Project 2

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Exercise 1

The purpose of this project is to examine how one can predict future expenses in order to be able to reserve against them. Specifically we wish to examine both the expected future payments as well as the variance within these payments. The data which we have been given is collected over several years and divided between two vastly different insurance products, henceforth refered to as product, or branch, 1 or 2. The final 10 years of the data can be aggregated in order to create the following two paid claims triangles. Lastly, since the claims triangle is limited to 10 development years the data has been truncated to accomodate this limit.

Table 1: Paid claims triangle for product 1

	1	2	3	4	5	6	7	8	9	10
1	14002196	20146708	21434023	22027415	22151985	22213380	22265789	22265789	22265789	22265789
2	7814759	11756938	12506719	12719773	12846004	12846004	12846004	12846004	12846004	
3	5181897	7401722	7820233	7922290	7922290	7940404	7940404	7940404		
4	5037120	7327216	7944307	8104325	8117003	8117003	8117003			
5	8042298	11453010	12085662	12166515	12212464	12274792				
6	6752949	10210348	10890964	11255347	11302949					
7	3715909	5176779	5580922	5689709						
8	6507705	9460226	10056047							
9	8386236	11910073								
10	6407931									

Table 2: Paid claims triangle of product 2

	1	2	3	4	5	6	7	8	9	10
1	3380661	11917434	19149141	23136198	25676759	26747939	27363824	27499046	27701642	27860156
2	4125276	14152392	21368586	25851401	28418708	29548445	30201830	30681144	30772972	
3	4388321	16013036	24759810	30271447	32403142	33733300	34205903	34665267		
4	3275256	11368463	16545811	19797983	21403379	22569878	23045976			
5	5981591	19410790	29558338	34979542	38445302	40182223				
6	4140485	13329441	20440916	24176081	26578838					
7	4282806	16150541	23746116	28208154						
8	3958824	13791706	20831932							
9	3775045	15531614								
10	4358136									

Using Mack's non-parametric CL approach we can predict the total cost of future payments per year as follows.

Table 3: Full claims triangle of type 1 predicted with CL

	1	2	3	4	5	6	7	8	9	10
1	14002196	20146708	21434023	22027415	22151985	22213380	22265789	22265789	22265789	22265789
2	7814759	11756938	12506719	12719773	12846004	12846004	12846004	12846004	12846004	12846004
3	5181897	7401722	7820233	7922290	7922290	7940404	7940404	7940404	7940404	7940404
4	5037120	7327216	7944307	8104325	8117003	8117003	8117003	8117003	8117003	8117003
5	8042298	11453010	12085662	12166515	12212464	12274792	12287377	12287377	12287377	12287377
6	6752949	10210348	10890964	11255347	11302949	11328296	11339910	11339910	11339910	11339910
7	3715909	5176779	5580922	5689709	5717088	5729908	5735783	5735783	5735783	5735783
8	6507705	9460226	10056047	10264529	10313922	10337051	10347649	10347649	10347649	10347649
9	8386236	11910073	12683551	12946506	13008805	13037977	13051345	13051345	13051345	13051345
10	6407931	9286944	9890068	10095108	10143686	10166433	10176857	10176857	10176857	10176857

Table 4: Full claims triangle of type 2 predicted with CL

	1	2	3	4	5	6	7	8	9	10
1	3380661	11917434	19149141	23136198	25676759	26747939	27363824	27499046	27701642	27860156
2	4125276	14152392	21368586	25851401	28418708	29548445	30201830	30681144	30772972	30949061
3	4388321	16013036	24759810	30271447	32403142	33733300	34205903	34665267	34840692	35040057
4	3275256	11368463	16545811	19797983	21403379	22569878	23045976	23315657	23433647	23567739
5	5981591	19410790	29558338	34979542	38445302	40182223	40973727	41453197	41662973	41901376
6	4140485	13329441	20440916	24176081	26578838	27747438	28294003	28625096	28769955	28934582
7	4282806	16150541	23746116	28208154	30831459	32187035	32821051	33205119	33373155	33564122
8	3958824	13791706	20831932	24963281	27284818	28484458	29045541	29385428	29534135	29703135
9	3775045	15531614	23591639	28270288	30899371	32257933	32893345	33278259	33446666	33638054
10	4358136	15380393	23361944	27995040	30598525	31943860	32573085	32954252	33121019	33310543

Lastly we wish to predict the future payments for each of the claims years, i.e. those on the vertical axes, and then combine them into a the total chain ladder reserve.

