ECE 2101L
Electrical Circuit Analysis II Laboratory

Lab 11
Transfer Function of AC Circuits

Prelab

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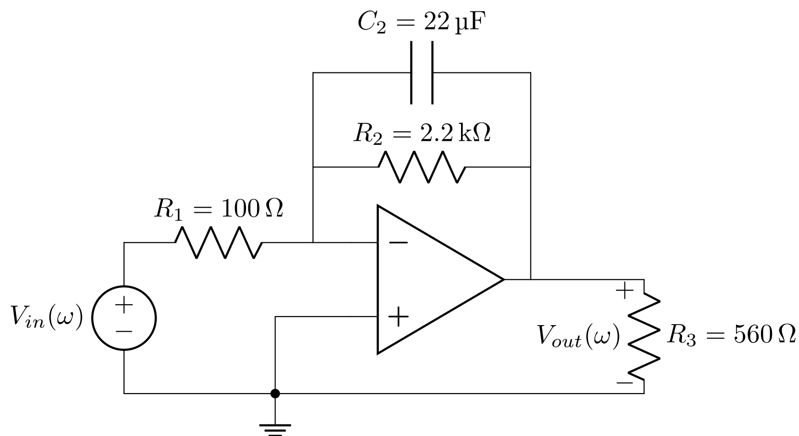
1 Theory of transfer function

According to *Fundamentals of Electric Circuits (2017)*, The transfer function $H(\omega)$ of a circuit is the frequency-dependent ratio of a phasor output $Y(\omega)$ (an element voltage or current) to a phasor input $X(\omega)$ (source voltage or current). [1] As $Y(\omega)$ may have a non-zero phase shift relative to $X(\omega)$, $H(\omega)$ is either real if they have the same phase, or complex otherwise. Given a transfer function $H(\omega)$ and an input voltage $V_{in}(\omega)$, the output voltage $V_{out}(\omega)$ can be calculated:

$$V_{out}(\omega) = H(\omega) \cdot V_{in}(\omega)$$

2 Transfer function of AC circuit with op amp



Assuming the op amp is ideal, the transfer function $H(\omega)$ can be determined as follow:

$$H(\omega) = \frac{V_{out}}{V_{in}} = -\frac{Z_2}{Z_1} = -\frac{R_2||Z_{C_2}}{R_1} = -\frac{\frac{1}{\frac{1}{R_2}+Z_{C_2}}}{R_1} = -\frac{1}{\frac{R_1}{R_2} + R_1 Z_{C_2}} = \frac{-\frac{R_1}{R_2} + R_1\omega C_2 j}{(\frac{R_1}{R_2})^2 + (R_1\omega C_2)^2}$$

$V_{out}(\omega)$ can then be determined as follow:

$$V_{out}(\omega) = \frac{-\frac{R_1}{R_2} + R_1\omega C_2 j}{(\frac{R_1}{R_2})^2 + (R_1\omega C_2)^2} V_{in}(\omega)$$

The magnitude of $H(\omega)$ can be determined as follow:

$$|H(\omega)| = \sqrt{\left(\frac{-\frac{R_1}{R_2}}{(\frac{R_1}{R_2})^2 + (R_1\omega C_2)^2}\right)^2 + \left(\frac{R_1\omega C_2}{(\frac{R_1}{R_2})^2 + (R_1\omega C_2)^2}\right)^2} = \frac{1}{\sqrt{(\frac{R_1}{R_2})^2 + (R_1\omega C_2)^2}}$$

The phase of $H(\omega)$ can be determined as follow:

$$arg\,(H(\omega)) = tan^{-1}\left(\frac{\frac{R_1\omega C_2}{(\frac{R_1}{R_2})^2+(R_1\omega C_2)^2}}{\frac{-\frac{R_1}{R_2}}{(\frac{R_1}{R_2})^2+(R_1\omega C_2)^2}}\right) = tan^{-1}\left(\frac{R_1\omega C_2}{-\frac{R_1}{R_2}}\right) = tan^{-1}\,(-R_2\omega C_2)$$

When $R_1 = 100\,\Omega$, $R_2 = 2.2\,\mathrm{k\Omega}$, $R_3 = 560\,\Omega$, $C_2 = 22\,\mathrm{\mu F}$, $\omega = 2\pi 60$ rad/s:

$$H(\omega) = \frac{1}{\sqrt{(\frac{R_1}{R_2})^2 + (R_1\omega C_2)^2}} \underline{/tan^{-1}\,(-R_2\omega C_2)}$$

$$H(2\pi 60) = \frac{1}{\sqrt{(\frac{100}{2200})^2 + ((100)(2\pi 60)(22)\times 10^{-6})^2}} \underline{/tan^{-1}\left(-(2200)(2\pi 60)(22)\times 10^{-6}\right)}$$

$$H(2\pi 60) = 1.203913\underline{/-86.8630^\circ}$$

References

[1] C.K. Alexander and M.N.O. Sadiku. *Fundamentals of Electric Circuits*. COLLEGE IE OVERRUNS. McGraw-hill Education, 2017. ISBN: 9781259251320. URL: https://books.google.com/books?id=e4-gjwEACAAJ.