Projeto Mathematical Ramblings

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Teorema de Pitágoras.

$(A-B) \perp (A-C) \;\Rightarrow\; \|B-C\|^2 = \|A-B\|^2 + \|A-C\|^2$

Demonstração:

$\langle (A-B),(A-C)\rangle = 0 \;\Rightarrow\; \langle A,A\rangle - \langle A,C\rangle - \langle B,A\rangle + \langle B,C\rangle = 0 \;\Rightarrow$

$\Rightarrow \langle A,A\rangle + \langle B,C\rangle = \langle A,C\rangle + \langle B,A\rangle$ (I)

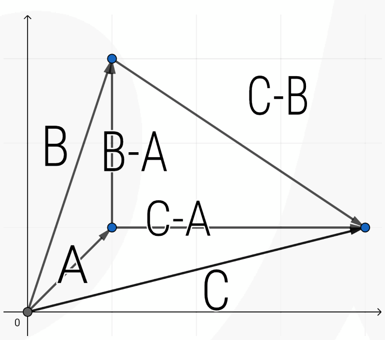
$\|B-C\|^2 = \langle (B-C),(B-C)\rangle = \langle B,B\rangle + \langle C,C\rangle - 2\langle B,C\rangle$ (II)

$(\|A-B\| + \|A-C\|)^2 = \langle (A-B),(A-B)\rangle + \langle (A-C),(A-C)\rangle + 2\|A-B\|\|A-C\| =$

$= \langle A,A\rangle + \langle B,B\rangle - 2\langle A,B\rangle + \langle A,A\rangle + \langle C,C\rangle - 2\langle A,C\rangle + 2\|A-B\|\|A-C\| \overset{(II)}{=}$

$\overset{(II)}{=} \|B-C\|^2 + 2\langle B,C\rangle + 2\langle A,A\rangle - 2\langle A,B\rangle - 2\langle A,C\rangle + 2\|A-B\|\|A-C\| \overset{(I)}{=} \|B-C\|^2 + 2\|A-B\|\|A-C\|$

Logo, $\boxed{\|B-C\|^2 = \|A-B\|^2 + \|A-C\|^2}$.



Quod Erat Demonstrandum.

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Última versão do documento (podem haver correções e/ou aprimoramentos): "bit.ly/mathematicalramblings_public".

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