INTRODUCCIÓN

Monday, January 28, 2019 11:14 AM

1] Libros

- Electricidad y Magnetismo
  - G. Jaramillo
- Electricidad y Magnetismo
  - Francis W. Sear
- Problemas inventados

2] Forma de evaluar

- 4 - exámenes | Cada 4 semanas
  - ↳ 7 o más | De los exámenes →
- 3 Tareas por clase | Se entregan el Lunes

Método de Análisis

Monday, January 28, 2019 11:52 AM

Fenómenos Físicos

↓

Identificar fenómenos físicos LC

↓

Clasificar fenómenos físicos CL

↓

Partes Involucradas MODELO FÍSICO LC

↓

Identificar Causas - Efectos CL

↓

Relacionar Causas y efectos CL

→

Establecer leyes que rigen el fenómeno CL

↓

MODELO MATEMÁTICO CL

↓

Probar el modelo LC

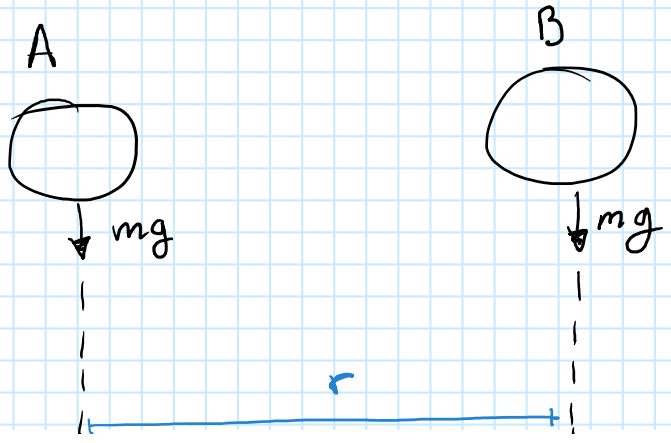
↓

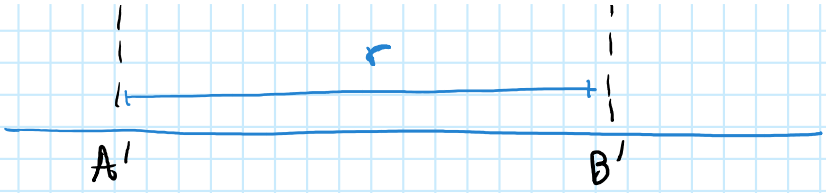
Es el error ≤ cierta tolerancia

- si → Modelo Bueno
- no → Repetir

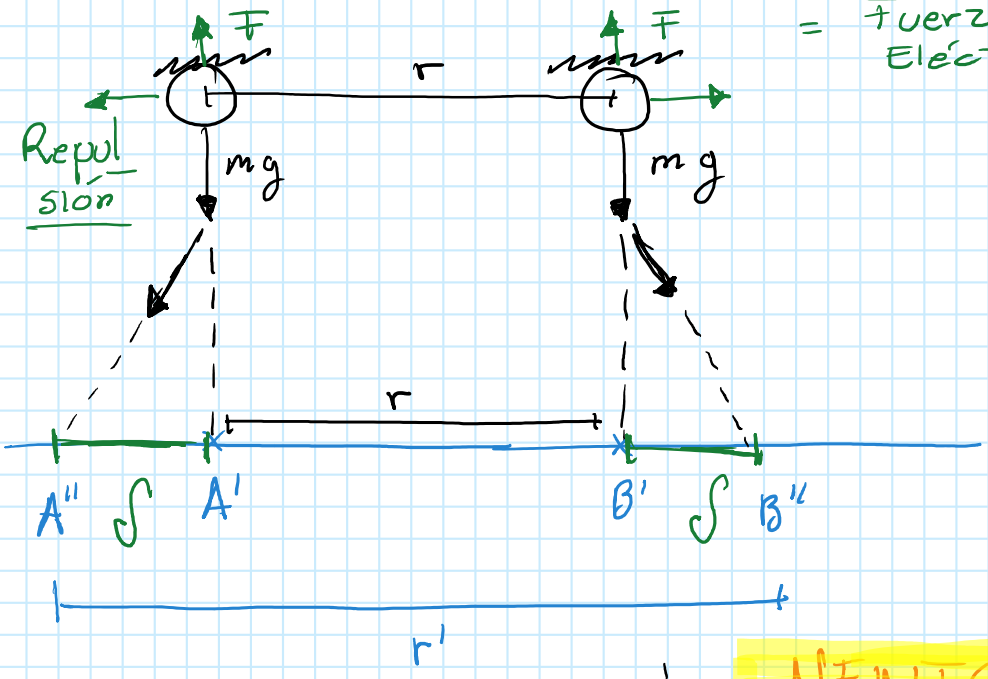
Ley de Coulumb

1]





2]



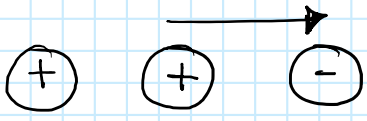
= Fuerza Eléctrica

CASO FROTADO

$\therefore r' = r + 2\delta$

- Atracción – Diferente cantidad
- Repulsión – Misma cantidad

Repulsión / Atracción



NEWTON

1. Permanece en mov o en reposo hasta otra fuerza
2. Directamente proporcional a la masa y aceleración
3. Acción – reacción

▶ NOTA

• Fuerza tangencial

- Fuerza tangencial
  - ↳ Naranjas - Lejos

2_Clase

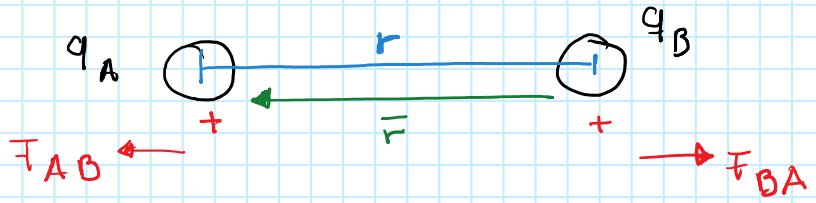
Wednesday, January 30, 2019 7:18 AM

- Clase #2
  30 - enero - 2019

Objetivo
1] Ley de Couluomb
2] Campo Eléctrico

Ley de Couloumb

- Carga electrica → "q"



q_{B_1}	q_{B2}	q_{Bm}
$q_{A_1} \to F_{A_1 1}$	$q_{A_1} \to F_{A_1 2}$	$q_{A_1} \to F_{A_1 m}$
$q_{A_2} \to F_{A_2 1}$	$q_{A_2} \to F_{A_2 2}$	$q_{A_2} \to F_{A_2 m}$
⋮	⋮	⋮
$q_{A_n} \to F_{A_n 1}$	$q_{A_n} \to F_{A_n 2}$	$q_{A_n} \to F_{A_n m}$

Experimento

$q_A \to F_A$

$F_A = f(q_A)$

$\therefore F_A \sim q_A$

$q_B \to F_A$

$F_A = f(q_B)$

$\therefore F_A \sim q_B$

$F \sim \frac{1}{r^2}$

* Inversamente con el cuadrado de la distancia

Ley de Couloumb

- $F \sim \frac{q_A q_B}{r^2}$

Modelo matemático

La fuerza de origen electrico entre 2 cuerpos cargos es directamente proporcional a la carga de los cuerpos e inversamente proporcional

r^2

Modelo matemático

carga de los cuerpos e inversamente proporcional al cuadrado de la distancia que los separa

$$F = K \frac{q_A \, q_B}{r^2}$$

$\bar{r} \triangleq$ vector ; $\bar{r}^2 \triangleq$ modulo al cuadrado

* Vector $\bar{r}$

Causa → Efecto

$$\bar{F}_{AB} = \underbrace{K \frac{q_A \, q_B}{r_{AB}^2}}_{\text{Escala (Positivo)}} \left[\bar{r}_{u_{AB}} \right]$$

Unidades / Sistemas

- $q \rightarrow$ couloumb
- $r \rightarrow$ m
- $F \rightarrow$ Newton

• MKS abs (racionalizado)

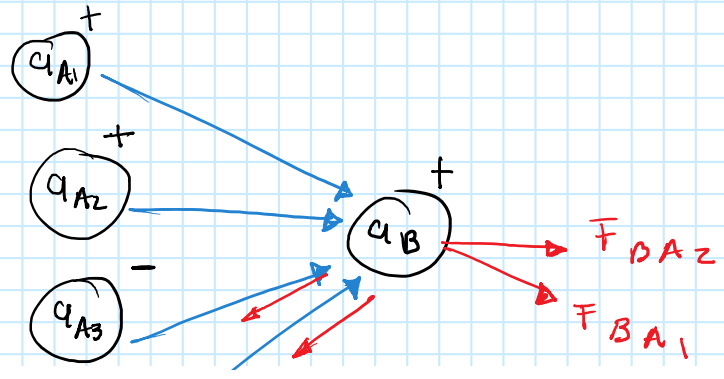
- Longitud → m
- Masa → kg
- Tiempo → s
- Couloumb → Q

} Fundamentales

- F → Newton } Compuesta

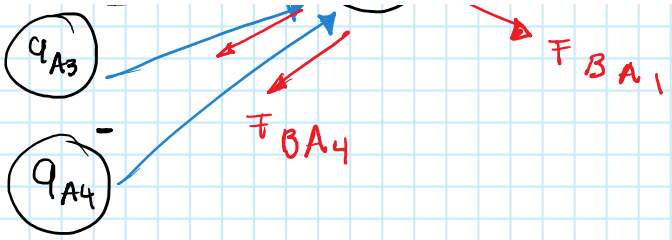
• $K = 9 \times 10^9 \left[\frac{\text{Newton} \; m^2}{\text{coulomb}^2} \right]$

• $K = \frac{1}{4\pi \varepsilon_0}$; $\varepsilon_0 = \frac{1}{4\pi K} = 8{,}85 \times 10^{-12}$



Principio de la superposición

Cuando en un fenómeno físico existe



meno físico existe proporcionalidad lineal entre la causa - efecto

↓

Principio de Superposición

El efecto resultante producido por una serie de causas es igual a la suma de los efectos producidos por cada una de ellas consideradas individualmente.

