A FAIR File Format for Mathematical Software

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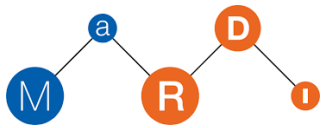
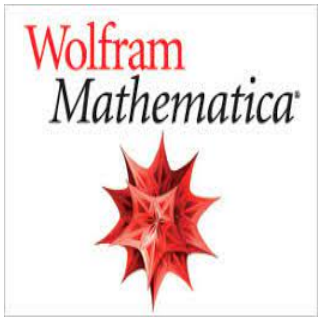
Joint work with M. Joswig and B. Lorenz

TU Berlin

ICMS Mathematical Research Data 2024-07-24



The Importance of Files for Computer Algebra



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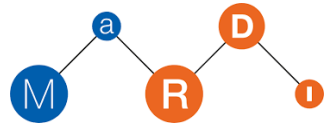

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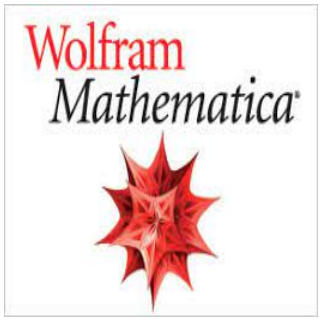

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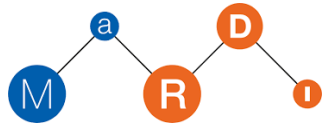
- Computations can be expensive.



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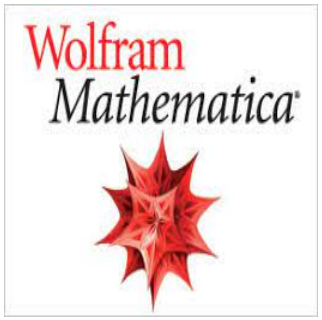

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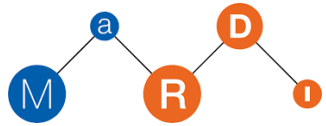

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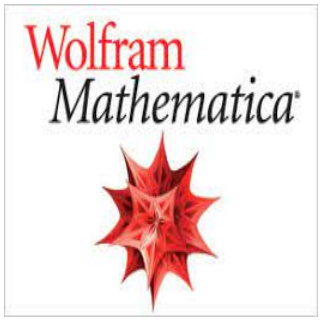

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- Computations can be expensive.
- Software changes often and requires maintenance.
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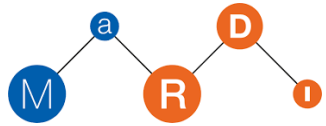

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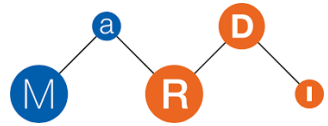

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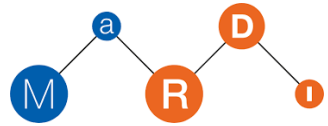
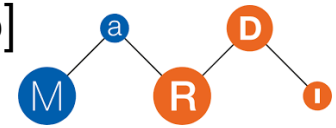


History of File Formats

```
* Class:          LP
* Rows:           5
* Columns:        2
* Format:         MPS
*
Name             unnamed#0
ROWS
 N  C0000000
   ...
 G  R0000003
COLUMNS
    x1   C0000000  2          R0000000  1
    x1   R0000001  -1
    x2   C0000000  3          R0000002  1
    x2   R0000003  -1
RHS
    B    C0000000  -1         R0000000  -1
    B    R0000001  -1         R0000002  -1
    B    R0000003  -1
BOUNDS
 FR BND         x1
 FR BND         x2
ENDATA
```

- The LP file format and the MPS file format. IBM [1970s] (industry standards)
- `Mathematica` Notebooks. Wolfram Mathematica [1988]
- `OpenMath` (tree structure). Mike Dewar [2000]
- `IPython 0.12` Interactive Browser Notebooks (Jupyter) [2011]
- `polymake` File Format. E. Gawrilow, S. Hampe, and M. Joswig [2016]

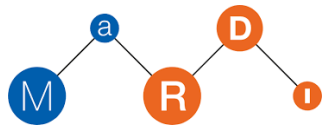


Storing Computer Algebra Data

- It's common to have multiple perspectives on an object in mathematics.
- While storing mathematical data a choice of perspective must me made.
- Such a choice might not be describeable in an email.

