4.1 Polinomio de Lagrange

Monday, March 25, 2019 11:20 AM

Interpolación con Incrementos Variables

- Interpolación { Obtención de nuevos puntos partiendo de un conjunto discreto de puntos

- Expresión Matemática: $$Y = \sum_{i=0}^{n} \left[\prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_i)}{(x_i - x_j)} \right] Y_i$$

$y = P(x)$	x
-1.472	$.2$
$-.656$	$.4$
$.496$	$.6$

// En este caso $x \rightarrow y$

$\leftarrow x_0 - y_0$

$\leftarrow x_1 - y_1$

$\leftarrow x_2 - y_2$

$$y = \frac{(0 - .656)(0 - .496)}{(-1.472 + .656)(-1.472 - .496)}(.2)$$

$$+$$

$$\left[\frac{(0 + 1.472)(0 - .496)}{(-.656 + 1.472)(-.656 - .496)} \right][.4]$$

$$+$$

$$\left[\frac{(0 + 1.472)(0 + .656)}{(.496 + 1.472)(.656)} \right][.6] \cong .5256$$

4.2 Polinomios Interpolantes

Monday, March 25, 2019 11:24 AM

Interpolación con Incrementos Constantes

x	y
x_0	y_0
x_1	y_1
x_2	y_2
$\vdots$	$\vdots$
x_n	y_n

x	y
x_0	y_0
$x_1 = x_0 + h$	y_1
$x_2 = x_1 + h$	y_2
$x_3 = x_2 + h$	y_3
$\vdots$	$\vdots$
$x_n = x_{n-1} + h$	y_n

$h \in \mathbb{R}$

x	y	Δy	$\Delta^2 y$
x_0	y_0	—	—
$x_1 = x_0 + h$	y_1	$a_1 = y_1 - y_0$	$b_0 = a_2 - a_1$
$x_2 = x_1 + h$	y_2	$a_2 = y_2 - y_1$	$b_1 = a_3 - a_2$
$x_3 = x_2 + h$	y_3	$a_3 = y_3 - y_2$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	
$x_n = x_{n-1} + h$	y_n	$a_n = y_n - y_{n-1}$	$b_n = a_n - a_{n-1}$

$$y_k = y_0 + \binom{k}{1} a_0 + \binom{k}{2} b_0 + \binom{k}{3} c_0 + \dots + \frac{k}{i} = \frac{k!}{(k-1)!\,i}$$

$$= y_0 + k\Delta y_0 + \frac{k(k-1)}{2!}\Delta^2 y_0 +$$

Ejemplo

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
-3	-51	—	—	—
-1	-11	40	—	—
1	-11	0	-40	—
3	-3	8	8	48
5	61	64	56	48
			104	48
				48

2 2 2 2

h

- Polinomio Grado 3

① Calcular valor de y cuando $x = 0.5$

① Calcular valor de y cuando $x = 0.5$

$$K = \frac{X_K - X_0}{h} = \frac{(.5) - (-1)}{2} = .75$$

$$Y_K = Y_0 + \binom{K}{1} a_0 + \binom{K}{2} b_0 + \dots$$

$$\frac{K}{i} = \frac{K!}{((K-i)!)(i!)} = \frac{K(K-1)(K-2)\dots(K-i+1)}{i!}$$

$$Y_K = Y_0 + K\Delta Y_0 + \frac{K(K-1)}{2!}\Delta^2 Y_0 + \frac{K(K-1)(K-2)}{3}\Delta^3 Y_0$$

FORMULAS

$$\rightarrow Y_{x=.5} = -11 + (.75)(0) + \frac{(.75)(.75-1)}{2!}(8)$$

$$+ \frac{(.75)(.75-1)}{3!}(48) = -9.875$$

Falta el -2

$$\rightarrow Y_{x=4} = -3 + (.5)(64) + \frac{(.5)(.5-1)}{2!}(104)$$

$$\rightarrow K = \frac{x_K - x_0}{h} = \frac{(4)-(3)}{2} = .5$$

$$+ \frac{(.5)(.5-1)(.5-2)}{3!}(48) = 19$$

→ Buscando el Polinomio $x_K = x$

$$K = \frac{x-(-1)}{2} = \frac{x+1}{2}$$

$$\therefore Y_{x=x} = -11 + \left(\frac{x+1}{2}\right)(0) + \frac{\left(\frac{x+1}{2}\right)\left(\frac{x+1}{2}-1\right)}{2!}(8)$$

$$+\frac{\left(\frac{x+1}{2}\right)\left(\frac{x+1}{2}-1\right)\left(\frac{x+1}{2}-2\right)}{3!}(48)=$$

