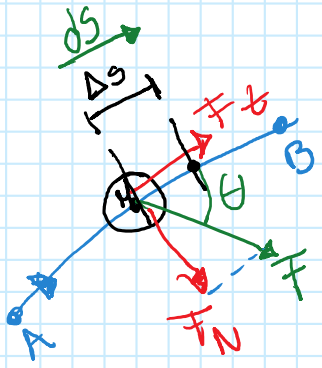
12_Clase

Monday, March 11, 2019 7:19 AM

Energía Es la habilidad latente o aparente para producir un cambio en condiciones existentes



$ds \triangleq$ Vector

$$Ft = ma = m\frac{dv}{dt} \qquad Ft = F\cos\theta;$$

$$F\cos\theta = m\frac{dv}{dt}$$

$$\underbrace{F\cos\theta\, ds}_{} = m\frac{dv}{dt}ds$$ velocidad

$$(F\cdot ds)\cos\theta = mv\,dv$$ $\triangleq$ 1° Ley Termodinámica

$$\int_a^b (F\cdot ds)(\cos\theta) = \int_a^b mv\,dv; \quad {}_aW_b = \frac{1}{2}mv_b^2 - \frac{1}{2}mv_a^2$$

NOTA: $v = \sqrt{2gy}$; $y = \frac{v^2}{2g}$

La habilidad que tiene es: $f(y,m)$ y $f(v^2,m)$

$$\therefore \; {}_aW_b = \text{Energía Cinética}_b - \text{Energía Cinética}_a = \Delta EnC\Big|_a^b$$

Energía Cinética

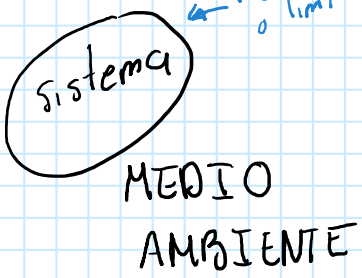
- Es aquella que posee un cuerpo en función de la magnitud de la velocidad.

Trabajo

- Es el producto de una fuerza por la distancia en que actua
- Actua $\triangleq$ Cambia la magnitud de la velocidad.

• El trabajo es una función de trayectoria, se requiere saber el recorrido.

Sistema



1° Ley de Termodinámica

• La máquina de movimiento perpétuo es imposible

// No puede crear energía

2° Ley de Termodinámica

• La máquina de movimiento perpétuo de segundo orden también es imposible

Estado del Sistema: Se define mediante sus propiedades. Y sus propiedades son independientes del camino seguido para llegar a ese valor.

Las propiedades son escalares.

Propiedades Extensivas: Dependen de la masa y volumen $f(m, v)$

Propiedad Intensiva: No depende de la masa y volumen

Trabajo: • El trabajo no es propiedad porque depende de recorrido. Como no es propiedad no se

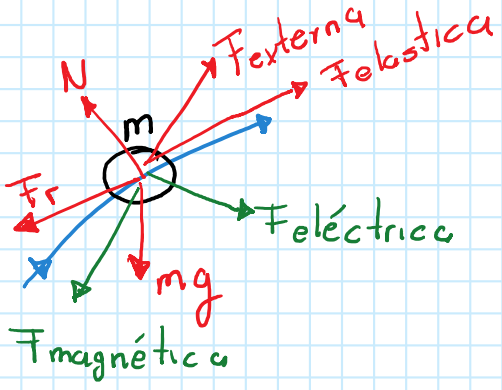
depende de recorrido, Como no es propiedad, no se puede poseer, solo se puede realizar

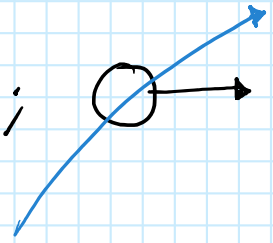
La otra es el calor

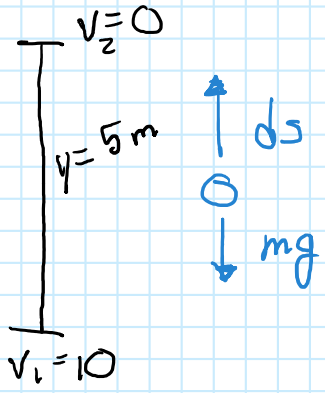
- Es una de las 2 formas mediante el sistema intercambia energía con el medio ambiente

∴ Es energía en transito.

Energía Potencial Es aquella que posee un cuerpo en función de su posición



;  ; ΣF ; Fresultante



$$W_2 = \int_1^2 F \cdot ds =$$

$$= \int_1^2 mg\, ds \cos 180^\circ =$$

$$\therefore W_2 = -mg\, Y$$

Trabajo El trabajo realizado por la resultante de un conjunto de fuerzas es

te de un conjunto de fuerzas es igual al cambio de la energía cinética

$$W_{FR} = \Delta E_{C}$$

Fuerza Conservativa $\triangleq$ El peso, es una fuerza que conserva la energía del sistema, cuando actua a lo largo de un ciclo

Fuerza no conservativa $\triangleq$ La fricción es una fuerza que cuando actua a lo largo de un ciclo no lo conserva

$$mg = f_{exclusiva}(x, y, z)$$

$$fr \neq f_{exculisiva}(x, y, z)$$

13_Clase

Wednesday, March 13, 2019 7:22 AM

- Fuerza Conservativa
  - Se debe calcular a lo largo de un ciclo
  - if $W=0$; Es conservativa
  - $\oint mg \cdot ds = 0$

- $F_{gravedad} = mg = G\frac{m_c m_T}{r^2}$; $g = G\frac{m_T}{r^2} = f(x,y,z)$
  (Conservativa)
- $F_{eléctrica} = K\frac{q_1 q_2}{r^2} = f(x,y,z)$
  (Conservativa)
- $F_{elástica} = k\,x = f(x,y,z)$
  (Conservativa)
- $F_{magnetica} = f(v) \neq f(x,y,z)$
  (No conservativa)
- Fuerza externa $\neq f(x,y,z)$
  (No conservativa)

- $F_{e\,Es} = (F_c + F_{NC})$

$$_aW_b = \int_a^b F_R \cdot ds = \int_a^b F_c \cdot ds + \int_a^b F_{NC} \cdot ds$$

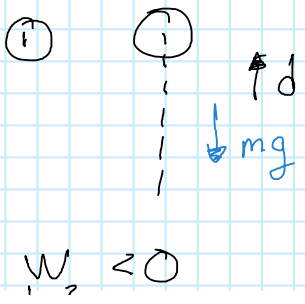
$$_aW_b = {}_aW_{b\,F_c} + {}_aW_{b\,F_{NC}}$$

$F_c = f_{exc}(x,y,z)$ | $W \rightarrow \Delta E_u$

$W_{F_c} = f(x,y,z)$ | $\therefore W_{F_c} \rightarrow \Delta E_u = F_{exc}(x,y,z)$

$E_u = f_{exc}(x,y,z) = E_u P$

$$\therefore {}_aW_b = \Delta E_u P \Big|_a^b = E_n P_b - E_n P_a$$



- $W_{F_R} = \Delta E_u C$
- $W_{F_C} = -\Delta E_u P$

$-\Delta_u P + F_{wc} = \Delta E_u C$

$W_{12} < 0$

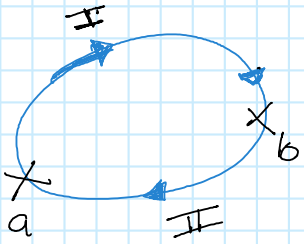
$\Delta E_u P|_2^1 < 0$

$-\Delta_u P + F_{wc} = \Delta E_u C$

$\therefore W_{FNC} = \Delta E_u C + \Delta E_u P$

E_n Mecánica $= E_u C + E_u P$

$\therefore W_{FNC} = \Delta E_N M$



$$\oint F_c \cdot ds = 0$$

$$\oint F_c \cdot ds = \underbrace{\int_a^b F_c \cdot ds}_{I} + \left(-\underbrace{\int_a^b F_c \cdot ds}_{II}\right) = 0$$

$$\therefore \underbrace{\int_a^b F_c \cdot ds}_{I} = \underbrace{\int_a^b F_c \cdot ds}_{II}$$

- El trabajo que realiza las fuerzas conservativas al ir de un punto a otro es igual a otro y es independiente del camino o trayectoria seguida.

