# Circuit Theory and Electronics Fundamentals

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1st Laboratory Report

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## Contents

# 1 Introduction

The objective of this laboratory assignment is to study a circuit containing only resistors and voltage/current sources, by determining all branch currents and all node voltages. The scheme of the circuit we're going to analyse can be seen in figure 1.
On the first hand, in subsection 2.1, a theoretical analysis of the circuit is presented, based on the mesh analysis method, used to determine all branch currents.
On the other hand, in subsection 2.2 we'll analyse the same circuit, using the node analysis method, in order to determine all node voltages.
To compare all these things, we present an error analysis in subsection 2.3.
In section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in subsections 2.1 and 2.2.
The conclusions of this study are outlined in section 4.

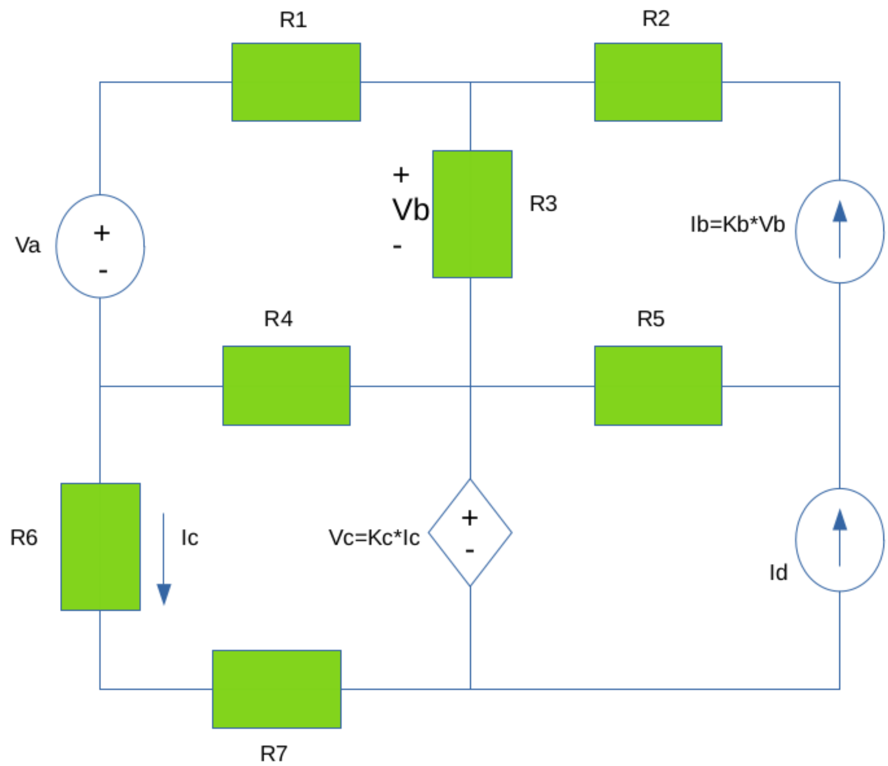


Figure 1: Circuit.

## 2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, to determine all mesh currents and nodes voltages. In Figure 2, conventions that will be used doing mesh and nodes analysis are shown.

