

## LDPC-CODES FRAME ERROR RATE AND TANNER'S GRAPH SPECTRUM CORRELATION RESEARCH

Kovsharov A.P., master student group M4239, University ITMO, St. Petersburg

Supervisor – Anokhina I.A., senior teacher, Department of Foreign Languages, University ITMO, St. Petersburg

Different error correction codes are used for data transmission through a noisy channel. In 1962 Robert G. Gallager introduced low density parity checks (LDPC) codes. However, due to computational complexity they have become popular only recently. These codes properties are close to turbo-codes, but decoding is easier and can be parallelized as shown by MacKay in his work. For example, WiMAX standard uses LDPC-codes.

LDPC-code as any other linear binary code can be set with check the matrix  $H$ . Rows of check matrix compose the basis of space which is orthogonal to the codewords space. A bipartite graph constructed on the matrix  $H$  as adjacency matrix is called Tanner's graph. Nodes corresponding to rows are called check nodes. Nodes corresponding to columns are called variable nodes.

Belief propagation algorithm, also known as sum-product message passing algorithm, is used for decoding. It is an iterative process. Variable nodes store logarithms of likelihood relations. In the first step, variable nodes send information (message) to connected check nodes containing the bit they believe to be the correct one for them. In the second step, every check node calculates a response to every connected variable node. The response message contains the bit that this node believes to be correct based on all information it has. If all check equations are fulfilled algorithm terminates otherwise these steps are repeated again. The maximum number of iterations used during research is 50. An important assumption for correctness of this algorithm is the absence of cycles in the graph. In most cases this assumption is not true but the algorithm works well anyway.

It is a well-known fact that with the increase of the shortest cycle length the frame error rate decreases. This happens due to the influence of the girth value on the number of iterations when the assumption stated above holds.

It is natural to guess that the success of decoding, except the length of the shortest cycle, also depends on the number of shortest cycles, and probably on the number of longer cycles. As a result, it is logical to analyze the correlation of frame error rate and distribution of cycle lengths in the graph – spectrum. However, it turns out that finding the distribution of simple cycles in the graph is a computationally intensive task. Besides, the message passing algorithm operates with something similar to closed paths, rather than objects isomorphic to simple cycles. These closed paths have a property not to traverse the same edge twice in a row. In other words, when a path approaches a vertex it cannot return to the previous vertex. Moreover, we should consider closed paths, which can be produced with a cycle shift from one another, to be equivalent. Thus, we analyze the frame error rate based on spectrum of closed paths in Tanner's graph (a set of numbers corresponding to the number of closed paths of particular lengths).


An algorithm for finding the spectrum within the time significantly shorter than the time needed for a simulation to determine the frame error rate has been developed. In this case, the demonstration of correlation between the code effectiveness and spectrum allows the search of most effective codes to be accelerated.

A CUDA decoder has been developed for comprehensive testing. The decoder allows fast transmission simulation of a large number of random codewords over a noisy channel for the determination of the frame error rate in case of fixed signal/noise ratio.

The lifting method of base matrices was used for building random codes. For example, for a 5 by 10 matrix can be transformed into a 100 by 200 matrix with an expansion parameter of 20. In a lifted matrix, every vertex of base matrix-graph gets 20 corresponding vertices. For the lifting purpose, the method needs a voltage assignment on edges to determine which new instances of vertices should be connected by an edge in the lifted graph.

Several thousand random matrices were created. The sequence of matrices with pairwise comparable spectrums of corresponding graphs was chosen. All matrices were divided into clusters according to the number of closed paths of shortest length excluding few matrices-exceptions. The value of signal/noise ratio sufficient to achieve the frame error rate of 0.001 was used for the comparison. Moreover, as was expected, this value was bigger in clusters with more closed paths with the shortest length. Consequently, the results of the LDPC code analysis can be used to improve the search and optimization of effective LDPC codes.


Магистрант гр. М4239

 Ковшаров А. П.

Руководитель

 Анохина И. А.

Зав. кафедрой ин. языков, Университет ИТМО

 Рябухина Ю. В.