# Serie de Fourier

Thursday, April4, 2019 7:14 PM

- Una función es periódica si sus imágenes se repiten en intervalos regulares en su dominio

$$f(t+mT) = f(t)$$

$$\text{frecuencia} = \frac{1}{\text{Período}} = \frac{1}{T}$$

Frecuencia Circular

$$\omega = \frac{2\pi}{T}$$

- Teorema de Fourier

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \operatorname{sen} n\omega t$$

$a_n \triangleq$ Componentes en fase

$b_n \triangleq$ Cuadratura de fase

// Partes

$$a_0 = \frac{2}{T}\int_d^{d+T} F(t)\,dt$$

$$a_n = \frac{2}{T}\int_d^{d+T} F(t)\cos n\omega t\,dt$$

$$b_n = \frac{2}{T}\int_d^{d+T} F(t)\operatorname{sen} n\omega t\,dt$$

- $\omega = 1 \rightarrow$ Unitaria

  $\hookrightarrow T = 2\pi$

  $\therefore \omega = \frac{2\pi}{2\pi} = 1$

- $\omega \neq 1 \rightarrow$ No unitaria

if $\omega = 1$

$$a_0 = \frac{1}{\pi}\int_d^{d+2\pi} F(t)\,dt$$

$$a_n = \frac{1}{\pi}\int_d^{d+2\pi} F(t)\cos nt\,dt$$

$$b_n = \frac{1}{\pi}\int_d^{d+2\pi} F(t)\operatorname{sen} nt\,dt$$

$$b_n = \frac{1}{\pi}\int_{d}^{d+2\pi} F(t)\,\text{sen}\,n t\,dt$$

- $F(t) = t^2 + t \qquad (-\pi < t < \pi)$

$F(t) = F(t+2\pi)$

// Período ; $T = 2\pi$ ; $\omega = 1$

$a_0, a_n, b_n$

$$a_0 = \frac{2}{T}\int_{d}^{d+T} F(t)\,dt = \frac{1}{\pi}\int_{-\pi}^{\cancel{-\pi+2\pi}\,\pi} (t^2+t)\,dt =$$

$$= \frac{1}{\pi}\left[\frac{t^3}{3} + \frac{t^2}{2}\right]\Bigg|_{-\pi}^{\pi} = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{n}\int_{-\pi}^{\pi} (t^2+t)\cos n t\,dt =$$

$$= \frac{1}{n}\left[\int_{-\pi}^{\pi} t^2 \cos n t\,dt + \int_{-\pi}^{\pi} t\cos n t\,dt\right] =$$

| | | | | | |
|---|---|---|---|---|---|
| $t^2$ | $+$ | $\cos n t$ | $t$ | $+$ | $\cos n t$ |
| $2t$ | $-$ | $\frac{1}{n}\text{sen}\,n t$ | $1$ | $-$ | $\frac{1}{n}\text{sen}\,n t$ |
| $2$ | $+$ | $-\frac{1}{n^2}\cos n t$ | $0$ | | $-\frac{1}{n^2}\cos n t$ |
| $0$ | | $-\frac{1}{n^3}\text{sen}\,n t$ | | | |

$$t^2\frac{1}{n}\text{sen}\,t + 2t\frac{1}{n^2}\cos n t - 2\frac{1}{n^3}\text{sen}\,n t$$

$$+$$

$$t\,\frac{1}{n}\,\text{sen}\,nt + \frac{1}{n^2}\cos nt$$

$$\therefore\ a_n = \frac{1}{\pi}\left[\cancelto{0}{\frac{t^2}{n}\,\text{sen}\,t} + \frac{2t}{n^2}\cos nt - \cancelto{0}{\frac{2}{n^3}\,\text{sen}\,nt} + \cancelto{0}{\frac{t}{n}\,\text{sen}\,nt} + \frac{1}{n^2}\cos nt\right]\Bigg|_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{2t}{n^2}\cos nt + \frac{1}{n^2}\cos nt\right]\Bigg|_{-\pi}^{\pi}$$

$$a_n = \frac{1}{\cancel{\pi}}\left[\frac{4\cancel{\pi}}{n^2}\cos(n\pi)\right]$$

**Nota** $\Big|\ \cos(n\pi) = (-1)^n \qquad \cos(x) = \cos(-x)$

$$\therefore\ a_n = \frac{4}{n^2}(-1)^n$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(t^2+t)\,\text{sen}\,nt\,dt$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} t^2\,\text{sen}\,nt\,dt + \int_{-\pi}^{\pi} t\,\text{sen}\,nt\,dt\right]$$

| | | | | | |
|---|---|---|---|---|---|
| $t^2$ | $+$ | $\text{sen}\,nt$ | $t$ | $+$ | $\text{sen}\,t$ |
| $2t$ | $-$ | $-\frac{1}{n}\cos nt$ | $1$ | $-$ | $-\frac{1}{n}\cos nt$ |
| $2$ | $+$ | $-\frac{1}{n^2}\,\text{sen}\,nt$ | $0$ | | $-\frac{1}{n^2}\,\text{sen}\,nt$ |
| $0$ | | $\frac{1}{n^3}\cos nt$ | | | |

$0$

$$= \frac{1}{\pi}\left[-\frac{t^2}{n}\cos nt + \cancel{\frac{2t}{n^2}\operatorname{sen} nt}^{\,0} + \frac{2}{n^3}\cos nt - \frac{t}{n}\cos nt + \cancel{\frac{1}{n^2}\operatorname{sen} nt}^{\,0}\right]\Bigg|_{-\pi}^{\pi} =$$

$$= \frac{1}{\pi}\left[-\frac{t^2}{n}\cos nt + \frac{2}{n^3}\cos nt - \frac{t}{n}\cos nt\right]\Bigg|_{-\pi}^{\pi} =$$

$$= \frac{1}{\pi}\left[\cancel{-\frac{\pi^2}{n}(-1)^n} + \cancel{\frac{2}{n^3}(-1)^n} - \frac{\pi}{n}(-1)^n + \cancel{\frac{\pi^2}{n}(-1)^n} - \cancel{\frac{2}{n^3}(-1)^n} \cancel{+} - \frac{\pi}{n}(-1)^n\right]$$

$$= \frac{1}{\cancel{\pi}}\left[-\frac{2\cancel{\pi}}{n}(-1)^n\right] = -\frac{2}{n}(-1)^n$$

// Serie

$t^2 + t \rightarrow T = 2\pi$

$\omega = 1$

T → Definido
- → Serie Fourier
- → Indefinido
- → Transformada Fourier

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \operatorname{sen} \omega t$$

$$= \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left[\frac{4}{n^2}(-1)^n \cos nt - \frac{2}{n}(-1)^n \operatorname{sen} nt\right]$$

$n = 1$ // Primer armónico

$$\therefore F(t) = \frac{\pi^2}{3} \underbrace{-4\cos t + 2\,\text{sen}\,t}_{n=1}$$

$$\underbrace{+\cos 2t - \text{sen}\,2t}_{n=2} \ldots + \underbrace{\frac{4}{n^2}(-1)^n \cos nt - \frac{2}{n}(-1)^n \text{sen}\,nt}_{n=n}$$

$$F(t) \Rightarrow \begin{cases} \underbrace{\cos t}_{n=1} + \underbrace{\cancel{2\cos 2t}}_{n=2} \to 0 + \underbrace{\cos 3t}_{n=3} \\ \underbrace{\text{sen}\,t}_{n=1} + \underbrace{\text{sen}\,2t}_{n=2} + \underbrace{\text{sen}\,3t}_{n=3} \end{cases}$$

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$$f(t) = \begin{cases} t \; ; & 0 \le t \le \frac{\pi}{2} \\ \pi/2 \; ; & \pi/2 \le t \le \pi \\ \pi - \frac{1}{2}t \; ; & \pi \le t \le 2\pi \end{cases}$$

$a_0$, $a_n$, $b_n$ y 6 armónicos

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} F(t)\,dt =$$

$$= \frac{1}{\pi}\left[\int_0^{\pi/2} t\,dt + \int_{\pi/2}^{\pi} \pi/2\,dt + \int_{\pi}^{2\pi} \pi - \frac{t}{2}\,dt\right]$$

$$= \frac{1}{\pi}\left[\left(\frac{t^2}{2}\right)\Big|_0^{\pi/2} + \left(\frac{\pi}{2}t\right)\Big|_{\pi/2}^{\pi} + \left(\pi t - \frac{t^2}{4}\right)\Big|_{\pi/2}^{\pi}\right] = \frac{5}{8}\pi$$

$$a_n = \frac{1}{\pi}\left[\int_0^{\pi/2} t\cos nt\,dt + \frac{\pi}{2}\int_{\pi/2}^{\pi} \cos nt\,dt + \right.$$

$$\left. + \pi\int^{2\pi} \cos nt\,dt - \frac{1}{2}\int^{2\pi} t\cos nt\,dt\right] =$$

$$+\pi\int_{\pi}^{2\pi}\cos nt\,dt - \frac{1}{2}\int_{\pi} t\cos nt\,dt\Big] =$$

$$\begin{array}{ll|l} t & \cos nt & \\ 1 & \frac{1}{n}\operatorname{sen} nt & \\ 0 & -\frac{1}{n^2}\cos nt & \end{array} \qquad \frac{t}{n}\operatorname{sen} nt + \frac{1}{n^2}\cos nt$$

$$= \left(\frac{t}{n}\operatorname{sen} nt + \frac{1}{n^2}\cos nt\right)\Big|_0^{\pi/2} + \frac{\pi}{2}\left(\frac{1}{n}\operatorname{sen} nt\right)\Big|_{\pi/2}^{\pi}$$

$$+\pi\left(\frac{1}{n}\operatorname{sen} nt\right)\Big|_{\pi}^{2\pi} - \frac{1}{2}\left(\frac{t}{n}\operatorname{sen} nt + \frac{1}{n^2}\cos nt\right)\Big|_0^{2\pi} =$$

$$= \quad n = 1, 2, 3, 4 \quad \begin{cases} n = \text{par} \to 2 \\ n = \text{impar} \to 1 \end{cases}$$

$$\cos 2n\pi = 1 \qquad \operatorname{sen} 2n\pi = 0$$

$$\therefore\ a_n = \begin{cases} \frac{1}{\pi n^2}\left((-1)^{\frac{n}{2}} - 1\right) & n = \text{par} \\ -\frac{2}{\pi n^2} & n \text{ impar} \end{cases}$$

$$b_n = \frac{1}{\pi}\Big[\int_0^{\pi/2} t\operatorname{sen} nt\,dt + \frac{\pi}{2}\int_{\pi/2}^{\pi}\operatorname{sen} nt\,dt$$

$$+ \pi\int_{\pi}^{2\pi}\operatorname{sen} nt\,dt - \frac{1}{2}\int_{\pi}^{2\pi} t\operatorname{sen} nt\,dt\Big]$$

$$\begin{array}{ll|l} t & \oplus \quad \operatorname{sen} nt & \\ 1 & \ominus \quad -\frac{1}{n}\cos nt & \end{array} = -\frac{t}{n}\cos nt + \frac{1}{n^2}\operatorname{sen} nt$$

$$\begin{array}{c|c} 1 & -\frac{1}{n}\cos nt \\ 0 & -\frac{1}{n^2}\operatorname{sen} nt \end{array} = -\frac{t}{n}\cos nt + \frac{1}{n^2}\operatorname{sen} nt$$

$$= \frac{1}{\pi}\left[ -\frac{t}{n}\cos nt + \frac{1}{n^2}\operatorname{sen} nt \Big|_0^{\pi/2} - \frac{\pi}{2n}\cos nt \Big|_{\pi/2}^{\pi} - \frac{\pi}{n}\cos nt \Big|_{\pi}^{2\pi} - \frac{1}{2}\left(-\frac{t}{n}\cos nt + \frac{1}{n^2}\operatorname{sen} nt \Big|_{\pi}^{2\pi}\right)\right] =$$

