Introducción

Tuesday, January 29, 2019 6:59 PM

Exámenes

- 19 marzo - 1° Parcial | Series Asociadas (5) → Ejercicios Impares
  ↳ Tema 1

- 23 mayo - 2° Parcial | Series Asociados (3) → Ejercicios Impares
  ↳ Tema 2 y 3

Calificación

- Promedio Exámenes Parciales } 7 como mínimo
- Puntos extras

1] Libros → 1 punto

① Libro 1: 22 marzo - 23:59

② Libro 2: 12 abril - 23:59

* Opinion de 3 cuartillas (per book)
* Plagio invalido
* Citas estilo APA

Formato

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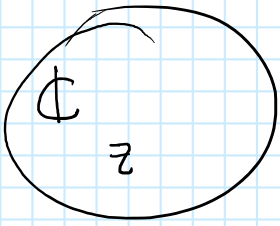
2] Ejercicios en clase o tareas → 1 punto

- 5 ejercicios (Al menos)
- Fecha de aplicación al azar
- Son del tema revisado

R. Num. Complejos

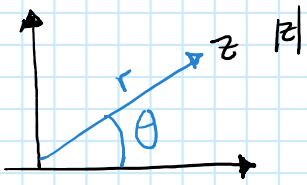
Thursday, January 31, 2019 7:09 PM



$z = Re(z) + Im(z)$

$z = x + yi$

- Geométrica



$r = |z| = \sqrt{x^2+y^2}$

$\theta = Arg\, z = \tan^{-1}\left(\frac{x}{y}\right)$

Formas

- binómica / binomial / afijo

$z = a + bi = x + yi$

- polar $\{ z = r\,cis\,\theta$
- trigonométrica $\begin{cases} z = r\,cis\,\theta = \\ = (r\cos\theta + i\,sen\,\theta) \end{cases}$
- exponencial $= re^{i\theta}$

Ejercicios (1)

a) $|z+3i| + |z-3i| = 10$

para todo $z \in \mathbb{C}$, $z = x+yi$

$$|(x+yi)+3i| + |(x+yi)-3i| = 10$$

$$|x + i(y+3)| + |x + i(y-3)| = 10$$

magnitud

$$\sqrt{x^2+(y+3)^2} + \sqrt{x^2+(y-3)^2} = 10$$

$$x^2 + (y+3)^2 = \left[10 - \sqrt{x^2+(y-3)^2}\right]^2$$

$$x^2 + (y+3)^2 = 100 - 20\sqrt{x^2+(y-3)^2} + \left[x^2+(y-3)^2\right]$$

$$\cancel{x^2} + \cancel{y^2} + 6y + \cancel{9} = 100 - 20\sqrt{x^2+(y-3)^2} + \cancel{x^2} + \cancel{y^2} - 6y + \cancel{9}$$

$$12y = 100 - 20\sqrt{x^2+(y-3)^2}$$

$$\therefore\ 6y = 50 - 10\sqrt{x^2+(y-3)^2}$$

$$6y - 50 = -10\sqrt{x^2+(y-3)^2}$$

$$(6y-50)^2 = \left[-10\sqrt{x^2+(y-3)^2}\right]^2$$

$$(6y-50)^2 = 100\left(x^2+(y-3)^2\right) \qquad \left(\tfrac{1}{2}\right)^2 = \tfrac{1}{4}$$

$$(3y-25)^2 = 25\left(x^2+(y-3)^2\right)$$

$$9y^2 - 150y + 625 = 25x^2 + 25(y^2 - 6y + 9)$$

$$9y^2 \cancel{-150y} + 625 = 25x^2 + 25y^2 \cancel{-150y} + 225$$

$$25x^2 + 16y^2 = 400 \ ; \quad \frac{x^2}{16} + \frac{y^2}{25} = 1$$

b) $9z^2 + 9\bar{z}^2 + 82z\bar{z} = 1600$

$$9(x+yi)^2 + 9(x-yi)^2 + 82(x+yi)(x-yi) = 1600$$

$$9(x^2 + 2xyi + yi^2) + 9(x^2 - 2xyi - y^2 + 82(x^2+y^2) = 1600$$

$i^2 = -1$

$$\therefore\ 9x^2 + \cancel{18xyi} - 9y^2 + 9x^2 \cancel{-18xyi} - 9y^2 + 82(x^2+y^2) = 1600$$

$$100x^2 + 64y^2 = 1600 \ ; \quad \frac{x^2}{16} + \frac{y^2}{25} = 1$$ //Circunferencia

c) $Re(z+i)^2 = 1$ //Parte real $\qquad i^2 = -1$

$$Re\left[z^2 + 2zi - 1\right] = 1$$

$$Re\left[(x+yi)^2 + 2(x+yi)i - 1\right] = 1$$

$$Re\left[x^2 + 2xyi - y^2 + 2xi - 2y - 1\right] = 1$$

Parte real //

$$x^2 - y^2 - 2y - 1 = 1$$

$$x^2 - (y^2 + 2y + 1) = 1$$

$$x^2 - (y+1)^2 = 1$$ // Hipérbola

NOTA

$z = x + yi$

$z = 0 + yi$; Imaginario puro

$z = x + 0i$; Real puro

$z = x + yi$

$\bar{z} = x - yi$

- $z + \bar{z} = (x + yi) + (x - yi)$

$$z + \bar{z} = 2x$$

$$\therefore x = \frac{z + \bar{z}}{2} \quad //$$

- $z - \bar{z} = (x + yi) - (x - yi)$

$$y = \frac{z - \bar{z}}{2i} \quad //$$

Ejercicio (2)

1] $25x^2 + 16y^2 = 400$

$$25\left(\frac{z+\bar{z}}{2}\right)^2 + 16\left(\frac{z-\bar{z}}{2i}\right)^2 = 400$$

$$25\left(\frac{z+\bar{z}}{2}\right)^2 + 16\left(\frac{z-\bar{z}}{2i}\right)^2 = 400$$

$$25\,\frac{(z+\bar{z})^2}{2^2} + 16\,\frac{(z-\bar{z})^2}{(2i)^2} = 400$$

$$25\,\frac{(z+\bar{z})^2}{4} + 16\,\frac{(z-\bar{z})^2}{-4} = 400$$

$$25\,(z+\bar{z})^2 - 16\,(z-\bar{z})^2 = 1600$$

$$25\,(z^2 + 2z\bar{z} + \bar{z}^2) - 16\,(z^2 - 2z\bar{z} + \bar{z}^2) = 1600$$

$$\therefore\ 9z^2 + 82z\bar{z} + 9\bar{z}^2 = 1600$$

2] $x - y = 1$

$$\frac{z+\bar{z}}{2} - \frac{z-\bar{z}}{2i} = 1\ ;$$

$$i(z+\bar{z}) - (z-\bar{z}) = 2i \quad \Big)\ 2i$$

$$(z+\bar{z}) + i(z-\bar{z}) = 2 \quad \Big)\ -i$$

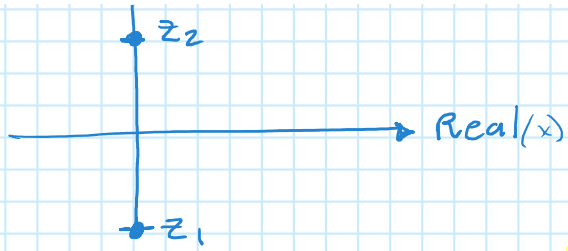
$$z(1+i) + \bar{z}(1-i) = 2$$

$$i^2 = -1$$

▶ Obtener ecuación e identificar lugar geométrico de los puntos z tales que el doble de su distancia al punto $z_1 = -3i$ es igual al punto $z_2 = 3i$

$$z_1 = x + yi = -3i = (\overset{x}{0}, \overset{y}{-3})$$

$$z_2 = x + yi = 3i = (0, 3)$$



$z_2 = x + yi = 3i = (0, 3)$

$|z - z_0| \triangleq$ Ecuación Distancia Núm. Complejos

$\therefore \quad 2|z - z_1| = |z - z_2|$

$2|z + 3i| = |z - 3i|$

$2|(x+yi) + 3i| = |(x+yi) - 3i|$

$2|x + i(y+3)| = |x + i(y-3)|$ Quitamos i

$2\sqrt{x^2 + (y+3)^2} = \sqrt{x^2 + (y-3)^2}$

$4\left(x^2 + (y+3)^2\right) = x^2 + (y-3)^2$

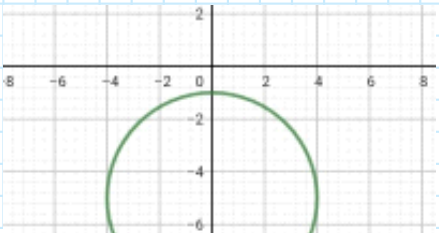
$4x^2 + 4y^2 + 24y + 36 = x^2 + y^2 - 6y + 9$

