Feature-variant Clustering Methods for Tolling Zone Definition and Their Impact on Distance-based Toll Optimization

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ABSTRACT
Real-time network control strategies such as congestion pricing have been used in a number of metropolitan areas around the world for traffic congestion mitigation. Advances in Global Satellite Navigation System technology have led to increasing interest in distance- or usage-based road pricing as an effective alternative to traditional facility-, cordon- and area-based pricing that typically rely on fixed infrastructure. We propose the use of feature-variant clustering methods, OPTICS and HDBSCAN*, as a systematic approach for tolling zone definition. Subsequently, we utilize a framework for predictive distance-based toll optimization to evaluate network performance for the various tolling zone definitions derived from the aforementioned feature-variant clustering methods. In this framework, for a specific tolling zone definition, tolling function parameters are optimized using a simulation-based Dynamic Traffic Assignment (DTA) model operating within a rolling horizon scheme. Predictive optimization is integrated with the guidance information generation. Behavioral models capture drivers' responses to the tolls in terms of trip cancellation, mode, route and departure time. For the evaluation of network performance we make use of the real-world Expressway and Major Arterials network of Singapore and were able to demonstrate improved effectiveness of distance-based toll optimization given tolling zone definitions derived from feature-variant clustering, compared to fixed cordon-based pricing, adaptive cordon-based pricing, as well as distance-based pricing with a tolling zone definition derived in an ad-hoc manner.

Keywords: Distance-based Road Pricing, Dynamic Toll Optimization, Density-based Clustering

INTRODUCTION

Traffic congestion is a pervasive problem world-wide resulting in significant costs to the commuter, economy and environment. According to the Victoria Transport Policy Institute's Urban Mobility Report (UMR) (*1*), congestion was estimated to waste 5.5 billion hours of time delay and 2.9 billion gallons of fuel in urban areas of the United States between 2000 and 2010. The UMR predicted that congestion cost will increase from \$121 billion (in 2011) to \$199 billion (in 2020). Mitigating congestion is always a high-priority task in any transportation networks to enhance the network reliability, driver's comfort and traffic safety. Road pricing is an important approach for congestion mitigation that encourages travelers to adjust all aspects of their behavior: number of trips, destination, mode of transport, time of day, route, and so on. Facility-based and area-based pricing typically depend on physical infrastructure such as gates or gantries to detect when drivers enter or leave the tolled area. Unfortunately, if there is a change in the charging facilities/areas, physical gantries are not so easily relocated. They also ignore the distance travelled within the tolling zone leading to inefficiency. The caveats associated with facility-/area-based pricing and the advancement of Global Satellite Navigation System technology have generated interest in usage-based tolling schemes wherein charges vary in terms of distance-traveled, time spent on traveling or time spent in congestion According to (*2*), time-spent-in-congestion-based pricing outperforms distance and time-based pricing because it achieves the greatest increase in network speeds. There is concern, however, that tolls based on time spent traveling and time spent in congestion would be unpredictable, and evidence shows that they potentially encourage unsafe driving (*3*). Distance-based pricing bypasses said concern while still solving the issues present in facility-/area-based pricing, such as inflexibility regarding gantry relocation and inefficient toll charges. This has led to distance-based pricing becoming an area of increasing interest in both theory and practice. It was directly tested in the London study (*4*) and recently, Singapore has been in the process of distance-based pricing implementation with tender awarded in 2016 (*5*). In past literature, while there has been extensive research into partitioning networks based on speed, flow and density data (*6–8*) in order to make use of the Network Fundamental Diagram (NFD) concept for traffic network management strategies, including congestion pricing (*9–12*), distance-based pricing had not been considered. Distance-based congestion pricing approaches were implemented on idealized networks (*13*), at the link level (*14*), by using nested regions, where only the inner region incurs toll charges, (*15*), or by using nested pricing cordons, where each cordoned zone is priced differently (*16*). Due to the increasing significance of distance-based road pricing in traffic network management and operations, this paper proposes the use of hierarchical density-based clustering methods as a systematic way to help define sets of tolling zones. The objective is to evaluate the impact these systematically derived tolling zone definitions have on traffic network performance, when used as input to a distance-based road pricing strategy optimization framework which generates dynamic, predictive road pricing strategies together with traffic guidance. The main contributions of this paper are as follows:

1. It presents the evaluation of the dynamic distance-based tolling optimization framework in the real-world network of Singapore expressways and major arterial roads under different tolling zone definitions, derived systematically from hierarchical density-based clustering methods.
2. The clustering methods used are feature-variant. In addition to location coordinates, link speeds and link marginal external costs inform the definition of sets of tolling zones for distance-based tolling optimization application.

3. It conducts a comparative study between dynamically optimized distance-based pricing and other popular road pricing schemes, including time-dependent and adaptive cordon-based pricing.

FRAMEWORK, MODEL AND PROBLEM DESCRIPTION

In this section, we describe our predictive distance-based toll optimization framework, the optimization problem formulation, the proposed feature-variant clustering methods for tolling zone definition and the algorithmic solution for the optimization problem.

Framework

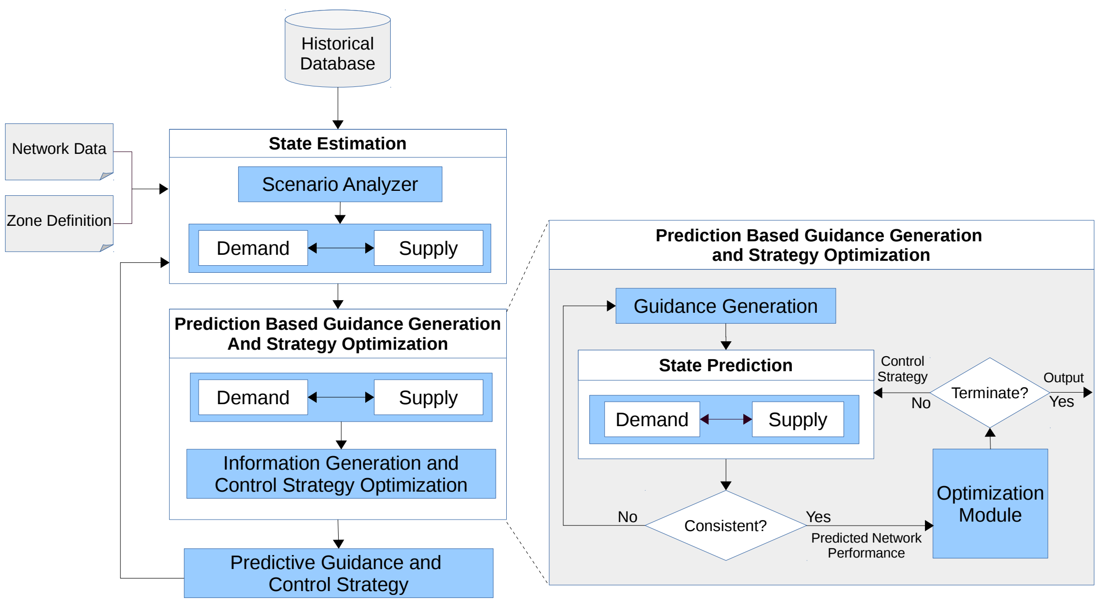


FIGURE 1: Framework for Prediction-based Guidance Generation and Distance-based Strategy Optimization

