Circuit Theory and Electronics Fundamentals

Aerospace Engineering, Técnico, University of Lisbon

4th Laboratory Report

João Martins 95806
Gonçalo Martins 95793
António Lopes 95771

May 23, 2021

Contents

1 Introduction

The goal of this laboratory assignment was to build a BandPass Filter (BPF) with the following specifications: central frequency at 1MHz and gain at central frequency at 40dB, using an Operational Amplifier. In our case, we chose to use the non-inverting amplifier, as the objetive was only to amplify the signal without inverting it. In order to optimize the amplifier, both the theoretical and simulation analysis were done. The architecture of the given circuit can be seen in the following picture.

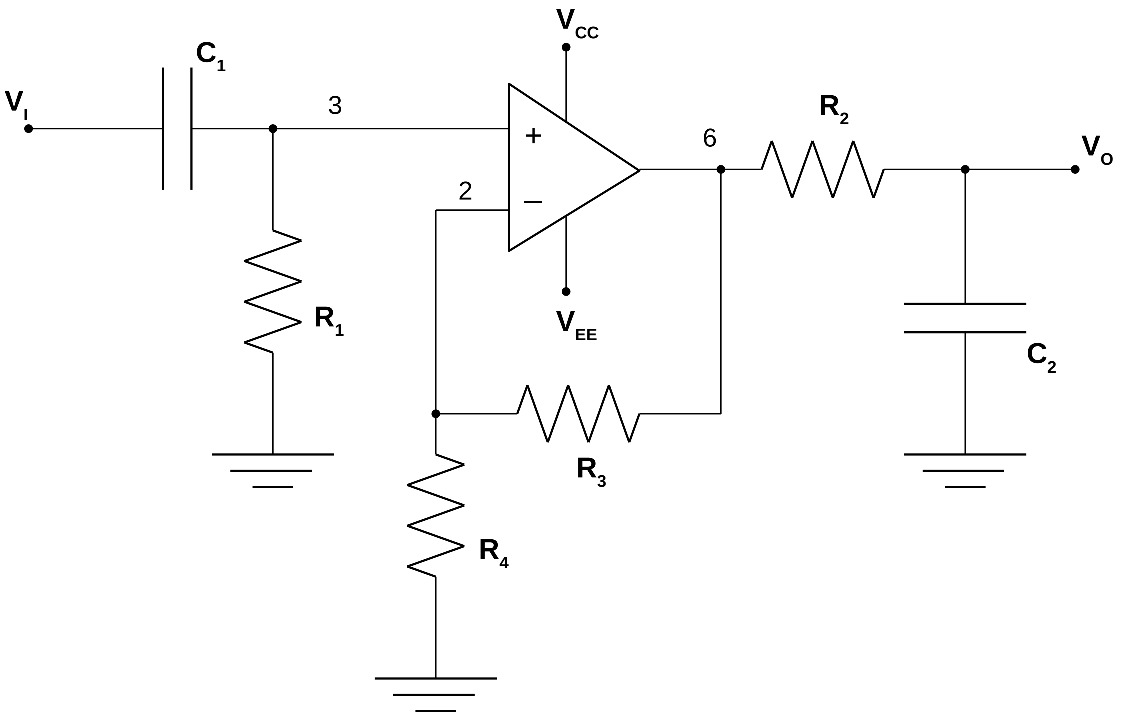


Figure 1: Band-Pass Filter

As seen in the picture above, in this circuit there were no sources used as the objetive of this assessment was to only build a circuit capable of amplifying any kind of signal and not a specific one. As such, v_I and v_o represent the input of the signal and its amplification, respectively. Through ngspice, we were capable of measuring the wanted gain in the passband, the central frequency and the input and output impedances at the central frequency. In octave, a simulation of the circuit was also done with the required simplifications of the circuit. Gain, input and output impedances and frequency response were also obtained and then compared to the previously calculated on ngspice.