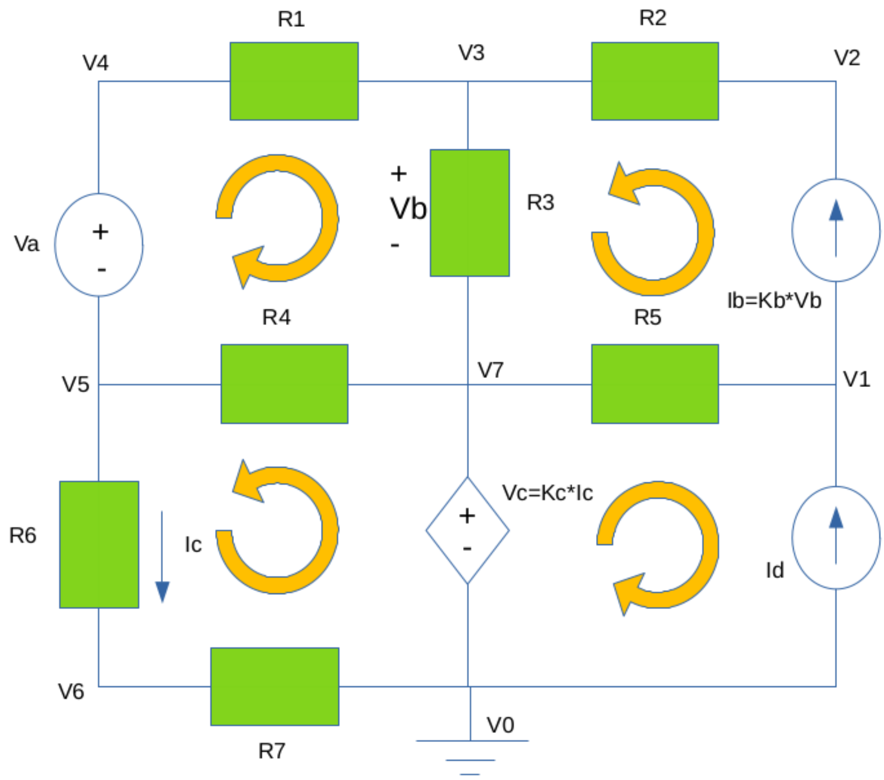


Figure 2: Circuit with nodes and currents convention

Also, the resistance, constants of the dependent sources and current values that were given by the *Python* program are shown in the following table.

| | |
|---|---|
| R1 | 1.006765e+00 kOhm |
| R2 | 2.033032e+00 kOhm |
| R3 | 3.033913e+00 kOhm |
| R4 | 4.003128e+00 kOhm |
| R5 | 3.131011e+00 kOhm |
| R6 | 2.093899e+00 kOhm |
| R7 | 1.017744e+00 kOhm |
| Id | 1.035361e+00 mA |
| Kc | 8.170065e+00 kOhm |
| Kb | 7.272764e+00 mS |
| Va | 5.223200e+00 V |

## 2.1 Mesh analysis

By inspection, the circuit has four meshes. In each one, the current flows like in Figure 2. Since there are four variables, $I_1$, $I_2$, $I_3$ and $I_4$, we'll need four linearly independent equations. To get them we'll apply KVL to mesh '1' and mesh '3' since they don't have current sources that hinder the job. the two equations will be

$$Va = R1 \cdot I_1 + R3(I_1 + I_2) + R4(I_1 + I_3) \tag{1}$$

$$Vc = R4(I_3 + I_1) + R6 \cdot I_3 + R7 \cdot I_3 \tag{2}$$

We can see now that in equation 2, we added a new variable, $Vc$, so we'll need to find another equation. Since we know by inspection that $Ic = I_3$, we can easily get,

$$Vc = Kc \cdot I_3 \tag{3}$$

Now we still need two other equations. We find one by analysing the dependent current source, the voltage drop in $R3$ and by detecting that $I_2 = Ib$,

$$\frac{I_b}{Kb} = Vb = (I_1 + I_2)R3 \cdot \Rightarrow \frac{I_2}{Kb} = (I_1 + I_2)R3. \tag{4}$$

Again, by inspection, we get $I_4 = -Id$ which finalises our equations. To solve these equations, we put them in matrix form and then use octave to solve the matrix equation in a much easier way.

$$\begin{bmatrix} R1 + R3 + R4 & R3 & R4 & 0 \\ R4 & 0 & R4 + R6 + R7 - Kc & 0 \\ R3Kb & R3Kb - 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ I3 \\ I4 \end{bmatrix} = \begin{bmatrix} Va \\ 0 \\ 0 \\ -Id \end{bmatrix}$$

The theoretical values are shown in the table below.

| | |
|---|---|
| I1 | 2.604926e-01 mA |
| I2 | -2.728588e-01 mA |
| I3 | 9.881466e-01 mA |
| I4 | -1.035361e+00 mA |

## 2.2 Node analysis

For the node analysis, we number the nodes as in Figure 2, as said. For this method we'll work with conductance instead of resistance, because it makes it easier to work with. We now have eight variables, so we'll have to find eight equations. Since we defined ground in node zero, we get the first equation, which is $V_0 = 0$. The next four equations, we get from doing KCL in nodes 1, 2, 3 and 6 since they don't have voltage sources. The equations are the following Node 1:

$$-Id + Ib - G5(V_7 - V_1) = 0. \tag{5}$$

Node 2:

$$-Ib + G2(V_2 - V_3) = 0. \tag{6}$$

Node 3:

$$-G2(V_2 - V_3) - G1(V_4 - V_3) - G3(V_3 - V_7) = 0. \tag{7}$$

Node 6:

$$G7(V_6 - V_0) - G6(V_5 - V_6) = 0. \tag{8}$$

From these equations, we can see that we added a new unknown variable, $Ib$. For that reason, we need to add another equation - we'll use the current source and the voltage drop in $R3$. Knowing that, we get

$$I_b = Kb \cdot Vb = Kb(V_3 - V_7) \Rightarrow Ib = Kb(V_3 - V_7). \tag{9}$$

We still need three equations. We'll get one from de voltage drop in $Va$, and another from the voltage drop in the dependent voltage source. These two equations are,

$$V_4 - V_5 = Va \tag{10}$$

and

$$V_7 - V_0 = Kc \cdot Ic = Kc \cdot G6(V_5 - V_6) \Rightarrow V_7 - V_0 = Kc \cdot G6(V_5 - V_6). \tag{11}$$

