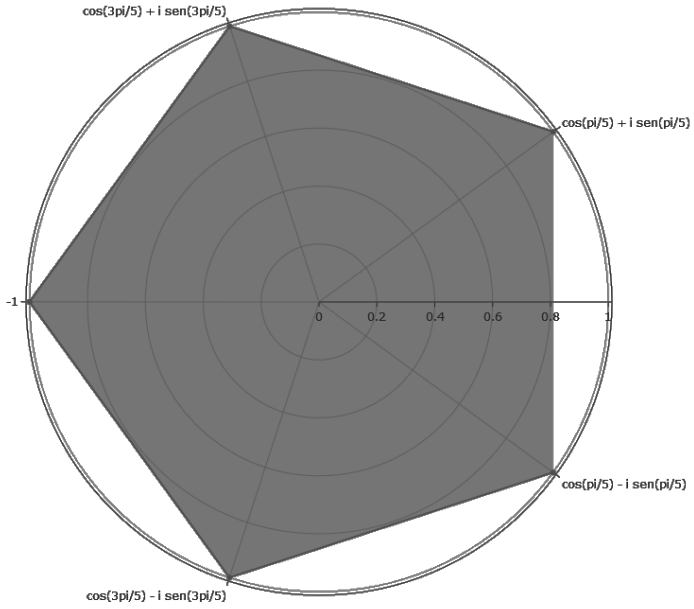
$$\int \frac{dx}{x^5+1}.$$

Afim de decompor $\frac{1}{x^5+1}$ em frações parciais, calculemos as raízes quintas de -1, que estão graficamente representadas abaixo:



Logo $\frac{1}{x^5+1} = \frac{Ax+B}{x^2-\left(2\cos\frac{\pi}{5}\right)x+1} + \frac{Cx+D}{x^2-\left(2\cos\frac{3\pi}{5}\right)x+1} + \frac{E}{x+1}.$ (I)

Donde, resolvendo o sistema:

$$\begin{cases} B+D+E=1 \\ \left[4-4\cos\left(\frac{3\pi}{5}\right)\right]A+\left[4-4\cos\left(\frac{3\pi}{5}\right)\right]B+\left[4-4\cos\left(\frac{\pi}{5}\right)\right]C+\left[4-4\cos\left(\frac{\pi}{5}\right)\right]D+\left[4-4\cos\left(\frac{3\pi}{5}\right)\right]\left[4-4\cos\left(\frac{\pi}{5}\right)\right]E=1 \\ \left[30-24\cos\left(\frac{3\pi}{5}\right)\right]A+\left[15-12\cos\left(\frac{3\pi}{5}\right)\right]B+\left[30-24\cos\left(\frac{\pi}{5}\right)\right]C+\left[15-12\cos\left(\frac{\pi}{5}\right)\right]D+\left[5-4\cos\left(\frac{3\pi}{5}\right)\right]\left[5-4\cos\left(\frac{\pi}{5}\right)\right]E=1 \\ \left[120-72\cos\left(\frac{3\pi}{5}\right)\right]A+\left[40-24\cos\left(\frac{3\pi}{5}\right)\right]B+\left[120-72\cos\left(\frac{\pi}{5}\right)\right]C+\left[40-24\cos\left(\frac{\pi}{5}\right)\right]D+\left[10-6\cos\left(\frac{3\pi}{5}\right)\right]\left[10-6\cos\left(\frac{\pi}{5}\right)\right]E=1 \\ \left[10+8\cos\left(\frac{3\pi}{5}\right)\right]A-\left[5+4\cos\left(\frac{3\pi}{5}\right)\right]B+\left[10+8\cos\left(\frac{\pi}{5}\right)\right]C-\left[5+4\cos\left(\frac{\pi}{5}\right)\right]D+\left[5+4\cos\left(\frac{3\pi}{5}\right)\right]\left[5+4\cos\left(\frac{\pi}{5}\right)\right]E=1 \end{cases},$$

obtemos:

$$\begin{cases} A = \dfrac{\left(24\cos^2\dfrac{\pi}{5} + 92\cos\dfrac{\pi}{5} + 200\right)\cos\dfrac{3\pi}{5} + 32\cos^2\dfrac{\pi}{5} + 170\cos\dfrac{\pi}{5} + 285}{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5}} \\ B = -\dfrac{\left(288\cos^2\dfrac{\pi}{5} + 420\cos\dfrac{\pi}{5} - 200\right)\cos\dfrac{3\pi}{5} + 480\cos^2\dfrac{\pi}{5} + 592\cos\dfrac{\pi}{5} - 285}{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5}} \\ C = -\dfrac{\left(24\cos\dfrac{\pi}{5} + 32\right)\cos^2\dfrac{3\pi}{5} + \left(92\cos\dfrac{\pi}{5} + 170\right)\cos\dfrac{3\pi}{5} + 200\cos\dfrac{\pi}{5} + 285}{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5}} \\ D = \dfrac{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(420\cos\dfrac{\pi}{5} + 592\right)\cos\dfrac{3\pi}{5} - 200\cos\dfrac{\pi}{5} - 285}{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5}} \\ E = -\dfrac{3}{\left(12\cos\dfrac{\pi}{5} + 20\right)\cos\dfrac{3\pi}{5} + 20\cos\dfrac{\pi}{5} + 30} \end{cases} . \text{ (II)}$$