//Evaluando

$$b_n = \frac{1}{\pi}\left[ -\frac{\pi}{2n}\cos\left(n\frac{\pi}{2}\right) + \frac{1}{n^2}\operatorname{sen}\left(n\frac{\pi}{2}\right) + 0 + 0 - \frac{\pi}{2n}\cos(n\pi) + \frac{\pi}{2n}\cos\left(\frac{n\pi}{2}\right) - \frac{\pi}{n}\cos(2n\pi) + \frac{\pi}{n}\cos(n\pi) + \frac{2\pi}{2n}\cos n2\pi - \frac{1}{n^2}\operatorname{sen} n2\pi - \left[\frac{\pi}{2n}\cos n\pi + \frac{1}{n^2}n\pi\right]\right]$$

$$\therefore b_n = \frac{1}{\pi}\left(\frac{1}{n^2}\operatorname{sen} n\frac{\pi}{2}\right)$$

$$b_n = \begin{cases} n \text{ par } 0 \\ n \text{ impar } n=1 \; ; \; \frac{1}{\pi}\left[\frac{1}{1^2}\operatorname{sen}\frac{\pi}{2}\right] = \frac{1}{\pi} \\ \qquad n=3 \; ; \; \frac{1}{\pi}\left[\frac{1}{9}\operatorname{sen}\frac{3\pi}{2}\right] = \frac{1}{9\pi} \end{cases}$$

$$n=3 \; ; \; \frac{1}{\pi}\left[\frac{1}{9}\,\text{sen}\,\frac{3\pi}{2}\right] = \frac{1}{9\pi}$$

- Cuando n es impar

$$\frac{(-1)^{\left(\frac{n-1}{2}\right)}}{\pi n^2}$$

## Obteniendo Serie de Fourier

$$F(t) = \frac{a_0}{2} + a_1 \cos nt + b_1 \,\text{sen}\, nt \quad n=1$$
$$+ a_2 \cos nt + b_2 \,\text{sen}\, nt \quad n=2$$
$$+ a_3 \cos nt + b_3 \cos nt \quad n=3$$

$$= \frac{5\pi}{16} - \frac{2}{\pi}\cos t + \frac{1}{\pi}\,\text{sen}\, t$$
$$- \frac{1}{2\pi}\cos 2t + 0$$
$$- \frac{2}{9\pi}\cos 3t - \frac{1}{9\pi}\,\text{sen}\, 3t$$

3 armónicos

# Método Alternativo

martes, 9 de abril de 2019 08:06 p. m.

$$a_n + jb_n = \frac{2}{T}\int_{\alpha}^{\alpha+T} f(t)\, e^{jn\omega t}\, dt$$

NOTACION FASORIAL

- $F(t) = t^2 + t$
- $T = 2\pi$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$= \frac{2}{T}\int_{-\pi}^{\pi}(t^2+t)e^{jnt}\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi}(t^2+t)\,e^{jnt}\,dt =$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{\pi} t^2 e^{jnt}\,dt + \int_{-\pi}^{\pi} t\, e^{jnt}\,dt\right]$$

| | | | | | |
|---|---|---|---|---|---|
| $t^2$ | $+$ | $e^{jnt}$ | $t$ | $+$ | $e^{jnt}$ |
| $2t$ | $-$ | $\frac{1}{jn}e^{jnt}$ | $1$ | $-$ | $\frac{1}{jn}e^{jnt}$ |
| $2$ | $+$ | $\frac{1}{j^2n^2}e^{jnt}$ | $0$ | | $\frac{1}{j^2n^2}e^{jnt}$ |
| $0$ | | $\frac{1}{j^3n^3}e^{jnt}$ | | | |

$$= \frac{1}{\pi}\left[\left(\frac{t^2}{jn}e^{jnt} - \frac{2t}{j^2n^2}e^{jnt} + \frac{2}{j^3n^3}e^{jnt}\right)\Bigg|_{-\pi}^{\pi} + \left(\frac{t}{jn}e^{jnt} - \frac{1}{j^2n^2}e^{jnt}\right)\Bigg|_{-\pi}^{\pi}\right] =$$

$$a_n + jb_n = \frac{1}{\pi}\left[\left(\frac{\pi^2+\pi}{jn}e^{jn\pi} - \frac{2\pi+1}{(jn)^2}e^{jn\pi} + \frac{2}{(jn)^3}e^{jn\pi}\right) - \left(\frac{\pi^2-\pi}{jn}e^{-jn\pi} - \frac{-2\pi+1}{(jn)^2}e^{-jn\pi} + \frac{2}{(jn)^3}e^{-jn\pi}\right)\right]$$

$$e^{jn\pi} = \cos(n\pi) + j\,\cancel{\operatorname{sen}(n\pi)}^{\,0} = (-1)^n$$

complejos

$$e^{jn\pi} = \cos(n\pi) + j\,\cancel{\text{sen}(n\pi)}^{\,0} = (-1)^n$$

$$e^{-jn\pi} = \cos(n\pi) - \cancel{j\,\text{sen}(n\pi)}^{\,0} = (-1)^n$$

$$= \frac{(-1)^n}{\pi}\left[\frac{\pi^2+\pi}{jn} - \frac{2\pi+1}{(jn)^2} + \frac{2}{(jn)^3} - \frac{\pi^2-\pi}{jn} + \frac{-2\pi+1}{(jn)^2} - \frac{2}{(jn)^3}\right]$$

complejo

$$(jn)^2 = j^2 n^2 = -n^2$$

$$(jn)^3 = j\,j^2 n^2 = -jn^2$$

$$\therefore = \frac{(-1)^n}{\pi}\left[\frac{\pi^2+\pi}{jn}\left(\frac{-j}{-j}\right) + \frac{2\pi+1}{n^2} + \frac{2}{-jn^3}\left(\frac{j}{j}\right) - \frac{\pi^2-\pi}{jn}\left(\frac{-j}{-j}\right) - \left(\frac{-2\pi+1}{n^2}\right) - \frac{2}{-jn^3}\left(\frac{j}{j}\right)\right]$$

$$= \left[\frac{-j(\pi^2+\pi)}{n} + \frac{2\pi+1}{n^2} + \frac{2j}{n^3} \frac{j(\pi^2-\pi)}{n} - \left(\frac{-2\pi+1}{n^2}\right) - \frac{2j}{n^3}\right] =$$

$$\therefore a_n + jb_n = \left[\frac{4\pi}{n^2} - \frac{j\pi^2 - j\pi + j\pi^2 - j\pi}{n}\right]$$

$$a_n = \frac{(-1)^n 4}{n^2}$$

$$b_n = -\frac{2}{n}(-1)^n$$

# Funciones pares e impares

jueves, 11 de abril de 2019 07:20 p. m.

- $F(t) = \frac{a_0}{2} + \sum_{i=1}^{\infty} a_n \cos n\omega t$

Función Par / Cosenos

Simétrico respecto a "y"

$\hookrightarrow a_n = \frac{4}{T} \int_0^{T/2} f(t) \cos n\omega t \, dt$

//Ejemplo

$$a_n = \frac{2}{\pi} \int_0^{\pi} t^2 \cos nt \, dt$$

| | | |
|---|---|---|
| $t^2$ | $+$ | $\cos nt$ |
| $2t$ | $-$ | $\frac{1}{n} \operatorname{sen} nt$ |
| $2$ | $+$ | $\frac{1}{n^2} \cos nt$ |
| $0$ | | $-\frac{1}{n^3} \operatorname{sen} nt$ |

$$= \frac{2}{\pi}\left[\frac{t^2}{n} \operatorname{sen} nt + \frac{2t}{n^2} \cos nt - \frac{2}{n^3} \operatorname{sen} nt\right]\Big|_0^{\pi} =$$

$$= \left[\cancel{\frac{\pi^2}{n} \operatorname{sen} n\pi}^{\,0} + \frac{2\pi}{n^2} \cos n\pi - \cancel{\frac{2}{n^3} \operatorname{sen} n\pi}^{\,0}\right] - \left[\cancel{0}\right] \to \text{Todos}$$

$$\therefore = \frac{2}{\cancel{\pi}}\left[\frac{2\cancel{\pi}}{n^2} \cos n\pi\right] = \frac{4}{n^2}(-1)^n$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} t^2 \, dt = \frac{2}{\pi} \frac{t^3}{3}\Big|_0^{\pi} = \frac{2\pi^2}{3}$$

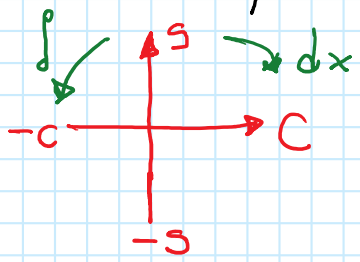
$\therefore$ Serie de Fourier

$$F(t) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}\left[\frac{4}{n^2}(-1)^n \cos nt\right]$$

