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Passivity-based control for bilateral teleoperation: A tutorial[☆]

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ABSTRACT

This tutorial revisits several of the most recent passivity-based controllers for nonlinear bilateral teleoperators with guaranteed stability properties. These schemes, which include scattering–based, damping injection and adaptive controllers, ensure asymptotic stability in multiple situations that range from constant to variable time-delays, with or without scattering transformation and with or without position tracking capabilities. Although all controllers exploit the basic property of passivity of the teleoperators, they have been developed invoking various analysis and design tools, which complicates their comparison and relative performance assessment. The objective of this paper is to present a unified theoretical framework—based on a *general* Lyapunov–like function—that, upon slight modification, allows to analyze the stability of all the schemes.

1. Introduction

A bilateral teleoperator is commonly referred as a system composed of five interconnected elements: a *human operator* that exerts torques on a *local manipulator* connected through a *communication channel* to a *remote manipulator* that interacts with an *environment*, such interaction is then reflected back to the operator. The application of such a system spans multiple fields, the most illustrative being space, underwater, medicine, and, in general, tasks in hazardous environments. The communications often involve large distances or impose limited data transfer between the local and the remote sites. Such situations can result in substantial delays between the time a command is sent by the operator and the time the command is executed by the remote manipulator, *idem* for the reflected interaction. These time-delays affect the overall stability of the teleoperator. Controlling these systems has become a highly active research field amongst engineering scientists. For a recent historical survey on this research line, the reader may refer to Hokayem and Spong (2006).

Several control schemes that ensure, with rigorous proofs, asymptotic stability of the teleoperator despite time-delays have been reported in the literature. Prominent among them are the ones that exploit the basic property of passivity of the teleoperators, that are referred here as passivity-based controllers. These schemes, which include scattering-based, damping injection and adaptive controllers, have been developed invoking various analysis and design tools, which complicates their comparison and relative performance assessment. The objective of this work is to propose a unified framework for the analysis of such controllers, providing a *general* Lyapunov-like function that can be tailored to fit different control schemes designed to deal with constant or variable time-delays, with or without the scattering transformation and with or without position tracking.

The tutorial covers eleven different controllers organized in three groups: (i) scattering-based, (ii) damping injection controllers and (iii) adaptive schemes. The first group gathers the scattering-based schemes that provide delay independent stability but, as originally designed, cannot guarantee position tracking (Anderson & Spong, 1989; Niemeyer & Slotine, 1991). In this group there are some schemes aimed at increasing tracking performance for constant and variable time-delays (Chopra, Spong, Ortega, & Barabanov, 2006; Namerikawa & Kawada, 2006; Nuño, Basañez, Ortega & Spong, 2009). The second group treats the Proportional Derivative and the Proportional plus damping injection controllers (PD + d and P + d, respectively), that ensure position tracking despite constant or variable time-delays (Nuño, Basañez, Ortega & Spong, 2009; Nuño, Ortega, Barabanov & Basañez, 2008), and the passive output interconnection schemes that asymptotically stabilize the teleoperator independent of the time-delays

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(Chopra & Spong, 2007a,c; Wang & Slotine, 2006). Finally, in the third group, the adaptive schemes estimate the physical parameters of the teleoperator in order to provide position tracking and delay independent stability (Chopra, Spong, & Lozano, 2008; Nuño, Ortega, & Basañez, 2010a; Polushin & Marquez, 2003). It should be mentioned that a unified framework for stability analysis of PD+d and P + d controllers using general Lyapunov-like functions has been presented in Nuño, Basañez, and Ortega (2009).

The tutorial does not treat controllers that do not deal with time-delays and those that have been designed using linearized teleoperator models such as the hybrid matrix approach, Llewellin's and Tsypkin's criteria, or other frequency domain techniques (Azorín, Reinoso, Aracil, & Ferre, 2004; Hastrudi-Zaad & Salcudean, 2001; Michels, van Assche, & Niculescu, 2005; Taoutaou, Niculescu, & Gu, 2003). These controller designs have been previously studied in Arcara and Melchiorri (2002), Arcara and Melchiorri (2004), Aziminejad, Tavakoli, Patel, and Moallem (2008) and Hastrudi-Zaad and Salcudean (2001).