The framework for predictive distance-based toll optimization, as can be seen in in Figure 1, is using DynaMIT2.0 - a simulation-based Dynamic Traffic Assignment (DTA) system developed at the MIT Intelligent Transportation Systems Lab, (*17, 18*). DynaMIT2.0 consists two main modules: state estimation and state prediction operating in a rolling horizon scheme. The state estimation module integrates real-time data obtained from traffic surveillance system, other information accessible in the historical database, the distance-based tolling zone definition and the translated demand/supply pattern from scenario analyzer to estimate the current network state through the complex demand-supply interactions. The state prediction module predicts traffic conditions over the prediction horizon and generates consistent guidance information, which will be disseminated to the travelers during the next roll period. The control strategy optimization and guidance generation process in DynaMIT2.0 is extended to optimize distance-based tolls. Within this process, the optimization module generates a series of candidate tolling function parameters which are to be evaluated on the basis of the desired objective. The state prediction and guidance generation module takes as input a candidate solution along with an initial guidance and predicts the future network conditions through the demand-supply simulators. This process yields simulated travel times, which will be combined with the previous guidance to generate an updated guidance, and the process continues iteratively until convergence. At this point, the generated guidance is considered consistent and the predicted network performance is used by the optimization module to evaluate the specified objective function. Based on the evaluated objective function value, the optimization module will generate a new set of control strategy candidates and the procedure will continue until optimality or a pre-specified set of termination criteria (e.g. specific time limit, number of iterations) have been achieved.

Problem Description

The transportation network is represented as a directed graph $G = (N, A)$, where N represents the set of n network nodes and A represents the set of m directed links. Let L be the number of tolling zones in the network G. Each tolling zone l is defined as a subset of A, i.e. $\forall l \in \{1,...,L\}, A_l \subseteq A$, and is associated with a tolling function $\phi_l(\theta_l^t, D_l)$ that maps the distance traveled within zone l, D_l to the toll amount, and θ_l^t is a vector of tolling function parameters in time interval t. The toll payable within any tolling zone is assumed to be bounded, i.e $\tau_{LB} \leq \phi_l(\theta_l^t, D_l) \leq \tau_{UB}, \forall l = 1,2...,L \forall t = 1,2,...,T$.

Let Δ be the the size of the state estimation interval (typically 5 minutes) and $H\Delta$, the length of the prediction horizon. Without loss of generality, we assume that the optimization horizon (the future period over which we wish to set the tolling function parameters) is the same as the prediction horizon. In addition, assume that the tolling function parameters are fixed over time intervals of size Δ and that the tolling intervals are aligned with the estimation intervals of the DTA system. Consider an arbitrary estimation interval $[t_0 - \Delta, t_0]$, let $\theta^h = (\theta_1^h, \theta_2^h ... \theta_L^h)$ represent the vector of toll function parameters for all tolling zones for the time period $[t_0 + (h-1)\Delta, t_0 + h\Delta]$ where $h = 1,...,H$. The vector of toll function parameters for the current optimization horizon is thus given by $\theta = (\theta^1, \theta^2, ..., \theta^H)$.

Consider the collection of vehicles $v = 1,...,V$ on the network during the prediction horizon $[t_0, t_0 + H\Delta]$. Let the travel time of vehicle v be represented by tt^v and the predictive guidance by $\mathbf{tt}^g = (\mathbf{tt_i^g}; \forall i \in A)$, where $\mathbf{tt_i^g}$ represents a vector of the time dependent link travel times (guidance) for link i . Note that the vehicle travel times $\mathbf{tt} = (tt^v; v = 1,...,V)$ are the result of the state prediction module of DynaMIT2.0 and thus cannot be written as an explicit function of the toll function parameters and predictive guidance. Thus, we characterize the complex relationship through a single constraint that represents the coupled demand and supply simulators of DynaMIT2.0 as:

$$G(\mathbf{x^p}, \gamma^{\mathbf{p}}, \mathbf{tt^g}, \theta) = \mathbf{tt} \tag{1}$$

Where $\mathbf{x^p}$, $\gamma^{\mathbf{p}}$ represent the forecast demand and supply parameters for the prediction horizon, and θ is the vector of tolling function parameters. The iterative procedure within the state prediction module ensures consistency between $\mathbf{tt^g}$ and $\mathbf{tt}$.

Pre-trip Behavioral Model with Elastic Demand

The response of users to the predictive distance-based tolls and travel time guidance is modeled at the pre-trip choice and en-route choice levels. The pre-trip choice model, as illustrated in Figure 2, includes decisions of trip cancellation, mode, departure time and path and is formulated as a nested logit model (for relevant notation, see Table 1). Thus, a driver may choose to change his/her