Fuerza Eléctrica

$$_aW_{bFe} = \int_a^b F_e \cdot ds = E_u P E_b - E_u P E_a =$$

$$E = \frac{1}{q} F \; ; \; F = Eq$$

$$\therefore -\int_a^b qE \cdot ds = \frac{E_u P_b}{q} - \frac{E_u P_a}{q}$$

$\frac{E_n P}{q} \triangleq$ Cuerpo que tiene cierta $= V \triangleq$ Potencial Eléctrico ; $V = \frac{EP}{q}$

$\frac{EP}{q}$ ≙ cuerpo que tiene cierta cantidad carga $= V$ ≙ Potencial Eléctrico ; $V = \frac{EP}{q}$

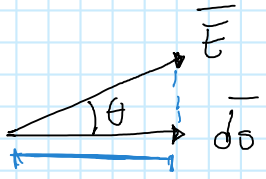
$$\text{Volt} = \frac{\text{Joules}}{\text{Coul}}$$

$$V_b - V_a = -\int_a^b E \cdot ds$$

$$V_{ba} = V_b - V_a \; ; \; V_{ab} = V_a - V_b \; ; \; V_{ba} = -V_{ab}$$

$$\therefore V_{ba} = -\int_a^b \bar{E} \cdot ds$$

2° Ley de Maxwell



$$\bar{E} \cdot \bar{ds} = |\bar{E}||\bar{ds}| \cos\theta \; ; \quad E \cdot ds = |\bar{E}_s||ds|$$

$$E \cos\theta = E_s$$

$$V_{ba} = -\int_a^b E_s \, ds \; ; \quad dv = -E_s \, ds \; ;$$

$$E_s = -\frac{dv}{ds} \qquad s = [x, y, z]$$

- $E_x = -\frac{dv}{dx}$
- $E_y = -\frac{dv}{dy}$
- $E_z = -\frac{dv}{dz}$

$$\bar{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$$

$$= -\left(\frac{dv}{dx}\hat{i} + \frac{dv}{dy}\hat{j} + \frac{dv}{dz}\hat{k}\right)$$

$$\nabla = -\left(\frac{\partial v}{\partial x}\hat{i} + \frac{\partial v}{\partial y}\hat{j} + \frac{\partial v}{\partial z}\hat{k}\right)$$

∇v es el mejor camino | Gradiente de la función potencial | $\therefore E = -\nabla v$

$E = -\nabla V$

$\nabla E = \frac{1}{\varepsilon_0}\rho$; $\nabla \cdot E = \nabla(-\nabla V) = \frac{1}{\varepsilon_0}\rho$

$\therefore \nabla^2 V = \frac{1}{\varepsilon_0}\rho$ ¦ Ecuación de Poisson

if $\rho = 0$

$\nabla^2 V = 0$ ¦ Ecuación de Laplace

Si se conoce

- V; $E = -\nabla \cdot V$
- $\nabla^2 V = \frac{1}{\varepsilon_0}\rho$

// Primera derivada $\triangleq$ Campo

// Segunda derivada $\triangleq$ Distribución Volumétrica Carga

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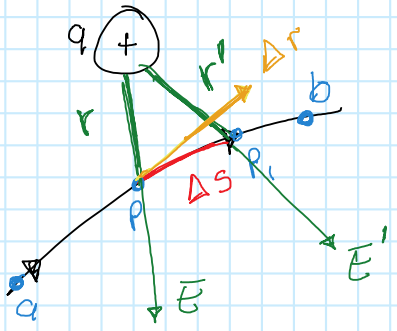
Caso Particular



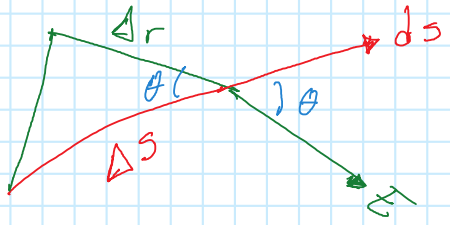
$V_{ab} = ?$

$r' = r + \Delta r$

Diferencia de Potencial ¦ Cantidad de energía por unidad de carga

$$V_{ba} = -\int_a^b E \cdot ds = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} ds \cos\theta' =$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$



$$\cos\theta' = \frac{\Delta r}{\Delta s}$$

$\therefore \lim \Delta s \to 0$; $\theta' \to \theta$; $\cos\theta' \to \cos\theta$

$$\lim_{\Delta s \to 0} \cos\theta' = \lim_{\Delta s \to 0} \frac{\Delta r}{\Delta s} ; \quad \cos\theta = \frac{dr}{ds}$$

// Regresando

$$-V_{ba} = -\frac{q}{4\pi\varepsilon_0}\left[-\left(\frac{1}{b}-\frac{1}{a}\right)\right]; \quad V_{ba} = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{b} - \frac{1}{4\pi\varepsilon_0}\frac{q}{a}\right)$$

if $b \to \infty$

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{b} \to 0$$

$$V_{a\infty} = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}; \quad V_a = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}$$

$$\therefore V_\infty = 0 \text{ [Volts]}$$

14_Clase

Wednesday, March 20, 2019 7:17 AM

1) Potencial Eléctrica 2) Capacitancia

- Fuerza Conservativa | Es aquella que conserva la energía a lo largo de un ciclo

→ $W_{FR} = \Delta E_u C$ → $W_{FC} = -\Delta E_u P$

→ $W_{FNC} = \Delta E_u C + \Delta E_u P$ | Energía Mecánica

$$W_{FR} = \Delta E_u C$$

$$\int F_r \cdot ds = \Delta E_u C \; ; \; F_r = F_c + F_{NC}$$

$$W_{Fc} = -\Delta_u P_1$$

$$W_{Fr} = W_{Fc} + W_{FNC}$$

$$\Delta E_u C = -\Delta_u P_1 + W_{FNC}$$

$$\therefore W_{FNC} = \Delta E_u C + \Delta E_u P$$

$$_aW_b \; F_c = {}_aW_b \; F_{eléctrica} = \int_a^b F_{elec} \cdot ds = -\Delta E_u P$$

$$E = \frac{1}{q} F \; ; \; F = Eq$$

$$\left(\frac{E_n P_a}{q} - \frac{E_n P_b}{q} \right) = -\int_a^b E \cdot ds$$

- Potencial lo tiene el punto

- $\frac{E_u P}{q} = V \left[\frac{Joul}{Coul} = Volt \right]$

- $V_b - V_a = V_{ba}$

$$V_{ba} = \int_a^b E \cdot ds = \frac{1}{4\pi\varepsilon_0} \left(\frac{q}{b} - \frac{q}{a} \right)$$

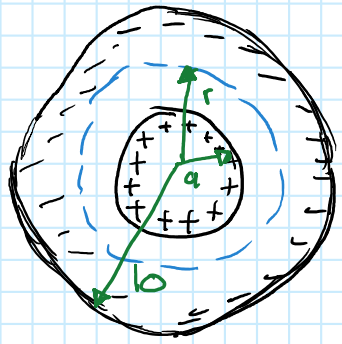
• $V_b - V_a = V_{ba}$ $\quad = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$

If $a \to \infty$

$$V_{b\infty} = V_b - V_\infty = \frac{1}{4\pi\infty}\frac{q}{b}$$

$V_\infty = 0$ absoluto $\qquad V_{\underline{\underline{1}}} = 0$ Relativo

▶ Esferas Concéntricas



I] $0 \le r \le a$; $E = 0$

II] $a \le r \le b$; $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

III] $b < r < \infty$; $E = 0$

▶ Región 1

$$V_{fe} = \int_e^f E \cdot ds$$

if $E = 0$

$V_{fe} = 0$

$\therefore V_e = V_f$

El interior de todo cuerpo cargado electricamente su campo es cero

▶ Región 3

$E = 0 \qquad V_y - V_x = 0$

$-\int E \cdot ds$

$\therefore V_y = V_x$

▶ Región 2

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad a \le r \le b$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \Big| \quad a \le r \le b$$

$$V_{ba} = -\int_a^b E \cdot ds \quad \Big| \quad E = [E_r, E_\theta, E_\varphi] \quad ds = [dr, d\theta, d\varphi]$$

- $E = f(r)$ • $E_\theta = E_\varphi = 0$

$\therefore \; E \cdot ds = E_r \, dr \cos\theta^\circ$

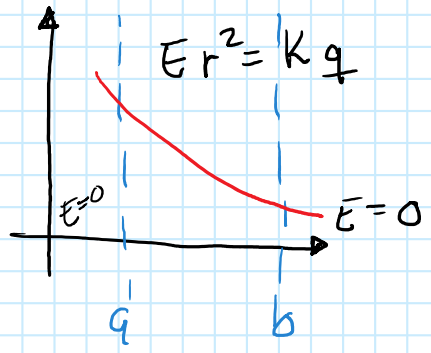
Final

$$V_{ba} = \int_a^b \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr =$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{b} - \frac{q}{a}\right)$$

$$\rightarrow V_{ba} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$$

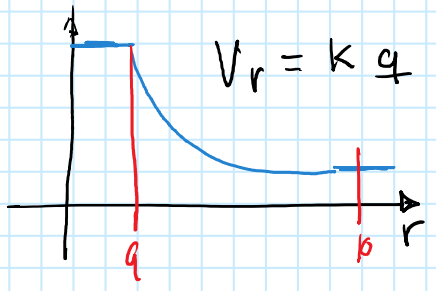
- Graficación



$$\rightarrow V_{ra} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} - \frac{q}{a}\right)$$

$$V_{ra} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a}\right)$$

- Es aquel lugar geométrico de pontos de igual potencial



// Ejercicio - Ejemplo

$$V_{hg} = -\int_g^h E \cdot ds = 0 \quad \Big| \quad E \perp ds = 0$$

$$\therefore V_{hg} = 0 \quad ; \quad V_h = V_g$$

Superficie Equipotencial | Es un lugar geométrico de puntos de igual potencial

- La superficie de la esfera internamente es equipotencial
  - ↳ El campo es perpendicular a la superficie.

Problemas Tarea

3.5]

$$W_{FNC} = \Delta E_u C + \Delta E_u P$$

$$F_r = F_{eléctrica}$$

$$F_{NC} = 0 \quad ; \quad W_{FNC} = 0$$

$$\therefore \Delta E_u C + \Delta_u P = 0$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + q V_f - q V_i = 0$$

$$V_i = 0$$

$$\rightarrow \frac{1}{2} m v_f^2 = -(q V_f - q V_i)$$

$$V = \sqrt{\frac{-2(q V_f - q V_i)}{m}} = \sqrt{\frac{-2 q V_{fi}}{m}}$$

if $q<0$; $V_{fi}>0$

$q>0$; $V_{fi}<0$

Capacitancia

- $C \sim q$
- $C \sim \frac{1}{V}$

- $C \sim \frac{q}{V}$

$$C = \frac{q}{V} \quad \left[\frac{Coul}{Volt}\right] = [Faradios]$$

→ Se define como la carga por unidad de voltaje y se mide en faradios

$$W_{FNC} = W_{FC} + W_{FNC}$$

$$F_e = F_{conservativa}$$

$$\int_a^b F_c \cdot ds = \Delta E_n P \Big|_a^b =$$

$$V = \frac{E_n P}{q} \quad ; \quad C = \frac{q}{V}$$

15_Clase

Monday, March 25, 2019 7:14 AM

• $V_{ba} = -\int_a^b E \cdot ds \longleftrightarrow E = -\nabla v$

• Altura es el potencial gravitacional

if $E = 0$

$\nabla v = 0$; $v = cte$

• La esfera tiene supeficies equipotenciales

$V_{zy} = \int_y^z E \cdot ds = 0$

→ Casos | ~~$|E| = 0$~~ ~~$|ds| = 0$~~ $E \perp ds$

• Porque el campo es perpendicular a la superficie de el cuerpo.

