Parametrizing Turbulent Flow in The Planetary Boundary Layer

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Goal

Use equation discovery methods to develop accurate and interpretable parametrizations for vertical turbulent fluxes in the planetary boundary layer.

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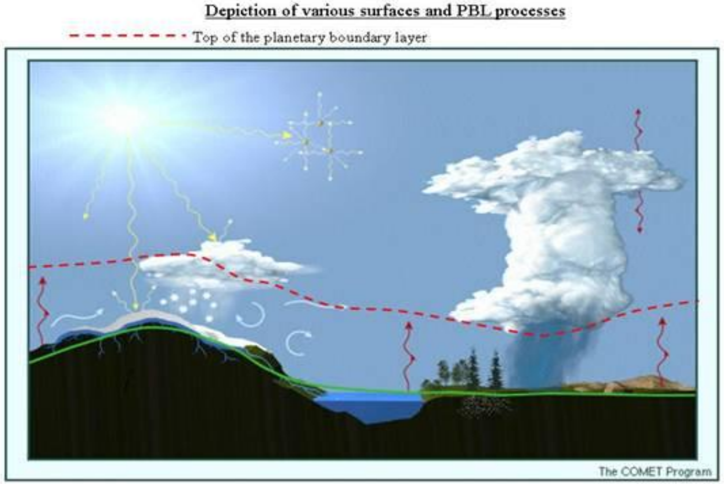
The Planetary Boundary Layer (PBL)

- Lowest layer of troposphere (~1 km)
    - Events are often too small to be resolved by climate models (~100km)
    - Directly affected by surface heating/cooling
    - Turbulent, well-mixed, unstable
    - Capped by a temperature "inversion"
- Vertical turbulent fluxes
    - Surface heat → buoyancy
    - Transport air and key quantities upward:
        - Pollution, heat, moisture, etc…

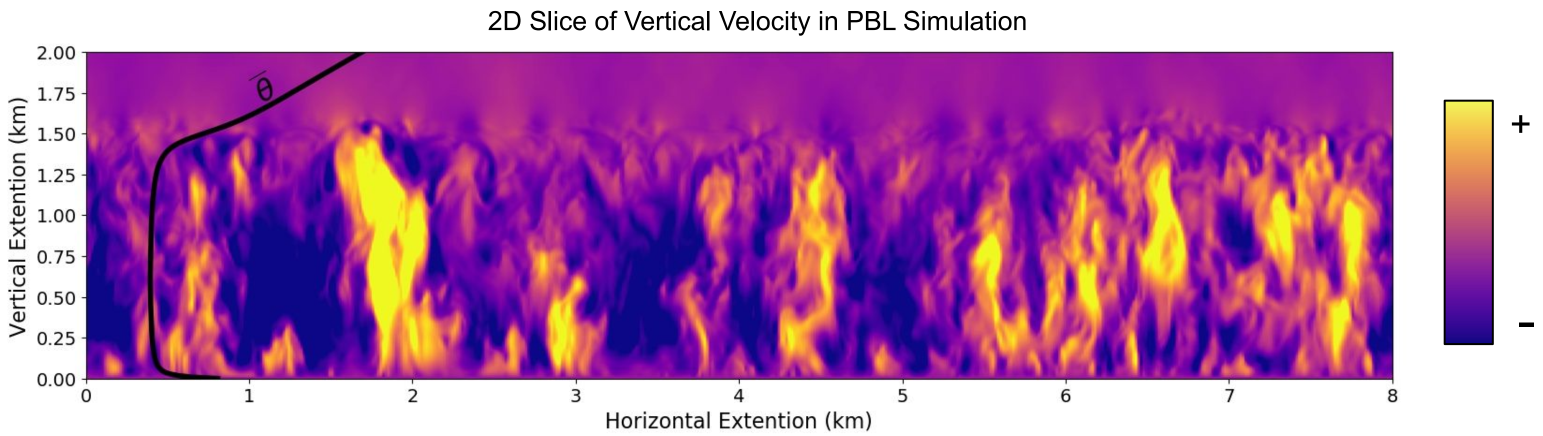
[1]



[2]

The Data

- Simulate PBL using Large Eddy Simulations (LES)
- Varying initial conditions: horizontal wind, surface heating, inversion strength
- Captures evolution of PBL at high resolution (24×24×6 m, 120 min)
- Coarse-grained and averaged down to vertical profile to remove noise
- Many variables and their higher order moments produced.



