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# Strategic Learning Approach to Region-based Dynamic Route Guidance

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*Abstract*— **In this paper we aim to implement an urban network-level traffic management scheme to mitigate congestion in urban areas by considering the effect of route choice at an aggregated level. We first present an implementation of a region-based dynamic traffic model. Then we present a strategic learning approach and a test case where we compare our approach to other routing strategies we have integrated within the region-based dynamic traffic model.**

*Index Terms* – route guidance, strategic learning, regret matching, urban-scale MFD

## I. Introduction

Drivers, either through the use of smartphone applications or vehicle-integrated navigation equipment, are able to receive up-to-date information, including map location, prevailing traffic state and expected travel times. Route guidance can benefit drivers individually, however, through coordination with traffic control centres, system-wide benefits can be obtained. Congestion during peak periods can be alleviated through a suggestion mechanism that will enable the distribution of vehicles along different paths with similar origin and destination locations. The application of this concept in large urban traffic networks is regrettably not a trivial problem, due to the large number of links, intersections and most importantly, drivers. Region-based route guidance can be considered as route guidance at an aggregated level. Any large urban traffic network can be appropriately partitioned into regions. The suggestion mechanism then presents a set of region sequences that drivers can follow from their origin region to their desired destination region. This should ideally lead to minimization of average vehicle delay and maximization of available resource use, in this case, the regions of the urban traffic network.

Geroliminis and Daganzo [1] posited that the well-known relationship from freeway traffic modeling, the Macroscopic Fundamental Diagram, can also exist for urban areas. The Urban-scale MFD can relate the number of vehicles within a certain region, called accumulation, to the product of average network flow and length, called production. A well-defined Urban-scale MFD can only exist under homogeneous distribution of congestion. Geroliminis and Sun [2], Mazloumian et al. [3], Daganzo et al. [4] have demonstrated the importance of network link density variability in acquiring a well-defined Urban-scale MFD. Additionally, work from [5], [6], [7], [8] indicates that network topology, traffic signal settings, route distribution and demand can actually affect the total output of a region. There are two approaches in mitigating these effects. One approach is to incorporate link density variability, a measure of congestion heterogeneity, in the relationship defined by the Urban-scale MFD, deriving the so-called Generalized MFD, as in [9], [10]. Another approach is to partition the urban traffic network into homogeneously congested regions with well-defined Urban-scale MFDs. Examples of different partitioning schemes resulting in homogeneously congested regions can be found in [11], [12].

The concept of Urban-scale MFD has been used extensively in research into perimeter control schemes, [13], [14], [15], [16], [17], [18], but to a lesser extent in routing applications, [19], [20], [21]. The objective of this paper is to implement a region-based dynamic traffic model and subsequently apply different routing strategies. We will compare our strategic learning approach to dynamic route guidance with several routing strategies, including periodically adjusted routing, which gives the shortest regional path for drivers to follow, based on prevailing traffic conditions and logit routing, which gives the Logit-based Dynamic Stochastic User Equilibrium, DSUE for brevity, representing a more realistic route choice behavior.

We propose a novel route guidance scheme which applies a strategic learning approach, Proxy Regret Matching, based on [22] and modified for a repeated game where players, representing the regions acting as origins in our network setup, compete for a region designated as the destination. Resulting routes can be considered as the Correlated Equilibria of the aforementioned repeated game, essentially a distribution over the available paths for each player/origin. This approach lends itself to region-based applications, rather than applications where each driver is considered a player, since the resulting distribution of paths can quite readily be interpreted as fractional flows from each origin region to each destination region.

Additional benefits of our region-based dynamic route guidance approach include the convergence in polynomial time for Correlated Equilibria and the decreased computational complexity due to the use of a region-

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based dynamic traffic model, rather than a detailed urban traffic network model, where each road and each intersection are modeled individually.
The rest of this paper is organized in the following manner. In Section II, we present the Network Transmission Model, a region-based dynamic traffic model that employs the Urban-scale MFD to accurately describe the traffic dynamics for homogeneously congested regions. In Section III, we present our strategic learning approach to region-based dynamic route guidance, Proxy Regret Matching. In Section IV we present our network setup. We also present alternative routing strategies. We then use our simulation results to compare the performance of our strategic learning approach to said strategies. In Section V we present our conclusions and possible future work.