Say we want to store:

$$p = 2y^3z^4 + (\mathbf{a} + 3)z^2 + 5\mathbf{a}y + 1$$



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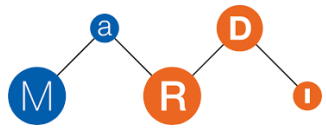
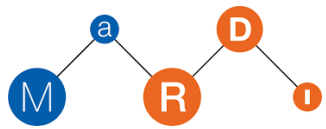
$$p = 2y^3z^4 + (\mathbf{a} + 3)z^2 + 5\mathbf{a}y + 1$$

- Some technicalities with the coefficients.
- Is y considered a coefficient of z?
- What is **a**?
- How can we guarantee the objects behave as expected on load?



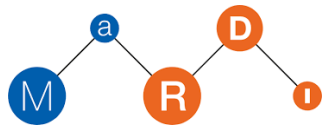
The `mrdi` File Format

- JSON file format.
- Similar to `polymake` format but generalizes to include algebraic data.
- Namespaces for semantic seperation.
- References stored with UUIDs.
- Prototype developed using `OSCAR`.

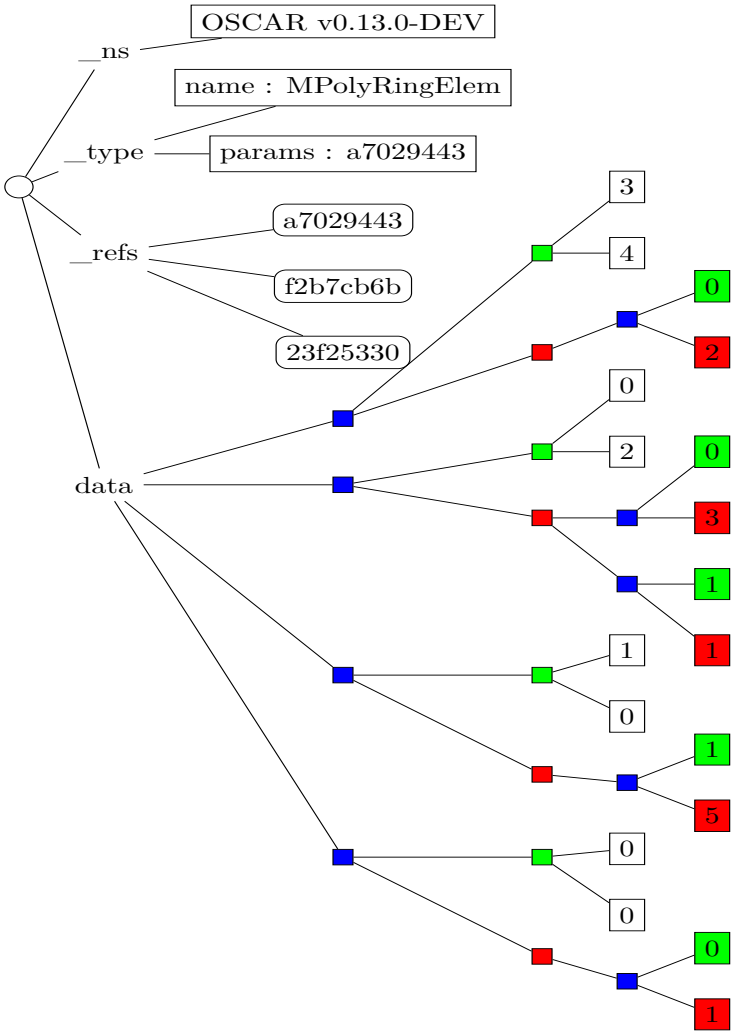


Demo

```
julia> using Oscar

julia> F = GF(7)
Prime field of characteristic 7

julia> L, a = finite_field(x^2 + 1)
(Finite field of degree 2 and characteristic 7, o)

julia> Lyz, (y, z) = L[:y, :z]
(Multivariate polynomial ring in 2 variables over L, FqMPolyRingElem[y, z])

julia> p = 2 * z^4 * y^3 + (a + 3) * z^2 + 5 * a * y + 1
2*y^3*z^4 + 5*o*y + (o + 3)*z^2 + 1

julia> q = z^2 + 3 * y
3*y + z^2

julia> save("p.mrdi", p)

julia> save("q.mrdi", q)

julia> Oscar.reset_global_serializer_state()
Dict{Base.UUID, Any}()

julia> load("p.mrdi") * load("q.mrdi")
6*y^4*z^4 + 2*y^3*z^6 + o*y^2 + (o + 2)*y*z^2 + 3*y + (o + 3)*z^4 + z^2
```