(Sumar Vector)

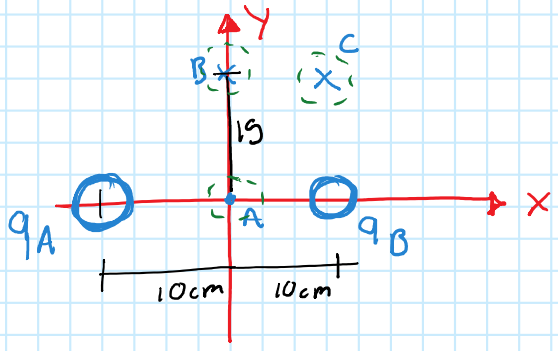
$$\therefore F_{\substack{resultante \\ BA}} = \sum_{i=1}^{n} F_{BAi} \; ; \; F = \sum_{i=1}^{n} k \frac{q q_i}{r_i^2} [r_{ui}]$$

- $F_{xi} = \sum_{i=1}^{n} F_{xi}$
- $F_{yi} = \sum_{i=1}^{n} F_{yi}$; $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ // Escalar
- $F_{zi} = \sum_{i=1}^{n} F_{zi}$
- $\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$; $\varphi = \tan^{-1}\left(\frac{F_z}{F_{xy}}\right)$ // Dirección

Carga Puntual

Es aquella que tiene toda su carga en un solo punto

- Problema Tarea



$q_A = 2 \times 10^{-8}$ Coulomb | $F_{PA} = ?$ $F_{PC} = ?$

$q_B = -3 \times 10^{-8}$ Coulomb | $F_{PB} = ?$

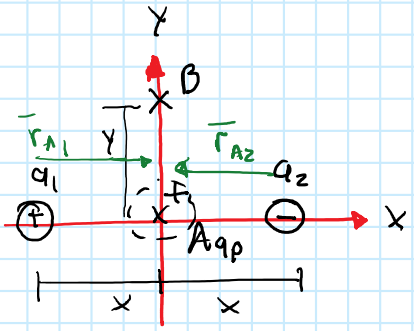
3_Clase

Wednesday, February 6, 2019 7:20 AM

Clase #3
6 -Feb- 2019

Objetivos
1. Ley de Coulomb
2. Campo electrico

Para "A"



$$F_{P_A} = F_{P_{A_1}} + F_{P_{A_2}}$$

$$F_{P_{A_1}} = R \frac{q_1 q_p}{(r_{1A})^2} [\hat{i}]$$

$$F_{P_{A_2}} = R \frac{q_2 q_p}{(r_{2A})^2}$$

- DATOS

$q_1 = 2 \times 10^{-8}$ Coul

$q_2 = -3 \times 10^{-8}$ Coul

$q_p = 4 \times 10^{-8}$ Coul

$x = 10$ cm

$y = 19$ cm

$$F_{P_{A_1}} = (9 \times 10^9) \frac{(2 \times 10^{-8})(4 \times 10^{-8})}{(10 \times 10^{-2})^2} \begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix} =$$

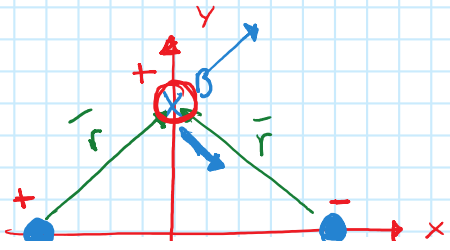
$$= 72 \times 10^{-4} \begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix} \text{ newtons}$$

$$F_{P_{A_2}} = (9 \times 10^9) \frac{(-3 \times 10^{-8})(4 \times 10^{-8})}{10 \times 10^{-4}} \begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix} =$$

$$= 108 \times 10^{-4} [\text{newtons}] \hat{i}$$

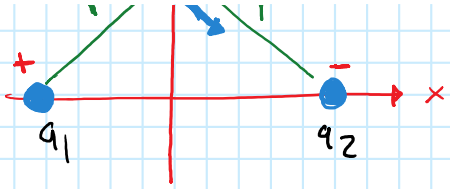
$$\therefore F_R = 180 \times 10^{-4} [\text{newtons}] \hat{i}$$

Para el punto "B"



$$F_R = F_{P_{B_1}} + F_{P_{B_2}}$$

$$F_{P_{B_1}} = R \frac{q_{1B} q_p}{r^2} [r_{u1B}]$$



$$F_{P_{B_1}} = R \frac{q_{1B}\, q_P}{r^2} [r_{u\,1B}]$$

$$F_{P_{B_2}} = R \frac{q_{2B}\, q_P}{r^2} [r_{u\,2B}]$$

$$F_{P_{B_1}} = F_{P_{B_1x}} [\hat{i}] + F_{P_{B_1y}} [\hat{j}]$$

$$F_{P_{B2}} = F_{P_{B2x}} [\hat{i}] + F_{P_{B2y}} [\hat{j}]$$

- $F_{P_{B_1x}} = F_{P_{B_1x}} \left(\frac{x}{r_{B_1}}\right) [\hat{i}]$

  $F_{P_{B_1y}} = F_{P_{B_1y}} \left(\frac{y}{r_{B_1}}\right) [\hat{j}]$

- $F_{P_{B_2x}} = F_{P_{B_2x}} \left(\frac{x}{r_{B_2}}\right) [-\hat{i}]$

  $F_{P_{B_2y}} = F_{P_{B_2y}} \left(\frac{x}{r_{B_2}}\right) [-\hat{j}]$

$$\therefore F_{P_{Bx}} = F_{P_{B_1x}} + F_{P_{B_2x}}$$

$$F_{P_{By}} = F_{P_{B_1y}} + F_{P_{2y}}$$

$$\therefore F_{P_B} = \sqrt{(F_{P_{Bx}})^2 + (F_{P_{By}})^2}$$

$$\theta = \tan^{-1}\left(\frac{F_{P_{By}}}{F_{P_{Bx}}}\right)$$

Campo Electrico: Aquel estado modificado del espacio en la vecindad de una particula cargada electricamente

Intensidad de Campo "E": $\bar{E} = \frac{1}{q_P} \bar{F}$ $\left[\frac{\text{Newtons}}{\text{Coulomb}}\right]$ //Asociado a un punto del espacio

$$F_{P_1} = k \frac{q_1 q_P}{r_{1P}^2} [r_{u1}]$$

$\bar{E} \rightarrow$ Punto

$\bar{F} \rightarrow$ Carga

$$E_A = F_{AP_1}\left(\frac{1}{q_A}\right) ; \quad E_{P_1} = \frac{1}{\cancel{q_P}} k \frac{q_1 \cancel{q_P}}{(r_{1P_1})^2}$$

$$E_{A_1} = K\frac{q_1}{(r_{1A})^2}[r_{u_{1A}}]$$

$$E_{A_2} = K\frac{q_2}{(r_{2A})^2}[r_{u_{1A}}]$$

$$\bar{E}_A = \bar{E}_{A_1} + \bar{E}_{A_2}$$

- Escalar $\frac{1}{q_p}$ por la fuerza ejercida en el punto a analizar

TAREA 3

E_A, E_B, E_C, E_D

$F = (q_p)(E)$

* Sacar intensidad eléctrica

Distribuciones de carga

1) Distribución volumétrica de carga o densidad volumétrica de carga:

$$\rho = \frac{q_f - q_I}{V_f - V_I} = \frac{\Delta q}{\Delta \tau}$$

vol $\rightarrow \tau$

$d\tau = dx\,dy\,d\tau$

$\lim \Delta\tau \rightarrow 0$

$$\therefore \rho = \frac{dq}{d\tau} \; ; \int_0^q dq = \int_0^\tau \rho\, d\tau$$

2) Distancia Superficial de Carga o densidad superficial de carga

$$\sigma = \frac{\Delta q}{\Delta A} \left[\frac{coul}{m^2}\right]$$

$$\sigma = \frac{\Delta q}{\Delta A} \quad \left[\frac{coul}{m^2}\right]$$

$$\lim \Delta A \to 0$$

$$\therefore \quad \sigma = \frac{dq}{dA} \quad ; \quad \int_0^q dq = \int_0^A \sigma \, dA$$

3) Distribución lineal de carga "λ"

$$\lambda = \frac{\Delta q}{\Delta l} \quad \left[\frac{Coul}{m}\right] \qquad \lim \Delta l \to 0$$

$$\therefore \quad \lambda = \frac{dq}{dl} \quad ; \quad \int_0^q dq = \int_0^l \lambda \, dl$$

4_clase

Monday, February 11, 2019 7:20 AM

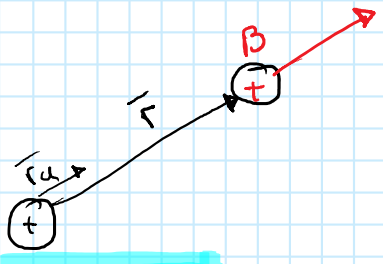
- Clase #4
- 11-Febrero-19
- Objetivos

- Distribución de carga
- Campos Electricos
- Dudas o Preguntas

¿Qué me indica el vector intensidad de Campo?