As seen in classes, an OP-AMP is a transistor based amplifier which is characterised by its high gain, high input impedance and low output impedance. For this laboratory assessment the group was given a list of components allowed to use and so the components used were all from this list.

2 Theoretical Analysis

In this section, the circuit shown in figure 1 is analysed theoretically.

The following analysis will be made considering the ideal model of an OP-AMP, where its gain is infinite, with an infinite input impedance and 0 output impedance. Pratically speaking an OP-AMP has different values for the properties refered above. These are the most important differences but, of course, they aren't the only ones. This differences will be talked about again when we compare the results and will also help to explain the differences encountered in Octave and NGspice.

This circuit will behave as a non-inverting amplifier and as a band-pass filter. The components responsible for this last feature are the capacitors, since the lower and higher cut off frequencies are calculated from this components like it will be shown below.

2.1 First Point

Our theoretical analysis starts by computing the gain and the input and output impedances at central frequency.

First we have to calculate the central frequency and for that we'll have to discover the lower and higher cut off frequencies, those will be the poles, using the characteristic equations. The formulae are,

$$w_{CutOff} = \frac{1}{R1C1} \tag{1}$$

$$w_{CutOff} = \frac{1}{R2C2} \tag{2}$$

As expected, the highest value will be the high cut off frequency, w_H, and the lowest will be the low cut off frequency, w_L. Then to discover the central frequency, w_0, we use

$$w_0 = \sqrt{w_L w_H} \tag{3}$$

Since we want a central frequency of $f_0 = 1000Hz$ ($w_0 = 2000\pi rad/s$), and from the previous equations we get the following relation

$$2000\pi = \frac{1}{\sqrt{R1C1R2C2}} \tag{4}$$

We can now choose the best values for the resistors and capacitors to get the smallest frequency deviation. We found that using a capacitor of 220nF and a parallel of capacitors of 220 for C2 and C1, respectively, and resistors of $1k\Omega$, would get the best results.

Lower Cut Off Frequency	4.545455e+03 rad/s
Higher Cut Off Frequency	9.090909e+03 rad/s
Central Frequency	6.428243e+03 rad/s

Table 1: All frequencies computed [rad/s]

Now since we know the central frequency we can calculate the gain at that frequency knowing the transfer function. We now show the formula used, the explanation of this formula will be made in the next subssection when we deduce the tranfer function.

$$Gain = |\frac{1}{1 + jR2C2w_0}\frac{jR1C1w_0}{1 + jR1C1w_0}(1 + \frac{R3}{R4})| \tag{5}$$

$$Gain_{dB} = 20log_{10}(Gain) \tag{6}$$

Gain	1.006667e+02
Gain	4.005771e+01 dB

Table 2: Computed Gains

To calculate the input and output impedances we just had to analise the circuit and knowing the input and output impedance of the OP-AMP we determine that the input impedance is equal to the impedance of R1 in series with C1 and the output impedance is equal to the impedance of R2 in parallel with C2.

$$Z_I = R1 + \frac{1}{jw_0C2} \tag{7}$$

$$Z_o = \frac{R2}{1 + jw_0C2R2} \tag{8}$$

Input Impedance	1.000000e+03 + -7.071068e+02j
Output Impedance	6.666667e+02 + -4.714045e+02j

Table 3: Computed impedances [Ω]

Now lets analise the frequency response of the circuit, and for that, we'll deduce the tranfer function of the circuit.

2.2 Second Point

Knowing that it is a non-inverting amplifier we know the following relation

$$\frac{v_6}{v_3} = 1 + \frac{R3}{R4} \tag{9}$$

We still don't have a relation between v_0 and v_I. To do that we have to relate them with v_6 and v_3.