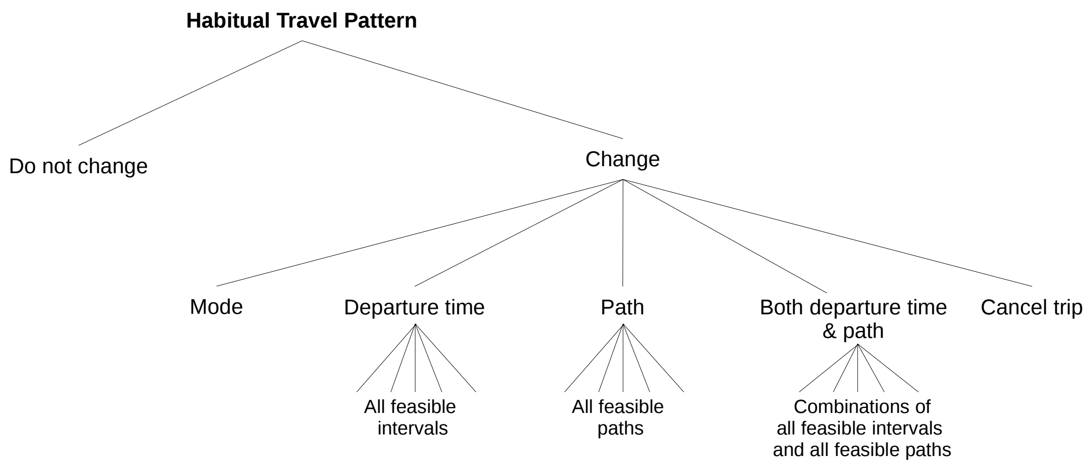


FIGURE 2: Pre-trip Behavior Model

TABLE 1: Pre-trip Behavioral Model - Abbreviations

Abbreviation	Variable
$acsCM$	Alternative Specific Constant for changing mode
$ascCT$	Alternative Specific Constant for canceling trip
$ascCDT_d$	Alternative Specific Constant for departing at time interval d
tc_m^v	money cost for traveling with another mode
tc_{dp}^v	distance-based toll for departing at d with path p
tc_p^v	distance-based toll for switching to path p
tt_m^v	travel time when traveling with another mode
tt_{dp}^g	predicted travel time for departing at d with path$p$
tt_p^g	predicted travel time for switching to path p
$at_{d'p'}^{hab}$	habitual arrival time for departing at habitual departure time interval d' and habitual path p'
at_{dp}^g	predicted arrival time for departing at departure time interval d and path p
β_c^v	coefficient for cost
β_t^v	coefficient for time
β_{early}	coefficient for early arrival
β_{late}	coefficient for late arrival
PS_p	path size variable
C_*	composite utility pertaining to additional variables including path length, number of left turns and number of signalized intersections
ε_*	random error term

habitual travel pattern (in response to the predictive guidance and tolls) in which case he/she may either change mode, cancel trip, change path or departure time, or a combination of the two. This results in elastic total demand w.r.t. traffic congestion. When it comes to mode choice, the only options we model are to drive or take public transit. The utility of change mode (CM) alternative for vehicle v is:

$$\cup^v(CM) = ascCM + \beta_c^v(tc_m^v) + \beta_t^v(tt_m^v) + \varepsilon_m \tag{2}$$

The utility of departing at time interval d and choosing path p of vehicle v is:

$$\begin{aligned}
\cup_{dp}^v &= ascCDT_{dp} + \beta_c^v(tc_{dp}^v) + \beta_t^v(tt_{dp}^g) \\
&\quad + \beta_{early}\max(at_{d'p'}^{hab} - at_{dp}^g, 0) + \beta_{late}\max(at_{dp}^g - at_{d'p'}^{hab}, 0) \\
&\quad + log(PS_p) + C_{dp} + \varepsilon_{dp} \\
where &: \\
tc_{dp}^v &= \sum_{l=1}^{L} \phi_l(\eta_{p,l}^v)
\end{aligned} \tag{3}$$

If there is a total of N combinations of path and departure time choices in the choice set, the alternative specific constant $ascCDT_d$ can only appear in $(N-1)$ utilities. The utility of the cancel trip (CT) alternative is:

$$\cup^v(CT) = ascCT + \varepsilon_{CT} \tag{4}$$

The probability of vehicle v choosing alternative c within the choice set C is given by:

$$P^v(c|C) = \frac{e^{\mathcal{V}_c^v/\vartheta}}{\sum_{a\in C} e^{\mathcal{V}_a^v}} \tag{5}$$

where ϑ is the scale parameter and $\mathcal{V}_c^v = \cup_c^v - \varepsilon_c$. The en-route choice model defines response of users in terms of path-choice to the toll and predictive travel time guidance. It is also formulated as a multinomial path size logit model where the utility of switching to path p is given by:

$$\begin{aligned}
\cup_p^v &= \beta_c^v(tc_p^v) + \beta_t^v(tt_p^g) + log(PS_p) + C_p + \varepsilon_p \\
where &: \\
tc_p^v &= \sum_{l=1}^{L} \phi_l(\eta_{p,l}^v)
\end{aligned} \tag{6}$$

The objective function for the optimization problem can be formulated from the consumers' perspective, producers' perspective or both. In this study, we adopt the objective of total social welfare (SW), defined as the sum of the consumer surplus and the producer surplus. Since we have simulated the choices of each traveler, the consumer surplus (CS) in this setting is simply the sum of the experienced utilities of each traveler. The producer surplus is the net toll revenue (TP) given by the total toll revenue (TR) minus the fixed costs (FC) and variable costs (VC). Thus, $SW = CS + TP = CS + (TR - FC - VC)$. Fixed costs are assumed to be 0. For the computation of total social welfare, the fraction of total toll revenue needed to cover toll operation costs needs to be identified. According to the report in (*19*), the cost of operating an electronic road user charge scheme based on distance driven should be minimal and experience in the United States and other countries suggests that the administrative and enforcement costs of collecting user fees would be in the range of 5% to 13% of collections. With this in mind, it is assumed that the toll operation costs occupy 10% of the total toll revenue. Thus, the variable cost component is assumed to be a portion of the total collected toll revenue with factor $\alpha < 1$. The total social welfare is given by:

$$\begin{aligned} SW &= CS + TP \\ &= CS + (TR - FC - VC) \\ &= \sum_{v=1}^{V} \frac{\cup^v}{|\beta_c^v|} + \left[(1-\alpha) \times \sum_{v=1}^{V} tc^v \right] \end{aligned} \tag{7}$$

The absolute value of β_c^v is used to express the consumer surplus in dollar equivalents. The dynamic distance-based toll optimization problem (DDTOP) in our context is formulated as a non-linear programming problem given in Equation 8. The decision variables are the vector of tolling function parameters for the optimization horizon period. The objective function is the total social welfare. The constraints are the DynaMIT2.0 system and upper and lower bounds on the toll values.

$$\begin{aligned} &\textbf{DDTOP} : \max_{\theta} \left[\sum_{v=1}^{V} \frac{\cup^v}{|\beta_c^v|} + (1-\alpha) \times \sum_{v=1}^{V} tc^v \right] \\ &\quad \textbf{s.t.} \\ &\quad G(\mathbf{x^p}, \gamma^{\mathbf{p}}, \mathbf{tt^g}, \theta) = \mathbf{tt} \\ &\quad \tau_{LB} \leq \phi_l(\theta_l^h, D_l^v) \leq \tau_{UB}, \forall v = 1,2,...,V; l = 1,2...,L; h = 1,2,...,H \end{aligned} \tag{8}$$

Tolling Zone Definition using OPTICS and HDBSCAN*

One of the inputs with significant impact to our distance-based tolling system performance is the tolling zone definition. This input specifies the number of tolling zones and the list of links for each zone. Each tolling function $\phi_l(\theta_l^h, D_l^v)$ corresponds to one tolling zone. To the authors' knowledge, no systematic approach to define distance-based road pricing zones can be found in current research and practice. For our experiments, we originally employed an ad-hoc approach to define distance-based tolling zone based on the gantry locations of the cordon-based road pricing system already in operation. In this approach, each distance-based tolling zone corresponds to one gantry and is defined as the area of the network which the vehicles could only access by passing through the gantry. This approach has a similar impact to the drivers' route choice as the gantry system but with added impact of distance to the final decision. While, with this approach, the impact of distance could be directly observed, by comparing the network conditions with cordon-based tolling to the conditions arising from distance-based tolling, it is unsuitable for cases with no pre-existing cordon-based road pricing systems and, most importantly, for large-scale urban network application. To address these shortcomings, we decided to investigate the potential use of clustering methods for tolling zone definition, to be used as input on our distance-based tolling system.

