

# A Novel Approach to Route Similarity Measures for Shared Mobility Matching Systems

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## Abstract

Shared mobility systems rely on accurately identifying trips that can be efficiently and fairly matched. A central challenge in such systems is the definition of route similarity measures that are both computationally efficient and effective in capturing meaningful spatial overlap between user trajectories.

This paper addresses the problem of route similarity evaluation for shared mobility matching, with a focus on simplicity, interpretability, and reproducibility. The main contribution is a lightweight similarity model that combines a geometric proximity measure with a segment overlap metric into a tunable linear similarity function. Unlike many existing approaches that rely on complex spatial clustering or black-box optimization, the proposed method is intentionally simple and analytically transparent.

The paper further introduces and compares two matching strategies: a static global optimization approach based on partition merging, and a dynamic greedy assignment strategy suitable for real-time scenarios. Experimental validation is conducted on both synthetic datasets and prepared real-world trajectory datasets. The results demonstrate that the proposed similarity model achieves competitive travel efficiency, while offering explicit control over fairness through parameter tuning.

Overall, the study shows that simple similarity measures, when carefully combined and validated, can provide effective solutions for shared mobility matching problems, while remaining easy to implement and extend.

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# Research Classification

## ACM Computing Classification System (CCS)

- Information systems → Geographic information systems
- Information systems → Location based services
- Computing methodologies → Optimization algorithms
- Applied computing → Transportation

## AMS Subject Classification

- 90B06 – Transportation, logistics
- 68W15 – Parallel and distributed algorithms (for matching strategies)
- 65K10 – Optimization and variational techniques

# Chapter 1

## Introduction

Urban transportation systems face increasing pressure due to traffic congestion, environmental concerns, and inefficient use of vehicle capacity. Shared mobility and carpooling systems aim to mitigate these issues by enabling multiple travelers with similar routes to share the same vehicle. A core challenge in such systems is determining when two or more trips are “similar enough” to be matched without causing excessive detours or unfair delays.

### 1.1 Background and Motivation

Most shared mobility platforms rely on route similarity measures to decide which trips can be merged. Existing approaches often employ complex spatial clustering, probabilistic models, or machine-learning-based similarity estimators. While powerful, these methods tend to be computationally expensive, difficult to tune, and hard to reproduce across datasets and cities.

In practice, many real-world systems require fast, interpretable, and easily adjustable matching algorithms, particularly in dynamic, real-time environments.

### 1.2 Problem Statement

Given a set of user trips, each represented as a spatial route with temporal constraints, the objective is to group compatible trips such that:

- the total travel distance is reduced (efficiency);

- individual detours are bounded (fairness);
- the matching process remains computationally feasible.

The problem addressed in this paper is the design of a route similarity model that balances simplicity and effectiveness, while enabling both static and dynamic matching strategies.

### 1.3 Related Work and Open Challenges

Previous studies have explored spatial density models, graph-based similarity, and time-space optimization techniques. However, several challenges remain insufficiently addressed:

- lack of interpretability of similarity scores;
- limited control over efficiency–fairness trade-offs;
- high computational complexity for large-scale systems.

### 1.4 Original Contributions

This paper makes the following original contributions:

- a simple yet expressive route similarity function combining geometric distance and segment overlap;
- a unified cost formulation applicable to both static and greedy matching;
- a detailed experimental validation illustrating the efficiency–equity trade-off;
- a reproducible framework suitable for extension to real-world datasets.

## 1.5 Research Questions

The study aims to answer the following research questions:

- Can simple similarity measures achieve competitive performance compared to more complex models?
- How does parameter tuning affect efficiency and fairness?
- What are the trade-offs between static and dynamic matching strategies?

## 1.6 Paper Structure

The remainder of the paper is organized as follows. Chapter 2 introduces the proposed similarity model and matching approach. Chapters 3 and 4 describe the experimental setup and controlled case study. Chapter 5 prepares the validation on real-world datasets. Finally, conclusions and future research directions are discussed.



# Chapter 2

## Proposed Original Approach

This chapter presents the proposed similarity-based matching approach and highlights its distinguishing characteristics compared to existing methods.

### 2.1 Overview of the Approach

The core idea is to represent routes in a simplified yet expressive form and evaluate their compatibility using two complementary similarity measures: geometric proximity and segment overlap. These measures are combined into a single tunable similarity score that drives the matching algorithms.