R= El vector de intensidad de campo en un punto me indica la fuerza que se ejercería sobre una particula cargada eléctricamente con una carga unitaria positiva al ser colocada en ese punto.

NOTA



$$\bar{E} = \left(\frac{1}{q}\right)\bar{F}$$

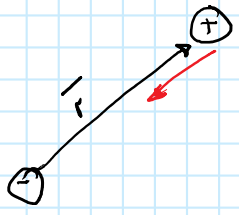
→ Si $q_p > 0$

∴ E y F tienen la misma dirección

→ Si $q_p < 0$

∴ E y F tiene la dirección contraria

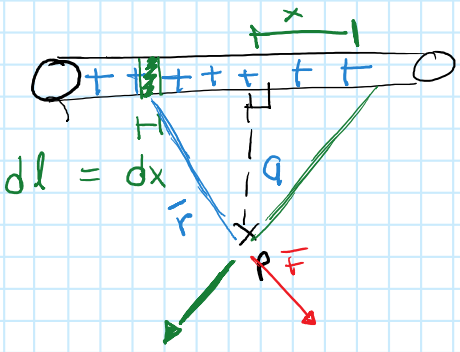
→ El campo eléctrico no depende de la carga de prueba



- Cuando el E es producido por una carga negativa va hacia adentro
- Cuando el E es producido por una carga positiva va hacia afuera

$$\bullet E_{B1} = K \frac{q_1}{r_{1B}^2} \cos\theta_1 = K\left(\frac{q_1}{r_{1B}^2}\right)\left(\frac{x}{r_{1B}}\right)\hat{i}$$

Conductores (Distribución carga)



A

$$\lambda = \frac{dq}{dl} = cte$$

$$dq = \lambda\, dl$$

$$\therefore E = K \frac{q}{r^2}[\bar{r}_u]$$

• if $q \to dq$

$$dE = K\frac{dq}{r^2} = K\frac{\lambda\, dl}{r^2}[\bar{r}_u]$$

$$dE_x = K\left(\frac{\lambda\, dy}{r^2}\right)\left(\frac{x}{r}\right)$$

$$dE_y = K\left(\frac{\lambda\, dx}{r^2}\right)\left(\frac{a}{r}\right)$$

$$E = E_x\hat{i} + E_y\hat{j}$$

$\rightarrow E_x = \int dE_x = 0$ // Sólo si esta en el centro

$$\rightarrow E_y = \int dE_y = \int K\left(\frac{\lambda\, dy}{r^2}\right)\left(\frac{a}{r}\right) =$$

$$= K\lambda a \int \frac{dx}{r^3}$$

$$\tan\theta = \frac{x}{a}\ ;\ x = a\tan\theta\ ;\ dx = a\sec^2\theta\, d\theta$$

$$\text{coseno} = \frac{a}{x}\ ;\ r^3 = a^3\sec^3\theta$$

$$\text{coseno} = \frac{a}{x} \; ; \quad r^3 = a^3 \sec^3\theta$$

$$E_y = K\lambda \cancel{a} \int \frac{\cancel{a} \sec^{\cancel{2}}\theta}{a^{\cancel{3}} \sec^{\cancel{3}}\theta} \, d\theta = \frac{1}{4\pi\varepsilon_0} \frac{\lambda}{a} \int_{\alpha}^{\beta} \cos\theta \, d\theta =$$

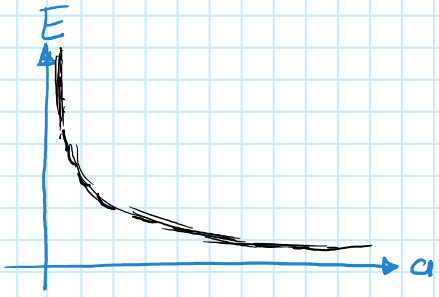
$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{\lambda}{a}\right)\left(\operatorname{sen}\beta - \operatorname{sen}\alpha\right) =$$

* Pero el conductor es muy largo

$$\therefore \quad \beta \to \frac{\pi}{2} \quad ; \quad \alpha = -\frac{\pi}{2}$$

$$\therefore \quad E_y = \left(\frac{1}{2\pi\varepsilon_0}\right)\left(\frac{\lambda}{a}\right)$$

* Campo eléctrico es perpendicular

- 

- $E\,a = \dfrac{\lambda}{2\pi\varepsilon_0} = cte$

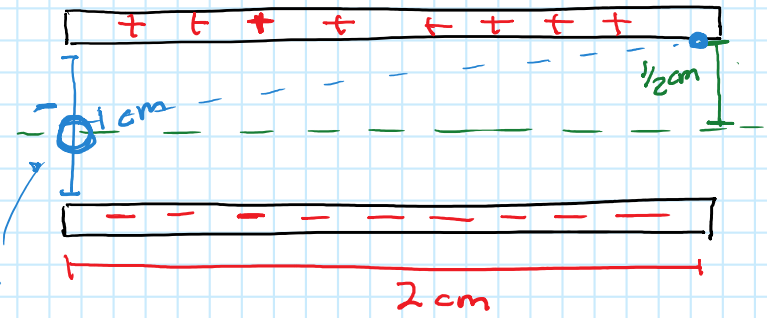
* $\varepsilon_0 = 8{,}8542 \times 10^{-12} \left[\dfrac{C^2}{Nm^2}\right]$

5_Clase

Wednesday, February 13, 2019 7:18 AM

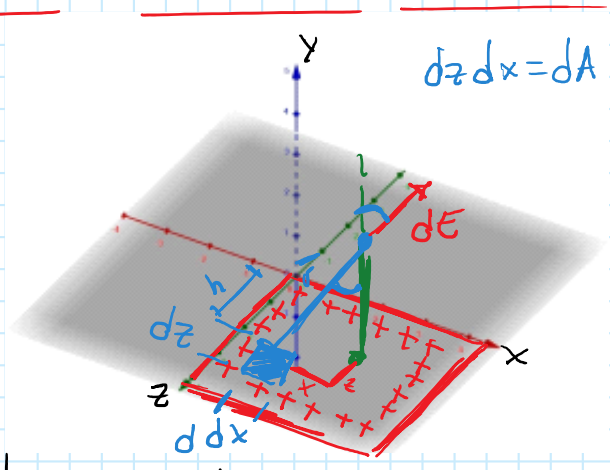
2.3]

$V_0 = 10^7 \frac{m}{seg}$



→ continua

- Campo eléctrico producido por una placa cargada electricamente



$E = ¿?$

$dE_x \; ; \; E_x = \int dE_x \overset{0}{=} 0$

$dE_y \; ; \; E_y = \int dE_y \neq 0$

$dE_z \; ; \; E_z = \int dE_z \overset{0}{=} 0$

$E = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$

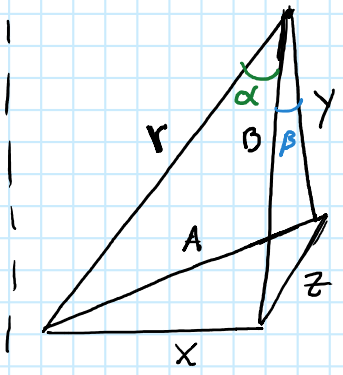
$\therefore E = E_y \hat{j}$

$\sigma = \frac{dq}{dA} \; ; \; dq = \sigma dA$

$E = k \frac{q}{r^2} \; [ru]$

$dE = k \frac{dq}{r^2} \; [ru]$

$dE_y = dE \cos\theta = k \frac{\sigma \, dx \, dz}{r^2} \frac{y}{r} \; [\hat{j}]$



$E_y = k\sigma y \iint \frac{dx\,dz}{r^3}$

$= k\sigma y \int dz \int \frac{dx}{r^3}$

- x con β

$$\tan\alpha = \frac{x}{\beta} \quad ; \quad x = \beta \tan\alpha$$

$$dx = \beta \sec^2\alpha \, d\alpha$$

$$\cos\alpha = \frac{\beta}{r} \quad ; \quad r^3 = \beta^3 \sec^3\alpha$$

// Sustituyendo

$$= K\sigma y \int dz \int \frac{\beta \sec^2\alpha \, d\alpha}{\beta^3 \sec^3\alpha} =$$

$$= K\sigma y \int \frac{dz}{\beta^2} \int \cos\alpha \, d\alpha =$$

$$= K\sigma y \int \frac{dz}{\beta^2} \operatorname{sen}\alpha \Big|_{\alpha_1}^{\alpha_2} =$$

$$\alpha_1 = -\frac{\pi}{2}$$

$$\alpha_2 = \pi/2$$

$$= K\sigma y \int \frac{dz}{\beta^2} \left[\cancel{1 - (-1)}\right]^{2} =$$

$$= 2K\sigma y \int \frac{dz}{\beta^2} =$$

- z con β

$$\tan\beta = \frac{z}{y} \quad ; \quad z = y \tan\beta \quad ; \quad dz = y \sec^2\beta \, d\beta$$

$$\cos\beta = \frac{y}{\beta} \quad ; \quad \beta^2 = y^2 \sec^2\beta$$

// Sustituyendo

$$E_y = 2K\sigma y \int \frac{y \sec^2\beta \, d\beta}{y^2 \sec^2\beta} =$$

$$= 2K\sigma y \cancel{\frac{1}{y}} \int d\beta = 2K\sigma \left[\beta_2 - \beta_1\right] =$$

