Determine os extremos absolutos, caso existam, da função $f(t) = t + \cot\left(\frac{t}{2}\right)$ no intervalo $\left[\frac{\pi}{4}, \frac{7\pi}{4}\right]$.

Resolução:

$$1 = \frac{2\tan\left(\frac{\pi}{8}\right)}{1 - \tan^2\left(\frac{\pi}{8}\right)} \;\Rightarrow\; \tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1 \;\wedge\; \tan\left(\frac{7\pi}{8}\right) = 1 - \sqrt{2}$$

$$f\left(\frac{\pi}{4}\right) = \frac{\pi}{4} + \frac{1}{\sqrt{2} - 1}$$

$$f\left(\frac{7\pi}{4}\right) = \frac{7\pi}{4} + \frac{1}{1 - \sqrt{2}}$$

$$f'(t) = 1 - \frac{1}{2} \cdot \csc^2\left(\frac{t}{2}\right)$$

$$\mathbb{U} = \left[\frac{\pi}{4}, \frac{7\pi}{4}\right] \;\wedge\; f'(t) = 0 \;\Rightarrow\; t = \frac{\pi}{2} \;\vee\; t = \frac{3\pi}{2}$$

$$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + 1$$

$$f\left(\frac{3\pi}{2}\right) = \frac{3\pi}{2} - 1$$

$$\frac{3\pi}{2} - 1 > \frac{\pi}{4} + \frac{1}{\sqrt{2} - 1} > \frac{7\pi}{4} + \frac{1}{1 - \sqrt{2}} > \frac{\pi}{2} + 1$$

Logo os pontos de extremos absolutos são $t = \frac{3\pi}{2} - 1$ e $t = \frac{\pi}{2} + 1$.

Documento compilado em Wednesday 12th March, 2025, 22:40, tempo no servidor.

Sugestões, comunicar erros: "a.vandre.g@gmail.com".