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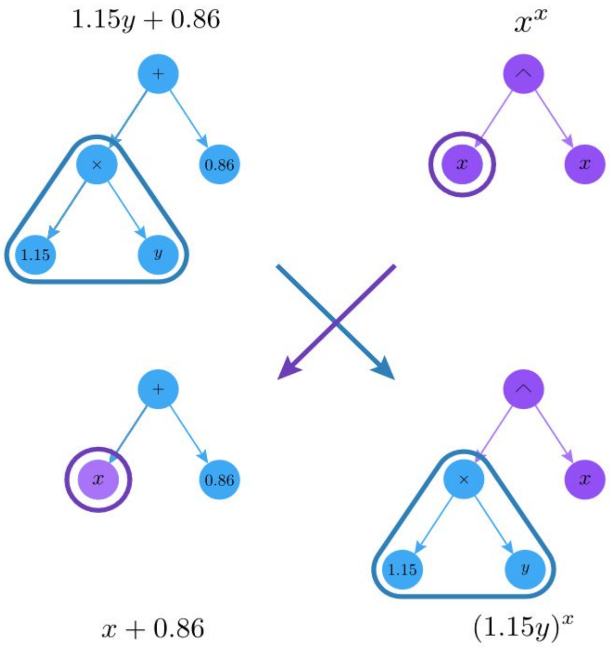
Use **equation discovery methods** to develop accurate and interpretable parametrizations for vertical turbulent fluxes in the planetary boundary layer.

Symbolic Regression (SR)

A machine learning task which aims to discover human-interpretable equations.

- Genetic algorithm + optimization task
- Combines operators (+,-,×,÷), basic functions (sin, cos, inv), and coefficients.
- Inputs: response variable, potential predictor variable(s)
- Output: Equation relating predictors and response
- Inherently very random, not guaranteed to converge or find correct equation



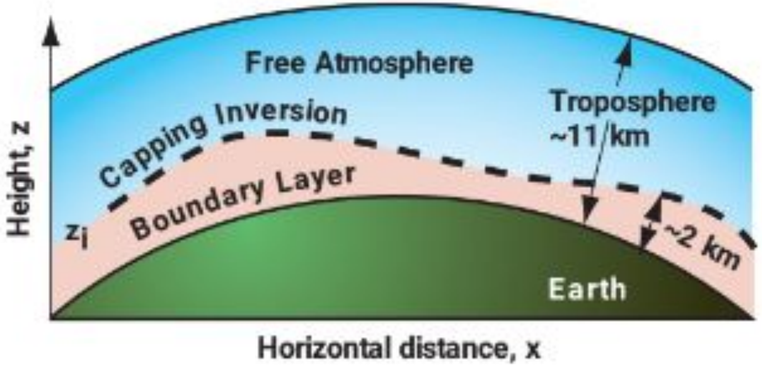
[3]

Goal

Use equation discovery methods to develop **accurate and interpretable parametrizations** for vertical turbulent fluxes in the planetary boundary layer.

Eq 1: Entrainment Velocity

- How does the PBL grow over time?
- At the capping inversion:
  - Inertia of rising air overshoots to the free troposphere
  - This causes mixing at the PBL
- Represented by $\frac{dh}{dt}$ or w_e
  - Large scale components in $\frac{dh}{dt}$ are not considered in our LES



[4]

