Método de Diferencias Finitas para Ecuaciones Diferenciales Parciales

PROFESOR:
GERARDO FLORES DELGADO

INTEGRANTES:
MURRIETA VILLEGAS ALFONSO
REZA CHAVARRIA SERGIO GABRIEL
VALDESPINO MENDIETA JOAQUIN

Clasificación de EDPs

$$A\frac{\partial^2 u}{\partial x^2} + B\,\frac{\partial^2 u}{\partial xy} + C\,\frac{\partial^2 u}{\partial y^2} = f\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right)$$

Para $\boldsymbol{x_0} \le \boldsymbol{x} \le \boldsymbol{x_f}\ , \boldsymbol{y_0} \le \boldsymbol{y} \le \boldsymbol{y_f}$ y con las condiciones de borde para un dominio rectangular

$$u(x, y_0) = b_{y_0}\,(x) \quad , \quad u(x, y_f) = b_{y_f}\,(x)$$

$$u(x_0, y) = b_{y_0}\,(y) \quad , \quad u(x_f, y) = b_{x_f}\,(y)$$

- Estas EDPs pueden ser clasificadas en 3 grupos:
  - EDP Elíptica: $B^2 - 4AC < 0$
  - EDP Parabólica: $B^2 - 4AC = 0$
  - EDP Hiperbólica: $B^2 - 4AC > 0$

Hiperbólica: $\frac{dC}{dt} - U\frac{dC}{dx} = 0$ *Ec. de advección*

$\frac{d^2C}{dt^2} - a\frac{d^2C}{dx^2} = 0$ *Ec. de onda*

Elíptica: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ *Ec. de Laplace*

Parabólica: $\frac{\partial C}{\partial t} - D\frac{\partial^2 C}{\partial x^2} = 0$ *Ec. de difusión*

EDP Elípticas

- Ecuación de Laplace $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$
- Ecuación Poisson $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

Diferencias Finitas para EDP

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{\Delta x^2}$$

$$\frac{\partial^2 u}{\partial y^2} \approx \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{\Delta y^2}$$

$$\frac{\partial u}{\partial y} \approx \frac{u_{i+1,j} - 2u_{i-1,j}}{2\Delta y}$$

$$\frac{\partial u}{\partial x} \approx \frac{u_{i+1,j} - 2u_{i-1,j}}{2\Delta x}$$

Discretización de Ecuación de Laplace

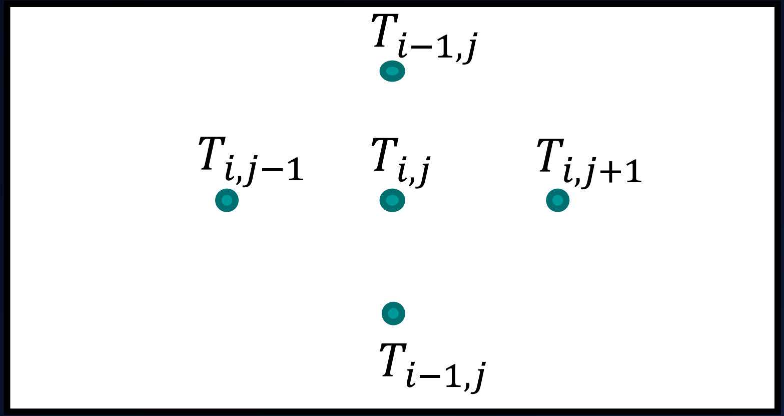
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad T(x, y) = T \qquad T\left(x_{i,j}, y_{i,j}\right) = T_{i,j}$$

-> Diferencias Finitas (Derivadas y evaluando)

$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{(\Delta x)^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{(\Delta y)^2} = 0 \qquad (\Delta x)^2 = (\Delta y)^2 = h$$