Clustering Methods

We propose the feature-variant implementation of two well-known hierarchical density-based clustering methods, OPTICS (*20*), (Ordering Points To Identify the Clustering Structure), and HDBSCAN (*21*), (Hierarchical Density-Based Spatial Clustering of Applications with Noise). While DBSCAN (*22*), (Density-Based Spatial Clustering of Applications with Noise), is the more established among density-based clustering methods, it is very sensitive to parameter selection and has difficulty in coping with clusters characterized by large inter-cluster density variability. OPTICS and HDBSCAN*, on the other hand, do not suffer from these caveats.

OPTICS, like DBSCAN, requires two parameters: ε, which describes the maximum distance radius around a particular data point and κ, describing the minimum number of data points used as density threshold for cluster assignment. Assume $X = \{x_1, x_2, ..., x_n\}$ a set of data points in a metric space (X, d). We consider a data point x to be a core point with respect to ε and κ if its ε-neighborhood $N_\varepsilon(x)$ contains a minimum of κ data points. Two core points x_i, x_j are ε-reachable with respect to ε and κ if they are both contained within each others ε-neighborhood. Two core points x_i, x_j are density-connected with respect to ε and κ if they are directly or transitively ε-reachable. A cluster C, with respect to ε and κ, is a non-empty maximal subset of X such that every pair of data points in C is density-connected. This definition of cluster results in the DBSCAN algorithm, upon which both OPTICS and HDBSCAN* are based. OPTICS also considers data points that are part of a more densely packed cluster, so each point is assigned a core distance that describes the distance to the κ-th nearest neighbor:

$$d_{\text{core}}^{\varepsilon,\kappa}(x) = \begin{cases} \text{Undefined} & \text{if } |N_\varepsilon(x)| < \kappa \\ \kappa\text{-th smallest distance to } N_\varepsilon(x) & \text{otherwise} \end{cases} \tag{9}$$

The reachability-distance of data point x_i from data point x_j is either the distance between x_i and x_j, or the core distance of x_i, whichever is bigger:

$$d_{\text{reach}}^{\varepsilon,\kappa}(x_i, x_j) = \begin{cases} \text{Undefined} & \text{if } |N_\varepsilon(x_i)| < \kappa \\ \max(d_{\text{core}}^{\varepsilon,\kappa}(x_i), d(x_i, x_j)) & \text{otherwise} \end{cases} \tag{10}$$

In our OPTICS implementation, we make use of a single global ε' value to extract a flat clustering. HDBSCAN* is essentially the same as OPTICS but parameter $\varepsilon = \infty$, whereas OPTICS simply constrains the range of ε. It also employs a different technique to extract a flat clustering, based on the stability of clusters. In the case of HDBSCAN*, we have:

$$d_{\text{core}}^{\kappa}(x_i) = \text{Distance of the} \kappa\text{-th nearest neighbor of } x_i \tag{11}$$

For a given fixed value κ, the mutual reachability distance, derived from the metric d, is defined, as follows:

$$d_{\text{mreach}}^{\kappa}(x_i, x_j) = \begin{cases} \max(d_{\text{core}}^{\kappa}(x_i), d_{\text{core}}^{\kappa}(x_j), d(x_i, x_j)) & \text{if } x_i \neq x_j \\ 0 & \text{if } x_i = x_j \end{cases} \tag{12}$$

HDBSCAN* generates a complete hierarchy of clusterings for a range of possible ε values, and thus for any fixed ε value , the clustering at that level in the hierarchy is going to be the clustering DBSCAN would give for that specific ε value. In DBSCAN, only core points belong to clusters.

If a point is not a core point it is considered noise and is not assigned to any cluster. Reachability-distance, both in the case of OPTICS and HDBSCAN*, captures this distinction by ensuring a point is not joined into a cluster until the DBSCAN ε value is such that the point is within the relevant distance of the other data points in the cluster and the point is a core point at that ε value. While the proposed clustering methods produce clusterings where some data points are considered noise, the nature of our problem, tolling zone definition, mandates the assignment of all data points to at least one of the resulting clusters. Therefore, a secondary assignment takes place, whereupon we employ a heuristic that assigns noise data points to previously assigned data points, based on minimum Euclidean distance.

Clustering Performance Metrics

We elected to use two standard internal evaluation indices for the clusterings produced by OPTICS and HDBSCAN*, the Silhouette Coefficient (SC) (*23*) and the Davies-Bouldin index (DB)(*24*). In short, SC measures two quantities, cohesion $a(x)$, which measures average distance between data points within the cluster and separation $b(x)$, which measures the minimum average distance of data points to other clusters. Then silhouette $s(x)$ is defined as:

$$s(x) = \frac{b(x) - a(x)}{max\{a(x), b(x)\}} \tag{13}$$

where $s(x) \in [-1, 1]$ measures at -1 for incorrect clustering, around 0 for overlapping clusters and 1 for highly dense clustering. SC is the average for all data points and can be calculated using:

$$SC = \sum_{i=1}^{N} s(x_i)/N \tag{14}$$

DB is a function of the ratio of intra-cluster scatter to inter-cluster separation. The intra-cluster scatter, S_l, also known as cluster diameter, describes the average of Euclidean distances of each individual data point belonging to cluster C and the cluster centroid. If we define $d_C^{l,m}$ as the inter-cluster centroid distance, then, for number of clusters w, DB can be calculated as follows:

$$\text{DB} = \frac{1}{w} \sum_{l=1}^{w} \max_{l \neq m} \left(\frac{S_l + S_m}{d_C^{l,m}} \right) \tag{15}$$

DB values closer to 0 indicate a better clustering result.