- $\beta_2 = \frac{\pi}{2}$
- $\beta_1 = -\frac{\pi}{2}$

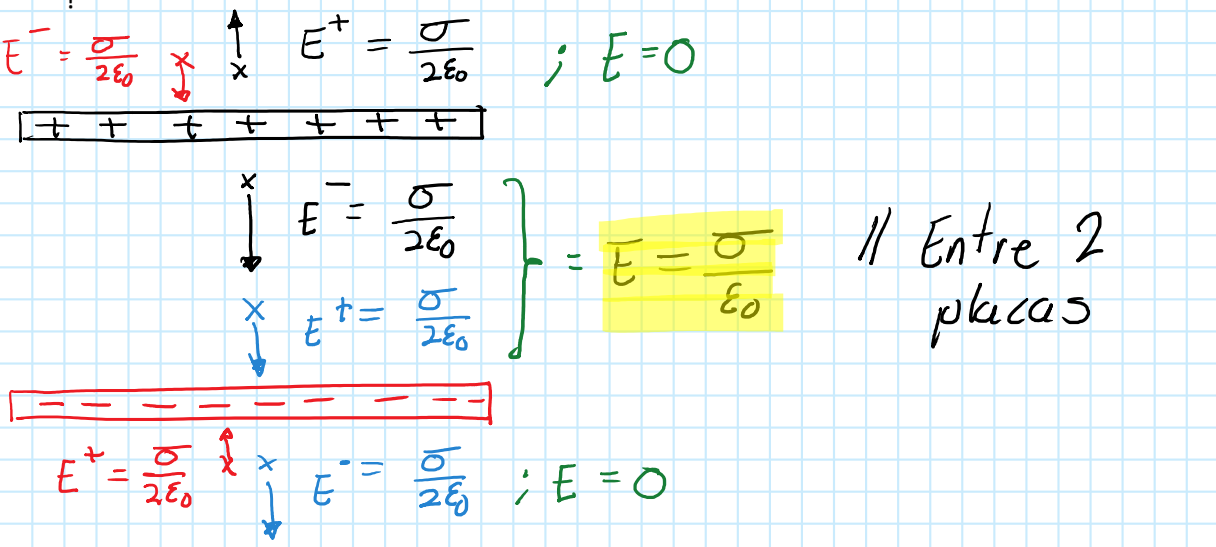
$$= 2K\sigma \left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] =$$

$$= 2K\sigma\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right] =$$

$$E_y = 2\pi K\sigma \left[\hat{j}\right] = \frac{2\pi\sigma}{4\pi\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}\left[\hat{j}\right]$$

?

$E^- = \frac{\sigma}{2\varepsilon_0}$ $\quad E^+ = \frac{\sigma}{2\varepsilon_0}$; $E = 0$



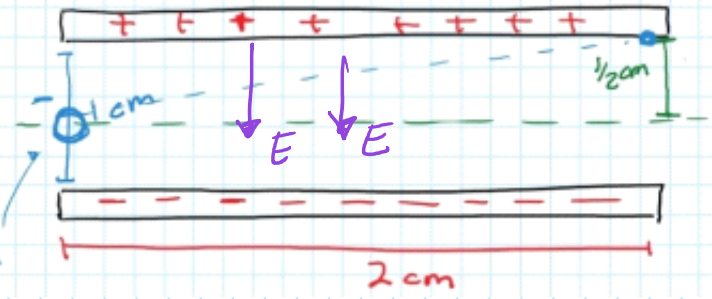
$$= E = \frac{\sigma}{\varepsilon_0}$$ // Entre 2 placas

; $E = 0$

→ Continua

2.3]

$V_0 = 10^7 \frac{m}{seg}$



$$E = \frac{1}{q_p} F \; ; \; F = q_p E \; ; \; F = q_p \frac{\sigma}{\varepsilon_0}\left[\hat{j}\right]$$

⊖ electron

- $V_x = \frac{dx}{dt}$ • $V_y = \frac{dy}{dt}$

$a_x = \frac{dV_x}{dt}$ $\quad a_y = \frac{dV_y}{dt}$

- $F_x = 0$; $F_y = q_p \frac{\sigma}{\varepsilon_0}$

- $F_x = 0$

$a_x = 0$

$\frac{dV_x}{dt} = 0$

$V_x = cte$

$dx = V_x dt$

$x = V_x t + \cancel{cte}^{0}$

$\therefore t = \frac{x}{V_x}$

$F_y = q_p \frac{\sigma}{\varepsilon_0}$

$F_y = m a_y$

$a_y = \frac{F_y}{m} = \frac{qE}{m} = ct$

$dV_y = a_y dt$

$V_{y2} - \cancel{V_{y0}}^{0} = a_y (t_2 - \cancel{t_0}^{0})$

$a_y = \frac{V_y}{t}$; $V_y = a_y t$

$dy = a_y t \, dt$

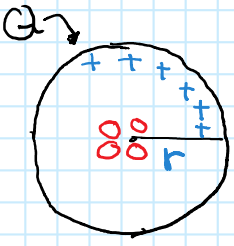
$y - \cancel{y_0}^{0} = a_y \frac{t^2}{2} \Big|_0$

$y = \frac{1}{2} a_y t^2$

$a_y = \frac{2y}{t^2} = \frac{qE}{m} = \frac{qr}{\varepsilon_0 m}$

$\therefore E = \frac{2ym}{qt^2} = \frac{2ym}{q \frac{x^2}{V_x^2}} = \frac{2ym V_x^2}{qx^2}$

- Esfera no conductora cargada electricamente



$q \sim r$

- Carga distribuida pero no uniformemente

$\rho = \frac{dq}{d\tau} \left[\frac{coul}{m^3}\right]$

$dq = \rho \, d\tau$

$\rho \sim r$

$\rho = cte \, r = Br$

$\rho \neq$ no es cte

la distribución varia respecto al radio

$$\rho = cte\, r = Br$$

d?

$$\therefore dq = Br\, d\tau \; ; \quad dq = \int_0^{\tau} Br\, d\tau \quad \text{// Esféricas}$$

$$d\tau = (dr)(r\,d\theta)(r'\,d\varphi)$$

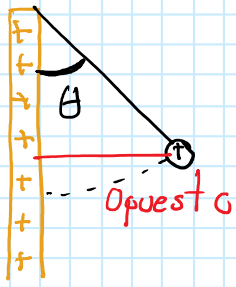
6_Clase

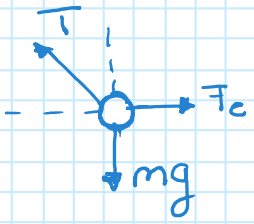
Monday, February 18, 2019 7:20 AM

19] Datos

- $m = .1$ gramos
- $q = 3 \times 10^{-9}$ Coul
- $l = 5$ cm
- $\sigma = 25 \times 10^{-7} \frac{coul}{m^2}$

Diagramas





- $\Sigma F_x = F_e - T\,sen\theta$; $F_e = T\,sen\theta$

$E = \frac{1}{q}F$; $F_e = (q)(E)$; $F_e = q\frac{\sigma}{2\varepsilon_0}$

$\therefore\ q\frac{\sigma}{2\varepsilon_0} = T\,sen\theta$

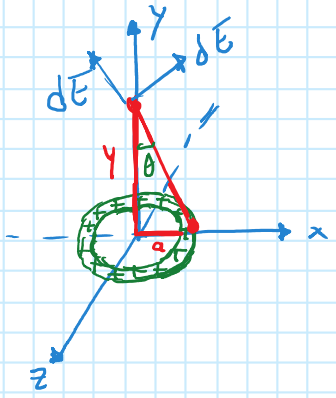
- $sen\theta = \frac{c.o}{hpo}$
- $sen\theta = \frac{x}{T}$
- $T\,sen\theta = x$

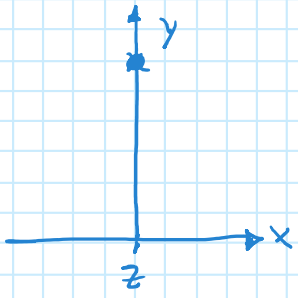
- $\Sigma F_y = -mg + T\cos\theta$; $mg = T\cos\theta$

$\therefore\ \tan\theta = \frac{\sigma v}{2\varepsilon_0 mg}$; $\theta = \tan^{-1}\frac{(3\times10^{-9})(25\times10^{-7})}{(2)(8.85\times10^{-12})(10^{-4})(9.81)}$

$\theta = 23.36°$

11]





