Seja $\langle f,g \rangle = \displaystyle\int_{-\pi}^{\pi} f(x)g(x)\ dx$.

Mostre que $\sin nx$ é ortogonal a $\cos mx$, $m,n \in \mathbb{Z}$.

Resolução:

$$P = \langle \sin nx\ ,\ \cos mx \rangle = \frac{1}{2}\left[\underbrace{\int_{-\pi}^{\pi} \sin(n+m)x\ dx}_{\alpha} + \underbrace{\int_{-\pi}^{\pi} \sin(n-m)x\ dx}_{\beta}\right]$$

Se $n = -m$,

$$P = \frac{\cancelto{0}{\alpha} + \beta}{2} = \underbrace{\left[-\frac{\cos(n-m)x}{2(n-m)}\right]\Bigg|_{-\pi}^{\pi}}_{\gamma} = 0.$$

Se $n = m$,

$$P = \frac{\alpha + \cancelto{0}{\beta}}{2} = \underbrace{\left[-\frac{\cos(n+m)x}{2(n+m)}\right]\Bigg|_{-\pi}^{\pi}}_{\theta} = 0.$$

Se $n \neq |m|$,

$P = \gamma + \theta = 0.$

C.Q.D.

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