ECE3715_CA2_1

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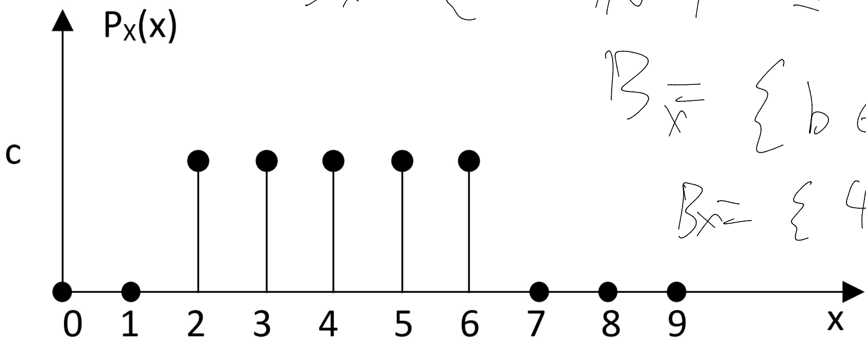


$S_X = \{x \in \mathbb{N} \mid 0 \le x \le 9\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$B_X = \{b \in S_X \mid 3 < b < 5\}$

$B_X = \{4\} \subset S_X$

$C_X = \{c \in S_X \mid 3 \le X < 5\}$

$C_X = \{3, 4\} \subset S_X$

a) $\sum_{x \in S_X} P_X(x) = 1$

$P_X(0) + \dots + P_X(9) = 1$

$0 + 0 + c + c + c + c + c + 0 + 0 + 0 = 1$

$5c = 1$

$c = \boxed{\frac{1}{5}}$

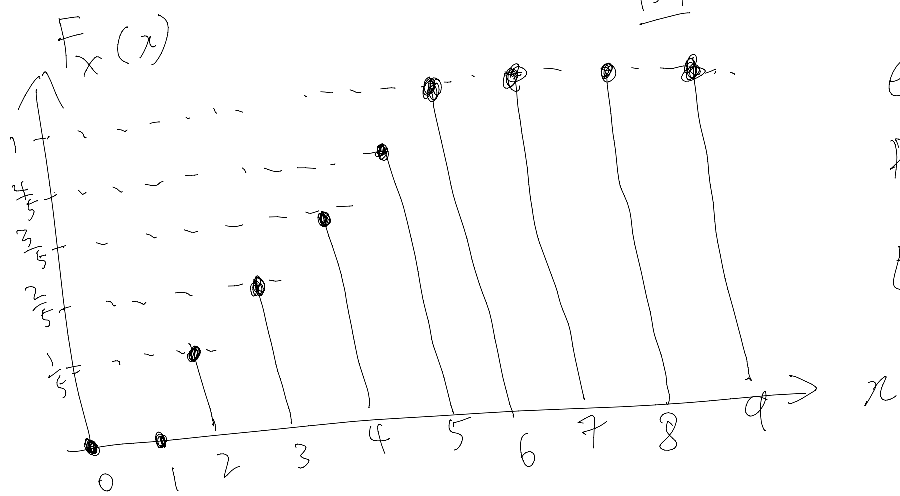
b) $P[B] = \sum_{x \in B_X} P_X(x)$

$= P_X(4) = c = \boxed{\frac{1}{5}}$

c) $P[C] = \sum_{x \in C_X} P_X(x)$

$= P_X(3) + P_X(4) = 2c = \boxed{\frac{2}{5}}$

d)



e) $E[X^2] = \sum_{x \in S_X} x^2 P_X[x]$

$P_X[x] = \begin{cases} \frac{1}{5}, & x = 2,3,4,5,6 \\ 0, & \text{otherwise} \end{cases}$

$E[X^2] = \sum_{x=2}^{6} \frac{1}{5} x^2 = \frac{1}{5}(2^2 + 3^2 + 4^2 + 5^2 + 6^2)$

$= \frac{1}{5}(90) = \boxed{18}$

ECE3715_CA2_2

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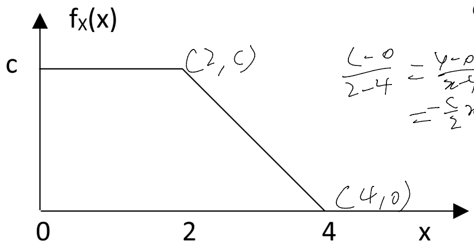

$\frac{c-0}{2-4} = \frac{y-0}{x-4}$
$= -\frac{c}{2}x + 2c$

b) $f_X(x) = \begin{cases} c, & 0 \le x \le 2 \\ \frac{c}{2}x - 2c, & 2 < x \le 4 \\ 0, & \text{otherwise} \end{cases} = \begin{cases} \frac{1}{3}, & 0 \le x \le 2 \\ \frac{1}{6}x - \frac{2}{3}, & 2 < x \le 4 \\ 0, & \text{otherwise} \end{cases}$

a) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$ $\boxed{C = \frac{1}{3}}$

$\int_0^2 c\,dx + \int_2^4 -\frac{c}{2}x + 2c\,dx = 1$

$[cx]_0^2 + \left[2cx - \frac{c}{4}x^2\right]_2^4 = 1$

$2c + 8c - 4c - 4c + c = 3c = 1$

f) $F_X(x) = \int_{-\infty}^{x} f_X(\hat{x})\,d\hat{x}$

$F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{3}, & 0 \le x < 2 \\ \frac{2}{3}x - \frac{x^2}{12} - \frac{1}{3} & 2 \le x < 4 \\ 1, & 4 < x \end{cases}$

c) $P[0 < X < 1] = \int_0^1 f_X(x)\,dx$
$= \int_0^1 \frac{1}{3}\,dx = \frac{1}{3}[x]_0^1 = \frac{1}{3}$

d) $X < 0$

e) $X > 4$

g) $E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx$
$= \int_0^2 \frac{x}{3}\,dx + \int_2^4 \frac{1}{6}x^2 - \frac{2}{3}x\,dx$

$= \frac{1}{3}[x]_0^2 + \frac{1}{18}[x^3]_2^4 - \frac{1}{3}[x^2]_2^4$

$= \frac{2}{3} + \frac{1}{18}(4^3 - 2^3) - \frac{1}{3}(4^2 - 2^2)$

$= \frac{2}{3} + \frac{56}{18} - \frac{4}{3} = \boxed{\frac{22}{9}} \approx 2.\overline{4}$