Project 2

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15 februari 2020

Exercise 1

```
## Warning: package 'knitr' was built under R version 3.5.3
```

```
## Warning: package 'tidyverse' was built under R version 3.5.3
```

```
## -- Attaching packages ---------------------------------------------------------------------------
```

```
## v ggplot2 3.1.0     v purrr   0.2.5
## v tibble  2.1.3     v dplyr   0.7.8
## v tidyr   0.8.2     v stringr 1.3.1
## v ggplot2 3.1.0     v forcats 0.3.0
```

```
## Warning: package 'tibble' was built under R version 3.5.3
```

```
## -- Conflicts ------------------------------------------------------------------------------------
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()
```

```
## Warning: The `printer` argument is deprecated as of rlang 0.3.0.
## This warning is displayed once per session.
```

```
## Warning in if (is.na(row.names)) row.names = has_rownames(x): the condition
## has length > 1 and only the first element will be used
```

```
## Warning in if (row.names) {: the condition has length > 1 and only the
## first element will be used
```

Table 1: Paid claims triangle

	1	2	3	4	5	6	7	8	9	10
1	17382857	32064142	40583164	45163613	47828744	48961319	49629613	49764835	49967431	50125945
2	11940035	25909330	33875305	38571174	41264712	42394449	43047834	43527148	43618976	
3	9570218	23414758	32580043	38193737	40325432	41673704	42146307	42605671		
4	8312376	18695679	24490118	27902308	29520382	30686881	31162979			
5	14023889	30863800	41644000	47146057	50657766	52457015				
6	10893434	23539789	31331880	35431428	37881787					
7	7998715	21327320	29327038	33897863						
8	10466529	23251932	30887979							
9	12161281	27441687								
10	10766067									

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```

```
## Warning in if (is.na(row.names)) row.names = has_rownames(x): the condition
## has length > 1 and only the first element will be used
```

Table 2: Full claims triangle predicted with CL

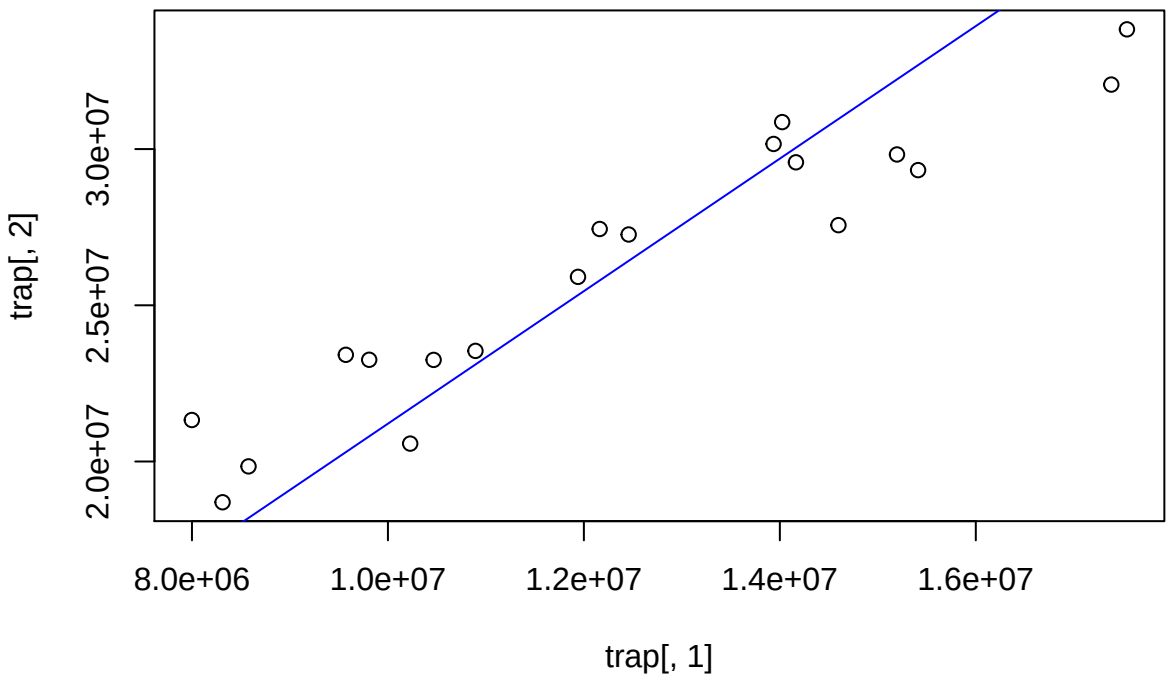
	1	2	3	4	5	6	7	8	9	10
1	17382857	32064142	40583164	45163613	47828744	48961319	49629613	49764835	49967431	50125945
2	11940035	25909330	33875305	38571174	41264712	42394449	43047834	43527148	43618976	43709224
3	9570218	23414758	32580043	38193737	40325432	41673704	42146307	42605671	42763727	42852206
4	8312376	18695679	24490118	27902308	29520382	30686881	31162979	31385330	31501761	31566938
5	14023889	30863800	41644000	47146057	50657766	52457015	53145436	53524633	53723196	53834349
6	10893434	23539789	31331880	35431428	37881787	39046296	39558721	39840976	39988776	40071513
7	7998715	21327320	29327038	33897863	35915920	37019997	37505830	37773437	37913567	37992010
8	10466529	23251932	30887979	34969100	37050931	38189899	38691085	38967149	39111708	39192630
9	12161281	27441687	35761444	40486478	42896778	44215451	44795714	45115335	45282702	45376392
10	10766067	22835114	29758252	33690105	35695794	36793104	37275959	37541927	37681198	37759160