$u=\left(\frac{x+1}{2}\right)$

$$=-11+\cancel{(u)(0)}^{\,0}+\cancel{\frac{(u)(u-1)}{2}(8)}^{\,4}$$

$$+\cancel{\frac{(u)(u-1)(u-2)}{6}48}^{\,8}=$$

$$=-11+4u^2-4u+(u^2-u)$$

Desarrollo

$$\begin{array}{r} u-2 \\ \hline u^3-u^2 \\ -2u^2+2u \\ \hline u^3-3u^2+2u \end{array}$$

$$=-11+u^3+u^2-4u$$

$$=-11+\left(\frac{x+1}{2}\right)^3+\left(\frac{x+1}{2}\right)^2-\cancel{4\left(\frac{x+1}{2}\right)}^{\,2}$$

$$=x^3-2x^2-x-9$$

4.3 Derivación numérica

Monday, March 25, 2019 11:25 AM

- Nos sirve para aproximar la derivada en un punto
- Se cuenta con datos equidistantes

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Diferencia progresiva o hacia adelante | $f'(x_j) \cong \dfrac{f(x_j+1) - f(x_j)}{h}$
- Diferencia central | $f'(x_j) \cong \dfrac{f(x_j+1) - f(x_j-1)}{2h}$
- Diferencia regresiva o hacia atrás | $f'(x_j) \cong \dfrac{f(x_j) - f(x_j-1)}{h}$

$$f'(x) = \frac{f(x_i) - f(x_i-1)}{h}$$ ← Regresiva

Derivadas

$$f''(x) = \frac{f(x_i) - 2f(x_i-1) + f(x_i-2)}{h^2}$$

$$f'''(x) = \frac{f(x_i) - 3f(x_i-1) + 3f(x_i-2) - f(x_i-3)}{h^3}$$

▶ Ejercicio: Obtener derivada de $f(x) = \ln x$ en $x = 5$ con un $h = .1$

x_K	$f(x_K)$	Df_K	D^2f_K	D^3f_K

x_k	$f(x_k)$	Df_k	D^2f_k	D^3f_k
4,7	1.5476			
4.8	1.5686			
4,9	1.5892			
→ 5	1.6094			
5.1	1.6292	$[\ln(5.1) - \ln 5]/.1$		
5.2	1.6487	$[\ln 5.2 - \ln 5.1]/.1$	$Df(5.2) - Df(5.1)/.1$	
5.3	1.6677	$[\ln 5.3 - \ln 5.2]/.1$	$Df(5.3) - Df(5.2)/.1$	$D^2f(5.3) - D^2f(5.2)/.1$

- Regresivo
- Progresiv

// Central

x_k	$f(x_k)$	Df_k	D^2f_k	D^3f_k

Esquemas de Derivación

- $f(x) = 2^x$ primera derivada en $x = 1.5$ utilizando diferencias centrales polinomio de segundo grado

a) $h = 1$

x	$f(x) = 2^x$
.5	1.4142
1.5	2.82843
2.5	5.65685

$$Y_1 = \frac{1}{2(1)}\left[-1.4142 + 5.65685\right]$$

$$Y \cong 2.12132$$

2.5	5.65685

$$y_1 \approx 2.12132$$

X	f(x)
1	2
1.5	2.82843
2	4

$$y_1 = \frac{1}{2(.5)}\left[-2 + 4\right]$$

$\therefore$ $h = 1$; $y_1' = 2.12132 + e(1)$

$h = .5$; $y_1' = 2 + e(.5)$

→ Faltaría polinomios (Fórmulas)

Extrapolación

→ Extrapolación de Richarson

$$I = \frac{4}{3} I(h_2) - \frac{1}{3} I(h_3)$$

- Estimar la primera derivada en $x = .5$ de

$$f(x) = .1x^4 - .15x^3 - .5x^2 - .25x + 1.2$$

$$D = \frac{4}{3} D(h_2) - \frac{1}{3} D(h_1) \qquad h_2 = \frac{h_1}{2}$$

//Resolviendo

$$D(h) = \frac{f(x+h) - f(x-h)}{2h} ;$$

$D(h_1)$

- $D(.5) = \dfrac{f(.5+.5) - f(.5-.5)}{2(.5)} = \dfrac{.2 - 1.2}{1} = -1$

$D(h_2)$

$2(.5)$

$D(h_2)$

$$\cdot\ D(.25) = \frac{f(.5+.25) - f(.5-.25)}{2(.25)} = \frac{.6363 - 1.103}{.5} = .9343$$

$$\therefore\ D = \frac{4}{3}(.9343) - \frac{1}{3}(-1)\ ;\ D = -.9125$$

$\cdot\ f(x) = 2^x$ primera derivada en $x = 1.5$

a) $h = .5$ $\qquad \cdot\ h_2 = \frac{h_1}{2}$

$$\cdot\ D(h) = \frac{f(x+h) - f(x-h)}{2} \qquad \cdot\ D = \frac{4}{3}D(h_2) - \frac{1}{3}D(h_1)$$