Notation. $\mathbb{R} := (-\infty, \infty), \mathbb{R}_{>0} := (0, \infty), \mathbb{R}_{\geq 0} := [0, \infty)$. $|\mathbf{x}|$ stands for the standard Euclidean norm of a vector $\mathbf{x} \in \mathbb{R}^n$. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty = \sup_{t\in[0,\infty)} |\mathbf{f}(t)|$, and the $\mathcal{L}_2$-norm $\|\mathbf{f}\|_2$ as $\|\mathbf{f}\|_2^2 = \int_0^\infty |\mathbf{f}(t)|^2 \mathrm{d}t$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively. The time argument of all functions will be omitted, e.g., $\mathbf{x} \triangleq \mathbf{x}(t)$, except for those which appear delayed, e.g., $\mathbf{x}(t - T(t))$. The argument of signals inside integrals will be omitted, it is supposed to be equal to the variable on the differential, unless otherwise noted, e.g., $\int_0^t \mathbf{x}\mathrm{d}\sigma \triangleq \int_0^t \mathbf{x}(\sigma)\mathrm{d}\sigma$. The subscript i stands for l and r, to denote local or remote manipulators, respectively.

2. Preliminaries

This section presents the dynamical model of the nonlinear teleoperator together with the assumptions and two *lemmata* needed for the stability analysis of the controllers covered in this paper.

2.1. Dynamic model of the teleoperator

The local and remote manipulators are modeled as a pair of n-Degrees of Freedom (DOF) serial links. Their corresponding nonlinear dynamics, together with the human operator and environment torques, are given by

$$\begin{aligned} \mathbf{M}_l(\mathbf{q}_l)\ddot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l + \mathbf{g}_l(\mathbf{q}_l) &= \boldsymbol{\tau}_h - \boldsymbol{\tau}_l^* \\ \mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{g}_r(\mathbf{q}_r) &= \boldsymbol{\tau}_r^* - \boldsymbol{\tau}_e, \end{aligned} \tag{1}$$

where $\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathbb{R}^n$ are the joint positions, velocities and accelerations; $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ are the inertia matrices; $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ are the Coriolis and centrifugal effects, defined using the Christoffel symbols of the first kind; $\mathbf{g}_i(\mathbf{q}_i) \in \mathbb{R}^n$ are the gravitational torques; $\boldsymbol{\tau}_i^* \in \mathbb{R}^n$ are the controllers; and $\boldsymbol{\tau}_h \in \mathbb{R}^n, \boldsymbol{\tau}_e \in \mathbb{R}^n$ the torques at the joints due to the forces exerted by the human and the environment. It is assumed that the manipulators are composed by actuated revolute joints[1] and that friction can be neglected. These dynamical models have the following important properties (Kelly, Santibáñez, & Loria, 2005; Nuño, Basañez, & Prada, 2009; Spong, Hutchinson, & Vidyasagar, 2005).

P1. The inertia matrix is lower and upper bounded, i.e., $0 < \lambda_m\{\mathbf{M}_i\}\mathbf{I} \leq \mathbf{M}_i(\mathbf{q}_i) \leq \lambda_M\{\mathbf{M}_i\}\mathbf{I} < \infty$.
P2. The Coriolis and inertia matrices are related as $\dot{\mathbf{M}}_i(\mathbf{q}_i) = \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + \mathbf{C}_i^\top(\mathbf{q}_i, \dot{\mathbf{q}}_i)$.
P3. The Coriolis torques are bounded as $|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i| \leq k_{c_i}|\dot{\mathbf{q}}_i|^2 \forall\, \mathbf{q}_i, \dot{\mathbf{q}}_i \in \mathbb{R}^n$ and $k_{c_i} \in \mathbb{R}_{>0}$.
P4. The Lagrangian dynamics are linearly parameterizable. Thus $\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i)\boldsymbol{\theta}_i$ where $\mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i) \in \mathbb{R}^{n\times p}$ are matrices of known functions and $\boldsymbol{\theta}_i \in \mathbb{R}^p$ are constant vectors of the manipulator physical parameters (link masses, moments of inertia, etc.).

