УНИВЕРСИТЕТ ИТМО

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«LDPC-codes frame error rate and Tanner's graph spectrum correlation research»

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CT Department

Content

Goals and Objectives

- Check hypothesis of correlation between number of short cycles and iterative decoding effectiveness
- Develop algorithm for Tanner's graph spectrum (number of fixed length cycles) calculation

Linear (n, k) code
G – generator matrix
H – paritiy check matrix
$G \cdot H^T = 0$ – paritiy check equation
LDPC-code – Low Density Parity Check code (small number of ones in H)

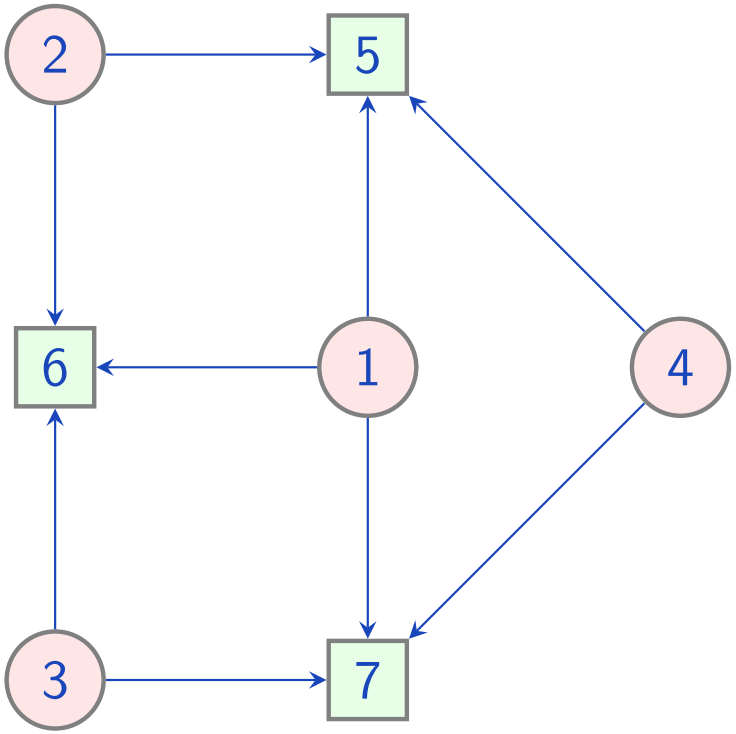
Example

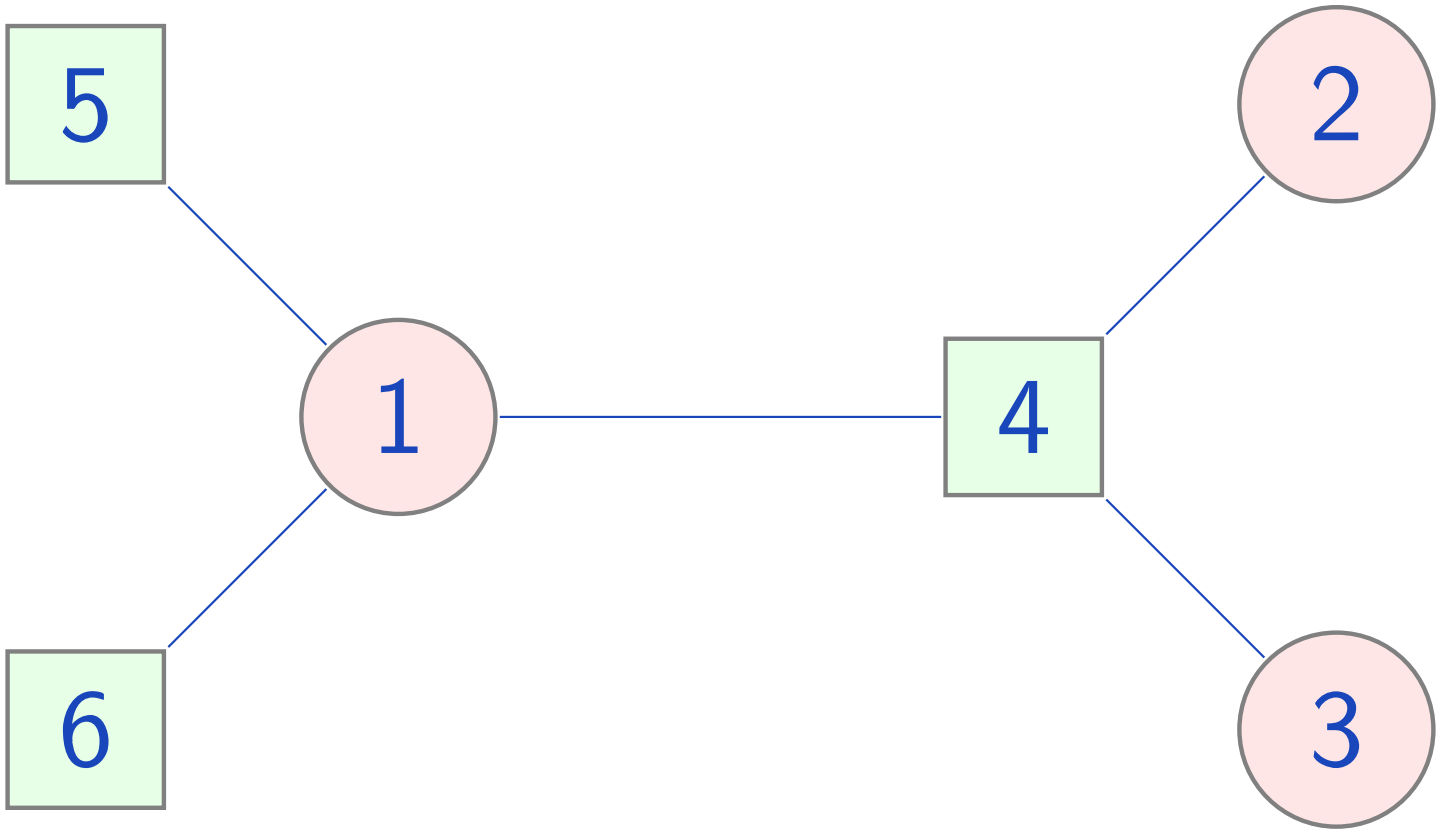
$$G = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

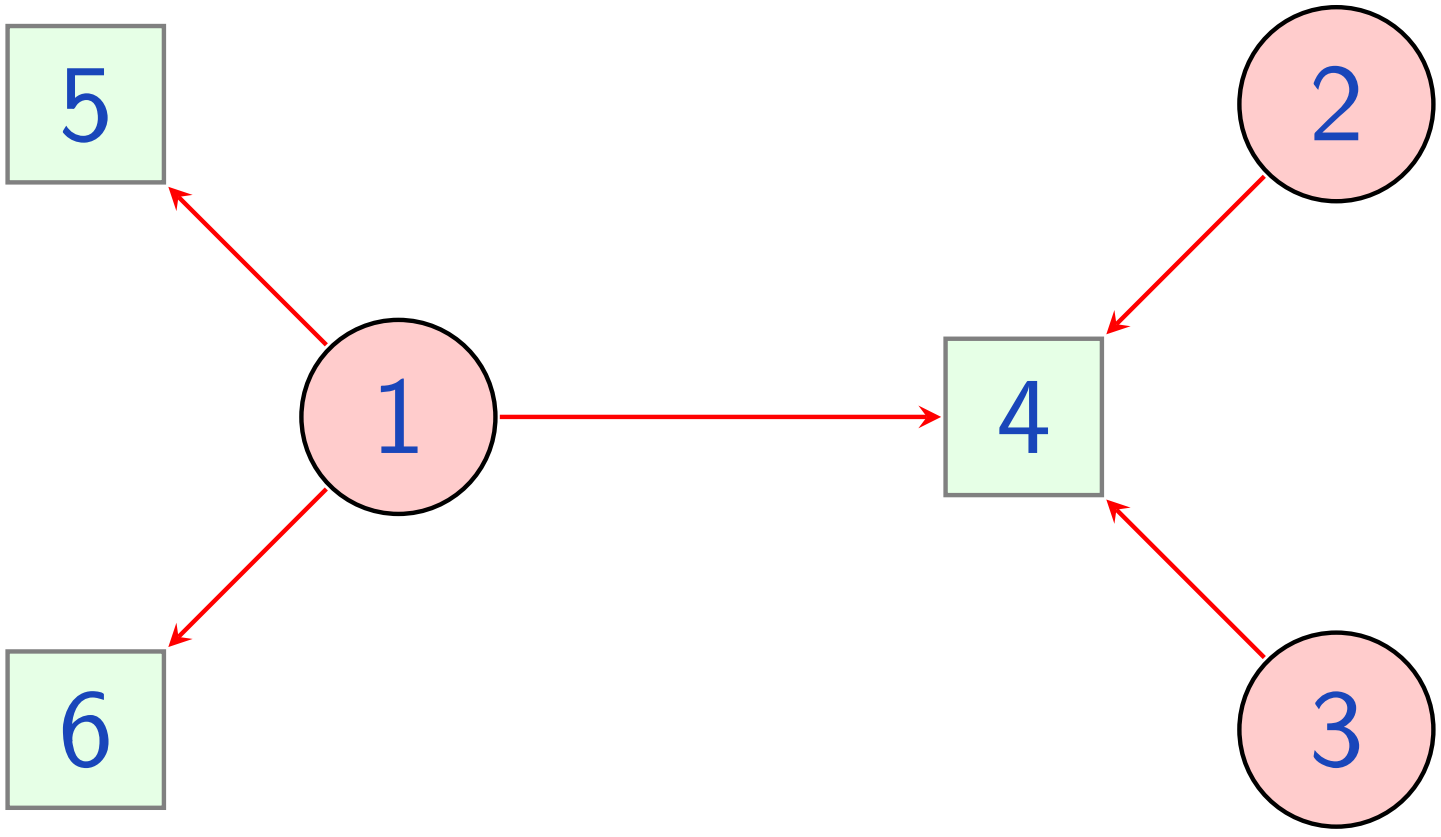
$$H = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