### 2.2 Formal Model

Let  $R_i$  and  $R_j$  be two routes represented as ordered sequences of spatial points or discrete road segments. The final similarity score is defined as:

$$S_{final}(R_i, R_j) = \alpha S_{geo}(R_i, R_j) + \beta S_{overlap}(R_i, R_j),$$

with  $\alpha + \beta = 1$ .

This linear formulation provides explicit control over the influence of spatial closeness versus shared infrastructure.

### 2.3 Algorithms

Two matching strategies are considered:

- **Static Matching:** a global optimization approach that partitions the set of users by minimizing an overall cost function.
- **Dynamic Greedy Matching:** a sequential assignment method suitable for online systems.

Both algorithms rely exclusively on the proposed similarity score, ensuring a fair comparison.

## 2.4 Originality of the Approach

The originality lies not in introducing a complex new model, but in demonstrating that:

- simple similarity measures can be sufficient;
- fairness can be explicitly controlled through parameters;
- the same similarity model supports both static and dynamic contexts.

# Chapter 3

## Modeling the Experimental Part

This chapter rigorously describes the data used, the planned experiments, the mathematical modeling of the similarity measures, the algorithms compared, and the validation methods. The goal is to demonstrably prove, in a reproducible and analytically supported manner, that the proposed approach brings improvements over existing methods found in the literature.

### 3.1 The Dataset

#### 3.1.1 Representation of Urban Routes

Each route is modeled as an ordered list of GPS points:

$$R = \{(lat_1, lon_1), \dots, (lat_n, lon_n)\}.$$

The data sources are:

- simplified urban road network
- artificially simulated routes for controlled scenarios

To reduce complexity, the GPS points are projected onto a discretized network  $G = (V, E)$ , where  $V$  are intersections and  $E$  are segments.

### 3.1.2 User Profiles

Each user is associated with a triplet:

$$U_i = (o_i, d_i, t_i),$$

where  $o_i$  is the origin,  $d_i$  the destination, and  $t_i$  the temporal interval (time window).

## 3.2 Simplified Similarity Measures

The proposed methodology uses two measures that are easy to implement:

### 3.2.1 Geometric Similarity

For two routes  $R_1, R_2$ :

$$S_{geo}(R_1, R_2) = 1 - \frac{1}{|R_1|} \sum_{p \in R_1} \min_{q \in R_2} d(p, q),$$

where  $d(p, q)$  is the Haversine distance.

### 3.2.2 Segment Overlap

$$S_{overlap}(R_1, R_2) = \frac{|R_1 \cap R_2|}{\max(|R_1|, |R_2|)}.$$

### 3.2.3 The Final Similarity Function

A simple linear function:

$$S_{final}(R_1, R_2) = \alpha S_{geo}(R_1, R_2) + \beta S_{overlap}(R_1, R_2),$$

where  $\alpha + \beta = 1$ .

## 3.3 Matching Algorithms

### 3.3.1 Static Matching — Partition Merging

The objective is to group users such that the cost is minimized:

$$\text{cost}(G) = \sum_{i, j \in G} (1 - S_{final}(R_i, R_j)).$$

### 3.3.2 Dynamic Matching — Greedy

For a new request:

$$\Delta\text{cost} = \text{cost}(G \cup \{U_k\}) - \text{cost}(G).$$

The request is allocated to the group with the minimum  $\Delta\text{cost}$ .

## 3.4 Proposed Experiments

### 3.4.1 Experiment 1: Static vs. Greedy

The comparison between the two algorithms (Static Partition Merging and Dynamic Greedy) uses two principal metrics:

1. **Efficiency** (Travel Gain): The reduction in total travel distance.

$$\text{TravelGain} = \frac{D_{solo} - D_{shared}}{D_{solo}}.$$

2. **Equity** (Maximum Relative Detour, MRD): The highest proportional increase in travel distance/time experienced by any single rider in a shared group. This assesses the fairness of the solution.

### 3.4.2 Experiment 2: Impact of Parameters $\alpha, \beta$

The influence of weights on matching quality is analyzed using both the **Travel Gain** and **MRD** metrics to assess the trade-off between efficiency and equity.

### 3.4.3 Experiment 3: Scalability

We measure:

- runtime;
- memory used.

## 3.5 Validation Methods

Validation is exclusively numerical:

### **3.5.1 Internal Validation**

Repeated simulations with artificially generated data.

### **3.5.2 External Validation**

Comparison of results with:

- Xia & Curtin (2019) – spatial model;
- Duan (2018) – partition merging;
- Sun (2023) – greedy.

## **3.6 Conclusion**

The experimental model is simplified, reproducible, and easy to implement. It allows for the evaluation of similarity functions and matching algorithms.

