

Teorema de Pitágoras.

$$(A - B) \perp (A - C) \Rightarrow \|B - C\|^2 = \|A - B\|^2 + \|A - C\|^2$$

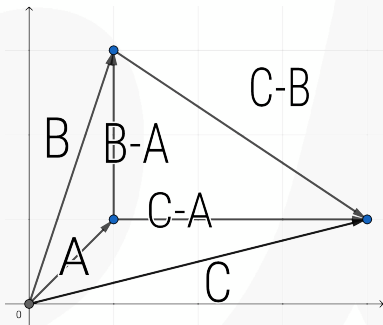
Demonstração:

$$\begin{aligned} \langle (A - B), (A - C) \rangle &= 0 \Rightarrow \langle A, A \rangle - \langle A, C \rangle - \langle B, A \rangle + \langle B, C \rangle = 0 \Rightarrow \\ &\Rightarrow \langle A, A \rangle + \langle B, C \rangle = \langle A, C \rangle + \langle B, A \rangle \quad (\text{I}) \end{aligned}$$

$$\|B - C\|^2 = \langle (B - C), (B - C) \rangle = \langle B, B \rangle + \langle C, C \rangle - 2\langle B, C \rangle \quad (\text{II})$$

$$\begin{aligned} (\|A - B\| + \|A - C\|)^2 &= \langle (A - B), (A - B) \rangle + \langle (A - C), (A - C) \rangle + 2\|A - B\|\|A - C\| = \\ &= \langle A, A \rangle + \langle B, B \rangle - 2\langle A, B \rangle + \langle A, A \rangle + \langle C, C \rangle - 2\langle A, C \rangle + 2\|A - B\|\|A - C\| \stackrel{(\text{II})}{=} \\ &\stackrel{(\text{I})}{=} \|B - C\|^2 + 2\langle B, C \rangle + 2\langle A, A \rangle - 2\langle A, B \rangle - 2\langle A, C \rangle + 2\|A - B\|\|A - C\| \stackrel{(\text{I})}{=} \|B - C\|^2 + 2\|A - B\|\|A - C\| \end{aligned}$$

Logo, $\boxed{\|B - C\|^2 = \|A - B\|^2 + \|A - C\|^2}.$



Quod Erat Demonstrandum.

Documento compilado em Thursday 13th March, 2025, 20:47, tempo no servidor.

Sugestões, comunicar erros: "a.vandre.g@gmail.com".

Licença de uso:  Atribuição-NãoComercial-CompartilhaIgual (CC BY-NC-SA).