$h_1 = .5 \qquad h_2 = .25$

1° $$D(.5) = \frac{f(1.5+.5) - f(1.5-.5)}{2(.5)} = 2$$

2° $$D(.25) = \frac{f(1.5+.25) - f(1.5-.25)}{2(.25)} = 1.97034$$

$$\therefore\ D = \frac{4}{3}(1.97034) - \frac{1}{3}(2) = 1.960453$$

Real $= 1.960516$

4.4 Integración numérica

Monday, March 25, 2019 11:26 AM

→ No se tiene una función
→ No tiene antiderivada

Razones por las que se usa

Método de la regla del trapecio

- $\int_1^2 \frac{1}{x^2}\,dx$, $n=5$
- $h = \frac{2-1}{5} = .2$

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + f(x_5)\right]$$

i	x_i	$f(x_i)$	Coef	$f(x_i)\cdot$coef
0	1	1	1	1
1	1.2	.64	2	1.388
2	1.4	.51	2	1.0204
3	1.6	.39	2	.7812
4	1.8	.3	2	.6172
5	2	.25	1	.25

→ $\Sigma = \frac{5.057}{9} = .5057$

$\therefore \int f(x)\,dx = .5057$

Método de Simpson

$\int_{-1}^{1} \sqrt[3]{1+\cos x}\,dx$; $n=4$; $h = \frac{2-1}{4} = .25$ → ≈ 1.01

i	x_i	$f(x_i)$	Coef	$f(x_i)\cdot$coef
0	1	.5487	1	1.1547
1	1.25	1.0957	4	4.382
2	1.5	1.023	2	2.046
3	1.75	.9357	4	3.74

$x_i = a + h\cdot i$

2	1.5	1.063	2	2.046
3	1.75	.9357	4	3.74
4	2	.8358	1	.8358

$\sum_n = 2.43224$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_5)\right]$$

$$\frac{.25}{3}\left(\quad\right) =$$

Cuadratura Gaussiano

$$\int_{-.8}^{1.5} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\,dx$$

$$z = \frac{2x - (a+b)}{b-a}$$

$$x = \frac{(b-a)}{2} z + \frac{(a+b)}{2}$$

$$dx = \frac{(b-a)}{2}\,dz$$

- $z(-.8) = \frac{2(-.8) - (.7)}{2.3} = -1$
- $z(1.5) = \frac{2(1.5) - (.7)}{2.3} = 1$

$$I = \frac{1}{\sqrt{2\pi}} \int_{-.8}^{1.5} e^{-\frac{1}{2}x^2}\,dx =$$

$$x = \left[\left(\frac{1.5 - (-.8)}{2}\right) z + \frac{1.5 + (-.8)}{2}\right]$$

$$dx = \left[\frac{1.5 - (-.8)}{2}\right] dz$$

$$\therefore I = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} \left[\frac{1.5 - (-.8)}{2}\right] e^{-\frac{1}{2}\left[\frac{1.5-(-.8)}{2} z + \frac{1.5 - .8}{2}\right]^2} dz$$

$$= \frac{2.3}{2\sqrt{2\pi}} \int_{-1}^{1} e^{-\frac{1}{8}(2.3z + .7)^2}\,dz$$

$$\int_a^b f(z)\,dz = W_1 f(z_1) + W_2 f(z_2) + \dots + W_n f(z_n)$$

VALORES TABLA

$W_1 = W_2 = 1 \qquad -z_1 = z_2 = .57773$

$f(z_1) = f(-.57773) = e^{-\frac{1}{8}[(2.3)(-.57773) + .7]^2} = .9519112$

$f(z_2) = f(.57773) = e^{-\frac{1}{8}[(2.3)(.57773) + .7]^2} = .5980$

$$f(z_2) = f(.57773) = e^{-\frac{1}{8}[(2.3)(.57773)+.7]^2} = .5980$$

$$\int_a^b f(z)\,dz = \frac{2.3}{2\sqrt{2\pi}}\left[(1)(.95191/2) + (-1)(.5980)\right] = .711105$$