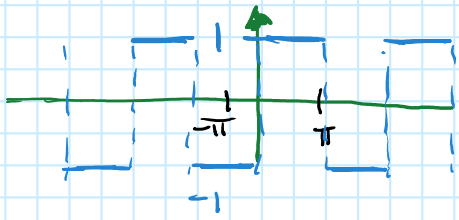
• $F(t) = \sum_{i=1}^{\infty} b_n \operatorname{sen} n\omega t$

Función Impar / Senos

Simétrico respecto a "x"




$\hookrightarrow b_n = \frac{4}{T} \int_0^{T/2} f(t) \operatorname{sen} n\omega t$




$T = 2\pi$

$\therefore$ Frecuencia $\triangleq \omega = \frac{2\pi}{T} = 1$

• $F(t) \begin{cases} -1 & -\pi < t < 0 \\ 1 & 0 < t < \pi \end{cases}$

$\therefore$ Función es impar

$f(t) = \sum_{n=1}^{\infty} b_n \operatorname{sen}(nt)$

$$b_n = \frac{2}{\pi} \int_0^{\pi} \operatorname{sen} nt \, dt =$$

$$= \frac{2}{\pi} \left[ -\frac{1}{n} \cos nt \right] \Big|_0^{\pi} = -\frac{2}{n\pi} \left[ \cos n\pi - 1 \right] =$$

$$= \frac{2}{n\pi} \left[ -1 + (-1)^n \right] =$$

$n = 1$ ; impar

$n = 2$ ; par

$b_n \begin{cases} n \text{ par} = 0 \\ n \text{ impar} = \frac{2}{n\pi} \end{cases}$

Serie con 5 armónicos

| $b_n$ | $n$ |
|---|---|
| $4/\pi$ | 1 |
| $4/3\pi$ | 3 |
| $4/5\pi$ | 5 |

; $f(t) = \frac{4}{\pi} \operatorname{sen} t + \frac{4}{3\pi} \operatorname{sen} 3t + \frac{4}{5\pi} \operatorname{sen} 5t + \frac{4}{7\pi} \operatorname{sen} 7t$ //

$4/5\pi \mid 5$

$$\frac{1}{5\pi}\,\text{sen}\,5t + \frac{4}{7\pi}\,\text{sen}\,7t$$

# -> Funciones de período T

jueves, 11 de abril de 2019 08:11 p. m.

- $T = 2$
- $\omega = \frac{2\pi}{T} = \pi$

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi t + b_n \operatorname{sen} n\pi t$$

Primeros 6 armónicos

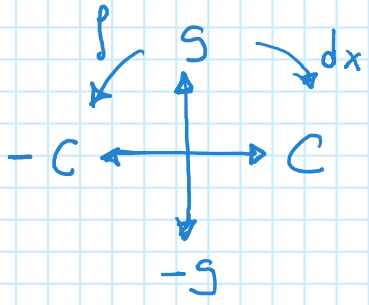
$$F(t) = \begin{cases} 3t & 0 < t < 1 \\ 3 & 1 < t < 2 \end{cases}$$

$$a_0 = \frac{2}{T}\int_0^2 f(t)\,dt = \int_0^2 f(t)\,dt = \int_0^1 3t\,dt + \int_1^2 dt =$$

$$= \frac{3t^2}{2}\Big|_0^1 + 3t\Big|_1^2 = \frac{3}{2} + 1 = \frac{9}{2}$$

$$a_n = \frac{2}{T}\int_0^2 f(t)\cos n\omega t\,dt = \int_0^1 3t\cos n\pi t + \int_1^2 3\cos n\pi t\,dt$$

| | | |
|---|---|---|
| $3t$ | $+$ | $\cos n\pi t$ |
| $3$ | $-$ | $\frac{1}{n\pi}\operatorname{sen} n\pi t$ |
| $0$ | | $-\frac{1}{(n\pi)^2}\cos n\pi t$ |




$$= \left[\frac{3t}{n\pi}\operatorname{sen} n\pi t + \frac{3}{(n\pi)^2}\cos n\pi t\right]\Big|_0^1 + \frac{3}{n\pi}\operatorname{sen} n\pi t\Big|_1^2 =$$

$$= \left[\cancelto{0}{\frac{3}{n\pi}\operatorname{sen} n\pi} + \frac{3}{(n\pi)^2}(-1)^n\right] - \left[0 + \frac{3}{(n\pi)^2}\right]$$

$$+ \left[\cancelto{0}{\frac{3}{n\pi}\operatorname{sen} 2n\pi} - \cancelto{0}{\frac{3}{n\pi}\operatorname{sen} n\pi}\right]$$

$$= \frac{3}{(n\pi)^2}\left[(-1)^n - 1\right] \begin{cases} n \text{ par} & 0 \\ n \text{ impar} & -\dfrac{6}{(n\pi)^2} \end{cases}$$

$$b_n = \int_0^1 3t \operatorname{sen} n\pi t \, dt + \int_1^2 3 \operatorname{sen} \pi n t \, dt =$$

| | | |
|---|---|---|
| $3t$ | $\operatorname{sen} n\pi t$ | |
| $3$ | $-\frac{1}{n\pi}\cos n\pi t$ | |
| $0$ | $-\frac{1}{(n\pi)^2}\operatorname{sen} n\pi t$ | |

$$= \left[-\frac{3t}{n\pi}\cos n\pi t + \frac{3}{(n\pi)^2}\operatorname{sen} n\pi t\right]\Bigg|_0^1 - \frac{3}{n\pi}\cos n\pi t\Bigg|_1^2 =$$

$$= \left[-\frac{3}{n\pi}\overset{(-1)^n}{\cancel{\cos n\pi}} + \underset{0}{\cancel{\frac{3}{(n\pi^2)}\operatorname{sen} n\pi}}\right] - \left[0 + \underset{0}{\cancel{\frac{3}{(n\pi)^2}\operatorname{sen}[n\pi(0)]}}\right]$$

$$-\frac{3}{n\pi}\overset{1}{\cancel{\cos n\pi 2}} + \frac{3}{n\pi}\overset{(-1)^n}{\cancel{\cos n\pi}} =$$

$$= -\frac{3}{n\pi}$$

// Serie de Fourier

$$F(t) = \frac{a_0}{2} + \left[a_1 \cos \pi t + b_1 \operatorname{sen} \pi t\right] + \left[\overset{0}{\cancel{a_2 \cos 2\pi t}} + b_2 \operatorname{sen} 2\pi t\right]$$

$$+ \left[a_3 \cos 3\pi t + b_3 \operatorname{sen} 3\pi t\right] + \left[\overset{0}{\cancel{a_4 \cos 4\pi t}} + b_4 \operatorname{sen} 4\pi t\right]$$

$$+\left[a_3 \cos 3\pi t + b_3 \operatorname{sen} 3\pi t\right] + \left[\cancel{a_4 \cos 4\pi t}^{\,0} + b_4 \operatorname{sen} 4\pi t\right]$$

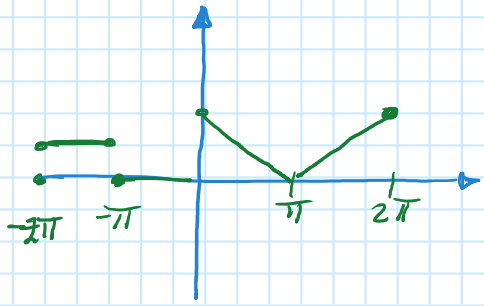
$$= \frac{9}{4} + \left[-\frac{6}{\pi^2}\cos t - \frac{3}{\pi}\operatorname{sen}\pi t\right] + \left[0 - \frac{3}{2\pi}\operatorname{sen}\pi t\right] +$$

$$+\left[-\frac{6}{9\pi^2}\cos 3\pi t - \frac{1}{\pi}\operatorname{sen} 3\pi t\right] + \left[0 - \frac{3}{4\pi}\operatorname{sen} 4\pi t\right] =$$

---

- Obtener Serie de Fourier

$$f(x) = \begin{cases} 1 & ; \quad -2\pi \le x \le -\pi \\ 0 & ; \quad -\pi \le x \le 0 \\ \pi - x & ; \quad 0 \le x \le \pi \\ x - \pi & ; \quad \pi \le x < 2\pi \end{cases}$$




- $T = 4\pi$ $\quad$ • $\omega = \frac{2\pi}{T} = \frac{2\pi}{4\pi} = \frac{1}{2}$

$$F(x) = \frac{a_0}{2} + a_1 \cos\frac{x}{2} + b_1 \operatorname{sen}\frac{x}{2} + \dots$$

---

$$a_0 = \frac{2}{T}\int_{-2\pi}^{2\pi} F(x)\,dx = \frac{1}{2\pi}\left[\int_{-2\pi}^{-\pi} dx + \cancel{\int_{-\pi}^{0} 0\,dx}^{\,0} + \int_0^{\pi}(\pi - x)\,dx + \int_{\pi}^{2\pi}(x-\pi)\,dx\right] =$$

$$= \frac{1}{2\pi}\left[x\Big|_{-2\pi}^{-\pi} + \left(\pi x - \frac{x^2}{2}\right)\Big|_0^{\pi} + \left(\frac{x^2}{2} - \pi x\right)\Big|_{\pi}^{2\pi}\right] =$$

$$= \boxed{\pi^2 + \pi}$$

$$= \left(\frac{\pi^2+\pi}{2\pi}\right)$$

$$a_n = \frac{1}{2\pi}\left[\int_{-2\pi}^{-\pi} \cos\frac{n}{2}x\,dx \;(0) + \int_0^{\pi} (\pi - x)\left(\cos\frac{n}{2}x\right)dx \;(1) + \int_{\pi}^{2\pi} (x-\pi)\left(\cos\frac{n}{2}x\right)dx \;(2)\right]$$

→ (1)

| | | |
|---|---|---|
| $\pi - x$ | $+$ | $\cos\frac{n}{2}x$ |
| $-1$ | $-$ | $-\frac{2}{n}\operatorname{sen}\frac{n}{2}x$ |
| $0$ | | $-\frac{4}{n^2}\cos\frac{n}{2}x$ |

$$= \left[(\pi - x)\left(\frac{2}{n}\operatorname{sen}\frac{n}{2}x\right) - \frac{4}{n^2}\cos\frac{n}{2}x\right]\Bigg|_0^{\pi} =$$