Lets find a relation between v_0 and v_6, applying the voltage divider we have

$$v_o = \frac{\frac{1}{jwC2}}{\frac{1}{jwC2+R2}}v_6 = \frac{1}{1 + jR2C2w} \tag{10}$$

Now to find a relation between v_3 and v_I, we'll also apply the voltage divider,

$$v_3 = \frac{R1}{\frac{1}{jwC1+R1}}v_I = \frac{jwR1C1}{1 + jR1C1w}v_I \tag{11}$$

Now that we have the relations that we needed and knowing that $s = jw$, we can combine the last three equations to get

$$\frac{v_o}{v_I} = T(s) = \frac{1}{1 + R2C2s}\frac{sR1C1}{1 + sR1C1}(1 + \frac{R3}{R4}) \tag{12}$$

This was the formula used to get the gain at the central frequency in the previous subsection. Doing $T_{dB}(s) = 20log_{dB}(|T(s)|)$ and $\phi = arg(T(s))$, we can plot for frequencies ranging from 10 Hz to 100MHz. We can see in the following figures this ploted in a logarithmic scale.

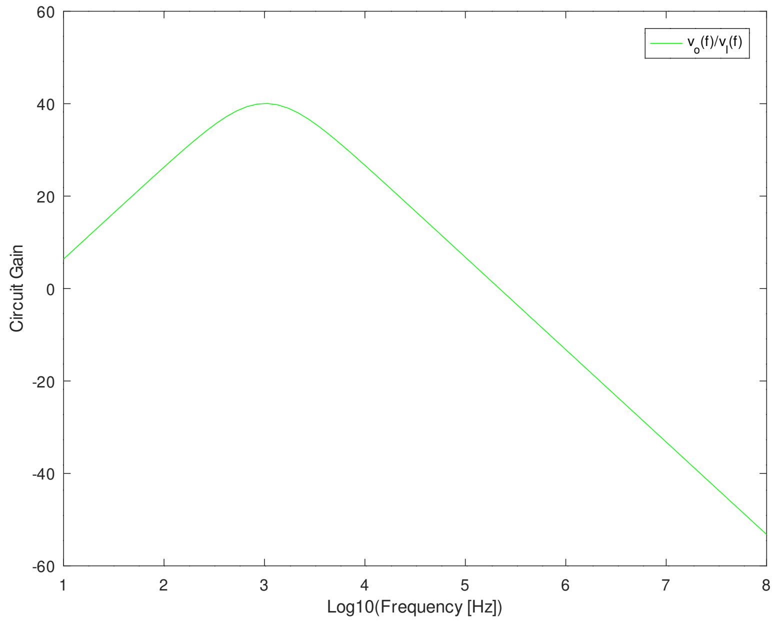


Figure 2: Frequency response [dB] ($\frac{V_o(f)}{V_I(f)}$)

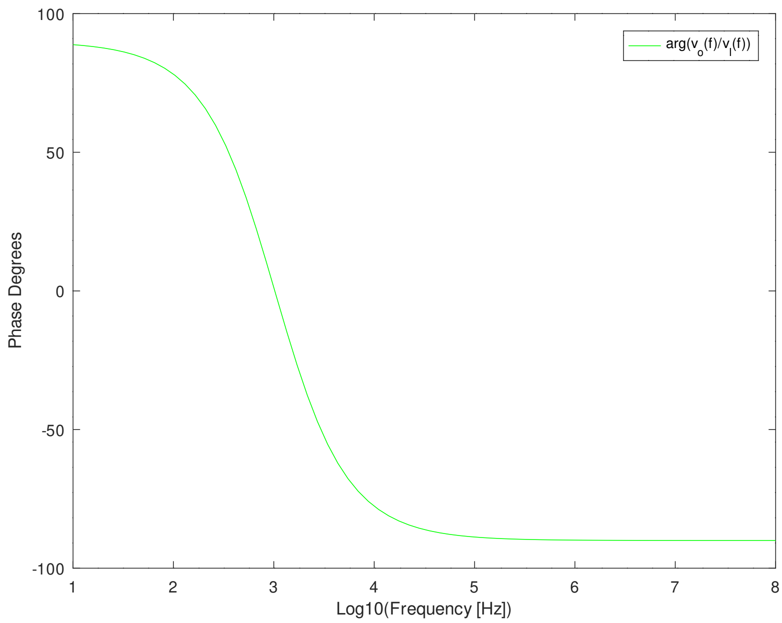


Figure 3: Frequency response [degrees]

To conclude this theoretical analysis we show now the frequency and gain deviation, the cost and the theoretical Merit and we also show the values of the components used.