- $F = K\frac{q_1 q_2}{r^2}[\bar{r}_u]$; $E = \frac{1}{q}F$

$E = K\frac{q_1}{r^2}[\bar{r}_u]$; $dq = dE$

$d\bar{E} = K\frac{dq}{r^2}[\bar{r}_u]$

$\lambda = \frac{dq}{dl}$ $\int_0^q dq$

$$E = k\frac{q_1}{r^2}[\bar{r}_u] \quad ; \quad dq = dE$$

$$dE = k\frac{\lambda\, dl}{r^2}[\bar{r}_u]$$

$$\lambda = \frac{dq}{dl} \quad \int_0^l dq$$

$$\lambda = cte$$

$$q = \lambda l$$

$$dq = \lambda\, dl$$

- $E = \int dE$

dE_x ; $E_x = \int dE_x = 0$

dE_y ; $E_y = \int dE_y \neq 0$

dE_z ; $E_z = \int dE_z = 0$

$$dE_y = dE\cos\theta =$$

$$\cos\theta = \frac{y}{r}$$

$$E_y = \int k\frac{\lambda\, dl}{r^2}\frac{y}{r}$$

$$E = \cancel{E_x\hat{i}} + E_y\hat{j} + \cancel{E_z\hat{k}}^{\,0}$$

$$\therefore E = k\frac{\lambda y}{r^3}\int_0^l dl = k\frac{\lambda y}{r^3}l$$

$$r^2 = a^2 + y^2$$

$$r^3 = (a^2+y^2)^{3/2}$$

$$E = \frac{\lambda y\, \cancel{2}\pi a}{\cancel{4}\pi\varepsilon_0 r^3} = \frac{\lambda y a}{2\varepsilon_0 r^3}$$

$$\therefore E = \frac{\lambda y a}{2\varepsilon_0 (a^2+y^2)^{3/2}}$$

Temperatura | Es un índice de la actividad interna de las moleculas

---

Esfera no conductora

$$\rho = \frac{dq}{d\tau} \quad \text{es variable}$$

- Carga Total – Q
- radio – a

$$dq = \rho\, d\tau \quad ; \quad \rho \sim r$$

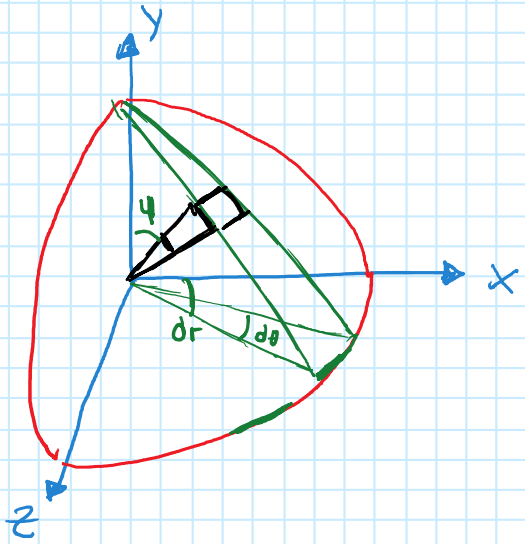
$dq = \rho \, d\tau \; ; \; \rho \sim r$

$\rho = cte \, r \; ; \; cte = B$

$\therefore \; \rho = Br$

$\therefore \; dq = Br \, d\tau \; ;$

$\int_0^Q dq = \int_0^a Br \, d\tau \; ; \; B = ¿?$



$d\tau = (r \, d\varphi)(dr)(r \, sen\theta \, d\theta)$

$\therefore \; Q = B \int_0^a r^3 dr \int_0^{\pi} sen\varphi \, d\varphi \int_0^{2\pi} d\theta = B \int_0^a r^3 dr \int_0^{\pi} sen\varphi \, d\varphi \, (2\pi)$

$= B \int_0^a r^3 dr \left(\cancel{-\cos\varphi \Big|_0^{\pi}} \right)^{2} (2\pi) = B \int_0^a r^3 dr \, (4\pi) =$

$= B \frac{r^4}{\cancel{4}} \Big|_0^a \cancel{4} \pi = B \, a^4 \pi$

$Q = B a^4 \pi \; ; \; B = \frac{Q}{\pi a^4}$ //

$\therefore \; \rho = Br = \left(\frac{Q}{\pi a^4} \right)(r) = \frac{Q}{\pi a^3}$ //

Campoo Eléctrico - Ley de Gauss

- $E \sim N$
- $E \sim \frac{1}{A}$

$E \sim \frac{N}{A} \; ; \; E = K \frac{N}{A} \; ; \; N = EA \; | \; E \perp A$

7_Clase

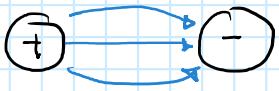
Wednesday, February 20, 2019 7:19 AM

- Objetivos

1] Lineas de fuerza
2] Ley de Gauss
3] Divergencia del campo

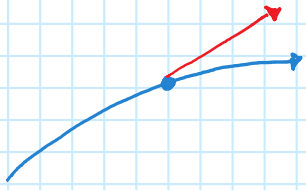
- Las lineas de fuerza indican el valor de intensidad de campo en un determinado punto.

- Los vectores intensidad de campo se alejan de las cargas positivas y se acercan a las negativas



- Para conocer la dirección del vector intensidad de campo, el vector intensidad de campo debe de tener la misma dirección de las lineas de fuerza en el punto considerado.
- Para que esto suceda el vector intensidad de campo debe ser tangente a las lineas de fuerza en el punto considerado

- Lineas de Fuerza
- Vector intensidad

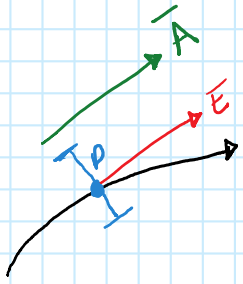


- La magnitud del vector intensidad de campo debe ser directamente proporcional a la cantidad de líneas de fuerza
- La magnitud del vector intensidad de campo también depende de que tan cercanas en

- La magnitud del vector intensidad de campo también depende de que tan agrupadas se encuentren las lineas de fuerza

Más Agrupadas → Área Menor → Mayor intensidad

Menos Agrupadas → Área Mayor → Menor intensidad



$E \sim N$ $\quad N \sim EA$

$E \sim \frac{1}{A}$ $\quad cte = 1$

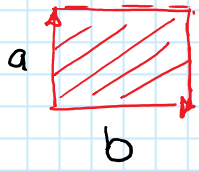
$\therefore E \sim \frac{N}{A}$ $\quad \therefore N = EA \; ; \; E = \frac{N}{A}$

- El área sobre la superficie donde se apoyan las lineas debe ser perpendicular a las lineas y a su vez al vector intensidad de campo

- Flujo Eléctrico – Φ_E | $N = \bar{E} \cdot A$

N = escalar $\quad$ A = vector

E = Vector



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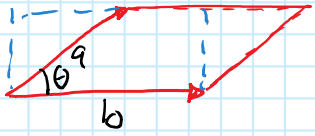
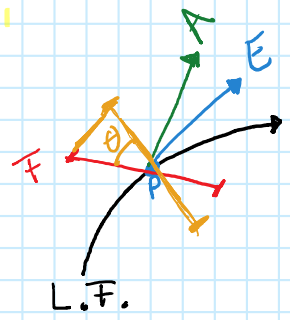


VECTOR ÁREA

$A = |a \times b| \hat{e}_1$ $\qquad A = |a||b| \operatorname{sen}\theta$



// No perpendicular

A proyectada $\triangleq$ A $\cos\theta$

$\therefore N = E$ A proyectada

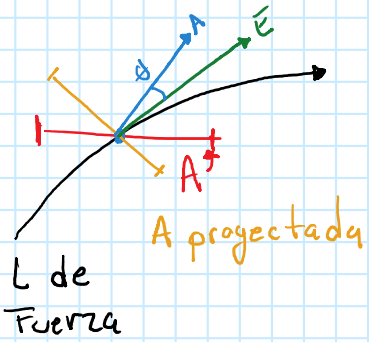
$= |E||A| \cos\theta$

$N = |E||A|$

L.F.

// No perpendicular

$$N = |E||A|$$



- Si E no es cte en toda la superficie

$$\Delta N = E \cdot \Delta A$$

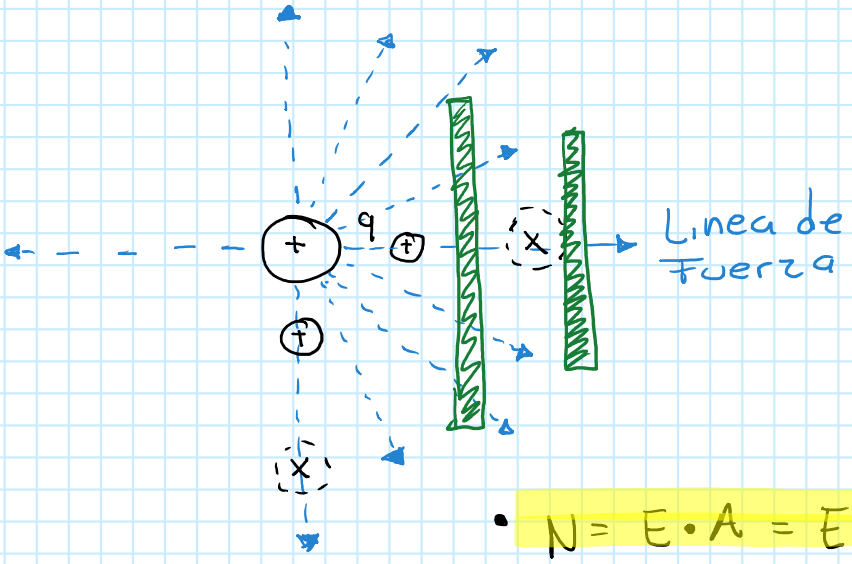
if $\lim \Delta A \to 0$

$$dN = E \cdot dA$$

$$N = \int_0^N dN = \int_0^A E \cdot dA$$

$$N = \int E \cdot dA \quad ; \quad E = \frac{dN}{dA_{proyectada}}$$

LEY DE GAUSS



- $N = E \cdot A = E A \cos\theta$

- Si la superficie es cerrada el número de lineas es el mismo:
  - → La carga que produce el campo está encerrada por la superficie cerrada

→ La carga que produce el campo está encerrada por la superficie cerrada.