$3x^2 + 3y^2 + 30y + 27 = 0$

$x^2 + y^2 + 10y = -9$

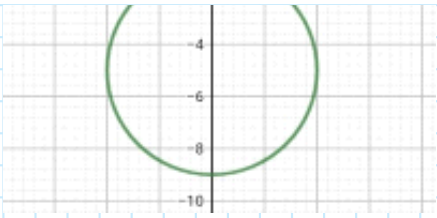
$x^2 + y^2 + 10y + 25 = 16$

$x^2 + (y+5)^2 = 16$ // Circunferencia



Conclusión

- $C(0, -5)$
- $r = 4$



- $C(0,-3)$
- $r = 4$

Fórmula de Moivre

$z = r\cos\theta + i\,r\,\text{sen}\,\theta$

- $\text{Re}(z) = x = r\cos\theta$
- $\text{Im}(z) = y = r\,\text{sen}\,\theta$

$[\cos\theta + i\,\text{sen}\,\theta]^n = \cos n\theta + i\,\text{sen}(n\theta)$

$(\cos\theta + i\,\text{sen}\,\theta)^4 = \cos 4\theta + i\,\text{sen}\,4\theta$

$(\cos\theta + i\,\text{sen}\,\theta)^2\,(\cos\theta + i\,\text{sen}\,\theta)^2 = \cos 4\theta + i\,\text{sen}\,4\theta$

$(\cos^2\theta + 2\cos\theta\, i\,\text{sen}\,\theta - \text{sen}^2\theta)$

$(\cos^2\theta + 2\cos\theta\, i\,\text{sen}\,\theta - \text{sen}^2\theta)$

$\cos^4\theta + 2\cos^3\theta\, i\,\text{sen}\,\theta - \text{sen}^2\theta\cos^2\theta$

$+ 2\cos^3\theta\, i\,\text{sen}\,\theta \cancel{+} 4\cos^2\theta\,\text{sen}^2\theta - 2\cos\theta\, i\,\text{sen}^2\theta$

$- \text{sen}^2\theta\cos^2\theta - 2\cos\theta\, i\,\text{sen}^3\theta + \text{sen}^4\theta =$

Triángulo de Pascal

$(\cos\theta + i\,\text{sen}\,\theta)^4 = \cos 4\theta + i\,\text{sen}\,4\theta$

1

1 1

2 → 1 2 1

3 → 1 3 3 1

4 → 1 4 6 4 1

$$(\cos\theta)^4(i\,\text{sen}\,\theta)^0 + 4(\cos\theta)^3(i\,\text{sen}\,\theta)^1 + 6(\cos\theta)^2(i\,\text{sen}\,\theta)^2$$

$$+ 4(\cos\theta)(i\,\text{sen})^3 + (\cos\theta)^0(i\,\text{sen}\,\theta)^4 = \cancel{*}$$

$$\cos^4\theta + 4\cos^3\theta\, i\,\text{sen}\,\theta - 6\cos^2\theta\,\text{sen}^2\theta - 4\cos\theta\, i\,\text{sen}^3\theta$$

$$+ \text{sen}^4\theta = \cos 4\theta + i\,\text{sen}\,4\theta$$

$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\,\text{sen}^2\theta + \text{sen}^4\theta$ ① Parte real

$\text{sen}\,4\theta = 4\cos^3\theta\,\text{sen}\,\theta - 4\cos\theta\,\text{sen}^3\theta$ ② Parte imaginario

R. Ejercicios

Tuesday, February 5, 2019 7:09 PM

1] Dados los siguientes números

$z_1 = -3+4i \qquad z_2 = 5-2i \qquad z_3 = \frac{3}{2} \qquad z_4 = 7i$

a) $(z_1 - z_2)\, z_3 =$

$= (-3+4i) - (5-2i)\ \frac{3}{2} =$

$= (-8+6i)\frac{3}{2} = -\frac{24}{2} + \frac{18}{2} i = -12+9i$ //

g) $\overline{(z_1+z_2)}^{-1} = (\overline{2+2i})^{-1} = (2-2i)^{-1} =$

$= \frac{1}{2-2i}\ \frac{(2+2i)}{(2+2i)} = \frac{2+2i}{8} = \frac{1}{4} + \frac{1}{4} i$ //

$$\begin{array}{l} 2+2i \\ \hline 4 - 4\cancel{i^2}^{-1} \\ 4+4 \end{array}$$

h) $z_1^2 z_3 = (-3+4i)^2\ \frac{3}{2} = (-7-24i)\frac{3}{2} =$

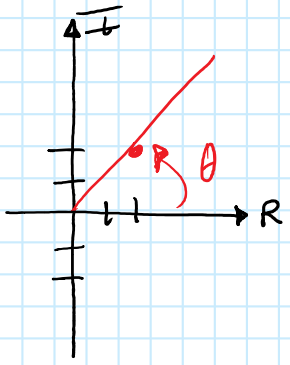
$$\begin{array}{r} -3+4i \\ \hline -12i + 16i^2 \\ +9 - 12i \\ \hline 9 - 24i + \cancel{16i^2}^{-1} \\ -16 \end{array}$$

$= -\frac{21}{2} - 36i$ //

2] Módulo, argumento, forma polar y forma trigonométrica

2] Modulo, argumento, forma polar y forma trigonométrica

a) $2+2i$



- Modulo $\overset{\Delta}{=} \sqrt{(2)^2+(2)^2} = \sqrt{8} = 2\sqrt{2}$

- Argumento $\overset{\Delta}{=} \measuredangle \tan\left(\frac{y}{x}\right) = \frac{\pi}{4} = 45°$

$\therefore$ Polar $\overset{\Delta}{=} 2\sqrt{2}\ \text{cis}(45°)$

- Trigonométrica $\overset{\Delta}{=} 2\sqrt{2}\cos 45° + 2\sqrt{2}\,\text{sen}\ 45°$

3] $\sqrt{3}\left(\cos\left(\frac{\pi}{6}\right) + i\,\text{sen}\left(\frac{\pi}{6}\right)\right) =$

$x = \sqrt{3}\cos\left(\frac{\pi}{6}\right) = \frac{3}{2}$

$y = \sqrt{3}\,\text{sen}\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\therefore\ z = \frac{3}{2} + i\frac{\sqrt{3}}{2}$

Expresar a la función

- $f(z) = z + \frac{1}{z}$, $z \in \mathbb{C}$
- $z = r\,\text{cis}\,\theta$

$$\therefore\ = r\cos\theta + ir\,\text{sen}\,\theta + \underbrace{\frac{1}{r\cos\theta + i\,r\,\text{sen}\,\theta}}$$

$$= \frac{1}{r\cos\theta + i\,r\,\text{sen}\,\theta}\left(\frac{r\cos\theta - i\,r\,\text{sen}\,\theta}{r\cos\theta - i\,\text{sen}\,\theta}\right)$$