2.2. General assumptions

The control schemes presented in this tutorial rely on the following assumptions:

A1. The human operator and the environment define passive, velocity to force, maps, that is, $\exists\, \kappa_i \in \mathbb{R}_{\geq 0}$, such that $E_h := -\int_0^t \dot{\mathbf{q}}_l^\top \boldsymbol{\tau}_h \mathrm{d}\sigma + \kappa_l \geq 0$ and $E_e := \int_0^t \dot{\mathbf{q}}_r^\top \boldsymbol{\tau}_e \mathrm{d}\sigma + \kappa_r \geq 0$, $\forall t \geq 0$. (Notice the signs, which are consistent with the standard power flow convention.)
A2. The gravitational torques are pre-compensated by the controllers $\boldsymbol{\tau}_i^*$, i.e., $\boldsymbol{\tau}_i^* = \boldsymbol{\tau}_i - \mathbf{g}_i(\mathbf{q}_i)$.

It is well-known that the communication channel imposes time-delays on the transmitted signals. These time-delays can be, in the simplest scenario, constant. However, when using, for example, a packet switched channel, such as the Internet, time-delays become variable. In this paper it is assumed that

A3. The constant time-delays T_i are known and the variable time-delays $T_i(t)$ have known upper bounds ${}^*T_i$, i.e., $0 \leq T_i(t) \leq {}^*T_i < \infty$.
A4. The variable time-delays do not grow or decrease faster than time itself, i.e., $|\dot{T}_i(t)| < 1$.

2.3. Convergence and boundedness lemmas

Lemma 1 provides the conditions under which the teleoperator's joint velocities and accelerations asymptotically converge to zero, and Lemma 2 gives an upper bound on the integral of delayed signals. The proof of Lemma 2 and its corollary can be found in Nuño, Basañez, Ortega and Spong (2009), Nuño, Basañez, Ortega and Spong (2008) and in Nuño, Ortega, Barabanov et al. (2008), Nuño, Ortega, Basañez and Barabanov (2008) and Nuño, Basañez and Ortega (2009), respectively.

Lemma 1. *Consider the teleoperator (1). Suppose* A2 *holds and* $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$. *Assume that* $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$ *and* $\boldsymbol{\tau}_i \in \mathcal{L}_\infty$. *Under these conditions* $|\dot{\mathbf{q}}_i| \to 0$ *as* $t \to \infty$. *Moreover, if additionally,* $\dot{\boldsymbol{\tau}}_i \in \mathcal{L}_\infty$ *then* $\ddot{\mathbf{q}}_i$ *are uniformly continuous, and* $|\ddot{\mathbf{q}}_i| \to 0$.

Proof. The assumption that $\dot{\mathbf{q}}_i, \boldsymbol{\tau}_i \in \mathcal{L}_\infty$ and Properties P1 and P3 ensure that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$. Thus, $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$, $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$, which implies that $|\dot{\mathbf{q}}_i| \to 0$ as $t \to \infty$.

To prove that $|\ddot{\mathbf{q}}_i| \to 0$ it suffices to show that $\ddot{\mathbf{q}}_i$ are uniformly continuous. Now, uniform continuity of $\ddot{\mathbf{q}}_i$ is implied by $\frac{\mathrm{d}}{\mathrm{d}t}\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$. Collecting for $\ddot{\mathbf{q}}_i$ in (1), after compensating the gravity effects, and then differentiating, two types of terms are recovered: the first term consists of $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{M}_i^{-1}(\mathbf{q}_i)$ times a bounded term, and the second term, the product of $\mathbf{M}_i^{-1}(\mathbf{q}_i)$ times the time derivative of the term in brackets. Using P2, $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{M}_i^{-1}$ is $\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{M}_i^{-1} = -\mathbf{M}_i^{-1}\dot{\mathbf{M}}_i\mathbf{M}_i^{-1} = -\mathbf{M}_i^{-1}[\mathbf{C}_i + \mathbf{C}_i^\top]\mathbf{M}_i^{-1}$. Properties P1, P3 and the boundedness of velocities, ensure that this term is bounded. The fact that $\dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ and the additional assumption that $\dot{\boldsymbol{\tau}}_i \in \mathcal{L}_\infty$ allows to show that the time derivative of $[\mathbf{C}_i\dot{\mathbf{q}}_i + \boldsymbol{\tau}_i]$ is bounded. Consequently, $\frac{\mathrm{d}}{\mathrm{d}t}\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ as required. □

[1] The restriction of *only* revolute joints can be relaxed for certain manipulators with revolute and prismatic joints, cf. Ghorbel, Srinivasan, and Spong (1998).