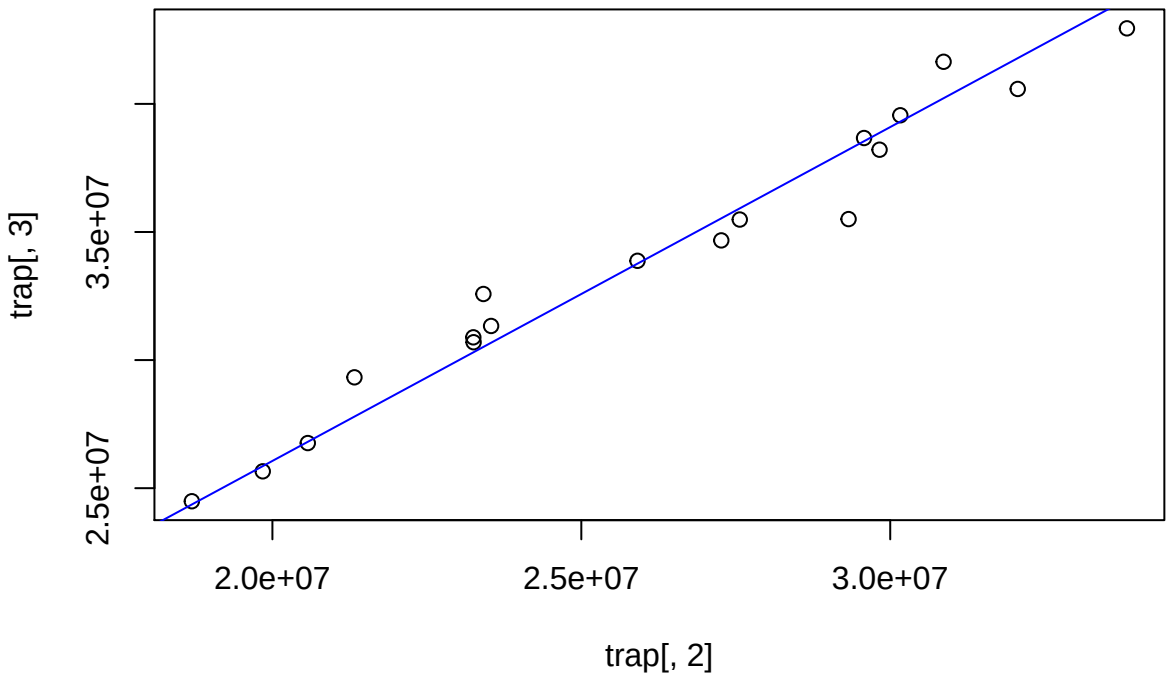
Exercise 2

We now want to check whether or not Mack's undrlying assumptions are met in our case. The assumptions are as follows.

1. $E[C_{i,k+1}|C_{i,1},...,Ci,k] = f_k C_{i,k}$
2. Independent accident years
3. $Var(C_{i,k+1}|C_{i,1},...,Ci,k) = \sigma_k^2 C_{i,k}$

We begin by examing whether or not the we have an approximate linear relationship between $C_{i,k}$ and $C_{i,k+1}$ for $i = 1,...,10$

