Public-Key Cryptography and Message Authentication

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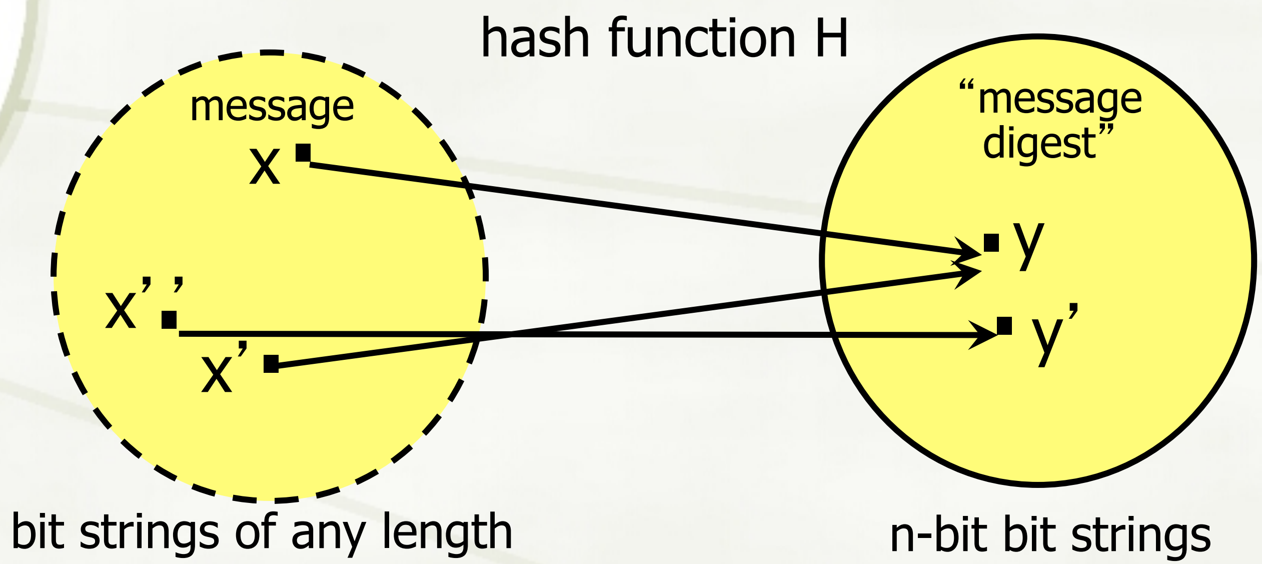
OUTLINE

- Secure Hash Functions and HMAC
- Public-Key Cryptography Principles
- Public-Key Cryptography Algorithms
- Approaches to Message Authentication
- Digital Signatures
- Key Management

Hash Functions: Main Idea



- Hash function H is a lossy compression function
  - Collision: H(x)=H(x') for some inputs x≠x'
- H(x) should look "random"
  - Every bit (almost) equally likely to be 0 or 1
- Cryptographic hash function must have certain properties

A simple hash function

	bit 1	bit 2	• • •	bit n
block 1	b_{11}	b_{21}		b_{n1}
block 2	b_{12}	b_{22}		b_{n2}
	⋮	⋮	⋮	⋮
block m	b_{1m}	b_{2m}		b_{nm}
hash code	C_1	C_2		C_n

- Hash code is bitwise XOR on the columns
- One-bit circular shift on the hash value after each block is processed would improve the code

Secure hash functions

- Purpose of the HASH function is to produce a ”fingerprint.
- Properties of a HASH function H :
  1. H can be applied to a block of data at any size
  2. H produces a fixed length output
  3. H(x) is easy to compute for any given x.
  4. For any given code h, it is computationally infeasible to find x such that H(x) = h
  5. For any given block x, it is computationally infeasible to find $y \neq x$ with H(y) = H(x).
  6. It is computationally infeasible to find any pair (x, y) such that H(x) = H(y)

Secure Hash Algorithm

- SHA originally designed by NIST & NSA in 1993
- was revised in 1995 as SHA-1
- US standard for use with DSA signature scheme
  - standard is FIPS 180-1 1995, also Internet RFC3174
  - nb. the algorithm is SHA, the standard is SHS
- based on design of MD4 with key differences
- produces 160-bit hash values
- In 2005 results on security of SHA-1 have raised concerns on its use in future applications