Tree Structure



$2y^3z^4$

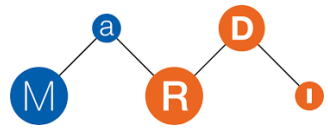
$(a+3)z^2$

$5ay$

1

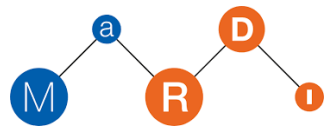
Example File Serialized with OSCAR


```
{
  "_ns": { "Oscar": [ "https://github.com/oscar-system/Oscar.jl", "1.0.0" ]  },
  "_type": {
    "name": "MPolyRingElem",
    "params": "869a359a-43d3-43f4-9821-0af9346be019"
  },
  "data": [[["3", "4"], [["0", "2"]]],
           [["0", "2"], [["0", "3"], ["1", "1"]]],
           [["1", "0"], [["1", "5"]]],
           [["0", "0"], [["0", "1"]]]],
  "_refs": {
    "152ac7bd-e85a-4b36-acc2-743ade2cad4f": {
      "data": { "base_ring": { "data": "7",  "_type": "FqField"},
                "symbols": ["x"] },
      "_type": "PolyRing"
    },
    "869a359a-43d3-43f4-9821-0af9346be019": {
      "data": {
        "base_ring": "a8309b96-caec-443c-bedb-e23bb0634c14",
        "symbols": [ "y", "z" ]
      },
      "_type": "MPolyRing"  },
    "a8309b96-caec-443c-bedb-e23bb0634c14": {
      "data": {
        "def_pol": {
          "data": [["0", "1"], ["2", "1"]],
          "_type": {
            "name": "PolyRingElem",
            "params": "152ac7bd-e85a-4b36-acc2-743ade2cad4f"
          }
        }
      },
      "_type": "FqField"
    }
  }
}
```

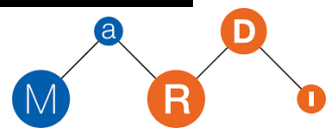
Namespaces

- We make no attempt to formalise the language of Mathematical Research Data.
- We rely on namespaces to pin the semantics to a specific software.
- Open source software provides example implementations for saving and loading.
- Freedom to upgrade encoding.
- Similar with PDF.



Serialized Toric Divisor

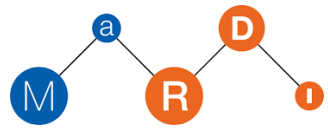

```
{
  "_ns": { ...
  "_type": { "name": "ToricDivisorClass", "params": "b66b610c-f21b-4b3d-8b1f-267e8e65ec7"  },
  "data": [ "5", "0", "0" ],
  "_refs": {
    "b66b610c-f21b-4b3d-8b1f-267e8e65ec7a": {
      "_type": "NormalToricVariety",
      "data": {
        "_ns": { "polymake": [ "https://polymake.org", "4.12" ] },
        "_type": "fulton::NormalToricVariety",
        "FAN_AMBIENT_DIM": 2,
        "PROJECTIVE": true,
        "MAXIMAL_CONES": [ ...  ],
        "PSEUDO_REGULAR": true,
        "COMPLETE": true,
        "DIVISOR": [ ...  ],
        "RAYS": [ [ "1", "0" ], [ "0", "1" ], [ "-1", "-1" ] ],
        ...
      }
    }
  }
}
```

Schema

Figure: `https://www.pexels.com/photo/plastic-shape-shorter-toy-11030155/`

- A schema defines a structure for data.



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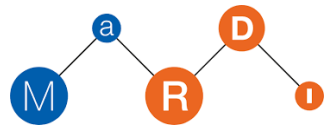

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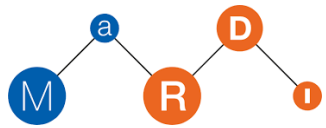

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- Schemata allow data to be validated before loading.

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- Adds structure to document based databases.

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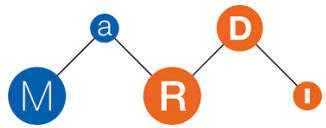
- `PolyDB`, Paffenholz [2017]

Data Collections

- Johnson Solids.
- Non-general type surfaces in $\mathbb{P}^4$.
- F-Theory models.
- Small Phylogenetic Trees. (Luis David Garcia-Puente, Jacob Porter)



Thank You!