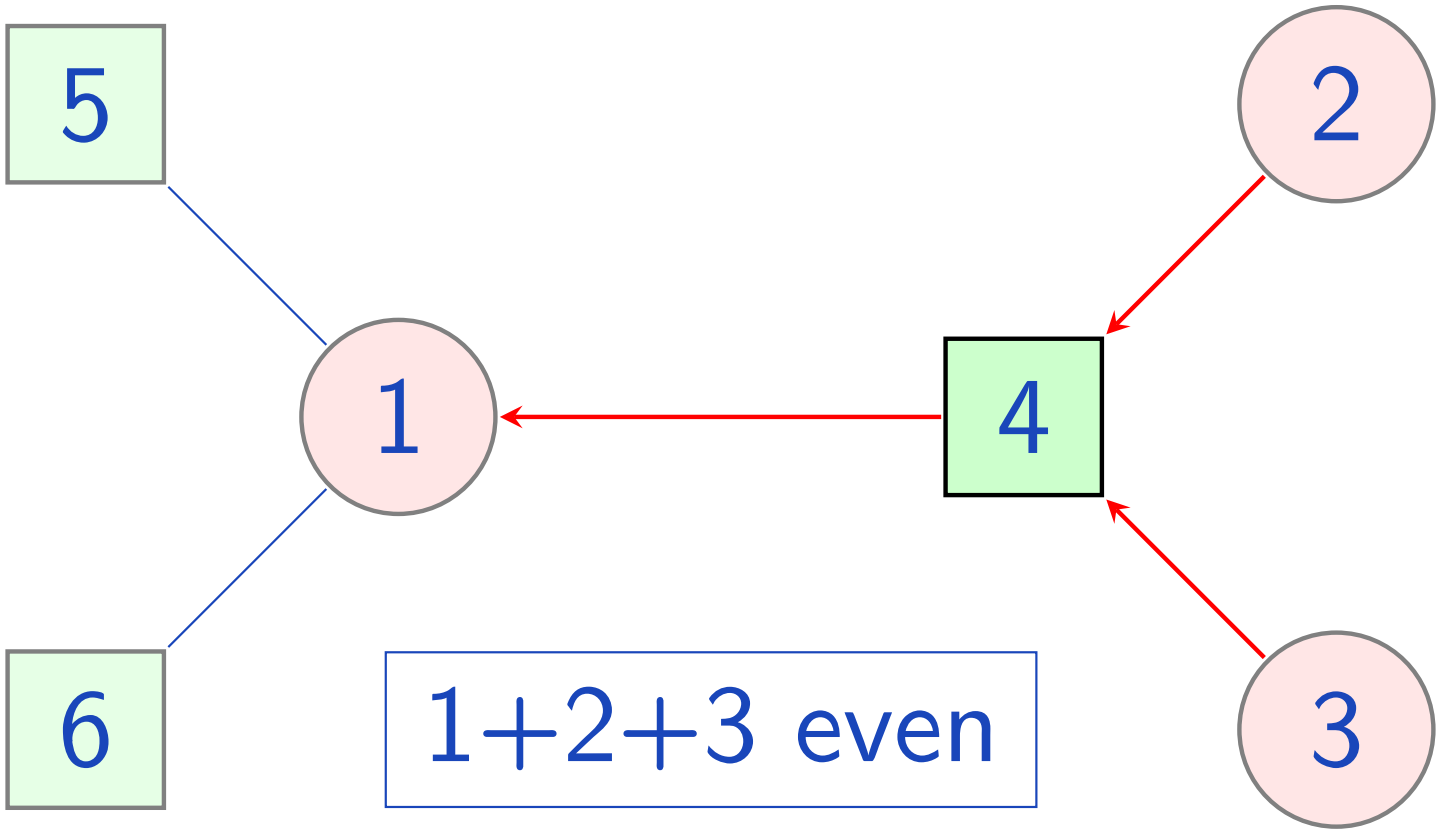
- Bit node - code symbol ○
- Parity node - parity equation □
- Line between if that bit is involved in that parity equation

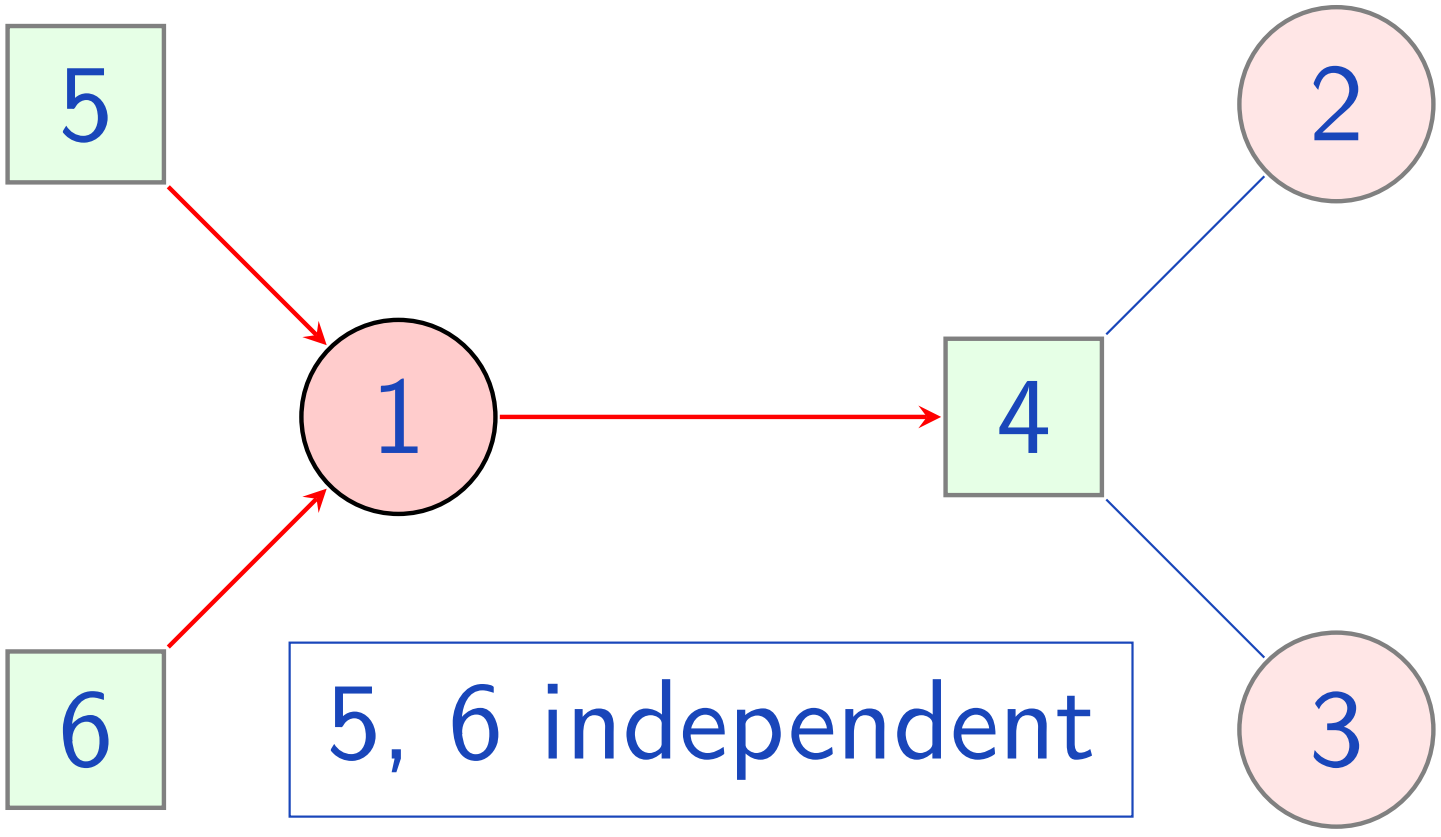
$$\begin{matrix} 1 & 2 & 3 & 4 \end{matrix}$$
$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix} \begin{matrix} 5 \\ 6 \\ 7 \end{matrix}$$

