The mrdi File Format

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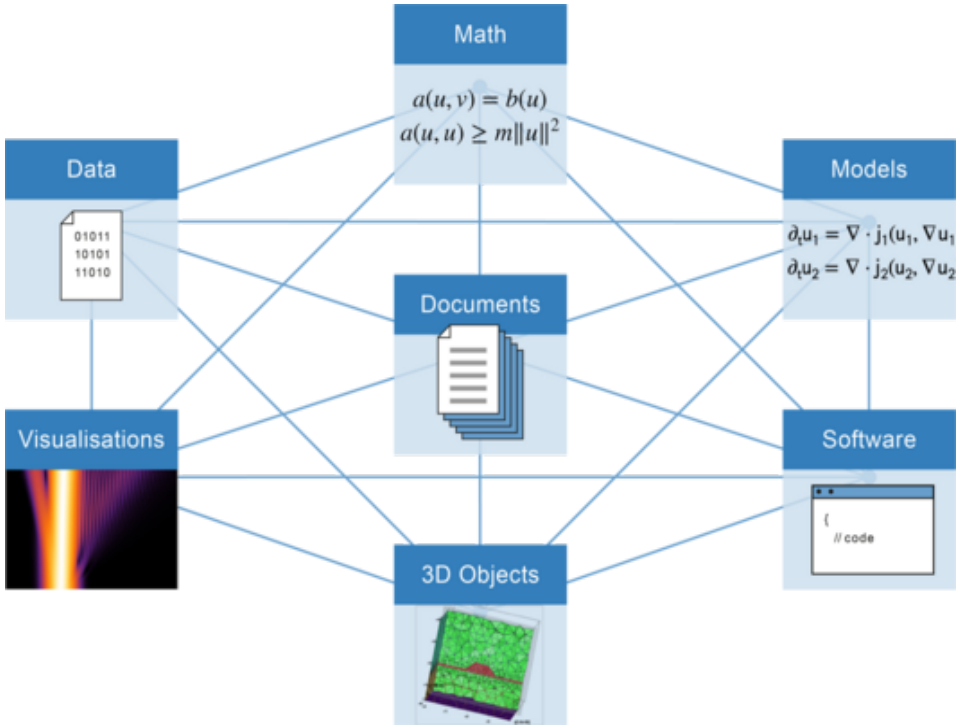
Overview

- MaRDI and the FAIR principles
- History of Files in Mathematical Software
- Technicalities with Algebraic Data
- Current Status of Prototype
- The File Format Specification
- Future Work

MaRDI and the FAIR Principles

- **Ma**thematics **R**esearch **D**ata Initiative.
- Develop a mathematical research data infrastructure.
- Set standards for confirmable workflows and certified mathematical research data.
- Provide services to both the mathematical and wider scientific community.
- **F**indable **A**ccessible **I**nteroperable **R**eusable [M. D. Wilkinson et al. 2016]



Alice and Bob

jupyter Alice_and_Bob Last Checkpoint: 30 minutes ago

File Edit View Run Kernel Settings Help

Code

[2]:
```
using Oscar
```

Alice's section of the Notebook

[3]:
```
Qxy, (x, y) = QQ[:x, :y]
I = ideal([x * y - 1, x^2 + y^2 -4])
gb = groebner_basis(I)
```

[3]:
```
Gröbner basis with elements
1 -> x*y - 1
2 -> x^2 + y^2 - 4
3 -> y^3 + x - 4*y
with respect to the ordering
degrevlex([x, y])
```

Bob's section of the Notebook

[24]:
```
F = fraction_field(Qxy)
y_value = 1 // x
eqs = [evaluate(p, [F(x), 1//x]) for p in gb]
Qz, z = QQ[:z]
uni_p = evaluate(numerator(eqs[2]), [z, z])
roots(uni_p)
```

[24]:
```
QQFieldElem[]
```

Storing Computer Algebra Data

- It's common to have multiple perspectives on an object in mathematics.
- While storing mathematical data a choice of perspective must me made.
- Such a choice might not be describeable in an email.

Say we want to store:

$$p = 2y^3z^4 + (\mathbf{a} + 3)z^2 + 5\mathbf{a}y + 1$$

Storing Computer Algebra Data

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Say we want to store:

$$p = 2y^3z^4 + (\mathbf{a} + 3)z^2 + 5\mathbf{a}y + 1$$

- Some technicalities with the coefficients.
- Is y considered a coefficient of z?
- What is **a**?
- How can we guarantee the objects behave as expected on load?

