Projeto Mathematical Ramblings

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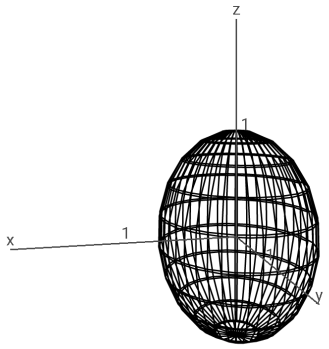
Encontre a área do elipsoide obtido pela rotação ao redor do eixo x da elipse $2x^2 + y^2 = 1$.

Resolução:



$y = \sqrt{1-2x^2}$, $y' = -\dfrac{x}{\sqrt{1-2x^2}}$

$A \;=\; 2\pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \sqrt{1-2x^2}\sqrt{1+\dfrac{x^2}{1-2x^2}}\; dx \;=\; 2\pi \int_{-1/\sqrt{2}}^{1/\sqrt{2}} \sqrt{1-x^2}\; dx$

Seja $x = \sin\theta$, $\theta \in [-\dfrac{\pi}{2}, \dfrac{\pi}{2}]$, $dx = \cos\theta\; d\theta$.

$A \;=\; \pi \int_{-\pi/4}^{\pi/4} \cos 2\theta \;+\; 1 \; d\theta \;=\; \pi(\dfrac{\sin 2\theta}{2} + \theta)|_{-\pi/4}^{\pi/4} = \boxed{\pi(1+\dfrac{\pi}{2})}$

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