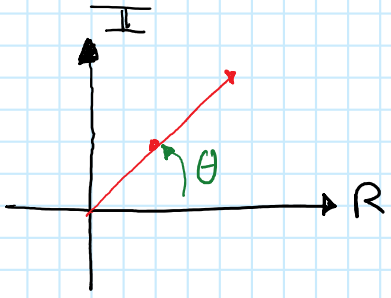
$$= \frac{r\cos\theta - i r \operatorname{sen}\theta}{r^2\cos^2\theta + \cancel{i r^2 \operatorname{sen}\theta\cos\theta} - \cancel{i r^2 \operatorname{sen}\theta\cos\theta} + r^2\operatorname{sen}^2\theta} =$$

$$= \frac{r\cos\theta - i r \operatorname{sen}\theta}{r^2\underbrace{(\cancel{\cos^2\theta + \operatorname{sen}^2\theta})}_{1}} = \frac{r\cos\theta - i r \operatorname{sen}\theta}{r^2}$$

$$= r\cos\theta + i r \operatorname{sen}\theta + \frac{r\cos\theta - i r \operatorname{sen}\theta}{r^2} =$$

$$= \cos\theta\left(r + \frac{1}{r}\right) + i \operatorname{sen}\theta\left(r + \frac{1}{r}\right)$$

▶ $z = 2 + 2i$



- Se tiene que limitar el argumento
- Argumento Principal: $\arg z = (\operatorname{Arg} z + 2k\pi)$

▶ Multiplicación (Form: Trigonométrica)

- $z = z_1 z_2$ $\begin{cases} r = r_1 r_2 \\ \theta = \theta_1 + \theta_2 \end{cases}$

▶ División

- $z = z_1 z_2$ $\begin{cases} r = \frac{r_2}{r_1} \\ \theta = \theta_1 - \theta_2 \end{cases}$

* Checar

F. Compleja

Tuesday, February 5, 2019 7:45 PM

• $F(z) = z^2 \qquad z \in \mathbb{C} \mid z = x + yi$

$$(x+yi)^2 = x^2 + 2ixy - y^2$$

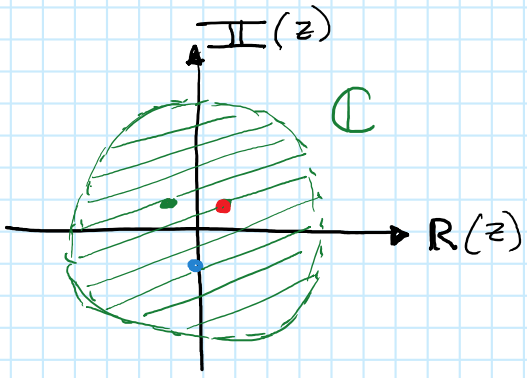
$$\therefore w \begin{cases} \bullet\ u(x,y) = x^2 - y^2 & \text{// Parte real} \\ \bullet\ v(x,y) = 2xy & \text{// Parte imaginaria} \end{cases}$$

$$\therefore F(z) = u(x,y) + iv(x,y)$$
$$= u(r,\theta) + iv(r,\theta)$$

Transformación o Mapeo

i) Determinar y hacer gráfica del dominio

a) $f(z) = \dfrac{1}{z^3 - 8i}$



// Función racional

$$\frac{R(z)}{Q(z)} \mid Q(z) \neq 0$$

$z = r\,\text{cis}\,\theta$

$z = x + yi$

$$z^3 - 8i \neq 0$$

$$D_{f(z)} = \{\mathbb{C} - z_1, z_2, z_3\}$$

$$z^3 = 8i \ ; \ z = \sqrt[3]{8i}$$

$$z = \sqrt[3]{8\,\text{cis}\,90} = \frac{2\,\text{cis}\,90 + 360k}{3}$$

$$z = \sqrt[3]{8 \operatorname{cis} 90} = \frac{2 \operatorname{cis} 90 + 360k}{3}$$

$\therefore$ • $z_1 = 2 \operatorname{cis} 30 = \sqrt{3} + i$

• $z_2 = 2 \operatorname{cis} 150 = \sqrt{3} - i$

• $z_3 = 2 \operatorname{cis} 270 = -i$

b) Escribir la función $f(z)$ en términos de w

$$F(z) = \frac{z+i}{z-i} \qquad w = u(x,y) + iv(x,y)$$

$$z = x + iy$$

$$\therefore F(z) = \frac{x+iy+i}{x+iy-i} = \frac{x+i(y+1)}{x+i(y-1)} \left(\frac{x-i(y-1)}{x-i(y-1)} \right) =$$

$$= \frac{x^2 + (y^2-1) + 2ix}{x^2 + (y-1)^2}$$

$$\begin{array}{l} x + i(y+1) \\ x - i(y-1) \\ \hline x^2 + x\,i(y+1) \\ \quad - x\,i(y-1) - \overset{-1}{\cancel{i^2}}(y+1)(y-1) \\ \hline x^2 + (y^2-1) + \underbrace{x\,i(y+1) - x\,i(y-1)}_{2ix} \end{array}$$

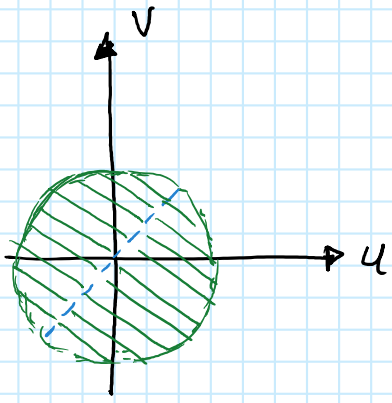
$$\therefore F(z) = \frac{x^2 + (y^2-1)}{x^2 + (y-1)^2} + i \frac{2x}{x^2 + (y-1)^2}$$

b) Obtener dominio

$$g(z) = \sqrt{9 - x^2 - y^2} + i\,\frac{x+y}{x-y}$$

- $u(x,y) = \sqrt{9 - x^2 - y^2} \in \mathbb{R}$
- $v(x,y) = \dfrac{x+y}{x-y} \in \mathbb{C}$

$$\left.\begin{array}{l} \therefore\ 9 - x^2 - y^2 \geq 0\ ;\ -x^2 - y^2 \geq -9\ ; \\ x^2 + y^2 \leq 9 \end{array}\right\} u$$



$$\therefore\ x - y \neq 0\ ;\ x \neq y \Big\} v$$

2] Identificar las familias de curvas

$f(z) = \dfrac{1}{(iz)}$ $\quad u(x,y) = \alpha \quad v(x,y) = \beta$

$\alpha, \beta \in \mathbb{R}$

$z \in \mathbb{C}\ ;\ z = x + yi$

$\rightarrow F(z) = u(x,y) + i\,v(x,y)$

$$\therefore f(z) = \frac{1}{i(x+yi)} = \frac{1}{ix + i^2 y} = \frac{1}{ix - y}$$

$$= \frac{1}{ix - y}\left(\frac{-y - ix}{-y - ix}\right) = \frac{-y - ix}{y^2 + x^2} =$$

$$\frac{-y - ix}{y^2 - \cancel{ixy} + \cancel{ixy} + x^2} = \underbrace{-\frac{y}{y^2+x^2}}_{u(x,y)} \overset{①}{} \underbrace{- i\frac{x}{y^2+x^2}}_{i\,v(x,y)} \overset{②}{}$$

$$\alpha = \frac{-y}{y^2+x^2} \qquad \beta = \frac{-x}{y^2+x^2}$$

$$(y^2+x^2)\alpha = -y$$

$$\alpha y^2 + \alpha x^2 = -y$$

$$\alpha y^2 + y + \alpha x^2 = 0$$

$$\underbrace{y^2 + \frac{y}{\alpha}}_{T.C.P} + x^2 = 0$$

$$\left(y + \frac{1}{2\alpha}\right)^2 + x^2 = \frac{1}{4\alpha^2}$$

$$-x = \beta x^2 + \beta y^2$$

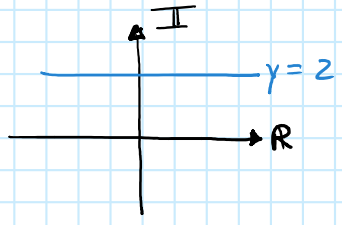
$$0 = \beta x^2 + x + \beta y^2$$

$$0 = x^2 + \frac{x}{\beta} + y^2$$

$$\left(x + \frac{1}{2\beta}\right)^2 + y^2 = \frac{1}{4\beta^2}$$

3] $w = -3y \operatorname{sen} x \cos x + i \cos^2 xy$

- Determinar en que mapea la recta de ecuación $y=2$ bajo dicha función



$$\xrightarrow{y=2}$$

$\rightarrow u(x,y) = -3y \operatorname{sen} x \cos x = -6 \operatorname{sen} x \cos x$