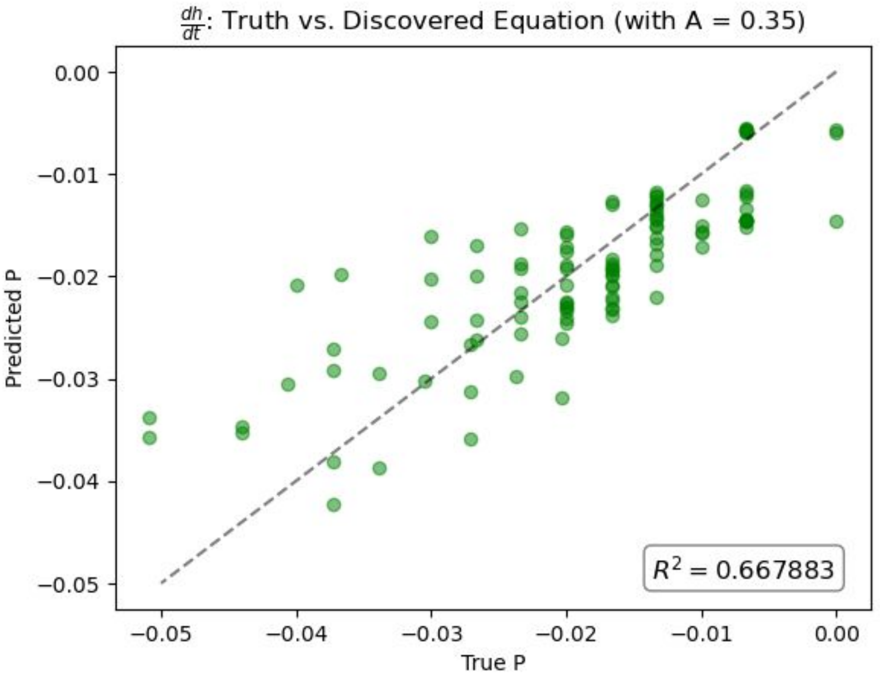
Eq 1: Discovering entrainment velocity

- Current parameterization:

$$\frac{dh}{dt} = A\frac{\overline{w'b'}_{\mathrm{sfc}}}{\frac{g}{\theta_0}\Delta\theta_\rho}$$

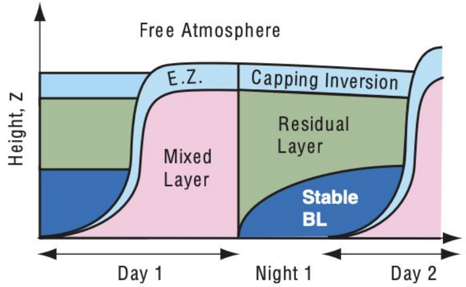
- Discovered equation:

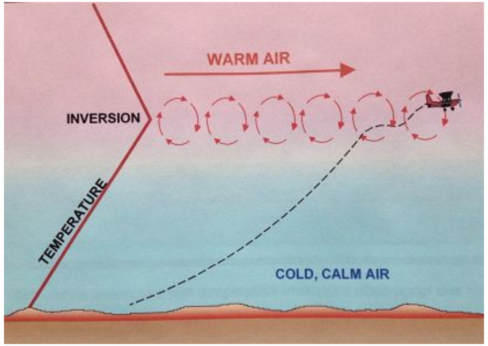
$$\frac{dh}{dt} = -10.751\frac{\overline{w'b'}_{\mathrm{sfc}}}{\Delta\theta_\rho} - 0.00076U_g$$



Eq 2: Inversion Layer Mass-Flux

- Calculation discrepancies
  - Some textbooks [7] say: $h^{-}\dfrac{d\theta_{ml}}{dt} = \overline{w\theta_{h^-}} + \overline{w\theta_{sfc}}$
  - Where: $\overline{w\theta_{h^-}} = w_e \Delta\theta_\rho$
- Looking for the mass flux in the inversion layer
  - Equation discovery struggles to adhere to physics & respect units





[5]

Eq 3: Heat Flux

We model the way the turbulent component of warm air vertically transports key quantities in the PBL ($\overline{w\theta}$)

First, the pressure redistribution term is found, as derived in [6]:

$$P = -\frac{1}{\rho_0}\overline{\theta\frac{\mathrm{d}p}{\mathrm{d}z}} = -C_1\frac{\overline{w\theta}}{\tau_1} - C_2\beta\overline{\theta^2} + C_3\sigma_w^2\frac{\mathrm{d}\Theta}{\mathrm{d}z}$$