## II. Network Transmission Model

### A. Notations and Terminologies

In order to describe region-based traffic dynamics, we make use of the Network Transmission Model, NTM for brevity. The NTM describes a multi-layer network that comprises of a regional and an urban traffic layer and takes into account the dynamics of a network as expressed through the Urban-scale MFD, as well as the limited capacity of the boundaries between regions [23].
The Network Transmission Model combines the concepts of Daganzo's Cell Transmission Model [24], CTM for brevity, and a node model by Jin et al. [25] and applies it to a model describing urban traffic network dynamics at a regional level. In short, the urban traffic network is partitioned into regions and we derive the Urban-scale MFD for each region. The interregional flow at each stage is determined by the Urban-scale MFD, separated in a demand and a supply. For the demand, the boundary capacity between two regions also comes into play.
We will now define the constant quantities of our network represented as a directed graph $G = (K, A)$, as follows:

- $\mathcal{A}$: set of arcs, $|\mathcal{A}| = l$, representing region boundaries
- $\mathcal{K}$: set of nodes,$|\mathcal{K}| = m$, representing homogeneous regions
- $\mathcal{K}_o \subset \mathcal{K}$: set of origins,$|\mathcal{K}_o| = o$
- $\mathcal{K}_d \subset \mathcal{K}$: set of destinations,$|\mathcal{K}_d| = d$
- $\forall \kappa \in \mathcal{K}, \forall b \in \mathcal{K}_o, \forall e \in \mathcal{K}_d$

$$\begin{aligned}\mathcal{P}_{b,e} := \; & \{(b, k_1, ..., k_w, e) \,|w \in \mathbb{N} \\ & \wedge \left[\forall i \in \{1, ..., w\}, k_i \in \mathcal{K} \setminus \{b, e\}\right] \\ & \wedge (b, k_1) \in \mathcal{A} \wedge (k_w, e) \in \mathcal{A} \\ & \wedge \left[\forall j \in \{1, ..., w-1\}, (k_j, k_{j+1}) \in \mathcal{A}\right] \\ & \wedge \left[\forall i, j \in \{1, ..., w\}, i \neq j \implies k_i \neq k_j\right]\},\end{aligned}$$

  set of all paths from origin $b$ to destination $e$
  - $\mathcal{P} := \mathcal{P}_b \bigcup \mathcal{P}_e$, set of all possible paths, where $\mathcal{P}_e := \bigcup_{b \in \mathcal{K}_o} \mathcal{P}_{b,e}$, $\mathcal{P}_b := \bigcup_{e \in \mathcal{K}_d} \mathcal{P}_{b,e}$
- $\forall i, j \in \mathcal{K}$
  - $C = \left[c_{i,j}\right]^{m \times m} \geq 0$, $c_{i,j} > 0 \iff (i, j) \in \mathcal{A}$, the boundary capacity matrix
  - $v_{i,f} \in \mathbb{R}_+$: average speed for region $i$ under free-flow conditions
  - $n_{i,crit} \in \mathbb{R}_+$: critical accumulation for region $i$
  - $Q_{i,crit} \in \mathbb{R}_+$: average network flow for region $i$ at capacity flow conditions
  - $\forall z \in \mathcal{Z}_i$, where $\mathcal{Z}_i$ the set of links in region $i$
    * $\lambda_z$: number of lanes on link $z$
    * $\Lambda_z$: length of link $z$