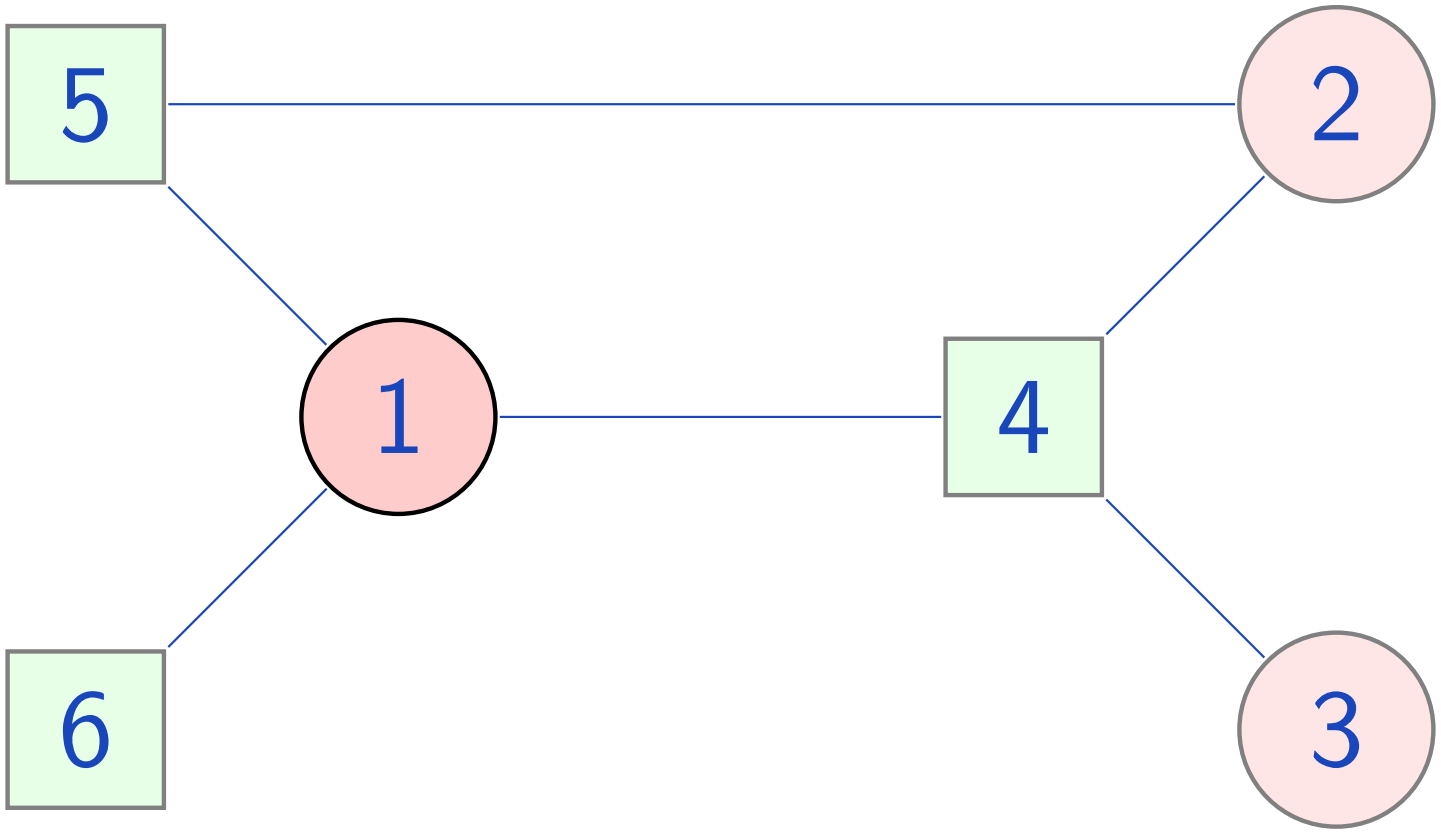


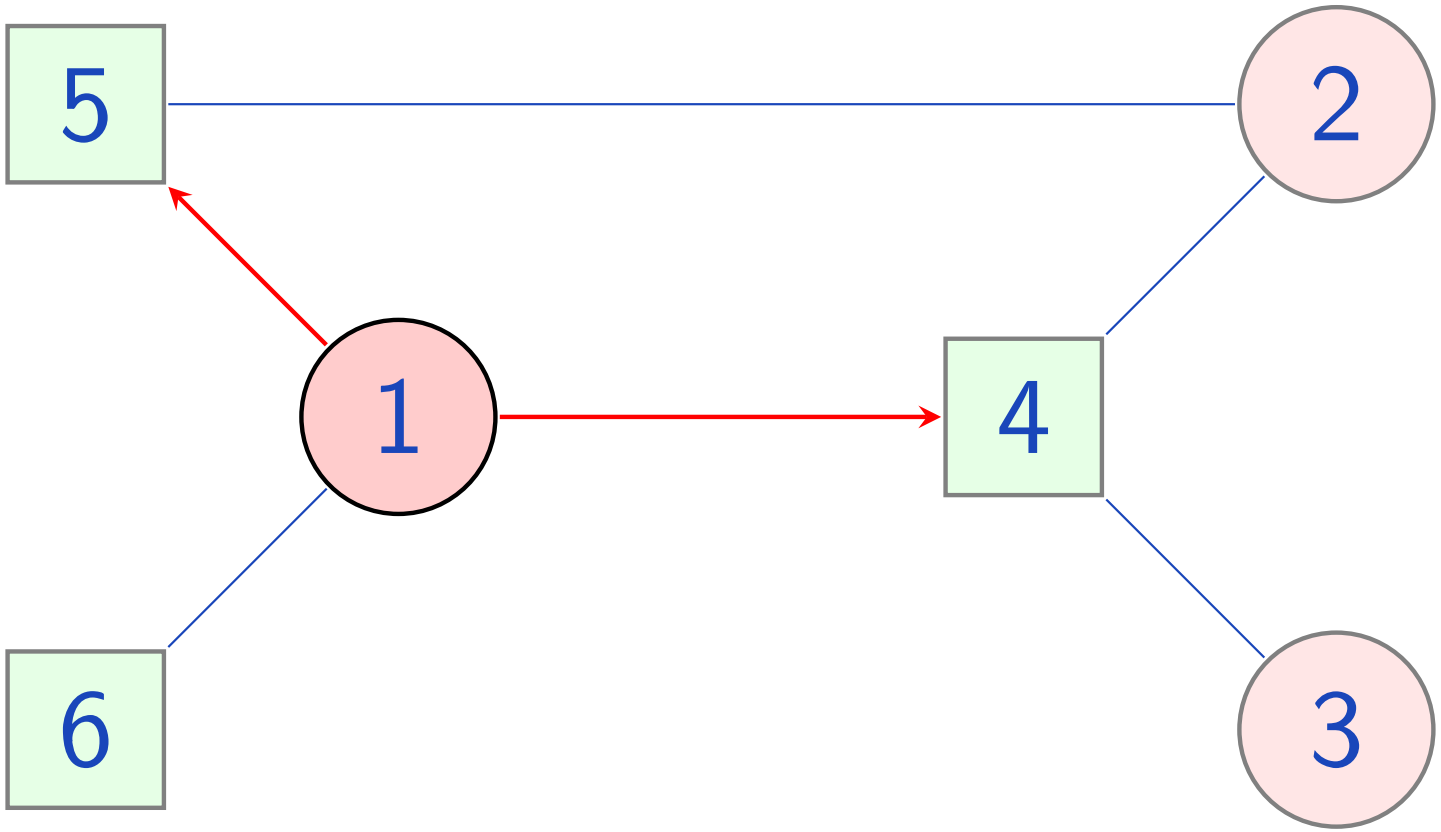

5
2
1
4
6
3

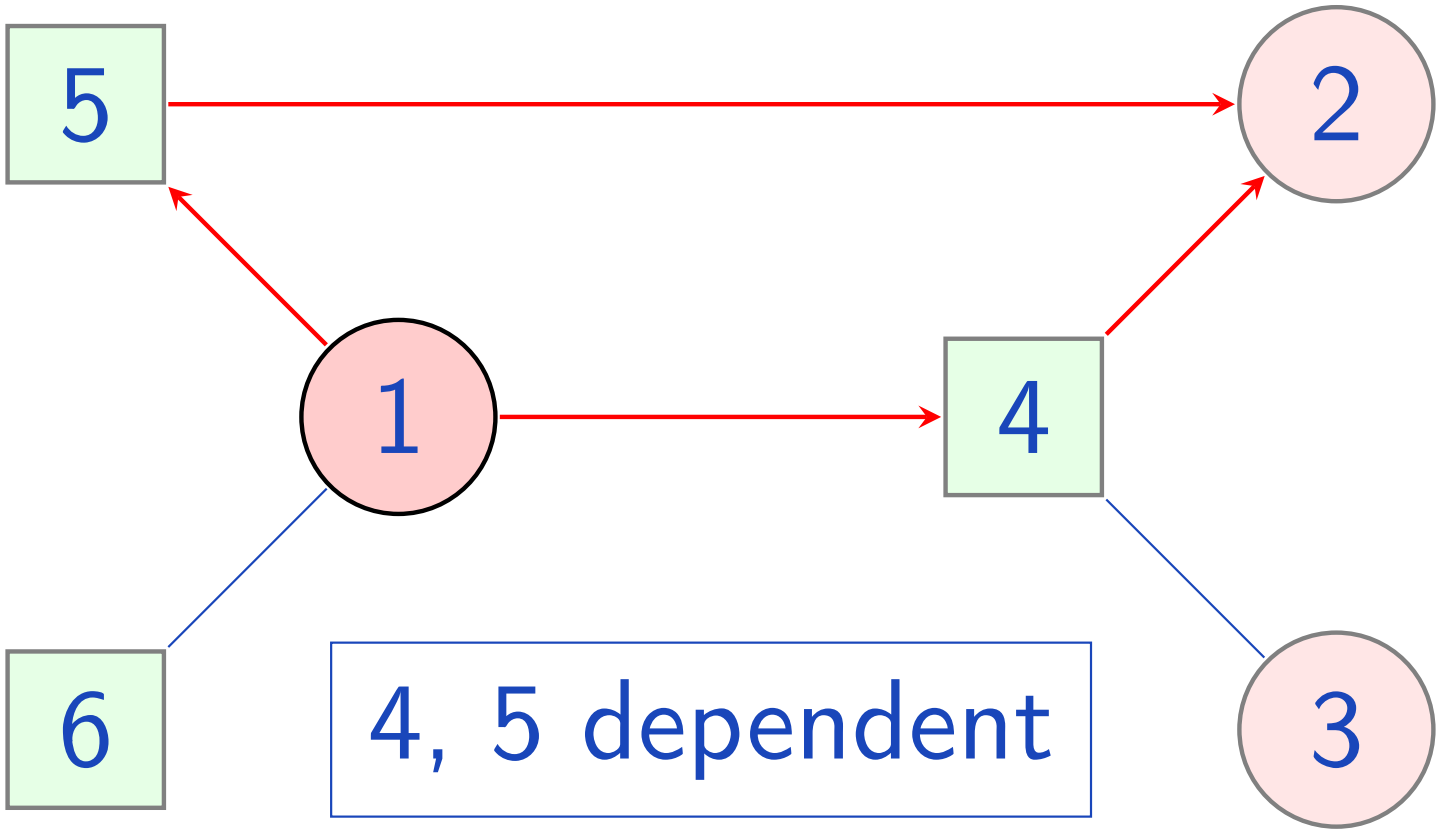

5
2
1
4
6
3


5
2
1
4
6
1+2+3 even
3

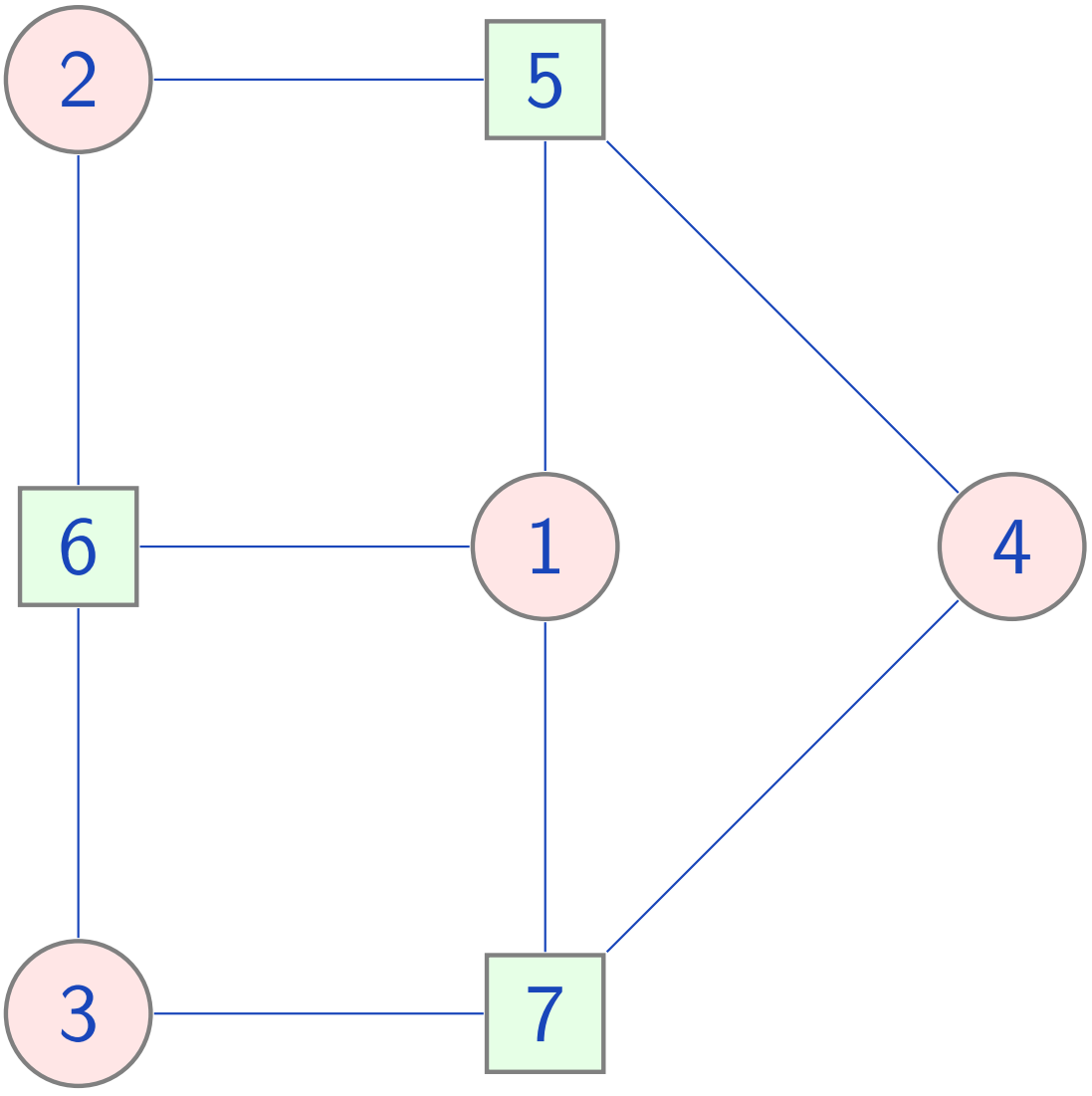

5
2
1
4
6