$\rightarrow v(x,y) = \cos^2 xy = \cos^2(2x)$

// Quitando x para tener plano $w(u,v)$

$u = -6 \operatorname{sen} x \cos x = -3 \operatorname{sen} 2x = u$

$w = \cos^2 2x$

if $u^2 = 9 \operatorname{sen}^2 2x$; $\operatorname{sen}^2 2x = \frac{u^2}{9}$

$w = \cos^2 2x$ $\qquad \cos^2 2x = w$

$\therefore \cancel{\operatorname{sen}^2 2x + \cos^2 2x}^{\,1} = \frac{u^2}{9} + w$ //

Tarea Moral*

b) $z_1 z_4 + z_3 z_4 =$

$= (-3+4i)(7i) + \frac{3}{2}(7i) =$

$= -21i + 28i^2 + \frac{21}{2} i$

$= -28 - \frac{21}{2} i$ //

c) $\overline{z_1 + z_4 - 5z_2}$

$(-3+4i) + 7i - 5(5-2i)$

$-3 + 4i + 7i - 25 + 10i$

$-28 + 21i$

$\therefore -28 - 21i$ //

f) $\overline{\overline{z_1 z_2}} = (-3+4i)(5-2i) =$

f) $\overline{\overline{z_1 z_2}} = (-3+4i)(5-2i) =$

$= -15+6i+20i-8i^2$

$= -7+26i$ //

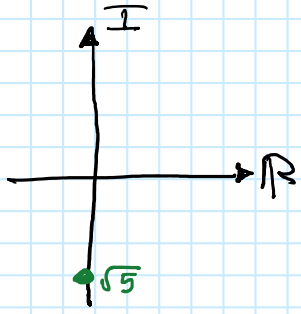
// Modulo, argumento, forma polar y trigonométrica

e) $-\sqrt{5} = -\sqrt{5}$

$|z_e| = \sqrt{-5} = \sqrt{5}$

$\theta = 180^\circ$

$\sqrt{5}\,\text{cis}(180^\circ) = \sqrt{5}\,\text{cis}\left(\frac{\pi}{2}\right)$

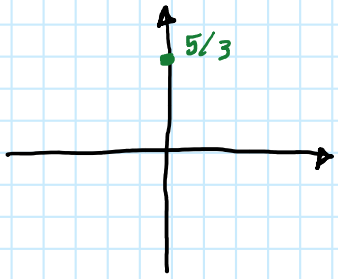


f) $\frac{5}{3}i$

$|z_f| = \sqrt{\frac{25}{9}} = \frac{5}{3}$

$\theta = 90^\circ$

$\therefore\ 5/3\ \text{cis}(\pi/2)$

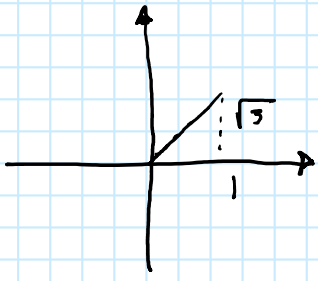


g) $\sqrt{3}+i$

$|z_g| = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{3+1} = 2$

$\theta = \text{ang}\tan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

$\therefore = 2\,\text{cis}(30^\circ) = 2\,\text{cis}\left(\frac{\pi}{6}\right)$



F. Racional, exponencial

Thursday, February 7, 2019 7:07 PM

$r(z) = \frac{P(z)}{Q(z)}$ $\quad D_r = \mathbb{C} - \{z \in \mathbb{C} \mid q(z) = 0\}$

$= \{z \in \mathbb{C} \mid q(z) \neq 0\}$

$P(z)$ y $Q(z)$ son polinomiales

$\operatorname{sen}(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

Función exponencial

$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$e^z = e^{x+iy} = e^x e^{yi}$ | $z \in \mathbb{C}$, $z = x + iy$

$\therefore e^{yi} = 1 + yi + \frac{(yi)^2}{2!} + \frac{(yi)^3}{3!} + \frac{(yi)^4}{4!}$

$= 1 + iy - \frac{y^2}{2!} - \frac{iy^3}{3!} + \frac{y^4}{4!} =$

$= 1 - \frac{y^2}{2!} + \frac{y^4}{4!} + \dots + i\left(y - \frac{y^3}{3!} + \dots\right)$

$\therefore e^{iy} = \cos y + i \operatorname{sen} y$

$e^z = e^{x+yi} = e^x e^{yi} = e^x(\cos y + i \operatorname{sen} y)$

1] Ejercicio

Dada la función $f(z) = e^{-z}$ | Identificar y dibujar las imágenes de las rectas

$x = 0$, $x = 1$, $y = 0$, $y = \frac{\pi}{4}$

$e^{-z} = e^{-(x+yi)} = e^{-x} e^{-yi} = e^{-x}[\cos(-y) + i \operatorname{sen}(-y)] =$

$= e^{-x}[\cos y - i \operatorname{sen} y] =$ // Propiedad trigonométrica

$F(z) = e^{-x} \cos y - i e^{-x} \operatorname{sen} y$

$u = e^{-x} \cos y$ // Reales

$v = -e^{-x} \operatorname{sen} y$ // Imaginarios

$w = u + v$

• **Caso 1** ; si $x = c$

$$u = e^{-c}\cos y \xrightarrow{\wedge 2} u^2 = e^{-2c}\cos^2 y$$
$$v = -e^{-c}\operatorname{sen} y \qquad v^2 = +e^{-2c}\operatorname{sen}^2 y$$

// $w = u + v$

$$u^2 + v^2 = e^{-2c}\cos^2 y + e^{-2c}\operatorname{sen}^2 y$$
$$= e^{-2c}\underbrace{(\cos^2 y + \operatorname{sen}^2 y)}_{1}$$
$$u^2 + v^2 = \underbrace{e^{-2c}}_{\text{es constante}}$$

→ if $c = 0$; $u^2 + v^2 = 1$
→ if $c = 1$; $u^2 + v^2 = e^{-2}$

} Para valores de x

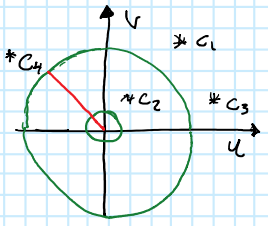
• **Caso 2** ; si $y = c$

$$u = e^{-x}\cos(c)$$
$$v = -e^{-x}\operatorname{sen}(c)$$

$$\frac{v}{u} = \frac{-e^{-x}\operatorname{sen}(c)}{e^{-x}\cos(c)} = -\cancel{e^{-x}}\,\cancel{e^{x}}\,\tan(c) =$$

$$\frac{v}{u} = -\tan(c) \; ; \; v = -\tan(c)\,u$$

→ if $c = 0$; $v = 0$
→ if $c = \frac{\pi}{4}$; $v = -u$



Seno y Coseno Complejos

$$\cos y = \frac{e^{yi} + e^{-yi}}{2} \qquad \operatorname{sen} y = \frac{e^{yi} - e^{-yi}}{2i}$$

1) $\operatorname{sen}^2 z + \cos^2 z = 1$

Demostración

$$\left(\frac{e^{iz} - e^{-iz}}{2i}\right)^2 + \left(\frac{e^{zi} + e^{-zi}}{2}\right)^2 = 1$$

$$\frac{(e^{iz} - e^{-iz})^2}{-4} + \frac{(e^{zi} + e^{-zi})^2}{4} = 1$$

$$\left[\left(e^{iz} - e^{-iz}\right)^2 + \left(e^{iz} + e^{-iz}\right)^2\right] = 1$$

$$\frac{\quad}{-4} + \frac{(\quad + \quad)}{4} = 1$$

$$\frac{1}{4}\left[-\left(e^{iz} - e^{-iz}\right)^2 + \left(e^{iz} + e^{-iz}\right)^2\right] = 1$$

$$\frac{1}{4}\left[-\left(e^{2iz} - 2 + e^{-2iz}\right) + \left(e^{2iz} + 2 + e^{-2iz}\right)\right] = 1$$

$$\frac{1}{4}\left[2 + 2\right] = 1$$

Coseno — Cartesiano

- $\cos(z)$ cuando $f(z) = u(x,y) + iv(x,y)$
- $\cos z = \dfrac{e^{zi} + e^{-zi}}{2}$ $\quad z \in \mathbb{C} \mid z = x + yi$

$$\cos(x+yi) = \frac{e^{(x+yi)i} + e^{-(x+yi)i}}{2} =$$

$$= \frac{e^{ix-y} + e^{-ix-y}}{2} = \frac{e^{-y}e^{xi} + e^{y}e^{-xi}}{2} =$$

$$= \frac{e^{-y}(\cos x + i\,\text{sen}\,x) + e^{y}(\cos x - i\,\text{sen}\,x)}{2} =$$

$$= \frac{e^{-y}\cos x}{2} + \frac{i\,e^{-y}\,\text{sen}\,x}{2} + \frac{e^{y}\cos x}{2} - \frac{i\,e^{y}\,\text{sen}\,x}{2} =$$

$$= \cos x\left(\frac{e^{-y} + e^{y}}{2}\right) + i\,\text{sen}\,x\left(\frac{e^{-y} - e^{y}}{2}\right)$$