Table 5: Chain Ladder Reserve

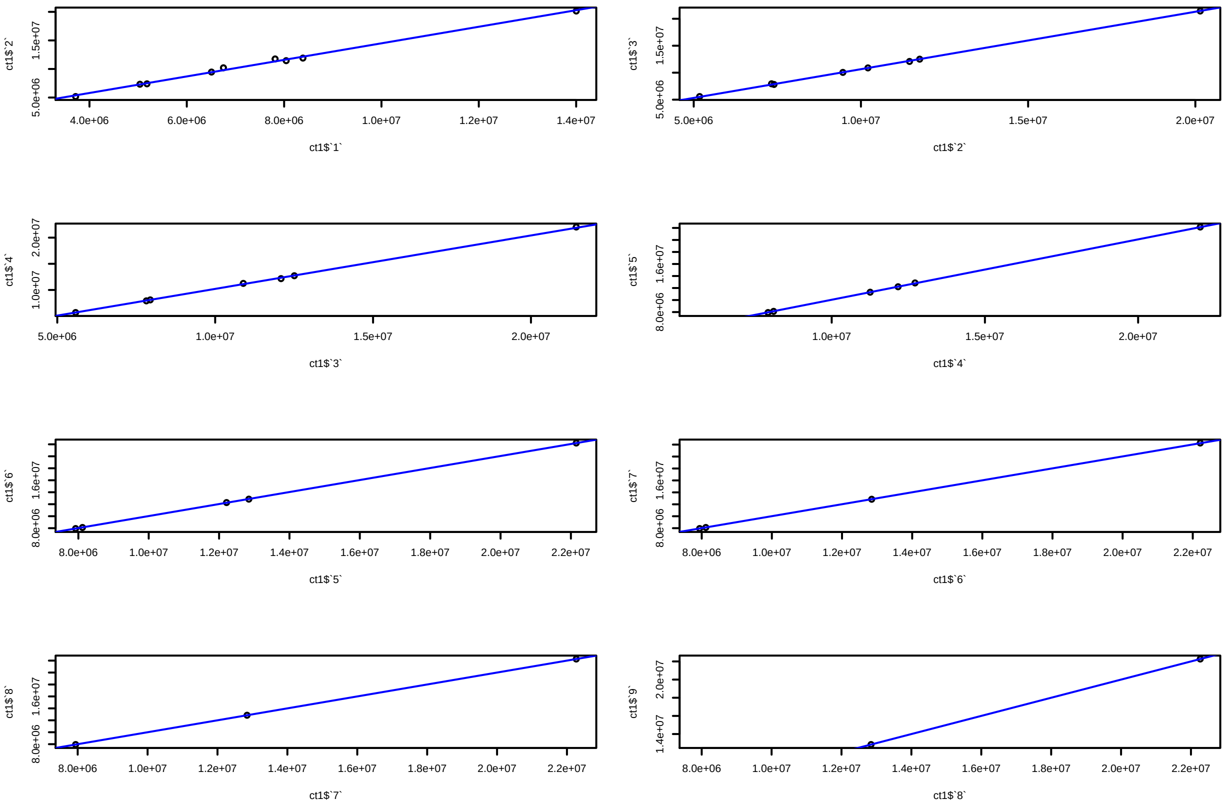
Product 1	5297420.03491499
Product 2	66433558.0748461