▶ Capacitancia $C = \frac{q}{v}$

$V_{ba} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)$; | $b > a$

$\frac{1}{b} < \frac{1}{a}$; $\left(\frac{1}{b} - \frac{1}{a}\right) < 0$

V_{ba} ; $V_b - V_a \longrightarrow V_{ba} = -V_{ab}$

$V_{ab} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$

$\therefore C_0 = \frac{q}{V_{ab}} = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$ //

C_0 en medio vacío

if $b \to \infty$

$C_0 = 4\pi\varepsilon_0 a$ //

// Depende del radio o radios de las esferas.

→ Con un dieléctrico entre "a" y "b"
// Es un aislante

$C = K_e C_0$ $\quad K_e \triangleq$ Depende del material dieléctrico

- $K_e = \frac{C}{C_0}$ // Adimensional Constante Dieléctrica
- $C_{mínima} = C_0$; $K_e = \frac{C_0}{C_0} = 1$
- $1 \leq K_e \leq \infty$

$$C = K_e \left(\frac{4\pi \varepsilon_0}{\frac{1}{a} - \frac{1}{b}} \right)$$

$K_e \triangleq$ Constante
$\varepsilon_0 \triangleq$ Constante

$$K_e \varepsilon_0 = \varepsilon \left[\frac{coul^2}{N \cdot m^2} \right] ; \quad C = \frac{4\pi \varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

Capacitancia

- De todo cuerpo depende de sus dimensiones y del medio en el que se establece su campo eléctrico.
- No depende de la carga ni de su voltaje

Capacitor

Es un dispositivo que permite almacenar energía mediante el establecimiento de un campo eléctrico.

Energía Almacenada

Energía Almacenada

- $EnP = ?$

$E_{neta} = EnP_F - EnP_I$ | $EnP_F = qV_F$

$V = \frac{EP}{q}$; $EnP = Vq$ | $EnP_I = qV_I$

$\therefore E_{neta} = q(V_F - V_I)$

$\Delta E_{neta} = q(\Delta V)$

$\rightarrow \lim_{\Delta V \to \infty}$

$dE_{neta} = q\,dV$ | $E_{neta} = \int q\,dV$

$C = \frac{q}{V}$; $q = CV$

$\therefore E_{neta} = \int_a^b CV\,dV$

$= \frac{1}{2} C V_{ba}^2$

Energía Almacenada $= \frac{1}{2} C V_{ba}^2$ [Joules] | $E_{neta} = \frac{1}{2} C_0 V_0^2$

$\rightarrow$ Energía Cinética $= \frac{1}{2} mg^2$

$E = \frac{1}{2}$ (Propiedad Extensiva) (Intensiva)^2 | Expresión General

- $k_e = \frac{V_0}{\ }$; $k_e = \frac{C}{\ }$ | $F = -\nabla V$

- $k_e = \frac{V_0}{V}$; $K_e = \frac{C}{C_0}$

$V_0 > V$ $\quad$ $V < V_0$

$E = -\nabla V$

$E_0 = -\nabla V_0$

$$\frac{\nabla V_0}{\nabla V} = \frac{V_0}{V} = \frac{E_0}{E} = k_e$$

$E < E_0$

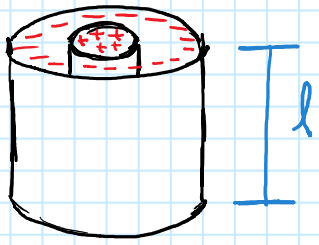
// Aumenta la Capacitancia

- Disminuye Campo eléctrico y Voltaje

16_Clase

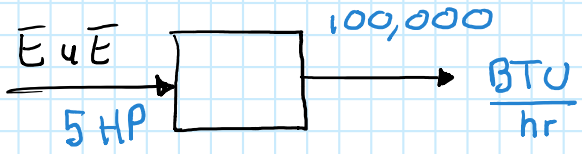
Wednesday, March 27, 2019 7:16 AM

- $\text{Energía}_{Alm} = \frac{1}{2} C V_{ab}^2$



- $V = \frac{E_n P}{q}$; $E_n P = V q$
- $\Delta E_n P = q V_{ab}$

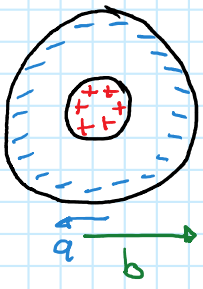
Potencial de un punto | La energía que tendría una carga positiva y unitaria en es punto



Kcal → BTU
Joules → KCal
KW → Joules

Hp → KW

Capacitancia



$$V_{ba} = -\int_a^b E \cdot ds$$

$$E_0 = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{r}$$

$$a \le r \le b$$

$$\frac{E_0}{E} = k_e$$

$$E = -\nabla v$$

$$\nabla \cdot E = \frac{1}{\varepsilon} \rho$$

$$\nabla^2 v = \frac{1}{\varepsilon} \rho$$

$$E_0 = k_e \varepsilon$$

$$\rightarrow E = \frac{E_0}{k_e} = \left(\frac{1}{2\pi\varepsilon_0 k_e}\right)\left(\frac{\lambda}{r}\right) ; \quad E = \frac{1}{2\pi\varepsilon} \frac{\lambda}{r}$$

- $E = [E_r, E_\theta, E_y]$
- $ds = [dr, r\,d\theta, d\varphi]$

$E = f(r)$; $E_\theta = E_y = 0$

$E = E(r)$; $E_\theta = E_y = 0$

$- \; ds = [dr, r\,d\theta, d\varphi]$

$\therefore E \cdot ds = E_r\, dr \cos\theta$

// Integrando

$$V_{ba} = -\int_a^b \frac{1}{2\pi\varepsilon}\frac{\lambda}{r}\,dr \; ; \; -V_{ba} = \int_a^b \frac{\lambda}{2\pi\varepsilon}\frac{1}{r}\,dr$$

$$-V_{ba} = \frac{\lambda}{2\pi\varepsilon}\left[\ln|r|\Big|_a^b\right] = \frac{\lambda}{2\pi\varepsilon}\left[\ln(b) - \ln(a)\right] =$$

$$= \frac{\lambda}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right)$$

$$\lambda = \frac{dq}{dl} \; ; \; \int dq = \lambda \int dl \; ; \; q = \lambda l \; ; \; \lambda = \frac{q}{l}$$

// Sustituyendo

$$-V_{ba} = V_{ab}$$

$$V_{ab} = \frac{q}{2\pi\varepsilon l}\ln\frac{b}{a} \; ; \qquad C = \frac{q}{V_{ab}}$$

$$\therefore C = \frac{2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)}$$

$$\frac{C}{C_0} = k_e \qquad C_0 = \frac{k_e}{C} \; ; \; C_0 = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$

if $b = 0$

$$\rightarrow V_{ar} = \frac{\lambda}{2\pi\varepsilon}\ln\left(\frac{r}{a}\right) \rightarrow \frac{V_{ar}}{V_{ab}} = \frac{\ln\left(\frac{r}{a}\right)}{\ln\left(\frac{b}{a}\right)}$$

$2\pi\varepsilon$ $\quad V_{ab}$ $\quad \ln\left(\frac{b}{a}\right)$

$$\rightarrow V_{ab} = \frac{\lambda}{2\pi\varepsilon} \ln\left(\frac{b}{a}\right)$$

$r_1 =$

$$V_{ar_1} = V_{ab}\left(\frac{L\frac{r_1}{a}}{L\frac{b}{a}}\right)$$

$r_2 =$ $\qquad V_{ar_2} =$

$$L\left(\frac{r_2}{a}\right) = L\left(\frac{b}{a}\right)\left(\frac{V_{ar_2}}{V_{ab}}\right)$$

$$Lr_2 - La = L\left(\frac{b}{a}\right)\left(\frac{V_{ar_2}}{V_{ab}}\right); \quad Lr_2 = La + L\left(\frac{b}{a}\right)\left(\frac{V_{ar_2}}{V_{ab}}\right)$$

$V_{ar_2} \leq V_{ab} \mid 0 \leq \delta \leq 1$

$$\delta = \frac{V_{ar_x}}{V_{ab}}$$

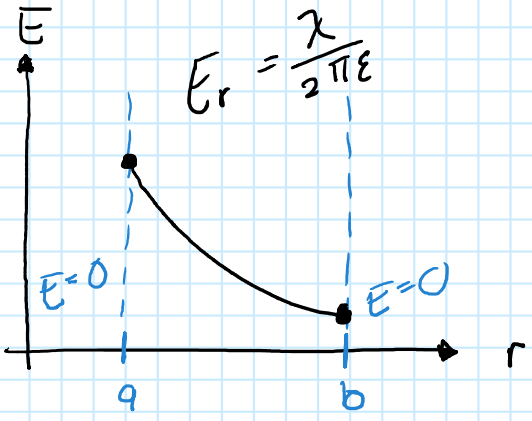
$$Lr_x = La + \left[L\frac{b}{a}\right]\left[\frac{\delta \cancel{V_{ab}}}{\cancel{V_{ab}}}\right]; \quad 1$$