DATOS PREVIOS

$$\cosh = \left(\frac{e^{-x} + e^{x}}{2}\right) \qquad \text{senh} = \left(\frac{e^{-x} - e^{x}}{2}\right)$$

$$\therefore \cos(x+yi) = \cos x\,(\cosh y) + \text{sen}\,x\,(\text{senh}\,x)$$

Funciones Hiperbólicas

- $\text{senh}\,z = \dfrac{e^{z} - e^{-z}}{2}$
- $\cosh z = \dfrac{e^{z} + e^{-z}}{2}$

$\cosh^2 z - \text{senh}^2 z = 1$

$\text{senh}(-z) = -\text{senh}(z)$

$\cosh(-z) = \cosh(z)$

1] Comprobar

$$\left|\cos\left(\underbrace{\underbrace{2\pi}_{x} + \underbrace{i\ln(2+\sqrt{3})}_{yi}}_{z}\right)\right| = 2$$

$z = x + yi$ $\qquad \cos(z) = \cos(x+yi) =$

$= \cos x(\cosh y)$

$\therefore\ x = 2\pi$ $\qquad - i\,\text{sen}\,x(\text{senh}\,y)$

$yi = i\ln(2+\sqrt{3})$

$|\cos(z)| = 2$;

$$\sqrt{(\cos x(\cosh y))^2 + (\text{sen}\,x\,\text{senh}\,y)^2} = 2$$

$$(\cos x\cosh y)^2 + (\text{sen}\,x\,\text{senh}\,y)^2 = 4$$

$\longrightarrow \text{sen}^2 x = 1 - \cos^2 x$

$\therefore\ \cos^2 x\cosh^2 y + (1-\cos^2 x)\,\text{senh}^2 y = 4$

$\cos^2 x\cosh^2 y + \text{senh}^2 y - \cos^2 x\,\text{senh}^2 y = 4$

$\cos^2 x(\underbrace{\cosh^2 y - \text{senh}^2 y}_{1}) + \text{senh}^2 y = 4$

$\cos^2 x + \text{senh}^2 y = 4$

$$\cos^2(2\pi) + \left(\frac{e^{y} - e^{-y}}{2}\right)^2 = 4$$

$$\underbrace{\cos^2(2\pi)}_{1} + \left(\frac{e^{\ln(2+\sqrt{3})} - \overbrace{e^{-\ln(2+\sqrt{3})}}^{\frac{1}{2+\sqrt{3}}}}{2}\right)^2 = 4$$

$$1 + \underbrace{\left[\frac{2+\sqrt{3} - \frac{1}{2+\sqrt{3}}}{2}\right]^2}_{3} = 4$$

2] Obtener las funciones $u(x,y)$ y $v(x,y)$ de

$\tan(z) = u(x,y) + v(x,y)$

$$\tan(z) = \frac{\text{sen}(z)}{\cos(z)} = \frac{\text{sen}\,x\cosh y + i\cos x\,\text{senh}\,y}{\cos x\cosh y - i\,\text{sen}\,x\,\text{senh}\,y} =$$

$$= \frac{\text{sen}\,x\cosh y + i\cos x\,\text{senh}\,y}{\cos x\cosh y - i\,\text{sen}\,x\,\text{senh}\,y}\left(\frac{\cos x\cosh y + i\,\text{sen}\,x\,\text{senh}\,y}{\cos x\cosh y + i\,\text{sen}\,x\,\text{senh}\,y}\right)$$

$$= \frac{(\operatorname{sen}x\cosh y)(\cos x\cosh y) + (\operatorname{sen}x\cosh y)(i\operatorname{sen}x\operatorname{senh}y) + (i\cos x\operatorname{senh}y)(\cos x\cosh y) - (\cos x\operatorname{sen}x\operatorname{senh}^2 y)}{(\cos x\cosh y)^2 + (\operatorname{sen}x\operatorname{senh}y)^2}$$

$$= \frac{(\operatorname{sen}x\cos x\cosh^2 y) + i\operatorname{sen}^2 x\cosh y\operatorname{senh}y + i\cos^2 x\operatorname{senh}y\cosh y - \cos x\operatorname{sen}x\operatorname{senh}^2 y}{(\cos x\cosh y)^2 + (\operatorname{sen}x\operatorname{senh}y)^2} =$$

$$= \frac{\operatorname{sen}x\cos x(\overset{1}{\cancel{\cosh^2 y - \operatorname{senh}^2 y}}) + i\cosh y\operatorname{senh}y(\overset{1}{\cancel{\operatorname{sen}^2 x + \cos^2 x}})}{(\cos x\cosh y)^2 + (\operatorname{sen}x\operatorname{senh}y)^2} =$$

$$= \frac{\operatorname{sen}x\cos x + i\cosh y\operatorname{senh}y}{(\cos x\cosh y)^2 + (\operatorname{sen}x\operatorname{senh}y)^2} = \Bigg\}$$ Ya es expresión

$$= \frac{\operatorname{sen}x\cos x + i\cosh y\operatorname{senh}y}{\cos^2 x\cosh^2 y + \underbrace{\cancel{\operatorname{sen}^2 x}}_{1-\cos^2 x}\operatorname{senh}^2 y} =$$

$$= \frac{\operatorname{sen}x\cos x + i\cosh y\operatorname{senh}y}{\cos^2 x\cosh^2 y + \operatorname{senh}^2 y - \cos^2 x\operatorname{senh}^2 y} =$$

$$= \frac{\operatorname{sen}x\cos x + i\cosh y\operatorname{senh}y}{\cos^2 x(\overset{1}{\cancel{\cosh^2 y - \operatorname{senh}^2 y}}) + \operatorname{senh}^2 y} =$$

$$= \frac{\operatorname{sen}x\cos x + i\cosh y\operatorname{senh}y}{\cos^2 x + \operatorname{senh}^2 y} =$$

$$u(x,y) = \frac{\operatorname{sen}x\cos x}{\cos^2 x + \operatorname{senh}^2 y} = \frac{\operatorname{sen}2x}{2(\cos^2 x + \operatorname{senh}^2 y)} \;//$$

$$v(x,y) = \frac{\cosh y\operatorname{senh}y}{\cos^2 x + \operatorname{senh}^2 y} \;//$$

3] Hallar todos las soluciones de la siguiente ecuación

$$\cos z = 2i\operatorname{sen}z$$ | Valores de z

$$\cos z = \frac{e^{zi} + e^{-zi}}{2}$$ | $z = x + yi$

$$\cos z = \frac{e^{zi} + e^{-zi}}{2} \qquad z = x + yi$$

$$\operatorname{sen} z = \frac{e^{zi} - e^{-zi}}{2}$$

$$\frac{e^{zi} + e^{-zi}}{2} = \cancel{2i}\left(\frac{e^{zi} - e^{-zi}}{\cancel{2i}}\right);$$

$$e^{zi} + e^{-zi} = 2\left(e^{zi} - e^{-zi}\right);$$

$$e^{zi} + e^{-zi} - 2e^{zi} + 2e^{-zi} = 0$$

$$3e^{-zi} - e^{zi} = 0 \;;\; -e^{+zi}\left(3e^{-zi} - e^{zi} = 0\right);$$

$$-3 + e^{2zi} = 0 \;;\; e^{2zi} = 3 + 0i$$

$$e^{2(x+yi)i} = 3 + 0i \;;\; e^{2xi - 2y} = 3 + 0i$$

$$e^{-2y} e^{2xi} = 3 + 0i \;;$$

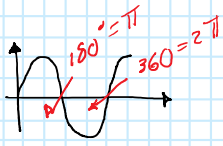
$$e^{-2y}\left(\cos 2x + i \operatorname{sen} 2x\right) = 3 + 0i$$

