

Matrix Determinant – Laplace and Sarrus Method

N.B

some of matrix picture might be good since it was generated from <https://www.symbolab.com/solver/matrix-vector-calculator> , meanwhile some of them are my handwriting.

<https://github.com/antoniusrobotsoft>

It has been so long since my last time playing with matrix. The last time I used matrix is when I was study about input and output optimization using leontief matrix. Since it has been so long, I decided to write an interesting topic about matrix called determinant.

What is matrix determinant ? Here is a short description from wikipedia:

In linear algebra, the determinant is a value that can be computed from the elements of a square matrix. The determinant of a matrix A is denoted \det , $\det A$, or $|A|$

First of all, it's only possible to find determinant of a matrix when a matrix has the same number of columns and rows. In order to calculate determinant of a matrix, there are many methods. There are 2 methods which I used frequently. They are : laplace method and sarrus method.

The most simple way to calculate a matrix determinant is when the matrix consists of 2 rows and 2 columns only.

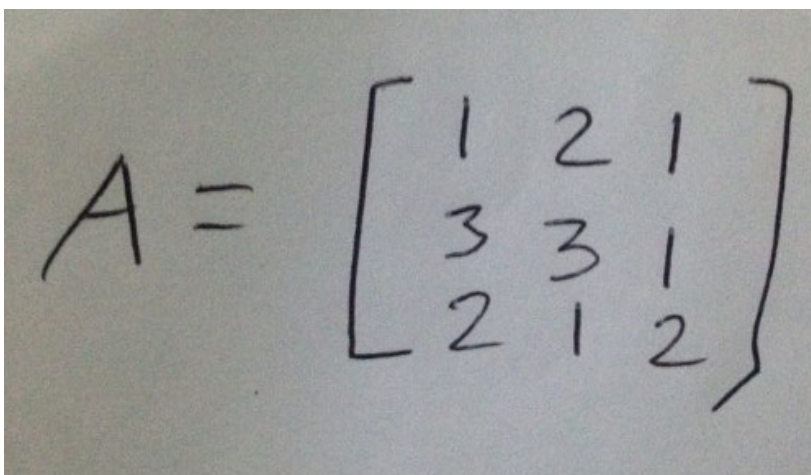
Example we have this matrix called U matrix :

$$\begin{pmatrix} 2 & 1 \\ 3 & 6 \end{pmatrix}$$

$$|U| = (2 * 6) - (1 * 3) = 9$$

Sarrus Method

Based on sarrus method : “3×3 matrix determinant” is the sum of the products of three diagonal north-west to south-east lines of matrix elements, minus the sum of the products of three diagonal south-west to north-east lines of elements. Suppose we have 3×3 matrix called A :



A handwritten 3x3 matrix A is shown on a piece of paper. The matrix is written as A = [[1, 2, 1], [3, 3, 1], [2, 1, 2]]. The elements are arranged in three rows and three columns, enclosed in large square brackets.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

To calculate $|A|$ of matrix A using sarrus method :

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 3 \\ 2 & 1 \end{matrix}$$

We added 2 columns from matrix beside our original matrix, so we get the sum of the products of three diagonal north-west to south-east lines of matrix elements :

$$(1 * 3 * 2) + (2 * 1 * 2) + (1 * 3 * 1)$$

Then we need to get the sum of the products of three diagonal south-west to north-east lines of elements :

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 3 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{matrix} 1 & 2 \\ 3 & 3 \\ 2 & 1 \end{matrix}$$

$$(2 * 3 * 1) + (1 * 1 * 1) + (2 * 3 * 2)$$

Based on above informations, we got this:

$$|A| = ((1 * 3 * 2) + (2 * 1 * 2) + (1 * 3 * 1)) - ((2 * 3 * 1) + (1 * 1 * 1) + (2 * 3 * 2))$$

$$|A| = (6 + 4 + 3) - (6 + 1 + 12) = -6$$

Laplace Method

Calculating determinant of 3×3 matrix using laplace is simple.

Based on literature, to calculate 3×3 matrix using laplace :

The Laplace formula for the determinant of a 3 × 3 matrix is

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

So the matrix is splitted into 3 small matrices which 2×2 matrix, where a,b and c are constants.

Those 3 small 2×2 matrices are permutation from this columns and rows:

$$\begin{vmatrix} d & e & f \\ g & h & i \end{vmatrix} :$$

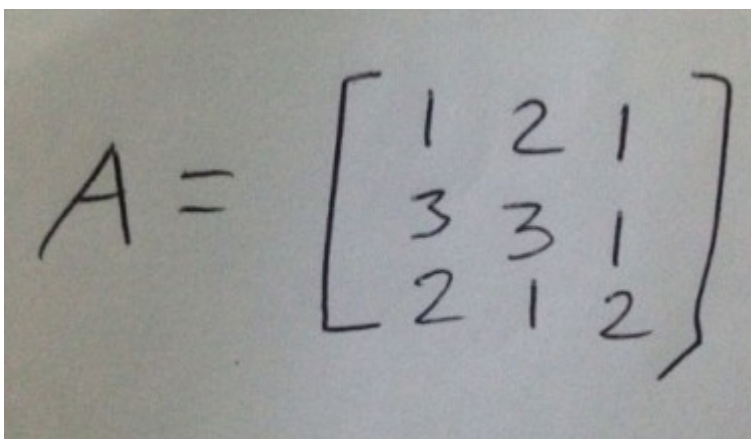
the 2×2 matrices :

$$\begin{vmatrix} e & f \\ h & i \end{vmatrix} :$$

$$\begin{vmatrix} d & f \\ g & i \end{vmatrix} :$$

$$\begin{vmatrix} d & e \\ g & h \end{vmatrix} :$$

Back again to A matrix:



A handwritten image showing a 3x3 matrix A. The matrix is written as A = [[1, 2, 1], [3, 3, 1], [2, 1, 2]].

$$A = \begin{bmatrix} 1^a & 2^b & 1^c \\ 3^d & 3^e & 1^f \\ 2^g & 1^h & 2^i \end{bmatrix}$$

$$a = 1, b = 2, c = 1$$

We got 2x2 permutation matrices :

$$\begin{bmatrix} \text{[redacted]} \\ \text{[redacted]} \\ 3^d & 3^e & 1^f \\ 2^g & 1^h & 2^i \end{bmatrix}$$

$$\begin{bmatrix} \text{[redacted]} \\ \text{[redacted]} \\ 3^d & 1^e & 1^f \\ 2^g & \text{[redacted]} & 2^i \end{bmatrix}$$

A handwritten 3x3 matrix is shown, enclosed in large square brackets. The top row is completely reded out. The second row contains the numbers 3 and 3, with a red 'd' below the first 3 and a red 'e' below the second 3. The third row contains the numbers 2 and 1, with a red 'g' below the 2 and a red 'h' below the 1. The third column is also reded out.

The handwritten calculation for the determinant of a 3x3 matrix is shown. It uses the first row expansion method. The first row is [1, 3, 1]. The calculation is: $|A| = 1 \cdot \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 3 \\ 2 & 1 \end{vmatrix}$

determinant of first matrix = 5

determinant of second matrix = 4

determinant of third matrix = -3

so we got : $(1 * 5) - (2 * 4) + (1 * -3) = 5 - 8 - 3 = -6$