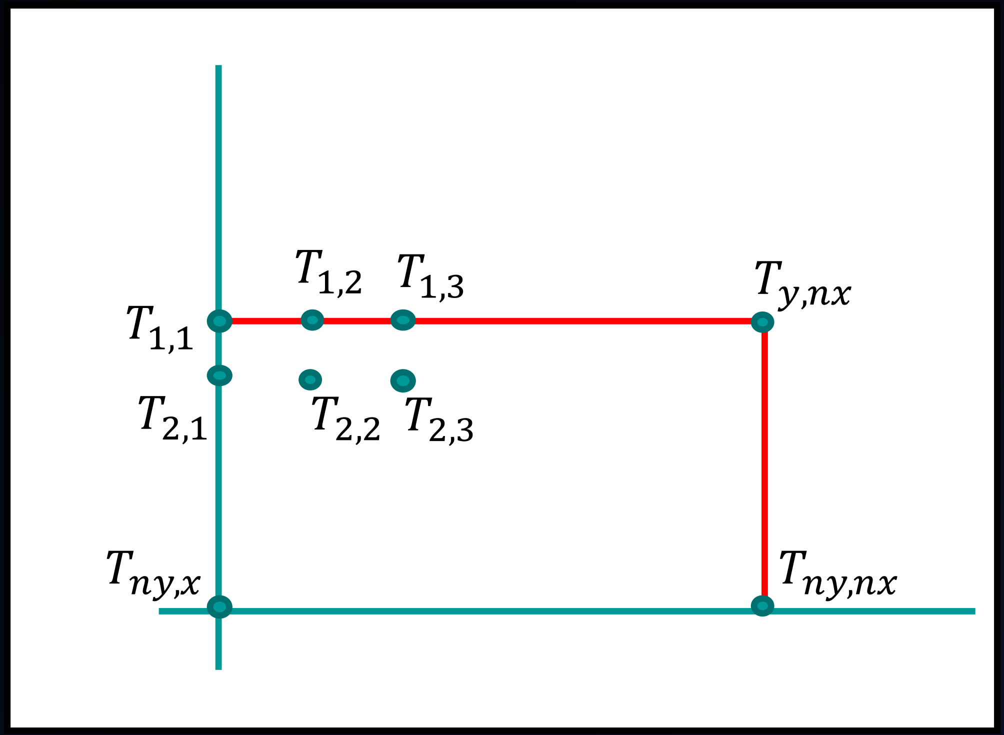
-> Simplificando denominador común

$$-4T_{i,j} + T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} = 0$$



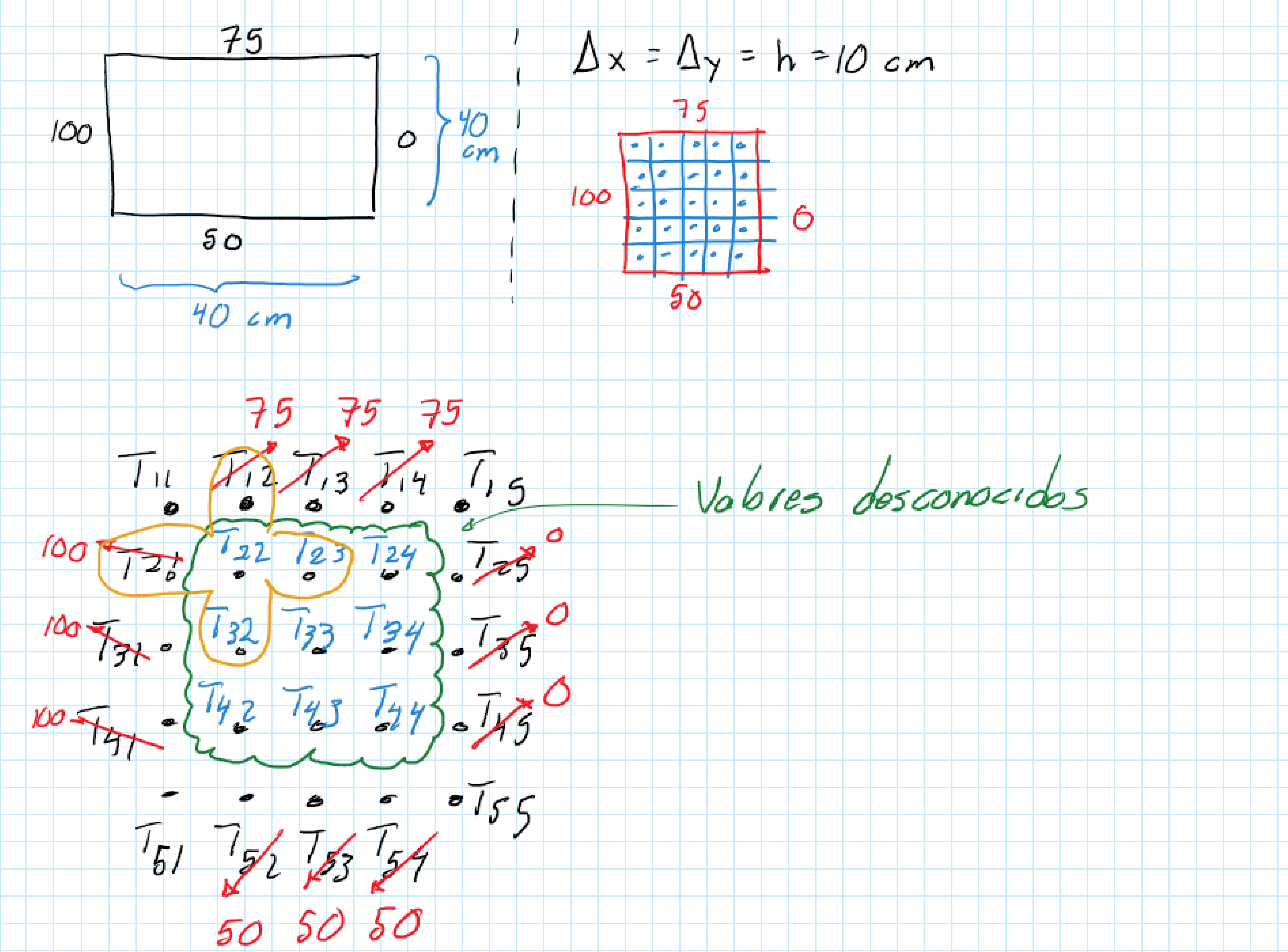
Discretización de Ecuación de Laplace

$$-4T_{i,j} + T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} = 0$$



$$T_{i,j} = \frac{T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1}}{4}$$

EJERCICIO - EJEMPLO



EJERCICIO - EJEMPLO

$$-4T_{i,j} + T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} = 0$$

$$-4T_{22} + T_{32} + T_{23} + \overset{175}{\cancel{100 + 75}}\ 0$$

$$-4T_{23} + T_{22} + T_{24} + T_{33} + 75 = 0$$

$$-4T_{24} + T_{23} + T_{34} + 75 = 0$$

$$-4T_{32} + T_{42} + T_{33} + T_{22} + 100 = 0$$

$$-4T_{33} + T_{23} + T_{34} + T_{43} + T_{32} = 0$$

$$-4T_{42} + T_{32} + T_{43} + \overset{150}{\cancel{100 + 50}} = 0$$