→ Simpson 1/3 | $\int_{x_i}^{x_f} f(x)\,dx = \frac{h}{3}\left[f(x_0) + f(x_n) + 4\sum \text{ord. con índice par} + 2\sum \text{ord. con índice impar}\right]$

→ Simpson 3/8 | $\int_{x_i}^{x_f} f(x)\,dx = \frac{3h}{8}\left[f(x_0) + f(x_n) + 2\sum \text{ord. con índice múltiplo 3} + 3\sum \text{otra ord.}\right]$

→ Ejercicio de clase

$$\int_{-.8}^{1.5} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\,dx \qquad n=4 \qquad h = \frac{1.5-(-.8)}{4} = .575$$

i	x_i	$f(x_i)$	Coeficientes	$f(x_i)\cdot$ Coeficientes
0	1	.2419	1	.2419
1	1.575	.11541	2	.23082
2	2.15	.03955	4	.1582
3	2.725	9.73×10^{-3}	2	.01946
4	3.3	1.72256×10^{-3}	4	6.89024×10^{-3}
5	3.875	2.1893×10^{-4}	1	2.1893×10^{-4}

$$\Sigma = .65718917$$

5.1 Método de la Serie de Taylor

Monday, March 25, 2019 11:26 AM

→ Ecuaciones Diferenciales de 1° Orden

- $f(x) = f(a) + \frac{f'(a)}{1!}(x-a)^1 + \frac{f''(a)(x-a)^2}{2!} + \dots$

→ Ejemplo 1

- Serie de Taylor en orden 2 para obtener $y(1)$ con la ecuación $y' = \cos(xy)$, con $h = .05$ y condición inicial: $y(0) = 1$

$x_0 = 0$

$y_0 = 1$

$h = .5$

$$y_{n+1} = y_n + h\cos(x_n y_n) + \frac{h^2}{2}\left(-\operatorname{sen}(x_n y_n)\right)\left(y_n + \cos(x_n y_n)\right)$$

$$\therefore y_1 = 1 + \underbrace{(.5)\cos(0)}_{.5} + \underbrace{\frac{(.5)^2}{2}\left(-\operatorname{sen}(0)\right)\left(1 + \cos 0\right)}_{0}$$

$$y_1 = 1.5$$

$$\therefore y_2 = 1.5 + (.5)\cos[(.5)(1.5)] + \frac{(.5)^2}{2}\left[-\operatorname{sen}[(.5)(1.5)]\right]\left[1.5 + \cos[(1.5)(.5)]\right]$$

$$= 1.67564$$

	x	y	
x_0	0	1	← y_0
x_1	.5	1.5	← y_1
x_2	1.0	1.6764	← y_2

→ Ejemplo 2

$$y(x_{n+1}) \simeq y_{n+1} = y_n + y'(x_n)h + \frac{1}{2}y''(x_n)h^2$$

DATOS

$y' = x + y$ $\quad x_0 = 0$, $y_0 = 0$ $\quad h = .1$

$$\therefore y_{n+1} = y_n + h(x_n + y_n) + \frac{h^2}{2}(x_n + y_n + \dots)$$

$$\therefore Y_{i+1} = Y_i + h\,(x_i + Y_i) + \frac{h^2}{2}\,(x_i + Y_i + 1)$$

$$Y_1 = 1 + (.1)(0+1) + \frac{(.1)^2}{2}\,(0+1+1) = 1.11$$

$$Y_2 = 1.11 + (.1)(.1+1) + \frac{(.1)^2}{2}\,(.1 + 1.11 + 1) = 1.24205$$

$$Y_3 = 1.24205 + (.1)(1.24205 + .2) + \frac{(.1)^2}{2}\,(.2 + 1.24205 + 1) = 1.34847$$

x debe incrementar respecto a "h"

5.2 Método de Euler modificado

Monday, March 25, 2019 11:27 AM

- Ecuación Diferencial Ordinarias
- Valores Iniciales

Ejercicio 1

$$\frac{dx}{dy} = x + \frac{y}{5}$$

- $y(0) = -3$
- $h = .2$
- Intervalo $[0, 1]$

n	x_n	y_n
0	0	-3
1	.2	$-3{,}12$
2	.4	$-3{,}2048$
3	.6	
4	.8	
5	1.0	

$$y_1 = -3 + (.2)\left(0 + \frac{-3}{5}\right) = -3.12$$

$$y_2 = -3{,}12 + (.2)\left(.2 + \frac{-3.12}{5}\right) = -3.2048$$

$$y_3 = -3{,}48 + (.2)\left(.4 + \frac{-3.48}{5}\right) =$$

// Euler modificado

$$y_1 = y_0 + \frac{h}{2}\left(f(x_0, y_0)\right) + f(x_1, y_1))$$

$$y_1 = -3 + \frac{.2}{2}\left(\left(0 + \frac{-3}{5}\right) + \left(.2 + \frac{-3.12}{5}\right)\right) = -3.1024$$

$$u_2 = -3.1024 + (.2)\left(.2 + \frac{-3.1024}{5}\right) = -3.1865$$

Ejercicio 2

$xy' + y = 2x; \quad y' = 2 - \frac{y}{x}$ | $y(1) = 0$ | Intervalo $[1, 1.5]$ | $h = .1$