R1	1.000000e+03
R2	1.000000e+03
R3	1.500000e+05
R4	1.000000e+03
C1	2.200000e-07
C2	1.100000e-07

Table 4: Values of the components used ([Ω],[F])

Central Frequency Deviation	2.308672e+01 Hz
Gain Deviation	5.771376e-02 dB
Cost	1.428600e+04 MU
Merit	3.024424e-06

Table 5: Frequency and gain deviation, cost and merit

It's important to note that the value for the resistor three was obtained using three resistor of $100k\Omega$, two in parallel with one in series.

3 Simulation

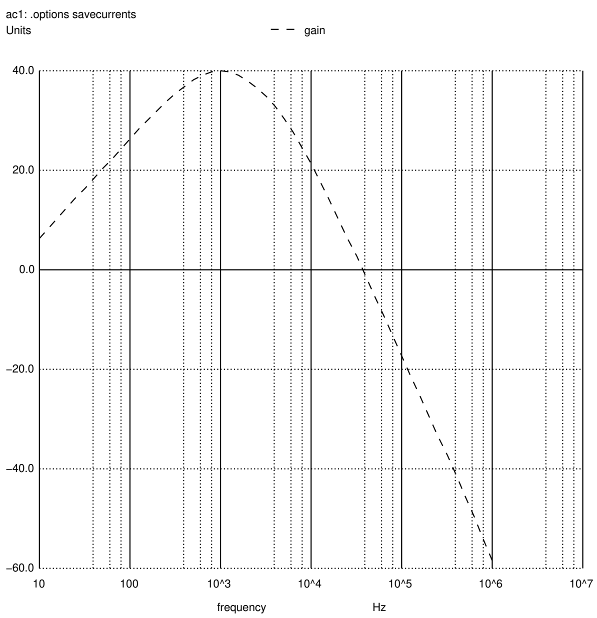
In this section, a brief description of the circuit modeled through NGspice is going to be presented and the values obtained on it are going to be compared with the ones obtained on Octave. The purpose of the simulation was to maximise the merit obtained and, as such, to obtain the values of the gain and central frequency. Furthermore, we'll analise the input and output impedances as well.

In order to do that, we started with the circuit previously provided, kept the OP-AMP (as it was supposed to be), and made some changes in the capacitors and resistors, as explained before. Next, both output gain and central frequency were measured, and the respective deviations were computed. The results are shown in the table bellow.

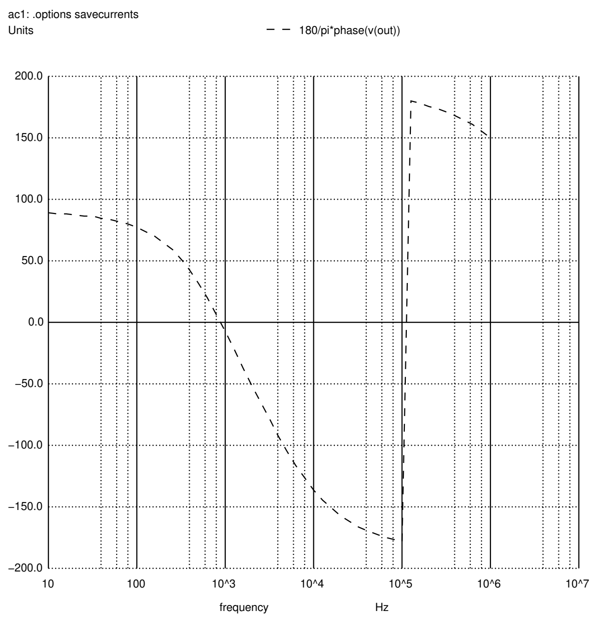
Gain	39.977 dB
Central Frequency	1000 Hz
Gain deviation	0.02295 dB
Central frequency deviation	0 Hz

Table 6: Output gain, central frequency and respective deviations ([dB],[Hz])

Following this, we analised the frequency responce of the circuit. In order to do this, we plotted both the gain (in dB) and the phase (in degrees), as functions of the frequency. The plots can be seen in the figures bellow.



