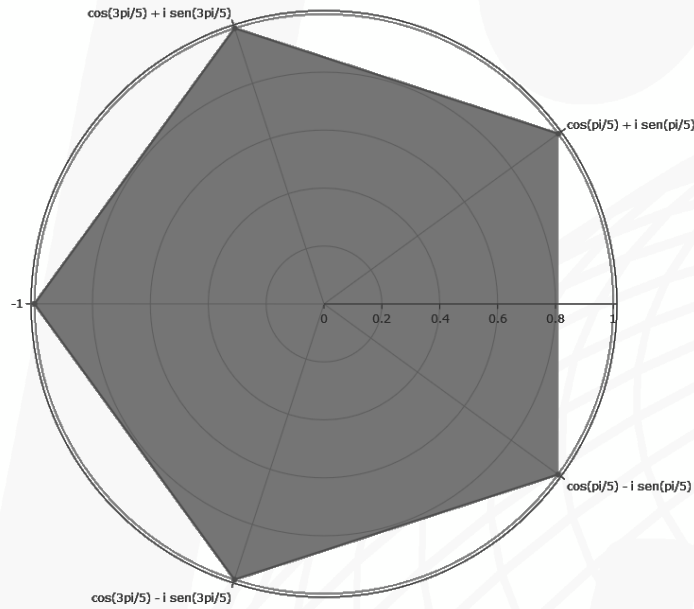


$$\int \frac{dx}{x^5 + 1}.$$

Afim de decompor $\frac{1}{x^5 + 1}$ em frações parciais, calculemos as raízes quintas de -1 , que estão graficamente representadas abaixo:



$$\text{Logo } \frac{1}{x^5 + 1} = \frac{Ax + B}{x^2 - \left(2 \cos \frac{\pi}{5}\right)x + 1} + \frac{Cx + D}{x^2 - \left(2 \cos \frac{3\pi}{5}\right)x + 1} + \frac{E}{x + 1}. \quad (\text{I})$$

Donde, resolvendo o sistema:

$$\begin{cases} B + D + E = 1 \\ \begin{bmatrix} 4 - 4 \cos \left(\frac{3\pi}{5} \right) \\ 30 - 24 \cos \left(\frac{3\pi}{5} \right) \\ 120 - 72 \cos \left(\frac{3\pi}{5} \right) \\ 10 + 8 \cos \left(\frac{3\pi}{5} \right) \end{bmatrix} A + \begin{bmatrix} 4 - 4 \cos \left(\frac{3\pi}{5} \right) \\ 15 - 12 \cos \left(\frac{3\pi}{5} \right) \\ 40 - 24 \cos \left(\frac{3\pi}{5} \right) \\ 5 + 4 \cos \left(\frac{3\pi}{5} \right) \end{bmatrix} B + \begin{bmatrix} 4 - 4 \cos \left(\frac{\pi}{5} \right) \\ 30 - 24 \cos \left(\frac{\pi}{5} \right) \\ 120 - 72 \cos \left(\frac{\pi}{5} \right) \\ 10 + 8 \cos \left(\frac{\pi}{5} \right) \end{bmatrix} C + \begin{bmatrix} 4 - 4 \cos \left(\frac{\pi}{5} \right) \\ 15 - 12 \cos \left(\frac{\pi}{5} \right) \\ 40 - 24 \cos \left(\frac{\pi}{5} \right) \\ 5 + 4 \cos \left(\frac{\pi}{5} \right) \end{bmatrix} D + \begin{bmatrix} 4 - 4 \cos \left(\frac{3\pi}{5} \right) \\ 5 - 4 \cos \left(\frac{3\pi}{5} \right) \\ 10 - 6 \cos \left(\frac{3\pi}{5} \right) \\ 5 + 4 \cos \left(\frac{3\pi}{5} \right) \end{bmatrix} E = \begin{bmatrix} 4 - 4 \cos \left(\frac{\pi}{5} \right) \\ 5 - 4 \cos \left(\frac{\pi}{5} \right) \\ 10 - 6 \cos \left(\frac{\pi}{5} \right) \\ 5 + 4 \cos \left(\frac{\pi}{5} \right) \end{bmatrix} \end{cases} \quad ,$$

obtemos:

$$\left\{ \begin{array}{l} A = \frac{\left(24\cos^2\frac{\pi}{5} + 92\cos\frac{\pi}{5} + 200\right)\cos\frac{3\pi}{5} + 32\cos^2\frac{\pi}{5} + 170\cos\frac{\pi}{5} + 285}{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}} \\ B = -\frac{\left(288\cos^2\frac{\pi}{5} + 420\cos\frac{\pi}{5} - 200\right)\cos\frac{3\pi}{5} + 480\cos^2\frac{\pi}{5} + 592\cos\frac{\pi}{5} - 285}{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}} \\ C = -\frac{\left(24\cos\frac{\pi}{5} + 32\right)\cos^2\frac{3\pi}{5} + \left(92\cos\frac{\pi}{5} + 170\right)\cos\frac{3\pi}{5} + 200\cos\frac{\pi}{5} + 285}{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}} \\ D = \frac{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(420\cos\frac{\pi}{5} + 592\right)\cos\frac{3\pi}{5} - 200\cos\frac{\pi}{5} - 285}{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}} \\ E = -\frac{3}{\left(12\cos\frac{\pi}{5} + 20\right)\cos\frac{3\pi}{5} + 20\cos\frac{\pi}{5} + 30} \end{array} \right. \cdot (II)$$