▸ Expresiones. Euler M.

- $x_{n+1} = x_n + h$
- $u_{n+1} = y_n + h \cdot f(x_n, y_n)$
- $y_{n+1} = y_n + \frac{h}{2}\left(f(x_n, y_n) + f(x_{n+1}, u_{n+1})\right)$

n	x_n	y_n	u_n
0	1	0	—
1	1.1	.1909	.2
2	1.2		
3	1.3		
4	1.4		
5	1.5		

5.3 Método de Runge - Kuta

Monday, March 25, 2019 11:28 AM

Runge - Kuta 2° y 4° Orden

1] Aplicar el método de Runge-Kuta de 4° Orden al problema de valor inicial

- $y' = x + y$
- $y(0) = 1$
- $H = .5$

// Expresiones Runge-Kuta 4°

- $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)(h)$
- $k_1 = f(x_i, y_i)$
- $k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h)$
- $k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h)$
- $k_4 = f(x_i + h, y_i + k_3 h)$

// Expresiones Runge-Kuta 2°

- $y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$
- $k_1 = (h) f(x_n, y_n)$

  $k_2 = (h) f(x_{n+1}, y_n + k_1)$

5.4 Aproximaciones Sistema de Ecuaciones Diferenciales

Monday, March 25, 2019 11:32 AM

// Forma para resolver

- $x' = f(t, x, y)$ | $x(t_0) = x_0$
- $y' = g(t, x, y)$ | $y(t_0) = y_0$

Ejemplo 1]

$x'(t) = xy + t$ | $x(0) = 1$ | $t = .1$

$y'(t) = ty + x$ | $y(0) = -1$ | $h = .1$

$t_0 = 0$

$$x_{k+1} = x_k + x'_k h + x''_k \frac{h^2}{2}$$

$$y_{k+1} = y_k + y'_k h + y''_k \frac{h^2}{2}$$

$x'_0 = (1)(-1) + 0 = -1$

$x''(t) = xy' + x'y + 1 =$

$= x(ty + x) + (xy + t)y + 1$

$= txy + x^2 + xy^2 + ty + 1$

$= (1)(0)(-1) + (1)^2 + (1)(-1)^2 + (0)(-1) + 1 = 3$

$y'_0 = (0)(-1) + 1 = 1$

$y'' = ty' + y + x' =$

$= t(ty + x) + y + xy + t =$

$= t^2 y + \cancel{tx} + y + xy + \cancel{t} =$

$= (-1) + (1) - 1 = -2$

$x_1 = x_0 + x'_0 h + x_0 \frac{h^2}{2} =$

$= 1 + (-1)(.1) + 3\left(\frac{.1^2}{2}\right) = .915$

$y_1 = y_1 + y'_1 h + y_0 \frac{h^2}{2} =$

$= -1 + (1)(.1) - 2\frac{(.1)^2}{2} = -.91$

$= 1 + (-1)(.1) + 3\left(\frac{.1^2}{2}\right) = .915$

$\therefore x(0.1) = .915$

$= -1 + (1)(.1) - 2\,\frac{(.1)^2}{2} = -.91$

$y(.1) = -.91$

Método de Euler Mejorado

$x'(t) = f(t, x, y) = xy + t \quad | \quad x(0) = 0 \quad | \quad t = .4$

$y'(t) = f(t, x, y) = x - t \quad | \quad y(0) = 1 \quad | \quad h = .2$

// Expresiones

$$x^*_{n+1} = x_n + (h)\, f(t_n, x_n, y_n)$$

$$y^*_{n+1} = y_n + (h)\, f(t_n, x_n, y_n)$$

$$x_{n+1} = x_n + \left(\frac{h}{2}\right) f(t_n, x_n, y_n) + f(t_{n+1}, x^*_{n+1}, y^*_{n+1})$$

$$y_{n+1} = y_n + \left(\frac{h}{2}\right) g(t_n, x_n, y_n) + g(t_{n+1}, x^*_{n+1}, y^*_{n+1})$$