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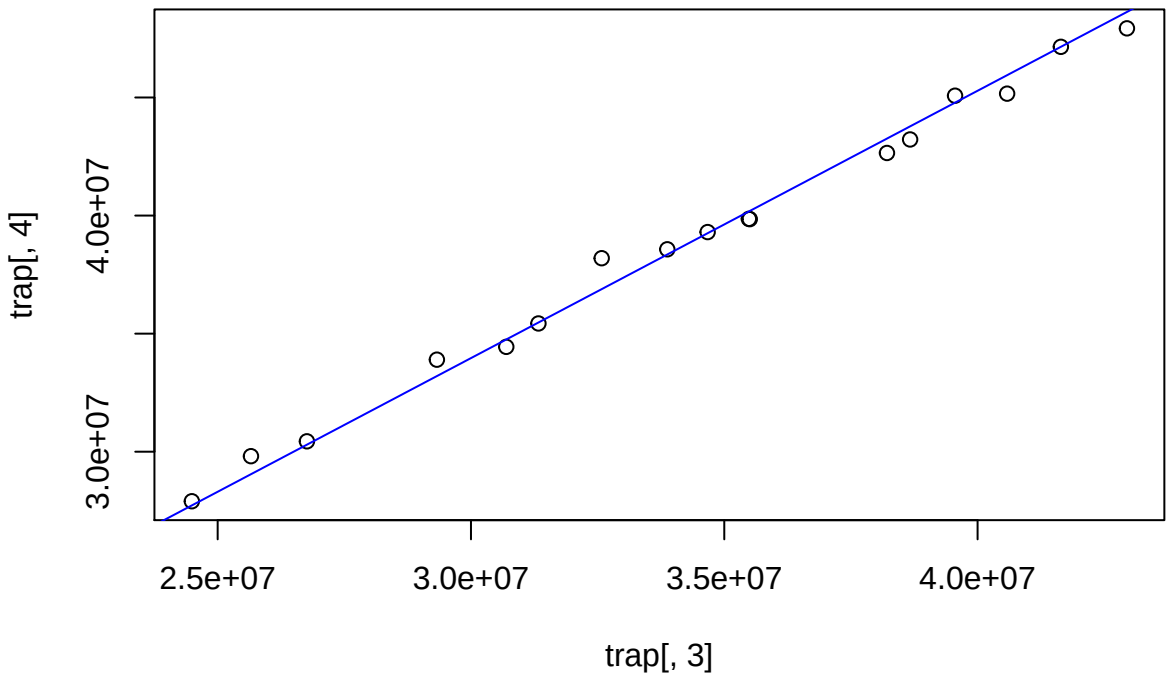
3.5e+07
2.0e+07
2.5e+07
3.0e+07
trap[, 2]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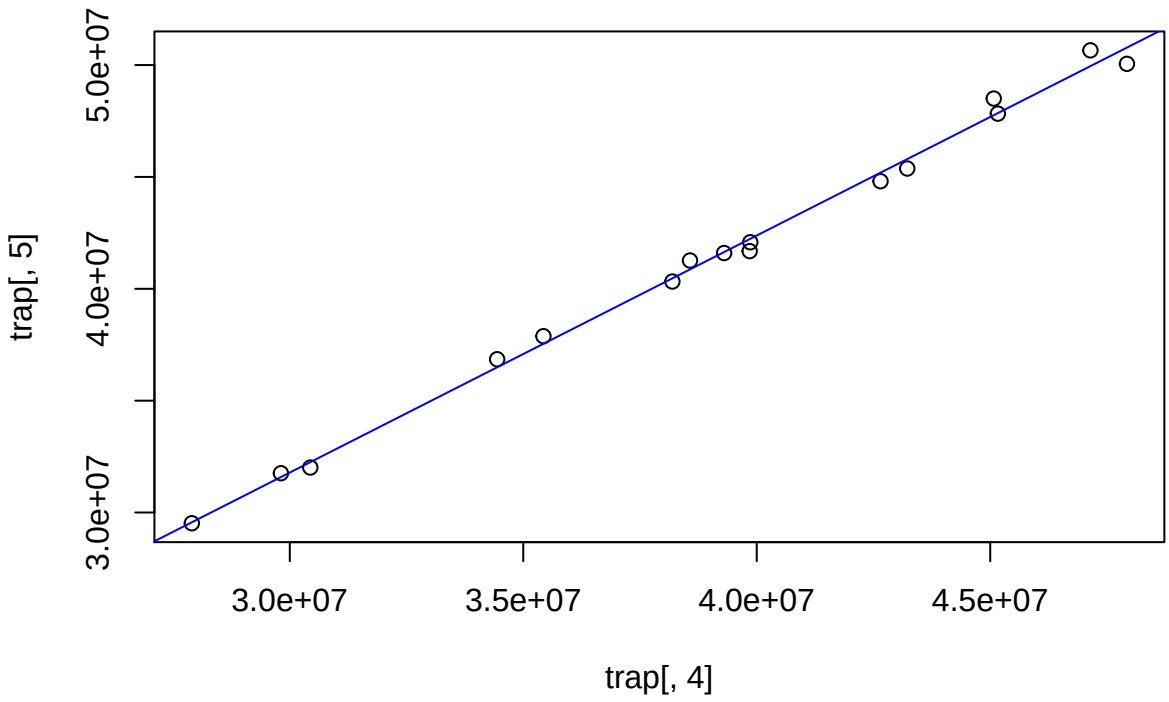
4.0e+07
2.5e+07
3.0e+07
3.5e+07
4.0e+07
trap[, 3]