→ Es independiente del tamaño de la superficie y de la forma de la superficie

$$N = \int_S E \cdot dA = \oint E \cdot dA$$

//Partícula puntual

$$E = K \frac{q}{r^2} [\bar{r}_q]$$

La superficie Cerrada es una esfera con centro en donde está la partícula cargada

$$N = \int E dA \cancel{\cos 0}^{1} = \int E dA = E \int_0^A dA = E A$$

$$N = \left(K \frac{q}{r_1^2}\right)\left(4\pi r_1^2\right) = \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{4\pi r_1^2}{}\right)\left(\frac{q}{r_1^2}\right) =$$

$$N = \frac{1}{\varepsilon_0} q$$

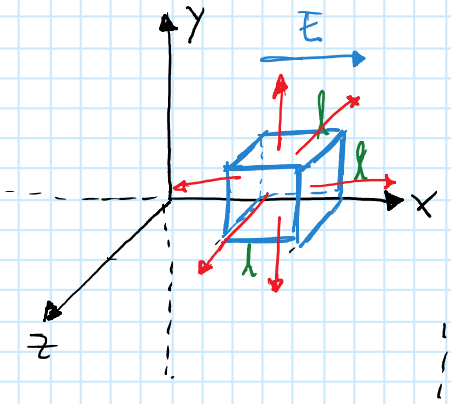
LEY DE GAUSS

El número neto de líneas de fuerza que cruzan una sup. cerrada es igual $\frac{1}{\varepsilon_0}$ la carga neta contenida o encerrada en ella

8_Clase

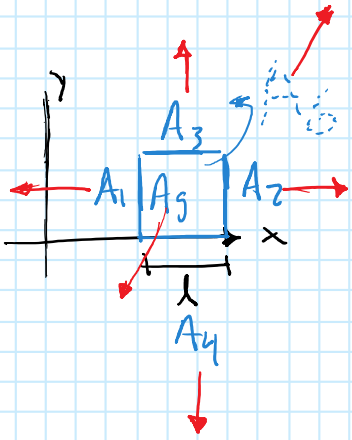
Monday, February 25, 2019 7:17 AM

2.16]



$x = 10$ cm | $E = 10^3 x^{1/3}$

$l = 10$ cm



$N = \int E \cdot dA =$

$dA_1 = A_1(-\hat{i}) + 0\hat{j} + 0\hat{k}$

$dA_2 = A_2(\hat{i}) + 0\hat{j} + 0\hat{k}$

$dA_3 = A_3(\hat{j})$ | $dA_6 = A_6(-\hat{k})$

$dA_4 = A_4(-\hat{j})$

$dA_5 = A_5(\hat{k})$

$\cdot E = E_x\hat{i} + 0\hat{j} + 0\hat{z}$

▸ $N_1 = \int E_1 \, dA = \int E_1 \cdot dA\,(-1) = E_1$

$N_1 = E_1 \int_{A_1} dA = -E_1 A_1 = -10^3(.1)^{1/3}\, 10^{-2} = -10(.464)$

$\therefore N_1 = -4.64$ [Lineas]

▸ $N_2 = \int E_2 \cdot dA = \int E_2 \cdot dA(1) = E_2 \int_A dA =$

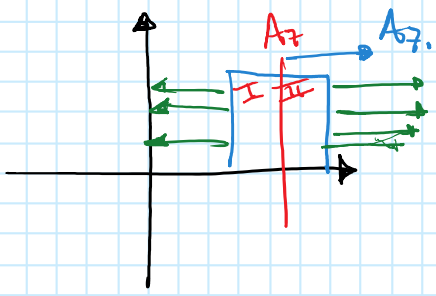
$= 10^3 (.2)^{1/3} (10^{-2}) = 10(.584) =$

$N_2 = \pm 5.84$ [Lineas]

• $N_3 = N_4 = N_5 = N_6 = 0$; $E \perp dA$

$\therefore N_R = N_1 + N_2 + \ldots + N_6 = 1.2$ [lineas]

$\rightarrow N = \frac{1}{\varepsilon_0} q \; ; \; q = \varepsilon_0 N \; ; \; q = 10.62 \times 10^{-12}$ [C]



$$N_7 = \int_{A_7} E_7 \cdot dA \, (+1) =$$

$$= E_7 \int_{A_7} dA \cdot 10^3 = E_7 (.15)^{\frac{1}{3}} \, 10^{-2} =$$

$$= 5.31 \text{ lineas}$$

Cubo 1

$N_I = N_7 + N_1 = (5.31) + (-4.64) = .67$ [lineas]

$q_I = \varepsilon_0 N_I \; ; \; q_I = 5.92 \times 10^{-12}$ [C]

$N_{II} = N_7 + N_2 = (5.84) + (-5.31) = .54$ [lineas]

$q_{II} = \varepsilon_0 N_{II} \; ; \; q_{II} = -4.7 \times 10^{-12}$ [C]

- $N_1 = -E_1 A_1$
- $N_2 = E_2 A_2$

$\Delta N = N_2 + N_1 \; ; \; \Delta N = E_2 A_2 - E_1 A_1$

But $A_2 = A_1$

$\therefore \Delta N = (E_2 - E_1) A \; ; \; \Delta N_x = \Delta E_x A \; ; \; \Delta x$

$\hookrightarrow \frac{\Delta N_x}{\Delta x} = \frac{\Delta E_x}{\Delta x} A$

if $\Delta x \rightarrow 0$

$\frac{dN_x}{} = \frac{dE_x}{} A \; ; \; dN_x = \frac{dE_x}{} A \, dx$

$$\frac{dN_x}{dx} = \frac{dE_x}{dx} A \; ; \; dN_x = \frac{dE_x}{dx} A\,dx$$

$$A\,dx = dVol \; ; \; dVol = d\tau$$

$$\therefore \; dN_x = \frac{dE_x}{dx} d\tau$$

$$\int dN_x = \int \frac{dE_x}{dx} d\tau \; ; \; N_x = \int_0^{N_x} dN_x$$

$$\rightarrow \; N_x = \int \frac{dE_x}{dx} d\tau$$

- $E = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$
- $N = N_x + N_y + N_z$

$$N = \int \left(\frac{dE_x}{dx} + \frac{dE_y}{dy} + \frac{dE_z}{dz} \right) d\tau = \frac{1}{\varepsilon_0} q$$

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$N = \int (\nabla \cdot E)\, d\tau = \frac{1}{\varepsilon_0} q$$

$\nabla \cdot E \triangleq$ Divergencia del (Campo) "E"

$$(\nabla \cdot E)\, d\tau = \frac{1}{\varepsilon_0} dq \; ; \; \nabla \cdot E = \frac{1}{\varepsilon_0} \frac{dq}{d\tau} = \frac{1}{\varepsilon_0} \rho$$

$$\therefore \; \rho = \varepsilon_0 (\nabla \cdot E)$$

$$\frac{dE_x}{dx} = \frac{1}{3}(10^3)\, x^{-\frac{2}{3}} \; ; \; \rho = \varepsilon_0 \left(\frac{1}{3}(10^3)\left(x^{-\frac{2}{3}}\right) \right)$$

TAREA

$\rightarrow E_x = 10^3\, x^{1/3}$, $E_y = 10^3\, y^{1/3}$, $E_z = 10^3\, z^{1/3}$

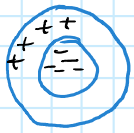
→ $E_x = 10^3\, x^{1/3}$, $E_y = 10^3\, y^{1/3}$, $E_z = 10^3\, z^{1/3}$

→ Cantidad de Líneas $\triangleq N$

→ Carga Neta $\triangleq q$

ρ_1	$P_1(.15, .05, .05)$
ρ_2	$P_2(.05, .05, .05)$
ρ_3	$P_3(.25, .15, .2)$

2.20]



$$E = \frac{1}{2\pi\varepsilon_0}\left(\frac{\lambda}{r}\right) \; ; \; a \le r \le b$$

// Sólo me afecta caso negativo

D.C.L

$F_{cf} = ma_n \leftarrow \bigcirc \rightarrow F_e = F_{cp}$

- $a_n = \omega^2 r$
- $F_e = F_{cp} = q^+ E$

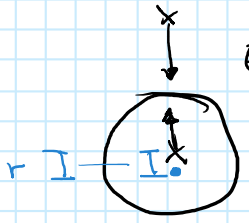
$$F_{cf} = F_{cp}$$

$$\omega^2 r\,(m) = q^+ \frac{1}{2\pi\varepsilon_0}\frac{+\lambda}{r}$$

$$\therefore\; \omega = \sqrt{\frac{q\lambda}{2\pi\varepsilon_0\, m r^2}} \; ; \; N = \frac{\omega}{2\pi}$$