OSCAR Demo

```
julia> using Oscar

julia> F = GF(7)
Finite field of characteristic 7

julia> Fx, x = F["x"]
(Univariate polynomial ring in x over GF(7), x)

julia> L, a = FiniteField(x^2 + x + 1)
(Finite field of degree 2 over GF(7), o)

julia> Lyz, (y, z) = L["y", "z"]
(Multivariate polynomial ring in 2 variables over GF(7^2), fqPolyRepMPolyRingElem[y, z])

julia> p = 2 * z^4 * y^3 + (a + 3) * z^2 + 5*a*y + 1
2*y^3*z^4 + 5*o*y + (o + 3)*z^2 + 1

julia> q = z^2 + 3*y
3*y + z^2

julia> save("p.mrdi", p)

julia> save("q.mrdi", q)

julia> load("p.mrdi") * load("q.mrdi")
6*y^4*z^4 + 2*y^3*z^6 + o*y^2 + (o + 2)*y*z^2 + 3*y + (o + 3)*z^4 + z^2
```

Example File Serialized with OSCAR


```
{
  "_ns": { "Oscar": [ "https://github.com/oscar-system/Oscar.jl", "0.13.0-DEV" ]  },
  "_type": {
    "name": "MPolyRingElem",
    "params": "869a359a-43d3-43f4-9821-0af9346be019"
  },
  "_refs": {
    "152ac7bd-e85a-4b36-acc2-743ade2cad4f": {
      "_type": "PolyRing",
      "data": { "base_ring": { "data": "7",  "_type": "Nemo.fpField"},
                "symbols": ["x"] }
    },
    "869a359a-43d3-43f4-9821-0af9346be019": {
      "_type": "MPolyRing",
      "data": {
        "base_ring": "a8309b96-caec-443c-bedb-e23bb0634c14",
        "symbols": [ "y", "z" ]
      }
    },
    "a8309b96-caec-443c-bedb-e23bb0634c14": {
      "_type": "fqPolyRepField",
      "data": {
        "def_pol": {
          "_type": {
            "name": "PolyRingElem",
            "params": "152ac7bd-e85a-4b36-acc2-743ade2cad4f"
          },
          "data": [["0", "1"], ["1", "1"], ["2", "1"]]
        }
      }
    }
  }
  "data": [[["3", "4"], [["0", "2"]]],
           [["0", "2"], [["0", "3"], ["1", "1"]]],
           [["1", "0"], [["1", "5"]]],
           [["0", "0"], [["0", "1"]]]]
}
```

Namespaces

```
polytope > $c = cube(4);

polytope > save($c, "~/c.poly");

polytope >
[2]+  Stopped                 polymake
vecchia@priort:~> julia-1.10

               _
   _       _ _(_)_     |  Documentation: https://docs.julialang.org
  (_)     | (_) (_)    |
   _ _   _| |_  __ _   |  Type "?" for help, "]?" for Pkg help.
  | | | | | | |/ _` |  |
  | | |_| | | | (_| |  |  Version 1.10.0 (2023-12-25)
 _/ |\__'_|_|_|\__'_|  |  Official https://julialang.org/ release
|__/                   |

julia> using Oscar
 -----    -----    -----      _      -----
|     |  |     |  |     |    | |    |     |
|     |  |        |         |   |   |     |
|     |   -----   |        |     |  |-----
|     |        |  |        |-----|  |   |
|     |  |     |  |     |  |     |  |    |
 -----    -----    -----   -     -  -     -

...combining (and extending) ANTIC, GAP, Polymake and Singular
Version 0.14.0 ...
 ... which comes with absolutely no warranty whatsoever
Type: '?Oscar' for more information
(c) 2019-2024 by The OSCAR Development Team

julia> load("./c.poly")
Polyhedron in ambient dimension 4
```

Schema

Figure:
https://www.pexels.com/photo/plastic-shape-shorter-toy-11030155/

- A schema defines a structure for data.

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- Schemata allow data to be validated before loading.

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- Schemata allow data to be validated before loading.
- Adds structure to document based databases.