The final equation will be from the the node seven, but since we don't easily know the current that flows through the dependent current source, we first need to find a equation for this current. For that, we'll analyse node 0, which allows us to get the equation

$$I = G7(V_6 - V_0) - Id \tag{12}$$

where $I$ is the current we are trying to find. Now, doing the node analysis for node 7, we get the equation

$$G5(V_7 - V_1) + G4(V_7 - V_5) - G3(V_3 - V_7) - G7(V_6 - V_0) + Id = 0. \tag{13}$$

We finaly have a sistem of linearly independent equations that we can solve, using the matrix form in octave.

$$\begin{bmatrix} 0 & G5 & 0 & Kb & 0 & 0 & 0 & -G5-Kb \\ 0 & 0 & G2 & -Kb-G2 & 0 & 0 & 0 & Kb \\ 0 & 0 & -G2 & G2+G1+G3 & -G1 & 0 & 0 & -G3 \\ -G7 & 0 & 0 & 0 & 0 & -G6 & G7+G6 & 0 \\ G7 & -G5 & 0 & -G3 & 0 & -G4 & -G7 & G5+G4+G3 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -KcG6 & KcG6 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix} = \begin{bmatrix} Id \\ 0 \\ 0 \\ 0 \\ -Id \\ Va \\ 0 \\ 0 \end{bmatrix}$$

The theoretical values are shown in the table below.

| | |
|---|---|
| V0 | -0.000000e+00 V |
| V1 | 1.216927e+01 V |
| V2 | 7.480974e+00 V |
| V3 | 8.035705e+00 V |
| V4 | 8.297960e+00 V |
| V5 | 3.074760e+00 V |
| V6 | 1.005680e+00 V |
| V7 | 8.073223e+00 V |

| Resistor | Expression | Value [A] | Error(%) |
|---|---|---|---|
| r1 | $I1$ | $2.604926e-4$ | $0$ |
| r2 | $I2$ | $-2.728588e-4$ | $7.329798416e-5$ |
| r3 | $I1+I2$ | $-1.23662e-5$ | $0$ |
| r4 | $I1+I3$ | $1.2486392e-3$ | $1.601743722e-5$ |
| r5 | $I2+I4$ | $-1.3082198e-3$ | $1.528795085e-5$ |
| r6 | $I3$ | $9.881466e-4$ | $0$ |
| r7 | $I3$ | $9.881466e-4$ | $0$ |

### 2.3 Error analysis

As *Octave*'s output (for the mesh analyses) is just the value of the circulation current in each mesh, we're going to compute the current value in each resistor, for the sake of comparison with *Ngspice*'s simulation results. We'll also compute the error associated to each current value output by *Ngspice*.

As we can observe, the error values are very small (in some cases even zero), and this happens because the precision used in *Octave* is higher than the one used in *Ngspice*.

If we take a look to the node voltage values, the situation is even better, since the precision of both programs is the same - the match is perfect.

# 3 Simulation

With the circuit solved using the mesh and node methods, it's necesssary to solve it experimentally. In order to simulate the real conditions, which we would have encountered in the laboratory, Ngspice was used. The results obtained from this simulation will take in account the energy dissipation due to the many processes that occur such as Joule effect on conductors. Using this software, we were able to verify the results obtained from the methods already described.

In order to describe the circuit to be studied, it is necessary to follow some guidelines. Firstly, it was needed to specify , for each component, which were its positive and negative nodes. Because of that, the current directions were defined accordingly to the ones seen in the picture describing the mesh and node methods. As such, the electric resistances and current sources are going to have the positive node in the location of the current input and the negative node in the current output. The voltage sources nodes were defined so that the nodes are in accordance with the pre established nodes. In the picture below, a brief representation of the interpertation of the circuit by the ngspice programme is presented.