5, 6 independent
3


5
2
1
4
6
3


5
2
1
4
6
3

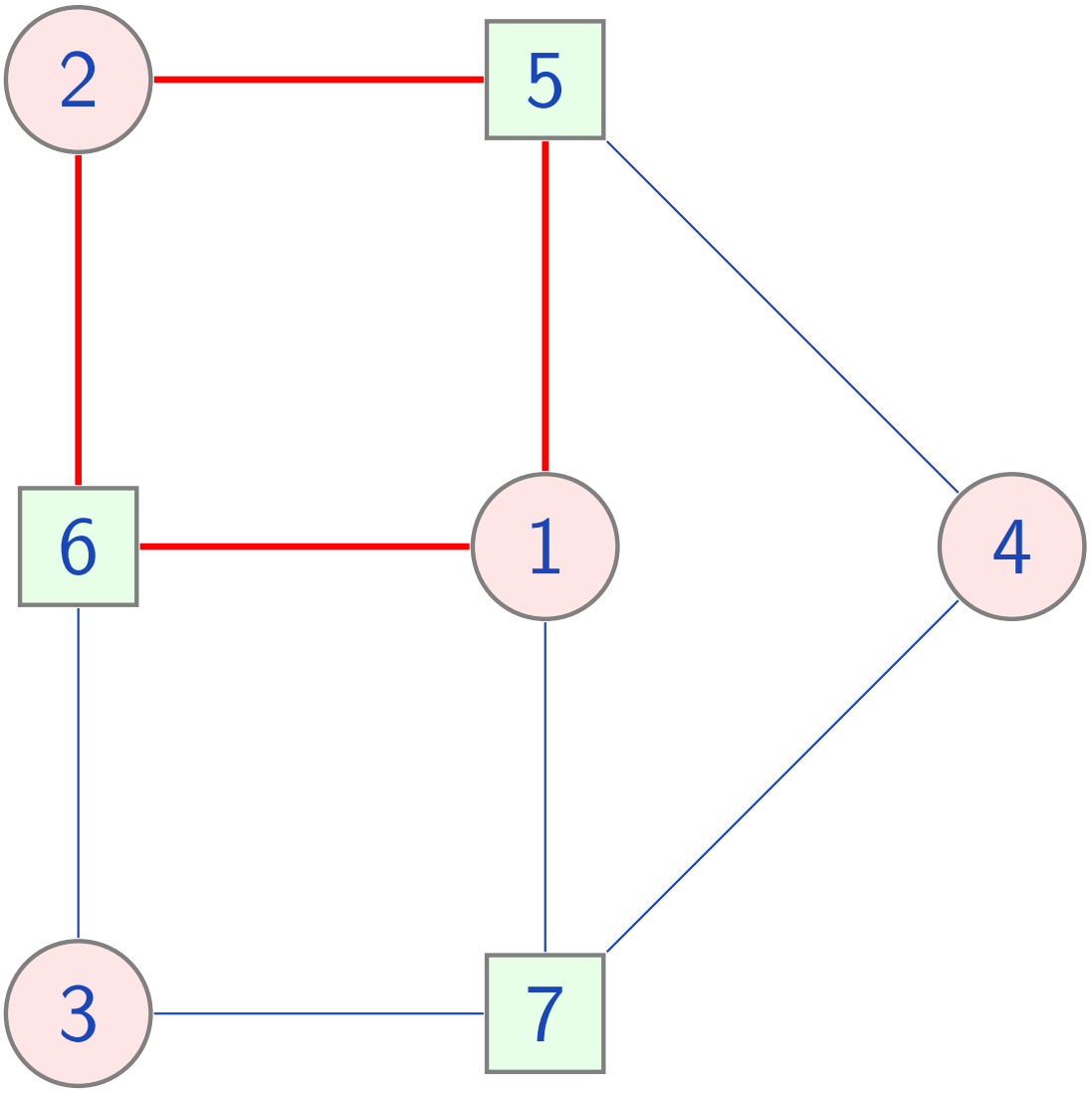

5
2
1
4
6
4, 5 dependent
3

Spectrum calc algorithm
Cycles



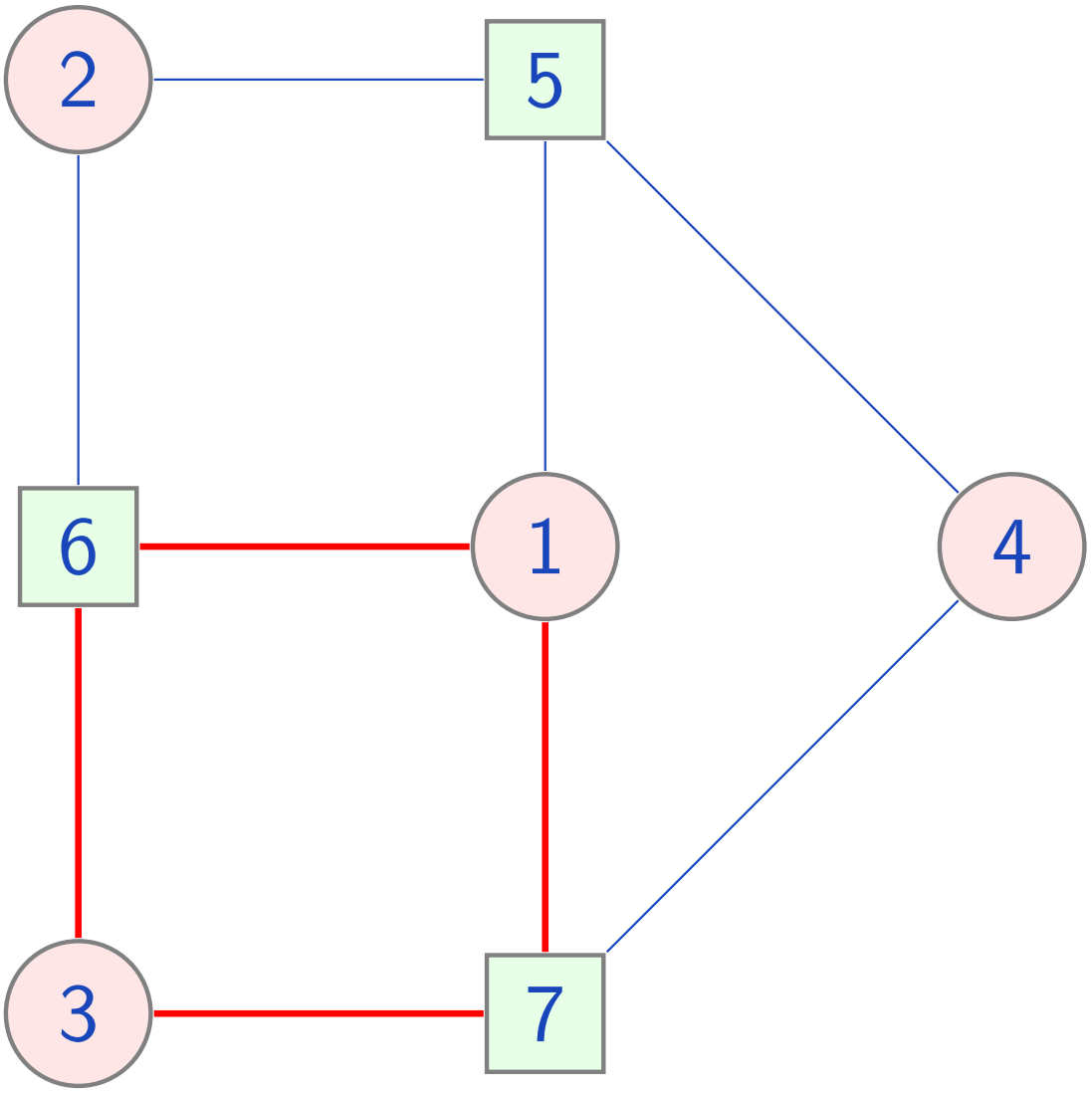
4: 0
6: 0
8: 0
10: 0
12: 0
14: 0
....

