1. Ways to select 8 students: $\frac{15!}{(15-8)!} = 259,459,200$

Ways to ask 8 questions: 15^8

Prob. no student will answer more than one: $\frac{259,459,200}{15^8} = 0.1012$

2. $100 \to 1000$: # of selected $= 5 \cdot 4 \cdot 5 = 100$

$1000 \to 10000$: # selected $= 5 \cdot 4 \cdot 7 \cdot 5 = 700$

$10000 \to 100000$: # selected $= 5 \cdot 4 \cdot 7 \cdot 6 \cdot 7 = 4200$

Total selected: $4200 + 700 + 100 = 5000$

Total integers in range $= 100{,}000$

$P[\text{selected integer}] = \frac{5000}{100{,}000} = 0.05$

If 5 meet criteria: $\binom{8}{5}(0.05)^5(0.95)^3 = 1.5 \times 10^{-5}$

3. A: $P(2+ \text{ dice show } 4+) = \binom{3}{2}\left(\frac{3}{6}\right)^2\left(\frac{3}{6}\right) + \binom{3}{3}\left(\frac{3}{6}\right)^3\left(\frac{3}{6}\right) = \frac{1}{2}$

$P(\text{same number}) = \frac{6}{6^3} = 1/36$

B

$P(A \cap B) = P(3\ 4s) + P(3\ 5s) + P(3\ 6s) = \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^3 = 1/72$

$P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{36} = \frac{1}{72} = P(A \cap B)$

Thus, events are independent

4. $\binom{4}{1}\binom{13}{5} = 5148$

Thus, $P[\text{Flush}] = \frac{5148}{\binom{52}{5}} = 0.001981$

Expected # of hands $= \frac{1}{0.001981} = 505$ hands

5. $P(\text{win 4 of 5} \mid \text{superstar}) = \binom{5}{4}(0.7)^4(0.3) = 0.36015$

$P(\text{win 4} \mid \text{no superstar}) = \binom{5}{4}(0.5)^5 = 0.15625$

Thus, $P(\text{win 4}) = 0.15625\left(\frac{1}{4}\right) + 0.36015\left(\frac{3}{4}\right) = 0.309175$

$P(\text{superstar played} \mid \text{win 4}) = \frac{0.36015 \cdot .75}{0.309175} = 0.8737$