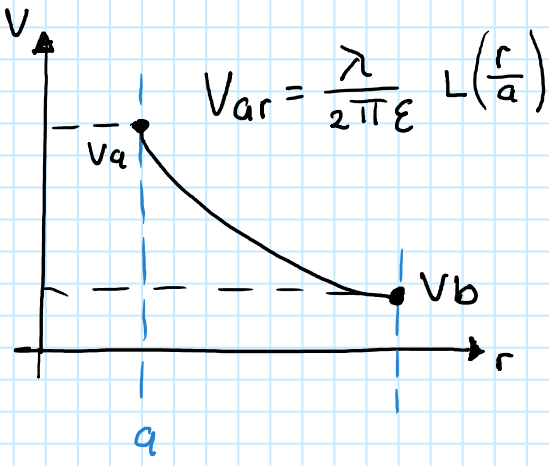
$$Lr_x = La + \left(L\frac{b}{a}\right)(\delta_x)$$

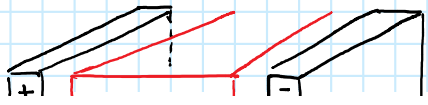
$V_{ab} = V_{ar_1} + V_{ar_2}$

$V_b = 0$

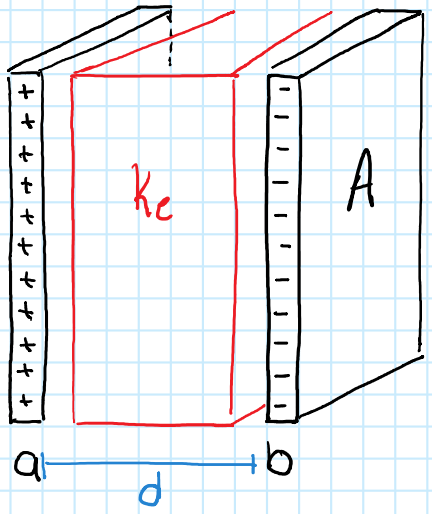
$\therefore V_{r_1} = V_{ab} - V_{ar_1}$







$E = \frac{\sigma}{\ }$



$$E_0 = \frac{\sigma}{\varepsilon_0}$$

$$\frac{E_0}{E} = K_e \; ; \; E = \frac{E_0}{K_e}$$

$$E = \frac{\sigma}{K_e \varepsilon_0} \; ; \; E = \frac{\sigma}{\varepsilon}\,\hat{i}$$

$$V_{ba} = -\int_a^b E \cdot ds$$

- $E = (E_x, E_y, E_z)$
- $ds = (dx, dy, dz)$
- $E = \hat{i} \qquad E_y = E_z = 0$

$$V_{ba} = -\int_a^b E \cdot dx \, \cancel{\cos 0}$$

$$V_{ba} = -E \int_a^b dx \; ; \; V_{ba} = -E\left[x_b - x_a\right]$$

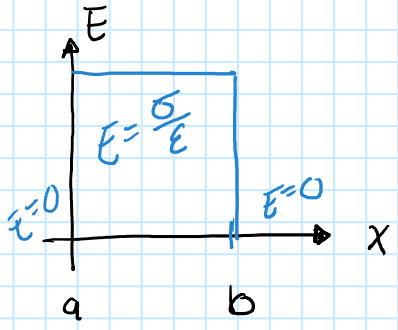
$$x_b - x_a = d \; ; \; V_{ba} = -Ed \; ; \; V_{ab} = Ed$$

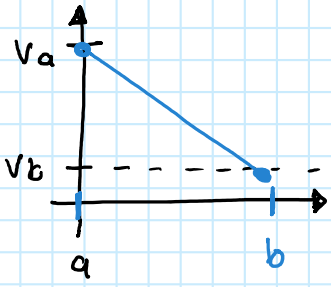
$$E = \frac{V_{ab}}{d} \left[\left(\frac{N}{C}\right) = \left(\frac{V}{m}\right)\right]$$

$$E = \frac{V_{ax_1}}{x_1} \; ; \; V_{ax_1} = Ex_1 \; ; \; V_a - V_{x_1} = Ex_1 \; ;$$

$$V_{x_1} = V_a - Ex_1$$

// Representación Gráfica





$$C = \frac{q}{V} = \frac{q}{Ed} = \frac{q}{\frac{\sigma}{\varepsilon} d} \; ; \; C = \frac{\cancel{q}}{\frac{\cancel{q}}{A\varepsilon} d} = \frac{\varepsilon A}{d}$$

$$C = \frac{\varepsilon A}{d}$$

$\frac{\sigma}{\varepsilon} d$ $\qquad$ $A\varepsilon$ $\qquad$ σ

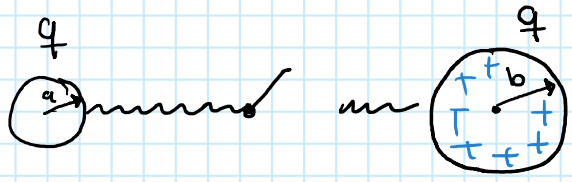
$$\sigma = \frac{dq}{dA} \; ; \; dq = \sigma dA \; ; \; q = \sigma A$$

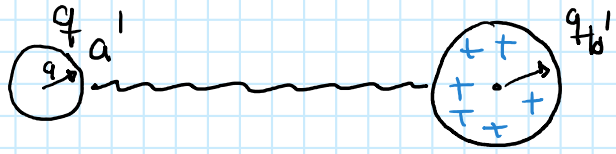
$$C = \frac{\varepsilon A}{d}$$

$$C_0 = \frac{\varepsilon_0 A}{d}$$

Capacitancia | Es una propiedad o característica intrínsica de los cuerpos no depende de la carga ni de su campo

---

- 

- 

$$E_{sup\,a} = E_{sup\,a} = E_{aa} + E_{ba}$$

$$E_{sup\,b} = E_{sup\,b} = E_{ba} + E_{bb}$$

$$V_a = V_{aa} + V_{ab}$$

$$V_b = V_{ba} + V_{bb}$$

$$\rightarrow V_{ab} = V_a - V_b$$

//Después (Cerrado)

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad \Big| \quad E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \Big| \quad a < b \qquad V_a > V_b$$

$$E_a > E_b$$

$E_{sup\,a'} =$

$E_{sup\,b'} =$

$V_a' =$

$V_b' =$

Se distribuye $= V_a' = V_b'$

$$V_{a'b'} = V_{ab'}$$

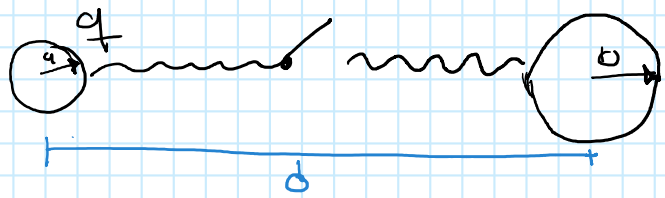
Nota | La superficie de un cuerpo internamente es equipotencial

$$\longrightarrow \quad q_a + q_b = q_a' + q_b'$$

$$\longrightarrow \quad \frac{q_a'}{a} = \frac{q_b'}{b}$$

17_Clase

Monday, April 1, 2019 7:20 AM



• $E_{sup\,a} = E_{aa} + E_{ab} =$

$= \frac{k q_a}{a^2} + \frac{k q_b}{d^2} =$

$= \frac{(9\times10^9)(10^{-8})}{10^{-4}} + (9\times10^9)\left(\frac{10^{-8}}{1}\right) =$

$= 9\times10^5 + 90 = \text{---} \left[\frac{N}{C}\right]$

• $E_{sup\,b} = E_{ba} + E_{bb}$

$= \frac{k q_a}{d^2} + \frac{k q_b}{b^2} =$

$= (9\times10^9)\left(\frac{10^{-8}}{1}\right) + (9\times10^9)\left(\frac{10^{-8}}{9\times10^{-4}}\right) =$

$= 90 + 225000 = \text{---} \left[\frac{N}{C}\right]$

• $V_a = V_{aa} + V_{ab} =$

$= k\frac{q_a}{a} + k\frac{q_b}{d} =$

$= (9\times10^9)\left(\frac{10^{-8}}{10^{-2}}\right) + (9\times10^9)\left(\frac{10^{-8}}{1}\right) =$

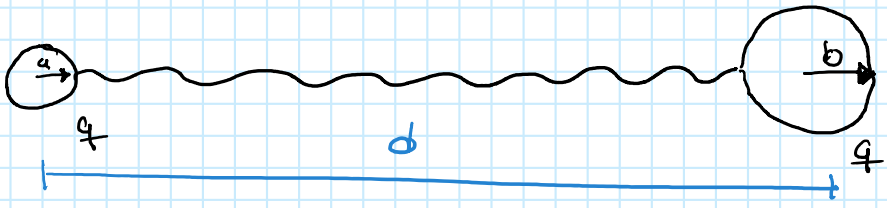
$= 9\times10^3 + 90 = \text{---} \left[\frac{N}{C}\right]$

• $V_b = V_{bb} + V_{ba} =$

$= k\frac{q_b}{b} + k\frac{q_a}{d} =$

$= (9\times10^9)\left(\frac{10^{-8}}{2\times10^{-2}}\right) + (9\times10^9)\left(\frac{10^{-8}}{1}\right) =$

$= 4500 + 90 = \text{---} \left[\frac{N}{C}\right]$



Equipotencial carga se dirige a la superficie

• El campo electrico es normal a la superficie. (Equipotencial)