→ (2)

| | |
|---|---|
| $x-\pi$ | $\cos\frac{n}{2}x$ |
| $1$ | $\frac{2}{n}\operatorname{sen}\frac{n}{2}x$ |
| $0$ | $-\frac{4}{n^2}\cos\frac{n}{2}x$ |

$$= \left[(x-\pi)\left(\frac{2}{n}\operatorname{sen}\frac{n}{2}x\right) + \frac{4}{n^2}\cos\frac{n}{2}x\right]\Bigg|_{\pi}^{2\pi}$$

→ (0) $= \frac{2}{n}\operatorname{sen}\frac{n}{2}x\Big|_{-2\pi}^{\pi}$

// Agrupando

$$= \frac{1}{2\pi}\left[\left(\frac{2}{n}\operatorname{sen}\frac{n}{2}\pi - \frac{2}{n}\operatorname{sen}\frac{n}{\cancel{2}}(-\cancel{2}\pi)\right) + \left(\cancel{(\pi-\pi)\left(\frac{2}{n}\operatorname{sen}\frac{n}{2}\pi\right)}^{\,0} - \frac{4}{n^2}\cos\frac{n}{2}\pi\right.\right.$$

$$- (\pi - 0)\left(\frac{2}{n}\operatorname{sen}\frac{n}{2}0\right) - \frac{4}{n^2}\cos\frac{n}{2}0\Big)$$

$$+\Big((2\pi - \pi)\left(\frac{2}{n}\operatorname{sen}\frac{n}{2}2\pi\right) + \frac{4}{n^2}\cos\frac{n}{2}2\pi$$

$$- (\pi - \pi)\left(\frac{2}{n}\operatorname{sen}\frac{n}{2}\pi\right) + \frac{4}{n^2}\cos\pi\Big)\Big] =$$

$$= \frac{1}{2\pi}\Big[\frac{2}{n}\operatorname{sen}\frac{n}{2}\pi + \frac{2}{n}\operatorname{sen} n\pi$$

$$-\frac{4}{n^2}\cos\frac{n}{2}\pi - \frac{4}{n^2}\cos\frac{n}{2}\pi \qquad -\frac{8}{n^2}\cos\frac{n}{2}\pi$$

$$+\frac{2\pi}{n}\operatorname{sen} n\pi - \frac{4}{n^2}\cos\frac{n}{2}0$$

$$+\frac{4}{n^2}\cos n\pi\Big] =$$

$$= \frac{1}{2\pi}\left[\frac{2}{n}\operatorname{sen}\frac{n\pi}{2} - \frac{8}{n^2}\cos\frac{n}{2}\pi - \frac{4}{n^2}(-1)^n\right] =$$

if $\begin{cases} n \overset{\Delta}{=} \text{par} \; ; \; n = 1 \\ n \overset{\Delta}{=} \text{impar} \; ; \; n = 2 \end{cases}$

// Miembros de la Serie de $a_n$

$a_1 = -\frac{1}{\pi}$ $\quad a_3 = \frac{1}{3\pi}$ $\quad a_5 = \frac{1}{-5\pi}$ $\quad a_7 = \frac{1}{7\pi}$

$a_2 = \frac{2}{\pi}$ $\quad a_4 = 0$ $\quad a_6 = \frac{4}{9\pi}$ $\quad a_8 = 0$

$$b_n = \frac{1}{2\pi}\Big[\overset{0}{\int_{-2\pi}^{-\pi}} \operatorname{sen}\frac{n}{2}x\,dx + \overset{1}{\int_0^{\pi}} (\pi - x)\left(\operatorname{sen}\frac{n}{2}x\right)dx +$$

$$\overset{2}{\int_{\pi}^{2\pi}}$$

$$\int_{-2\pi}^{0} \qquad \int_{\pi}^{2\pi} (x-\pi)\left(\operatorname{sen}\frac{n}{2}x\right)dx \Big] =$$

②

$$\begin{array}{lll|l} \pi - x & \oplus & \operatorname{sen}\frac{n}{2}x & = \Big[(\pi-x)\left(-\frac{2}{n}\cos\frac{n}{2}x\right) \\ -1 & - & -\frac{2}{n}\cos\frac{n}{2}x & -\frac{4}{n^2}\operatorname{sen}\frac{n}{2}x\Big]\Big|_{0}^{\pi} \\ 0 & & -\frac{4}{n^2}\operatorname{sen}\frac{n}{2}x & \end{array}$$

$$\begin{array}{lll|l} x-\pi & & \operatorname{sen}\frac{n}{2}x & = \Big[(x-\pi)\left(-\frac{2}{n}\cos\frac{n}{2}x\right) \\ 1 & & -\frac{2}{n}\cos\frac{n}{2}x & +\frac{4}{n^2}\operatorname{sen}\frac{n}{2}x\Big]\Big|_{\pi}^{2\pi} \\ 0 & & -\frac{4}{n^2}\operatorname{sen}\frac{n}{2}x & \end{array}$$

$$= -\frac{2}{n}\cos\frac{n}{2}x\Big|_{-2\pi}^{-\pi}$$

// Agrupando

$$= \frac{1}{2\pi}\Big[\Big(\cancel{(\pi-\pi)\left(-\frac{2}{n}\cos\frac{n}{2}\pi\right)}^{0} - \frac{4}{n^2}\operatorname{sen}\frac{n}{2}\pi$$

$$-(\pi)\left(-\frac{2}{n}\cancel{\cos\frac{n}{2}0}\right) - \cancel{\frac{4}{n^2}\operatorname{sen}\frac{n}{2}0}^{0}\Big)$$

$$+\Big((\pi)\left(-\frac{2}{n}\cos\frac{n}{2}2\pi\right) + \frac{4}{n^2}\operatorname{sen}\frac{n}{2}2\pi$$

$$-\cancel{(0)\left(-\frac{2}{n}\cos\frac{n}{2}0\right)}^{0} + \frac{4}{n^2}\operatorname{sen}\left(\frac{n}{2}\pi\right)\Big)$$

$$+\left(-\frac{2}{n}\cos\frac{n}{2}-\pi + \frac{2}{n}\cos\frac{n}{2}-2\pi\right)\Big] =$$

$$= \frac{1}{2\pi}\Big[-\cancel{\frac{4}{n^2}\operatorname{sen}\frac{n}{2}\pi} + \frac{2\pi}{n} - \frac{2\pi}{n}\underbrace{\cos n\pi}_{(-1)^n} - \cancel{\frac{4}{n^2}\operatorname{sen}\frac{n\pi}{2}}$$

$$2\pi \Big[ \cancel{\frac{4}{n^2}\operatorname{sen}\frac{n}{2}\pi} + \frac{2}{n} - \frac{2}{n}\underbrace{\cos n\pi}_{(-1)^n} \cancel{\frac{4}{n^2}\operatorname{sen}\frac{n}{2}}$$

$$+ \cancel{\frac{4}{n^2}\operatorname{sen} n\pi} - \frac{2}{n}\cos\frac{n\pi}{2} + \frac{2}{n}\underbrace{\cos n\pi}_{(-1)^n} \Big] =$$

$$= \frac{1}{2\pi}\left[ -\frac{8}{n^2}\operatorname{sen}\frac{n\pi}{2} - \frac{2}{n}\cos\frac{n\pi}{2} + \frac{2}{n}(-1)^n - \frac{2\pi}{n}(-1)^n + \frac{2\pi}{n} \right]$$

if $\begin{cases} n=1 \; ; & b_1 = -\frac{5}{\pi} + 2 = \frac{-5+2\pi}{\pi} \\ n=2 \; ; & b_2 = \frac{1}{\pi} \end{cases}$

→ Resultado de la Serie

$$F(x) = \frac{1}{4} + \frac{\pi}{4} - \frac{1}{\pi}\cos\frac{x}{2} + \left(-\frac{5+2\pi}{\pi}\operatorname{sen}\frac{x}{2}\right) + \frac{2}{\pi}\cos x + \frac{1}{\pi}\operatorname{sen} x + \frac{1}{3\pi}\cos\frac{3x}{2} + \dots$$

# Fase y amplitud

jueves, 25 de abril de 2019 07:22 p. m.

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos n\omega t + b_n \operatorname{sen} n\omega t \right)$$

- $\cos n\omega t = \dfrac{e^{in\omega t} + e^{-in\omega t}}{2}$

- $\operatorname{sen} n\omega t = \dfrac{e^{in\omega t} - e^{-in\omega t}}{2i}$

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \frac{a_n}{2} e^{in\omega t} + \frac{a_n}{2} e^{-in\omega t}$$

$$+ \left(\frac{-i}{-i}\right) \frac{b_n}{2i} e^{in\omega t} - \left(\frac{-i}{-i}\right) \frac{b_n}{2i} e^{-in\omega t}$$

$$\rightarrow F(t) = \underbrace{\frac{a_0}{2}}_{C_0} + \sum_{n=1}^{\infty} \left( \underbrace{\frac{1}{2}(a_n - ib_n) e^{in\omega t}}_{C_n^-} + \underbrace{\frac{1}{2}(a_n + ib_n) e^{-in\omega t}}_{C_n^+} \right)$$

$$C_n^+ = C_n^-$$

$\therefore$ Serie de Fourier Complejo

$$F(t) = C_0 + \sum_{n=1}^{\infty} C_n e^{in\omega t}$$

$$C_n = \frac{1}{T} \int_{-p}^{p} F(t) e^{-in\omega t} dt$$

1] Obtener la serie compleja de Fourier de $F(t)$

- $F(t) = \cos\frac{1}{2}t$ | $T = 2\pi$
- $(-\pi < t < \pi)$ | $\therefore \omega = \frac{2\pi}{T} = 1$

$$\therefore F(t) = \frac{1}{T}\int_{-p}^{p} F(t)e^{-in\omega t}\,dt = \frac{1}{2\pi}\int_{-\pi}^{\pi}\cos\frac{1}{2}t\, e^{-int} =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{e^{i\frac{1}{2}t} + e^{-i\frac{1}{2}t}}{2}\right)\left(e^{-int}\right)dt =$$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi} e^{i\frac{1}{2}t - int} + e^{-i\frac{1}{2}t - int}\,dt =$$

$$= \frac{1}{4\pi}\left[\int_{-\pi}^{\pi} e^{(\frac{1}{2}-n)it}\,dt + \int_{-\pi}^{\pi} e^{-it(\frac{1}{2}+n)}\,dt\right] =$$

$$= \frac{1}{4\pi}\left[\int_{-\pi}^{\pi} e^{(2n-1)-i\frac{t}{2}}\,dt + \int_{-\pi}^{\pi} e^{-(2n+1)\,i\frac{t}{2}}\right]dt =$$

$$= \frac{1}{4\pi}\left[\frac{(-i)}{(-i)}\frac{2}{(2n-1)(-i)}\, e^{-i\frac{t}{2}(2n-1)}\Big|_{-\pi}^{\pi} - \frac{2}{(i)(2n-1)}\frac{(-i)}{(-i)}\, e^{-i\frac{t}{2}(2n+1)}\Big|_{-\pi}^{\pi}\right] =$$

$$= \frac{1}{4\pi}\left[\frac{2i}{\ldots} e^{-int + i\frac{t}{2}} + \frac{2i}{\ldots} e^{in\frac{t}{2} - i\frac{t}{2}}\Big|^{\pi}\right] =$$

$$= \frac{1}{4\pi}\left[\frac{2i}{(2n-1)} e^{-int + i\frac{t}{2}} + \frac{2i}{(2n+1)} e^{in\frac{t}{2} - i\frac{t}{2}}\right]_{-\pi}^{\pi} =$$