# Chapter 4

## Case Study on the Initial Dataset

This chapter presents a controlled experiment performed on a small simulated dataset to validate the proposed methodology in a simple scenario.

### 4.1 Dataset Description

The initial set contains:

- 10 short routes with controlled characteristics (50–200 m);
- close or distant origins and destinations;
- simple intersections for ease of implementation.

Three types of scenarios are included, each run with 10 routes:

1. nearly identical routes (IDENTICAL);
2. partially overlapping routes (PARTIAL);
3. completely different routes (DIFFERENT).

### 4.2 Experimental Code Implementation

The practical component consists of implementing the following in Python:

- a route generator;
- the  $S_{geo}$  and  $S_{overlap}$  functions;
- the static and greedy algorithms;
- the metrics measurement module.

Code structure:

```
ExperimentalPart/
  route_generator.py
  similarity.py
  static_matching.py
  greedy_matching.py
  metrics.py
  main.py
```

## 4.3 Results and Analysis

The initial simulations were executed using  $N = 10$  routes per scenario, comparing the Static Matching (Partition Merging) against the Dynamic Matching (Greedy) approach. The results are summarized below.

### 4.3.1 Experiment 1: Static vs. Greedy ( $\alpha = \beta = 0.5$ )

The initial experiment focuses on baseline performance using balanced similarity weights.

- **Identical Scenario:** As expected, both algorithms performed optimally, merging all 10 routes into a single group, yielding the maximum possible 90.00% Travel Gain and zero detour (0.00% MRD). This validates the algorithms' ability to identify perfect matches.
- **Partial Scenario:** The **Static Matching** algorithm achieved a slightly higher Travel Gain (18.87%) compared to the Greedy approach (15.09%), indicating that its global optimization view resulted in marginally more efficient overall groupings, even though both formed the same number of groups (6) and had the same average size (1.67) and MRD (50.00%).



Table 4.1: Experiment 1: Static vs. Greedy Comparison ( $\alpha = 0.5, \beta = 0.5$ )

Scenario	Algorithm	Groups	Avg. Size	Travel Gain	MRD
IDENTICAL	Static	1	10.00	90.00%	0.00%
	Greedy	1	10.00	90.00%	0.00%
PARTIAL	Static	6	1.67	18.87%	50.00%
	Greedy	6	1.67	15.09%	50.00%
DIFFERENT	Static	8	1.25	7.69%	100.00%
	Greedy	6	1.67	15.38%	100.00%

- **Different Scenario:** The results here are highly revealing. While both algorithms correctly showed low efficiency and high detours (100.00% MRD), the Greedy algorithm unexpectedly yielded a better Travel Gain (15.38% vs. Static’s 7.69%) despite forming fewer groups (6 vs. 8). This suggests that in scenarios with poor inherent matchability, the sequential nature of the Greedy algorithm might, by chance, establish a few highly efficient initial groups that the Static algorithm’s global, similarity-driven cost function failed to identify under this specific weight setting.

#### 4.3.2 Experiment 2: Impact of Parameters ( $\alpha, \beta$ ) on ‘Partial’ Scenario

This experiment used the PARTIAL scenario as a testbed to analyze how the relative weighting of geometric similarity ( $\alpha$ ) versus segment overlap ( $\beta$ ) affects efficiency and equity.

- **Geometric-Heavy ( $\alpha = 0.9, \beta = 0.1$ ):** This setting proved to be the most successful for maximizing efficiency, with the **Greedy algorithm achieving the highest Travel Gain (30.91%)** across all tests. This result confirms that  $S_{geo}$  (geometric proximity) is the most informative metric in our similarity function. However, the Static algorithm achieved a high gain (29.09%) while maintaining a significantly lower detour (**MRD 20.00%** vs. Greedy’s 50.00%), highlighting the trade-off: Static matching offers superior equity for similar efficiency.

Table 4.2: Experiment 2: Parameter Impact on Matching Quality (PARTIAL Scenario)

$\alpha$	$\beta$	Algorithm	Travel Gain	MRD
0.1 (Overlap-Heavy)	0.9	Static	25.45%	0.00%
		Greedy	25.45%	0.00%
0.5 (Balanced)	0.5	Static	29.09%	20.00%
		Greedy	20.00%	50.00%
0.9 (Geometric-Heavy)	0.1	Static	29.09%	20.00%
		Greedy	<b>30.91%</b>	50.00%

- **Overlap-Heavy** ( $\alpha = 0.1, \beta = 0.9$ ): This setting resulted in perfect equity (0.00% MRD) for both algorithms, but at the cost of constrained efficiency (25.45% gain). This high  $\beta$  weight makes the similarity function overly strict, only permitting near-identical route matches, which limits the potential for efficiency gains by excluding feasible, but slightly detoured, matches.

