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RESEARCH OF CORRELATION BETWEEN FRAME ERROR RATE AND TANNER’S GRAPH SPECTRUM FOR LDPC-CODES

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Different error correction codes are used for data transmission through noisy channel. Low density parity checks codes or LDPC-codes were introduced by Gallager in 1962 but due to computational complexity become popular only recently. MacKay showed in his work that these code’s properties are close to turbo-codes but decoding are easier and can be parallelized. One of the examples of LDPC-codes usage – WiMAX standard.

LDPC-code as any other linear binary code can be set with check matrix H. Rows of check matrix compose basis of space which is orthogonal to codewords space. Bipartite graph constructed on matrix H as adjacency matrix called Tanner’s graph. Nodes corresponding to rows called check nodes. Nodes corresponding to columns called variable nodes.

Belief propagation algorithm, also known as sum-product message passing algorithm, was used for decoding. It’s an iterative process. Variable nodes keep logarithm of likelihood relation. In the first step, variable nodes send information (message) to connected check nodes containing the bit they believe to be the correct one for them. In the second step, every check node calculates a response to every connected variable node. The response message contains the bit that this node believes to be correct based on all information it has. If all check equations are fulfilled algorithm terminates otherwise these steps repeat again. Maximum number of iterations used during research is 50. Important assumption for correctness of this algorithm is absence of cycles in graph. Of course, in most cases it’s not true but algorithm works well anyway.

It's not hard to notice and it's well known that as length of shortest cycle(girth) increases then frame error rate decreases. The reason for this is that as girth increases then more iterations hypothesis of check equations independence (absence of cycles) holds.

It’s natural to guess that success of decoding except length of shortest cycle also depends on number of shortest cycles and probably on number of longer cycles. So, it’s good idea to analyze correlation of frame error rate and distribution of cycle lengths in graph – spectrum. However, it turns out that finding distribution of simple cycles in graph is computationally intensive task. Besides that, message passing algorithm rather operate with something similar to closed paths than objects isomorphic to simple cycles. These closed paths have property to not go by the same edge twice in a row. In other words, when path comes to vertex it can’t go immediately back. Other than that, we should consider closed paths which can be produced with cycle shift from one another to be equivalent. So, we will analyze frame error rate based on spectrum of closed paths in Tanner’s graph (set of numbers corresponding to numbers of closed paths of particular lengths).

The algorithm for finding spectrum in time significantly lower than time needed for simulation to determine frame error rate was developed. In this case demonstration of correlation between code effectiveness and spectrum will allow to speed up search of most effective codes.

CUDA decoder was developed for comprehensive testing. The main aim of this decoder is to allow fast simulation of transmission of big number of random codewords over noisy channel for determination of frame error rate in case of fixed signal/noise ratio. This decoder allowed to test significantly bigger number of codes during search of correlation.

The method of lifting of base matrices was used for building random codes. So, for example, for matrix of size 5 by 10 it can get matrix of size 100 by 200 with expansion parameter 20. In lifted matrix, every vertex of base matrix-graph gets 20 corresponding vertices. For purpose of lifting we need, so called, voltage assignment on edges for determination which of new instances of vertices should be connected by edge in lifted graph.

Several thousands of random matrices were created. Sequence of matrices with pairwise comparable spectrum of corresponding graphs were chosen. It was shown that with small number of matrices-exceptions all matrices were divided to clusters according to number of closed paths of shortest length. Value of signal/noise ratio sufficient to get frame error rate of 0.001 were used for comparison. More than that, as expected, this value was bigger in clusters with bigger number of shortest length closed paths. Consequently, described algorithm of analysis of LDPC codes can be used for improvement of search and optimization of effective LDPC codes.

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