For a simulation time $T_H$ $(s)$ and sample interval $T_s$ $(s)$, we have simulation horizon $H = \lfloor T_H / T_s \rfloor$. Then set $\mathcal{H} = [0, 1, ..., H-1]$ represents the division of horizon $H$ in discrete time steps $h \in \mathcal{H}$.
Making use of the derived Urban-scale MFDs, we model the interregional traffic dynamics. In each region $i \in \mathcal{K}$ , with $\mathcal{K}$ the set of all regions, the accumulation $n_i$ and the production $Q_i$ are defined as the weighted average density and flow of all links in region $i$ respectively. Dixit and Radwan [26] as well as more recent results from Knoop et al. [20] have shown that the MFD approximation derived by Drake et al. [27] is suitable for region-based approaches. Now we can define the dynamic quantities of our network:

- $\forall i \in \mathcal{K}, z \in \mathcal{Z}_i, e \in \mathcal{K}_d$
  - $n_z(h)$: number of vehicles on link $z \in \mathcal{Z}_i$ at step $h$
  - $n_i(h) = \dfrac{\sum_{z \in \mathcal{Z}_i} (\lambda_z \Lambda_z n_z(h))}{\sum_{z \in \mathcal{Z}_i} \lambda_z \Lambda_z}$, accumulation in region $i$ i at step $h$
  - $n_{i,e}(h)$: $e$ destination-specific accumulation in region $i$ at step $h$
  - $Q_i(n_i(h)) = n_i(h) v_{i,f} e^{\left(-\frac{1}{2}\left(\frac{n_i(h)}{n_{i,crit}}\right)^2\right)}$, approximation function to the Urban-scale MFD, according to [27]
  - $v_i(n_i(h)) = v_{i,f} e^{\left(-\frac{1}{2}\left(\frac{n_i(h)}{n_{i,crit}}\right)^2\right)}$, average network speed approximation function , according to [27].

  As in [24], the minimum of the boundary capacity, $c_{i,j}$, the demand, $Z_{i,j}$ and the supply, $P_j$, gives us the flow from region $i$ to all regions $j$ such that $(i, j) \in \mathcal{A}$. An important distinction from the CTM, when accumulation exceeds the critical value, regional demand decreases, accounting for possible internal traffic jams which limit the outflow.

- $\forall e \in \mathcal{K}_d, \forall i, j \in \mathcal{K}, (i, j) \in \mathcal{A}$

- $P_j(h) = \begin{cases} Q_{j,crit} & \text{if } n_j(h) \leq n_{j,crit} \\ Q_j(n_j(h)) & \text{if } n_j(h) > n_{j,crit} \end{cases}$ representing the supply, similar to [24]
- $\beta_{i,j,e}(h)$: $e$ destination-specific splitting rates from region $i$ to $j$ at step $h$
- $Z_{i,j}(h) = \sum_{e \in \mathcal{K}_d} \left( \beta_{i,j,e}(h) \frac{n_{i,e}(h)}{n_i(h)} Q_i(n_i(h)) \right)$, unrestricted demand from region $i$ to $j$ at step $h$
- $\tilde{Z}_{i,j}(h) = \min\{Z_{i,j}(h), c_{i,j}\}$, effective demand from region $i$ to $j$ at step $h$
- $Z_j(h) = \sum_{i \in \mathcal{K}, (i,j) \in \mathcal{A}} \tilde{Z}_{i,j}(h)$, total demand to region $j$ at step $h$
- $\epsilon_j(h) = \min_{j \in \mathcal{K}, (i,j) \in \mathcal{A}} \left\{ \frac{P_j(h)}{Z_j(h)}, 1 \right\}$, the flow proportion that can travel into region $j$
- $\chi_i(h) = \min_{j \in \mathcal{K}, (i,j) \in \mathcal{A}} \{\epsilon_j(h)\}$, the minimum of the exit flow proportions
- $q_{i,j}(h) = \chi_i(h) \tilde{Z}_{i,j}(h)$, flow from region $i$ to $j$ at step $h$

Flow proportion $\chi_i(k)$ will be the same for all demands $Z_{i,j}(h)$, where $i, j \in \mathcal{K}, (i,j) \in \mathcal{A}$, under the assumption that traffic cannot freely transition from region to region, due to spillback events on the boundaries as well as congestion within region $i$. If the flow is limited by the supply, region $j$ receives flow proportional to the demands converging to $j$. The flow $q_{i,j}(h)$, which is assumed constant between time steps, is effectively the minimum of supply $P_j(h)$ and demand $Z_{i,j}(h)$. Finally, we derive the state equations for our dynamic model:

- $n_i(h+1) = \sum_{e \in \mathcal{K}_d} n_{i,e}(h+1)$, where $n_{i,e}(h+1) = n_{i,e}(h) + \frac{T_s}{\sum_{z \in \mathcal{Z}_i} \Lambda_z} \left( \sum_{j \in \mathcal{K}, (i,j) \in \mathcal{A}} q_{j,i,e}(h) - \sum_{j \in \mathcal{K}, (i,j) \in \mathcal{A}} q_{i,j,e}(h) \right)$

## III. Region-based Dynamic Route Guidance

We propose a strategic learning algorithm called Proxy Regret Matching for dynamic region-based route guidance. We consider a repeated game where players, in this case Origin regions, are competing for different endpoints, in this case Destination regions. The actions are probability distributions over the route choices available for each Origin region. The aforementioned route choices belong to the sets of $k$-shortest paths found by Yen's algorithm [28], which are updated every 5 minutes according to the prevailing traffic conditions for each region in the network.

### A. Assumptions

Certain assumptions are made regarding dynamic region-based route guidance in the network:

- In each vehicle, driver-integrated (smartphone applications) or vehicle-integrated navigation equipment (GPS device) is assumed to be present. The driver is able to receive traffic state information and store a digital representation of the urban traffic network, as well as experienced travel times and expected travel times for available regional routes. A similar scheme for individual network links is used in [29].
- We assume that each departing driver can receive real-time pre-trip information about the prevailing traffic conditions, in 5-minute intervals.
- We assume that each driver completely follows the route assigned to them, selected from the alternative $k$-shortest routes provided at their Origin region.

These assumptions are considered to hold for large metropolitan areas and are expected to be commonplace for most urban areas in the near future.

### B. Repeated Game

In a repeated game, each player tracks their past payoffs and computes the empirical average payoff for each action. Let the Origins be the players competing for each of the Destinations. A repeated game in a traffic context can then be defined in the following manner, $\forall e \in \mathcal{K}_d$:

*Definition 1:* A repeated game tuple $\langle \mathcal{K}_o, \mathcal{P}_e, U, h \rangle$ where

- $\mathcal{K}_o$: is the set of players representing the regions designated as Origins
- $\mathcal{P}_e := \bigcup_{b \in \mathcal{K}_o} \mathcal{P}_{b,e}$, representing the set of joint actions, where $P_{b,e}$ represents the set of actions available to player $b \in \mathcal{K}_o$. These can be coded as $e$ destination-specific route choices.
- $U_b : \mathcal{P}_e \to \mathbb{R}$ represents the utility function for player $i$ dependent on the joint action of the players
- $h$ represents the current stage of our repeated game

### C. Regret Matching

Regret matching algorithms aim to minimize the players' respective regret for their selected actions [22]. Regret in this context refers to the difference in utility of playing a particular alternative action instead of their currently selected action, given that other players' selected actions are fixed. Players select their actions by drawing from a probability distribution which is proportional to their positive regrets. Regret matching algorithms provide guarantees for convergence of the empirical probability distribution to the set of correlated equilibria. This is realized in our dynamic region-based route guidance by incorporating into the empirical probability distribution every player/region's utility/performance during each play of a repeated game. After convergence to the set of correlated equilibria, a probability distribution on the set of all possible strategies, we can consider the solution as the splitting rates for the aggregated flow from Origin $b \in \mathcal{K}_o$ to Destination $e \in \mathcal{K}_d$.