(a) Gain as function of frequency



(b) Phase as funtion of frequency

The next step was to simulate the input impedance. The obtained value can be seen bellow - as one might confirm, this value is high, which is a requirement to have low voltage degradation at the input.

Zin	998.999 + -7.2972 j

Table 7: Input impedance of the circuit [Ω]

Besides having a high input impedance, it is also important to have a low output impedance, in order to have low voltage degradation at the output as well. In order to calculate this value, we needed a different setup, which explains the two *.net* files in *sim* folder. The value of the output impedance is shown bellow, and it is very low, as we can see.

Zo	0.20916 + -14.4657 j

Table 8: Output impedance of the circuit [Ω]

Finally, in order to evaluate the relation cost/quality of our solution, the figure of merit is presented in the next table.

Cost	14286 MU
Merit	0.00304992

Table 9: Figure of merit

4 Comparison

Now a comparison between the theoretical analysis and the experimental analysis results is done and the results discussed conserning its accuracy and discrepancy. For the theoretical analysis as seen in the previous section an operating point analysis is made in order to calculate the needed values.

Concerning the central frequency, we were able to obtain it using the function measure of ngspice. In the theoretical analysis it was calculated using the high and low cut off frequencies. Because of this the values are going to differ as we can see below.

Calculus	Value
Gain	39.977 dB
Central Frequency	1000 Hz
Gain deviation	0.02295 dB
Central frequency deviation	0 Hz

Table 10: Central frequency [Hz] and respective gain [dB]. (Ngspice)

Name	Value
Gain	4.005771e+01 dB
Central Frequency	1.023087e+03 Hz
Gain Deviation	5.771376e-02 dB
Central Frequency Deviation	2.308672e+01 Hz

Table 11: Central frequency [Hz] and respective gain [dB]. (Octave)

Also the input and output impedances were calculated in both analysis are now compared below in the tables shown.

Calculus	Value [Ohm]
Zin	998.999 + -7.2972 j
Zo	0.20916 + -14.4657 j

Table 12: Circuit impedances. Variables are expressed in Ohm.(Ngspice)

Name	Value [Ohm]
Input Impedance	1.000000e+03 + -7.071068e+02j
Output Impedance	6.666667e+02 + -4.714045e+02j

Table 13: Circuit impedances. Variables are expressed in Ohm.(Octave)

As we can see the values are close and the differences that exist may be explained because of the approximations done in the amplifier model, as no IOS and VOS current and voltage were considered which means that only the ideal model of the transistor was considered. These sources are put in place as the resistance in the circuits' cables are bigger than zero which means that the current on the inputs of the amplifier isn't going to be null as we considered. Also the OP-Amp model used in the theoretical analysis consists of many simplifications which explain the differences shown above.

5 Conclusion

In this laboratory assignment, the goal was to create a circuit with the fuction of a Band-Pass Filter, reaching the looked after gain for the central frequency of it whilst minimizing its cost. To do that theoretical and experimental analysis were conducted and the results studied.

Even though the model used in the octave script is a good approximation of a real Op amplifier, as the results between ngspice and octave differ we can conclude that the model used isn't perfect. This may be explained because of the approximations used in the amplifier model (no current in the inputs of the amplifier and less the variables controlling it on the theroretical model when compared to the ngspice model), taking into account that these are non linear components.

In conclusion, with this laboratory assignment we were able to understand more deeply the functioning of Op-Amps and its applicability on different devices. Remembering the goal of reaching the needed gain for the given frequency, we can say we are pretty satisfied with our results as we were able to achieve these goals and also improve on the merit obtained.