Spectrum calc algorithm
Cycles



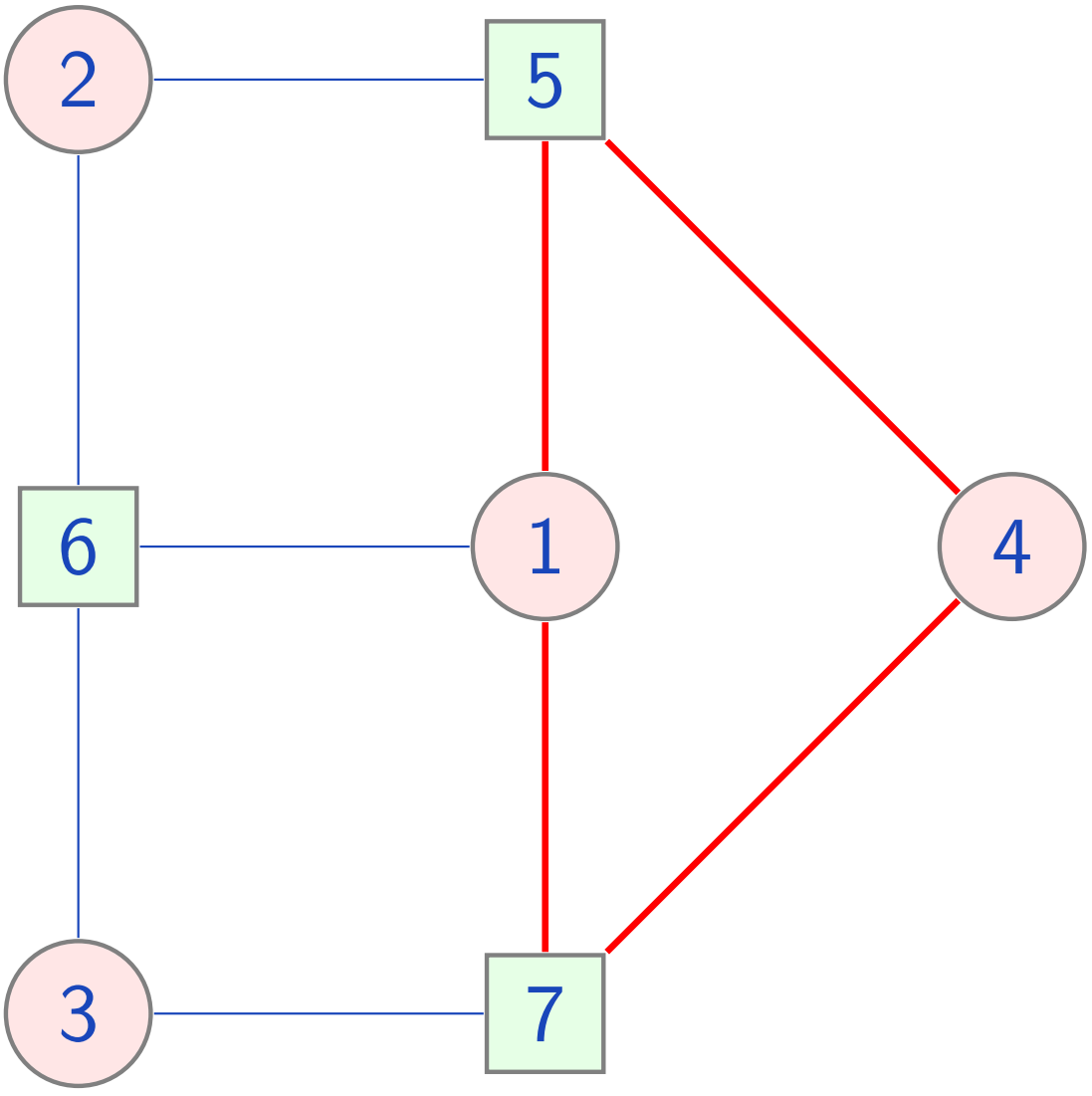
4: 1
6: 0
8: 0
10: 0
12: 0
14: 0
....

Spectrum calc algorithm
Cycles



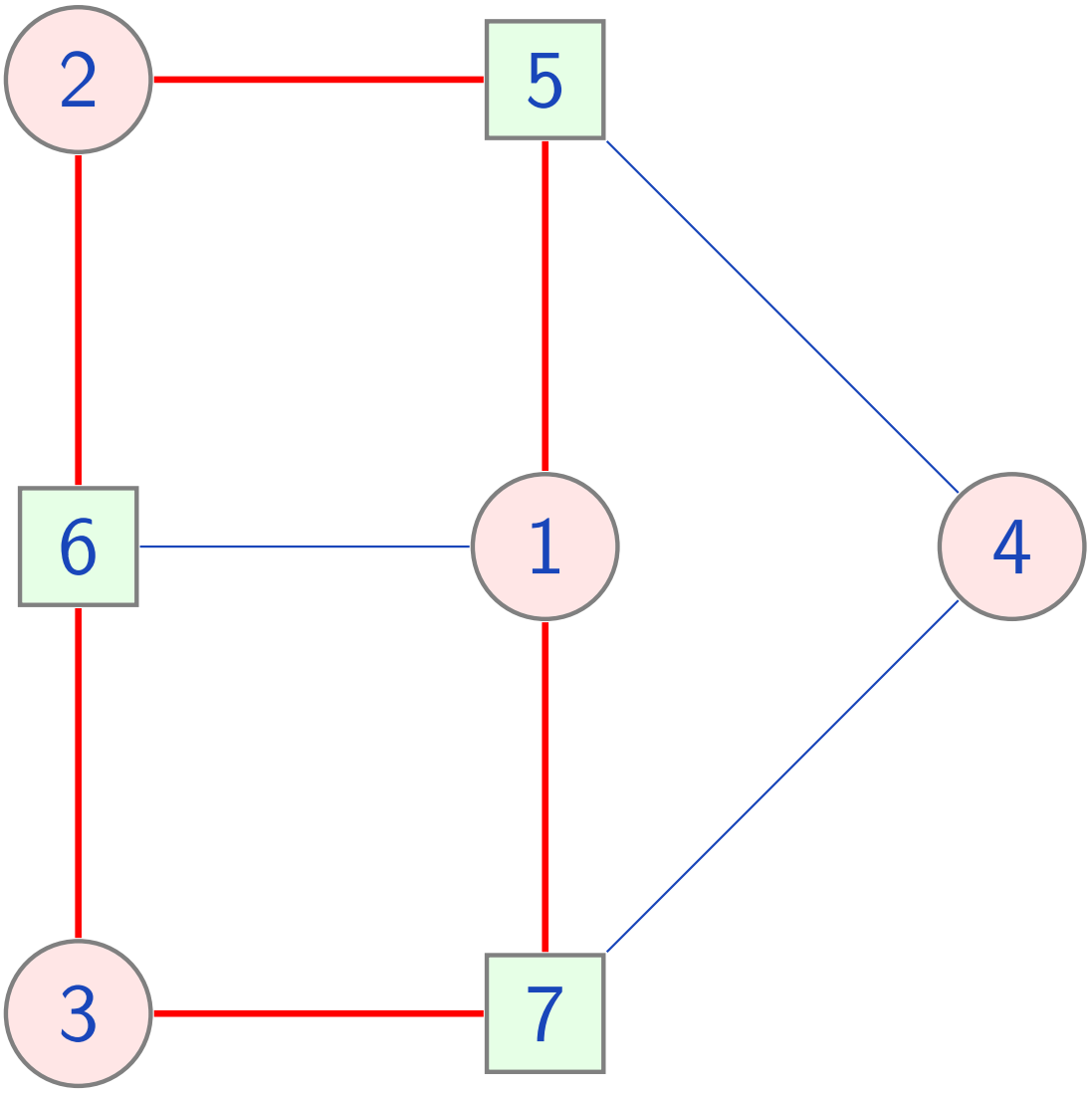
4: 2
6: 0
8: 0
10: 0
12: 0
14: 0
....

