Calcular $I = \int_0^a \sqrt{a^2 - x^2}\, dx$, $a > 0$.

$$I = a \int_0^a \sqrt{1 - \left(\frac{x}{a}\right)^2}\, dx$$

Seja $y = \frac{x}{a}$. $dy = \frac{dx}{a}$

$$I = a^2 \int_0^1 \sqrt{1 - y^2}\, dy$$

Seja $y = \sin\theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. $dy = \cos\theta\, d\theta$.

$$I = a^2 \int_0^{\pi/2} \cos^2\theta\, d\theta = a^2 \int_0^{\pi/2} \frac{(\cos 2\theta) + 1}{2} d\theta = a^2 \int_0^{\pi/2} \cos 2\theta\, d\theta + \frac{a^2\pi}{4}$$

Seja $\varphi = 2\theta$. $d\varphi = 2d\theta$.

$$I = \frac{a^2}{2} \cancelto{0}{\int_0^{\pi} \cos\varphi\, d\varphi} + \frac{a^2\pi}{4} = \boxed{\frac{a^2\pi}{4}}$$

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