

Projeto Mathematical Ramblings

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Livro senos e cossenos explícitos.

$$\sin \frac{\pi}{2} = 1$$

$$\cos \frac{\pi}{2} = 0$$

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\sin \frac{\pi}{8} = \frac{\sqrt{\sqrt{2}-1}}{2^{\frac{3}{4}}}$$

$$\cos \frac{\pi}{8} = \frac{\sqrt{\sqrt{2}+1}}{2^{\frac{3}{4}}}$$

$$\sin \frac{\pi}{16} = \frac{\sqrt{2^{\frac{3}{4}} - \sqrt{\sqrt{2}+1}}}{2^{\frac{7}{8}}}$$

$$\cos \frac{\pi}{16} = \frac{\sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}{2^{\frac{7}{8}}}$$

$$\sin \frac{\pi}{32} = \frac{\sqrt{2^{\frac{7}{8}} - \sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}}{2^{\frac{15}{16}}}$$

$$\cos \frac{\pi}{32} = \frac{\sqrt{2^{\frac{7}{8}} + \sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}}{2^{\frac{15}{16}}}$$

$$\sin \frac{\pi}{64} = \frac{\sqrt{2^{\frac{15}{16}} - \sqrt{2^{\frac{7}{8}} + \sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}}}{2^{\frac{31}{32}}}$$

$$\cos \frac{\pi}{64} = \frac{\sqrt{2^{\frac{15}{16}} + \sqrt{2^{\frac{7}{8}} + \sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}}}{2^{\frac{31}{32}}}$$

$$\sin \frac{\pi}{128} = \frac{\sqrt{2^{\frac{31}{32}} - \sqrt{2^{\frac{15}{16}} + \sqrt{2^{\frac{7}{8}} + \sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}}}}{2^{\frac{63}{64}}}$$

$$\cos \frac{\pi}{128} = \frac{\sqrt{2^{\frac{31}{32}} + \sqrt{2^{\frac{15}{16}} + \sqrt{2^{\frac{7}{8}} + \sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}}}}{2^{\frac{63}{64}}}$$

$$\sin \frac{\pi}{256} = \frac{\sqrt{2^{\frac{63}{64}} - \sqrt{2^{\frac{31}{32}} + \sqrt{2^{\frac{15}{16}} + \sqrt{2^{\frac{7}{8}} + \sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}}}}}}{2^{\frac{127}{128}}}$$

$$\cos \frac{\pi}{256} = \frac{\sqrt{2^{\frac{63}{64}} + \sqrt{2^{\frac{31}{32}} + \sqrt{2^{\frac{15}{16}} + \sqrt{2^{\frac{7}{8}} + \sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}}}}}}{2^{\frac{127}{128}}}$$

$$\sin \frac{\pi}{512} = \frac{\sqrt{2^{\frac{127}{128}} - \sqrt{2^{\frac{63}{64}} + \sqrt{2^{\frac{31}{32}} + \sqrt{2^{\frac{15}{16}} + \sqrt{2^{\frac{7}{8}} + \sqrt{2^{\frac{3}{4}} + \sqrt{\sqrt{2}+1}}}}}}}}}}{2^{\frac{255}{256}}}$$

$$\cos \frac{\pi}{512}$$

\sin

$$\cos \frac{\pi}{1024}$$

$$\sin \frac{\pi}{2048}$$

$$\cos \frac{\pi}{2048}$$

$$\sin \frac{\pi}{4096}$$

$$\cos \frac{\pi}{4096}$$

$$\sin \frac{\pi}{8192}$$

$$\cos \frac{\pi}{819}$$

$$\sin \frac{\pi}{1638}$$

$$\cos \frac{\pi}{1638}$$

$$\sin \frac{\pi}{32768} = \frac{\sqrt{2 \frac{8191}{8192} - \sqrt{2 \frac{4095}{4096} + \sqrt{2 \frac{2047}{2048} + \sqrt{2 \frac{1023}{1024} + \sqrt{2 \frac{511}{512} + \sqrt{2 \frac{255}{256} + \sqrt{2 \frac{127}{128} + \sqrt{2 \frac{63}{64} + \sqrt{2 \frac{31}{32} + \sqrt{2 \frac{15}{16} + \sqrt{2 \frac{7}{8} + \sqrt{2 \frac{3}{4} + \sqrt{\sqrt{2}+1}}}}}}}}}}}}}}}}}}}}}}}{2 \frac{16383}{16384}}$$