① $e^{-2y} \cos 2x = 3$

② $\cancel{i}\, e^{-2y} \operatorname{sen} 2x = 0\cancel{i}$

→ ② ; $e^{-2y} \operatorname{sen} 2x = 0$

$e^{-2y} > 0 \;;\; \operatorname{sen} 2x = 0$



$2x = \operatorname{sen}^{-1}(0) \;;\; x = \frac{\operatorname{sen}^{-1}(0)}{2}$

$\therefore\; x = \frac{n\pi}{2} \;\Big|\; n = \pm 1, 2, \ldots$

→ ① ;

$e^{-2y} \cos 2x = 3$

$\therefore e^{-2y} = 3 \Longleftarrow$

$-2y = \ln(3)$

$y = -\frac{1}{2}\ln(3)$

Analizando "$\cos 2x$"

$n = 0 \;;\; e^{-2y} \cancel{\cos(0)}^{\,1} = 3$

$n = \text{par} \;;$ ✓✓

$n = \text{impar}$ ××

$$z = x + iy = \frac{n\pi}{2} - \frac{1}{2}\ln(3)$$

Ejercicio

Tuesday, February 12, 2019 7:05 PM

Ejercicio

Sea $F(z) = \dfrac{2z-5}{z+4}$ obtener los valores de z para los casos

1] $F(z) = 3i$ // $z = x + yi$

$$\frac{2z-5}{z+4} = 3i \; ; \; 2z-5 = 3i(z+4)$$

$$2z-5 = 3iz + 12i$$

$$2z - 3iz = 12i + 5$$

$$z(2-3i) = 12i+5 \; ; \; z = \frac{12i+5}{2-3i}\left(\frac{2+3i}{2+3i}\right) =$$

$$z = \frac{(12i+5)(2+3i)}{4+9} = \frac{(12i+5)(2+3i)}{13} =$$

$$z = \frac{-26}{13} + \frac{39}{13}i = -2+3i$$

$$\begin{array}{r} 12i+5 \\ 3i+2 \\ \hline 24i \quad 10 \\ 15i \\ -36 \\ \hline -26+39i \end{array}$$

2] $F(z) = z$

$$z = \frac{2z-5}{z+4} \; ; \; z(z+4) = 2z-5$$

$$z^2 + 4z - 2z + 5 = 0$$

$$z^2 + 2z + 5 = 0$$

$$z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4(1)(5)}}{2} =$$

$$= -1 \pm \frac{\cancel{\sqrt{4-20}}}{2} \quad \cancel{\sqrt{16}} \to -4 \qquad = -1 \pm 2i$$

$$z_1 = -1 + 2i$$

$$z_2 = -1 - 2i$$

F. Hiperbólicas

Tuesday, February 19, 2019 7:10 PM

- $\operatorname{senh} z = \dfrac{e^{z} - e^{-z}}{2}$ • $\cosh z = \dfrac{e^{z} + e^{-z}}{2}$

Propiedades

1] $\operatorname{senh}(z+w) = \operatorname{senh}(z)\cosh(w) + \operatorname{senh}(w)\cosh(z)$

$$= \left(\frac{e^{z} - e^{-z}}{2}\right)\left(\frac{e^{w} + e^{-w}}{2}\right) + \left(\frac{e^{w} - e^{-w}}{2}\right)\left(\frac{e^{z} + e^{-z}}{2}\right) =$$

$$= \frac{1}{4}\left[\left(e^{z} - e^{-z}\right)\left(e^{w} + e^{-w}\right) + \left(e^{w} - e^{-w}\right)\left(e^{z} + e^{z}\right)\right] =$$

//

$$\begin{array}{l} e^{z} - e^{-z} \\ e^{w} + e^{-w} \\ \hline e^{w+z} - e^{-z+w} + e^{-w+z} - e^{-z-w} \end{array}$$

$$\begin{array}{l} e^{w} - e^{-w} \\ e^{z} + e^{-z} \\ \hline e^{w+z} - e^{-w+z} + e^{-z+w} - e^{-w-z} \end{array}$$

$$= \frac{1}{4}\Big(\underline{e^{w+z}} - \cancel{e^{-z+w}} + \cancel{e^{-w+z}} \underline{- e^{-z-w}}$$

$$+ \underline{e^{w+z}} - \cancel{e^{-w+z}} + \cancel{e^{-z+w}} \underline{- e^{-w-z}}\Big) =$$

$$= \frac{1}{4}\left(2e^{w+z} - 2e^{-w-z}\right) = \frac{e^{w+z} - e^{-w-z}}{2}$$

$$= \frac{e^{w+z} - e^{-(w+z)}}{2} = \underline{\operatorname{senh}(z+w)}$$

// Segunda forma

$$\operatorname{senh}(z+w) = \frac{e^{z+w} - e^{-(z+w)}}{2} = \frac{e^{z}e^{w} - e^{-z}e^{-w}}{2} =$$

$$= e^{-i(iz)}e^{-i(iw)} - e^{i(iz)}e^{i(iw)}$$

$$= \frac{e^{-i(iz)}\, e^{-i(iw)} - e^{i(iz)}\, e^{i(iw)}}{2} =$$

$\longrightarrow$ $e^{yi} = \cos y + i \operatorname{sen} y \qquad e^{-yi} = \cos y - i \operatorname{sen} y$

$$= \frac{\cos(iz) - i \operatorname{sen}(iz)(\cos iw - \operatorname{sen} iw) - (\cos iz + \operatorname{sen} iz)(\cos iw + \operatorname{sen} iw)}{2} =$$

$$= \frac{(\cosh z + \operatorname{senh} z)(\cosh w + \operatorname{senh} w) - (\cosh z - \operatorname{senh} z)(\cosh w - \operatorname{senh} w)}{2} =$$

$$= \frac{1}{2}\left(\cancel{\cosh z \cosh w} + \operatorname{senh} z \cosh w + \cosh z \operatorname{senh} w + \underline{\operatorname{senh} z \operatorname{senh} w}\right)$$

$$- \left(\cancel{\cosh z \cosh w} - \operatorname{senh} z \cosh w + \cancel{\operatorname{senh} z \operatorname{senh} w} - \cosh z \operatorname{senh} w\right)$$

$$= (\operatorname{senh} z \cosh w + \cosh z \operatorname{senh} w) \;//$$

▸ Propiedades

- $\cosh^2 z - \operatorname{senh}^2 z = 1$
- $\operatorname{senh}(-z) = -\operatorname{senh} z$ • $\cosh(-z) = \cosh z$
- $\operatorname{sen}(iz) = i \operatorname{senh} z$ • $\cosh(iz) = \cosh z$

F. Logarítmica

Tuesday, February 19, 2019 7:44 PM

→ Donde $w = u + iv$

si $z = e^{w}$ existe $w = \ln(z)$

$$z = re^{i\theta}$$

$$re^{i\theta} = e^{u+iv} = e^{u} e^{iv}$$

$$\therefore r = e^{u} \qquad e^{i\theta} = e^{iv}$$

$$\ln(r) = u \qquad i\theta = iv \; ; \; v = \theta$$

→ $\ln(z) = w = u + vi$;

$$\ln(z) = w = \ln(r) + \theta$$

$$\ln(z) = \ln|z| + i \operatorname{Arg} z \begin{cases} \theta + 2n\pi \\ n = 0 \end{cases}$$

$$\ln(z) = \ln|z| + i(\theta + 2n\pi) \mid n = 0, 1, 2, \ldots$$

//Multivaluada

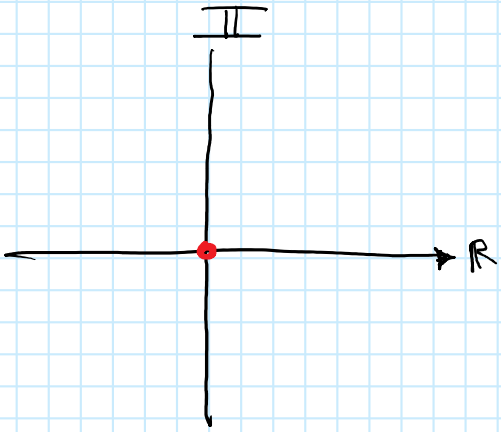
Propiedades Logaritmos Naturales $\in \mathbb{C}$