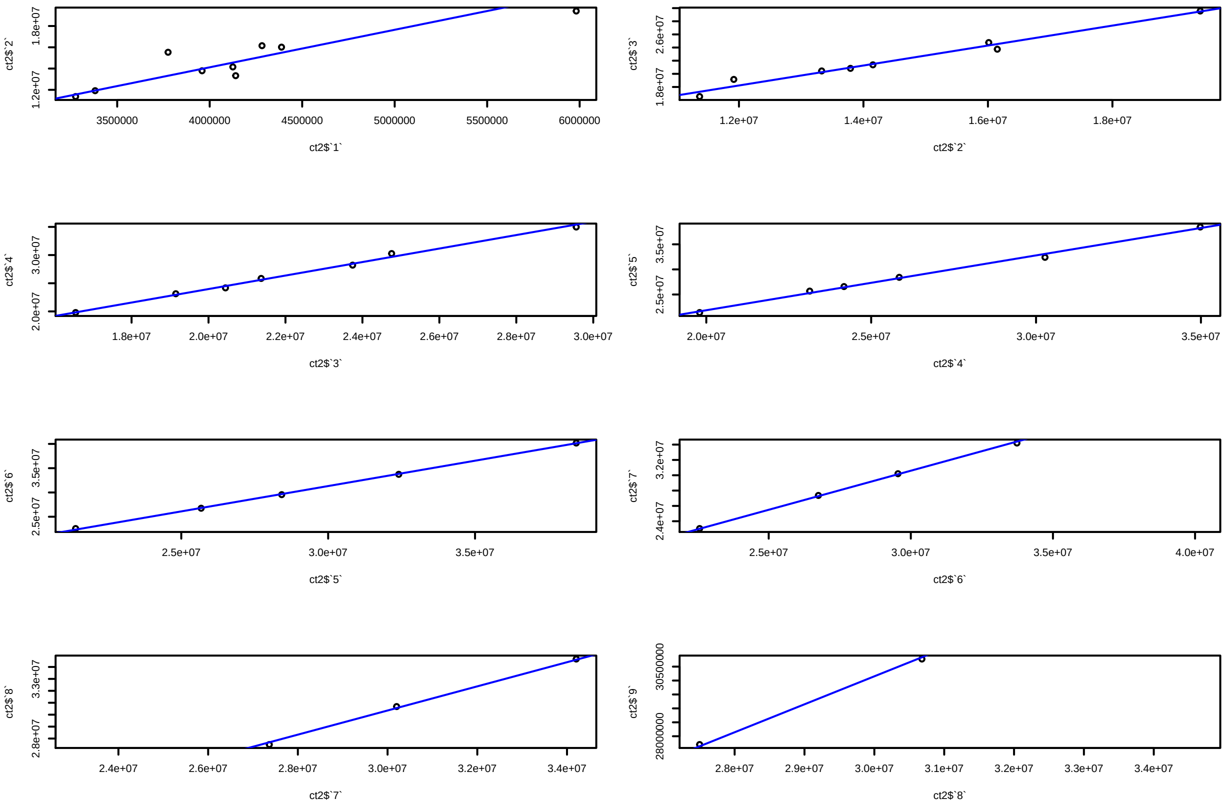
Exercise 2

We now want to check whether or not Mack's underlying assumptions are met in our case. The assumptions are as follows.

1. $E[C_{i,k+1}|C_{i,1},...,Ci,k] = f_k C_{i,k}$
2. Independent accident years
3. $Var(C_{i,k+1}|C_{i,1},...,Ci,k) = \sigma_k^2 C_{i,k}$

We begin by examing whether or not the we have an approximate linear relationship between $C_{i,k}$ and $C_{i,k+1}$ for $i = 1,...,10$.


ct1$`2`
5.0e+06
1.5e+07
4.0e+06
6.0e+06
8.0e+06
1.0e+07
1.2e+07
1.4e+07
ct1$`1`
ct1$`3`
5.0e+06
1.5e+07
5.0e+06
1.0e+07
1.5e+07
2.0e+07
ct1$`2`
ct1$`4`
1.0e+07
2.0e+07
5.0e+06
1.0e+07
1.5e+07
2.0e+07
ct1$`3`
ct1$`5`
8.0e+06
1.6e+07
1.0e+07
1.5e+07
2.0e+07
ct1$`4`
ct1$`6`
8.0e+06
1.6e+07
8.0e+06
1.0e+07
1.2e+07
1.4e+07
1.6e+07
1.8e+07
2.0e+07
2.2e+07
ct1$`5`
ct1$`7`
8.0e+06
1.6e+07
8.0e+06
1.0e+07
1.2e+07
1.4e+07
1.6e+07
1.8e+07
2.0e+07
2.2e+07
ct1$`6`
ct1$`8`
8.0e+06
1.6e+07
8.0e+06
1.0e+07
1.2e+07
1.4e+07
1.6e+07
1.8e+07
2.0e+07
2.2e+07
ct1$`7`
ct1$`9`
1.4e+07
2.0e+07
8.0e+06
1.0e+07
1.2e+07
1.4e+07
1.6e+07
1.8e+07
2.0e+07
2.2e+07
ct1$`8`



We note that the linear approximation seems to hold. Secondly we wish to examine the weighted residuals $\frac{C_{i,k+1}-C_{i,k}f_k}{C_{i,k}}$ in order to see if we have randomness, i.e. no systematic behaviours within the years.