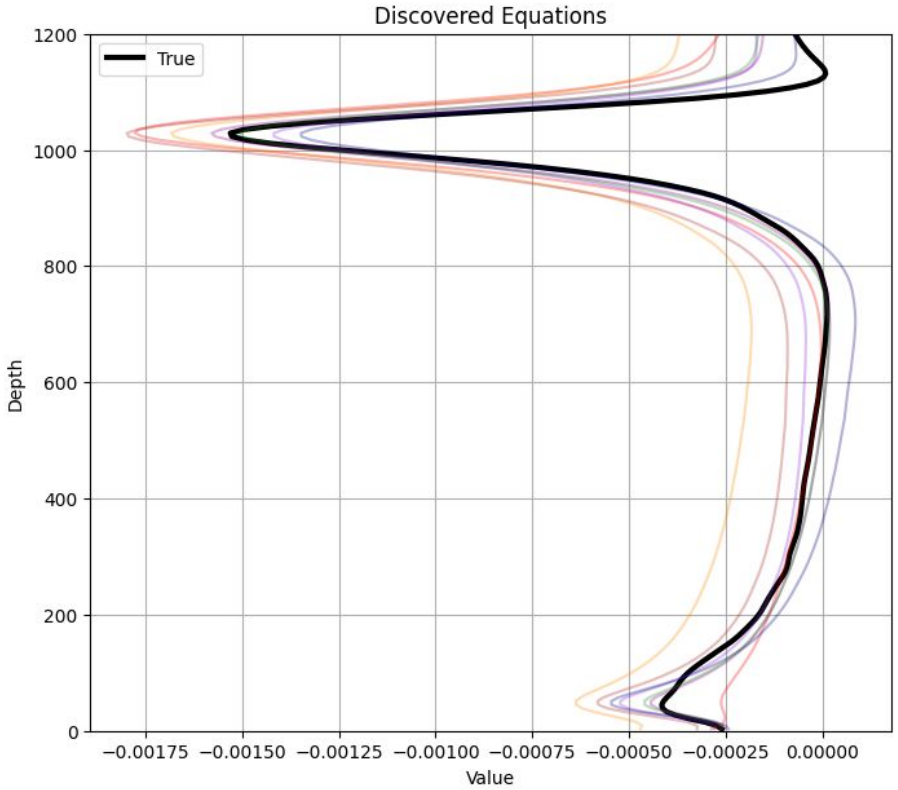
Then, the resulting coefficients C_1, C_2, C_3 are plugged into a parametrization for $\overline{w\theta}$ that is dependent on them.

Goals:

1. Verify the functional form for *P*, using the above predictors and response
2. If correct, compare discovered coefficients to theoretical/typical estimates
3. Plug coefficients in to test to see how well $\overline{w\theta}$ is parametrized.

