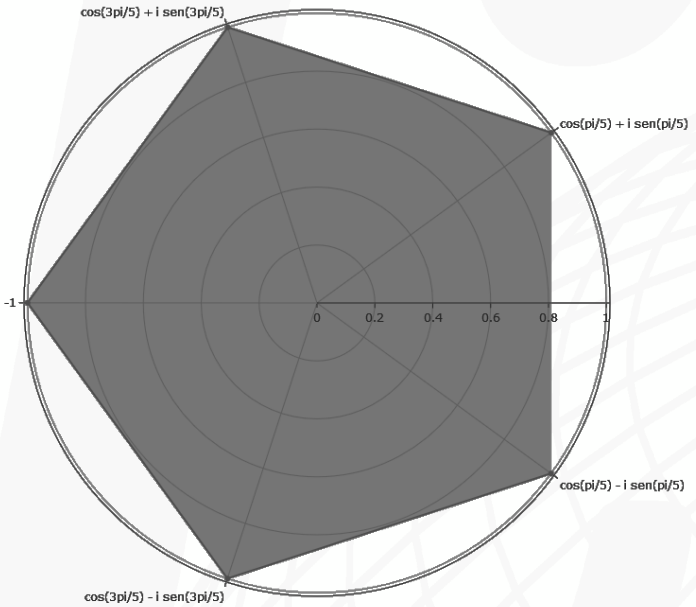
$$\int \frac{dx}{x^5+1}.$$

Afim de decompor $\frac{1}{x^5+1}$ em frações parciais, calculemos as raízes quintas de $-1$, que estão graficamente representadas abaixo:




Logo $\frac{1}{x^5+1} = \frac{Ax+B}{x^2-\left(2\cos\frac{\pi}{5}\right)x+1} + \frac{Cx+D}{x^2-\left(2\cos\frac{3\pi}{5}\right)x+1} + \frac{E}{x+1}$. (I)

Donde, resolvendo o sistema:

$$\begin{cases} B+D+E=1 \\ \left[4-4\cos\left(\frac{3\pi}{5}\right)\right]A+\left[4-4\cos\left(\frac{3\pi}{5}\right)\right]B+\left[4-4\cos\left(\frac{\pi}{5}\right)\right]C+\left[4-4\cos\left(\frac{\pi}{5}\right)\right]D+\left[4-4\cos\left(\frac{3\pi}{5}\right)\right]\left[4-4\cos\left(\frac{\pi}{5}\right)\right]E=1 \\ \left[30-24\cos\left(\frac{3\pi}{5}\right)\right]A+\left[15-12\cos\left(\frac{3\pi}{5}\right)\right]B+\left[30-24\cos\left(\frac{\pi}{5}\right)\right]C+\left[15-12\cos\left(\frac{\pi}{5}\right)\right]D+\left[5-4\cos\left(\frac{3\pi}{5}\right)\right]\left[5-4\cos\left(\frac{\pi}{5}\right)\right]E=1 \\ \left[120-72\cos\left(\frac{3\pi}{5}\right)\right]A+\left[40-24\cos\left(\frac{3\pi}{5}\right)\right]B+\left[120-72\cos\left(\frac{\pi}{5}\right)\right]C+\left[40-24\cos\left(\frac{\pi}{5}\right)\right]D+\left[10-6\cos\left(\frac{3\pi}{5}\right)\right]\left[10-6\cos\left(\frac{\pi}{5}\right)\right]E=1 \\ \left[10+8\cos\left(\frac{3\pi}{5}\right)\right]A-\left[5+4\cos\left(\frac{3\pi}{5}\right)\right]B+\left[10+8\cos\left(\frac{\pi}{5}\right)\right]C-\left[5+4\cos\left(\frac{\pi}{5}\right)\right]D+\left[5+4\cos\left(\frac{3\pi}{5}\right)\right]\left[5+4\cos\left(\frac{\pi}{5}\right)\right]E=1 \end{cases},$$