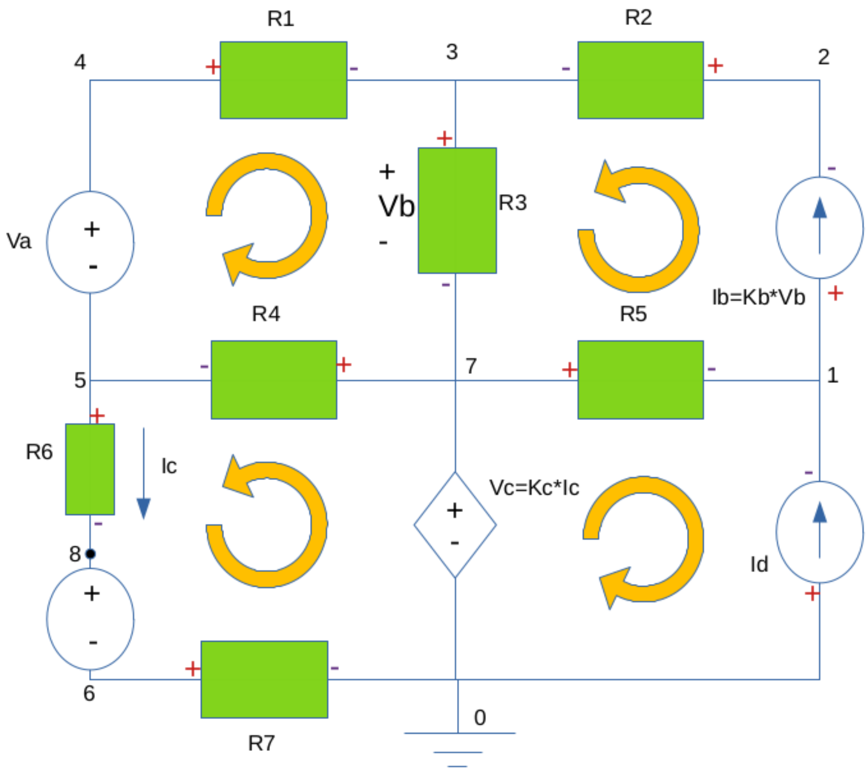


Figure 3: Circuit with voltage control source - current flow and voltage difference representation

Apart from this, it was also necessary to establish the values of the resistance, current and voltage associated with the resistors and independent current and voltage sources. The dependent sources, Vc and Ib (Hc and Gb in ngspice because of the symbolic representation in the program), were respectively current controlled voltage source and voltage controlled current source. Because of that, kc and kb were in kOhm and mS as the script indicated. In the case of Vc, it was needed to indicate a control voltage source, Ve which, even though it doesn't exist, is going to measure the current Ic, the control current, passing through it. This voltage control has a null potential so it doesn't interfere with the rest of the circuit. The nodes of Ve were placed in

the circuit so that the current input is in the positive node of the voltage source and the output of the current is in the negative node of the source. In the case of Ib, it was needed to introduce the nodes in which the potential difference is going to be read (again the positive node in first place and then the negative one).
Finally, a table with the values obtained is asked every time the program is run, in order to obtain the looked for data as we can see below.

| | |
|---|---|
| @gb[i] | -2.72859e-04 |
| @id[current] | 1.035361e-03 |
| @r1[i] | 2.604926e-04 |
| @r2[i] | -2.72859e-04 |
| @r3[i] | -1.23662e-05 |
| @r4[i] | 1.248639e-03 |
| @r5[i] | -1.30822e-03 |
| @r6[i] | 9.881466e-04 |
| @r7[i] | 9.881466e-04 |
| v(1) | 1.216927e+01 |
| v(2) | 7.480974e+00 |
| v(3) | 8.035705e+00 |
| v(4) | 8.297960e+00 |
| v(5) | 3.074760e+00 |
| v(6) | 1.005680e+00 |
| v(7) | 8.073223e+00 |
| v(8) | 1.005680e+00 |

# 4 Conclusion

In this laboratory assignment the goal of determining node voltages as well as branch currents has been achieved. Both of the analyses have been performed from the values given by the *Python* program - theoretically using the *Octave* maths tool and by circuit simulation using the *Ngspice* tool. As we mentioned in subsection 2.3, the error is very small in branch current values (even zero in some cases) due to lower precision used by *Ngspice*, and it assumes the value zero for all the node voltage values due to the fact that both *Octave* and *Ngspice* used the same precision. Taking this into account, we can safely say that the simulation results matched the theoretical results pretty well. The reason for this match is the fact that this is a circuit containing only linear components, so the theoretical and simulation models cannot differ. Although the match is almost perfect, we have to say that if the data were obtained in a real situation, this result would be very hard to achieve, since we had to take into acount other systhematic and accidental errors that might have occurred, as well as phenomena like joule effect in the conductors.