$V_a = V_b$ // Cambian las cargas

$\therefore V_a' = V_b'$; $V_a' = k\frac{q_a'}{a} + k\frac{q_b'}{d} = 9\times10^{11}\, q_a'$

$V_b' = k\frac{q_b'}{b} + k\frac{q_b'}{d} = 9\times10^{11}\, q_b'$

$$V_a' = K\frac{q_a'}{a} \quad \Big| \quad \cancel{K}\frac{q_a'}{a} = \cancel{K}\frac{q_b'}{b} \quad ; \quad q_a + q_b = q_a' + q_b' = 2q$$

$$V_b' = K\frac{q_b'}{b} \quad \Big| \quad q_a' = \left(\frac{a}{b}\right)(q_b')$$

$$\left(\frac{a}{b}\right) q_b' + q_b = 2q \quad ; \quad q_b'\left(1 + \frac{a}{b}\right) = 2q$$

$$\therefore\ q_b' = \frac{2q}{1+\frac{a}{b}} \qquad \therefore\ q_a' = \left(\frac{a}{b}\right)(q_b')$$

$$= 4/3\ 10^{-8}\ [C] \qquad = \frac{2}{3}\ 10^{-8}\ [C]$$

→ Voltajes

$$V_a' = 9\times10^9 \frac{\left(\frac{2}{3}\ 10^{-8}\right)}{10^{-2}} = 6000\ [V]$$

$$V_b' = 9\times10^9 \frac{\left(\frac{4}{3}\times10^{-8}\right)}{2\times10^{-2}} = 6000\ [V]$$

→ Campos Electricos

$$E_a' = 9\times10^9 \frac{\left(\frac{2}{3}\ 10^{-8}\right)}{10^{-4}} = 6\times10^5 \left[\frac{V}{m}\right]$$

Menor radio, mayor campo

$$E_b' = 9\times10^9 \frac{\left(\frac{4}{3}\times10^{-8}\right)}{(2\times10^{-2})^2} = 3\times10^5 \left[\frac{V}{m}\right]$$

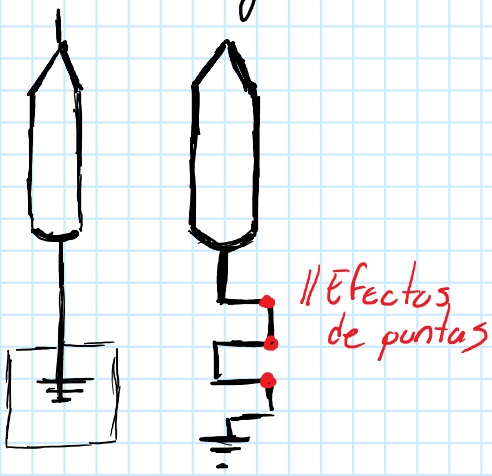
- $E_a' > E_b'$
- $q_a' < q_b'$
- $V_a' = V_b'$

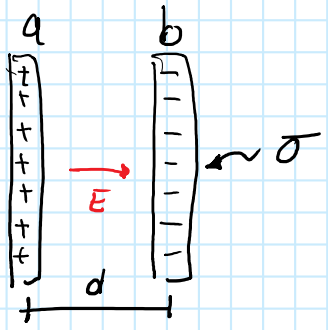
Conclusión: En un cuerpo cargado electricamente el campo eléctrico es mayor en aquellos puntos de radio curvatura menor o de curvatura mayor

Conc mayor

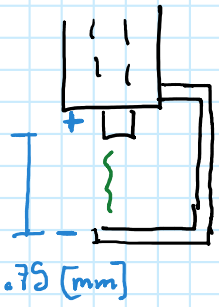
- $E_{max(Aire)} = 3 \times 10^6 \left[\frac{V}{m}\right]$

- Para Rayos





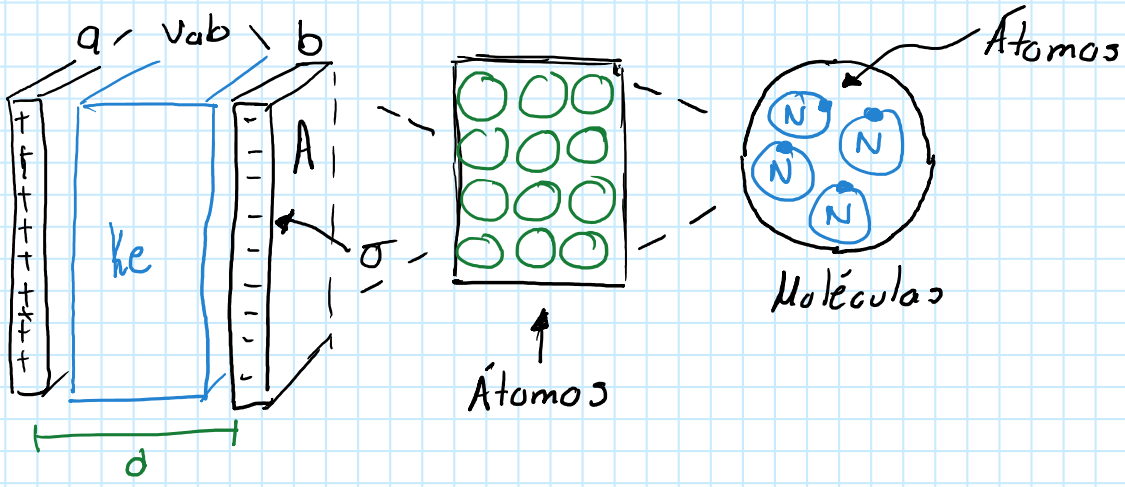
$E = -\nabla V$; $E = \frac{V}{\varepsilon} = \frac{V_{ab}}{d}$

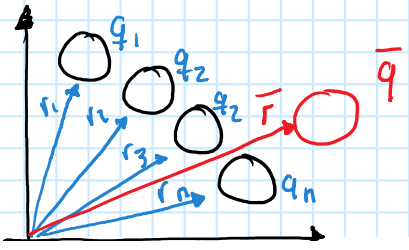


$E_{max} = 3 \times 10^6 \left[\frac{V}{m}\right]$

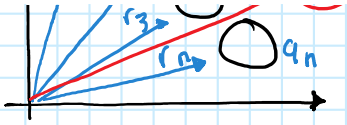
- $E_{max} = \frac{3 \times 10^6}{1\ m} = \frac{3 \times 10^3}{10^{-3}\ m}$

Propiedades Dieléctricos



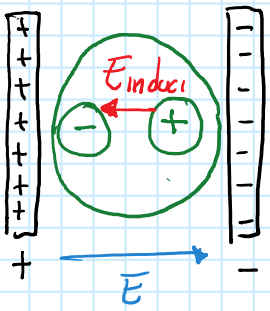


- Centro de gravedad eléctrico



- $\bar{q} = \sum_{i=1}^{n} q_i$; $\bar{r} = \dfrac{\sum_{i=1}^{n} r_i q_i}{\sum_{i=1}^{n} q_i}$ | First Electric Moment
- $\bar{r}q = \sum_{i=1}^{n} r_i q_i$

// Coinciden los centros de gravedad positivo y negativo
∴ Es una molécula no polarizada



// Se desplazaron centros de gravedad

→ Las moléculas en estado natural son moléculas no polarizados.

- Esto significa que los centros de gravedad eléctrico y negativo coinciden.
- When a polarized molecule is colocada bajo la influencia de un campo electréctico se polariza and if it was previosly polarazid it keeps equal but with inverse (reorienta) the polarazized.
- la polarización implica que se produzca un campo eléctrico nuevo denominado "Campo Eléctrico Inducido", todo esto como consecuencia del desplazamiento de los centros de gravedad eléctricos.
- El campo eléctrico inducido es opuesto al

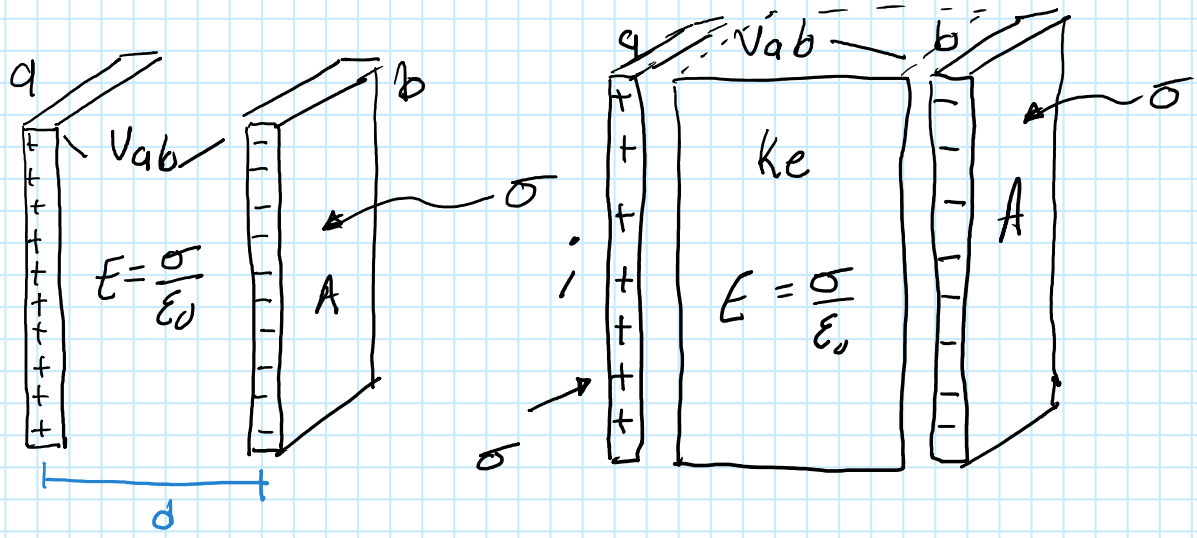
- El campo eléctrico inducido es opuesto al campo eléctrico que le dio origen.