De (I) obtemos:

$$\frac{1}{x^5+1} = \frac{A}{2}\cdot\frac{2x - 2\cos\dfrac{\pi}{5}}{x^2 - \left(2\cos\dfrac{\pi}{5}\right)x + 1} + \frac{B + A\cos\dfrac{\pi}{5}}{\sin^2\dfrac{\pi}{5}}\cdot\frac{1}{\left(\dfrac{x - \cos\dfrac{\pi}{5}}{\sin\dfrac{\pi}{5}}\right)^2 + 1} + \frac{C}{2}\cdot\frac{2x - 2\cos\dfrac{3\pi}{5}}{x^2 - \left(2\cos\dfrac{3\pi}{5}\right)x + 1} + \frac{D + C\cos\dfrac{3\pi}{5}}{\sin^2\dfrac{3\pi}{5}}\cdot\frac{1}{\left(\dfrac{x - \cos\dfrac{3\pi}{5}}{\sin\dfrac{3\pi}{5}}\right)^2 + 1} + \frac{E}{x+1}. \text{ (III)}$$

Substituindo (II) em (III):

$$\int \frac{dx}{x^5+1} \overset{x \neq \cos\frac{\pi}{5}}{=} \frac{\left(24\cos^2\dfrac{\pi}{5} + 92\cos\dfrac{\pi}{5} + 200\right)\cos\dfrac{3\pi}{5} + 32\cos^2\dfrac{\pi}{5} + 170\cos\dfrac{\pi}{5} + 285}{2\left[\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5}\right]}\log\left|x^2 - \left(2\cos\frac{\pi}{5}\right)x + 1\right| +$$

$$\frac{\cos\dfrac{\pi}{5}\left[\left(24\cos^2\dfrac{\pi}{5} + 92\cos\dfrac{\pi}{5} + 200\right)\cos\dfrac{3\pi}{5} + 32\cos^2\dfrac{\pi}{5} + 170\cos\dfrac{\pi}{5} + 285\right]}{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5}} - \frac{\left(288\cos^2\dfrac{\pi}{5} + 420\cos\dfrac{\pi}{5} - 200\right)\cos\dfrac{3\pi}{5} + 480\cos^2\dfrac{\pi}{5} + 592\cos\dfrac{\pi}{5} - 285}{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5}} -$$

$$\left(\sin\frac{\pi}{5}\right)\cdot\arctan^{-1}\frac{x - \cos\dfrac{\pi}{5}}{\sin\dfrac{\pi}{5}}$$

$$-\frac{\left(24\cos\dfrac{\pi}{5} + 32\right)\cos^2\dfrac{3\pi}{5} + \left(92\cos\dfrac{\pi}{5} + 170\right)\cos\dfrac{3\pi}{5} + 200\cos\dfrac{\pi}{5} + 285}{2\left[\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5}\right]}\log\left|x^2 - \left(2\cos\frac{3\pi}{2}\right)x + 1\right| +$$

$$\frac{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(420\cos\dfrac{\pi}{5} + 592\right)\cos\dfrac{3\pi}{5} - 200\cos\dfrac{\pi}{5} - 285}{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5} - \dfrac{\cos\dfrac{3\pi}{5}\left(\left(24\cos\dfrac{\pi}{5} + 32\right)\cos^2\dfrac{3\pi}{5} + \left(92\cos\dfrac{\pi}{5} + 170\right)\cos\dfrac{3\pi}{5} + 200\cos\dfrac{\pi}{5} + 285\right)}{\left(288\cos\dfrac{\pi}{5} + 480\right)\cos^2\dfrac{3\pi}{5} + \left(720 - 288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5} - 480\cos^2\dfrac{\pi}{5} - 720\cos\dfrac{\pi}{5}}} +$$

$$\left(\sin\frac{3\pi}{5}\right)\cdot\arctan^{-1}\frac{x - \cos\dfrac{3\pi}{5}}{\sin\dfrac{3\pi}{5}}$$

$$-\frac{3\log|x+1|}{\left(12\cos\dfrac{\pi}{5} + 20\right)\cos\dfrac{3\pi}{5} + 20\cos\dfrac{\pi}{5} + 30} + c$$

$c \in \mathbb{R}$

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