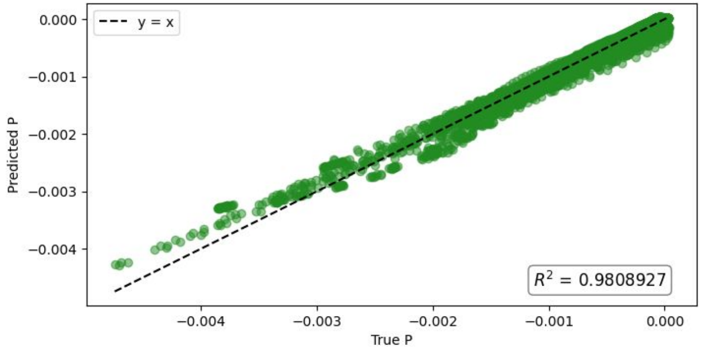
Eq 3: Goals 1 & 2

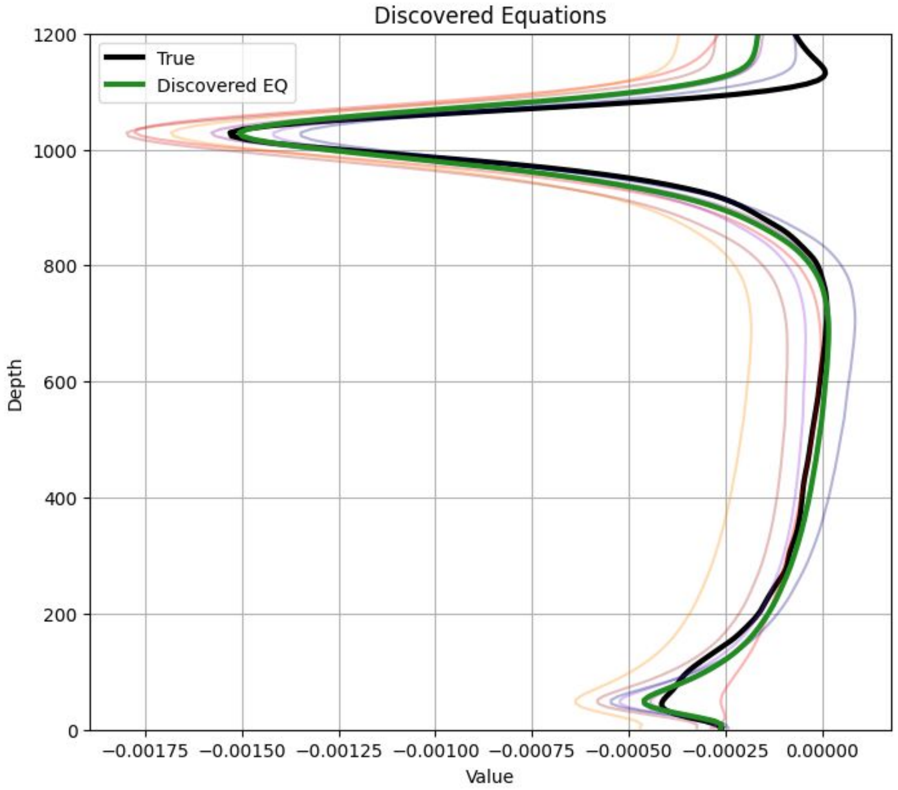
- Rediscovery of eq for *P* is successful after lots of tuning: functional form appears correct



Eq 3: Goals 1 & 2

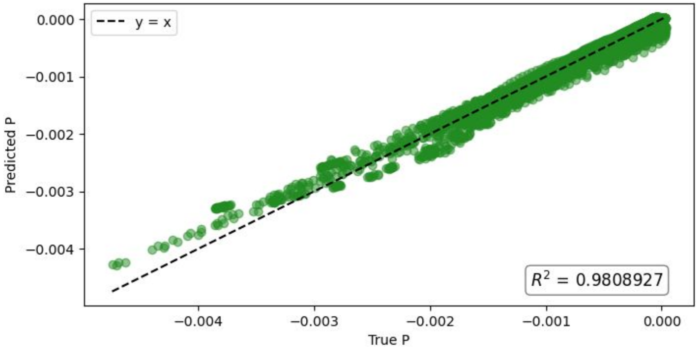
- Rediscovery of eq for *P* is successful after lots of tuning: functional form appears correct
- Final eq provides excellent fit.

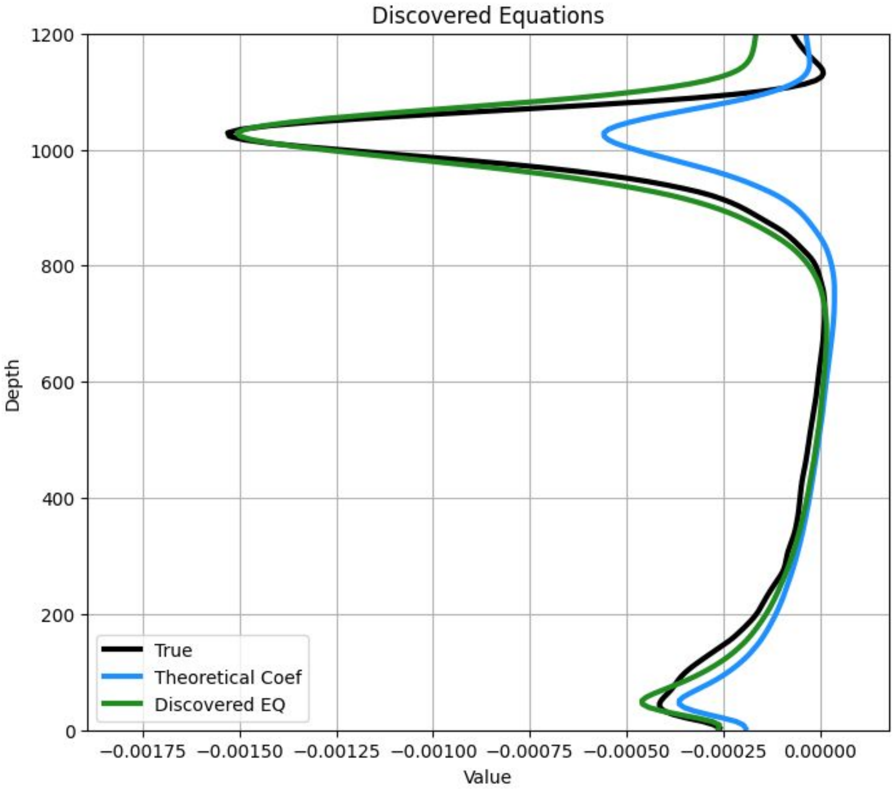




Eq 3: Goals 1 & 2

- Rediscovery of eq for *P* is successful after lots of tuning: functional form appears correct
- Final eq provides excellent fit.
- Coefficients differ strongly from theoretical/typical values