- $\ln(zw) = \ln(z) + \ln(w) \mid z, w \in \mathbb{C} - \{0\}$
- $\ln\left(\frac{z}{w}\right) = \ln(z) - \ln(w)$
- $\ln(z^{r}) = r \ln z$

1] Obtener el dominio de la función

$$f(z) = \ln\left[z^3 - z^2 + (1+i)z\right]$$

$$D_{f(z)} = \mathbb{C} - \{z_1, z_2, z_3\}$$



$$z^3 - z^2 + (1+i)z$$

$$z(z^2 - z + (1+i))$$

$$\therefore z_1 = 0$$

$$\underset{A}{z^2} \underset{B}{- z} + \underbrace{(1+i)}_{C}$$

Raices cuadradas

$$w = a + bi$$

$$w = \pm\left[x + i\,\mathrm{sign}(b)\,y\right] =$$

$$= \pm\left[\sqrt{\tfrac{1}{2}\left(a + \sqrt{a^2+b^2}\right)} + i\,\underbrace{\mathrm{sign}(b)}_{\text{signo}}\sqrt{\tfrac{1}{2}\left(-a + \sqrt{a^2+b^2}\right)}\right] =$$

//Sustituyendo

//Normal

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2} = \frac{1 \pm \sqrt{1 - 4(1)(1+i)}}{2} =$$

$$= \frac{1 \pm \sqrt{1 - 4 - 4i}}{2} = \frac{1 \pm \sqrt{-3-4i}}{2}$$

raíz

$$-3 - 4i = a + bi \;;\quad a = -3 \quad b = -4$$

// Compleja

$$= \pm\left[\sqrt{\tfrac{1}{2}\left(-3 + \sqrt{9+16}\right)} - i\sqrt{\tfrac{1}{2}\left(3 + \sqrt{25}\right)}\right] =$$

$$= \pm\left[\sqrt{\tfrac{1}{2}(2)} - i\sqrt{\tfrac{1}{2}(8)}\right] = \pm(1-2i)$$

$$= \begin{array}{l} +1-2i \\ -1+2i \end{array}$$

// Regresando

$$= \frac{1 \pm \sqrt{-3-4i}}{2} \begin{cases} \dfrac{1+(1-2i)}{2} = z_2 \\ \dfrac{\cancel{1-1}+\cancel{2}i}{\cancel{2}} = i = z_3 \end{cases}$$

$$\therefore D_{f(z)} = \mathbb{C} - \{0, i, 1-i\}$$

2] Demostrar propiedad

$$\ln(zw) = \ln|zw| + i\,\mathrm{Arg}(zw);$$

NOTA $|zw| = |z||w|$

$$\ln(zw) = \ln(|z||w|) + i\left[\mathrm{Arg}(z) + \mathrm{Arg}(w)\right]$$

$$= \underbrace{\ln|z| + i\,\mathrm{Arg}(z)}_{\ln(z)} + \underbrace{\ln|w| + i\,\mathrm{Arg}(w)}_{\ln(w)}$$

3] Calcular el $\ln(-1-i)(1-i)$ donde el argumento toma valores de $[0, 2\pi)$

$$\rightarrow \ln(-1-i)(1-i) = \ln(-1-i) + \ln(1-i) =$$

$\rightarrow \ln(-1-i)(1-i) = \ln\underbrace{(-1-i)}_{z} + \ln\underbrace{(1-i)}_{w} =$

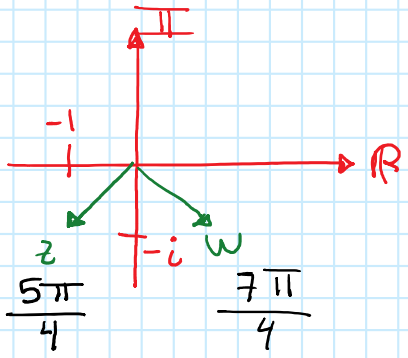
- $|z| = \sqrt{1+1} = \sqrt{2}$
- $|w| = \sqrt{2}$

- $\mathrm{Arg}(z) = \theta$

$\rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right) = \frac{5\pi}{4}$

- $\mathrm{Arg}(w) = \theta$

$\rightarrow \theta = \frac{7\pi}{4}$



$= \left[\ln\sqrt{2} + i\frac{5\pi}{4}\right] + \left[\ln\sqrt{2} + i\frac{7\pi}{4}\right] =$

$= 2\ln\sqrt{2} + i\left(\frac{5\pi}{4} + \frac{7\pi}{4}\right) =$

Pasos Mortales $\rightarrow$ $= 2\ln\sqrt{2} + i(3\pi) =$

$= \ln 2 + i\,\pi$ //

$\ln(\sqrt{2})^2 = \ln 2$

$i(3\pi) = i(\pi)$

F. Potencia

Tuesday, February 19, 2019 8:29 PM

- Si z y $w \in \mathbb{C} \mid z, w \neq 0$

$\rightarrow z^{w} = e^{w \ln z}$

- Propiedades

$\rightarrow z^{w_1} z^{w_2} = z^{w_1 + w_2}$ $\rightarrow \dfrac{z^{w_1}}{z^{w_2}} = z^{w_1 - w_2}$

Caso 1]

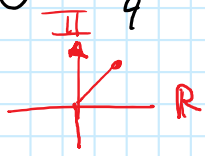
Ejercicio 1

- $(1+i)^3$ $z = 1+i$ $w = 3$

- $$\begin{array}{r} (1+i) \\ (1+i) \\ \hline 1 + 2i - 1 \end{array} = \begin{array}{r} (2i) \\ (1+i) \\ \hline 2i - 2 \end{array}$$

$(1+i)^3 = 2i - 2$

- $e^{3[\ln(1+i)]} = e^{3(\ln|z| + i(\theta + 2n\pi))}$ $\begin{cases} |z| = \sqrt{2} \\ \theta = \frac{\pi}{4} \end{cases}$



$= e^{3 \ln \sqrt{2} + i \frac{3\pi}{4} + i 6n\pi} =$

$= e^{3 \ln \sqrt{2}} e^{i \frac{3\pi}{4}} \cancel{e^{i 6n\pi}} \cancel{e^{i 2n\pi}} =$

$= e^{3 \ln \sqrt{2}} \left[\cancel{\cos\left(\frac{3\pi}{4}\right)}^{-\frac{1}{\sqrt{2}}} + i \cancel{\operatorname{sen}\left(\frac{3\pi}{4}\right)}^{\frac{1}{\sqrt{2}}} \right] =$

$$= 2\sqrt{2}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = -2+2i$$

$$Pr\left[(1+i)^3\right] = -2+2i$$

Ejercicio 2

- $(1+i)^{-3} = \frac{1}{(1+i)^3} = \frac{1}{-2+2i}\left(\frac{-2-2i}{-2-2i}\right) = \frac{-2-2i}{8} =$

$$-\frac{1}{4} - \frac{1}{4}i$$

- $z^{\omega} = e^{-3\ln z} = e^{-3\left(\ln\sqrt{2} + i\left(\frac{\pi}{4} + 2n\pi\right)\right)} =$

$\omega = -3$
$z = 1+i$

$$= e^{-3\ln\sqrt{2}}\, e^{-\frac{3\pi}{4}i}\, \cancel{e^{-6n\pi i}}^{\,1} =$$

$$= e^{-3\ln\sqrt{2}}\left(\cos\left(\frac{3\pi}{4}\right) + i\,\mathrm{sen}\left(\frac{3\pi}{4}\right)\right) =$$

$$= \frac{1}{2\sqrt{2}}\left(-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) = \frac{1}{4} + i\frac{1}{4}$$