We now define the average Regret of player $i$ for $y \in P_i$ to $z \in P_i$ at stage $h$

$$M_i^h(y,z) = max\left\{\frac{\sum_{\eta \le h: p_i^h = y}\left[U_i^\eta(z, p_{-i}^\eta) - U_i^\eta(y, p_{-i}^\eta)\right]}{h}, 0\right\} \quad (1)$$

which showcases the potential loss in utility for not having selected action $z$ every time that action $y$ was selected in the past. Regret Matching algorithms can be memory intensive, since, each player must keep track of the strategies of all players at every period of play, as well as being able to compute their own utility for changing their strategies during their past plays [22].

### D. Proxy Regret Matching

Consider now that each player, after each period of play, knows only their own set of actions and their received utility but do not know what is their utility function. They are not aware of what game is being played, i.e., the number of players, those players' respective actions and payoffs. These are the assumptions a modified Regret Matching algorithm, called Proxy Regret Matching, makes.

The average Proxy Regret of player $i$ for $y \in P_i$ to $z \in P_i$ at stage $h$

$$\hat{M}_i^h(y,z) = max\left\{\frac{1}{h}\left[\sum_{\eta \le h: p_i^\eta = z} \frac{\sigma_i^\eta(y)}{\sigma_i^\eta(z)} U_i^\eta(p^\eta) - \sum_{\eta \le h: p_i^\eta = y} U_i^\eta(p^\eta)\right], 0\right\} \quad (2)$$

with $\sum_{y=1}^{p_i} \sigma_i^\eta(y) = 1$, where $\sigma_i^\eta(y)$ denotes the play probability for player $i$ and $\eta$ denotes the history of plays up to stage $h$. After calculating the estimated average Proxy Regret, the player adaptively updates the probability of selecting actions to achieve higher utility. If the player selects action $y$ at stage $h$, the probability of selecting action $z$ at stage $h+1$ is approximately proportional to the average Proxy Regret from $y$ to $z$.

The play probabilities of player $i$ at stage $h+1$ are assigned as follows

$$\sigma_i^{h+1}(z) = \left(1 - \frac{\delta}{h^\gamma}\right) min\left\{\frac{\hat{M}_i^h(y,z)}{\mu}, \frac{1}{p_i - 1}\right\} + \frac{\delta}{h^\gamma p_i}, \; z \neq y, z \in P_i \quad (3)$$

and

$$\sigma_i^{h+1}(y) = 1 - \sum_{w \in P_i: w \neq x} \sigma_i^{h+1}(w) \quad (4)$$

where $\delta \in [0,1]$, $\gamma < 0.25$ and inertia parameter $\mu$ large enough to guarantee that $\sigma_i^{h+1}(y) > 0$

## IV. Experiments

A diamond-shaped grid network is considered for implementation of the NTM. The network consists of 16 regions, with an area of 25 $km^2$ (5x5) for each region, as shown in Fig. 1

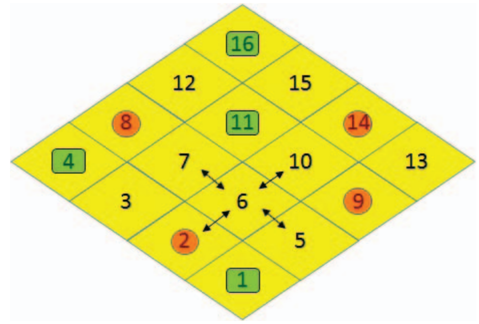


Fig. 1. Layout of the 4x4 regional network, where blue squares and red dots denote Origins and Destinations, respectively.

The homogeneous regions are each described by an Urban-scale MFD with critical accumulation $n_{i,crit} = 25$ veh/km, region network length $L_i = 10$ km and free flow speed $v_{i,f} = 100$ km/h, $\forall i \in \mathcal{K}$. Capacity along the boundaries is $c_{i,j} = 2000$ veh/h/km, $\forall i, j \in \mathcal{K}, (i,j) \in \mathcal{A}$. For each region $i$, all regions $j$ such that $(i,j) \in \mathcal{A}$ are defined as the ones that are in the northwest-southeast or northeast-southwest directions with respect to the location of the region $i$. We introduce disturbance to the demand through time-varying factor multiplication to increase the level of realism of our simulations. The simulation horizon $H$=45 min or 2700s. The sample time $T_s$=10s.