$$\cos \frac{\pi}{32768} = \frac{\sqrt{2 \frac{8191}{8192} + \sqrt{2 \frac{4095}{4096} + \sqrt{2 \frac{2047}{2048} + \sqrt{2 \frac{1023}{1024} + \sqrt{2 \frac{511}{512} + \sqrt{2 \frac{255}{256} + \sqrt{2 \frac{127}{128} + \sqrt{2 \frac{63}{64} + \sqrt{2 \frac{31}{32} + \sqrt{2 \frac{15}{16} + \sqrt{2 \frac{7}{8} + \sqrt{2 \frac{3}{4} + \sqrt{\sqrt{2}+1}}}}}}}}}}}}}}}}}}}}}}}{2 \frac{16383}{16384}}$$

$$\sin \frac{\pi}{65536} = \frac{\sqrt{2 \frac{16383}{16384} - \sqrt{2 \frac{8191}{8192} + \sqrt{2 \frac{4095}{4096} + \sqrt{2 \frac{2047}{2048} + \sqrt{2 \frac{1023}{1024} + \sqrt{2 \frac{511}{512} + \sqrt{2 \frac{255}{256} + \sqrt{2 \frac{127}{128} + \sqrt{2 \frac{63}{64} + \sqrt{2 \frac{31}{32} + \sqrt{2 \frac{15}{16} + \sqrt{2 \frac{7}{8} + \sqrt{2 \frac{3}{4} + \sqrt{\sqrt{2}+1}}}}}}}}}}}}}}}}}}}}}}}{2 \frac{32767}{32768}}$$

$$\cos \frac{\pi}{65536} = \frac{\sqrt{2 \frac{16383}{16384} + \sqrt{2 \frac{8191}{8192} + \sqrt{2 \frac{4095}{4096} + \sqrt{2 \frac{2047}{2048} + \sqrt{2 \frac{1023}{1024} + \sqrt{2 \frac{511}{512} + \sqrt{2 \frac{255}{256} + \sqrt{2 \frac{127}{128} + \sqrt{2 \frac{63}{64} + \sqrt{2 \frac{31}{32} + \sqrt{2 \frac{15}{16} + \sqrt{2 \frac{7}{8} + \sqrt{2 \frac{3}{4} + \sqrt{\sqrt{2}+1}}}}}}}}}}}}}}}}}}}}}}}{2 \frac{32767}{32768}}$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\sin \frac{\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\cos \frac{\pi}{12} = \frac{\sqrt{\sqrt{3}+2}}{2}$$

$$\sin \frac{\pi}{24} = \frac{\sqrt{2-\sqrt{\sqrt{3}+2}}}{2}$$

$$\cos \frac{\pi}{24} = \frac{\sqrt{\sqrt{\sqrt{3}+2}+2}}{2}$$

$$\sin \frac{\pi}{48} = \frac{\sqrt{2-\sqrt{\sqrt{\sqrt{3}+2}+2}}}{2}$$

$$\cos \frac{\pi}{48} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{3}+2}+2}+2}}{2}$$

$$\sin \frac{\pi}{96} = \frac{\sqrt{2-\sqrt{\sqrt{\sqrt{\sqrt{3}+2}+2}+2}}}{2}$$

$$\cos \frac{\pi}{96} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{3}+2}+2}+2}+2}}{2}$$

$$\sin \frac{\pi}{192} = \frac{\sqrt{2 - \sqrt{\sqrt{\sqrt{\sqrt{3} + 2} + 2} + 2} + 2}}{2}$$

$$\cos \frac{\pi}{192} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{3}+2}+2}+2}+2}+2}+2}{2}$$

$$\sin \frac{\pi}{384} = \frac{\sqrt{2 - \sqrt[7]{\sqrt[6]{\sqrt[5]{\sqrt[4]{\sqrt[3]{\sqrt{3+2+2+2+2+2}}}}}}}}}{2}$$

$$\cos \frac{\pi}{384} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{3+2+2+2+2+2+2}}}}}}}}{2}$$

$$\sin \frac{\pi}{768} = \sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt{3+2+2+2+2+2}}}}}^2}}$$

$$\cos \frac{\pi}{768} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{3}+2+2+2+2+2+2+2}}}}}}}}{2}$$

[illegible]

[illegible]

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[illegible]

$$\sin \frac{\pi}{6144} = \sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt{3+2+2+2+2+2+2+2+2+2}}}}} }}}$$

$$\cos \frac{\pi}{6144} = \sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt[2]{\sqrt{3+2+2+2+2+2+2+2+2+2}}}}}}}$$

[illegible]

[illegible]

[illegible]

$$\cos \frac{\pi}{24576} = \frac{\sqrt{2}\sqrt{\sqrt{3}+2+2+2+2+2+2+2+2+2+2+2+2+2+2+2}}{2}$$

