Projeto Mathematical Ramblings

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Calcular $I = \displaystyle\int_0^a \sqrt{a^2 - x^2}\ dx$, $a > 0$.

$$I = a\int_0^a \sqrt{1 - \left(\frac{x}{a}\right)^2}\ dx$$

Seja $y = \dfrac{x}{a}$. $dy = \dfrac{dx}{a}$

$$I = a^2\int_0^1 \sqrt{1 - y^2}\ dy$$

Seja $y = \sin\theta$, $-\dfrac{\pi}{2} \leq \theta \leq \dfrac{\pi}{2}$. $dy = \cos\theta\ d\theta$.

$$I = a^2\int_0^{\pi/2} \cos^2\theta\ d\theta \ = \ a^2\int_0^{\pi/2} \frac{(\cos 2\theta) + 1}{2} d\theta \ = \ a^2\int_0^{\pi/2} \cos 2\theta\ d\theta + \frac{a^2\pi}{4}$$

Seja $\varphi = 2\theta$. $d\varphi = 2d\theta$.

$$I = \cancelto{0}{\frac{a^2}{2}\int_0^{\pi} \cos\varphi\ d\varphi} + \frac{a^2\pi}{4} \ = \boxed{\frac{a^2\pi}{4}}$$

Documento compilado em Friday 17^{th} December, 2021, 10:45, tempo no servidor.

Última versão do documento (podem haver correções e/ou aprimoramentos): "bit.ly/mathematicalramblings_public".

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