### A. Region-based Routing

The high-level Urban-scale MFD-based model presented above will be used to simulate the evolution of traffic and develop routing strategies on a macroscopic level. Our urban traffic network is partitioned into homogeneously congested regions. Then we implement an allocation of traffic flows through different regional routes so as to minimize total travel time and improve the rate of arrival for the respective destinations.

For simplicity, we assume that vehicle trips start from the center of each region and that any vehicle traveling from their Origin to their respective Destination will only traverse each region once. Subsequently, $\forall i, j \in \mathcal{K}, (i,j) \in \mathcal{A}$, we find the $\kappa$ shortest paths using Yen's algorithm [28] from region $i$ to region $j$. Then we use the shortest of the $\kappa$ paths to implement fixed and periodically adjusted routing.

*1) Fixed routing:* In the case of fixed routing, we calculate the travel times for the shortest paths and retain these shortest paths for the duration of our simulation. Fixed routing will be our so-called uncontrolled case, against which, all subsequently presented approaches will be compared.

*2) Periodically adjusted routing:* In the case of periodically adjusted routing, we update the shortest path set for every region from the travel times based on the prevailing average speeds $v_i(h)$ per region $i \in \mathcal{K}$ at step $h$.

*3) Multinomial Logit route choice:* For a more realistic route choice, we also implement the multinomial logit choice model [30]. This approach provides a Logit-based DSUE for our network at every step. We assign a probability to each alternate path $\mu$ with travel time $TT_\mu$ from a set of $\kappa$ shortest paths as follows:

$$P(\mu|\kappa) = \frac{e^{\theta TT_\mu}}{\sum_{\nu \in \kappa} e^{\theta TT_\nu}} \tag{5}$$

where $\theta$ is a scale parameter. It should be noted that the resulting probabilities are sensitive to the units (h,min,s) selected for the travel times. In our case study, $\theta = \frac{1}{6}$. When applying the multinomial logit route choice model, as in the case of periodically adjusted routing, we update the shortest path set for every region from the travel times based on the prevailing average speeds $v_i(h)$. The vehicle accumulation results for respective routing approaches can be seen in Fig. 2.

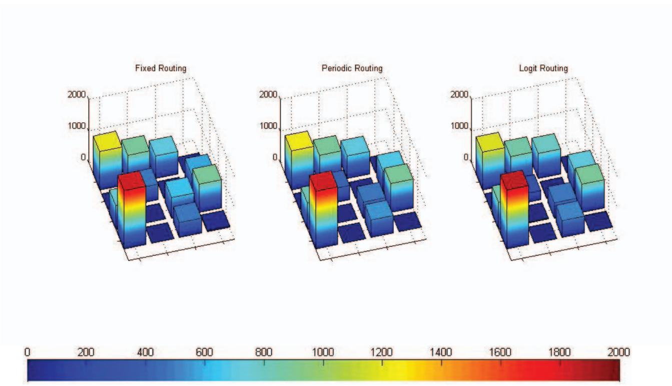


Fig. 2. Vehicle accumulation results per region for Fixed Routing, Periodic Routing and Logit Routing, at the end of the simulation

### *B. Comparison of Routing Strategies with Dynamic Region-based Route Guidance*

In this section we will compare the different routing strategies we integrated in our example of the Network Transmission Model with Proxy Regret Matching Dynamic Region-based Route Guidance. We define 2 performance metrics:

- $\mathcal{C}_{\text{AVD}} = T_s \sum_{h=0}^{H-1} \sum_{i \in \mathcal{K}} \left( \sum_{z \in \mathcal{Z}_i} \lambda_z \Lambda_z n_i(h) \right)$, the AVD for all regions
- $\mathcal{C}_{\text{v}}(H-1) = \sum_{i,j \in \mathcal{K}} \left( v_i(H-1) - v_j(H-1) \right)^2$ average Speed Variability for all regions at the end of the simulation horizon $H-1$