Clustering Method Results

The network features we opted to use, in addition to location coordinates, are link speed and link marginal external cost. While speed has often been used as one of the features for spatial clustering of traffic networks (*25*), marginal external cost has not been considered before as a feature. Marginal external cost has been used at the link level for first-best toll pricing studies. For large road networks, calculating marginal external costs for individual links, at each time interval, can be extremely onerous, which limits the potential for real-time large-scale practical applications (*26*). Our innovation lies in the fact that we make use of link marginal external costs, averaged over a predetermined period, which inform the definition of appropriate tolling zones for distance-based tolling optimization application, rather than calculating tolls for each link throughout the peak

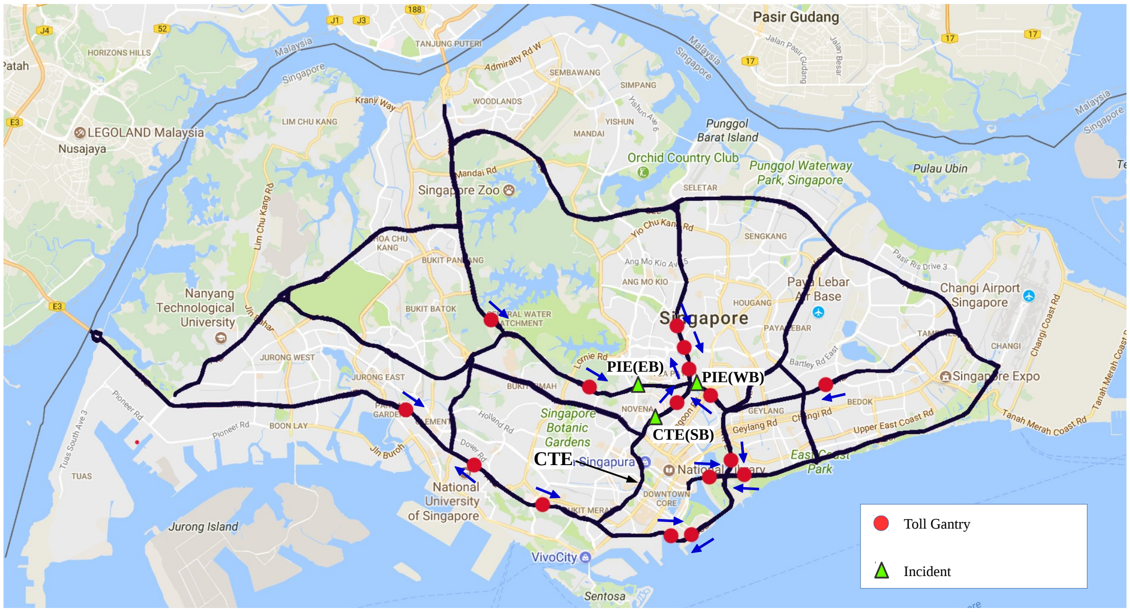


FIGURE 3: The Network of Singapore Expressways and Major Arterials

period. This allows for the practical implementation of our distance-based tolling optimization approach on large-scale road networks. While getting the link speed is quite straightforward, such is not the case for the link marginal external cost. The link flow-speed-density relationship used in DynaMIT2.0 is as follows:

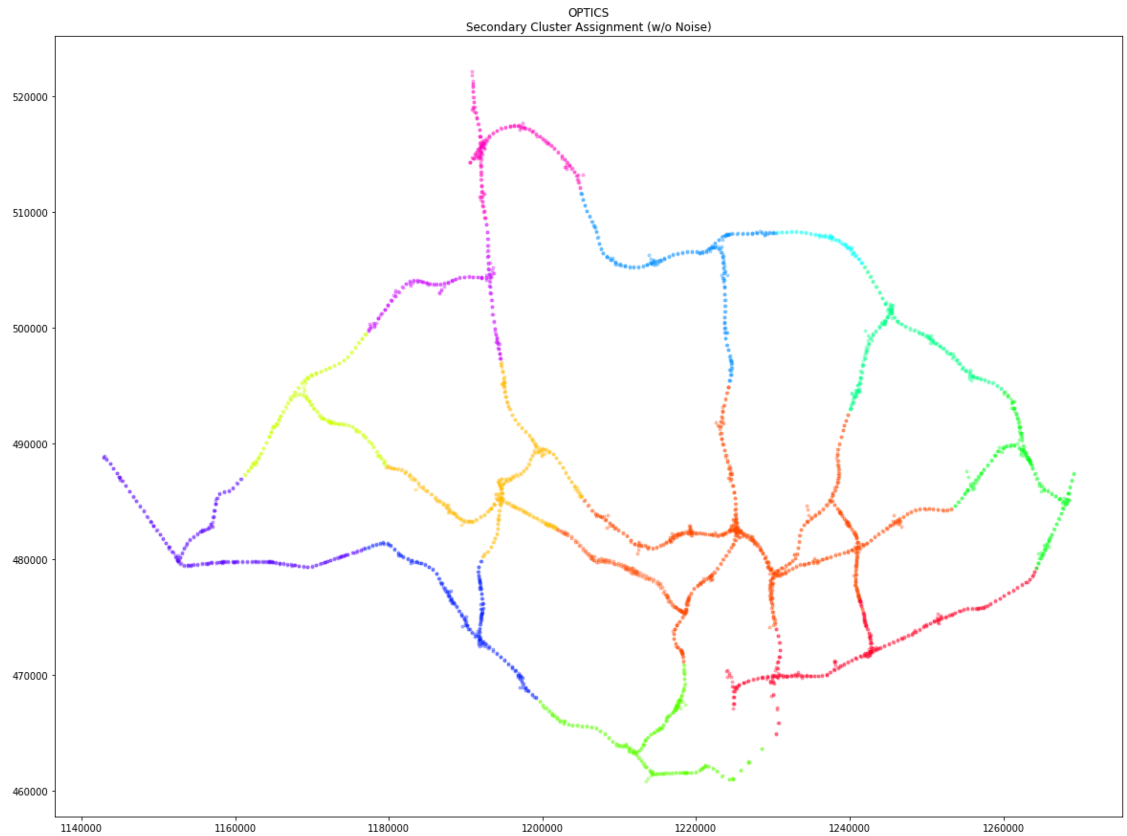
$$q = k \cdot v_f \left[1 - \left(\frac{k}{k_{jam}} \right)^{\chi} \right]^{\psi} \tag{16}$$

Where χ, ψ are model parameters and q, v_f, k, k_{jam} represent flow, free flow speed, density and jam density respectively. From the literature (*26*), marginal external cost for link i can be calculated as:

$$\delta_i = q \cdot \left(\frac{dt}{dq} \right) \tag{17}$$

Assuming s represents link length, from Equations 16 and 17 we have:

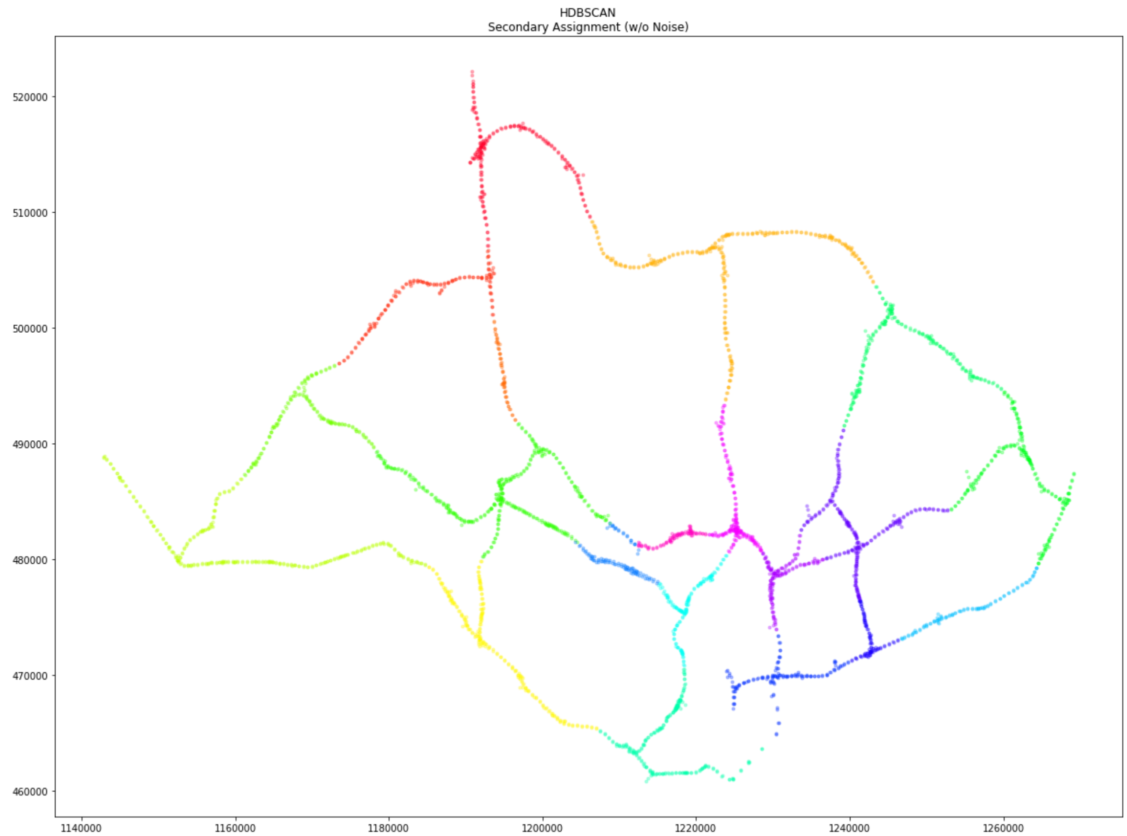
$$\delta_i = -q \cdot s \cdot \left(\frac{\chi \psi k_{jam} \left(\left(\frac{v}{v_f} \right)^{1/\psi} - 1 \right)}{\left(\left(1 - \frac{v}{v_f} \right)^{1/\psi} \right)^{-1/\chi} \left(\chi \psi \left(\frac{v}{v_f} \right)^{1/\psi} - \chi \psi + \left(\frac{v}{v_f} \right)^{1/\psi} \right)} \right) \tag{18}$$



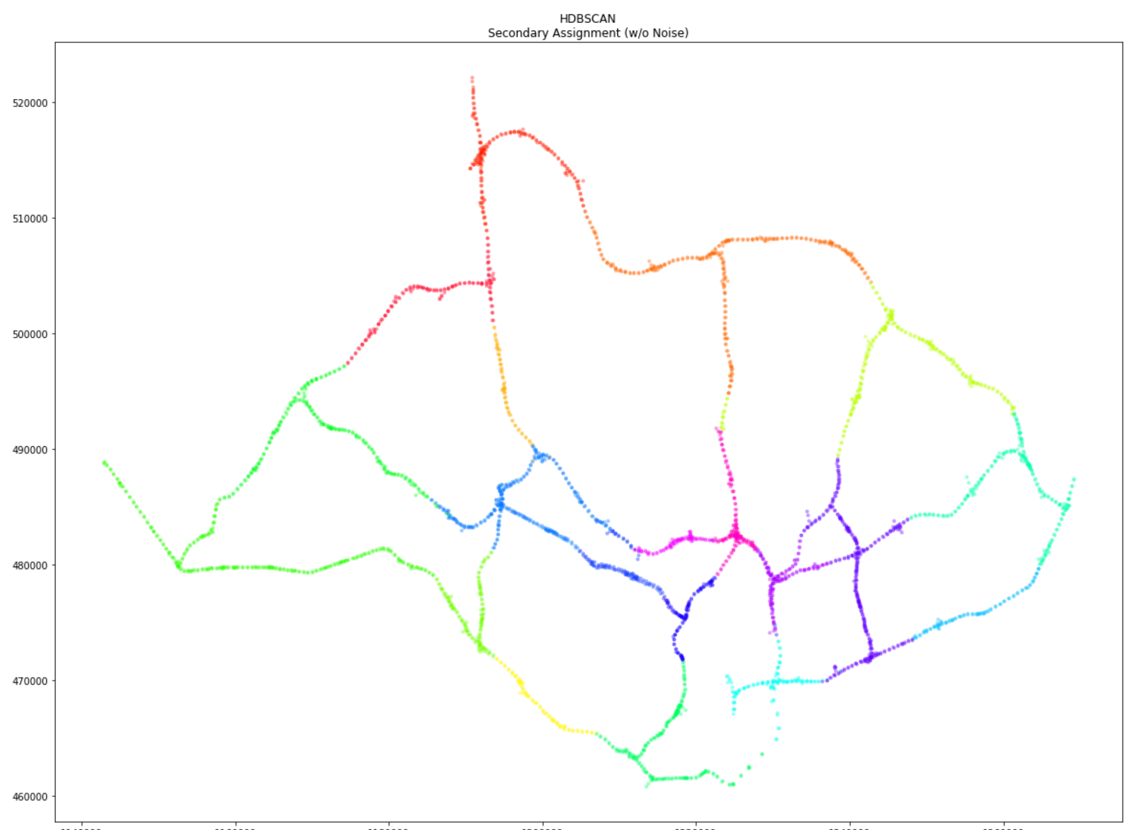
(a) 13 zones derived using OPTICS for feature SPD, with SC=0.339, DB=0.804

(b) 11 zones derived using OPTICS for feature MEC, with SC=0.355, DB=0.759

FIGURE 4: Tolling zone definitions and clustering performance results from using OPTICS



(a) 20 zones derived using HDBSCAN* for feature SPD, with SC=0.394, DB=0.744



(b) 20 zones derived using HDBSCAN* for feature MEC, with SC=0.393, DB=0.756

FIGURE 5: Tolling zone definitions and clustering performance results from using HDBSCAN*

Certain requirements have to be met, in order to guarantee low computational overhead, feasibility and ultimately, practical application potential of this framework. First and foremost, the number of tolling zones has to be kept low, to allow for feasible practical implementation. All resulting clusters must be spatially compact, i.e., not have links in completely disparate locations belong to the same cluster. Tolling zone definitions may not change throughout the simulation horizon, to limit computational overhead and subsequently ensure the feasibility of a practical implementation for large-scale networks.

