Resolva a equação $x^3 + 3x + 1 = 0$ pelo método de Cardano-Tartaglia

Resolução:

$uv = 1$

$(u - v)^3 + 3(u - v) + 1 = 0$

$u^3 - 3u^2v + 3uv^2 - v^3 + 3u - 3v + 1 = 0$

$u^3 - 3u^2 \cdot \dfrac{1}{u} + 3u \cdot \dfrac{1}{u^2} - \dfrac{1}{u^3} + 3u - \dfrac{3}{u} + 1 = 0$

$u^3 - 3u + \dfrac{3}{u} - \dfrac{1}{u^3} + 3u - \dfrac{3}{u} + 1 = 0$

$u^3 - \dfrac{1}{u^3} + 1 = 0$

$u^6 + u^3 - 1 = 0$

Tomando $U = u^3$:

$U^2 + U - 1 = 0$

$U = \dfrac{-1 \pm \sqrt{1+4}}{2}$

$U' = \dfrac{-1+\sqrt{5}}{2} \;\Rightarrow\; u' = \sqrt[3]{\dfrac{-1+\sqrt{5}}{2}} \;\Rightarrow\; v' = \dfrac{1}{\sqrt[3]{\dfrac{-1+\sqrt{5}}{2}}}$

$U'' = \dfrac{-1-\sqrt{5}}{2} \;\Rightarrow\; u'' = \sqrt[3]{\dfrac{-1-\sqrt{5}}{2}} \;\Rightarrow\; v'' = \dfrac{1}{\sqrt[3]{\dfrac{-1+\sqrt{5}}{2}}}$

$x' = u' - v' = \sqrt[3]{\dfrac{-1+\sqrt{5}}{2}} - \sqrt[3]{\dfrac{2}{-1+\sqrt{5}}}$

$x'' = u' - v' = \sqrt[3]{\dfrac{-1-\sqrt{5}}{2}} - \sqrt[3]{\dfrac{2}{-1-\sqrt{5}}}$