Projeto Mathematical Ramblings

https://sites.google.com/site/mathematical ramblings/ Resolva a equação $x^3 + 3x + 1 = 0$ pelo método de Cardano-Tartaglia

Resolução:

uv = 1

$$(u-v)^3 + 3(u-v) + 1 = 0$$

$$u^3 - 3u^2v + 3uv^2 - v^3 + 3u - 3v + 1 = 0$$

$$u^3 - 3u^2 \cdot \frac{1}{u} + 3u \cdot \frac{1}{u^2} - \frac{1}{u^3} + 3u - \frac{3}{u} + 1 = 0$$

$$u^3 - 3u + \frac{3}{u} - \frac{1}{u^3} + 3u - \frac{3}{u} + 1 = 0$$

$$u^3 - \frac{1}{u^3} + 1 = 0$$

$$u^6 + u^3 - 1 = 0$$
Tomando $U = u^3$:
$$U^2 + U - 1 = 0$$

$$U = \frac{-1 \pm \sqrt{1 + 4}}{2}$$

$$U' = \frac{-1 + \sqrt{5}}{2} \implies u' = \sqrt[3]{\frac{-1 + \sqrt{5}}{2}} \implies v' = \frac{1}{\sqrt[3]{\frac{-1 + \sqrt{5}}{2}}}$$

$$U'' = \frac{-1 - \sqrt{5}}{2} \implies u'' = \sqrt[3]{\frac{-1 - \sqrt{5}}{2}} \implies v'' = \frac{1}{\sqrt[3]{\frac{-1 + \sqrt{5}}{2}}}$$

 $x' = u' - v' = \sqrt[3]{\frac{-1 + \sqrt{5}}{2}} - \sqrt[3]{\frac{2}{-1 + \sqrt{5}}}$

 $x'' = u' - v' = \sqrt[3]{\frac{-1 - \sqrt{5}}{2}} - \sqrt[3]{\frac{2}{-1 - \sqrt{5}}}$