obtemos:

$$\begin{cases} A = \dfrac{\left(24\cos^2\dfrac{\pi}{5}+92\cos\dfrac{\pi}{5}+200\right)\cos\dfrac{3\pi}{5}+32\cos^2\dfrac{\pi}{5}+170\cos\dfrac{\pi}{5}+285}{\left(288\cos\dfrac{\pi}{5}+480\right)\cos^2\dfrac{3\pi}{5}+\left(720-288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5}-480\cos^2\dfrac{\pi}{5}-720\cos\dfrac{\pi}{5}} \\ B = -\dfrac{\left(288\cos^2\dfrac{\pi}{5}+420\cos\dfrac{\pi}{5}-200\right)\cos\dfrac{3\pi}{5}+480\cos^2\dfrac{\pi}{5}+592\cos\dfrac{\pi}{5}-285}{\left(288\cos\dfrac{\pi}{5}+480\right)\cos^2\dfrac{3\pi}{5}+\left(720-288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5}-480\cos^2\dfrac{\pi}{5}-720\cos\dfrac{\pi}{5}} \\ C = -\dfrac{\left(24\cos\dfrac{\pi}{5}+32\right)\cos^2\dfrac{3\pi}{5}+\left(92\cos\dfrac{\pi}{5}+170\right)\cos\dfrac{3\pi}{5}+200\cos\dfrac{\pi}{5}+285}{\left(288\cos\dfrac{\pi}{5}+480\right)\cos^2\dfrac{3\pi}{5}+\left(720-288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5}-480\cos^2\dfrac{\pi}{5}-720\cos\dfrac{\pi}{5}} \\ D = \dfrac{\left(288\cos\dfrac{\pi}{5}+480\right)\cos^2\dfrac{3\pi}{5}+\left(420\cos\dfrac{\pi}{5}+592\right)\cos\dfrac{3\pi}{5}-200\cos\dfrac{\pi}{5}-285}{\left(288\cos\dfrac{\pi}{5}+480\right)\cos^2\dfrac{3\pi}{5}+\left(720-288\cos^2\dfrac{\pi}{5}\right)\cos\dfrac{3\pi}{5}-480\cos^2\dfrac{\pi}{5}-720\cos\dfrac{\pi}{5}} \\ E = -\dfrac{3}{\left(12\cos\dfrac{\pi}{5}+20\right)\cos\dfrac{3\pi}{5}+20\cos\dfrac{\pi}{5}+30} \end{cases} . \text{ (II)}$$

De (I) obtemos:

$$\frac{1}{x^5+1} = \frac{A}{2}\cdot\frac{2x-2\cos\dfrac{\pi}{5}}{x^2-\left(2\cos\dfrac{\pi}{5}\right)x+1}+\frac{B+A\cos\dfrac{\pi}{5}}{\sin^2\dfrac{\pi}{5}}\cdot\frac{1}{\left(\dfrac{x-\cos\dfrac{\pi}{5}}{\sin\dfrac{\pi}{5}}\right)^2+1}+\frac{C}{2}\cdot\frac{2x-2\cos\dfrac{3\pi}{5}}{x^2-\left(2\cos\dfrac{3\pi}{5}\right)x+1}+\frac{D+C\cos\dfrac{3\pi}{5}}{\sin^2\dfrac{3\pi}{5}}\cdot\frac{1}{\left(\dfrac{x-\cos\dfrac{3\pi}{5}}{\sin\dfrac{3\pi}{5}}\right)^2+1}+\frac{E}{x+1}. \text{ (III)}$$