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- Is possible to define recursive structure.
- Schemata allow data to be validated before loading.
- Adds structure to document based databases.
- PolyDB, Paffenholz [2017]

File Format Specification

```
julia> mrdi_schema = Schema(JSON.parsefile(schema_path))
A JSONSchema

julia> jsondict = JSON.parsefile(polynomial_path)
Dict{String, Any} with 4 entries:
  "data"  => Any[Any[Any["3", "4"], Any[Any["0", "2"]]], Any[Any["0", "2"], Ar
  "_ns"   => Dict{String, Any}("Oscar"=>Any["https://github.com/oscar-system/(
  "_type" => Dict{String, Any}("name"=>"MPolyRingElem", "params"=>"869a359a-43
  "_refs" => Dict{String, Any}("869a359a-43d3-43f4-9821-0af9346be019"=>Dict{St

julia> validate(mrdi_schema, jsondict)
```

```
{
  "$id": "https://oscar-system.github.io/schemas/data.json",
  "$schema": "https://json-schema.org/draft/2020-12/schema",
  "type": "object",
  "properties": {
    "_ns": { "type": "object" },
    "_type": {"oneOf": [
      {"type": "string"},
      {"type": "object",
       "properties": {
         "name": {"type": "string"},
         "params": {"$ref": "#/$defs/data"}
       }}
    ]},
    "data": {"$ref": "#/$defs/data"},
    "_refs": {
      "type": "object",
      "patternProperties":{
        "^[0-9a-fA-F]{8}-([0-9a-fA-F]{4}-){3}[0-9a-fA-F]{12}$":
        {"$ref": "#"}
      }
    }
  },
  "required": ["_type"],
  "$defs": {
    "data": {
      "oneOf": [
        {"type": "string"},
        {"type": "object",
         "patternProperties": {
           "[a-z]": { "$ref": "#/$defs/data" }
         }
        },
        {"type": "array", "items": {"$ref": "#/$defs/data"}}
      ]
    }
  }
}
```

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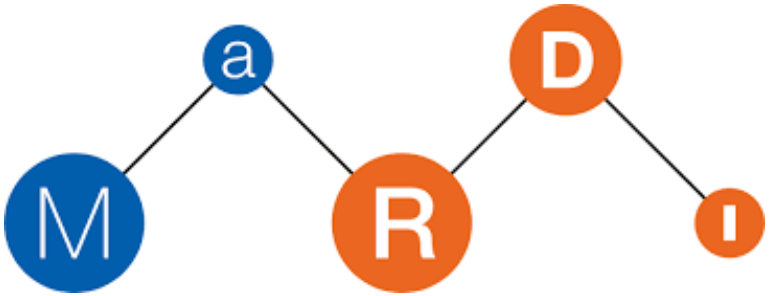

Benefits of Julia

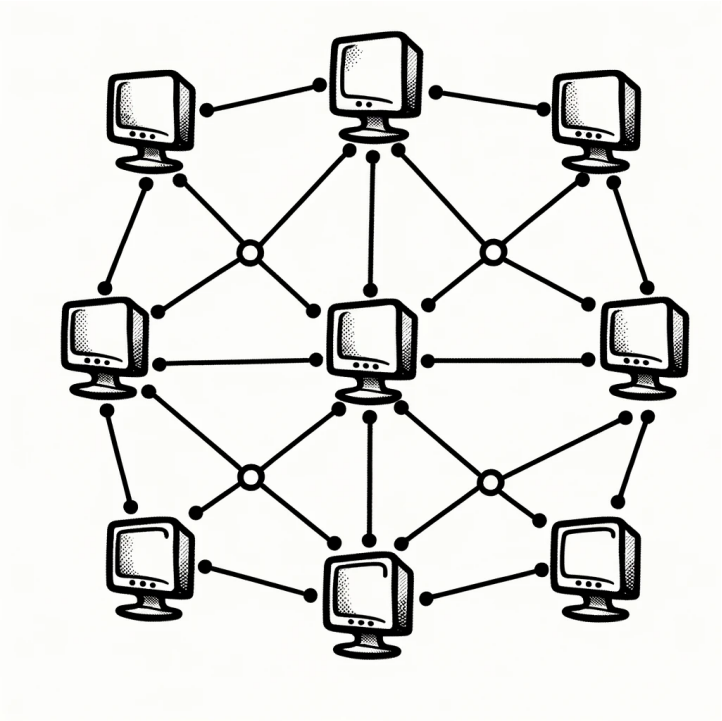

Figure: ChatGPT

```julia
process_ids = addprocs(5)

@everywhere using Oscar

channels = Oscar.params_channels(Union{Ring, MatSpace})

Qx, x = QQ["x"]
F, a = number_field(x^2 + x + 1)
MR = matrix_space(F, 2, 2)

Oscar.put_params(channels, Qx)
Oscar.put_params(channels, F)
Oscar.put_params(channels, MR)

c = [MR([a^i F(1); a a + 1]) for i in 1:5]
dets = pmap(det, c)
total = reduce(*, dets)

@test total == F(4)
```

End

Thank you!