Spectrum calc algorithm
Cycles



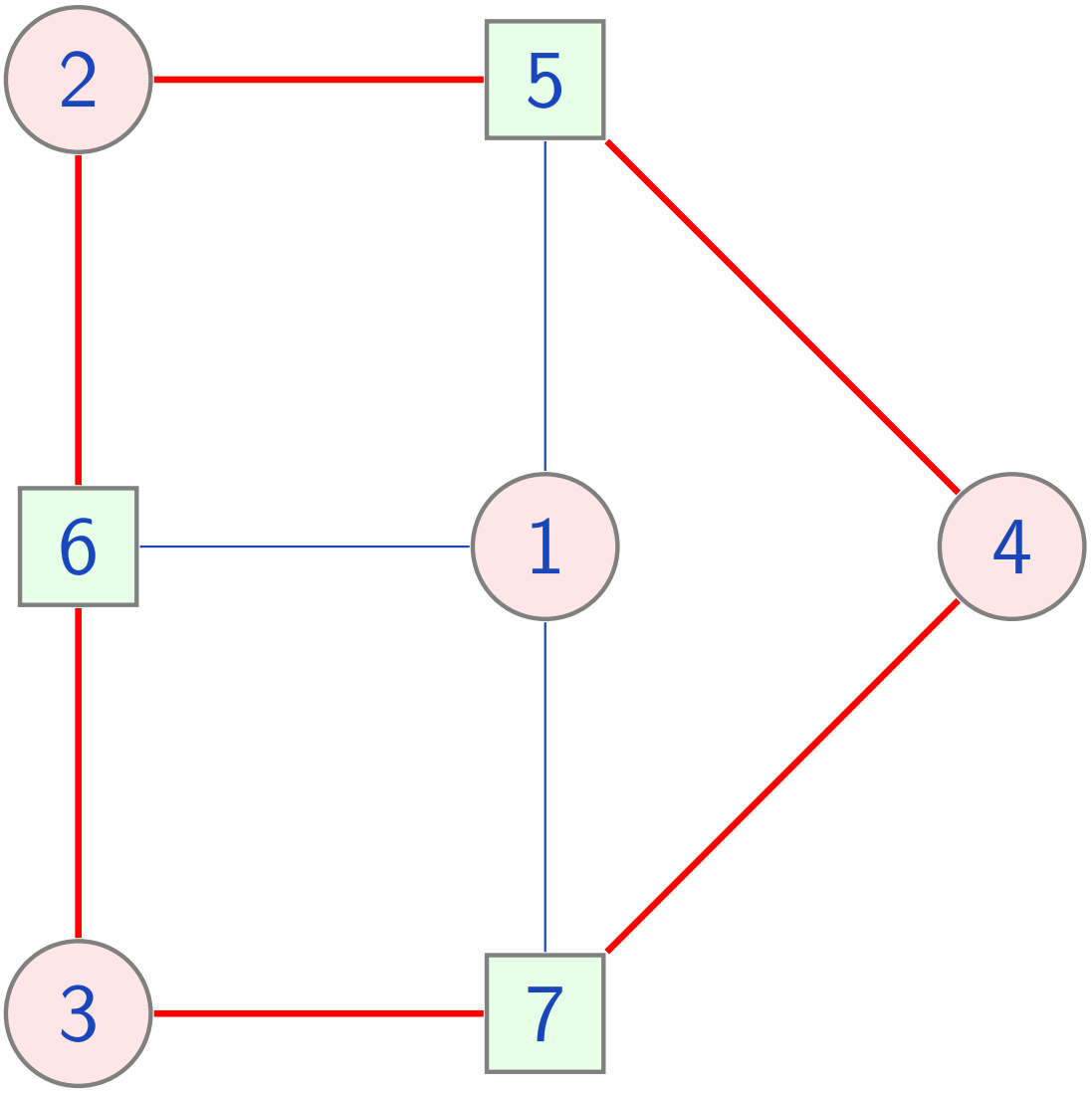
4: 3
6: 0
8: 0
10: 0
12: 0
14: 0
....

Spectrum calc algorithm
Cycles



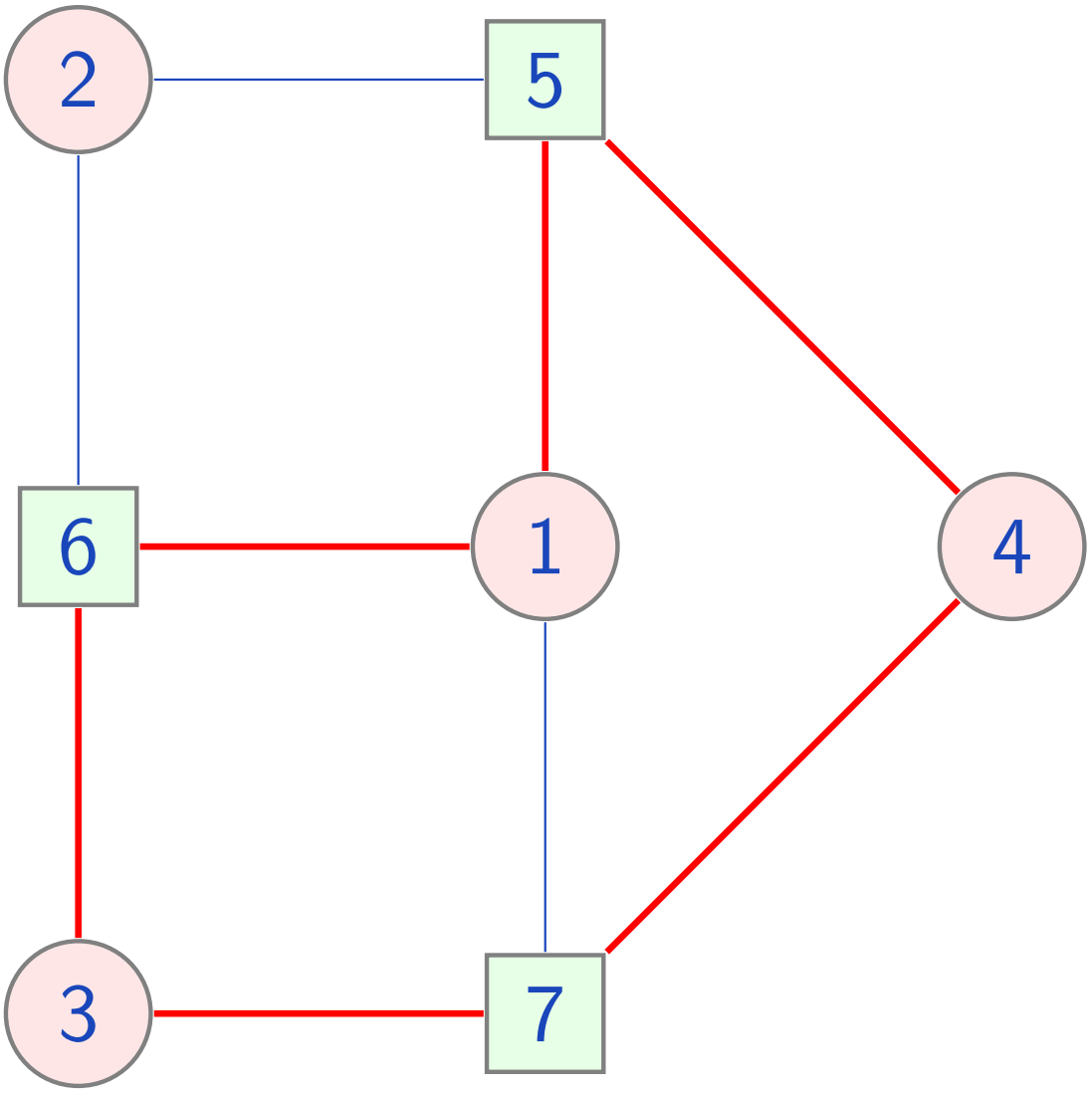
4: 3
6: 1
8: 0
10: 0
12: 0
14: 0
....

Spectrum calc algorithm
Cycles



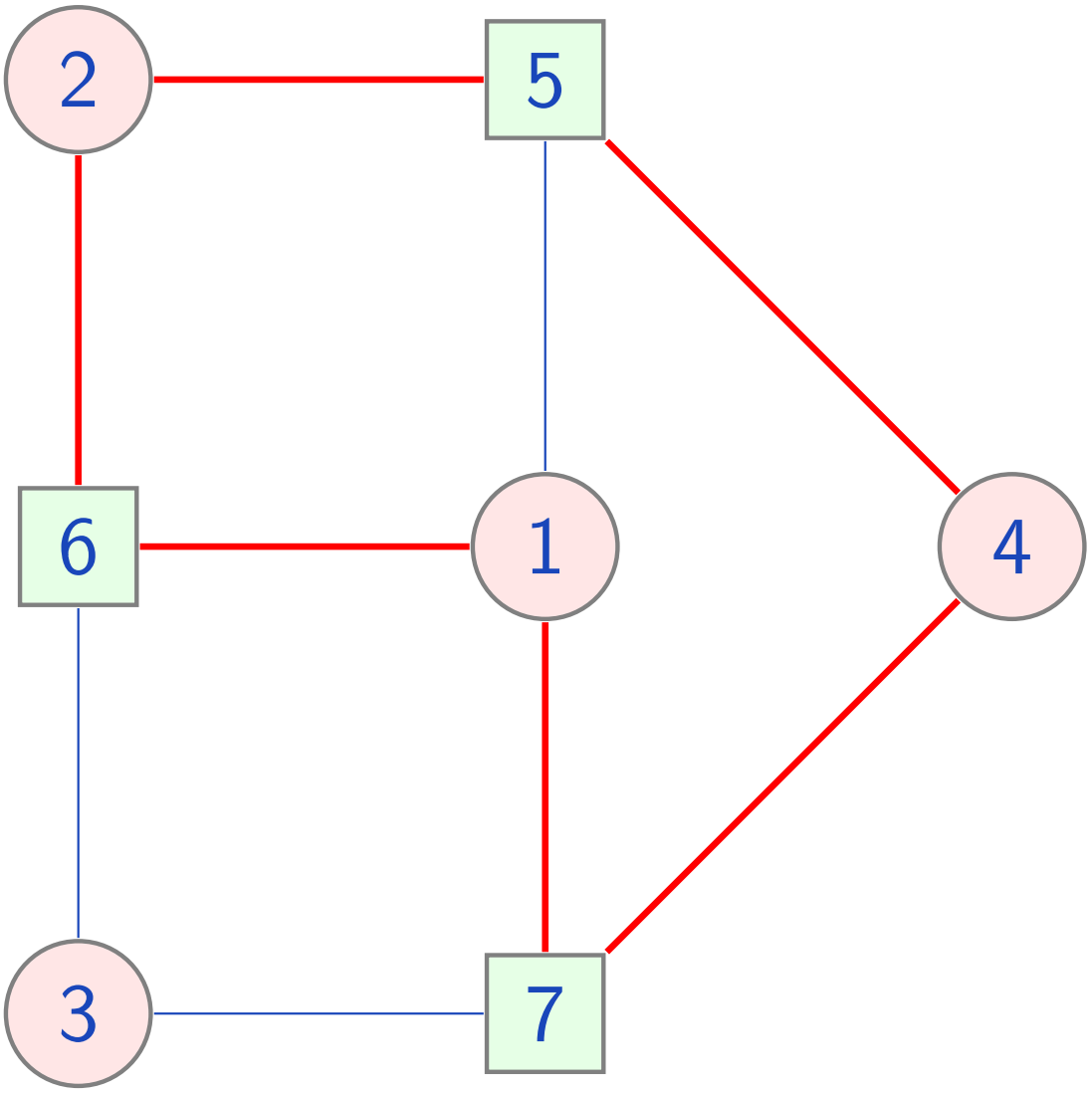
4: 3
6: 2
8: 0
10: 0
12: 0
14: 0
....