$x_1^* = 0 + (.2)[(0)(1) + 0] = 0$

$y_1^* = 1 + (.2)(0 - 0) = 1$

$x_1 = 0 + \frac{(.2)}{2}(0 + .2) = .02$

$y_1 = 1 + \left(\frac{.2}{2}\right)(0 + (-.2)) = .98$

$x_2^* = (.02) + (.2)[(.02)(.98) + 0.2] = .06392$

$y_2^* = (.98) + (.2)[.02 - .2] = .944$

$$x_2 = .02 + \left(\frac{.2}{2}\right)\left((.02)(.98) + .2 + (.06392)(.944)\right) + .4 =$$

$$= 0.08799$$

$$y_2 = .98 + \left(\frac{.2}{2}\right)\left(.02 - .2\right) + \left(.06392 - .4\right) =$$

$$= 0.928$$

$$t = .4$$

Método Euler

Expresiones

$$x_{k+1} = x_k + h x'_k$$

$$y_{k+1} = y_k + h y'_k$$

- Resolver

$$y'' + yy' - y^2 = t + 1 \quad \Big| \quad y(0) = 0, \; y'(0) = 1 \quad \Big| \quad h = .5, \; y(1) = ¿?$$

// Cambio de variable

$$y' = x$$

$$x' = t + 1 - yy' + y^2$$

$$x_1 = 1 + (.5)\, x'(0) = 1 + (.5)\left(0 + 1 - (0)(1) + (0)^2\right) = 1.5$$

$$y_1 = 0 + (.5)(1) = .5$$

Método de Runge-Kutta

$$x_{k+1} = x_k + \frac{h}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$
$$y_{k+1} = y_k = \frac{h}{6}(m_1 + 2m_2 + 2m_3 + m_4)$$

Expresiones de las ecuaciones

$$l_1 = x' = f(t_k, x_k, y_k)$$
$$l_2 = f\left(t_k + \frac{h}{2}, x_k + l_1\frac{h}{2}, y_k + l_1\frac{h}{2}\right)$$
$$l_3 = f\left(t_k + \frac{h}{2}, x_k + l_2\frac{h}{2}, y_k + l_2\frac{h}{2}\right)$$
$$l_4 = f(t_k + h, x_k + l_3 h, y_k + l_3 h)$$

Coeficientes delos sistemas.

5.5_Diferencias finitas

lunes, 29 de abril de 2019 12:38 p. m.

Diferencia Númerica → Diferencias finitas

* Aproximaciones mediante polinomios

Aplicar ecuación discretizada ⟹ Sistema de ecuaciones ⟹ Resultado aproximado

Pasos a realizar

Tipos de Diferencias → Se desarrollan de la Serie de Taylor

1) Hacia Adelante
2) Hacia Atrás
3) Diferencias Centrales

1] 2° Derivada

//Datos

- $f(x) = e^{2x}$
- $h = .1$
- $x_0 = 1.1$
- $f''(x) = 36.1$

→ D.F.P.

$f(x_0) = f(1.1) = 9.025$

$f(x_0+h) = f(1.1+.1) = 11.023$

$f(x_0+2h) = f(1.1+(2)(.1)) = 13.463$

$f(x_0+3h) = 16.445$

//Progresiva de 2 y 3 puntos

$$f''(x_0) = \frac{f(x_0) - 2f(x_0+h) + f(x_0+2h)}{h^2}$$

$$f''(x_0) = \frac{2f(x_0) - 5f(x_0+h) + 4f(x_0+2h) - f(x_0+3h)}{h^2}$$

→ Diferencias Finitas Regresivas

$$f''(x_0) = \frac{f(x_0-h) - (2)f(x_0) + f(x_0+h)}{h^2}$$

2] Ecuaciones de Orden Superior

- $\frac{\partial^2 y}{\partial x^2} = \frac{y_{i-1} - 2y_i + y_{i+1}}{\Delta x^2}$
- $\frac{\partial y}{\partial x} = \frac{y_{i+1} - y_{i-1}}{\Delta x}$

