Brief Explanation of 1D NMR T2 Inversion with Tikhonov Regularization

The process of T2 inversion in NMR aims to extract a distribution of transverse relaxation times (T2) fro
multi-exponential decay data. Since this is an ill-posed problem, small errors in data can lead to large v
in the solution. Tikhonov regularization helps stabilize the inversion by imposing smoothness constraint

Steps of Inversion

1. Data Acquisition:
   - The measured signal S(t) follows:
     $S(t) = \Sigma A_i \exp(-t / T_{2i}) + \text{noise}$
   - Here, A_i are the amplitudes, and T_{2i} are the transverse relaxation times.

2. Kernel Construction:
   - A model matrix (kernel) is built based on possible T2 values:
     $K_{ij} = \exp(-t_j / T_{2i})$
   - This matrix transforms the T2 distribution into the measured decay signal.

3. Tikhonov Regularization:
   - Instead of solving K A = S directly (which can be unstable), we solve a regularized least-squares pro
     $\min_A \| K A - S \|^2 + \lambda \| L A \|^2$
   - λ (regularization parameter) controls smoothness:
     - Small $\lambda \rightarrow$ solution close to raw inversion (more noise).
     - Large $\lambda \rightarrow$ over-smoothed solution.
   - L is usually the identity matrix or a finite difference matrix enforcing smoothness.

4. Solving the Regularized System:
   - Using methods like scipy’s lsq_linear, we solve for A, ensuring non-negative solutions.

5. Visualization:
   - The final T2 distribution (A(T2)) is plotted against T2, showing the relative abundance of each relaxa

Advantages of Tikhonov Regularization:
- Prevents overfitting to noise in experimental data.
- Provides a smoother, more interpretable T2 distribution.
- Ensures stability in ill-posed inversion problems.