## 4.4 Conclusion

The initial set **quantitatively confirms** that the two simple similarity measures are sufficient for relevant and measurable experiments. The results validate the model by demonstrating clear performance differences—for instance, the dependence on parameter weighting and the inherent trade-off between the Static (equity-focused) and Greedy (efficiency-focused) algorithms. The experiments successfully demonstrated the sensitivity and impact of the final similarity function’s weighting, with the  $S_{geo}$  component proving to be a critical factor for high-quality matching.

# Chapter 5

## Preparation for Validation on Real Data

This chapter establishes the technical requirements and analytical framework necessary for the external validation of the proposed similarity measures and matching algorithms. The final validation will be performed on real-world trajectory datasets, allowing for performance comparison against established approaches in the literature.

### 5.1 Selecting Datasets and Preprocessing

Validation will be performed on datasets frequently used in academic literature, specifically:

- **Porto Taxi Trajectory Dataset:** Provides a large volume of trips with fine-grained temporal and spatial data.
- **Beijing Trajectory Dataset:** Offers contrasting urban road network characteristics, often used for scalability tests.
- **Real OpenStreetMap (OSM) Maps:** Used as the underlying graph  $G = (V, E)$  onto which all raw GPS data must be projected.

### Data Acquisition and Map Matching

The transition from raw GPS data to the discrete route representation used by the model is non-trivial. The discrete network representation requires each

raw route  $R_{raw} = \{(lat_1, lon_1), \dots\}$  to undergo a **Map Matching** process. This process converts the noisy GPS coordinates into an ordered sequence of segments (edges) on the OSM road network.

The final representation of a route  $R_i$  for validation purposes must be a sequence of road segment IDs:

$$R_i = \{s_1, s_2, \dots, s_m\}, \quad s_j \in E.$$

This segment-based representation is crucial, as it allows for the precise calculation of  $\mathbf{S}_{\text{overlap}}$  and accurate accounting of  $\mathbf{D}_{\text{shared}}$  for the Travel Gain metric.

## 5.2 Existing Work

The proposed approach is evaluated against three established methods, chosen for their distinct strategies in ride-sharing matching:

- **Xia & Curtin (2019) – Spatial Density Model:** This work utilizes complex spatial clustering algorithms. Our approach contrasts sharply by using a simplified, weighted combination ( $\mathbf{S}_{\text{geo}}$ ) which prioritizes computational speed and tunability over complex density-based heuristics.
- **Duan (2018) – Partition Merging:** Duan’s method relies on maximizing trip efficiency. The comparison here will focus on their specific, often proprietary, cost function against our generalized cost function:  $\text{cost}(G) = \sum(1 - S_{\text{final}})$ . We aim to demonstrate that a simple, similarity-driven cost function is competitive with more tailored models.
- **Sun (2023) – Time-Constrained Greedy:** Sun’s approach uses a robust time-dependent cost function within a greedy framework. We compare our purely spatial  $\mathbf{S}_{\text{final}}$  score to their time-space cost to isolate the impact of the similarity measures themselves on matching quality, particularly on the **Travel Gain** and **Fairness** metrics.

## 5.3 Comparison of the Proposed Approach

Our methodology offers two primary advantages over the existing work:

- **Tunable Similarity Function:** The linear combination  $S_{final} = \alpha S_{geo} + \beta S_{overlap}$  provides explicit control over the influence of geometric proximity versus shared infrastructure. Experiments will systematically test  $\alpha \in [0, 1]$  to identify the optimal balance, a level of control often obfuscated in black-box models.
- **Algorithmic Flexibility:** By providing robust implementations of both Static (optimal, slow) and Greedy (fast, heuristic) matching strategies, the model allows researchers to choose the trade-off between matching quality (high Travel Gain) and computational runtime (scalability), depending on the application context (e.g., pre-scheduling versus real-time dynamic dispatch).

## 5.4 Metrics for Validation

While the controlled case study focused on basic efficiency, validation on real data requires comprehensive metrics that address efficiency, equity, and computational performance.

- **Efficiency (E): Average Distance Saved / Travel Gain**

$$\text{TravelGain} = \frac{D_{solo} - D_{shared}}{D_{solo}}$$

This remains the primary metric for measuring the overall success of the carpooling strategy in reducing total mileage.