Lemma 2. *Assume* A3 *holds for variable time-delays. Then, for any vector signals* $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $\alpha \in \mathbb{R}_{>0}$

$$-\int_0^t \mathbf{x}^\top(\sigma) \int_{-T(\sigma)}^0 \mathbf{y}(\sigma+\theta)\mathrm{d}\theta\mathrm{d}\sigma \le \frac{\alpha}{2}\|\mathbf{x}\|_2^2 + \frac{{}^*T^2}{2\alpha}\|\mathbf{y}\|_2^2.$$

Corollary 3. *Lemma* 2 *holds for constant time-delays with* $T = {}^*T$.

3. A general Lyapunov-like function

This part of the paper presents a *general* Lyapunov-like function intended to unify the stability analysis of different controllers. In all cases this function is not a *bona fide* Lyapunov Function (LF); it may fail to be positive definite or its derivative may not be non-positive. However, it can be employed to find the conditions under which velocities and position errors remain bounded and converge to zero. Interestingly, for scattering-based methods, it exactly coincides with the sum of the energy functions of all the elements in the system—providing a rigorous physical interpretation to these popular schemes.

Define $V : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_{\ge 0} \to \mathbb{R}$

$$V(\mathbf{q}_i, \dot{\mathbf{q}}_i, t) = V_1(\mathbf{q}_i, \dot{\mathbf{q}}_i) + V_2(t) + V_3(\mathbf{q}_i, t). \tag{2}$$

The function V_1 is a weighted sum of the kinetic energy of the local and remote manipulators and it is given by

$$V_1(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \frac{\beta_l}{2}\dot{\mathbf{q}}_l^\top \mathbf{M}_l(\mathbf{q}_l)\dot{\mathbf{q}}_l + \frac{\beta_r}{2}\dot{\mathbf{q}}_r^\top \mathbf{M}_r(\mathbf{q}_r)\dot{\mathbf{q}}_r, \tag{3}$$

by Property P1, it is positive definite and radially unbounded in $\dot{\mathbf{q}}_i$ for any $\beta_i \in \mathbb{R}_{>0}$. Differentiating (3), using P2 and A2, after evaluation along (1), yields

$$\dot{V}_1(\mathbf{q}_i, \dot{\mathbf{q}}_i) = -\beta_l\dot{\mathbf{q}}_l^\top[\boldsymbol{\tau}_l - \boldsymbol{\tau}_h] - \beta_r\dot{\mathbf{q}}_r^\top[\boldsymbol{\tau}_e - \boldsymbol{\tau}_r], \tag{4}$$

that is a restatement of the well-known passivity property of robot manipulators (Ortega & Spong, 1989). $V_2(t)$ is the weighted sum of the energies supplied (or extracted) by the human and the environment, which we recall, were assumed to define passive operators. That is

$$V_2(t) = \beta_l E_h + \beta_r E_e, \tag{5}$$

which by A1, is non-negative. Combining (4) and $\dot{V}_2$, yields $\dot{V}_1 + \dot{V}_2 = -\beta_l\dot{\mathbf{q}}_l^\top\boldsymbol{\tau}_l + \beta_r\dot{\mathbf{q}}_r^\top\boldsymbol{\tau}_r$.

The functional V_3 is determined by the controller, that establishes the coupling between the local and remote manipulators. By suitably selecting this term, it will be possible to carry out the stability analysis of the various control schemes, as is spelled out in the following sections.