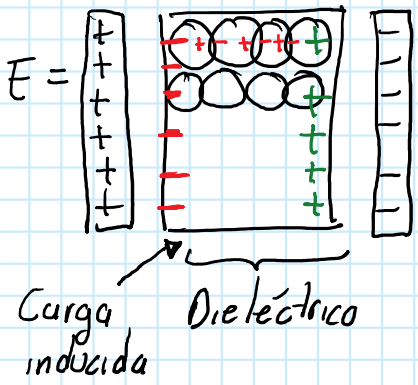
18_Clase

Wednesday, April 3, 2019 7:19 AM

→ Propiedades de dieléctrico | Condiciones de Frontera | Redes de Capacitores

Disminuye Campo → Disminuye (Voltaje) → Aumenta Capacitancia





→ $E_{nuevo} = E_{original} + E_{inducido}$;

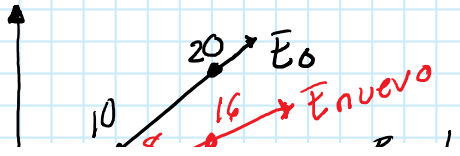
$E_n = E_0 + E_i$;

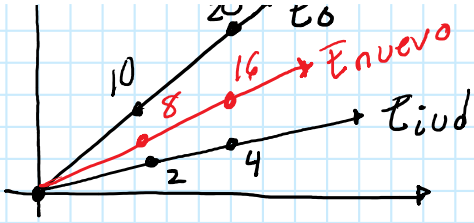
$E_0 = \frac{\sigma}{\varepsilon_0}\hat{i}$ $\quad E_i = \frac{\sigma}{\varepsilon_0}-\hat{i}$

$\therefore E = \frac{\sigma}{\varepsilon_0}[\hat{i}] + \frac{\sigma}{\varepsilon_0}[-\hat{i}]$

- $\sigma\hat{i} = \bar{D} \triangleq$ Vector desplazamiento
- $\sigma-\hat{i} = \bar{P} \triangleq$ Vector polarización

$\therefore \varepsilon_0 E = \bar{D} + \bar{P}$





if $\sigma = 0$; $E_0 = 0$; $E_i = 0$; $\sigma_i = 0$; $E = 0$

if $\sigma = \nearrow$; $E_0 = \nearrow$; $E_i = \nearrow$; $\sigma_i = \nearrow$; $E = \nearrow$

$\therefore \sigma_i \leq \sigma$ // Es igual si es conductor

Campo Resultante

$E \sim \sigma_i$

$\sigma_i = cte\, E$; $cte = \eta$; $E\eta = \sigma_i$

• $\eta = f(\text{material})$; $\eta \triangleq$ Suceptividad Electrica

$E_0 = \frac{\sigma}{\varepsilon_0}$; $\sigma = \varepsilon_0 E_0$; $\sigma_i = \eta E$

$\therefore \eta = \left[\frac{Coul^2}{N - m^2}\right]$

$\rightarrow$ $E = \frac{\sigma}{\varepsilon_0}[\hat{i}] + \frac{\sigma_i}{\varepsilon_0}[-\hat{i}]$;

$\varepsilon_0 E = \sigma[\hat{i}] + \eta E[-\hat{i}]$; $[\varepsilon_0 + \eta] E \hat{i} = \sigma \hat{i}$

$|E| = \frac{\sigma}{\varepsilon_0 + \eta}$; $E = \frac{\sigma}{\varepsilon_0 \underbrace{\left(1 + \frac{\eta}{\varepsilon_0}\right)}_{\text{adimensional}}}$

$\left(1 + \frac{\eta}{\varepsilon_0}\right) \triangleq cte$

$1 + \frac{\eta}{\varepsilon_0} = 1$; $1 \leq \left(1 + \frac{\eta}{\varepsilon_0}\right) < \infty$ $\therefore$ $K_e = 1 + \frac{\eta}{\varepsilon_0}$

$1 \leq K_e < \infty$

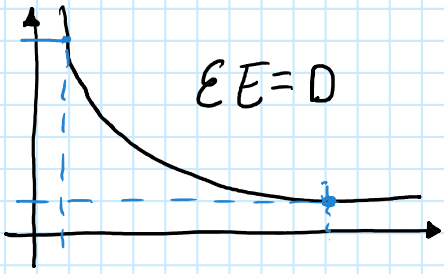
ε_0 ε_0

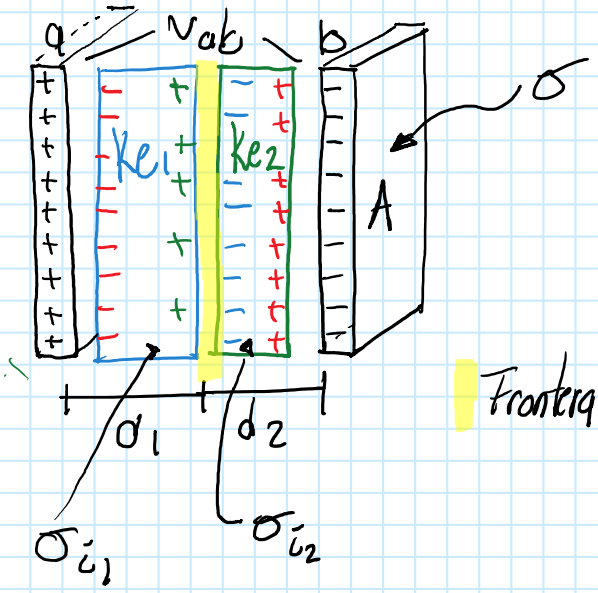
$$1 \le K_e < \infty$$

$$E = \frac{\sigma}{\varepsilon_0 K_e} \qquad \varepsilon_0 K_e = \varepsilon \quad ; \quad E = \frac{\sigma}{\varepsilon}$$

$$E = \frac{\sigma}{\varepsilon}[\hat{i}] \quad ; \quad \varepsilon E = D$$

$$\sigma \hat{i} = D$$



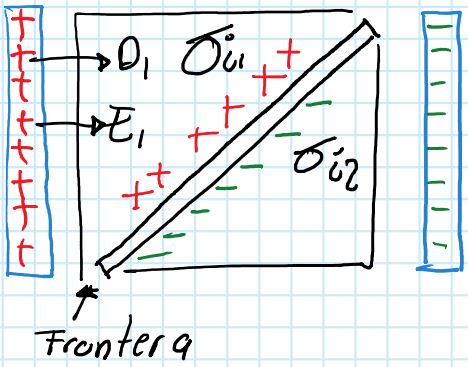


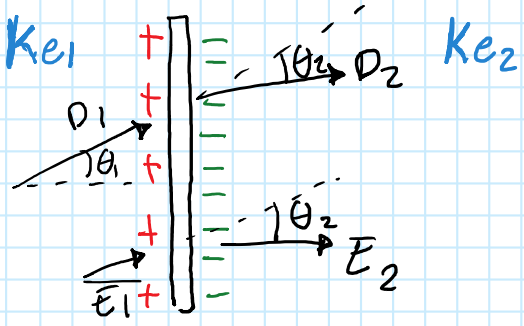
DATOS

K_{e_1} ; $\eta_1 = (K_{e_1} - 1)\varepsilon_0$

K_{e_2} ; $\eta_2 = (K_{e_2} - 1)\varepsilon_0$

- $E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{K_e \varepsilon_0}$ | $\bar{D} = \varepsilon \bar{E}$, $\varepsilon > 0$;



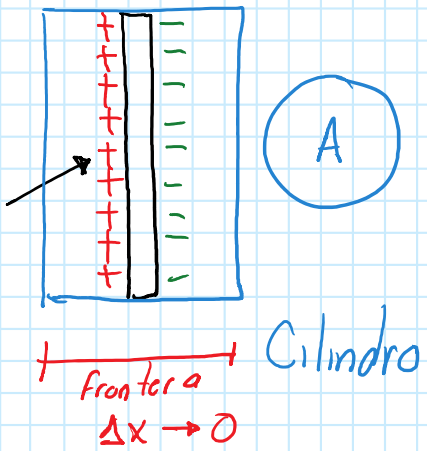


$\sigma_{i_1} \neq \sigma_{i_2}$

$\therefore$ Desequilibrio

DATOS		Incog	
	D_1		D_2
	E_1		E_2
	θ_1		θ_2

Condiciones de Frontera



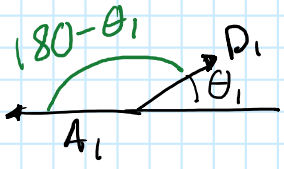
$$N = \frac{1}{\varepsilon} q = \oint_{S.C} E \cdot dA \; ;$$

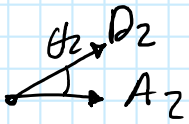
$$\oint_{S.C} \varepsilon E \cdot dA = q \; ; \; \oint_{S.C} D \cdot dA = q_{en\,C}$$

$$\therefore \oint_{S.C} D \cdot dA = \int_{T_1} D_1 \cdot dA + \int_{T_2} D_2 \cdot dA + \int_{Cara} D_3 \cdot dA = q_{en\,C}$$

// Para estar en la frontera

$\Delta x \to 0$; $q_{en\,C} \to 0$





Area de la Cara $\to 0$

$|D_1| \triangleq cte$

$|D_2| \triangleq cte$

$$\therefore \int D_1 \cos(180-\theta_1) \cdot dA + \int D_2 \cos\theta_2 \cdot dA + 0 = 0$$

$$D_1 - (\cos\theta_1) \int_{T_1} dA + D_2 (\cos\theta_2) \int_{T_2} dA = 0$$

$$-D_1 \cos\theta_1 A_1 + D_2 \cos\theta_2 A_2 = 0 \; ;$$

$$D_1 \cos\theta_1 A_1 = D_2 \cos\theta_2 A_2 \qquad A_1 = A_2$$

→ La componente del vector desplazamiento perpendicular a la frontera es igual en ambos

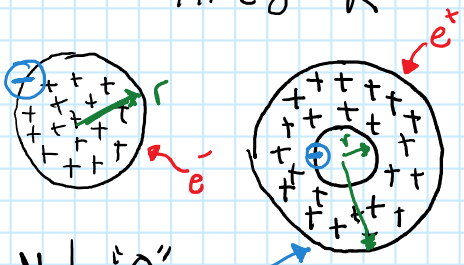
la componente del vector desplazamiento perpendicular a la frontera es igual en ambos lados de ella

19_Clase

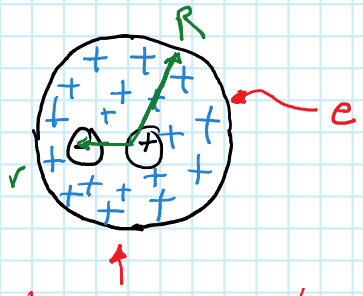
lunes, 8 de abril de 2019 07:27 a. m.

