

Calcular $I = \int_0^a \sqrt{a^2 - x^2} \, dx, a > 0.$

$$I = a \int_0^a \sqrt{1 - \left(\frac{x}{a}\right)^2} \, dx$$

$$\text{Seja } y = \frac{x}{a}. \, dy = \frac{dx}{a}$$

$$I = a^2 \int_0^1 \sqrt{1 - y^2} \, dy$$

$$\text{Seja } y = \sin \theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}. \, dy = \cos \theta \, d\theta.$$

$$I = a^2 \int_0^{\pi/2} \cos^2 \theta \, d\theta = a^2 \int_0^{\pi/2} \frac{(\cos 2\theta) + 1}{2} d\theta = a^2 \int_0^{\pi/2} \cos 2\theta \, d\theta + \frac{a^2 \pi}{4}$$

$$\text{Seja } \varphi = 2\theta. \, d\varphi = 2d\theta.$$

$$I = \frac{a^2}{2} \int_0^\pi \cos \varphi \, d\varphi + \frac{a^2 \pi}{4} = \boxed{\frac{a^2 \pi}{4}}$$

Documento compilado em Thursday 13th March, 2025, 20:48, tempo no servidor.

Sugestões, comunicar erros: "a.vandre.g@gmail.com".

Licença de uso:    Atribuição-NãoComercial-CompartilhaIgual (CC BY-NC-SA).