$$-4T_{43} + T_{33} + T_{42} + T_{44} + 50 = 0$$

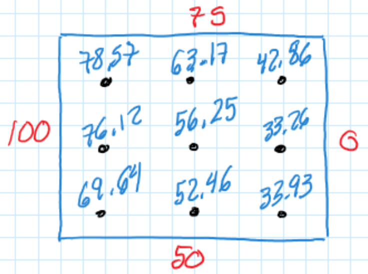
$$-4T_{44} + T_{34} + T_{43} + 50 = 0$$

//Simplificando en forma matricial

$$\begin{array}{ccccccccc} T_{22} & T_{23} & T_{24} & T_{32} & T_{33} & T_{34} & T_{42} & T_{43} & T_{44} \end{array}$$

$$\begin{bmatrix} -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 \end{bmatrix} \begin{bmatrix} T_{22} \\ T_{23} \\ T_{24} \\ T_{32} \\ T_{33} \\ T_{34} \\ T_{42} \\ T_{43} \\ T_{44} \end{bmatrix} = \begin{bmatrix} -175 \\ -75 \\ -75 \\ -100 \\ 0 \\ 0 \\ -150 \\ -50 \\ -50 \end{bmatrix}$$

//Resultado



Referencia

1. Douglas Burden, Richard. Faires.Análisis Numérico. 2002.

2. Curtis F. Gerald.Análisis numérico. Segunda edición, 1991.

3. Discretización: https://www.youtube.com/watch?v=Jv0RwYKxQ10&list=PL9UvoBWjA8MCaEAYIaA9mDmXVA1fYJ4b7&index=7