Substituindo (II) em (III):

$$\begin{aligned}
\int\frac{dx}{x^5+1} \overset{x\neq\cos\frac{\pi}{5}}{=}\ & \frac{\left(24\cos^2\frac{\pi}{5}+92\cos\frac{\pi}{5}+200\right)\cos\frac{3\pi}{5}+32\cos^2\frac{\pi}{5}+170\cos\frac{\pi}{5}+285}{2\left[\left(288\cos\frac{\pi}{5}+480\right)\cos^2\frac{3\pi}{5}+\left(720-288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5}-480\cos^2\frac{\pi}{5}-720\cos\frac{\pi}{5}\right]}\log\left|x^2-\left(2\cos\frac{\pi}{5}\right)x+1\right|+ \\
& \frac{\cos\frac{\pi}{5}\left[\left(24\cos^2\frac{\pi}{5}+92\cos\frac{\pi}{5}+200\right)\cos\frac{3\pi}{5}+32\cos^2\frac{\pi}{5}+170\cos\frac{\pi}{5}+285\right]}{\left(288\cos\frac{\pi}{5}+480\right)\cos^2\frac{3\pi}{5}+\left(720-288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5}-480\cos^2\frac{\pi}{5}-720\cos\frac{\pi}{5}}-\frac{\left(288\cos^2\frac{\pi}{5}+420\cos\frac{\pi}{5}-200\right)\cos\frac{3\pi}{5}+480\cos^2\frac{\pi}{5}+592\cos\frac{\pi}{5}-285}{\left(288\cos\frac{\pi}{5}+480\right)\cos^2\frac{3\pi}{5}+\left(720-288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5}-480\cos^2\frac{\pi}{5}-720\cos\frac{\pi}{5}}- \\
& \left(\sin\frac{\pi}{5}\right)\cdot\arctan^{-1}\frac{x-\cos\frac{\pi}{5}}{\sin\frac{\pi}{5}} \\
& -\frac{\left(24\cos\frac{\pi}{5}+32\right)\cos^2\frac{3\pi}{5}+\left(92\cos\frac{\pi}{5}+170\right)\cos\frac{3\pi}{5}+200\cos\frac{\pi}{5}+285}{2\left[\left(288\cos\frac{\pi}{5}+480\right)\cos^2\frac{3\pi}{5}+\left(720-288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5}-480\cos^2\frac{\pi}{5}-720\cos\frac{\pi}{5}\right]}\log\left|x^2-\left(2\cos\frac{3\pi}{2}\right)x+1\right|+ \\
& \frac{\left(288\cos\frac{\pi}{5}+480\right)\cos^2\frac{3\pi}{5}+\left(420\cos\frac{\pi}{5}+592\right)\cos\frac{3\pi}{5}-200\cos\frac{\pi}{5}-285}{\left(288\cos\frac{\pi}{5}+480\right)\cos^2\frac{3\pi}{5}+\left(720-288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5}-480\cos^2\frac{\pi}{5}-720\cos\frac{\pi}{5}-\frac{\cos\frac{3\pi}{5}\left(\left(24\cos\frac{\pi}{5}+32\right)\cos^2\frac{3\pi}{5}+\left(92\cos\frac{\pi}{5}+170\right)\cos\frac{3\pi}{5}+200\cos\frac{\pi}{5}+285\right)}{\left(288\cos\frac{\pi}{5}+480\right)\cos^2\frac{3\pi}{5}+\left(720-288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5}-480\cos^2\frac{\pi}{5}-720\cos\frac{\pi}{5}}}+ \\
& \left(\sin\frac{3\pi}{5}\right)\cdot\arctan^{-1}\frac{x-\cos\frac{3\pi}{5}}{\sin\frac{3\pi}{5}} \\
& -\frac{3\log|x+1|}{\left(12\cos\frac{\pi}{5}+20\right)\cos\frac{3\pi}{5}+20\cos\frac{\pi}{5}+30}+c
\end{aligned}$$

$c \in \mathbb{R}$

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Última versão do documento (podem haver correções e/ou aprimoramentos):
"bit.ly/mathematicalramblings_public".

Sugestões, comunicar erros: "a.vandre.g@gmail.com".