Recordando:

- $e^{in\pi} = \cos n\pi + i \operatorname{sen} n\pi = (-1)^n$
- $e^{-in\pi} = \cos n\pi - i \operatorname{sen} n\pi = (-1)^n$
- $e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i \operatorname{sen}\frac{\pi}{2} = i$

// Reduciendo

$$= \frac{i}{2\pi}\left[\frac{1}{(2n-1)}(-1)^n i + \frac{1}{(2n+1)}(-1)^n i - \frac{1}{(2n-1)}(-1)^n(-i) - \frac{1}{(2n+1)}(-1)^n i\right] =$$

$$\therefore C_n = \frac{-1}{2\pi}\left[\frac{(-1)^n}{(2n-1)} - \frac{(-1)^n}{(2n+1)} + \frac{(-1)^n}{(2n-1)} - \frac{(-1)^n}{(2n+1)}\right] =$$

$$= \frac{-(-1)^n}{\pi}\left[\frac{1}{2n-1} - \frac{1}{2n+1}\right] =$$

$$\rightarrow C_n = \frac{-(-1)^n}{\pi}\left[\frac{2n+1-2n+1}{(4n^2-1)}\right] = \frac{-2(-1)^n}{\pi(4n^2-1)}$$

// Obteniendo $a_0$

$$a_0 = \frac{2}{T}\int_{d}^{d+T} f(t)\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos\frac{1}{2}t\,dt =$$

$$= \frac{2}{\pi} \operatorname{sen} \tfrac{1}{2} t \Big|_{-\pi}^{\pi} = \frac{2}{\pi}\left[\operatorname{sen}\frac{\pi}{2} - \operatorname{sen}\frac{-\pi}{2}\right] = \frac{4}{\pi}$$

$$\therefore\ C_0 = \frac{a_0}{2} = \frac{\frac{4}{\pi}}{2} = \frac{2}{\pi}$$

$\therefore$ Serie Fourier Trigonométrica

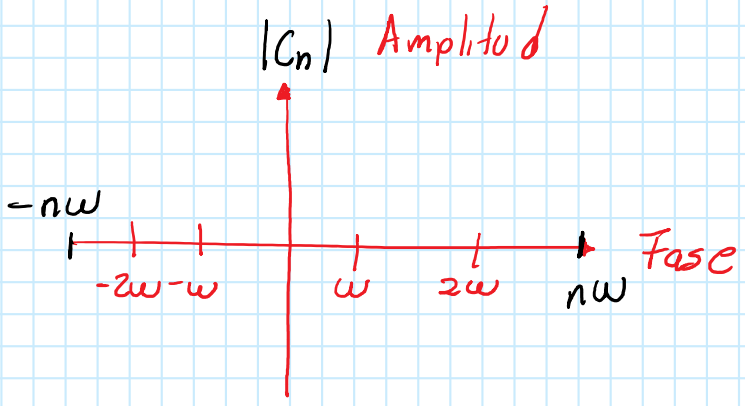
$$F(t) = \frac{2}{\pi} + \sum_{n=-\infty}^{\infty} \frac{-2(-1)^n}{n(4n^2-1)} e^{int}$$

---

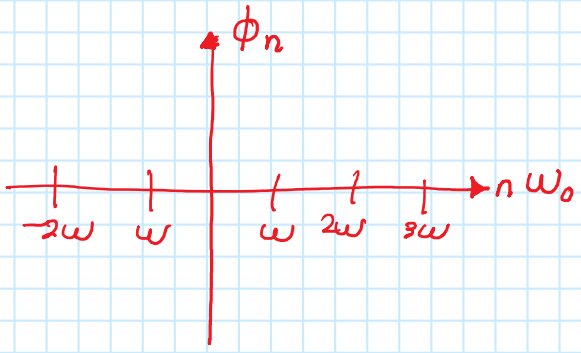
## Teorema de Fourier

3 gráficas
- $F(t)$ // Función
- $\sum_{n=1}^{0}$ // Serie
- $+$ // Espectros

- Espectro de frecuencia en amplitud (Discreta)
  - discreta ; $T \to$ Finita
  - continua ; $T \to \infty$




- Espectro de frecuencia de fase (Discreta)



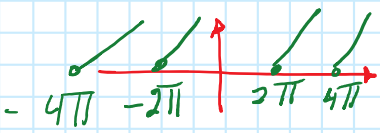
1) Obtener espectro de frecuencia de fase y amplitud (Discreta)

$F(t) = t$

$T \triangleq \text{Periodo} = 2\pi$

$\omega \triangleq \text{Frecuencia} = \frac{2\pi}{T} = 1$

$$F(t) = C_0 + \sum_{n=-\infty}^{\infty} C_n e^{int}$$




$$C_n = \frac{1}{T}\int_{-P}^{P} F(t)\, e^{-in\omega t}\, dt = \frac{1}{2\pi}\int_0^{2\pi} t\, e^{-int}\, dt =$$

| | |
|---|---|
| $t$ | $e^{-int}$ |
| $1$ | $\frac{1}{-in} e^{-int}$ |
| $0$ | $\frac{1}{(in)^2} e^{-int}$ |

$$= \left[(t)\frac{1}{-in} e^{-int} - \frac{1}{n^2} e^{-int}\right]\Bigg|_0^{2\pi} =$$

$$= \frac{2\pi}{-in} e^{-in2\pi} - \frac{1}{(in)^2} e^{-in2\pi}$$

$$- 0 - \frac{1}{n^2} e^{-in0} =$$

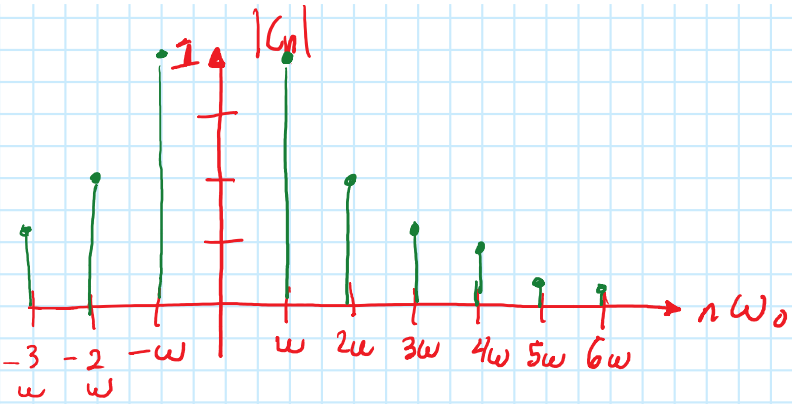
$$= \frac{2\pi}{-in} e^{-in2\pi} - \frac{1}{(in)^2} e^{-in2\pi} - \frac{1}{n^2} =$$

$i^2 = -1 \; - = +$

$$e^{-in2\pi} = \cos(2n\pi) - i\,\text{sen}\, 2n\pi = 1$$

$$= \frac{1}{2\pi}\left[-\frac{2\pi}{in} + \frac{1}{n^2} - \frac{1}{n^2}\right] = -\frac{1}{in}\left(\frac{-i}{-i}\right) = \frac{i}{n}$$

$C_1 = i$




$$C_n = \frac{i}{n}$$

$$|C_n| = \left|\frac{i}{n}\right| = \frac{1}{|n|}$$

• Punto $\left(n\omega_0, \frac{1}{|n|}\right)$

| $n$ | $C_n$ |
|---|---|
| 1 | 1 |
| 2 | 1/2 |
| 3 | 1/3 |
| 4 | 1/4 |
| 5 | 1/5 |

▶ $C_0 = \frac{a_0}{2}$

$$a_0 = \frac{2}{T}\int^{2\pi} f(t)\,dt = \frac{2}{2\pi}\int_0^{2\pi} t\,dt = \frac{1}{\pi}\,\frac{t^2}{2}\Big|_0^{2\pi} =$$

$$= 2\pi$$

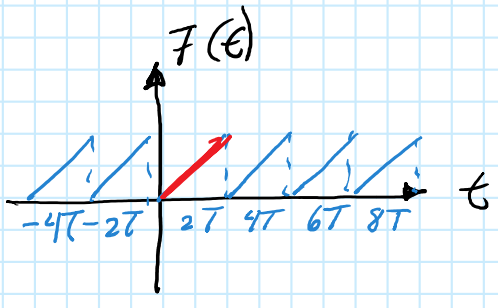
$$\therefore C_0 = \frac{2\pi}{2} = \pi$$

# Fase y amplitud

martes, 30 de abril de 2019 07:39 p. m.