trap[, 5]
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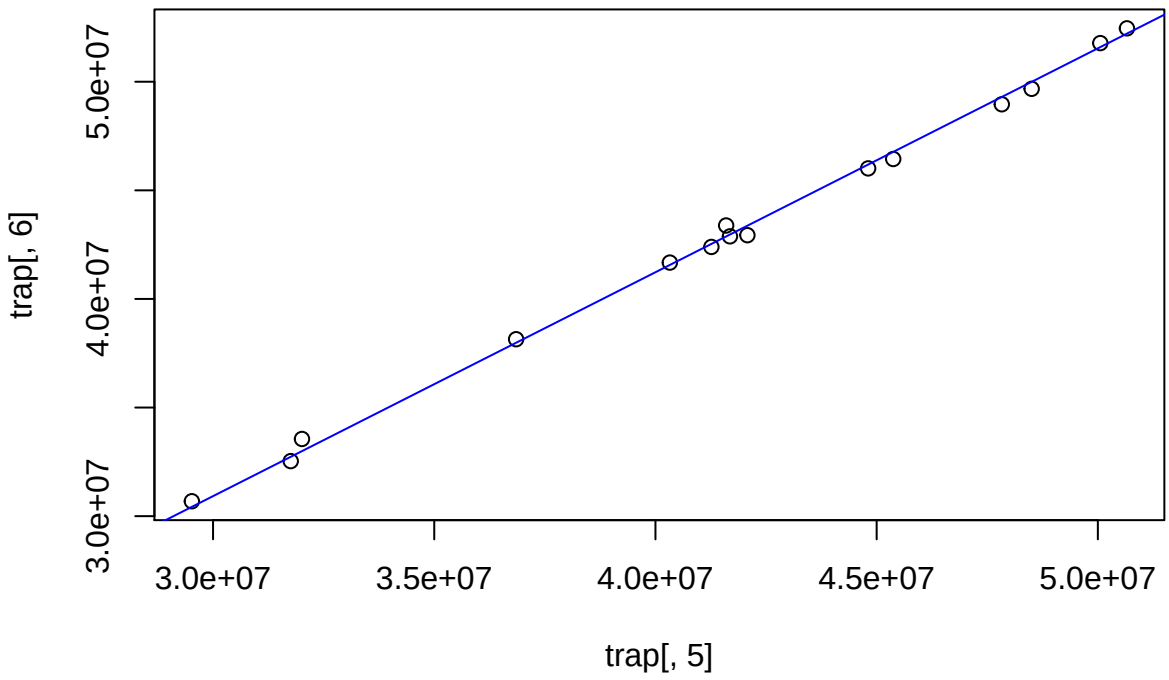
5.0e+07
3.0e+07
3.5e+07
4.0e+07
4.5e+07
trap[, 4]


trap[, 6]
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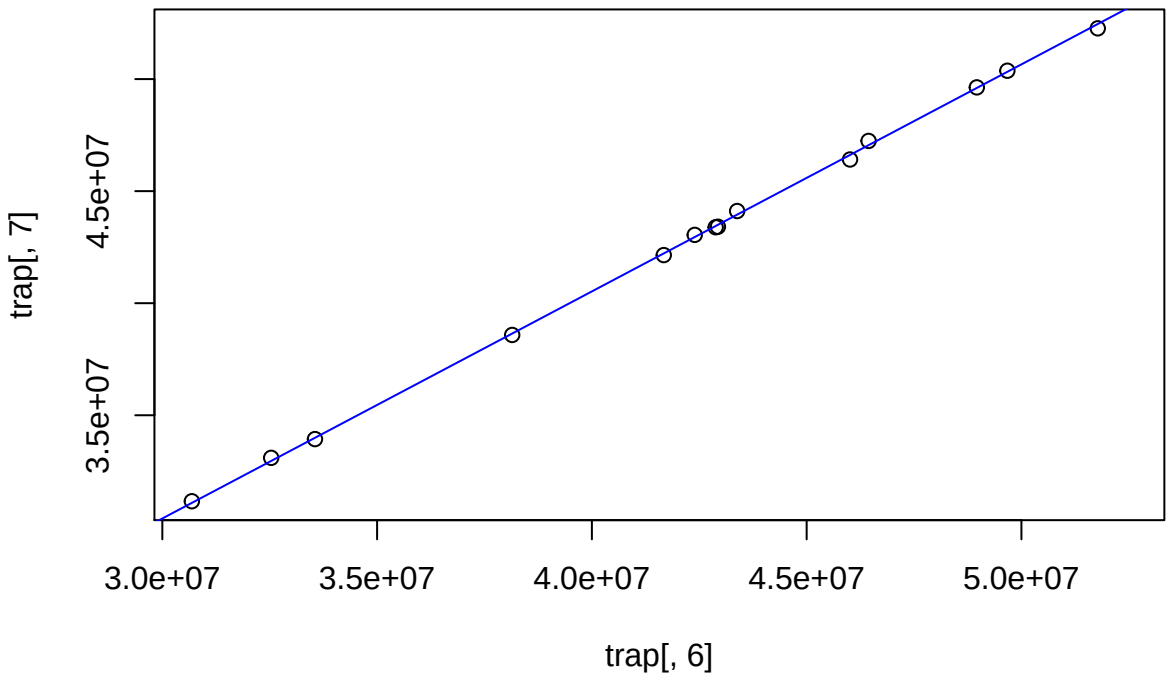
3.0e+07
3.5e+07
4.0e+07
4.5e+07
5.0e+07
trap[, 5]