4. Scattering-based schemes

The key idea behind the scattering-based controllers is to render passive the communication channel by emulating the behavior of an electrical lossless transmission line.[2] As explained below, this objective is achieved transmitting through the (delayed) communication channel, instead of the forces and velocities, their corresponding scattering waves. The design is completed interconnecting the line with a simple PI controller, which is also passive. This ground-breaking approach was first proposed by Anderson and Spong (1989) and, due to its physical appeal and robustness, it has dominated the field.

[2] The reader may refer to de Rinaldis, Ortega, and Spong (2006) and Ortega, de Rinaldis, Spong, Lee, and Nam (2004) for the application of the dual idea, that is, transform a real transmission line into pure delays, and for a compensation method aimed at eliminating wave reflections in long cable interconnections.

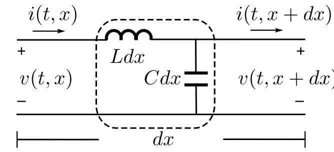

Fig. 1. Schematics of an element of a lossless transmission line.

The performance of this scheme was later improved by Niemeyer and Slotine (1991) and Niemeyer (1996), which proved that, with a suitable selection of the controller gains, the virtual line impedance can be adjusted to match the termination impedances, hence avoiding wave reflections. To further improve performance, several variations of the scheme have been reported: transmitting *wave* integrals (Niemeyer & Slotine, 2004; Nuño, Basañez, & Ortega, 2007; Ortega, Chopra, & Spong, 2003), *wave* filtering (Tanner & Niemeyer, 2005), *wave* prediction (Munir & Book, 2002), and power scaling (Secchi, Stramigioli, & Fantuzzi, 2007, 2008), amongst others. It should be underscored that the first Lyapunov-based proof, of the asymptotic stability of the teleoperator, has been proposed by Anderson and Spong (1992).

4.1. Transforming a bilateral communication delay into a transmission line

Before going through the stability analysis of these schemes, the fundamentals of a transmission line are introduced. A lossless electrical transmission line, of length ℓ, can be modeled as an infinite series of elements composed by inductances and capacitances between the two conductors (Fig. 1). Each element represents an infinitesimally short segment of the transmission line. The Telegrapher's equations model the behavior of this transmission line, and are given by

$$\frac{\partial i(t,x)}{\partial x} = -C\frac{\partial v(t,x)}{\partial t}; \qquad \frac{\partial v(t,x)}{\partial x} = -L\frac{\partial i(t,x)}{\partial t}, \tag{6}$$

where $v(t,x)$ and $i(t,x)$ are the voltage and the current associated to the spatial coordinate $x \in [0,\ell]$. The energy stored in the transmission line corresponds to the energy in all the elements along the length ℓ, that is $E(t) = \int_0^\ell [Cv^2(t,x) + Li^2(t,x)]\mathrm{d}x$, its time derivative, using (6) and integration by parts, yields

$$\dot{E}(t) = v(t,0)i(t,0) - v(t,\ell)i(t,\ell), \tag{7}$$

which proves that the transmission line defines a passive (actually, lossless) operator. Note that, if the port variables are interconnected (in a power preserving way) with passive operators, passivity is preserved. In the context of teleoperation, these interconnections are the controller and the terminations with the local and remote manipulators. See Duindam, Macchelli, Stramigioli and Bruyninckx (2009) and Stramigioli, van der Schaft, Maschke, and Melchiorri (2002) for further details on this port viewpoint of control.

To transform the communication delay into a transmission line we proceed as follows. First, define the so-called scattering variables

$$\begin{bmatrix} s^+(t,x) \\ s^-(t,x) \end{bmatrix} = \mathbf{T}\begin{bmatrix} v(t,x) \\ i(t,x) \end{bmatrix} \tag{8}$$

where $\mathbf{T} = \begin{bmatrix} 1 & Z_0 \\ 1 & -Z_0 \end{bmatrix}$ and $Z_0 = \sqrt{\frac{L}{C}}$ is the impedance of the transmission line. As is well-known, the scattering variables satisfy

$$\begin{bmatrix} s^+(t,\ell) \\ s^-(t,\ell) \end{bmatrix} = \begin{bmatrix} s^+(t-T,0) \\ s^-(t+T,0) \end{bmatrix}, \tag{9}$$

where $T = \ell\sqrt{LC}$ is the propagation delay.