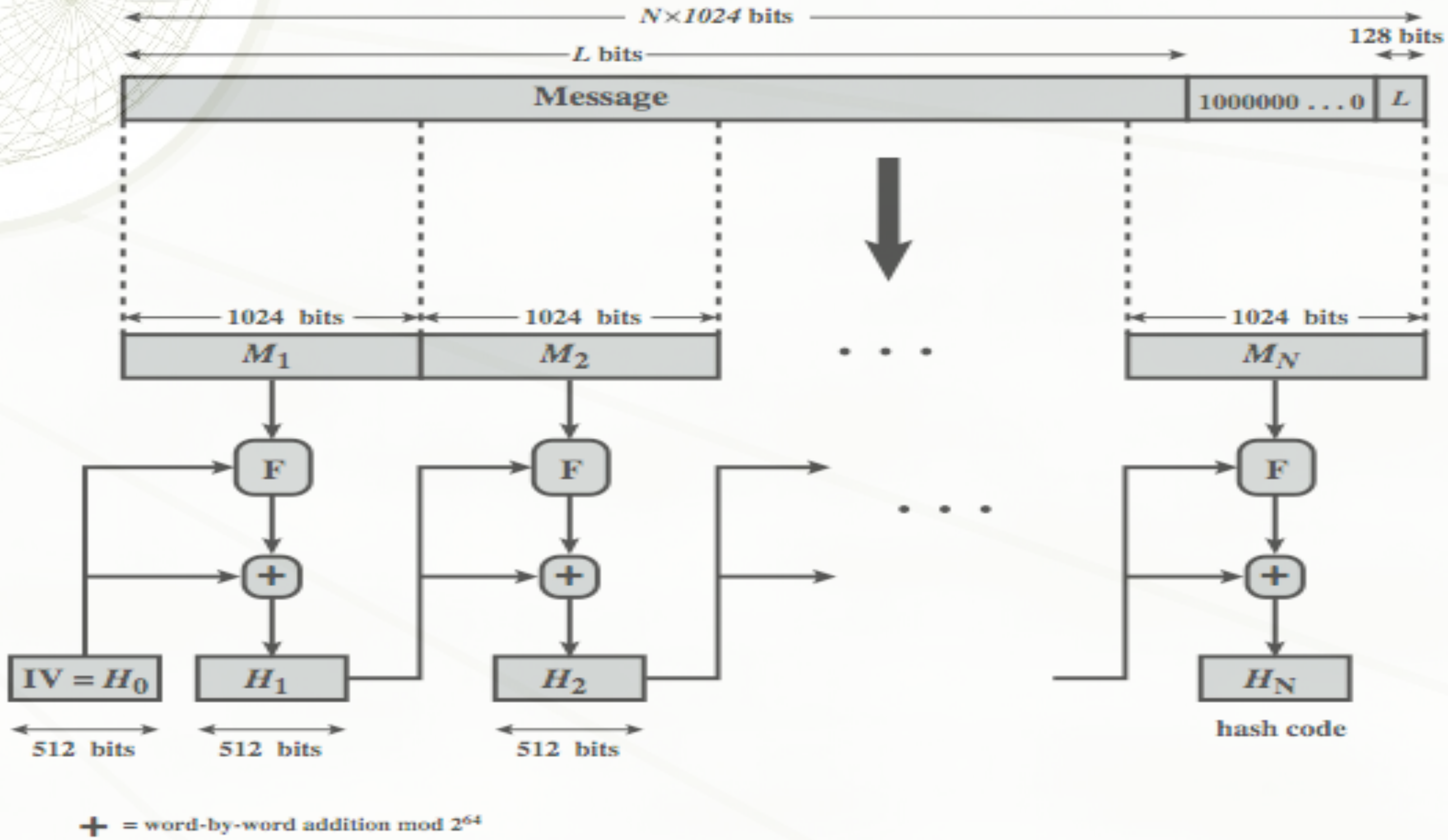
Revised Secure Hash Algorithm

- NIST issued revision FIPS 180-2 in 2002
- adds 3 additional versions of SHA
  - SHA-256, SHA-384, SHA-512
- designed for compatibility with increased security provided by the AES cipher
- structure & detail is similar to SHA-1
- hence analysis should be similar
- but security levels are rather higher

SHA versions

	SHA-1	SHA-224	SHA-256	SHA-384	SHA-512
Message digest size	160	224	256	384	512
Message size	$< 2^{64}$	$< 2^{64}$	$< 2^{64}$	$< 2^{128}$	$< 2^{128}$
Block size	512	512	512	1024	1024
Word size	32	32	32	64	64
Number of steps	80	64	64	80	80

SHA-512 overview



SHA-512 compression function

- heart of the algorithm (the Merkle-Damgård construction)
- processing message in 1024-bit blocks
- consists of 80 rounds
  - updating a 512-bit buffer
  - using a 64-bit value derived from the current message block and a round constant based on cube root of first 80 prime numbers

One-Way

- Intuition: hash should be hard to invert
  - "Preimage resistance"
  - Let $h(x')=y\in\{0,1\}^n$ for a random x'
  - Given random y, it should be hard to find any x such that h(x)=y
- How hard?
  - Brute-force: try every possible x, see if h(x)=y
  - SHA-1 (common hash function) has 160-bit output
    - Suppose we have hardware that do 2^{30} trials a pop
    - Assuming 2^{34} trials per second, can do 2^{89} trials per year
    - Will take 2^{71} years to invert SHA-1 on a random image

"Birthday Paradox"

- T people
- Suppose each birthday is a random number taken from K days (K=365) – how many possibilities?
  - K^T - samples with replacement
- How many possibilities that are all different?
  - $(K)_T = K(K-1)\ldots(K-T+1)$ - samples without replacement
- Probability of no repetition?
  - $(K)_T/K^T \approx 1 - T(T-1)/2K$
- Probability of repetition?
  - $O(T^2)$

Collision resistance

- Should be hard to find $x \neq x'$ such that $h(x)=h(x')$
- Brute-force collision search is $O(2^{n/2})$, <u>not</u> $O(2^n)$
  - n = number of bits in the output of hash function
  - For SHA-1, this means $O(2^{80})$ vs. $O(2^{160})$
- Example:
  - In a group of people, how many are required to have a 50% chance of two people having the same birthday?
  - Perhaps surprisingly the answer is only 23 even if there are 365 days on a year.
  - If there are more then 60 people the probability for a collision is almost 100%

SHA-1 weakness

- In 2005 Chinese cryptographer Xiaoyun Wang found collision-finding attacks that require only 2^{63} operations, rather than the 2^{80} operations stated by the birthday attack. Such an attack is feasible for a very well-funded adversary.
- Consequently, for many applications one needs to look for stronger hash functions. The SHA-2 family, including SHA-224, SHA-256 and SHA-512 already exists, but they are based on similar ideas as SHA-1.
- It seems that new ideas are needed in the design of hash functions, as well as careful analysis of what properties are needed for various applications.

SHA-3, the new kid in the block

- On November 2, 2007, NIST announced a public competition to develop a new cryptographic hash algorithm.
- Deadline for proposals was October 31, 2008.
  14 candidates selected for 2nd round in 2009,
  6 for 3rd round in 2010.
- The winner, Keccak, announced on October 2, 2012.
- Keccak is now officially SHA-3; see www.nist.gov/hash-competition.