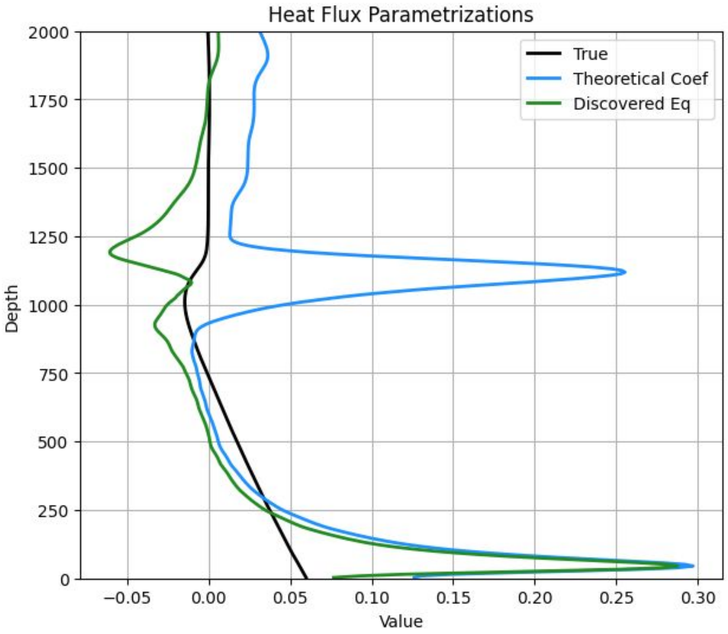




Eq 3: Goal 3

When coefficients are plugged back into $\overline{w\theta}$ parametrization, performance is bad..

- Discrepancy most likely from data issues, or extra dependency on large scale forcings.
- A second stage SR on the coefficients shows that C_1 may be dependent on U_g.
- This makes sense: horizontal wind likely speeds up mixing.



Conclusions

Our Contributions:

- Equation rediscovery: w_e , P
- Equation improvement: better coefficients & dependencies on large scale forcings
- Attention drawn to horizontal wind
- Github repo for reproducibility + future work

Future Directions:

- Include more simulations with varying initial conditions
- Refine the second stage SR for finding coefficient dependence on large scale forcings
- Building in physics/consideration of units
- One big limitation of equation discovery is reliance on local information, bring in non-locality (function space or inputs)

References

1. Britannica. (n.d.). *Smog*. Retrieved from https://www.britannica.com/science/smog
2. NOAA. (n.d.). *Planetary Boundary Layer*. Retrieved from https://www.esrl.noaa.gov/research/themes/pbl/
3. Cranmer, M. (2023). *Interpretable Machine Learning for Science with PySR and SymbolicRegression.jl*. arXiv. Retrieved from https://arxiv.org/abs/2305.01582
4. Stull, R. (2000). *Meteorology for Scientists and Engineers, A Technical Companion Book to C. Donald Ahrens' Meteorology Today* (2nd ed., p. 69).
5. National Weather Service. (n.d.). *Turbulence*. Retrieved from https://www.weather.gov/source/zhu/ZHU_Training_Page/turbulence_stuff/turbulence/turbulence.htm
6. Ghannam, K., Duman, T., Salesky, S. T., Chamecki, M., & Katul, G. (2017). The non-local character of turbulence asymmetry in the convective atmospheric boundary layer. *Quarterly Journal of the Royal Meteorological Society, 143*(702), 494-507. Wiley Online Library.
7. Siebesma, A. P., Bony, S., Jakob, C., & Stevens, B. (Eds.). (2020). *Clouds and Climate: Climate science's greatest challenge*. Cambridge University Press. https://doi.org/10.1017/9781107447738

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