//Ejercicio

$$\cancel{\frac{\partial^2 y}{\partial x^2}} \rightarrow y'' + 4y = 8\cos(2x)$$

$y(0) = 0$

$y(10) = -18.26$

$$\frac{y_{i-1} - 2y_i + y_{i+1}}{\Delta x^2} + 4y = 8\cos(2x)$$

//Datos: $x\ [0, 10]$ $\Delta x = 2$ $y_0 = 0$

① $i = 1$ $x = 2$

$$\cancel{y_0}^{\,0} + (4 \cdot 2^2 - 2)\, y_1 + y_2 = 8 \cdot (2)^2 \cos(2 \cdot 2)$$

$$\therefore 14\, y_1 + y_2 = 20.9166$$

② $i = 2$ $x = 4$

$$y_1 + (4 \cdot 2^2 - 2)\, y_2 + y_3 = (8)(2)^2 \cos(2 \cdot 4)$$

$$\therefore y_1 + 14 y_3 + y_4 = -4.6560$$

$\therefore y_1 + 14y_3 + y_4 = -4.6560$

③ $y_2 + 14y_3 + y_4 = -27.003$

④ $y_3 + 14y_2 = -12.3862$

⑤ $y_5 = -18.26$ // Es directa

// Sistema de ecuaciones

$y_0 = 0$ $y_1 = -1.4874$ $y_2 = -.0929$ $y_3 = -1.8683$

$y_4 = -0.7513$ $y_5 = -18.26$

5.6 El problema de valores en la frontera

viernes, 3 de mayo de 2019 12:14 p. m.

→ Método del disparo

$$y'' = f(t, y, y'') \; ; \; y(a) = \alpha \qquad y(b) = \beta$$

Expresión (Secante) $\quad x_{k+1} = x_k - \dfrac{E(x_k)(x_k - x_{k-1})}{E(x_k) - E(x_{k-1})}$

// Ejemplo

- $y'' = -\frac{2}{t}(y' - 2) \; ; \; y(\frac{1}{2}) = 3 \qquad y(1) = 3$

- $E(x) = Y_x(b) - \beta = Y_x(1) - 3 =$

// $Y_x(t)$ es solución del PVI asociado

$$y'' = -\frac{2}{t}(y' - 2) \qquad y(\tfrac{1}{2}) = 3 \qquad y'(\tfrac{1}{2}) = x$$

// Proponiendo valores de la pendiente

$x = 0 \rightarrow Y_x(1) = 3.5 \; ; \; E(0) = 3.5 - 3 = \frac{1}{2}$

$x = -1 \rightarrow Y_x(1) = 3.25 \; ; \; E(-1) = 3.25 - 3 = \frac{1}{4}$

// Método de la secante

$$x_1 = \frac{x_0 - E(x_0)(x_0 - x_{-1}}{E(x_0) - E(x_{-1})} =$$

$$= -1 - \frac{1}{4}(-1 - 0)$$

$$= -1 - \frac{\frac{1}{4}(-1-0)}{\frac{1}{4}-\frac{1}{2}} = \underline{-2}$$

//Al resolver PVI con $\alpha - x_1 = \underline{-2}$

$x = -2 \longrightarrow y_x(1) = 3$; $E(-2) = \underline{0}$

$$y(t) \longrightarrow y_x(t) = 2t - \frac{1}{t}\left(\frac{2-x}{4}\right) + \left(\frac{2+x}{4}\right)$$

6.1 Clasificación de Ecuaciones derivadas parciales

viernes, 3 de mayo de 2019 12:16 p. m.

- Ecuaciones Lineales
  - Exponente 1
  - Variables dependientes (sumas o restas)

$\frac{\partial^2 u}{\partial x^2} + xu = 0$ // Lineal

$\frac{\partial^2 u}{\partial x^2} + uu = 0$ // No Lineal

- Ecuación Homogénea: Exponentes iguales en las variables

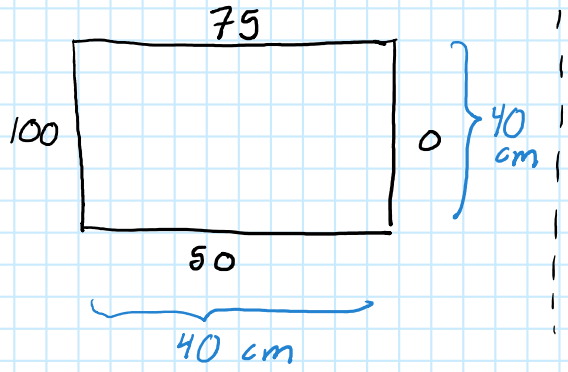
- Clasificación de las EDP

EDP Lineal 2° Orden

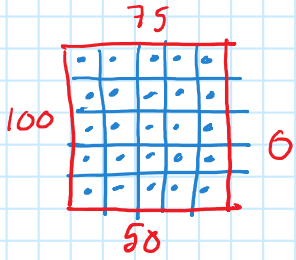
- Elíptica $\Rightarrow B^2 - 4AC < 0$
- Hiperbólica $\Rightarrow B^2 - 4AC > 0$
- Parabólicas $\Rightarrow B^2 - 4AC = 0$