trap[, 7]
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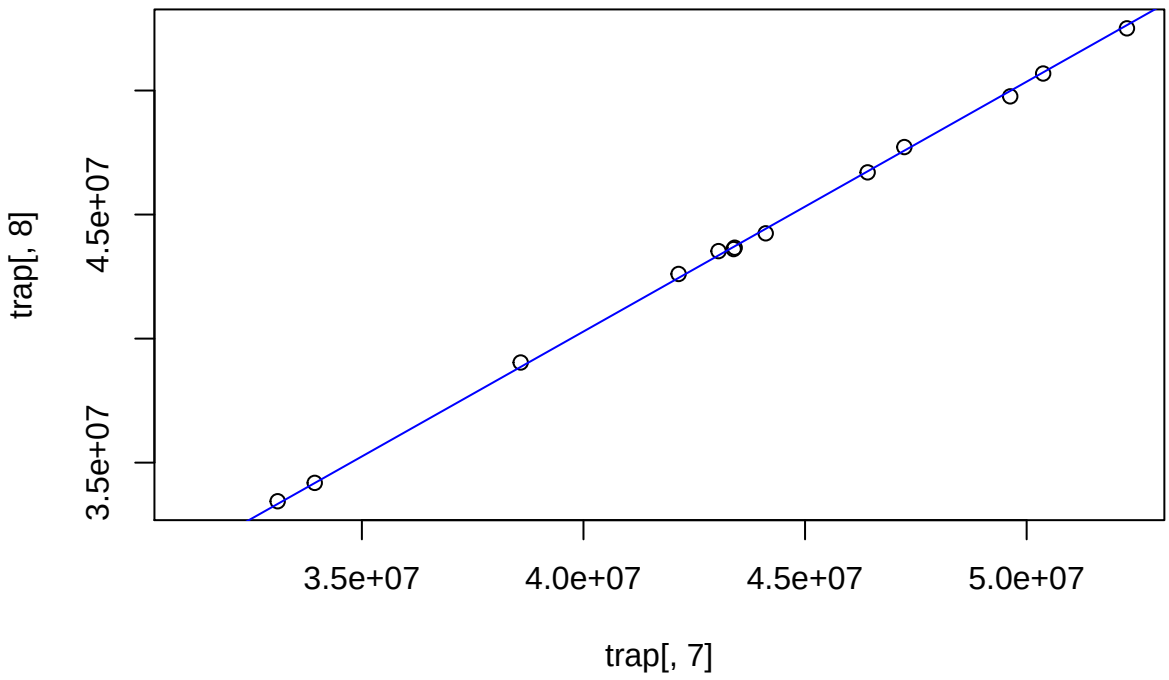
3.5e+07
3.0e+07
3.5e+07
4.0e+07
4.5e+07
5.0e+07
trap[, 6]