Different secure hash functions

	SHA-1	MD5	RIPEMD-160
Digest length	160 bits	128 bits	160 bits
Basic unit of processing	512 bits	512 bits	512 bits
Number of steps	80 (4 rounds of 20)	64 (4 rounds of 16)	160 (5 paired rounds of 16)
Maximum message size	2^{64}-1 bits	∞	∞

Keyed hash functions as Message Authentication Code

- want a MAC based on a hash function
  - because hash functions are generally faster
  - crypto hash function code is widely available
- hash includes a key along with message
- original proposal:

```
KeyedHash = Hash(Key|Message)
```

  - some weaknesses were found with this
- eventually led to development of HMAC

HMAC design objectives

- use, without modifications, hash functions
- allow for easy replaceability of embedded hash function
- preserve original performance of hash function without significant degradation
- use and handle keys in a simple way.
- have well understood cryptographic analysis of authentication mechanism strength

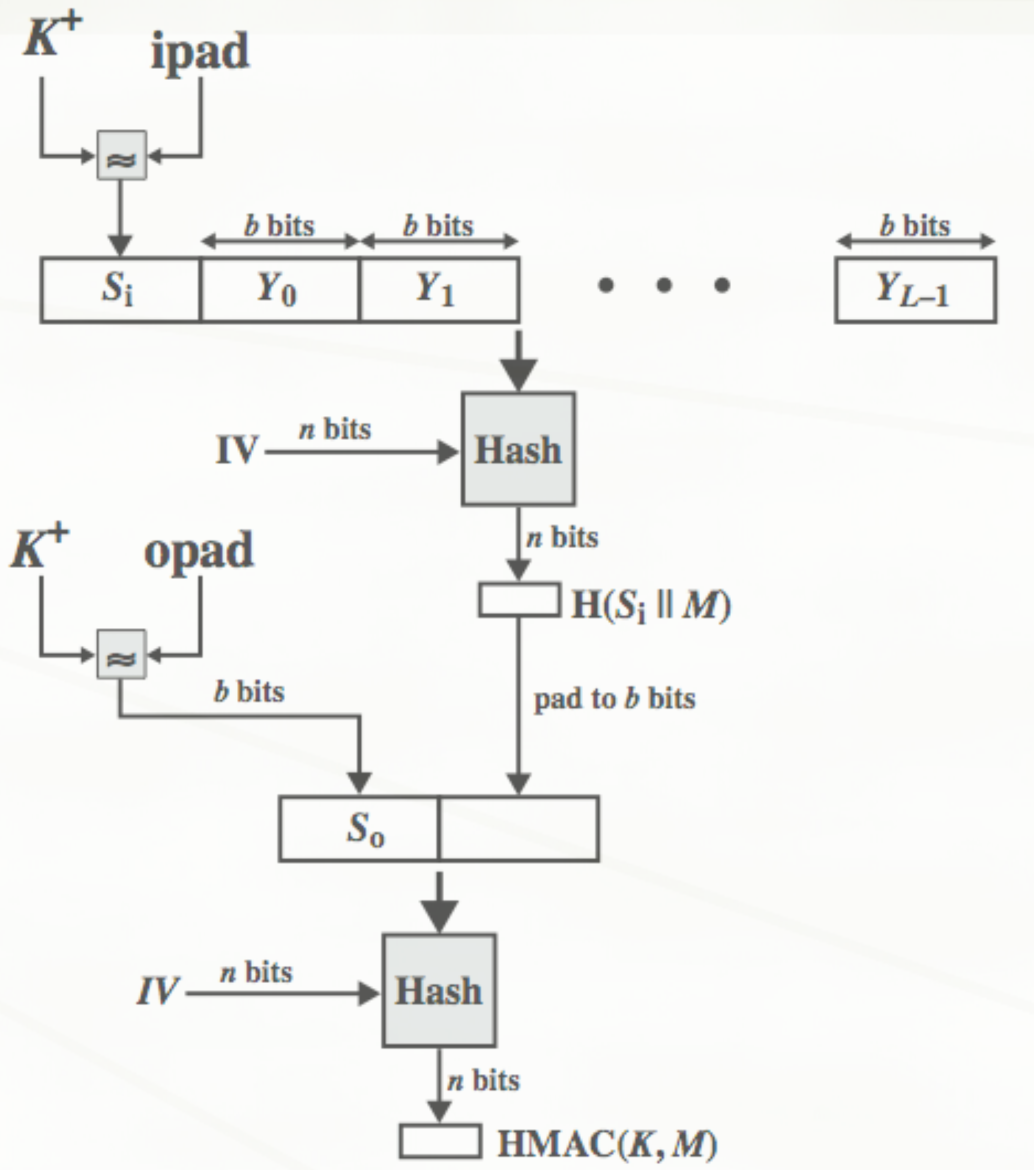
HMAC

- specified as Internet standard RFC2104
- uses hash function on the message:

  $$HMAC_K(M) = \mathrm{Hash}[(K^+ \text{ XOR opad}) \,||\, \mathrm{Hash}[(K^+ \text{ XOR ipad}) \,||\, M)]\,]$$

  - where K^+ is the key padded out to size
  - `opad`, `ipad` are specified padding constants
- overhead is just 3 more hash calculations than the message needs alone
- any hash function can be used
  - eg. MD5, SHA-1, RIPEMD-160, Whirlpool

HMAC overview



HMAC security

- proved security of HMAC relates to that of the underlying hash algorithm
- attacking HMAC requires either:
  - brute force attack on key used
  - birthday attack (but since keyed, would need to observe a very large number of messages)
- choose hash function used based on speed verses security constraints

Private-Key Cryptography (revisited)

- traditional **private/secret/single key** cryptography uses **one** key
- shared by both sender and receiver
- if this key is disclosed communications are compromised
- also is **symmetric**, parties are equal
- hence does not protect sender from receiver forging a message & claiming is sent by sender

Public-Key Cryptography the new idea

- probably most significant advance in the 3000 year history of cryptography
- uses **two** keys – a public & a private key
- **asymmetric** since parties are **not** equal
- uses clever application of number theoretic concepts to function
- complements **rather than** replaces private key crypto

Why Public-Key Cryptography?