Spectrum calc algorithm
Cycles



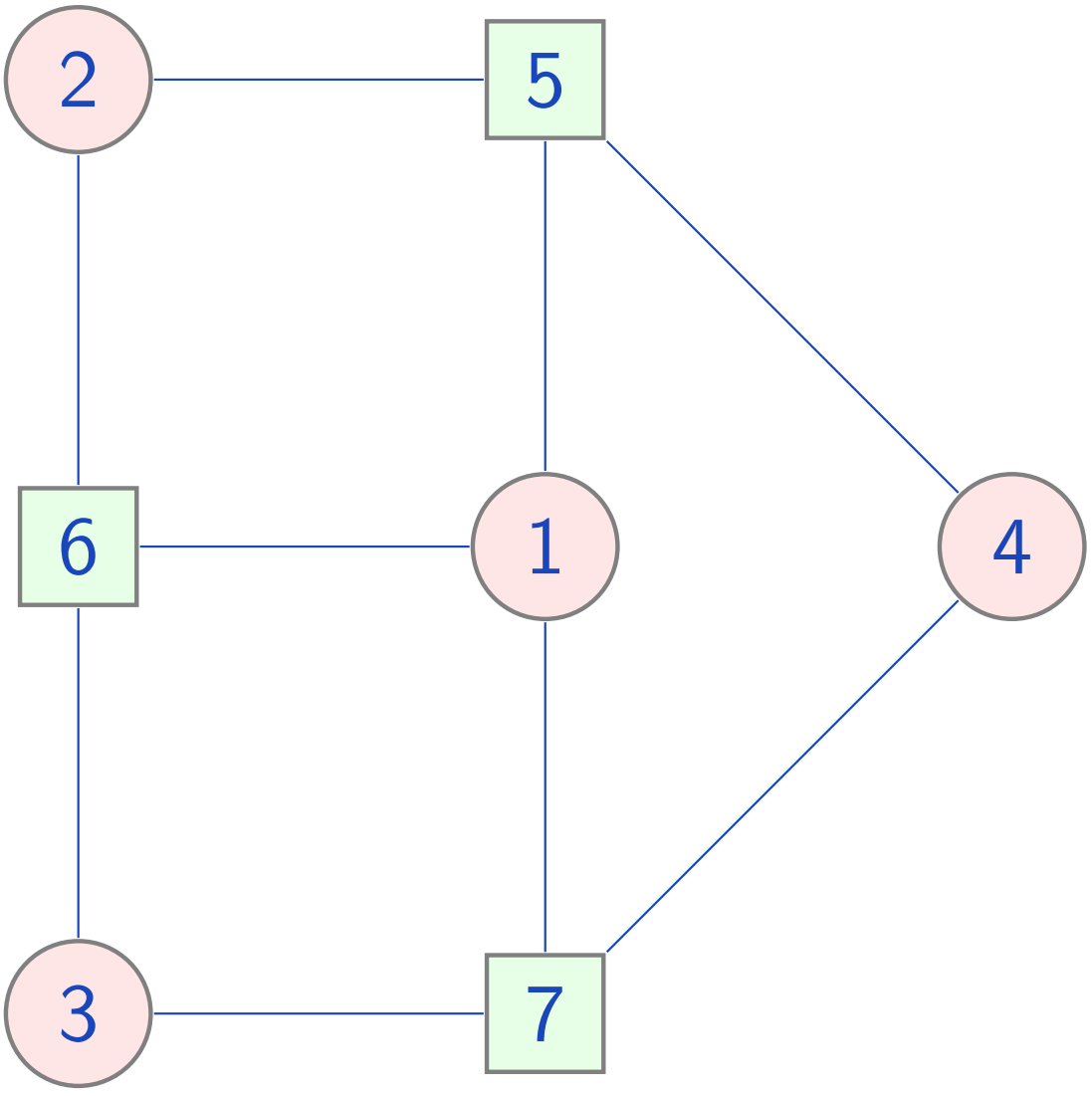
4: 3
6: 3
8: 0
10: 0
12: 0
14: 0
....

Spectrum calc algorithm
Cycles



4: 3
6: 4
8: 0
10: 0
12: 0
14: 0
....

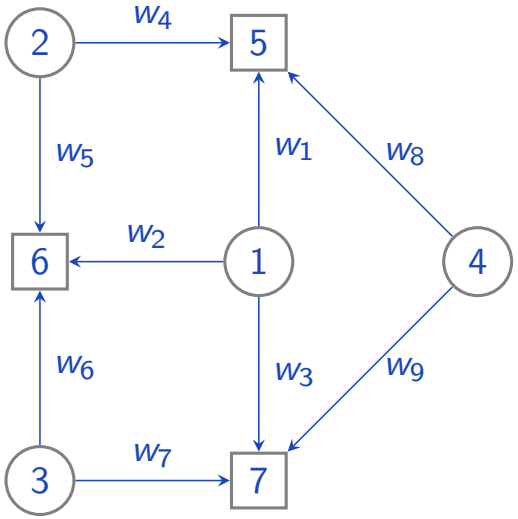
Spectrum calc algorithm
Cycles



4: 3
6: 4
8: 6
10: 12
12: 29
14: 48
....

Spectrum calc algorithm
State, transitions

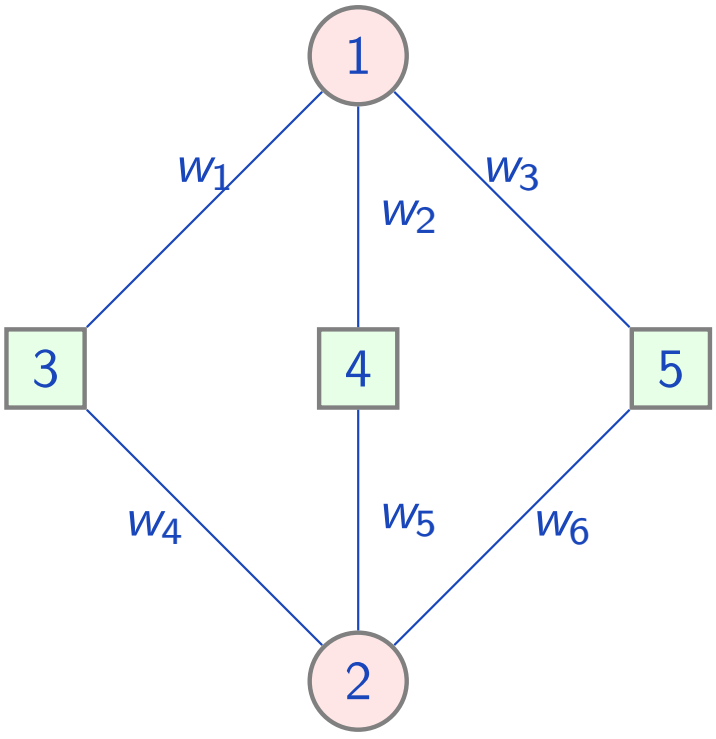
State - directed edge w_i



$$A = A_+ A_- = \begin{pmatrix} 0 & 0 & 0 & 0 & \omega_{45} & 0 & 0 & 0 & \omega_{89} \\ 0 & 0 & 0 & \omega_{54} & 0 & 0 & \omega_{67} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_{76} & 0 & \omega_{98} & 0 \\ 0 & \omega_{12} & \omega_{13} & 0 & 0 & 0 & 0 & 0 & \omega_{89} \\ \omega_{21} & 0 & \omega_{23} & 0 & 0 & 0 & \omega_{67} & 0 & 0 \\ \omega_{21} & 0 & \omega_{23} & \omega_{54} & 0 & 0 & 0 & 0 & 0 \\ \omega_{31} & \omega_{32} & 0 & 0 & 0 & 0 & 0 & \omega_{98} & 0 \\ 0 & \omega_{12} & \omega_{13} & 0 & \omega_{45} & 0 & 0 & 0 & 0 \\ \omega_{31} & \omega_{32} & 0 & 0 & 0 & \omega_{76} & 0 & 0 & 0 \end{pmatrix}$$