• Obtener
1. Espectro de amplitud
2. Espectro de fase
3. Representación compleja de Fourier

$$F(t) = \frac{2t}{T} \quad (0 < t < 2T)$$




//Resolviendo

• $\omega = \frac{2\pi}{T} = \frac{2\pi}{2T} = \frac{\pi}{T}$

$$F(t) = C_0 + \sum_{n=-\infty}^{\infty} C_n e^{in\omega t} = C_0 + \sum_{n=-\infty}^{\infty} C_n e^{in\frac{\pi}{T}t}$$

$$C_n = \frac{1}{T}\int_0^{2T} F(t)\, e^{-in\omega t}\, dt = \frac{1}{2T}\int_0^{2T} \frac{2t}{T}\, e^{-in\frac{\pi}{T}t}\, dt =$$

$$= \frac{1}{T^2}\int_0^{2T} t\, e^{-in\frac{\pi}{T}t}\, dt =$$

$t \qquad e^{-\frac{in\pi}{T}t}$

$1 \qquad \frac{T}{-in\pi} e^{-\frac{in\pi}{T}t}$

$0 \qquad -\frac{T^2}{n^2\pi^2} e^{-\frac{in\pi}{T}t}$

$$= \frac{1}{T^2}\left[\frac{T}{-in\pi}\, t\, e^{-\frac{in\pi}{T}t} + \frac{T^2}{n^2\pi^2} e^{-\frac{in\pi}{T}t}\right]\Bigg|_0^{2T} =$$

$$= \frac{1}{n^2\pi^2} e^{-i\frac{n\pi}{T}t}$$

$$= \left[ \frac{1}{-in\pi T} t\, e^{-i\frac{n\pi}{T}t} + \frac{1}{n^2\pi^2} e^{-i\frac{n\pi}{T}t} \right]\Bigg|_0^{2T} =$$

// Evaluando $\quad C_n = \frac{2}{-in\pi} = \frac{2i}{n\pi}$ //

// Obteniendo $\quad C_0 = \frac{a_0}{2}$

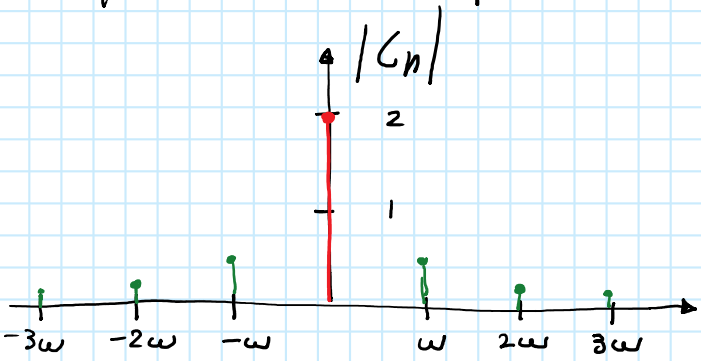
$$a_0 = \frac{1}{T}\int_0^{2\pi} \frac{2t}{T}\, dt = \frac{1}{T^2}\int_0^{2\pi} 2t\, dt = \frac{1}{T^2} t^2 \Big|_0^{2T} =$$

$$= \frac{1}{T^2}\left[\frac{(2T)^2}{1}\right] = 4 \qquad \therefore C_0 = \frac{4}{2} = 2$$

▶ Espectro de Amplitud

$$C_n = \frac{2i}{n\pi}$$

$$|C_n| = \frac{2}{|n\pi|}$$

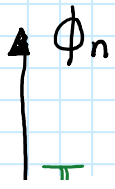



// Evaluaciones

| $n$ | $\lvert C_n \rvert$ |
|---|---|
| $-4$ | $1/2\pi$ |
| $-3$ | $2/3\pi$ |
| $-2$ | $1/\pi$ |
| $-1$ | $2/\pi$ |
| $0$ | $2$ |
| $1$ | $2/\pi$ |
| $2$ | $1/\pi$ |

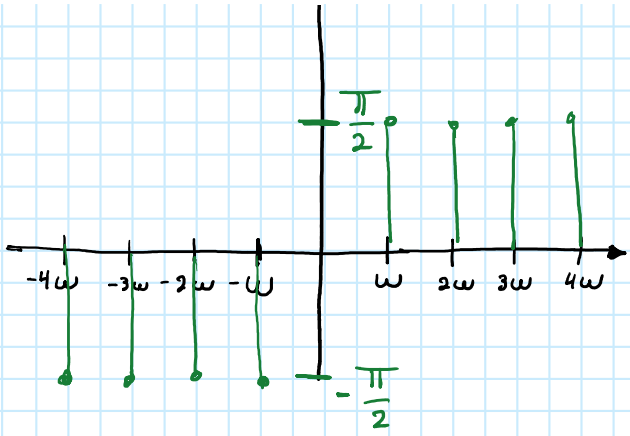
▶ Espectro de Fase

$$\Phi_n = \operatorname{Arg} C_n$$

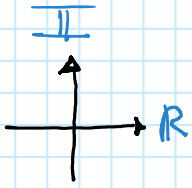



→ NOTA:

- $\operatorname{Arg} z = \theta = \tan^{-1}\left(\frac{x}{y}\right)$
- La que expresa la posición en la




- $\text{Arg } z = \theta = \tan\left(\frac{\bar{y}}{\bar{x}}\right)$
- Lo que expresa la posición en la que cambian los armónicos

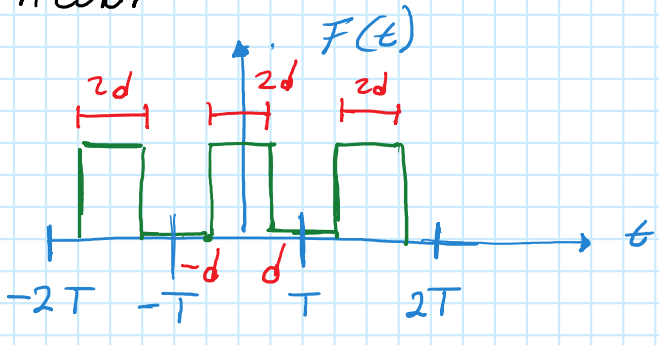



$$C_n = \frac{2i}{n\pi} \qquad y = \frac{2i}{n\pi}$$

---

- Determine la forma compleja de Fourier del tren tren infinito periódico (Periodo $2T$) de pulsos idénticos rectángulares idénticos de magnitud $A$ y duración $2d$.
- Obtenga el espectro de frecuencia discreta en el caso particular

- $d = \frac{1}{10}$
- $T = \frac{1}{2}$




// Obteniendo función (Tren infinito de pulsos)

$$f(t): \begin{cases} 0; & -T \le t < -d \\ A; & -d \le t \le d \\ 0; & d < t \le T \end{cases}$$

- $F(t) = C_0 + \sum_{n=-\infty}^{\infty} C_n e^{in\omega t}$
- $\omega = \frac{2\pi}{T} = \frac{2\pi}{2T} = \frac{\pi}{T}$

$$C_n = \frac{1}{\quad} \int^{d} A\, e^{-i n \frac{\pi}{T} t}\, dt =$$

// Podemos omitir los que son 0

$$C_n = \frac{1}{T}\int_{-d}^{d} A\, e^{-i n \frac{\pi}{T} t}\, dt =$$

// Podemos omitir los que son 0

$$= \frac{A}{2\cancel{T}}\left[\frac{\cancel{T}}{-i n \pi}\, e^{-i n \frac{\pi}{T} t}\right]\Bigg|_{-d}^{d} =$$

$$= \frac{A}{2}\left[-\frac{1}{i n \pi}\, e^{-i n \frac{\pi}{T} d} + \frac{1}{i n \pi}\, e^{i n \frac{\pi}{T} d}\right] =$$

$$= \frac{A}{2}\left[\frac{e^{i n \frac{\pi}{T} d} - e^{-i n \frac{\pi}{T} d}}{i n \pi}\right] =$$

$$= \frac{A}{n\pi}\left[\operatorname{sen}\left(\frac{n \pi d}{T}\right)\right] =$$

$$\frac{\operatorname{sen}(c)}{c} \triangleq \text{Seno Cardinal}$$

$$= \frac{\frac{A}{n\pi}\operatorname{sen}\left(\frac{n\pi d}{T}\right)}{\frac{n\pi d}{T}} = \frac{Ad}{T}\operatorname{sinc}\left(\frac{n\pi d}{T}\right)$$

// Función Sinc

$$\operatorname{sinc} t : \begin{cases} \frac{\operatorname{sen} t}{t} & ; t \neq 0 \\ 1 & ; t = 0 \end{cases}$$

$$f(t) = C_0 + \sum_{n=-\infty}^{\infty} \frac{Ad}{T}\operatorname{sinc}\left(\frac{n\pi d}{T}\right) e^{i n \frac{\pi}{T} t}$$

$$C_0 = \frac{a_0}{2}$$

$$a_0 = \frac{2}{T}\int_{-T}^{T} f(t)\, dt = \frac{1}{T}\int_{-d}^{d} A\, dt = \frac{A}{T} t \Big|_{-d}^{d} =$$

$$= \frac{1}{T}\left[Ad - (-Ad)\right] = \frac{2Ad}{T}$$

$$= \frac{1}{T}\left[Ad - (-Ad)\right] = \frac{2Ad}{T}$$

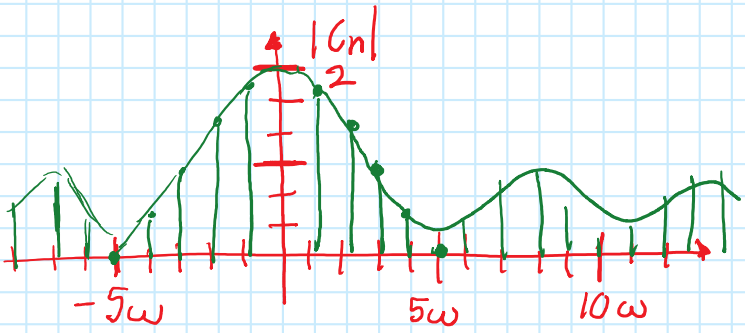
$$\therefore C_0 = \frac{\cancel{2}Ad}{\frac{T}{\cancel{2}}} = \frac{Ad}{T}$$

// Serie Fourier

$$\therefore F(t) = \frac{Ad}{T} + \sum_{n=-\infty}^{\infty} \frac{Ad}{T} \operatorname{sinc}\left(\frac{n\pi d}{T}\right) e^{i n \frac{\pi}{T} t}$$

// Buscando con valores $d = \frac{1}{10}$ $T = \frac{1}{2}$

$$\therefore F(t) = \frac{A}{5} + \sum_{n=-\infty}^{\infty} \frac{A}{5} \operatorname{sinc}\left(\frac{n\pi}{5}\right) e^{i 2n\pi t}$$




if $A = 10$

- $n = 1$

$$|C_n| = \left|\frac{A}{5} \operatorname{sinc}\left(\frac{\pi}{5}\right)\right| = (2)\left|\frac{\operatorname{sen}\left(\frac{\pi}{5}\right)}{\frac{\pi}{5}}\right| = 1.87$$

- $n = 2$

$$|C_n| = (2)\left(\frac{\operatorname{sen}\left(\frac{2\pi}{5}\right)}{\frac{2\pi}{5}}\right) = 1.5136$$