- developed to address two key issues:
  - **key distribution** – how to have secure communications in general without having to trust a KDC with your key
  - **digital signatures** – how to verify a message comes intact from the claimed sender
- public invention due to Whitfield Diffie & Martin Hellman at Stanford Uni in 1976
  - known earlier in classified community

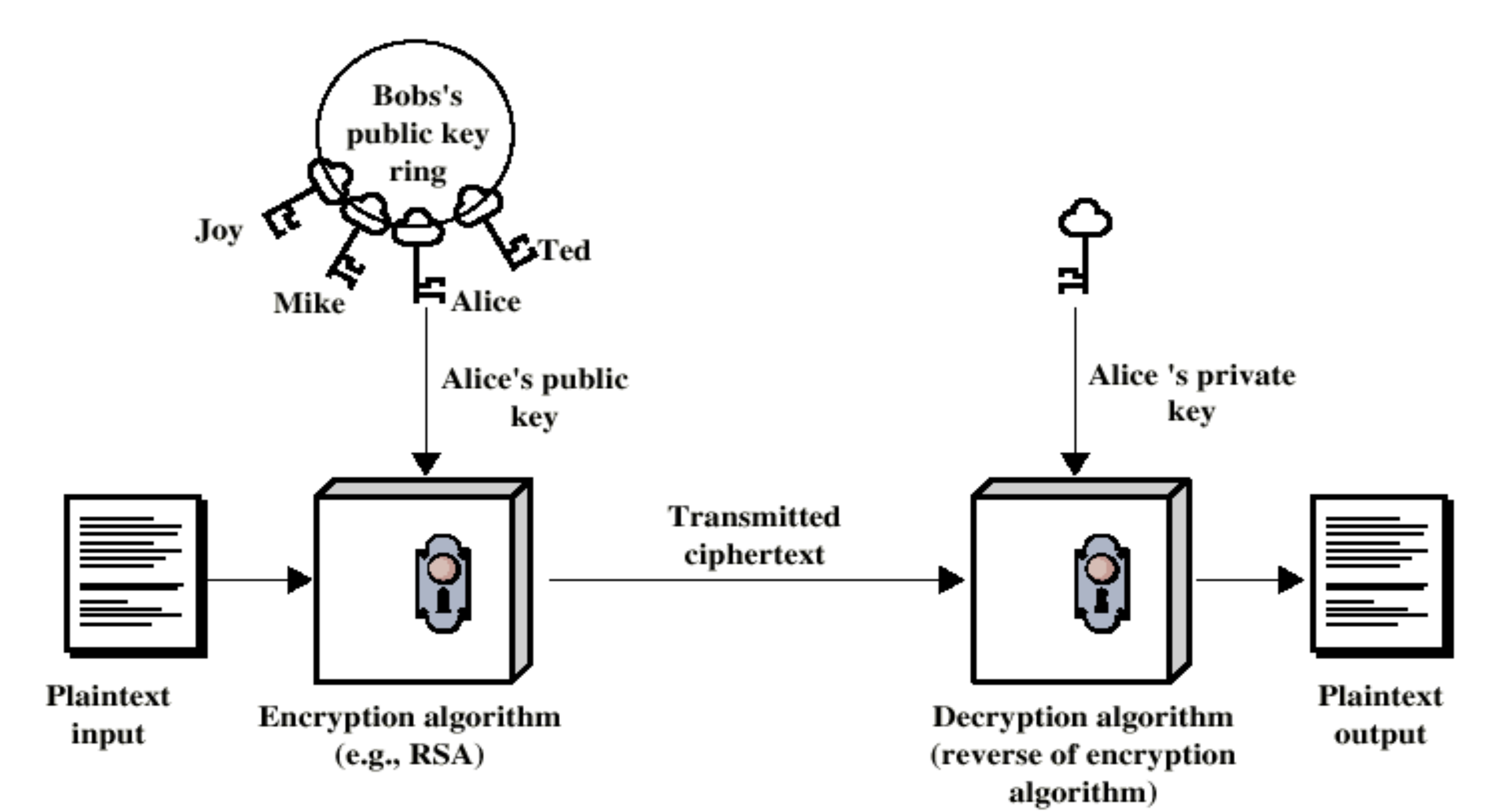
Public-Key Cryptography

- **public-key/two-key/asymmetric** cryptography involves the use of **two** keys:
  - a **public-key**, which may be known by anybody, and can be used to **encrypt messages**, and **verify signatures**
  - a related **private-key**, known only to the recipient, used to **decrypt messages**, and **sign** (create) **signatures**
- **infeasible to determine private key from public**
- is **asymmetric** because
  - those who encrypt messages or verify signatures **cannot** decrypt messages or create signatures