7.4] $F = \frac{1}{4\pi\varepsilon_0}\frac{e^2 r}{R^3}$

Centro Gravedad Eléctrica



Carga Neta "0"



Tiende ⊖ hacia el centro (Centro de gravedad eléctrico)

LEY DE GAUSS

El número neto de lineas de fuerza que cruzan una sup. cerrada es igual $\frac{1}{\varepsilon_0}$ por la carga neta contenida o encerrada en ella

$N = \frac{1}{\varepsilon_0} q$

$N' = \varepsilon_0 N = q$

$\rho = \frac{dq}{d\tau}$; $\int_0^Q dq = \int_0^\tau \rho\, d\tau$; $Q = \rho\tau$

$\therefore \rho = \frac{Q}{\tau}$ $\tau = \frac{4}{3}\pi R^3$; $\rho = \frac{e}{\frac{4}{3}\pi R^3}$

$\rho = \frac{e'}{\frac{4}{3}\pi r^3}$ *

Porque cambia el radio

LEY DE COULOMB

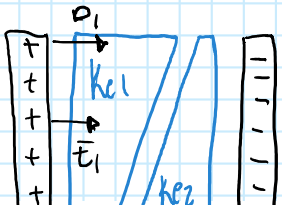
La fuerza de origen electrico entre 2 cuerpos cargos es directamente proporcional a la carga de los cuerpos e inversamente proporcional al cuadrado de la distancia que los separa

$\therefore \frac{e}{r^3} = \frac{e'}{R^3}$; $e' = \left(\frac{e}{R^3}\right)(r^3)$

$F = \frac{1}{4\pi\varepsilon_0}\frac{e e'}{r^2}$; $F = \frac{1}{4\pi\varepsilon_0}\left(\frac{e}{\cancel{r^2}}\right)\left(\frac{e}{R^3}\right)(\cancel{r^3})$

$\therefore F = \frac{1}{4\pi\varepsilon_0}\left(\frac{e^2 r}{R^3}\right)$

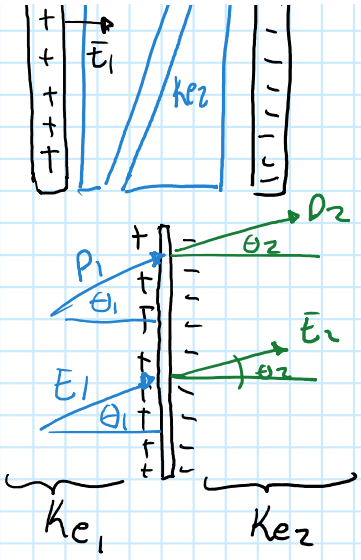
[7.2



$D = \varepsilon E$ | $P = \eta E$

$|D| = \sigma$ | $|P| = \sigma_i$

$D = \sigma_i$ | $P = \sigma_i [-\hat{i}]$



$|D| = \sigma_i$ | $|P| = \sigma_i$

$D = \sigma_i$ | $P = \sigma_i[-\hat{i}]$

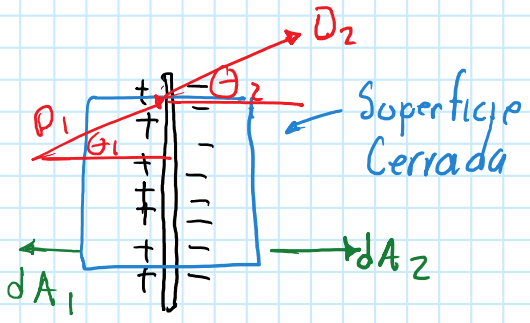
LEY GAUSS

$$N = \oint E \cdot dA = \frac{1}{\varepsilon} q$$

$$\oint_{S.C} \varepsilon E \cdot dA = q$$

$$\therefore \oint D \cdot dA = q$$

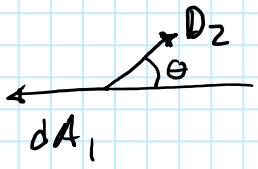
// Empleando Cilindro



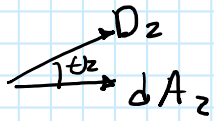
$$\oint_{S.C} D \cdot dA = \int_{T_1} D_1 \cdot dA + \int_{T_2} D_2 \cdot dA + \int_{S.LAT} D_3 \cdot dA =$$

$q_{neta} \to 0 \quad \Delta x \to 0$

$$\therefore \int_{T_1} D_1 \cdot dA + \int_{T_2} D_2 \cdot dA = 0$$



$$\to \int_{T_1} D_1 (\cos 180 - \theta_1) dA_1 + \int_{T_2} D_2 (\cos \theta_1) dA_2 = 0$$



$|D_1| \triangleq cte$

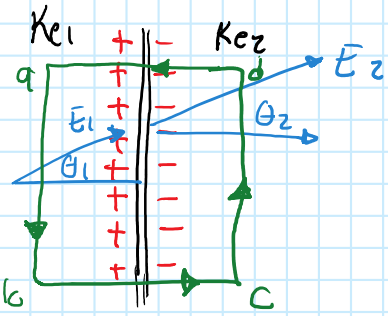
$|D_2| \triangleq cte$

$$\therefore (-D_1)(\cos\theta_1)\int dA_1 + D_2 \cos\theta \int dA_2 = 0$$

$$\underbrace{D_1 \cos\theta_1 A_1}_{\text{Componente del vector desplazamiento}} = D_2 \cos\theta A_2$$

La componente perpendicular a la frontera es igual en ambos lados

Ecuación de Maxwell



$\sigma_{i_1} \neq \sigma_{i_2}$

• $V_{ba} = -\int_a^b E \cdot ds$ | $\oint E \cdot ds = 0$

$V_{aa} = V_a - V_a = 0$

• $E = -\nabla V$; En un ciclo $\nabla V \to 0$

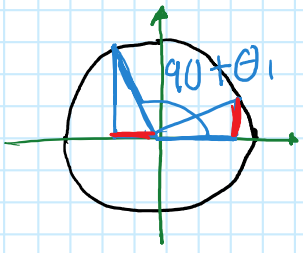
$$\oint E \cdot ds = \int_a^b E_1 \cdot ds + \int_b^c E_2 \cdot ds + \int_c^d E_3 \cdot ds + \int_d^a E_4 \cdot ds = 0$$

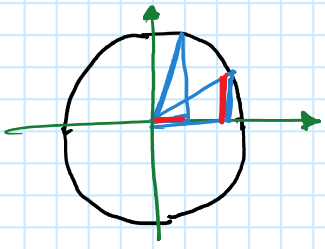
// Para estar en la frontera | $\Delta x \to 0$ l_{bc} y $l_{ba} \to 0$

$$\int_b^c E \cdot ds \quad y \quad \int_d^a E \cdot ds \to 0$$

$$\therefore \int_a^b E_1 \cdot ds + \int_c^d E_2 \cdot ds = 0$$

$$\int_a^b E_1 \cos(90+\theta_1)\, ds + \int_c^d E_2 \cos(90-\theta_2)\, ds = 0$$





$\cos(90+\theta_1) = -\operatorname{sen}\theta_1$ | $\cos(90-\theta_2) = \operatorname{sen}\theta_2$

$$-E_1 \operatorname{sen}\theta_1 \cancel{(S_b - S_a)} + E_2 \operatorname{sen}\theta_2 \cancel{(S_d - S_c)} = 0$$

$$(S_b - S_a) = (S_d - S_a)$$

$$\therefore E_1 \operatorname{sen}\theta_1 = E_2 \operatorname{sen}\theta_2 \quad //$$

$$\therefore \; E_1 \operatorname{sen} \theta_1 = E_2 \operatorname{sen} \theta_2$$

- La componente del vector intensidad de campo eléctrico paralela a la frontera es igual a ambos lados de ella.