The data set used for clustering is collected from the network of Singapore Expressways and Major Arterial Roads as illustrated in Figure 3. This network has 939 nodes, 1157 links, 3906 segments, 10954 lanes and 4120 Origin-Destination pairs. After taking into consideration the requirements described above, with careful parameter tuning, we were able to produce 4 tolling zone definitions, 2 using the link speed feature, referred to as SPD, and 2 using the average link marginal external cost feature, referred to as MEC, for OPTICS and HDBSCAN* respectively. The tolling zone definitions for each clustering method and feature, as well as the clustering performance index results, are presented in Figures 4,5.

Solution

Due to the highly non-linear nature of the objective function of the DDTOP, with no guarantee of convexity, as well as the obvious difficulty to define closed analytical expressions that represent the drivers' dynamic route choice decisions as a function of the overall network traffic state and the developed tolling strategy, we decided to adopt heuristic-based algorithms that could provide near optimal solutions.

In the current implementation, we adopt a Genetic Algorithm (GA) approach to optimize the parameters of distance-based tolling functions. The algorithm starts with a random population of N control strategy individuals, which form the initial population or parent population. Each control strategy individual is a vector of distance-based toll function parameters for different road pricing zones in the network during the current optimization horizon: $\theta = (\theta^1, \theta^2, ..., \theta^H)$. The Optimization Module then sends this population of control strategies to the State Prediction module of DynaMIT2.0 for evaluation with the predefined objective function. The strategies in the initial population is then ranked based on the values of objective function (in our case, the total social welfare). From this N-individual parent population, genetic operators (such as tournament selection, SBX crossover, polynomial mutation) are applied to generate another N-individual child population. The newly generated children are then evaluated with DynaMIT2.0 State Prediction. After that they are merged with the parent population to form a bigger population of $2N$ individuals and out of which, N best individuals are selected to be the parent population for the next generation. The algorithm iteratively moves on until some stopping conditions are met. The stopping condition could be the computation budget in term of time, the number of generations/iterations or the minimum improvement in the objective function.

In order to speed up the optimization, parallel computing is employed in which the evaluations of different control strategy individuals are conducted in parallel, each in one processing unit, which is expected to bring the execution time closer to real-time performance.

EXPERIMENTS AND RESULTS

Experiment Settings

In order to investigate the impact tolling zone definitions, which are derived from feature-variant clustering methods, have on distance-based road pricing control strategy performance, the experiments are conducted in the network illustrated in Figure 3. Three incidents from the historical database of Land Transport Authority of Singapore are also simulated to further emphasize the impact of non-recurrent congestion on the network conditions. The selected incidents include 2 major incidents on the PIE (East Bound and West Bound) and one major incident on the CTE (South Bound) as also shown in Figure 3. The simulation period is from 07:00-11:00 covering the morning peak. The simulation interval is 5 minutes and the prediction horizon is 30 minutes. Since the objective function is total welfare, the comparison is only meaningful if it is done for a specific population of travelers including the ones who cancel or change mode. Therefore, the statistics are calculated for the population of vehicles with habitual departure time within 07:00-10:00 (these drivers may later change the departure time in response to the traffic conditions). The last one hour of simulation (10:00-11:00) is the cooling down period without toll optimization to make sure all the vehicles with habitual departure time in 07:00-10:00 finish their trips and produce necessary statistics. The benchmarking is done over 8 scenarios, which are briefly described on Table 2. The base scenario **S0** is **No Pricing**, meaning it is without any sort of pricing and its

TABLE 2: Brief summary of the 8 scenarios

Scenarios	Description
S0	**No Pricing**, used as a baseline for all comparisons
S1	**Time-dependent cordon-based pricing**, based on real toll settings
S2	**Adaptive cordon-based pricing**, adaptive tolls for fixed gantries
S3	**Dynamically optimized distance-based pricing (ad-hoc)**
S4	**Dynamically optimized distance-based pricing (OPTICS-SPD)**
S5	**Dynamically optimized distance-based pricing (OPTICS-MEC)**
S6	**Dynamically optimized distance-based pricing (HDBSCAN*-SPD)**
S7	**Dynamically optimized distance-based pricing (HDBSCAN*-MEC)**

results will be used as a baseline for all comparisons. The first scenario **S1** is with **time-dependent cordon-based pricing**. The tolls for this scenario are collected from the real toll settings of the Singapore Expressway Network including 16 gantries, whose location is illustrated in Figure 3. The second scenario **S2** is **adaptive cordon-based pricing** in which, tolls for fixed gantry locations are optimized. The third scenario **S3** is **dynamically optimized distance-based pricing**, where tolling zone definition is derived in an **ad-hoc** manner, using the fixed gantry locations as a starting point and containing the area of the network which the vehicles could only access by passing through each gantry. The fourth scenario **S4** is **dynamically optimized distance-based pricing (OPTICS-SPD)**, where tolling zone definition is derived from speed data using OPTICS. The fifth scenario **S5** is **dynamically optimized distance-based pricing (OPTICS-MEC)**, where tolling zone definition is derived from MEC data using OPTICS. The sixth scenario **S6** is **dynamically optimized distance-based pricing (HDBSCAN*-SPD)**, where tolling zone definition is derived from speed data using HDBSCAN*. Finally, the seventh scenario **S7** is **dynamically optimized distance-based pricing (HDBSCAN*-MEC)**, where tolling zone definition is derived from MEC data using HDBSCAN*. The objective function for all scenarios is to maximize the total social welfare (SW), however, it would be interesting to compare additional performance indices such as the consumer surplus (CS) and average travel time (TT), through which we can better observe the impact on individual travelers, in money terms and travel time respectively.

Results

The network performance, in terms of SW, CS and TT, can be found on Table 3. Furthermore, performance improvement (%) over the base scenario S0 is illustrated on Figure 6. As is evident from the results, tolling zone definitions with more than 16 zones, which is the number of zones derived in an ad-hoc manner for S3, can have a very positive impact on distance-based tolling strategy performance. S7, with 20 zones, shows the highest performance improvement among all scenarios, with over 30% improvement for SW, almost 20% improvement for CS and over 23% improvement for TT. It should be noted that, even with relatively high number of tolling zones, performance improvement results may also vary. S6, also with 20 zones, shows only moderate improvement of about 21% for SW, 12% for CS and 16% for TT, which is comparable to the performance improvement of S3. This can be attributed to the type of data used as a feature. In this case, HDBSCAN* produces clustering results with higher positive impact on distance-based tolling optimization, when MEC, rather than speed data are used. S4, with 13 zones, presents with more than 25% improvement over the base scenario S0 for SW, more than 16% for CS and over 20% improvement for TT, which even surpasses S6, with 20 zones. In this case, OPTICS performs better when speed data are used. A low number of tolling zones, however, can lead to unsatisfactory results. Such is the case of S5, with 11 zones, which only manages to improve performance by about 10% for SW, 5% for CS and 8% for TT. This can be explained by the fact that, if one observes the clustering result from MEC data using OPTICS on Figure 4b, it consists of a very large cluster encompassing most approaches to the south-east, where Singapore's Central Business District is located, and all other clusters are much smaller and positioned at the perimeter of the one large cluster. This clustering result is counter-intuitive, as the intra-cluster density variability contained within this large cluster is expected to be high, while the optimization module will provide the same tolling function parameter values for the entire cluster. One could argue that, since this is distance-based pricing, travel distance also plays a role, on the actual toll paid by each vehicle. However, since the simulation horizon covers the morning peak period, a high percentage of vehicles has the Central Business District as their destination, hence travel distances within the large cluster should be homogeneous.

