# Fundamentals of Computer Graphics

Wrap-up


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# Exercises

- Rank of a map
- ICP

# Meshes

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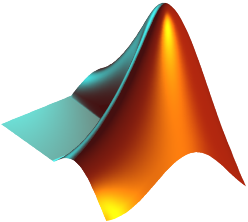

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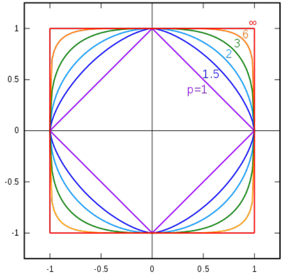

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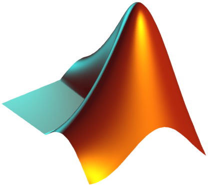
What about 3D shapes?



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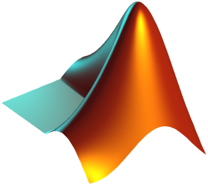
...still, we can treat them as restrictions of $\mathbb{R}^3$ and apply vector space operations on them (e.g. to do basic shape interpolation, extrapolation, caricaturization and smoothing)



# Correspondence

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$$\mathbf{S} = t\mathbf{P}_1 + (1-t)\mathbf{P}_2 \quad \text{s.t. } \mathbf{S1} = \mathbf{1}\,, \mathbf{S}^\top \mathbf{1} = \mathbf{1}$$

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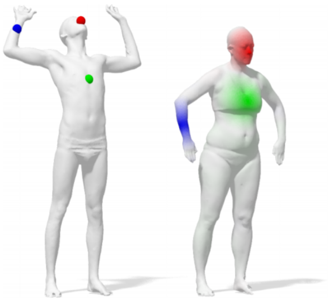

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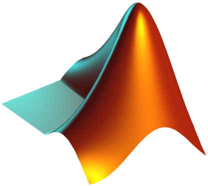

# Exercise: Rank of a map

$f : \mathcal{X} \to \mathbb{R}$

$$g = Tf$$

$g : \mathcal{Y} \to \mathbb{R}$

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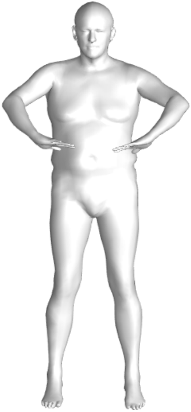
$$\mathbf{g} = \mathbf{T}\mathbf{f}$$



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ortho. basis $\mathbf{A}$

$$\mathbf{g} = \mathbf{T}\mathbf{f}$$

$g : \mathcal{Y} \to \mathbb{R}$

ortho. basis $\mathbf{B}$

# Exercise: Rank of a map

$f : \mathcal{X} \to \mathbb{R}$

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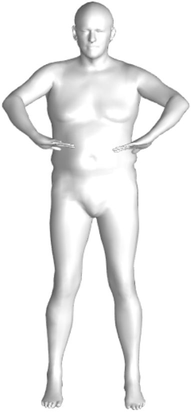
$$(\mathbf{B}\mathbf{B}^\top \mathbf{g}) = \mathbf{T}(\mathbf{A}\mathbf{A}^\top \mathbf{f})$$



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$$\mathbf{B}^\top \mathbf{g} = (\mathbf{B}^\top \mathbf{T} \mathbf{A}) \mathbf{A}^\top \mathbf{f}$$

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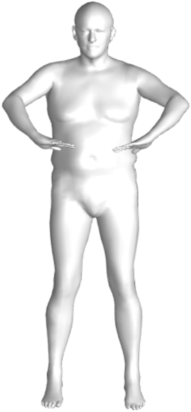
# Exercise: Rank of a map

$f : \mathcal{X} \to \mathbb{R}$

ortho. basis $\mathbf{A}$

$$\mathbf{B}^\top \mathbf{g} = \mathbf{C}\mathbf{A}^\top \mathbf{f}$$

matrix $\mathbf{C} = \mathbf{B}^\top \mathbf{T}\mathbf{A}$
is map $T$ wrt $\mathbf{A}, \mathbf{B}$ bases



$g : \mathcal{Y} \to \mathbb{R}$

ortho. basis $\mathbf{B}$

# Exercise: Rank of a map

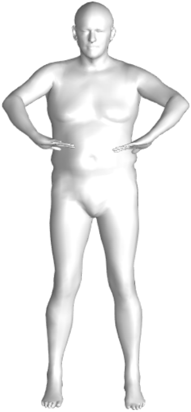
$f : \mathcal{X} \to \mathbb{R}$

ortho. basis $\mathbf{A}$

$$\mathbf{g} = \mathbf{BCA}^\top \mathbf{f}$$

matrix $\mathbf{C} = \mathbf{B}^\top \mathbf{TA}$
is map $T$ wrt $\mathbf{A}, \mathbf{B}$ bases

matrix $\mathbf{T} = \mathbf{BCA}^\top$
goes back to the std bases



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# Teaser exercise

Let $G = (V, E)$ be the graph representing mesh connectivity

Denote by $\mathbf{v}_i \in \mathbb{R}^3$ the location of vertex $i$ in space

Denote by $d_i$ the valence of vertex $i$

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Denote by $d_i$ the valence of vertex $i$

$$\mathbf{v}_i - \frac{1}{d_i} \sum_{j:(i,j)\in E} \mathbf{v}_j = 0$$

- When is the equation above satisfied?
- Write the equation using matrix notation s.t. $\mathbf{AV} = \mathbf{0}$
- What does $\mathbf{A}$ do on functions?