Concluyendo

$$E_1 \operatorname{sen} \theta_1 = E_2 \operatorname{sen} \theta_2 \quad ; \quad \frac{E_1}{D_1} \tan\theta_1 = \frac{E_2}{D_2} \tan\theta_2$$

$$D_1 \cos\theta_1 = D_2 \cos\theta_2$$

$$D_1 = \varepsilon_1 E_1 = K_{e_1} \varepsilon_0 E_1 \quad ; \quad \frac{\cancel{E_1}}{K_{e_1} \cancel{\varepsilon_0} \cancel{E_1}} \tan\theta_1 = \frac{\cancel{E_2}}{K_{e_2} \cancel{\varepsilon_0} \cancel{E_2}} \tan\theta_2$$

$$D_2 = \varepsilon_2 E_1 = K_{e_2} \varepsilon_0 E_2$$

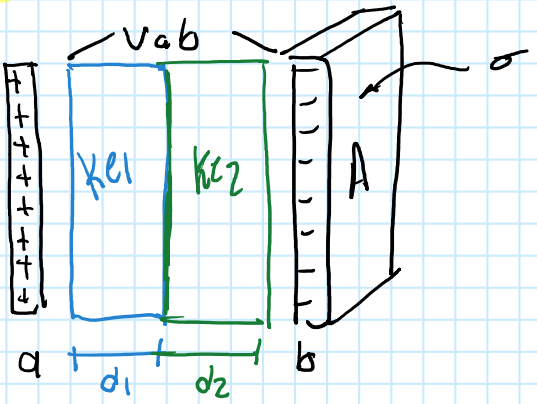
$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{K_{e_1}}{K_{e_2}}$$

20_Clase

miércoles, 10 de abril de 2019 07:22 a. m.

$$V_{ab}A = K\left(\frac{q_1}{a} + \frac{q_2}{b}\right)$$

Condiciones de frontera



DATOS	Contestar	
$d_1 = 3mm$	$E_1 =$	$P_1 =$
$d_2 = 7mm$	$E_2 =$	$P_2 =$
$A = 10cm^2$	$D_1 =$	$\eta_1 =$
$Ke_1 = 2$	$D_2 =$	$\eta_2 =$
$Ke_2 = 3$	$\sigma =$	$C =$
$V_{ab} = 100$ [V]	$\sigma_{i1} =$	$V_{af} =$
	$\sigma_{i2} =$	$V_{fb} =$

→ Condiciones de frontera

Frontera y-z

- $D_{1x} = D_1 \cos\theta_1$
- $D_{1y} = D_1 \operatorname{sen}\theta_1$
- $D_{2x} = D_2 \cos\theta_2$
- $D_{2y} = D_2 \operatorname{sen}\theta_2$

$D_1 = D_{1x}\begin{bmatrix}1\\0\end{bmatrix} + D_{2y}\begin{bmatrix}1\\0\end{bmatrix}$

$D_2 = D_{2x}\begin{bmatrix}1\\0\end{bmatrix} + D_{2y}\begin{bmatrix}1\\0\end{bmatrix}$

C.F | $D_{1x} = D_{2x}$

- $E_{1x} = E_1 \cos\theta_1$
- $E_{1y} = E_1 \operatorname{sen}\theta_1$
- $E_{2x} = E_2 \cos\theta_2$
- $E_{2y} = E_2 \operatorname{sen}\theta_2$

- $E_1 = E_{1x}\begin{pmatrix}1\\0\end{pmatrix} + E_{1y}\begin{bmatrix}1\\0\end{bmatrix}$
- $E_2 = E_{2x}\begin{bmatrix}1\\0\end{bmatrix} + E_{2y}\begin{bmatrix}1\\0\end{bmatrix}$

NOTAS

$D_1 = \varepsilon_1 E_1 = Ke\,\varepsilon_0 E_1$

$D_2 = \varepsilon_2 E_2 = Ke\,\varepsilon_0 E_2$

- $D_{1y} = 0$; $E_{1y} = 0$

if $E_{1y} = 0$; $E_y = 0$

- $D_{2y} = 0$; E_{2y} 0

y $D_{1y} = 0$

//Resumen

- $D_1 = D_{1x}\begin{bmatrix}1\\0\end{bmatrix}$
- $D_2 = D_{2x}\begin{bmatrix}1\\0\end{bmatrix}$
- $E_1 = E_{1x}\begin{bmatrix}1\\0\end{bmatrix}$
- $E_2 = E_{2x}\begin{bmatrix}1\\0\end{bmatrix}$

C.F $\left| \begin{array}{l} D_1 x = D_2 x \\ D_1 x = D_1 \\ D_2 x = D_2 \end{array} \right.$ $\therefore D = D_1 = D_2$

$\hookrightarrow |D| = \sigma$

- $D_1 = K_{e_1} \varepsilon_0 E_1$

  $D_2 = K_{e_2} \varepsilon_0 E_2$; $K_{e_1} E_1 = K_{e_2} E_2$; $E_1 = \frac{K_{e_2}}{K_{e_1}} E_2$

- $V_{ab} = V_a + V_{fb}$
- $E = \frac{V_{ab}}{d}$; $E_1 = \frac{V_{af}}{d_1}$ $E_2 = \frac{V_{fb}}{d_2}$

- $V_{fb} = E_2 d_2$
- $V_{af} = E_1 d_1$; $V_{ab} = E_1 d_1 + E_2 d_2$

$$E_2 = \frac{V_{ab}}{\left(\frac{K_{e_2}}{K_{e_1}}\right) d_1 + d_2} \begin{bmatrix} 1 \\ i \end{bmatrix} ; \quad E_1 = \frac{K_{e_2}}{K_{e_1}} E_2 \begin{bmatrix} 1 \\ i \end{bmatrix}$$

- $D_1 = K_{e_1} \varepsilon_0 E_1 \begin{bmatrix} 1 \\ i \end{bmatrix}$
- $D_2 = K_{e_2} \varepsilon_0 E_2 \begin{bmatrix} 1 \\ i \end{bmatrix}$

$D_1 = D = D_2$; $K_e = 1 + \frac{\eta}{\varepsilon_0}$

$|D| = \sigma$

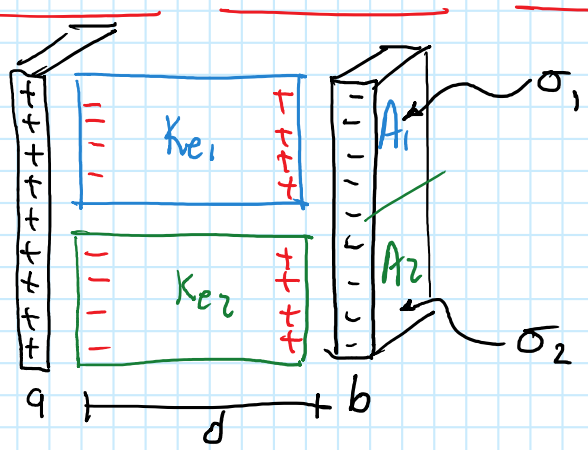
- $\eta_1 = (K_{e_1} - 1)(\varepsilon_0)$ • $\eta_2 = (K_{e_2} - 1)(\varepsilon_0)$ $\Big|$ $|P_1| = \sigma_{i_1}$
- $P_1 = \eta_1 E_1 \begin{bmatrix} 1 \\ i \end{bmatrix}$ • $P_2 = \eta_2 E_2 \begin{bmatrix} -1 \\ i \end{bmatrix}$ $\Big|$ $|P_2| = \sigma_{i_2}$

// Capacitancia

- $C = \frac{q}{V_{ab}} = \frac{\sigma A}{V_{ab}}$ $\sigma = \frac{q}{A}$; $q = \sigma A$

- $E_{almc} = \frac{1}{2} C V_{ab}^2$

• $E_{alm} = \frac{1}{2} C V_{ab}^2$



DATOS

$V_{ab} = 100\ V$

$A_1 = 4\ cm^2$

$A_2 = 6\ cm^2$

$d = 10\ cm$

$K_{e_1} = 2$

$K_{e_2} = 3$

Incognitas

$D_1 =$ $\quad \eta_1 =$

$D_2 =$ $\quad \eta_2 =$

$E_1 =$ $\quad C =$

$E_2 =$ $\quad E_{alm} =$

$\sigma_1 =$ $\quad \theta_2$

$\sigma_2 =$

$P_1 =$

$P_2 =$

$\sigma_{i1} =$

$\sigma_{i2} =$

$A_1 + A_2 = A$ | $q_1 = \sigma_1 A_1$

$q_1 + q_2 = q$ | $q_2 = \sigma_2 A_2$

$\therefore\ q = (\sigma_1 A_1) + (\sigma_2 A_2)$

//Despejando

• $D_{1y} = D_{2y}$ | Como $E_1 \perp$ Sup. Placa

$E_{1x} = E_{2x}$ | $E_{1y} = 0$; $E_{2y} = 0$

• $D_1 = K_{e_1} \varepsilon_0 E_1$ $\quad D_2 = K_{e_2} \varepsilon_0 E_2$

• $D_1 = D_{1x}[\hat{i}] + D_{1y}[\hat{j}]$ $\quad D_2 = D_{2x}[\hat{i}] + D_{2y}[\hat{j}]$

• $E_1 = E_{1x}[\hat{i}] + E_{2y}[\hat{j}]$ $\quad E_2 = E_{2x}[\hat{i}] + E_{2y}[\hat{j}]$

• $E_1 = E_{1x}$

• $E_2 = E_{2x}$; $E_1 = E_2 = E$

$E = \frac{V_{ab}}{d}$

$D_1 \rightarrow \sigma_1$

$D_2 \rightarrow \sigma_2$

$\eta_1 = (K_e - 1)\varepsilon_0$

$\eta_2 = (K_e - 1)\varepsilon_0$

$$E = \frac{V_{ab}}{d}$$

$$\eta_1 = (k_e - 1)\varepsilon_0$$

$$\eta_2 = (k_e - 1)\varepsilon_0$$

- $P_1 = \eta_1 E_1 = \eta_1 E$
- $P_2 = \eta_2 E_2 = \eta_2 E$