← Miércoles

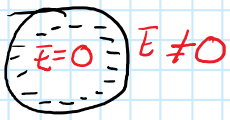
- $0 \le r \le a$; $E = 0$
- $a \le r \le b$; $E = \frac{1}{2\pi\varepsilon_0}\left(\frac{\lambda}{r}\right)$



$E = \frac{1}{r}\frac{\lambda}{2\pi\varepsilon_0}$ | Afuera

$E \neq 0$ | Dentro

$E = 0$



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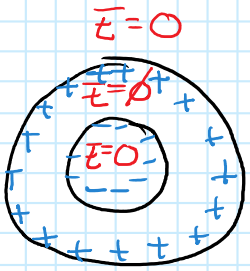
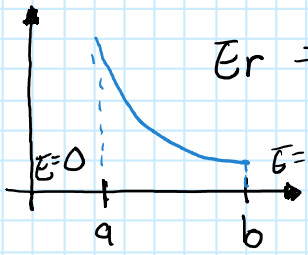




$$E_r = \frac{\lambda}{2\pi\varepsilon_0} = cte$$

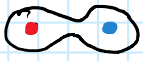
- Cables Blindados

-

Aislantes

} Partes que llevan la parte positiva y negativa

- Coaxial

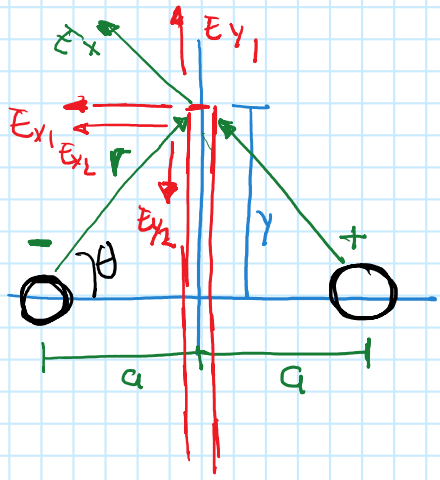
- 

Versión anterior

9_Clase

Wednesday, February 27, 2019 7:51 AM

• Lenguajes de Gauss



$$\oplus \cdot E_1 = \frac{q}{r^2}\frac{1}{4\pi\varepsilon_0}\left[\hat{e}_1\right]$$

$$\ominus \cdot E_2 = \frac{-q}{r^2}\frac{1}{4\pi\varepsilon_0}\left[\hat{e}_1\right]$$

- $E_{x_1} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\cancel{\cos\theta}\ \frac{a}{r}\left[-\hat{i}\right]$
- $E_{y_1} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{y}{r}\left[\hat{j}\right]$
- $E_{x_2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{a}{r}\left[\hat{i}\right]$
- $E_{y_2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{y}{r}\left[-\hat{j}\right]$

$$E_x = \frac{1}{2\pi\varepsilon_0}\frac{qa}{r^3}\left[-\hat{i}\right]$$

$$E_y = 0$$

$$\therefore E = E_x + E_y = \frac{1}{2\pi\varepsilon_0}\frac{qa}{r^3}$$

PASOS

① E ③ $\cdot$

② dA ④ $\int$

2) $A = \pi y^2$

$$dA = \pi\, 2y\, dy\left[\hat{i}\right]$$

3) $$E \cdot dA = \left(\frac{1}{\cancel{2\pi}\varepsilon_0}\frac{qa}{r^3}\right)\cdot\left(\cancel{\pi}\, 2y\, dy\right) =$$

$$= \frac{qa}{\varepsilon_0}\left(\frac{y\,dy}{r^3}\right)$$

4) $$N = \frac{qa}{\varepsilon_0}\int\frac{y\,dy}{r^3}$$

Sustitución Trigonométrica

$\tan\theta = \frac{y}{a}$; $y = a\tan\theta$

• $dy = a\sec^2\theta\, d\theta$

- $dy = a\sec^2\theta \, d\theta$
- $r = a\sec\theta$

$$= \frac{qa}{\varepsilon_0}\int \frac{a\tan\theta \; a\sec^2\theta \, d\theta}{a^3 \sec^3\theta} = \frac{q}{\varepsilon_0}\int \frac{\tan\theta}{\sec\theta}\, d\theta =$$

$$= \frac{q}{\varepsilon_0}\int \operatorname{sen}\theta \, d\theta = \frac{q}{\varepsilon_0} - \cos\theta \Big|_0^\theta = \frac{q}{\varepsilon_0}\left(-\cos\theta + \cos 0\right) =$$

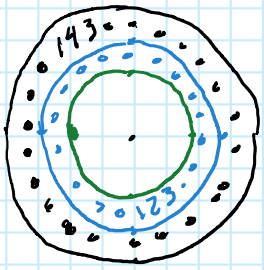
$$= \frac{q}{\varepsilon_0}\left(\cancel{\cos 0}^{\,1} - \cos\theta\right) = \frac{q}{\varepsilon_0}\left(1 - \cos\theta\right)$$

Ley Gauss - Campos Eléctricos

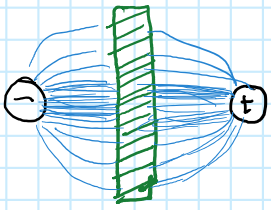
- La superficie de Gauss es una superficie imaginaria y puede coincidir con una real
- La superficie debe contener el punto donde deseamos calcular el campo
- El calculo de la superficie de esa área, debe ser sencillo, por lo que debe seleccionarse una superficie "adecuada"
- La magnitud del campo en todos los puntos debe ser constante en la superficie
- El vector intensidad de campo y el superficie siempre deben formar el mismo ángulo, por lo que los vectores campo siempre deben de ser perpendiculares a la superficie.

10_Clase

Monday, March 4, 2019 7:16 AM



- $N = \frac{q}{\varepsilon_0}(1 - \cos\theta)$



- if $\begin{cases} \theta = 60^\circ \\ q = 8.85 \times 10^{-9} [C] \end{cases}$

$\therefore N = 500$ [Lineas]

- if $\theta = 50^\circ$

$\therefore N = 357$ [Lineas]

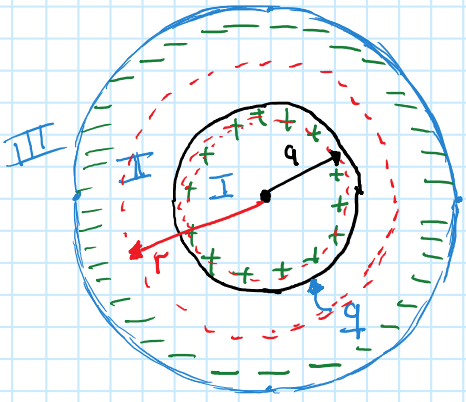
143 Lineas

- if $\theta = 40^\circ$

$\therefore N = 234$ [Lineas]

123 Lineas

Esfera Conductora – Ley de Gauss



I] $0 \le r \le a$

II] $a \le r \le b$

III] $b \le r \le \infty$

- Superficie de Gauss

I] Carga Neta $= 0$; $\oint E \cdot dA = \frac{1}{\varepsilon_0} q$; $\oint \bar{E} \cdot \bar{dA} = 0$

$|E| = 0$ o $\cancel{|dA| = 0}$ o $\cancel{E \perp dA}$ | Casos para "0"

$\therefore E = 0$

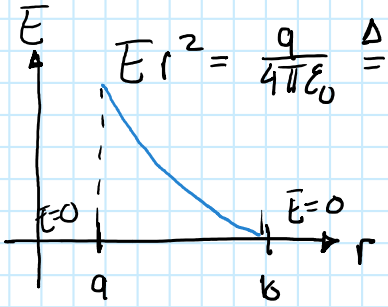
III] $q = q^+ q^- = 0$ // Mismo caso pero con negativas

II) $q_{Neta} = q$

$$\oint E \cdot dA = \frac{1}{\varepsilon_0} q \; ; \; \oint E \cdot dA \cos\theta = \frac{1}{\varepsilon_0} q \; ;$$

$$E \oint dA = \frac{1}{\varepsilon_0} q \; ; \; EA = \frac{1}{\varepsilon_0} q \; ; \; E = \frac{1}{\varepsilon_0} \frac{q}{A}$$

→ $A \triangleq$ Área de una esfera ; $E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$ | As a puntual carga*



$$Er^2 = \frac{q}{4\pi\varepsilon_0} \triangleq Cte$$

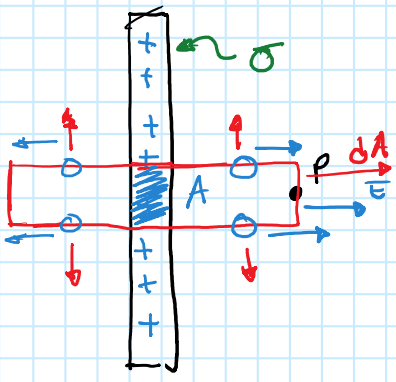
Conclusiones

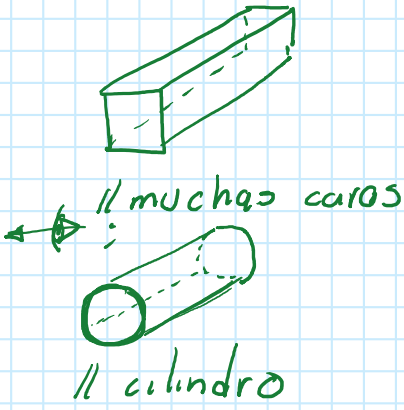
- Menor cantidad de lineas mientras nos alejamos
- Si aumento superficie, también disminuye en las lineas