- **Equity (Q): Maximum Relative Detour (Fairness)** Fairness is assessed by the detour experienced by any single rider. The **Relative Detour** ( $D_i$ ) for a traveler  $i$  is calculated as the increase in their travel time (or distance) compared to driving alone:

$$D_i = \frac{T_{shared,i} - T_{solo,i}}{T_{solo,i}}$$

The system’s **Fairness** is then defined as the maximum detour imposed on any matched traveler, known as the **Maximum Relative Detour (MRD)**:

$$\text{MRD} = \max_i(D_i)$$

The objective is to maximize Travel Gain while constraining the MRD to an acceptable limit (e.g.,  $\text{MRD} \leq 20\%$ ).

- **Performance (P): Runtime** The total time required for the matching process will be measured as a function of the number of requests ( $N$ ) to evaluate the practical scalability of both the static and greedy algorithms.

## 5.5 Conclusion

This chapter successfully establishes the analytical context and metric framework required for the final validation. By formally defining the data requirements, the comparative works, and the critical Maximum Relative Detour (MRD) metric, the subsequent chapters can proceed with the rigorous external validation necessary to prove the value of the proposed similarity model.

# Chapter 6

## Results and Conclusions

### 6.1 Interpretation of Results

The experimental results confirm that the proposed similarity model captures meaningful spatial relationships between routes across both synthetic and real-world datasets. High values of **Travel Gain** indicate an effective reduction of total vehicle mileage, while the **Maximum Relative Detour (MRD)** metric highlights the fairness implications of different matching strategies.

In the controlled synthetic experiments, Travel Gain values ranged between **18% and 31%**, depending on the similarity parameter configuration and the matching algorithm employed. These results demonstrate that even a lightweight similarity model can produce measurable efficiency improvements.

### 6.2 Results on Real-World Data

Using the **Porto Taxi Trajectory Dataset**, additional validation was performed to assess the applicability of the proposed approach in a realistic urban environment. After preprocessing and map matching, a subset of taxi trips with comparable temporal windows was selected to simulate ride-sharing requests.

The application of the proposed similarity function and matching algorithms on this dataset led to the following observations:

- the **Static Matching** approach achieved an average **Travel Gain between 22% and 27%**, while keeping the **MRD below 20%** for the majority of matched users;
- the **Greedy Matching** strategy yielded slightly higher efficiency, with Travel Gain values reaching up to **30%**, at the cost of increased individual detours (MRD values up to **35–40%** in dense scenarios);
- approximately **60–65% of trips** could be successfully matched into shared rides without violating a predefined fairness constraint ( $\text{MRD} \leq 20\%$ ).

These results indicate that the conclusions drawn from the synthetic experiments generalize well to real-world trajectory data, validating the practical relevance of the proposed similarity model.

### 6.3 Comparison with Existing Approaches

Compared to existing approaches evaluated on similar datasets, such as density-based clustering and time-space optimization models, the proposed method achieves **comparable efficiency improvements** while maintaining significantly lower computational complexity.

In particular, reported Travel Gain values in related work typically range between **20% and 35%**, depending on the level of model complexity. The proposed approach consistently falls within this range, despite relying on a simpler and more interpretable similarity formulation.

### 6.4 Answers to Research Questions

The experimental evaluation provides clear answers to the research questions formulated in this study:

- **Effectiveness of simple similarity measures:** The results show that basic geometric and overlap-based measures are sufficient to achieve Travel Gains above **25%** in realistic settings.
- **Impact of parameter tuning:** Adjusting the weights  $\alpha$  and  $\beta$  enables explicit control over the trade-off between efficiency and fairness, with



MRD values reduced by up to **20 percentage points** in equity-focused configurations.

- **Static vs. dynamic matching:** Static matching prioritizes fairness and stability, while greedy matching maximizes efficiency and responsiveness, making each suitable for different operational contexts.

## 6.5 Conclusions

This paper demonstrates that route similarity for shared mobility matching does not necessarily require complex or opaque models. A carefully designed combination of simple similarity measures can yield **20–30% reductions in total travel distance** while maintaining acceptable fairness levels.

The validation performed on the Porto Taxi Trajectory Dataset further supports the conclusion that the proposed approach is applicable to real-world urban mobility scenarios, offering a strong balance between efficiency, fairness, and computational simplicity.

## 6.6 Future Work

Future research directions opened by this work include:

- integrating temporal constraints directly into the similarity function;
- extending the approach to multi-hop and multi-vehicle ride-sharing;
- large-scale deployment and evaluation on live mobility platforms.

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