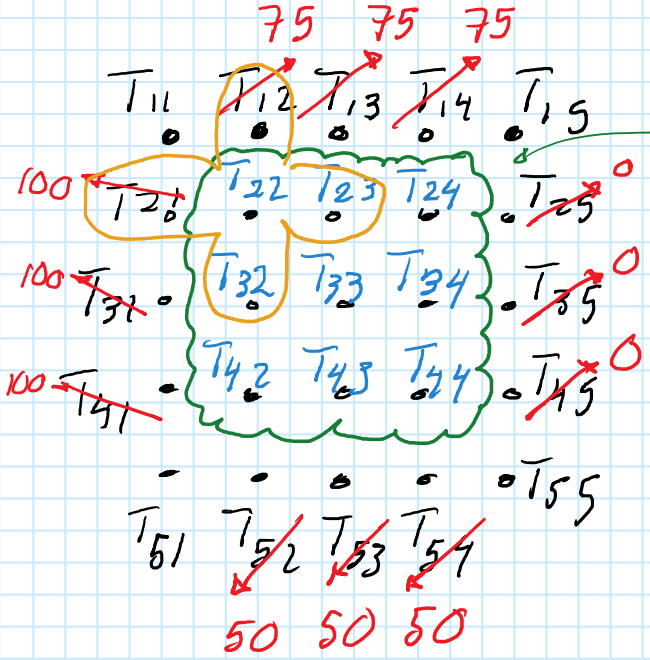
6.2 Ecuaciones Diferenciales de derivadas parciales - diferencias finitas

viernes, 3 de mayo de 2019 12:17 p. m.



$\Delta x = \Delta y = h = 10$ cm





Valores desconocidos

//Retomando la Ecuación de Laplace discretizada

$$-4T_{ij} + T_{i+1,j} + T_{i-1,j} + T_{i,j+1} + T_{i,j-1} = 0$$

$$-4T_{22} + T_{32} + T_{23} + 100 + 75 \; 0$$

175

$$-4T_{23} + T_{22} + T_{24} + T_{33} + 75 = 0$$

$$-4T_{24} + T_{23} + T_{34} + 75 = 0$$

$$-4T_{32} + T_{42} + T_{33} + T_{22} + 100 = 0$$

$$-4T_{33} + T_{23} + T_{34} + T_{43} + T_{32} = 0$$

$$-4T_{42} + T_{32} + T_{43} + \overset{150}{\cancel{100 + 50}} = 0$$

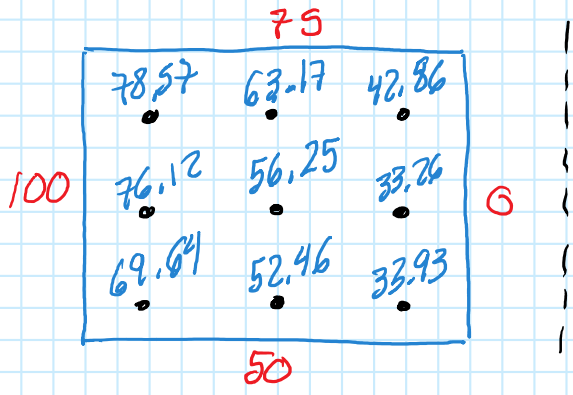
$$-4T_{43} + T_{33} + T_{42} + T_{44} + 50 = 0$$

$$-4T_{44} + T_{34} + T_{43} + 50 = 0$$

//Simplificando en forma matricial

$$\begin{array}{c}
\begin{array}{ccccccccc} T_{22} & T_{23} & T_{24} & T_{32} & T_{33} & T_{34} & T_{42} & T_{43} & T_{44} \end{array} \\
\begin{bmatrix}
-4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & -4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & -4 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & -4 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & -4 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & -4 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & -4 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -4
\end{bmatrix}
\end{array}
\begin{bmatrix} T_{22} \\ T_{23} \\ T_{24} \\ T_{32} \\ T_{33} \\ T_{34} \\ T_{42} \\ T_{43} \\ T_{44} \end{bmatrix}
=
\begin{bmatrix} -175 \\ -75 \\ -75 \\ -100 \\ 0 \\ 0 \\ -150 \\ -50 \\ -50 \end{bmatrix}$$

//Resultado



6.3 Solución de Ecuaciones Diferenciales - método diferencias finitas

viernes, 3 de mayo de 2019 12:19 p. m.

SERIE

lunes, 6 de mayo de 2019 12:35 p. m.