TABLE 3: Performance metrics SW, CS and TT for **S1, S2, S3, S4, S5, S6, S7**

	Scenarios						
Metrics	$S1$	$S2$	$S3$	$S4$	$S5$	$S6$	$S7$
SW ($)	395197.8	501978.6	636316.9	761938.1	309681.0	630773.1	905603.2
CS ($)	329423.9	406217.8	476413.7	498388.9	158131.3	359741.1	586320.2
TT (s)	608.5	586.3	564.6	532.9	614.5	559.4	512.5

CONCLUSIONS AND FUTURE WORK

In this paper we investigated the impact tolling zone definitions, derived from feature-variant hierarchical density-based clustering methods, have on traffic network performance, when used as

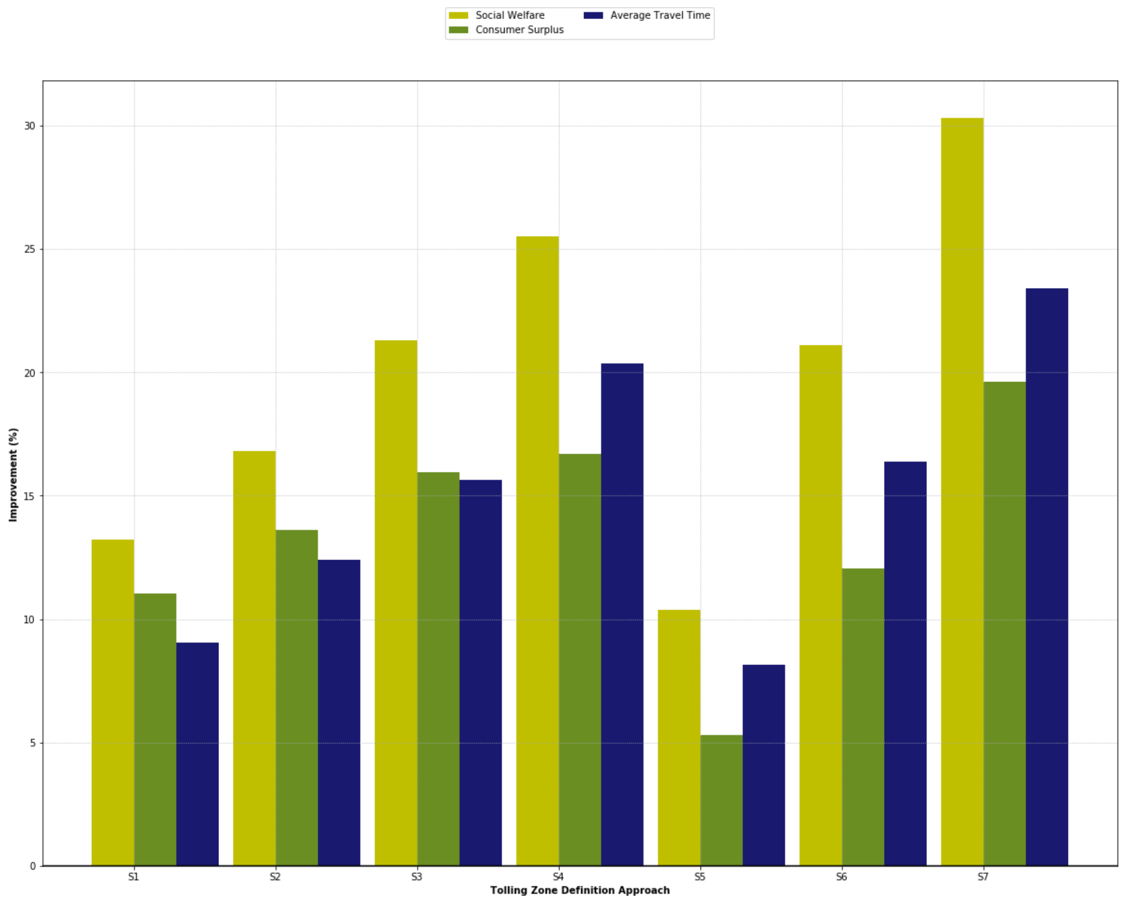


FIGURE 6: Performance improvement for SW, CS and TT over **S0** for scenarios **S1,S2,S3,S4,S5,S6,S7**

input for a predictive distance-based toll optimization framework. We evaluated this framework's performance against other popular road pricing schemes, including adaptive cordon-based pricing, on the real-world Expressways and Major Arterials network of Singapore. We determined that the best network performance comes from the use of a tolling zone definition derived from link marginal external cost data (MEC), using HDBSCAN*. When using a tolling zone definition derived from link speed data, using HDBSCAN*, performance was better than time-dependent and adaptive cordon-based pricing schemes and comparable to that of a tolling zone definition derived in an ad-hoc manner. While we were able to get very good performance from the use of a tolling zone definition derived from link speed data (SPD), using OPTICS, we also got the worst performance when we used a tolling zone definition derived from link marginal external costs (MEC), using OPTICS. This warrants further investigation, however, HDBSCAN* has proven to be more robust in providing a tolling zone definition, which, when used as input to our predictive distance-based toll optimization framework, has a positive impact on network performance. In conclusion, we have determined that the best choice to systematically derive a tolling zone definition, for this particular network data set, is the HDBSCAN* clustering method.

In future work, we aim to investigate the possibility of implementing OPTICS and HDBSCAN*, as well as other appropriate clustering algorithms, to define sets of tolling zones on data sets from urban traffic networks, with additional features (density, speed, marginal external cost or any combination thereof), and evaluate their impact on network performance when used as part of the distance-based tolling optimization framework.

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AUTHORS CONTRIBUTION

The authors confirm contribution to the paper as follows: study conception and design: A.F. Lentzakis, R. Seshadri, A.P. Akkinepally, V. Vu, M. Ben-Akiva; analysis and interpretation of results: A.F. Lentzakis, R. Seshadri, A.P. Akkinepally; draft manuscript preparation: A.F. Lentzakis, R. Seshadri, A.P. Akkinepally. All authors reviewed the results and approved the final version of the manuscript.

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