

Calcule a área da elipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Resolução:

$$\begin{aligned}\frac{A}{2} &= \int_{-a}^a \sqrt{b^2 - b^2\left(\frac{x}{a}\right)^2} dx = ab \int_{-1}^1 \sqrt{1 - u^2} du = -ab \int_{\pi}^0 \sin^2 \theta d\theta = \\ &= -ab \int_{\pi}^0 \frac{1 - \cos(2\theta)}{2} d\theta = -\frac{ab\theta}{2} \Big|_{\pi}^0 + \frac{ab \sin(2\theta)}{4} \Big|_{\pi}^0 = ab \frac{\pi}{2}\end{aligned}$$

$$\boxed{A = ab\pi}$$

$$x = au, \quad u = \cos \theta, \quad \theta \in [0, \pi]$$

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