The Importance of Symbolic Data Types

Antony Della Vecchia

Technische Universität Berlin (MaRDI TA1 Computer Algebra)

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Expect the Unexpected

"This was unexpected because while I expected numerical error could be possible in the constraints" . . . " I did not expect a completely incorrect result"

- Github User

Overview

Convex Hull Definition

Definition

A set is S is convex if for any two points $p_1, p_2 \in S$ the line between them lies entirely in S. The convex hull of a finite point set V is the intersection of all convex sets which contains V.

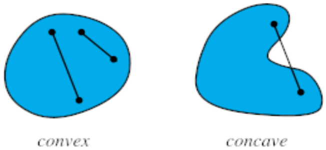


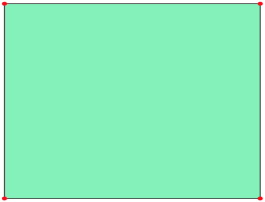
Figure: Weisstein, Eric W. "Concave." From MathWorld–A Wolfram Web Resource. `mathworld.wolfram.com/Concave.html`

Convex Hull Descriptions

Convex hulls can be described uniquely in multiple ways

- By their set of vertices.
- By a system of inequalities.

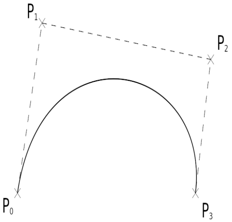
These properties make convex hulls computationally "nice". That is, they are easy for a computer to describe them and there exists algorithms to go between these descriptions.



Where to Find Convex Hulls

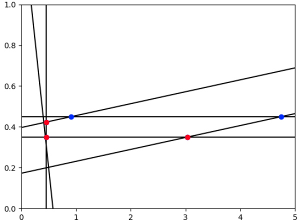
Convex hulls play a very important role in computational geometry and arise naturally in scientific computations.

- Optimization over a set of linear constraints (simplex algorithm).
- Intersections of bezier curves.
- The state space of any quantum system.
- Approximations of convex bodies.



A Real World Example

The following example was taken from an open github issue from CDDLib and can be found here. `https://github.com/cddlib/cddlib/issues/67`



- The red points were returned from the convex hull computation.
- The blue points were missing.
- Although finding the right precision may help, it is not apriori clear what that precision is.

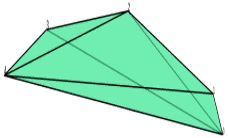
Constructed Example Part 1

Definition

The moment curve $\mu_n \to \mathbb{R}^n$ is defined as $\tau \to (\tau, \ldots, \tau^n)$. A cyclic polytope is the convex hull of points on the moment curve.

Remark

Notice that since any $n+1$ vertices lie in a distinct supporting hyperplane, each facet is an n-simplex.



Constructed Example Part 2

```
$P=cyclic(3,20);
```

```
for (my $i=0; $i<5; ++$i) {
  my $Q=new Polytope<Float>(INEQUALITIES=>$P->FACETS,REL_INT_POINT=>$P->REL_INT_POINT,BOUNDED=>1,FULL_DIM=>1);
  my $V=$Q->VERTICES;
  print $i, ":", $V->rows(), " ";
  $P=new Polytope<Float>(POINTS=>$V);
}
```

```
0:20 1:43 2:68 3:96 4:118
```

Constructed Example Part 3

```
$C=cyclic(3,20);
$P=product($C,$C);
```

```
$P->FACETS;
```

Click here for additional output

```
print $P->N_FACETS;
```

```
72
```

```
for (my $i=0; $i<5; ++$i) {
  my $Q=new Polytope(INEQUALITIES=>$P->FACETS,REL_INT_POINT=>$P->REL_INT_POINT,BOUNDED=>1,FULL_DIM=>1);
  my $V=$Q->VERTICES;
  print $i, ":", $V->rows(), " ";
  $P=new Polytope(POINTS=>$V);
}
```

```
0:400 1:400 2:400 3:400 4:400
```

```
include("qhull.rules");
prefer "qhull";
```

```
for (my $i=0; $i<5; ++$i) {
  my $Q=new Polytope<Float>(INEQUALITIES=>$P->FACETS,REL_INT_POINT=>$P->REL_INT_POINT,BOUNDED=>1,FULL_DIM=>1);
  my $V=$Q->VERTICES;
  print $i, ":", $V->rows(), " ";
  $P=new Polytope<Float>(POINTS=>$V);
}
```

```
0:400 1:59 2:1 3:1 4:1
```

Click here for additional output

```
print $P->VERTICES;
```

```
1 0 0 0 0 0 0
```

Exact Computations

Possible alternatives to working with floating points would be to consider these data types.

- rationals
- multi precision float (integers, rationals, complex numbers)
- arb, acb (arbitrary real ball, arbitrary complex ball)
- nmod, nf_elem, ...
- fmpz_poly, fmpq_mpoly
- ...

Working with these types is possible within OSCAR, and we are close to releasing a version where computing convex hulls over ordered fields (for example finite extensions of $\mathbb{Q}$).

Shift of Perspective on Mathematical Computations

- The previous slide gives a glimpse into what MaRDI TA1 (Computer Algebra) is concerned with.
- The biggest tradeoff is speed (a lot time goes into multiplication).
- The focus turns to finding the combinatorics and storing the data.
- Finding a data format has been a first step. (IR)

End

Thank You!

Ewgenij Gawrilow and Michael Joswig

polymake: a framework for analyzing convex polytopes. Polytopes—combinatorics and computation

(Oberwolfach, 1997), 43–73, DMV Sem., 29, Birkhäuser, Basel, 2000. MR1785292 (2001f:52033)