All years with more than 6 points, the threshold suggested by Mack, seem to showcase random behaviour and no clear signs of any systematic deviations.

Lastly we want to examine whether or not we have any calender year effects. An example of a scenario where the assumption of independent years might be violated is if there is an overhaul of the way claims are handled which comes into effect during a specific year.

```
## [[1]]
##        7        9        5        3        1        8        4        2
## 1.393139 1.420193 1.424097 1.428381 1.438825 1.453696 1.454644 1.504453
##        6
## 1.511984
##
## [[2]]
##        6        4        9        3        2        7        8        5
## 1.055239 1.056542 1.062982 1.063773 1.063897 1.066659 1.078068 1.084219
##
## [[3]]
##        7        5        4        9        6        3        8
## 1.006690 1.013050 1.017035 1.019493 1.020142 1.027685 1.033457
##
## [[4]]
##        6        7        8        9        4        5
## 1.000000 1.001564 1.003777 1.004229 1.005655 1.009924
```

```
##
## [[5]]
##        6        8        7        5        9
## 1.000000 1.000000 1.002286 1.002772 1.005104
##
## [[6]]
##        7        8        9        6
## 1.000000 1.000000 1.000000 1.002359
##
## [[7]]
## 7 8 9
## 1 1 1
##
## [[8]]
## 8 9
## 1 1
##
## [[9]]
## 9
## 1
```

```
## [[1]]
##        6        5        2        4        8        1        3        7
## 3.219295 3.245088 3.430653 3.471015 3.483789 3.525179 3.649012 3.771019
##        9
## 4.114286
##
## [[2]]
##        5        8        3        9        6        7        4        2
## 1.455413 1.470298 1.509892 1.510468 1.522779 1.533516 1.546228 1.606817
##
## [[3]]
##        8        7        9        6        3        4        5
## 1.182730 1.183407 1.187906 1.196556 1.208211 1.209785 1.222604
##
## [[4]]
##        6        7        8        5        9        4
## 1.070419 1.081089 1.099080 1.099310 1.099386 1.109809
##
## [[5]]
##        6        7        5        9        8
## 1.039753 1.041050 1.041718 1.045179 1.054501
##
## [[6]]
##        8        9        7        6
## 1.014010 1.021094 1.022112 1.023026
##
## [[7]]
##        7        9        8
## 1.004942 1.013429 1.015870
##
## [[8]]
##        9        8
## 1.002993 1.007367
```

Table 6: Ultimate claim amounts over observed years for branch 1

Year	UCA
0	149073425
1	149623460
2	149699237
3	149687965
4	149613291
5	149669595
6	149659761
7	149652191
8	149652191

```
##
## [[9]]
##        9
## 1.005722
```

From the previous figures it looks as if we might have some annual dependencies, they are however not significant enough to make us deviate from the use of Mack's non-parametric CL.

Exercise 3

In this exercise we wish to examine the variance parameter for the last development year (i.e. development year 10). We can do this by implementing the formulas presented in Mack's paper. First we have that

$$Var(R) = \sum_{i=2}^{I} Var(C_{i,I}) + C_{i,I} \left(\sum_{j=i+1}^{I} C_{j,I} \right) \sum_{k=I+1-i}^{I-1} \frac{2\sigma_k^2}{f_k^2 \sum_{n=1}^{I-k} C_{n,k}}$$

where

$$Var(C_{i,I}) = C_{i,I}^2 \sum_{k=I+1-i}^{I-1} \frac{\sigma_k^2}{f_k^2} \left(\frac{1}{C_{i,k}} + \frac{1}{\sum_{j=1}^{I-k} C_{j,k}} \right)$$

and where we estimate σ_k^2 by

$$\hat{\sigma}_k^2 = \frac{1}{I-k-1} \sum_{j=1}^{I-k} C_{j,k} \left(\frac{C_{j,k+1}}{C_{j,k} - \hat{f}_k} \right)^2$$

and

$$\hat{f}_k = \frac{\sum_{j=1}^{I-k} C_{i,k+1}}{\sum_{j=1}^{I-k} C_{i,k}}$$

as previously.

By doing this we get the following ERROR :((

Table 7: Ultimate claim amounts over observed years for branch 2

Year	UCA
0	286830634
1	289014336
2	287807470
3	289468788
4	288615446
5	289142567
6	289111674
7	288964958
8	288761436

Exercise 5

Exercise 5