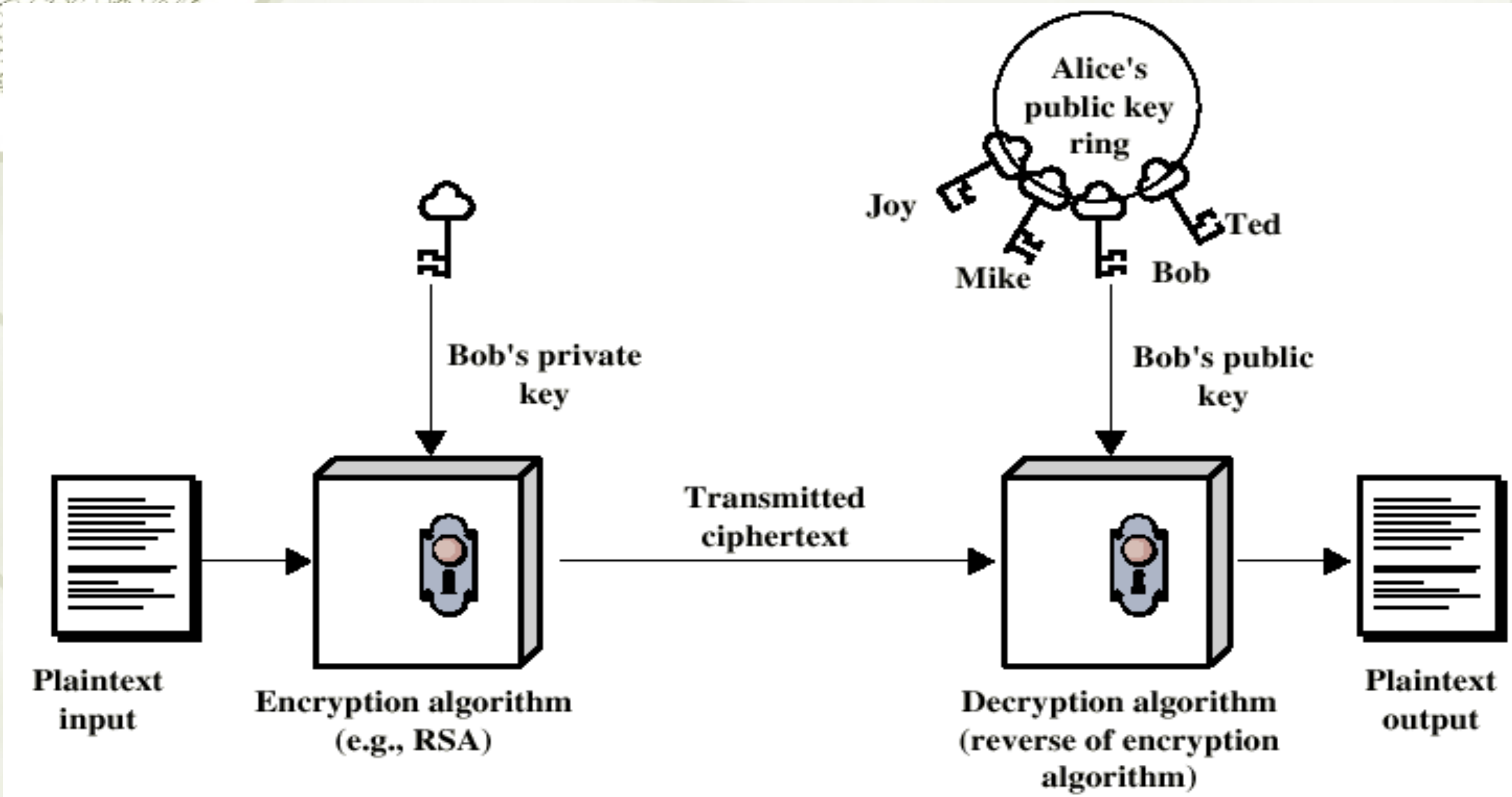
Public-Key Cryptography principles

- The use of two keys has consequences in: key distribution, confidentiality and authentication.
- The scheme has six ingredients
  - Plaintext
  - Encryption algorithm
  - Public and private key
  - Cipher text
  - Decryption algorithm

Encryption using Public-Key System



Authentication using Public-Key System



Applications for Public-Key Cryptosystems

- Three categories:
  - **Encryption/decryption:** The sender encrypts a message with the recipient's public key.
  - **Digital signature:** The sender ”signs” a message with its private key.
  - **Key exchange:** Two sides cooperate two exchange a session key.

Requirements for Public-Key Cryptography

1. Computationally easy for a party B to generate a pair (public key KU_b, private key KR_b)
2. Easy for sender to generate cipher text:
$$C = E_{KUb}(M)$$
3. Easy for the receiver to decrypt cipher text using private key:
$$M = D_{KRb}(C) = D_{KRb}[E_{KUb}(M)]$$

Requirements for Public-Key Cryptography

4. Computationally infeasible to determine private key (KR_b) knowing public key (KU_b)
5. Computationally infeasible to recover message M, knowing KU_b and cipher text C
6. Either of the two keys can be used for encryption, with the other used for decryption:

$$M = D_{KRb}[E_{KUb}(M)] = D_{KUb}[E_{KRb}(M)]$$

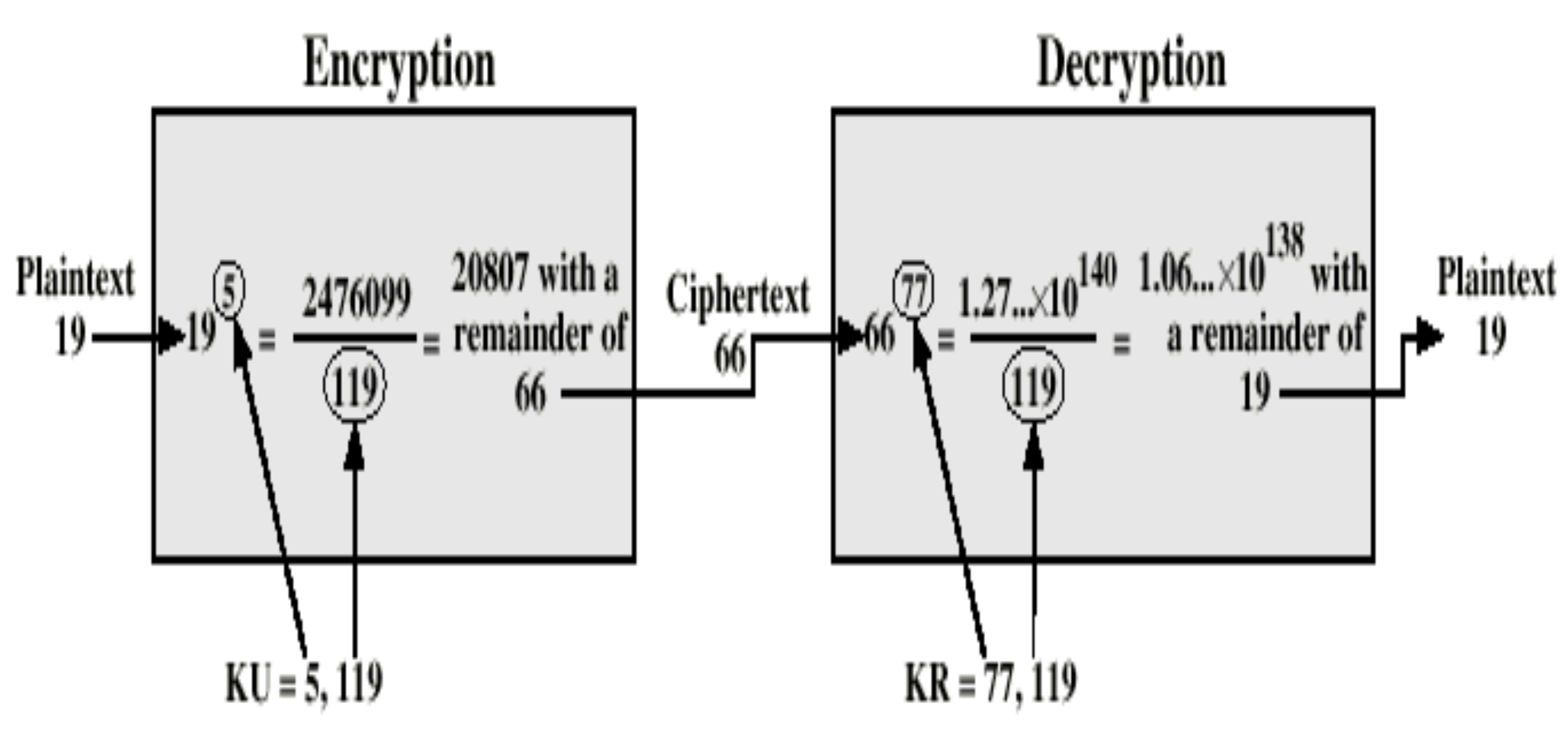
Public-Key Cryptographic algorithms

- RSA and Diffie-Hellman
  - **RSA** - Ron Rivest, Adi Shamir and Len Adleman at MIT, in 1977.
    - RSA is a block cipher
    - The most widely implemented
  - **Diffie-Hellman**
    - Exchange a secret key securely
    - Compute discrete logarithms

The RSA algorithm - key generation

1.	Select *p*,*q*	*p* and *q* both prime
2.	Calculate	$n = p \times q$
3.	Calculate	$\Phi(n) = (p-1)(q-1)$
4.	Select integer *e*	$\gcd(\Phi(n), e) = 1; 1 < e < \Phi(n)$
5.	Calculate *d*	$d = e^{-1} \bmod \Phi(n)$
6.	Public Key	KU = {e,n}
7.	Private key	KR = {d,n}

Example of RSA algorithm



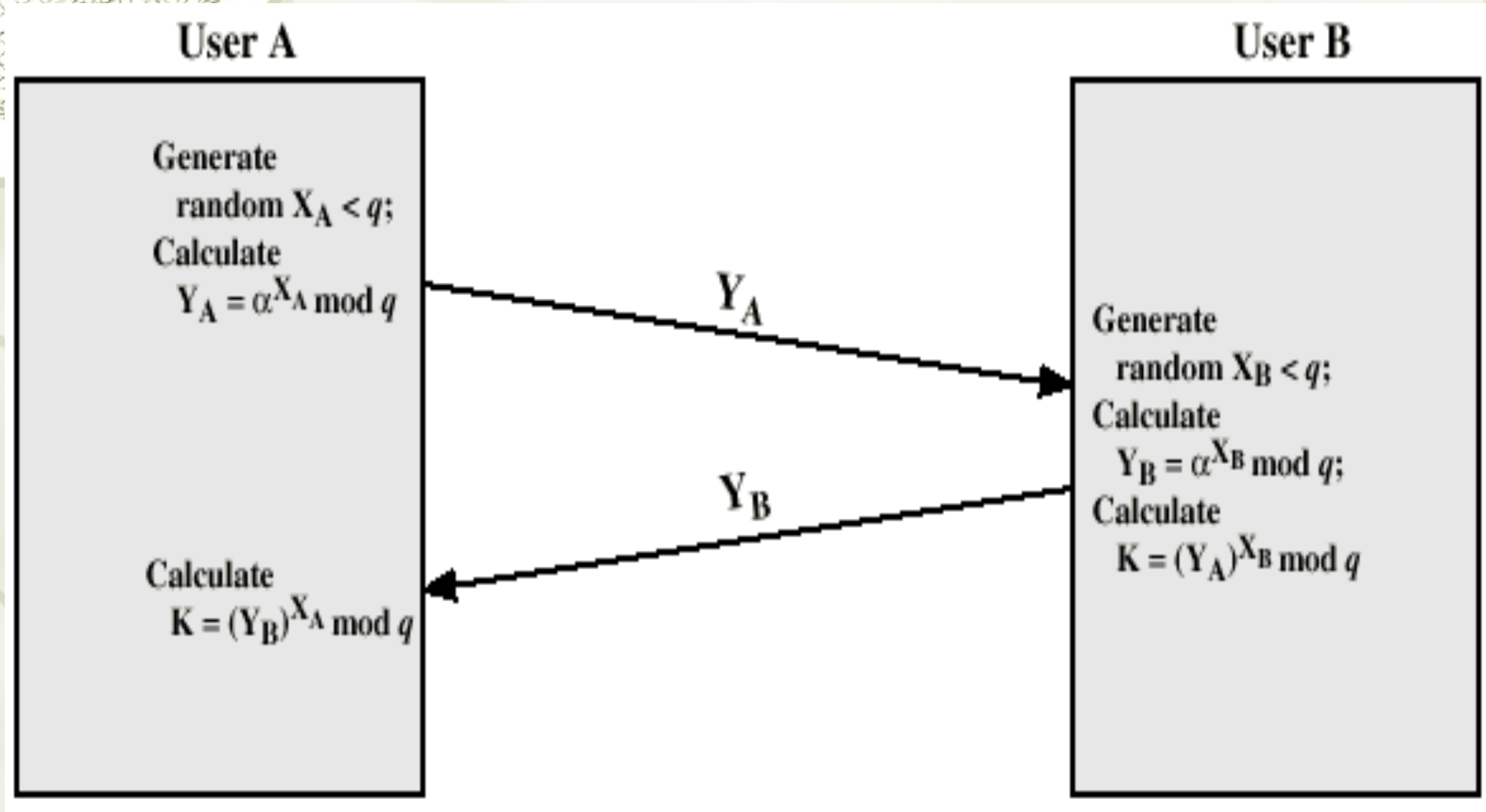
The RSA algorithm - encryption

- Plain text: $M<n$
- Cipher text: $C = M^e \pmod{n}$

The RSA algorithm - decryption

- Cipher text: C
- Plain text: $M = C^d \pmod{n}$

Diffie-Hellman key exchange



Other Public-Key cryptographic algorithms

- Digital Signature Standard (DSS)
  - Makes use of the SHA-1
  - Not for encryption or key exchange
- Elliptic-Curve Cryptography (ECC)
  - Good for smaller bit size
  - Low confidence level, compared with RSA
  - Very complex
- ElGamal
  - an asymmetric key encryption algorithm based on the Diffie-Hellman key exchange