The average vehicle delay can be found using the total number of vehicles arriving at the region over a defined time interval $T_s$. The speed variability metric measures how evenly traffic load is distributed and, implicitly, demonstrates the phenomenon of build up of congestion to capacity in some regions, while other regions receive very little traffic. It should be noted that we also introduced 50% non-compliance as a behavioral characteristic of drivers in our route guidance scheme. Non-compliance was implemented by simply selecting each OD-pair $(b, e), \forall b \in \mathcal{K}_o, e \in \mathcal{K}_d$ and multiplying the respective $e$ destination-specific accumulations at each step $h$, $n_{b,e}(h)$, by the selected non-compliance percentage $\omega$. Subsequently we get two types of $e$ destination-specific accumulations,

$$n^{CO}_{b,e}(h) = (1-\omega) n_{b,e}(h)) \tag{6}$$
$$n^{NC}_{b,e}(h) = \omega n_{b,e}(h) \tag{7}$$

representing compliant and non-compliant accumulations respectively. For compliant $e$ destination-specific accumulations $n^{co}_{b,e}(h)$, we apply the Proxy Regret Matching method as before and for non-compliant $e$ destination-specific accumulations $n^{nc}_{b,e}(h)$, we apply logit routing. Logit routing has been used in literature to model drivers' route choice when there is uncertainty in their perception of travel time [31] which, in our case, can induce drivers to reject the route guidance provided.

For Fixed Routing (FR), Periodically Adjusted Routing (PAR), Logit Routing (LR), Proxy Regret Matching (PRM) and PRM with non-compliance (PRMNC) we have: We now define Fixed Routing (FR) as the uncontrolled case, and compare it to the remaining Routing Strategies as follows:

| | FR | PAR | LR | PRM | PRMNC |
|---|---|---|---|---|---|
| $C_{AVD}$ | 1.5232 | 1.5196 | 1.5208 | 1.5156 | 1.5206 |
| $C_v$ | 8.5190 | 8.4541 | 8.3666 | 6.6510 | 7.1397 |

$$\mathcal{C}_{\text{TOT}} = w \frac{\mathcal{C}_{\text{AVD,g}}}{\mathcal{C}_{\text{AVD,FR}}} + (1-w) \frac{\mathcal{C}_{\text{v,g}}}{\mathcal{C}_{\text{v,FR}}}$$

where $g \in \{PAR, LR, PRM, PRMNC\}$ and $w$ an appropriate weight to denote the importance we place on the respective performance measures

| *w=0.5* | FR | PAR | LR | PRM | PRMNC |
|---|---|---|---|---|---|
| $C_{TOT}$ | 1.0000 | 0.9950 | 0.9902 | 0.8878 | 0.918 |

Vehicle accumulation results for respective region-based route guidance approaches can be seen below

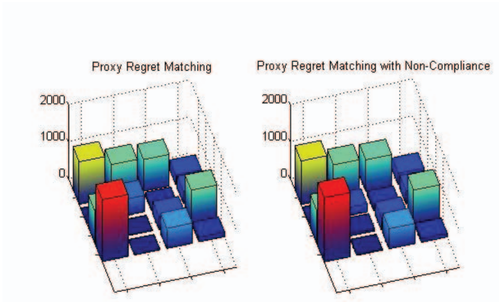


Fig. 3. Vehicle accumulation results per region for Proxy Regret Matching (PRM) and PRM with non-compliance, at the end of the simulation

## V. Conclusions and Future Work

As is evident from the above table, we get at least 11% improvement in our composite index $C_{TOT}$ performance and even with 50% non-compliance on the drivers' part, we still get an 8.8% improvement, compared to the uncontrolled case, i.e. Fixed Routing. In the figure that follows, we can visually confirm the more uniform distribution of vehicle accumulation in the example network at the end of the simulation, both for Proxy Regret Matching and Proxy Regret Matching with non-compliance on the drivers' part. Therefore, we believe that there is value in further examining strategic learning algorithm integration in Dynamic Region-based Route Guidance schemes.

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