$\begin{array}{c} \textbf{Projeto Mathematical Ramblings} \\ \text{mathematical ramblings.blogspot.com} \end{array}$

$$\int \frac{dx}{x^4 + 1}.$$

$$\frac{1}{x^4+1} = \frac{1}{(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)} = \frac{1}{4} \cdot \frac{-\sqrt{2}x+2}{x^2-\sqrt{2}x+1} + \frac{1}{4} \cdot \frac{\sqrt{2}x+2}{x^2+\sqrt{2}x+1}$$

$$\int \frac{dx}{x^4 + 1} = \frac{-1}{2} \int \frac{\sqrt{2}x - 2}{(\sqrt{2}x - 1)^2 + 1} dx + \frac{1}{2} \int \frac{\sqrt{2}x + 2}{(\sqrt{2}x + 1)^2 + 1} dx = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{v + 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{v + 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{v + 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{v + 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{v + 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{v + 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{\sqrt{2}}{4} \int \frac{u - 1}{v^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} dv = \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}{u^2 + 1} du + \frac{-\sqrt{2}}{4} \int \frac{u - 1}$$

$$= \boxed{\frac{-\sqrt{2}}{8}\log[(\sqrt{2}x-1)^2+1] + \frac{\sqrt{2}}{8}\log[(\sqrt{2}x+1)^2+1] + \frac{\sqrt{2}}{4}\arctan(\sqrt{2}x-1) + \frac{\sqrt{2}}{4}\arctan(\sqrt{2}x+1) + c}$$

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Última versão do documento (podem haver correções e/ou aprimoramentos): "bit.ly/mathematicalramblings_public".

Sugestões, comunicar erros: "a.vandre.g@gmail.com".



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