[illegible]

$$\cos \frac{\pi}{49152} = \sqrt[2]{\sqrt{3+2+2+2+2+2+2+2+2+2+2+2}}^2$$

[illegible]

$$\cos \frac{\pi}{98304} = \sqrt{2\sqrt{3+2+2+2+2+2+2+2+2+2+2+2+2+2+2}}$$

$$\cos \frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$$

$$\sin \frac{\pi}{10} = \frac{\sqrt{3-\sqrt{5}}}{2^{\frac{3}{2}}}$$

$$\cos \frac{\pi}{10} = \frac{\sqrt{\sqrt{5}+5}}{2^{\frac{3}{2}}}$$

$$\sin \frac{\pi}{20} = \frac{\sqrt{2^{\frac{3}{2}} - \sqrt{\sqrt{5} + 5}}}{2^{\frac{5}{4}}}$$

$$\cos \frac{\pi}{20} = \frac{\sqrt{\sqrt{\sqrt{5}+5}+2^{\frac{3}{2}}}}{2^{\frac{5}{4}}}$$

$$\sin \frac{\pi}{40} = \frac{\sqrt{2^{\frac{5}{4}} - \sqrt{\sqrt{5+5+2^{\frac{3}{2}}}}}}{2^{\frac{9}{8}}}$$

$$\cos \frac{\pi}{40} = \frac{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}}}}{2^{\frac{9}{8}}}$$

$$\sin \frac{\pi}{80} = \frac{\sqrt{2^{\frac{9}{8}} - \sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}}}}}}{2^{\frac{17}{16}}}$$

$$\cos \frac{\pi}{80} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}}}}}}{2^{\frac{17}{16}}}$$

$$\sin \frac{\pi}{160} = \frac{\sqrt{2^{\frac{17}{16}} - \sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}}}}}}}}{2^{\frac{33}{32}}}$$

$$\cos \frac{\pi}{160} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}} + 2^{\frac{17}{16}}}}}}}}{2^{\frac{33}{32}}}$$

$$\sin \frac{\pi}{320} = \frac{\sqrt{2^{\frac{33}{32}} - \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}} + 2^{\frac{17}{16}}}}}}}}}}{2^{\frac{65}{64}}}$$

$$\cos \frac{\pi}{320} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}} + 2^{\frac{17}{16}} + 2^{\frac{33}{32}}}}}}}}}}{2^{\frac{65}{64}}}$$

$$\sin \frac{\pi}{640} = \frac{\sqrt{2^{\frac{65}{64}} - \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}} + 2^{\frac{17}{16}} + 2^{\frac{33}{32}}}}}}}}}}}}{2^{\frac{129}{128}}}$$

$$\cos \frac{\pi}{640} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}} + 2^{\frac{17}{16}} + 2^{\frac{33}{32}} + 2^{\frac{65}{64}}}}}}}}}}}}{2^{\frac{129}{128}}}$$

$$\sin \frac{\pi}{1280} = \frac{\sqrt{2^{\frac{129}{128}} - \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}} + 2^{\frac{17}{16}} + 2^{\frac{33}{32}} + 2^{\frac{65}{64}}}}}}}}}}}}}}{2^{\frac{257}{256}}}$$

$$\cos \frac{\pi}{1280} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}} + 2^{\frac{17}{16}} + 2^{\frac{33}{32}} + 2^{\frac{65}{64}} + 2^{\frac{129}{128}}}}}}}}}}}}}}{2^{\frac{257}{256}}}$$

$$\sin \frac{\pi}{2560} = \frac{\sqrt{2^{\frac{257}{256}} - \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}} + 2^{\frac{17}{16}} + 2^{\frac{33}{32}} + 2^{\frac{65}{64}} + 2^{\frac{129}{128}}}}}}}}}}}}}}}}{2^{\frac{513}{512}}}$$

$$\cos \frac{\pi}{2560} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}}} + 2^{\frac{5}{4}} + 2^{\frac{9}{8}} + 2^{\frac{17}{16}} + 2^{\frac{33}{32}} + 2^{\frac{65}{64}} + 2^{\frac{129}{128}} + 2^{\frac{257}{256}}}}}}}}}}}}}}}}{2^{\frac{513}{512}}}$$

$$\sin \frac{\pi}{5120} = \frac{\sqrt{2^{\frac{513}{512}} - \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}+2^{\frac{5}{4}}+2^{\frac{9}{8}}+2^{\frac{17}{16}}+2^{\frac{33}{32}}+2^{\frac{65}{64}}+2^{\frac{129}{128}}+2^{\frac{257}{256}}}}}}}}}}}}}}}}}}}}}{2^{\frac{1025}{1024}}}}$$