trap[, 8]
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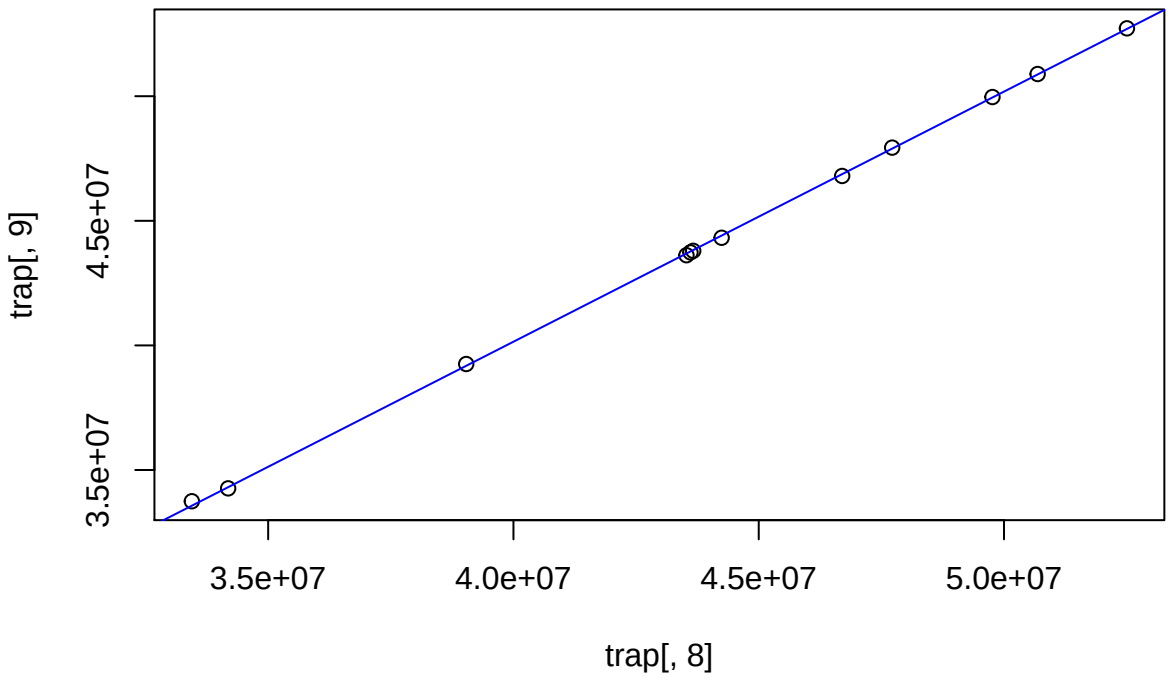
3.5e+07
3.5e+07
4.0e+07
4.5e+07
5.0e+07
trap[, 7]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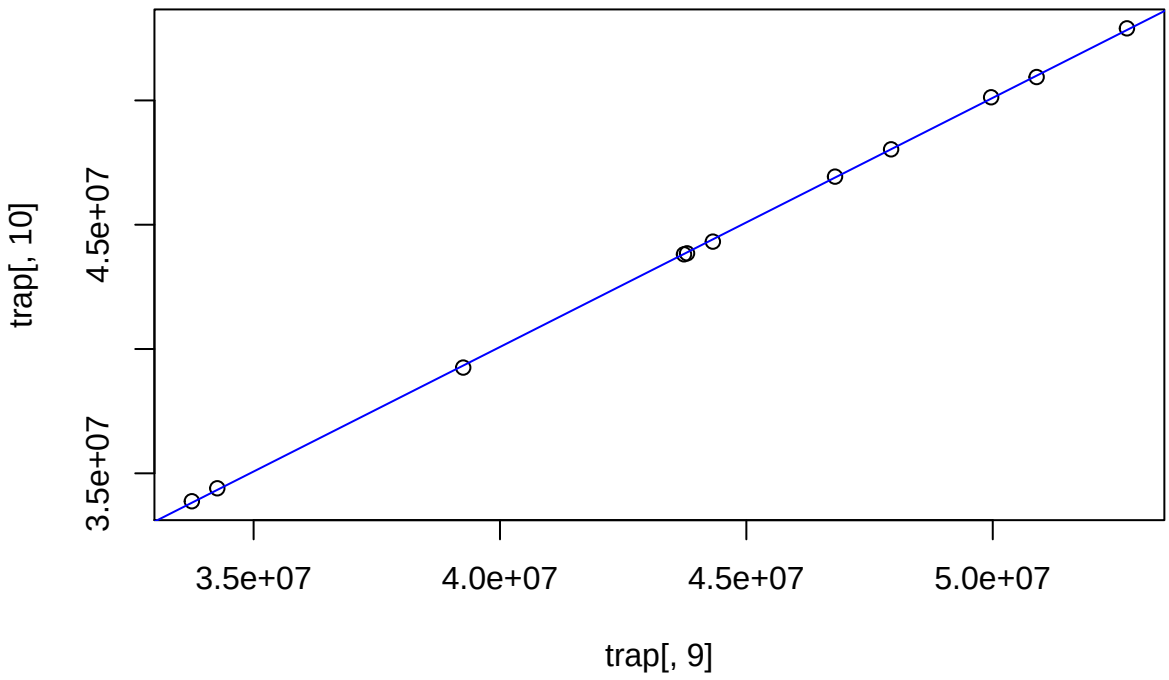
3.5e+07
4.0e+07
4.5e+07
5.0e+07
trap[, 8]