$$a_0^i = (\underbrace{0, 0, ..., 0}_{i \text{ times}}, 1, 0, ..., 0)$$

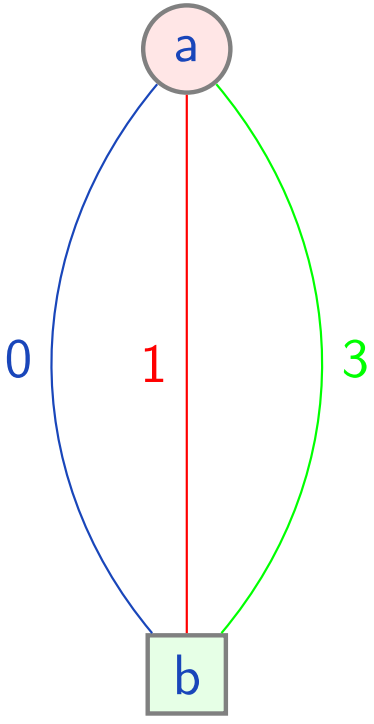
$$a_{2L}^i = a_0^i \cdot A^L$$

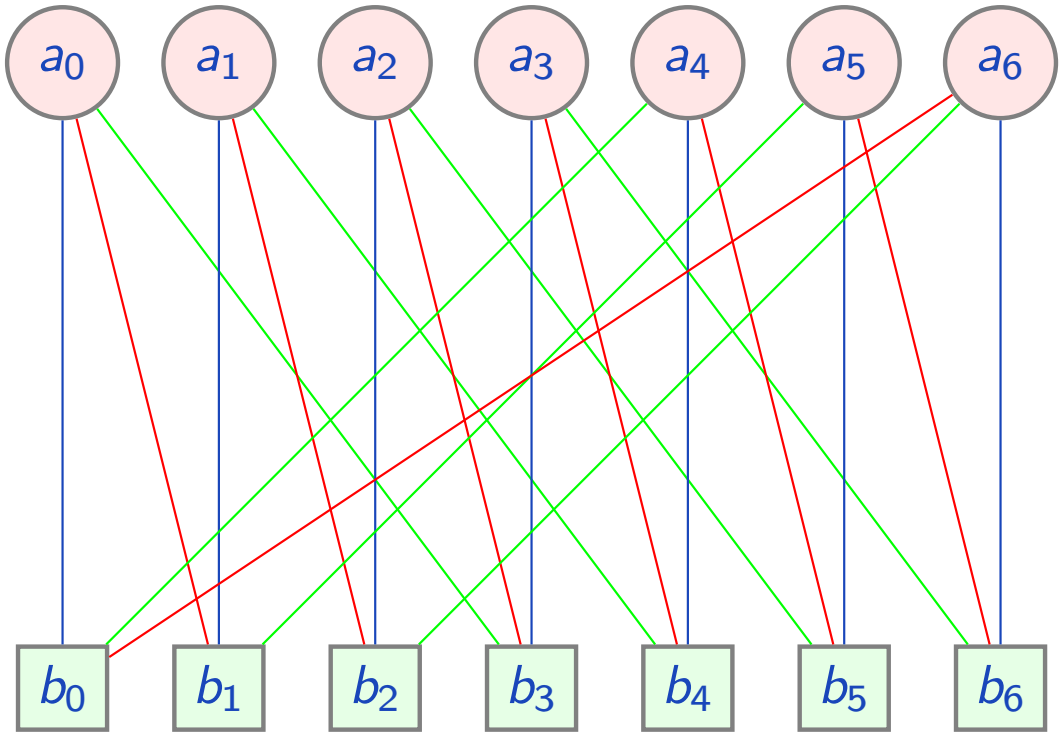


$w_1 w_4 w_5 w_2 w_1 w_4 w_5 w_2$ - 4 times

$w_1 w_4 w_5 w_2 w_1 w_4 w_6 w_3$ - 8 times

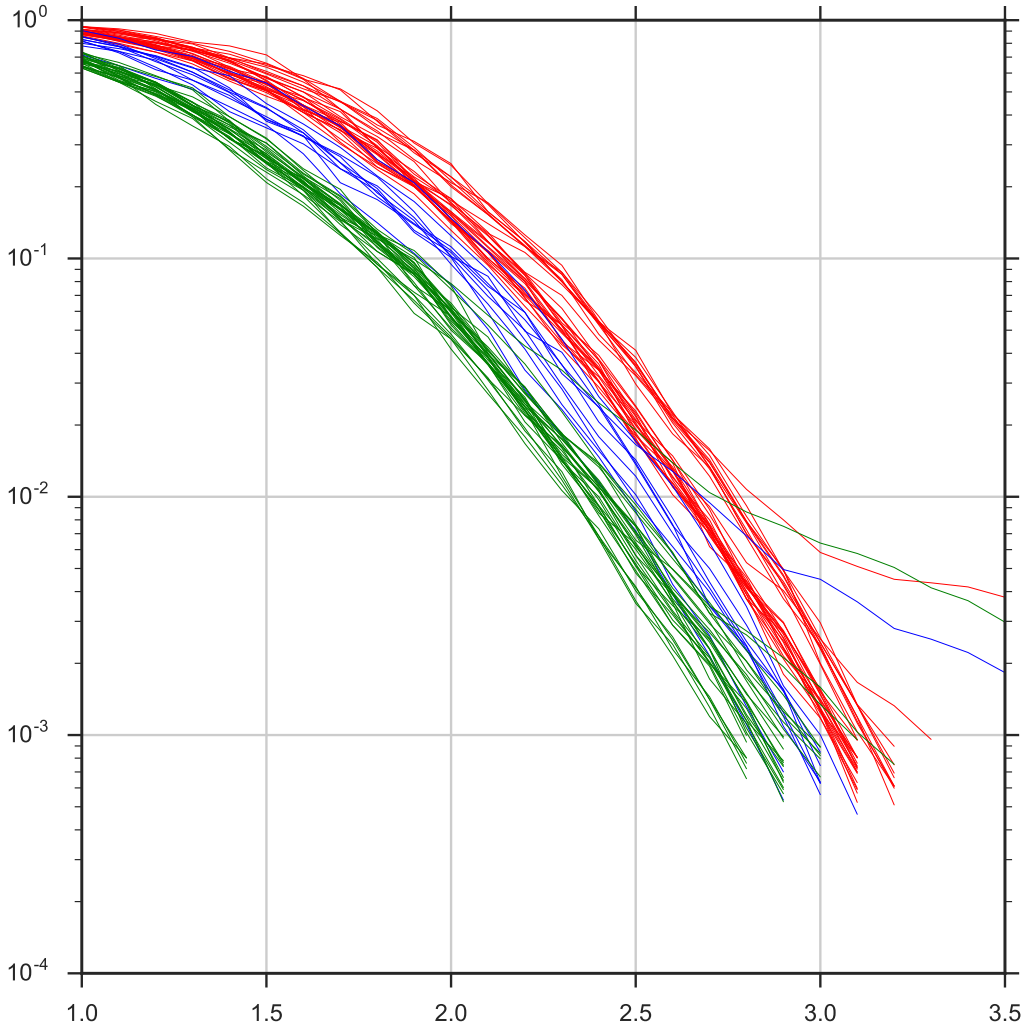
Spectrum calc algorithm
Lifting, weights



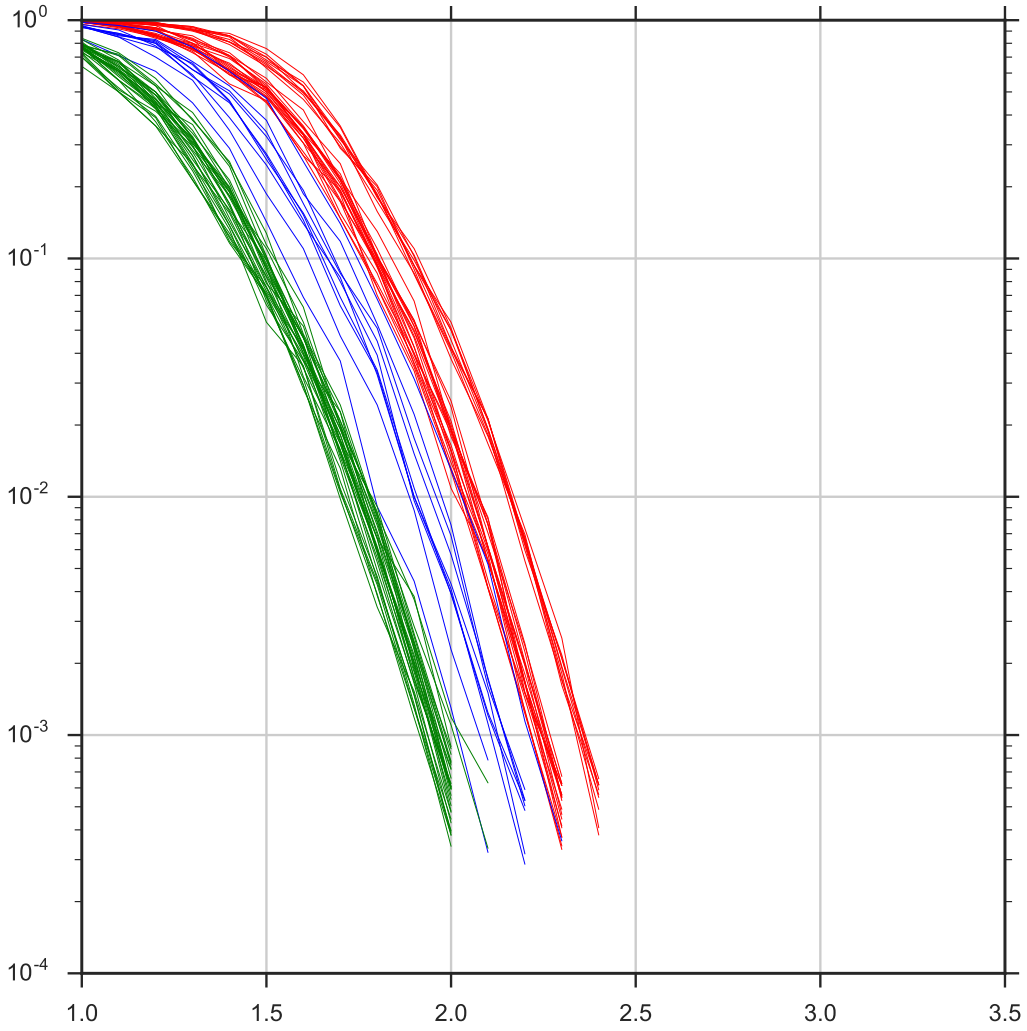


$O(M \cdot S \cdot E^3)$

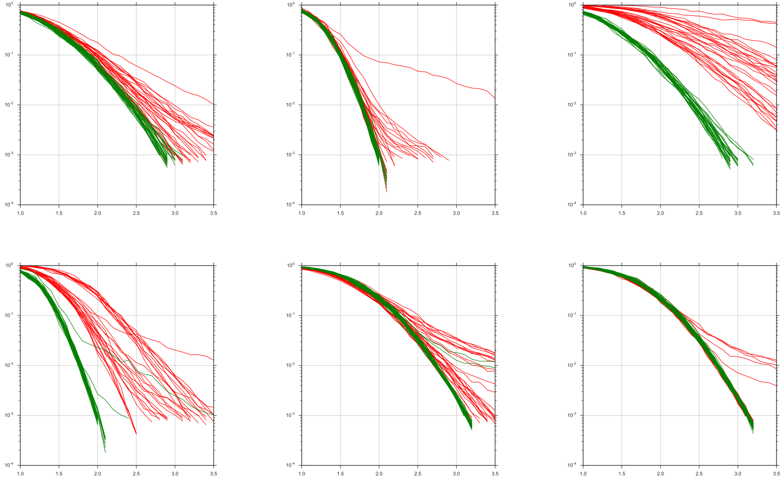
Experiments
Richardson-Urbanke 4x8 576



Experiments
Richardson-Urbanke 4x8 2304



Experiments



Conclusion

- Experiments confirmed correlation and showed that codes with lower number of short cycles has lower frame error rate.
- Computationally effective algorithm for Tanner's graph spectrum calculation was developed and can be used for good codes search acceleration