Plano Cargado

- $E = \frac{\sigma}{2\varepsilon_0}$

// Superficie de Gauss debe ser → simétrica





LEY GAUSS

1) Tener superficie imaginaria
   - Cerrada
   - Debe tener el punto
   - Encerrar la carga

* Magnitud de E cte en toda la superficie

- $\oint E \cdot dA = \frac{1}{\varepsilon_0} q$

$$\sigma = \frac{q}{A} \; ; \; \sigma = \frac{dq}{dA} \; ; \; \sigma dA = dq \; ; \; q = \int \sigma dA \; ;$$

$$\sigma = \frac{q}{A} \; ; \; \sigma = \frac{dq}{dA} \; ; \; \sigma dA = dq \; ; \; q = \int \sigma dA \; ;$$

$$q = \sigma A \; ; \; \sigma \overset{\Delta}{=} cte$$

$$\bullet \oint_{S.C} E \cdot dA = \int_{T_1} E_1 \cdot dA_1 + \int_{T_2} E_2 \cdot dA_2 + \cancel{\int_{Lateral} E_3 \cdot dA_3}^{\,0} =$$

$$= \frac{1}{\varepsilon_0} \sigma A$$

$$* \int_{Lateral} E_3 \cdot dA_3 = 0$$

$$* \int_{T_1} E_1 \cdot dA \cancel{\cos\theta}^{\,1} = E_1 \int_{T_1} dA = E_1 A_1$$

$$* \int_{T_2} E_2 \cdot dA \cancel{\cos\theta}^{\,1} = E_2 \int_{T_2} dA = E_2 A_2$$

//Al ser a la misma distancia

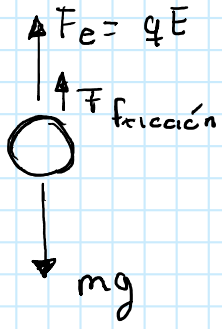
$\therefore \int dA = A$

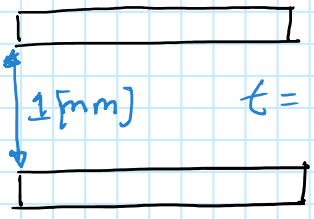
$$\therefore E_1 A_1 + E_2 A_2 = \frac{1}{\varepsilon_0} \sigma A$$

$$E = |E_1| = |E_2| \; ; \quad A_1 = A_2$$

$$\therefore 2 E \cancel{A} = \frac{1}{\varepsilon_0} \sigma \cancel{A} \qquad E = \frac{\sigma}{2\varepsilon_0}$$

D.C.L





$t = 27.4$ seg

$\bullet E = 2.37 \times 10^3 \left[\frac{N}{C}\right]$

- 1° instante
  $v = 3.6496 \times 10^{-5}$
- 2° instante
  $v = 0$

// Realizando

$$\Sigma F_y = F_{fricción} + F_e - mg = 0 \;;$$

$$F_e = mg - \cancel{F_{fricción}}^{\,0} \;;\quad F_e = qE \;;$$

$$\therefore \rightarrow qE = mg - F_{fricción} \qquad \left\{ (q - q') = \frac{F_{fricción}}{E} \right.$$

$$\rightarrow q'E = mg$$

- Stokes | $F_{fricción} = 6\pi \eta r V$

$$\rho = (\rho_{aceite} - \rho_{aire}) \cdot v \cdot g$$

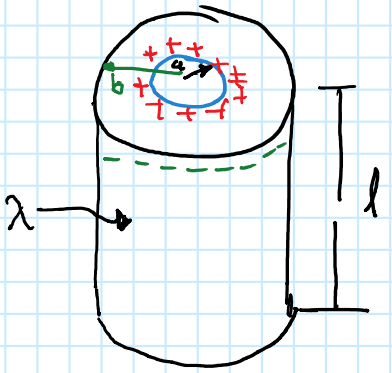
$$\therefore r = \sqrt{\frac{9\eta v}{2g(\rho_{aceite} - \rho_{aire})}} = 6.052 \times 10^{-7} [mm]$$

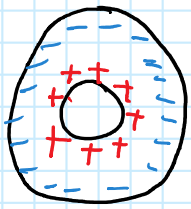
$$\therefore (q - q') = \frac{3.16216 \times 10^{-19}}{1.6 \times 10^{-19}} \approx 2e^- \;//$$

11_Clase

Wednesday, March 6, 2019 7:20 AM

1) Ley de Gauss 2) Potencial eléctrico
3) Fuerzas Conservativas 4) Energía 5) Trabajo





I) $0 \le r \le a$; $E=0$
II) $a \le r \le b$
III) $b \le r \le \infty$; $E=0$

I] $q_{neta}=0$

$\oint E \cdot dA = \frac{1}{\varepsilon_0} q_{neta}$; $\oint E \cdot dA = 0$; $E=0$

$|E|=0$ $\cancel{dA=0}$ $\cancel{E \perp dA}$

III] $q_{neta} = ({}^{+}\lambda l) + ({}^{-}\lambda l) = 0$

$\lambda = \frac{dq}{dl}$; $\int \lambda dl = \int dq$ $\lambda = cte$; $E=0$

↳ $q = \lambda l$

$\oint E \cdot dA = 0$

II] $\oint E \cdot dA = \frac{1}{\varepsilon_0} q = {}^{+}\lambda l$

$E = (E_r, E_\theta, E_y)$ ⋮ $E \triangleq$ Campo tiene dirección radial

$dA = (dA_r, r\,d\theta, dy)$ ⋮

• $E_\theta = 0$ • $E_y = 0$; ∴ $E \cdot dA = E_r\, dA_r$

→ $\oint E \cdot dA = \cancel{\int_{T_1} E \cdot dA}^{0} + \cancel{\int_{T_2} E \cdot dA}^{0} + \int_{SLAT} E \cdot dA =$

$$\rightarrow \oint E \cdot dA = \cancel{\int_{T_1} E \cdot dA} + \cancel{\int_{T_2} E \cdot dA} + \int_{SLAT} E \cdot dA =$$

$$= \int E \cdot dA \cancel{\cos\theta}^{\,1} = \frac{1}{\varepsilon_0} q_{enc}$$

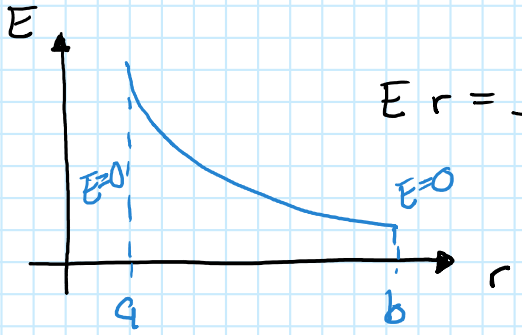
$$q_{enc} = l\lambda$$

① Contener punto
② Cerrada y simétrica

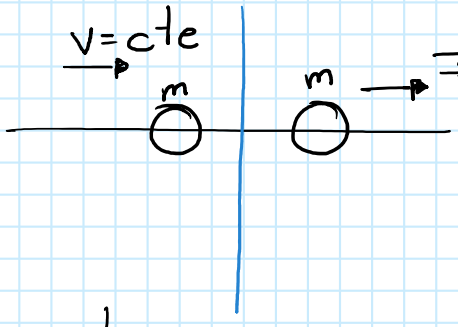
$$= E \int dA = \frac{1}{\varepsilon_0} l\lambda \; ; \quad A = 2\pi r l$$

$$= EA = \frac{1}{\varepsilon_0} l\lambda \; ;$$

$$= E\, 2\pi r l = \frac{l\lambda}{\varepsilon_0} \; ; \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

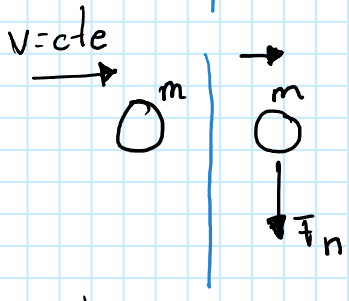


$$E r = \frac{\lambda}{2\pi\varepsilon_0} = cte$$

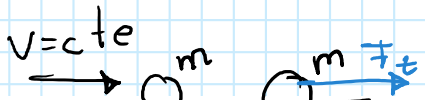


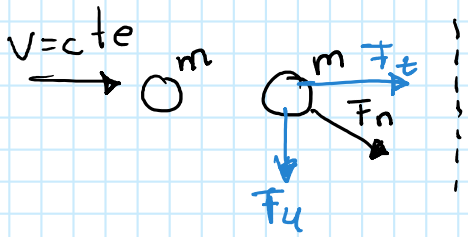
$$\vec{F}_t = m a_t = m \frac{dv}{dt}$$

$\vec{V} \triangleq$ Vector $\begin{cases} \bullet \text{Magnitud} \checkmark \\ \bullet \text{Dirección} \times \end{cases}$ — Var



- magnitud = cte ×
- Dirección – Var ✓





$$E = \frac{F}{q_p}$$

$$N = \frac{q}{\varepsilon_0} \; ; \; q = N \varepsilon_0$$

$$\rho = \varepsilon_0 (\nabla \cdot E)$$

- Energía: Es la habilidad latente o aparente para producir un cambio en condiciones existentes