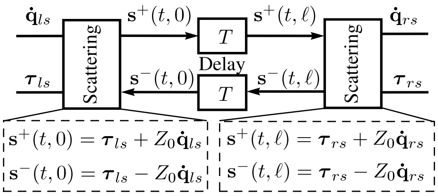

Fig. 2. Emulation of the transmission line transmitting the scattering variables.

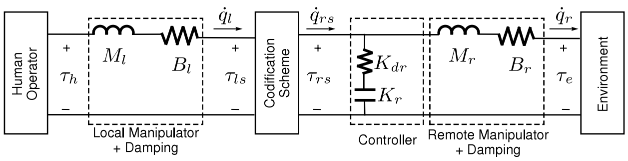

Fig. 3. Electrical analog of a 1-DOF linear mechanical teleoperator controlled by the classical scattering scheme.

From (8) and (9) it is clear that the objective is achieved transmitting from the local side the signal $s^+(t,0)$ and from the slave side $s^-(t,\ell)$ and then, use (8), to reconstruct the voltages and currents. This is depicted in Fig. 2, where the standard analogy $v \leftrightarrow \tau$ and $i \leftrightarrow \dot{q}$ has been used and the subindex s, which denotes scattered signal, has been added.

4.2. Classical scattering scheme

Once the virtual transmission line has been created Anderson and Spong (1989) proposed to complete the design with a PI controller in the remote side and damping injection terms on both sides. To preserve the physical interpretation, an electrical analog of the overall system is depicted in Fig. 3,[3] for the 1-DOF linear case. The damping terms correspond to resistors with coefficients $B_i > 0$. The PI controller is an RC circuit, whose dynamics is obtained from Kirchhoff's voltage law as $\tau_{rs} = -K_r\int_0^t(\dot{q}_r - \dot{q}_{rs})\mathrm{d}\sigma - K_{dr}(\dot{q}_r - \dot{q}_{rs})$, where the new "current" $\dot{q}_{rs}$ is defined. The latter is computed from the first equation of (9), that is,

$$\begin{aligned} s^+(t,\ell) &= \tau_{rs} + Z_0\dot{q}_{rs} \\ &\equiv s^+(t-T,0) = \tau_{ls}(t-T) + Z_0\dot{q}_{ls}(t-T). \end{aligned} \tag{10}$$

Finally, the local control signal τ_{ls} is obtained from the second equation of (9), namely

$$\begin{aligned} s^-(t,0) &= \tau_{ls} + Z_0\dot{q}_{ls} \\ &\equiv s^-(t-T,l) = \tau_{rs}(t-T) - Z_0\dot{q}_{rs}(t-T). \end{aligned} \tag{11}$$

It should be underscored that this was the first controller that, using Lyapunov stability analysis, ensured an asymptotically stable behavior of the local and remote velocities despite constant time-delays (Anderson & Spong, 1989, 1992).

Proposition 4 (*Anderson & Spong, 1992*)**.** *Consider the teleoperator in (1) controlled by*

$$\boldsymbol{\tau}_l = \boldsymbol{\tau}_{ls} + B_l\dot{\mathbf{q}}_l; \qquad \boldsymbol{\tau}_r = \boldsymbol{\tau}_{rs} - B_r\dot{\mathbf{q}}_r \tag{12}$$

with $\boldsymbol{\tau}_{rs} = -K_r\tilde{\mathbf{q}}_r - K_{dr}\dot{\tilde{\mathbf{q}}}_r$, where $\tilde{\mathbf{q}}_r = \int_0^t(\dot{\mathbf{q}}_r - \dot{\mathbf{q}}_{rs})\mathrm{d}\sigma$, and $\boldsymbol{\tau}_{ls}$ and $\dot{\mathbf{q}}_{rs}$ are obtained from (11) and (10), respectively. Then, under assumptions A1 and A2 , for $Z_0, K_{dr}, K_r, B_i \in \mathbb{R}_{>0}$ and any constant time-delay T, $|\dot{\mathbf{q}}_i| \to 0$ as $t \to \infty$.