trap[, 10]
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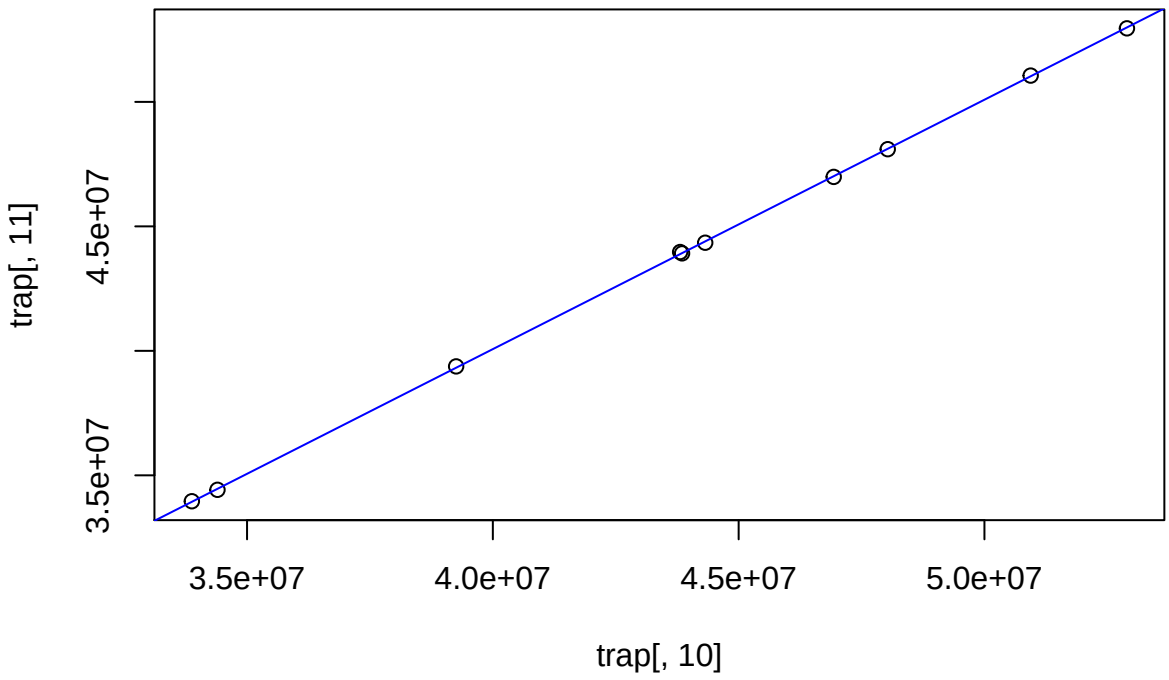
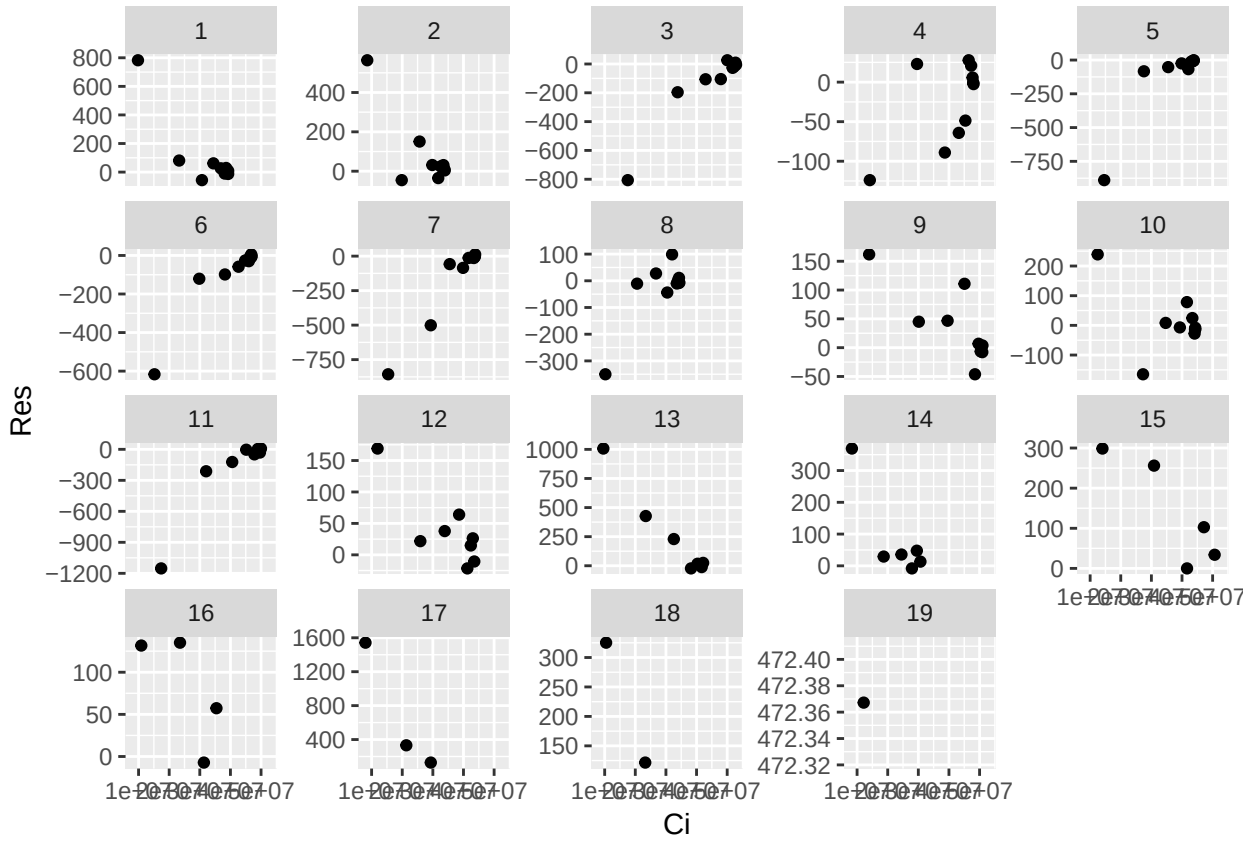
3.5e+07
4.0e+07
4.5e+07
5.0e+07
trap[, 9]



We note that the linear approximation seems to hold. We now want to plot the weighted residuals.



All years with more than 6 points, as suggested by Mack, seem to showcase random behaviour and no signs of any systematic deviations.