De (I) obtemos:




$$\frac{1}{x^5 + 1} = \frac{A}{2} \cdot \frac{2x - 2\cos\frac{\pi}{5}}{x^2 - \left(2\cos\frac{\pi}{5}\right)x + 1} + \frac{B + A\cos\frac{\pi}{5}}{\sin^2\frac{\pi}{5}} \cdot \frac{1}{\left(\frac{x - \cos\frac{\pi}{5}}{\sin\frac{\pi}{5}}\right)^2 + 1} + \frac{C}{2} \cdot \frac{2x - 2\cos\frac{3\pi}{5}}{x^2 - \left(2\cos\frac{3\pi}{5}\right)x + 1} + \frac{D + C\cos\frac{3\pi}{5}}{\sin^2\frac{3\pi}{5}} \cdot \frac{1}{\left(\frac{x - \cos\frac{3\pi}{5}}{\sin\frac{3\pi}{5}}\right)^2 + 1} + \frac{E}{x + 1}. \quad (III)$$

Substituindo (II) em (III):

$$\begin{aligned} \int \frac{dx}{x^5 + 1} \quad x \neq \cos\frac{\pi}{5} &= \frac{\left(24\cos^2\frac{\pi}{5} + 92\cos\frac{\pi}{5} + 200\right)\cos\frac{3\pi}{5} + 32\cos^2\frac{\pi}{5} + 170\cos\frac{\pi}{5} + 285}{2\left[\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}\right]} \log\left|x^2 - \left(2\cos\frac{\pi}{5}\right)x + 1\right| + \\ &\frac{\cos\frac{\pi}{5}\left[\left(24\cos^2\frac{\pi}{5} + 92\cos\frac{\pi}{5} + 200\right)\cos\frac{3\pi}{5} + 32\cos^2\frac{\pi}{5} + 170\cos\frac{\pi}{5} + 285\right]}{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}} - \frac{\left(288\cos^2\frac{\pi}{5} + 420\cos\frac{\pi}{5} - 200\right)\cos\frac{3\pi}{5} + 480\cos^2\frac{\pi}{5} + 592\cos\frac{\pi}{5} - 285}{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}} \\ &\quad \left(\sin\frac{\pi}{5}\right) \cdot \arctan -1 \frac{x - \cos\frac{\pi}{5}}{\sin\frac{\pi}{5}} \\ &- \frac{\left(24\cos\frac{\pi}{5} + 32\right)\cos^2\frac{3\pi}{5} + \left(92\cos\frac{\pi}{5} + 170\right)\cos\frac{3\pi}{5} + 200\cos\frac{\pi}{5} + 285}{2\left[\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}\right]} \log\left|x^2 - \left(2\cos\frac{3\pi}{5}\right)x + 1\right| + \\ &\frac{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(420\cos\frac{\pi}{5} + 592\right)\cos\frac{3\pi}{5} - 200\cos\frac{\pi}{5} - 285}{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}} - \frac{\cos\frac{3\pi}{5}\left[\left(24\cos\frac{\pi}{5} + 32\right)\cos^2\frac{3\pi}{5} + \left(92\cos\frac{\pi}{5} + 170\right)\cos\frac{3\pi}{5} + 200\cos\frac{\pi}{5} + 285\right]}{\left(288\cos\frac{\pi}{5} + 480\right)\cos^2\frac{3\pi}{5} + \left(720 - 288\cos^2\frac{\pi}{5}\right)\cos\frac{3\pi}{5} - 480\cos^2\frac{\pi}{5} - 720\cos\frac{\pi}{5}} \\ &+ \frac{\left(\sin\frac{3\pi}{5}\right) \cdot \arctan -1 \frac{x - \cos\frac{3\pi}{5}}{\sin\frac{3\pi}{5}}}{\left(\sin\frac{3\pi}{5}\right) \cdot \arctan -1 \frac{x - \cos\frac{3\pi}{5}}{\sin\frac{3\pi}{5}}} \\ &- \frac{3\log|x + 1|}{\left(12\cos\frac{\pi}{5} + 20\right)\cos\frac{3\pi}{5} + 20\cos\frac{\pi}{5} + 30} + c \\ c &\in \mathbb{R} \end{aligned}$$

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