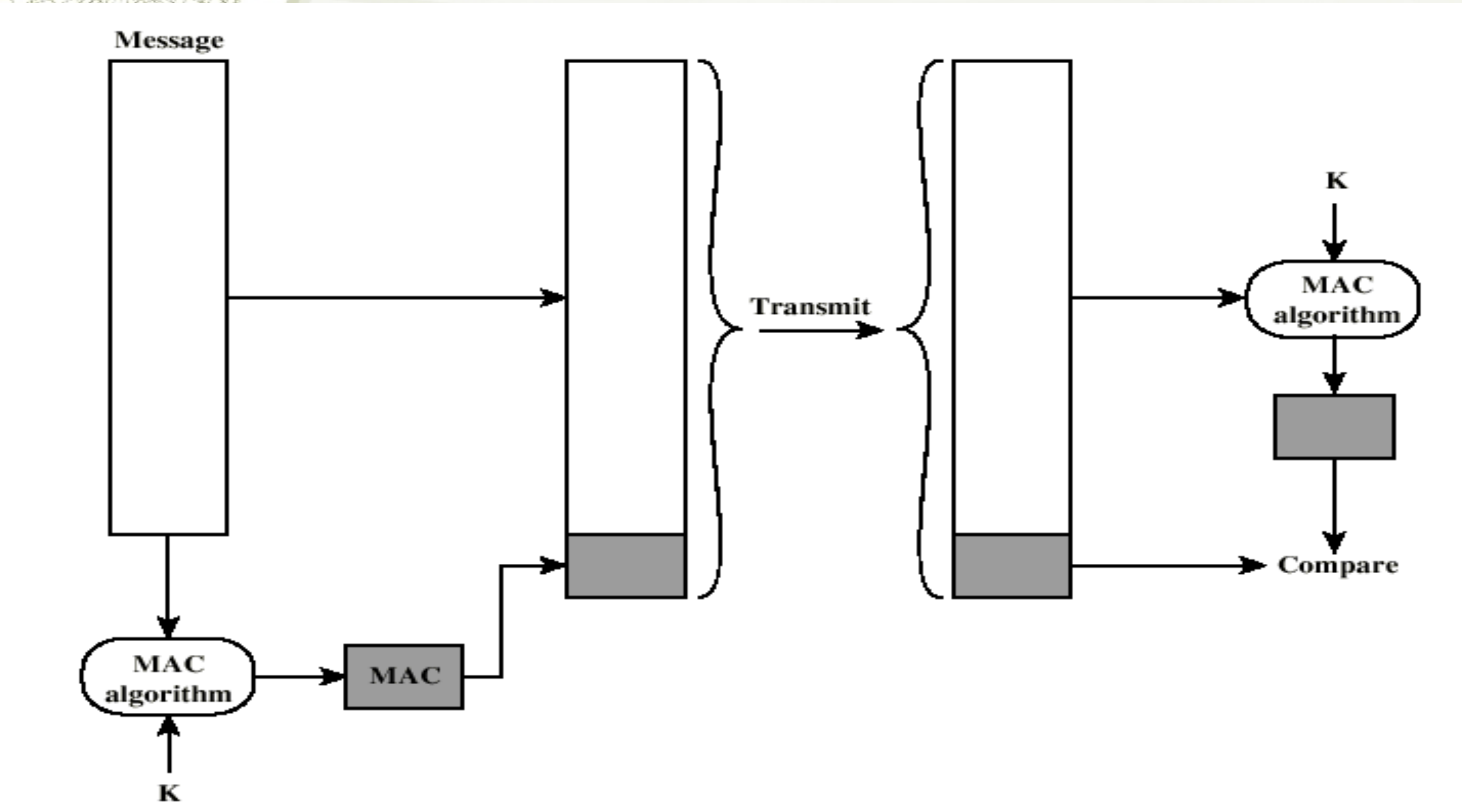
Authentication

- Requirements - must be able to verify that:
  1. Message came from apparent source or author,
  2. Contents have not been altered,
  3. Sometimes, it was sent at a certain time or sequence.
- Protection against active attack (falsification of data and transactions)