- $n = 3$ $|C_n| = 1.00910$

$n = 4$ $|C_n| = .467744$

$n = 5$ $|C_n| = 0$

$n = 6$

$n = 7$

$n = 8$

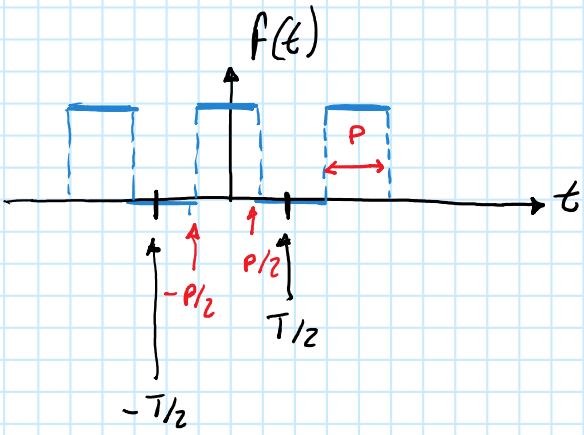
$n = 9$

| | | | | | |
|---|---|---|---|---|---|
| $n=6$ | $n=7$ | $n=8$ | $n=9$ | | |
| $\lvert C_n \rvert = \cancel{-}.311$ | $\lvert C_n \rvert = \cancel{-}.432$ | $\lvert C_n \rvert = \cancel{-}.378$ | $\lvert C_n \rvert = \cancel{-}.207$ | | |
| $n=10$ | $n=11$ | $n=12$ | $n=13$ | $n=14$ | $n=15$ |
| $\lvert C_n \rvert = 0$ | $= .17$ | $\lvert C_n \rvert = .2522$ | $= .2328$ | $\lvert C_n \rvert = .1336$ | $\lvert C_n \rvert = 0$ |

# Transformada de Variables

jueves, 9 de mayo de 2019 07:15 p. m.

- Serie de Fourier nos permite obtener una representación en el dominio de funciones periódicas $f(t)$.
- ¿Dominio de la frecuencia de funciones no periódicas?




//Espacio "frecuencia"

$$F(t) = C_0 + \sum_{n=-\infty}^{\infty} C_n e^{i\omega n t}$$

$t \to \infty$ ∴ Espectro Continuo

- Integral de Fourier

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

// La solución de la integral es la transformada de Fourier

$$\frac{1}{\pi} \int_{-\infty}^{\infty} A(\omega) \cos \omega t + B(\omega) \operatorname{sen} \omega t \, d\omega \quad \ldots (-\infty < t < \infty)$$

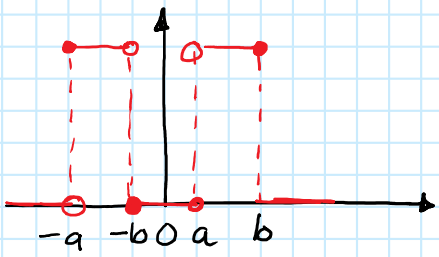
- Transformada de Fourier

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

// Ejercicio

$$\begin{cases} 0 ; & -\infty < t < -a \end{cases}$$

// Ejercicio




$$f(t) = \begin{cases} 0 \,; & -\infty < t < -a \\ 1 \,; & -a \le t < -b \\ 0 \,; & -b \le t \le b \\ 1 \,; & b < t \le a \\ 0 \,; & a < t < \infty \end{cases}$$

$$F(\omega) = \cancel{\int_{-\infty}^{-a} 0}^{\,0} + \int_{-a}^{-b} 1\, e^{-i\omega t}\, dt + \cancel{\int_{-b}^{b} 0}^{\,0} + \int_{b}^{a} 1\, e^{-i\omega t}\, dt + \cancel{\int_{a}^{\infty} 0}^{\,0}$$

$$= \int_{-a}^{-b} e^{-i\omega t}\, dt + \int_{b}^{a} e^{-i\omega t}\, dt =$$

$$= -\frac{1}{i\omega} e^{-i\omega t}\Big|_{-a}^{-b} - \frac{1}{i\omega} e^{-i\omega t}\Big|_{b}^{a} =$$

$$= -\frac{1}{i\omega}\left[e^{i\omega b} - e^{i\omega a}\right] - \frac{1}{i\omega}\left[e^{-i\omega a} - e^{-i\omega b}\right] =$$

$$= -\frac{1}{i\omega}\left[e^{i\omega b} - e^{i\omega b} - e^{i\omega a} + e^{i\omega a}\right] =$$

$$= -\frac{2}{\omega}\left[\frac{e^{i\omega b} - e^{i\omega b} - e^{i\omega a} + e^{i\omega a}}{2i}\right] =$$

$$= -\frac{2}{\omega}\left[\operatorname{sen}(\omega b) - \operatorname{sen}(\omega a)\right] \;//$$

$$\rightarrow \operatorname{sen} z = \frac{e^{zi} - e^{-zi}}{2i}$$

# Transformada de Fourier

martes, 14 de mayo de 2019 07:09 p. m.

- Obtener la Transformada de Fourier

$$F(t)\begin{cases} 1-|t| ; & |t|\le 1 \\ 0 ; & |t|>1 \end{cases} \Rightarrow F(t):\begin{cases} 1+t ; & -1\le t<0 \\ 1-t ; & 0\le t\le 1 \\ 0 ; & |t|>1 \end{cases}$$

$$F(\omega)=\int_{-\infty}^{\infty} F(t)\, e^{-i\omega t}\, dt =$$

$$= \cancel{\int_{-\infty}^{-1} 0\, e^{-i\omega t}\, dt}^{\,0} + \int_{-1}^{0}(1+t)\, e^{-i\omega t}\, dt + \int_{0}^{1}(1-t)\, e^{-i\omega t}\, dt$$

$$+ \cancel{\int_{1}^{\infty} 0\, e^{-i\omega t}\, dt}^{\,0} =$$

$$= \int_{-1}^{0}(1+t)\, e^{-i\omega t}\, dt + \int_{0}^{1}(1-t)\, e^{-i\omega t}\, dt =$$

$$\begin{array}{lll} 1+t & e^{-i\omega t} & \\ 1 & -\frac{1}{i\omega}\, e^{-i\omega t} & \oplus \\ 0 & -\frac{1}{\omega^2}\, e^{-i\omega t} & \ominus \end{array}$$

$$= (1+t)\left(-\frac{1}{i\omega}\, e^{-i\omega t}\right) + \left(\frac{1}{\omega^2}\, e^{-i\omega t}\right) =$$

$$= (1-t)\left(-\frac{1}{i\omega}\, e^{-i\omega t}\right) - \left(\frac{1}{\omega^2}\, e^{-i\omega t}\right)$$

$$= \left[-\frac{1+t}{i\omega}\, e^{-i\omega t} + \frac{1}{\omega^2}\, e^{-i\omega t}\right]\Bigg|_{-1}^{0}$$

$$+$$

$$\left[-(1-t)\, e^{-i\omega t} - \frac{1}{\omega^2}\, e^{-i\omega t}\right]\Bigg|_{0}^{1} =$$

$$\left[-\frac{(1-t)}{i\omega}e^{-i\omega t}-\frac{1}{\omega^2}e^{-i\omega t}\right]\Bigg|_0^1 =$$

$$= \cancel{-\frac{1}{i\omega}} + \frac{1}{\omega^2}\left(1-e^{-i\omega}\right) + \cancel{\left(\frac{1}{i\omega}\right)} - \frac{1}{\omega^2}\left(e^{-i\omega}-1\right) =$$

$$= \frac{2}{\omega^2} - \frac{e^{-i\omega}-e^{-i\omega}}{\omega^2}$$

2 ← sale junto $\frac{1}{\omega^2}$

2 ← completa

Argumento

$$= \frac{2}{\omega^2} - \left(\frac{2}{\omega^2}\right)\cos(\omega)$$

## ▶ Propiedades de la T. Fourier

| $f(t)$ | $F(\omega)$ |
|---|---|
| $a f(t) + b g(t)$ | $a F(\omega) + b G(\omega)$ |
| $f(t-t_0)$ | $e^{-i\omega t_0} F(\omega)$ |
| $e^{i\omega_0 t} f(t)$ | $F(\omega-\omega_0)$ |
| $f(at)$ | $\frac{1}{\lvert a\rvert} F\left(\frac{\omega}{a}\right)$ |
| $f(-t)$ | $F(-\omega)$ |
| $F(t)$ | $2\pi f(-\omega)$ |
| $f(t)\cos(\omega_0 t)$ | $\frac{1}{2}\left[F(\omega+\omega_0)+F(\omega-\omega_0)\right]$ |
| $f(t)\,\mathrm{sen}(\omega_0 t)$ | $\frac{i}{2}\left[F(\omega+\omega_0)-F(\omega-\omega_0)\right]$ |
| $f^{(n)}(t)\ ,\ n\in\mathbb{N}$ | $(i\omega)^n F(\omega)$ |
| $t^n f(t)\ ,\ n\in\mathbb{N}$ | $i^n F^n(\omega)$ |
| $\int_{-\infty}^{t} f(\tau)\,d\tau$ | $\frac{1}{i\omega} F(\omega)$ |
| $(f*g)(t)$ | $F(\omega)G(\omega)$ |
| $f(t)g(t)$ | $\frac{1}{2\pi}(F*G)(\omega)$ |

← Linealidad

← Desplazamiento (time)

← Corrimiento en (Frecuencia)

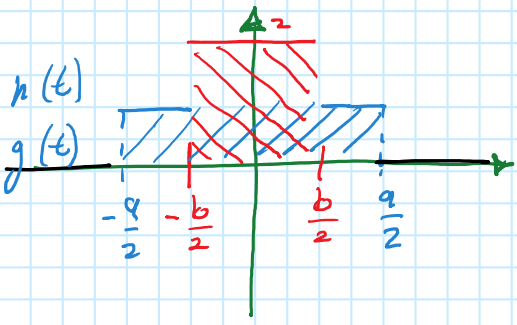
← Escalado

1] Usando "Linealidad" apliqué Fourier.

