Sejam $W_1, \ldots, W_n$ subespaços, mostre que $L(W_1, \ldots, W_n) = W_1 + \cdots + W_n$.

Definamos $L(W_1, \ldots, W_n) = L(W_1 + \cdots + W_n)$.

Obviamente $W_1 + \cdots + W_n \subset L(W_1, \ldots, W_n)$. (I)

Sejam w um elemento de $L(W_1, \ldots, W_n)$, e $a_i,\ i \in \mathbb{N}_m$ escalares,

$$w = \sum_i a_i \sum_{j=1}^{n} w_j = \sum_{j=1}^{n} w_j \sum_i a_i,\ w_j \in W_j.$$

Tomando quaisquer a_i's tais que $\displaystyle\sum_i a_i = 1$, $w = \displaystyle\sum_{j=1}^{n} w_j$ que é um elemento de $\displaystyle\sum_{j=1}^{n} W_j$. Logo

$L(W_1, \ldots, W_n) \subset W_1 + \cdots + W_n$. (II)

(I) $\wedge$ (II) $\Rightarrow\ L(W_1, \ldots, W_n) = W_1 + \cdots + W_n$

Quod Erat Demonstrandum.

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