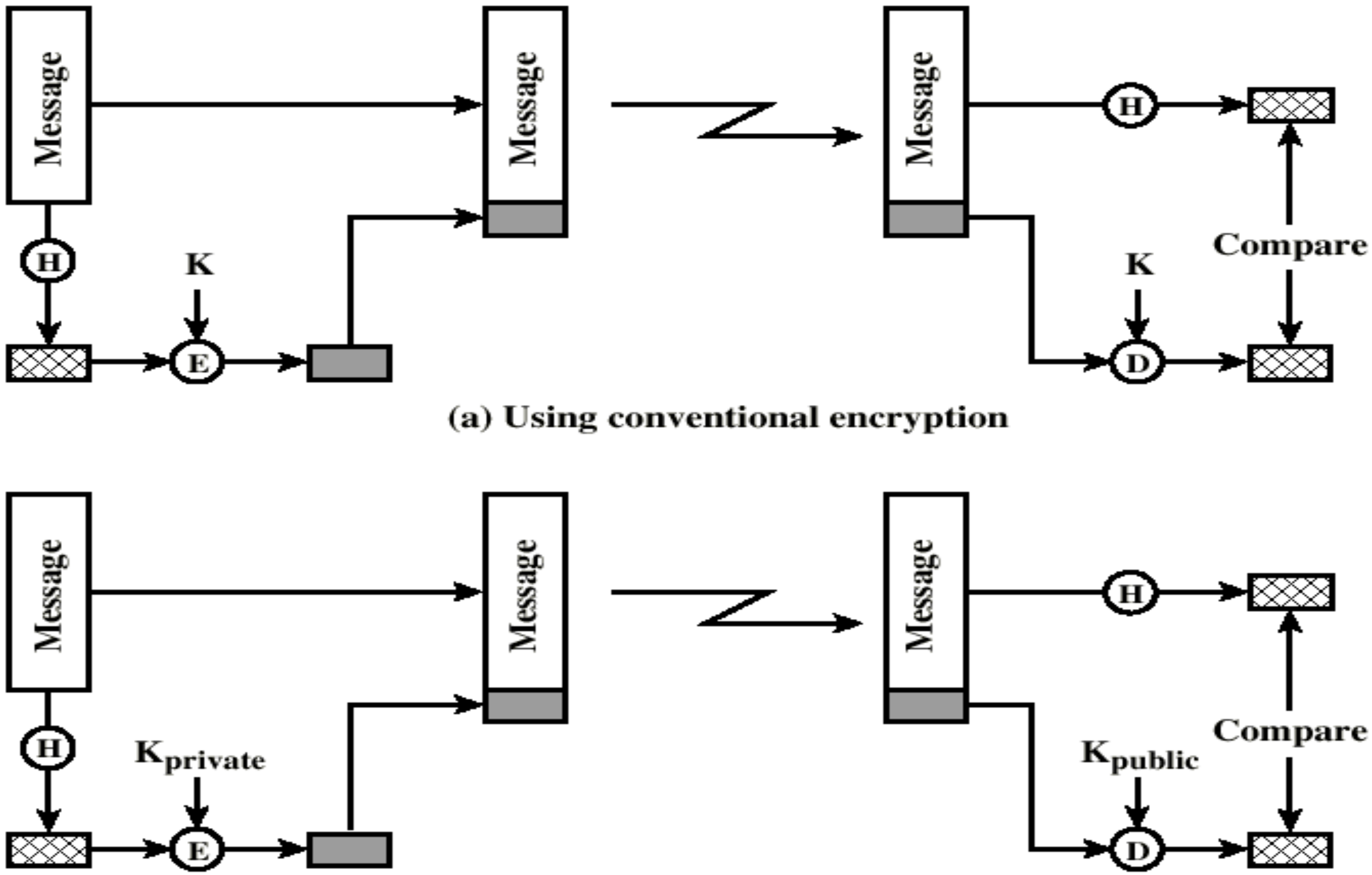
Approaches to Message Authentication

- Authentication Using Conventional Encryption
  - Only the sender and receiver should share a key
- Message Authentication without Message Encryption
  - An authentication tag is generated and appended to each message
- Message Authentication Code
  - Calculate the MAC as a function of the message and the key. MAC = F(K, M)

Authentication using a Message Authentication Code



One-way hash function

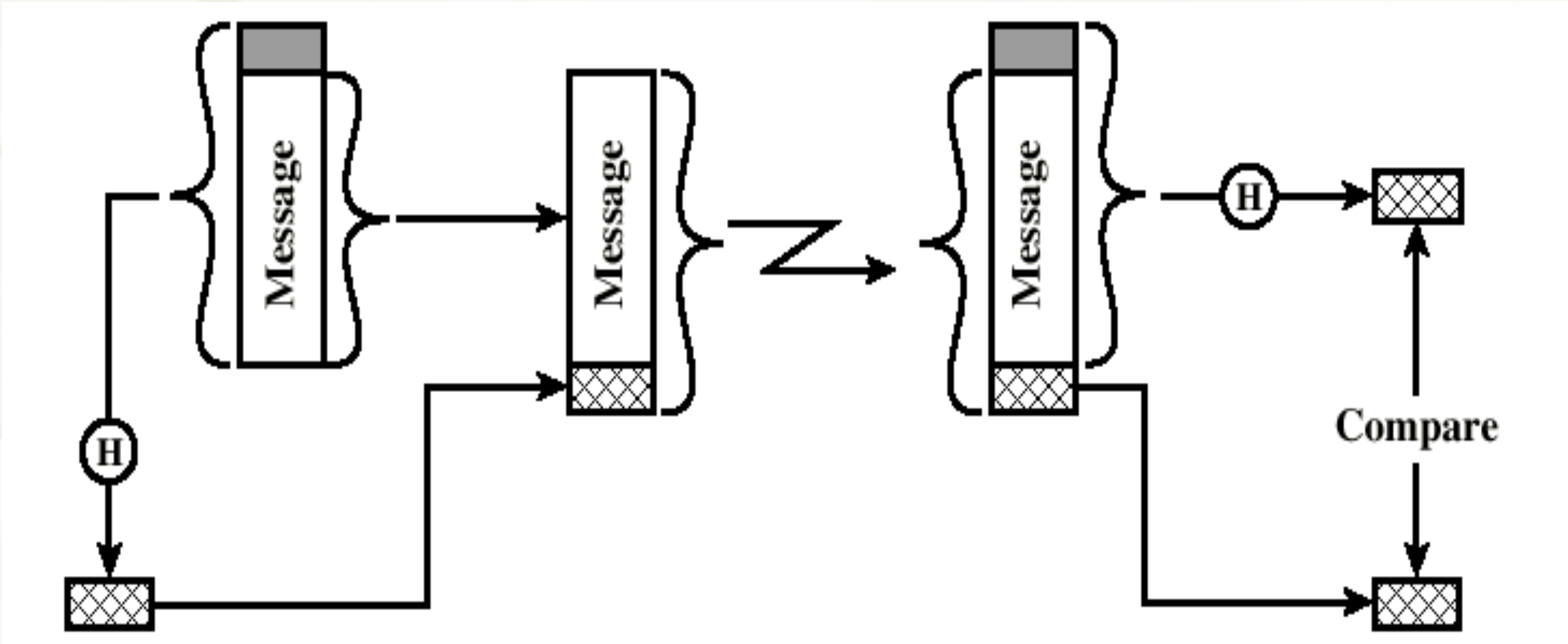


(a) Using conventional encryption

(b) Using public-key encryption

One-way hash function

- Secret value is added before the hash and removed before transmission.



Key management
Public-Key certificate use