Tarea Moral! 3) $(i)^{2/3}$ 4) $i^{\sqrt{2}}$ 5) i^{i}

3] $(i)^{2/3}$

- $|z| = \sqrt{1} = 1$
- $\theta = 90^\circ = \frac{\pi}{2}$

$$e^{\frac{2}{3}\ln(z)} = e^{\frac{2}{3}\left(\ln(1) + i\left(\frac{\pi}{2} + 2\pi n\right)\right)} =$$

$$= e^{\frac{2}{3}\ln(1) + i\frac{\pi}{3} + \frac{4}{3}\pi n i} = e^{\frac{2}{3}\ln(1)}\, e^{i\frac{\pi}{3}}\, \cancel{e^{\frac{4}{3}\pi n i}}^{\,1}$$

$$= \cancel{1^{2/3}}^{\,1}\left(\cos(\pi/3) + i\,\mathrm{sen}(\pi/3)\right) =$$

$$= \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \quad \text{// Valor principal}$$

4] $i^{\sqrt{2}}$ $\quad |z| = 1 \quad \theta = 90°$

$$e^{\sqrt{2}\ln(z)} = e^{\sqrt{2}\left(\ln(1) + i\left(\frac{\pi}{2} + 2\pi n\right)\right)} =$$

$$= e^{\sqrt{2}\ln(1) + \sqrt{2}\, i\left(\frac{\pi}{2} + 2\pi n\right)} = \cancel{e^{\sqrt{2}\ln(1)}}^{\,1}\, \underbrace{e^{\sqrt{2}\, i\frac{\pi}{2}}}\, \cancel{e^{2\pi n}}^{\,1} =$$

$$= e^{\sqrt{2}\, i\frac{\pi}{2}} \quad ; \quad \text{infinidad de valores} \quad //$$

5] $i^{i} =$ $\quad z = 1 \quad \theta = 90$

$$e^{i\ln(z)} = e^{i\left(\ln(1) + i\left(\frac{\pi}{2} + 2\pi n\right)\right)} =$$

$$= e^{i\ln(1)}\, e^{-\pi/2}\, e^{-2\pi n} = \text{¿?}$$

F. Trigo. Inversas

Thursday, February 21, 2019 7:31 PM

- $\operatorname{Arg} z = \theta \leftarrow Pr(z)$
- $Pr(z) + N\,Pr(z) = \theta + 2n\pi i$

- $n = 0, 1, 2, 3 \ldots$
- $(0, 2\pi)$

Ejercicio

1] $\operatorname{sen}^{-1}(z) = -i \ln\left(iz + \sqrt{1-z^2}\right)$

- $\operatorname{sen} z = \dfrac{e^{zi} - e^{-zi}}{2i}$
- $\ln(z) = \ln|z| + i(\theta + 2n\pi)$

- $w = F(z) = \operatorname{sen}(z)$

$\therefore w = \operatorname{sen}^{-1}(z)$

si y sólo si

$\operatorname{sen} w = (z)$

- $z = \operatorname{sen} w$

$z = \dfrac{e^{wi} - e^{-wi}}{2i}$

$\rightarrow \left[2i(z) = e^{wi} - e^{-wi}\right] e^{wi}$

$e^{wi}(2i(z)) = e^{wi + wi} - e^{-wi + wi}$

$2iz\, e^{wi} = e^{2wi} - \cancel{e^{0}}^{\,1}$;

$\left(-e^{2wi} + 2iz\, e^{wi} + 1 = 0\right)(-1)$;

$\underbrace{e^{2wi}}_{A} \underbrace{- 2iz}_{B} e^{wi} \underbrace{- 1}_{C} = 0$

$e^{wi} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$e^{wi} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{\cancel{2}iz \pm \cancel{2}\sqrt{-z^2+1}}{\cancel{2}}$$

// Sustituyendo

$$e^{wi} = \frac{2iz \pm \sqrt{-4z^2 - 4(1)(-1)}}{2} =$$

$$= \frac{2iz \pm \sqrt{-4z^2 + 4}}{2} = iz \pm \sqrt{1 - z^2} =$$

$$\left(e^{wi} = iz \pm \sqrt{1 - z^2}\right) \ln$$

$$wi = \ln\left(iz \pm \sqrt{1 - z^2}\right);$$

$$w = \frac{\ln\left(iz \pm \sqrt{1 - z^2}\right)}{i}\left(\frac{i}{i}\right) =$$

$$w = -i \ln\left(iz \pm \sqrt{1 - z^2}\right)$$

Funciones Trigonométricas Inversas

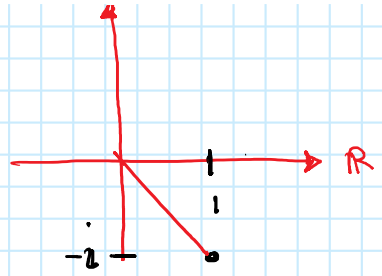
- $\operatorname{sen}^{-1}(z) = -i \ln\left[iz + (1 - z^2)^{1/2}\right]$
- $\cos^{-1}(z) = -i \ln\left[z + (1 - z^2)^{1/2}\right]$

Ejercicios

1) $(1-i)^{\sqrt{2}\, i}$

1] $(1-i)^{\sqrt{2}\,i}$

- $z = 1-i$; $|z| = \sqrt{1^2+1^2} = \sqrt{2}$
- $w = \sqrt{2}\,i$



$\theta = \frac{7\pi}{4}$

$$z^w = e^{w \ln z} = e^{\sqrt{2}\,i\left(\ln\sqrt{2} + i\left(\frac{7\pi}{4} + 2n\pi\right)\right)} =$$

$$= e^{\sqrt{2}\,i \ln\sqrt{2}}\ \underbrace{e^{-\sqrt{2}\left(\frac{7\pi}{4}\right)}}_{\in \mathbb{R}}\ \underbrace{e^{-\sqrt{2}\,(2n\pi)}}_{\in \mathbb{R}} =$$

n-valores

Infinidad de valores//

2] $(1-i)^{i}$

- $z = 1-i$ $\quad |z| = \sqrt{2}$
- $w = i$
- $\theta = \frac{7\pi}{4}$

$$e^{w \ln z} = e^{i\left[\ln\sqrt{2} + i\left(\frac{7\pi}{4} + 2\pi n\right)\right]} =$$

$$= e^{i \ln\sqrt{2}}\ \underbrace{e^{-\frac{7\pi}{4}}\ e^{-2\pi n}}_{\text{Reales - infinidad de valores}}$$

SERIE

Tuesday, February 26, 2019 7:51 PM

14] Calcular todos los valores de $\ln[2 \operatorname{cis} 210°]$ es decir $a+bi \mid a,b \in \mathbb{R}$

$\rightarrow \ln(2 \operatorname{cis} 210°) = \ln z$

- $z = 2 \operatorname{cis} 210 = 2[\cos 210° + i \operatorname{sen} 210°] =$

$$= 2\left[-\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right] = -\sqrt{3} - i$$

$$|z| = \sqrt{(\sqrt{3})^2 + 1} = 2$$

$$\ln(z) = \ln|z| + i(\operatorname{Arg} z + 2n\pi)$$

$$= \ln|2| + i\left(\underbrace{\frac{7}{6}\pi}_{\text{Argumento}} + 2n\pi\right)$$

$$= \ln(2) + \frac{7}{6}\pi i$$

13] • Calcular $\cosh\left(-1 - \frac{11\pi}{6} i\right)$, forma $a+bi$

$$\cosh(z) = \frac{e^{z} + e^{-z}}{2} = \frac{e^{-1-\frac{11\pi}{6}i} + e^{1+\frac{11\pi}{6}i}}{2} =$$

$$= \frac{e^{-1} e^{-\frac{11\pi}{6}i} + e^{1} e^{\frac{11\pi}{6}i}}{2} =$$

$$= \frac{e^{-1}\left[\cos\left(\frac{11\pi}{6}\right) + \operatorname{sen}\left(\frac{11\pi}{6}\right)\right] + e\left[\cos\left(\frac{11\pi}{6}\right) + \operatorname{sen}\left(\frac{11\pi}{6}\right)\right]}{2}$$