Seja $\langle f,g\rangle = \int_{-\pi}^{\pi} f(x)g(x)\ dx$.

Mostre que $\sin nx$ é ortogonal a $\cos mx$, $m,n\in\mathbb{Z}$.

Resolução:

$$P = \langle \sin nx\ ,\ \cos mx\rangle = \frac{1}{2}\left[\underbrace{\int_{-\pi}^{\pi}\sin(n+m)x\ dx}_{\alpha} + \underbrace{\int_{-\pi}^{\pi}\sin(n-m)x\ dx}_{\beta}\right]$$

Se $n=-m$,

$$P = \frac{\cancelto{0}{\alpha}+\beta}{2} = \underbrace{\left[-\frac{\cos(n-m)x}{2(n-m)}\right]\Bigg|_{-\pi}^{\pi}}_{\gamma} = 0.$$

Se $n=m$,

$$P = \frac{\alpha+\cancelto{0}{\beta}}{2} = \underbrace{\left[-\frac{\cos(n+m)x}{2(n+m)}\right]\Bigg|_{-\pi}^{\pi}}_{\theta} = 0.$$

Se $n\neq |m|$,

$$P = \gamma+\theta = 0.$$

C.Q.D.

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