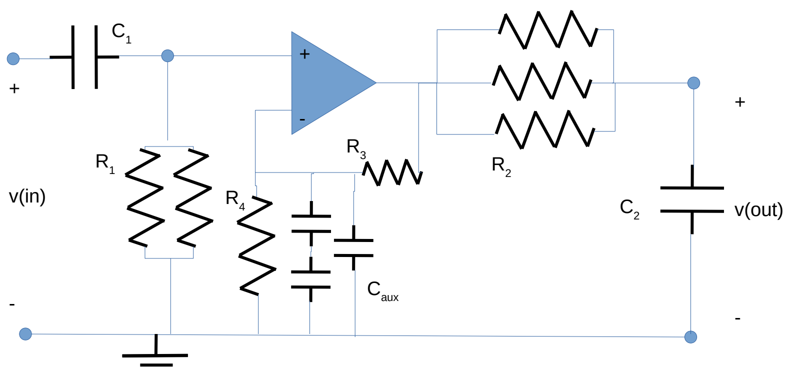


Figure 6: Improved circuit

Name	Value	Units
R_1	500	Ω [Ohms]
R_2	30000	Ω [Ohms]
R_3	100000	Ω [Ohms]
R_4	1000	Ω [Ohms]
C_1	220	ηF [ηFarads]
C_2	220	ηF [ηFarads]
C_{aux}	1.5	μF [μFarads]

Table 9: Constants Values [3]for the improved circuit

The results that come from these modifications are shown below:

Name	Value	Units
AV	40.774	dB
fO	1028.02	Hz

Table 10: Improved Results [4]

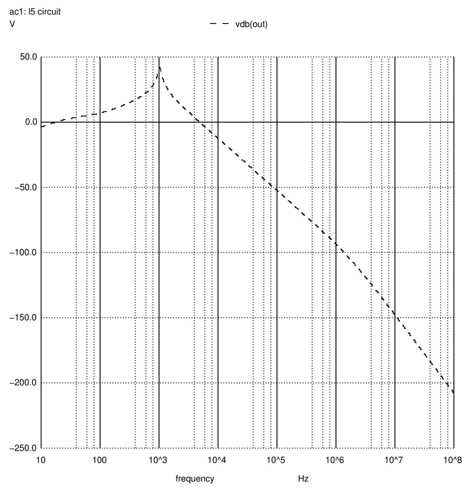


Figure 7: Gain Frequency Response

With these changes, we managed to decrease massively the output impedance. With this configuration, the circuit is much more prepared to be connected to a greater range of loads, which increases its utility in practical terms. Another plus are the better results, as shown in table 10, when compared to the ones previously obtained (in table 7).

[3]Note that these values reflect and summarise the composition shown in Figure 6 and, as so, some components may actually reflect a certain composition of multiple ones.
[4]Gain is measured at a frequency of 1000Hz.

Name	Value [Ω]
Input Imp	500.706 + -725.317 j
Output Imp	17.5247 + -723.225 j

Table 11: Input and Output Impedances

5 Improved Merit Results

Name	Value	Units
AV dev	9.31975	
Freq dev	28.016	Hz
Cost	13459.5	MU
Merit	1.98996E-06	

Table 12: Improved Merit Results

With the improvements made, even though the central frequency was slightly worse, the benefit of the increase in the gain excelled making the merit figure rise, giving us the results shown in table 12.

6 Final Conclusion and General Notes

As a conclusion, we can state that, unlike the previous two lab assignments, there is a degree of similarity between both analysis, in terms of precision. This was rather surprising, for we are dealing with a high-complexity component - the OPAMP - at least in terms of architecture. But, when we analyse it in a macroscopical point of view, it's rather predictable and stable - at least in this circuit.

Overall, the laboratory gave us the opportunity to deepen our knowledge with OPAMPs and Active 2nd-order filters. We understood it as an extension of the second lab, where we analysed 1st-order RC circuits.

It is also worth noting that the concept of the merit figure proves itself to be extremely important, because it narrows the gap between an academic point of view and an industrial/engineering approach. It made us understand that there are many factors involved in the construction of a circuit, some of them being the cost, the space availability and the customers' needs. Another limiting factor was the fixed values and quantities for each of the components, which made us have to think more thoroughly and inventively about how and where we would use them. A complete circuit that takes into account all these factors is most of the times really hard to achieve, which leads us to sacrifice some characteristics in favour of some more important others, achieving a balance between them all.