$$\cos \frac{\pi}{5120} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}+2^{\frac{5}{4}}+2^{\frac{9}{8}}+2^{\frac{17}{16}}+2^{\frac{33}{32}}+2^{\frac{65}{64}}+2^{\frac{129}{128}}+2^{\frac{257}{256}}+2^{\frac{513}{512}}}}}}}}}}}}}}}}}}}{2^{\frac{1025}{1024}}}}$$

$$\sin \frac{\pi}{10240} = \frac{\sqrt{2^{\frac{1025}{1024}} - \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}+2^{\frac{5}{4}}+2^{\frac{9}{8}}+2^{\frac{17}{16}}+2^{\frac{33}{32}}+2^{\frac{65}{64}}+2^{\frac{129}{128}}+2^{\frac{257}{256}}+2^{\frac{513}{512}}}}}}}}}}}}}}}}}}}{2^{\frac{2049}{2048}}}}$$

$$\cos \frac{\pi}{10240} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}+2^{\frac{5}{4}}+2^{\frac{9}{8}}+2^{\frac{17}{16}}+2^{\frac{33}{32}}+2^{\frac{65}{64}}+2^{\frac{129}{128}}+2^{\frac{257}{256}}+2^{\frac{513}{512}}+2^{\frac{1025}{1024}}}}}}}}}}}}}}}}}}}{2^{\frac{2049}{2048}}}}$$

$$\sin \frac{\pi}{20480} = \frac{\sqrt{2^{\frac{2049}{2048}} - \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}+2^{\frac{5}{4}}+2^{\frac{9}{8}}+2^{\frac{17}{16}}+2^{\frac{33}{32}}+2^{\frac{65}{64}}+2^{\frac{129}{128}}+2^{\frac{257}{256}}+2^{\frac{513}{512}}+2^{\frac{1025}{1024}}}}}}}}}}}}}}}}}}}{2^{\frac{4097}{4096}}}}$$

$$\cos \frac{\pi}{20480} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}+2^{\frac{5}{4}}+2^{\frac{9}{8}}+2^{\frac{17}{16}}+2^{\frac{33}{32}}+2^{\frac{65}{64}}+2^{\frac{129}{128}}+2^{\frac{257}{256}}+2^{\frac{513}{512}}+2^{\frac{1025}{1024}}+2^{\frac{2049}{2048}}}}}}}}}}}}}}}}}}}{2^{\frac{4097}{4096}}}}$$

$$\sin \frac{\pi}{40960} = \frac{\sqrt{2^{\frac{4097}{4096}} - \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}+2^{\frac{5}{4}}+2^{\frac{9}{8}}+2^{\frac{17}{16}}+2^{\frac{33}{32}}+2^{\frac{65}{64}}+2^{\frac{129}{128}}+2^{\frac{257}{256}}+2^{\frac{513}{512}}+2^{\frac{1025}{1024}}+2^{\frac{2049}{2048}}}}}}}}}}}}}}}}}}}{2^{\frac{8193}{8192}}}}$$

$$\cos \frac{\pi}{40960} = \frac{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}+2^{\frac{5}{4}}+2^{\frac{9}{8}}+2^{\frac{17}{16}}+2^{\frac{33}{32}}+2^{\frac{65}{64}}+2^{\frac{129}{128}}+2^{\frac{257}{256}}+2^{\frac{513}{512}}+2^{\frac{1025}{1024}}+2^{\frac{2049}{2048}}+2^{\frac{4097}{4096}}}}}}}}}}}}}}}}}}}{2^{\frac{8193}{8192}}}}$$




$$\sin \frac{\pi}{81920} = \frac{\sqrt{2^{\frac{8193}{8192}} - \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{5+5+2^{\frac{3}{2}}+2^{\frac{5}{4}}+2^{\frac{9}{8}}+2^{\frac{17}{16}}+2^{\frac{33}{32}}+2^{\frac{65}{64}}+2^{\frac{129}{128}}+2^{\frac{257}{256}}+2^{\frac{513}{512}}+2^{\frac{1025}{1024}}+2^{\frac{2049}{2048}}+2^{\frac{4097}{4096}}}}}}}}}}}}}}}}}}}{2^{\frac{16385}{16384}}}}$$

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Última versão do documento (podem haver correções e/ou aprimoramentos):
["bit.ly/mathematicalramblings_public"](https://bit.ly/mathematicalramblings_public).

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