Lastly we want to examine whether or not we have any calender year effects. An example of a scenario where the assumption of independent years might be violated is if there is an overhaul of the way claims are handled.

```
## [[1]]
##       11        5        7        3        6        8        4       16
## 1.844584 1.888253 1.902904 1.928537 1.962896 2.011539 2.088097 2.160915
##        9       12       10       15       18       14       19        2
## 2.164397 2.169954 2.188784 2.200802 2.221551 2.249138 2.256480 2.313555
##        1       13       17
## 2.370927 2.446627 2.666343
##
## [[2]]
##        8       12        4       11        7        6        3        9
## 1.210636 1.265687 1.269471 1.271618 1.281084 1.287342 1.292967 1.300823
##        5       13       15       10        2       19       17       16
## 1.307399 1.307456 1.309935 1.311377 1.319939 1.328405 1.331018 1.349283
##       18       14
## 1.375093 1.391432
##
## [[3]]
##       13        5        8        6        3        9        7       18
## 1.112866 1.115950 1.116178 1.117806 1.121973 1.122418 1.123323 1.130843
##       17       12       10       14       16       11       19        4
## 1.132121 1.133523 1.137437 1.138622 1.139329 1.139560 1.155857 1.161856
```

```
##       15
## 1.172305
##
## [[4]]
##        6       10        7        9       11        8       16       17
## 1.044376 1.046048 1.049775 1.050573 1.051588 1.055617 1.055813 1.057991
##       13       14        5       19       15        4       18       12
## 1.058437 1.059011 1.065267 1.069158 1.069833 1.069965 1.074486 1.076045
##
## [[5]]
##        9        8       15       13        6       10       16       11
## 1.020340 1.023523 1.023680 1.024107 1.024511 1.026830 1.027378 1.028729
##       17        7        5       19       18       14       12
## 1.033435 1.034406 1.035131 1.035518 1.039515 1.042893 1.048229
##
## [[6]]
##       11        8       10       18       13        6       12       16
## 1.008739 1.009388 1.011119 1.011341 1.011390 1.011476 1.011805 1.013649
##       14       17       19       15        9        7
## 1.014092 1.015412 1.015515 1.016842 1.017185 1.017389
##
## [[7]]
##       17       16        9       13       11       15       12       14
## 1.002725 1.002979 1.004647 1.005079 1.005822 1.006210 1.006224 1.007317
##       10        8       19       18        7
## 1.010168 1.010404 1.010899 1.011134 1.011774
##
## [[8]]
##       17       13       19       15       14       12       16       18
## 1.001715 1.002104 1.002110 1.002313 1.002880 1.003011 1.004045 1.004071
##       10       11        8        9
## 1.004135 1.004544 1.005544 1.009049
##
## [[9]]
##        9       18       17       13       15       12       14       19
## 1.000000 1.000000 1.000978 1.001218 1.001733 1.001990 1.002875 1.003172
##       11       10       16
## 1.003324 1.003867 1.003952
##
## [[10]]
##       19       17       12       15       13       14       18       11
## 1.000531 1.000569 1.001042 1.001123 1.001384 1.001524 1.002277 1.002429
##       10       16
## 1.003106 1.003648
```

We might have some seasonal dependence idk??

Exercise 3

From the paper by Mack we have that

$$Var(C_{i,I}) = C_{i,I}^2 \sum_{k=I+1-i}^{I-1} \frac{\sigma_k^2}{f_k^2} \left(\frac{1}{C_{i,k}} + \frac{1}{\sum_{j=1}^{I-k} C_{j,k}} \right)$$

where we estimate σ_k^2 by

$$\hat{\sigma}_k^2 = \frac{1}{I-k-1} \sum_{j=1}^{I-k} C_{j,k} \left(\frac{C_{j,k+1}}{C_{j,k} - \hat{f}_k} \right)^2$$

which is an unbiased estimator.

Table 3: Reserve risk for coming years

	Year	Reserve Risk	Reserve
10	1	1.498830e+10	50125945
9	2	2.992001e+10	43709224
8	3	5.987413e+10	42852206
7	4	6.456256e+10	31566938
6	5	1.663411e+11	53834349
5	6	2.088943e+11	40071513
4	7	4.183250e+11	37992010
3	8	8.154027e+11	39192630
2	9	2.965815e+12	45376392
1	10	1.571915e+13	37759160

```
## Warning in if (is.na(row.names)) row.names = has_rownames(x): the condition
## has length > 1 and only the first element will be used
```

```
## Warning in if (row.names) {: the condition has length > 1 and only the
## first element will be used
```

Table 4: Ultimate claim amounts over observed years

UCA
434153077
438530632
438155393
439698940
438864727
439489538
439444448
439451113
439208255
439138088
439078290

Looks pretty good to me.

Exercise 4

```
## Warning in cbind(rep(0, 10), truect[, 1:10]): number of rows of result is
## not a multiple of vector length (arg 1)
```