$\int_a : |t| > \frac{a}{}$

$$f(t): \begin{cases} 0; & |t| > \frac{a}{2} \\ 1; & \frac{b}{a} < |t| < \frac{a}{2} \\ 2; & |t| < \frac{b}{2} \end{cases} \quad ; \; a > b > 0$$

$$f(t): \begin{cases} 0; & \left(-\infty, -\frac{a}{2}\right) \\ 0; & \left(\frac{a}{2}, \infty\right) \\ 1; & \left[-\frac{a}{2}, -\frac{b}{2}\right) \\ 1; & \left(\frac{b}{2}, \frac{a}{2}\right] \\ 2; & \left[-\frac{b}{2}, \frac{b}{2}\right] \end{cases}$$




// Separando la función ; $f(t) = g(t) + h(t)$

$$g(t) = \begin{cases} 0; & |t| > \frac{a}{2} \\ 1; & |t| < \frac{a}{2} \end{cases} \qquad h(t) = \begin{cases} 0; & |t| > \frac{b}{2} \\ 1; & |t| < \frac{b}{2} \end{cases}$$

$$G(\omega) = \int_{-\infty}^{-a/2} 0 + \int_{-a/2}^{a/2} (1)\, e^{-i\omega t}\, dt + \int_{a/2}^{\infty} 0 = \int_{-a/2}^{a/2} e^{-i\omega t}\, dt$$

$$= -\frac{1}{i\omega} e^{-i\omega t} \Big|_{-a/2}^{a/2} =$$

$$H(\omega) = \int_{-b/2}^{b/2} e^{-i\omega t}\, dt = -\frac{1}{i\omega} e^{-i\omega t} \Big|_{-b/2}^{b/2}$$

// Juntando

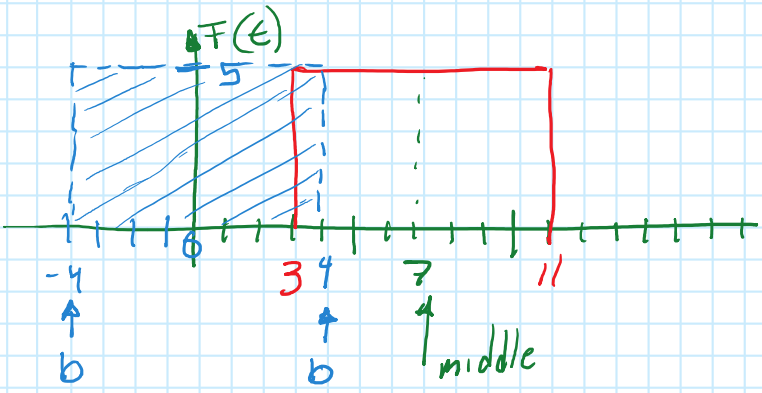
$$= \cancel{-}\frac{1}{i\omega}\left[\left(\cancel{-}e^{-i\omega\frac{b}{2}} \cancel{-} e^{i\omega\frac{b}{2}}\right) + \left(\cancel{-}e^{-i\omega\frac{a}{2}} \cancel{-} e^{i\omega\frac{a}{2}}\right)\right] =$$

$$= \frac{2}{\cancel{i}\omega}\left[\left(\frac{e^{i\omega\frac{b}{2}} - e^{-i\omega\frac{b}{2}}}{2i}\right) + \left(\frac{e^{i\omega\frac{a}{2}} - e^{-i\omega\frac{a}{2}}}{2i}\right)\right] =$$

$$= \frac{2}{\omega}\left[\operatorname{sen}\left(\omega\frac{b}{2}\right) + \operatorname{sen}\left(\omega\frac{a}{2}\right)\right]$$

2] $F(t) = 5\left[1 + (t-3) - 1 + (t-11)\right]$

$$\underbrace{-3 \quad -11}_{8}$$




→ Función Desplazada | $F(t) = (t-7)$

$$\therefore H(\hat{\omega}) = e^{i\omega a} F(\omega) = e^{i\omega a}\frac{2K}{\omega}\operatorname{sen}(a\omega)$$

$$= \left(e^{-i\omega 7}\right)\left(\frac{2(5)}{\omega}\operatorname{sen}(4\omega)\right) =$$

a ≙ Desp. del medio

b ≙ Fin de la función

3] $F(t) = 5\, e^{-3(t-5)^2}$

$g(t) = F(t-5)$

$$e^{-at^2} \longrightarrow \sqrt{\frac{\pi}{a}}\, e^{-\frac{\omega^2}{4a}}$$

$$G(\hat{\omega}) = e^{i\omega a} F(\hat{\omega}) = \left(e^{-i\omega 5}\right)\left(5\sqrt{\frac{\pi}{3}}\, e^{-\frac{\omega^2}{12}}\right)$$

$$G(\hat{\omega}) = e^{i\omega 4} F(\hat{\omega}) = \left(e^{-i\omega 3}\right)\left(5\sqrt{\frac{\pi}{3}}\, e^{-\frac{\omega^2}{12}}\right)$$

4] $$F(t) = \frac{1}{t^2+6t+13} = \frac{1}{(t+3)^2+4} = \frac{1}{(t+3)^2+2^2}$$

Desplazada $\rightarrow a$ $\quad a$

$$g(t) = F(t+3)$$

$$G(\hat{\omega}) = e^{i\omega a} F(\hat{\omega}) =$$

$$\frac{1}{a^2+t^2} \longrightarrow \frac{\pi}{a} e^{-a|\omega|}$$

$$= \left(e^{i\omega 3}\right)\left(\frac{\pi}{2} e^{-2|\omega|}\right)$$

5] $$F(t) = 3e^{-4|t|}\cos(2t) \qquad \frac{1}{2}\left[F(\omega+\omega_0) + F(\omega-\omega_0)\right]$$

$$e^{-a|t|} = \frac{2a}{a^2+\omega^2} = 3\,\frac{(2)(4)}{(4)^2+\omega^2} = \frac{24}{16+\omega^2}$$

$$\longrightarrow F(\hat{\omega}) = \frac{1}{2}\left[\frac{24}{16+(\omega+2)^2} + \frac{24}{16+(\omega-2)^2}\right]$$

6] $$F(t) = H(t-k)\, e^{-\frac{t}{4}}$$

guardar (Desplazamiento)

$$T = t-K$$

$$t = T+K$$

$$\therefore F(t) = H(T)\, e^{-\frac{T+K}{4}} = H(T)\, e^{-\frac{T}{4}}\, e^{-\frac{K}{4}}$$

$$= \left(e^{-\frac{K}{4}}\right) H(T)\, e^{-\frac{T}{4}} =$$

$$= \left(e^{-\frac{K}{4}}\right)\left(\frac{1}{\frac{1}{4}+i\omega}\right) = \left(e^{-\frac{K}{4}}\right)\left(\frac{4}{1+i\omega}\right)\underbrace{\left(e^{-i\omega K}\right)}_{\text{Original}}$$

// Me cerró la

// Me cerró la
boca xd

# Anti-transformada

martes, 21 de mayo de 2019 07:14 p. m.

Tiempo $\rightarrow$ Frecuencia

Anti-transformada

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{i\omega t}\, d\omega$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{i\omega t}\, dt$$

1] $F^{-1}\left[\dfrac{e^{2i\omega}}{5+i\omega}\right]$

- $F(\omega) = \underbrace{\left(e^{2i\omega}\right)}_{\text{Desplazamiento en } a} \underbrace{\left(\frac{1}{5+i\omega}\right)}_{\text{Sin desplazamiento}}$

$a$

$F(t+a)$

$\dfrac{1}{a+i\omega}$

$u(t)\, e^{-at} =$

$u(t)\, e^{-5t}$

$$F(t) = u(t+2)\, e^{-5(t+2)}$$

13] $F(\omega) = \dfrac{2}{4\omega^2 - 10i\omega - 6}$

$$= \frac{2}{(2\omega - 3i)(2\omega - 2i)} = \frac{A}{2\omega - 3i} + \frac{B}{2\omega - 2i} =$$

$$2 = (A)(2\omega - 2i) + (B)(2\omega - 3i)$$

- if $\omega = i$

$-iB = 2$ ; $B = \dfrac{2}{-i}\left(\dfrac{i}{i}\right)$ ; $B = 2i$

- if $\omega = 0$

$$-i\,(i) \qquad \frac{}{}$$

- if $\omega = 0$

$$2 = 2iA + \cancel{(2i)(-3i)}^{\,6}$$

$$A = -2i$$

$$\therefore F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(-\frac{2i}{2\omega-3i} + \frac{2i}{2\omega-2i}\right)e^{i\omega t}\,dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{-2i}{2\omega-3i}\left(\frac{i}{i}\right)e^{i\omega t}\,dt + \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{2i}{2\omega-2i}\right)\left(\frac{i}{i}\right)e^{i\omega t}\,dt$$

$$= \frac{1}{2\pi}\left[\int_{-\infty}^{\infty}\frac{\cancel{2}^{\,1}}{\cancel{2}\omega i+\underset{\frac{\cancel{2}}{2}}{3}}e^{i\omega t}\,dt \;\cancel{+}\; - \int_{-\infty}^{\infty}\frac{\cancel{+}\cancel{2}^{\,1}}{\cancel{2}\omega i+\cancel{2}}e^{i\omega t}\,dt\right. =$$

$a = 3/2$ $\qquad$ $a = 1$

$$= \frac{1}{2\pi}\,u(t)\,e^{-3/2\,t} \qquad - \frac{1}{2\pi}\left(u(t)\,e^{t}\right)$$

$$= \frac{1}{2\pi}\left[u(t)\left(e^{-\frac{3}{2}t} - e^{t}\right)\right]$$

12) $$F(\omega) = \frac{-2}{(1+3i\omega)(3+3i\omega)}$$

$$-2 = (A)(3+3i\omega) + (B)(1+3i\omega)$$

- if $\omega = i$

$$-2 = (B)\left(1 - 3\cancel{(i)(-i)}^{\,1}\right) \;;\; B = 1 \qquad \therefore A = -1$$

$$\therefore \mathcal{F}^{-1}\left[F(\omega)\right] = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(-\frac{1}{1+3i\omega} + \frac{1}{3+3i\omega}\right)e^{i\omega t}\,dt =$$

$$= \frac{1}{6\pi}\int_{-\infty}^{\infty} -\frac{1}{\frac{1}{3}+i\omega}\,e^{i\omega t}\,dt + \frac{1}{1+i\omega}\,e^{i\omega t}\,dt =$$

$a = \frac{1}{3}$ $\qquad$ $a = 1$

$$= \frac{1}{6\pi}\left[u(t)\left(e^{-t} - e^{-\frac{1}{3}t}\right)\right]$$