Proof. Consider $V_3 = \int_0^t(\dot{\mathbf{q}}_{ls}^\top\boldsymbol{\tau}_{ls} - \dot{\mathbf{q}}_{rs}^\top\boldsymbol{\tau}_{rs})\mathrm{d}\sigma + \frac{K_r}{2}|\tilde{\mathbf{q}}_r|^2$ (notice that, in view of (7), V_3 is the sum of the energy in the transmission line, plus the magnetic energy stored in the capacitor). Using (8) and (9), setting $\beta_i = 1$ in (3) and (5), and with this V_3, the general functional V (2) is positive semi-definite and radially unbounded in $\dot{\mathbf{q}}_i$ and $\tilde{\mathbf{q}}_r$. Its time derivative along (1), (11) and (12) and using A2 is $\dot{V} = -B_l|\dot{\mathbf{q}}_l|^2 - B_r|\dot{\mathbf{q}}_r|^2 - K_{dr}|\dot{\tilde{\mathbf{q}}}_r|^2$. The fact that $V \geq 0$ and $\dot{V} \leq 0$ implies that $V \in \mathcal{L}_\infty$. Thus $\dot{\mathbf{q}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, $\dot{\tilde{\mathbf{q}}}_r \in \mathcal{L}_2$ and $\tilde{\mathbf{q}}_r \in \mathcal{L}_\infty$. Hence, $\boldsymbol{\tau}_i \in \mathcal{L}_\infty$. From (1) it can be shown that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$. Finally, Barbălat's lemma allows to state that $|\dot{\mathbf{q}}_i| \to 0$. □

4.3. Symmetric impedance matching

Wave reflection is a well-known phenomenon in transmission lines that deforms the transmitted signals and degrades the performance. It occurs when the impedance of the line termination is different from the line impedance. In the case of teleoperators this effect can be observed looking at the behavior of $\dot{\mathbf{q}}_{rs}$. The substitution of $\boldsymbol{\tau}_{rs}$ on (10), and this in its turn in (8) and (9) yields the delay differential equation $\dot{\mathbf{q}}_{rs} + \frac{Z_0 - K_{dr}}{Z_0 + K_{dr}}\dot{\mathbf{q}}_{rs}(t-2T) = f\left(\mathbf{q}_{rs}, \dot{\mathbf{q}}_{ls}, \int\dot{\mathbf{q}}_{rs}\right)$, where f is some functional relation. The coefficient $\frac{Z_0 - K_{dr}}{Z_0 + K_{dr}}$ is called the reflection coefficient and, if it is different from zero, $\dot{\mathbf{q}}_{rs}$ exhibits large oscillations (called wave reflections), degrading the performance of the overall system. Selecting $K_{dr} = Z_0$ the aforementioned effect disappears, and this is the underlying idea behind the *impedance matching* proposed in Niemeyer (1996) and Niemeyer and Slotine (1991).

Since the transmission line is virtual, the coefficient Z_0 can be arbitrarily selected. Hence, it is more convenient to work with a normalized implementation of (8), that is

$$\begin{bmatrix} \mathbf{s}_i^+ \\ \mathbf{s}_i^- \end{bmatrix} = \frac{1}{\sqrt{2b}}\begin{bmatrix} \mathbf{I} & b\mathbf{I} \\ \mathbf{I} & -b\mathbf{I} \end{bmatrix}\begin{bmatrix} \boldsymbol{\tau}_{is} \\ \dot{\mathbf{q}}_{is} \end{bmatrix} \tag{13}$$

where b is the *virtual* impedance and $\mathbf{I}$ is the $n \times n$ identity matrix. This ubiquitous transformation has also been used in other fields: in networked control and in port-Hamiltonian systems to obtain delay-independent stability (Chopra & Spong, 2007c; Fantuzzi, Secchi, & Stramigioli, 2008; Matiakis, Hirche, & Buss, 2006; Stramigioli et al., 2002); in nonlinear interconnected agents to synchronize their outputs (Chopra & Spong, 2005, 2006); and in balancing of lossless systems (van der Schaft, 2008).

Similarly to the derivations above, using (13) and (9) it can be proved that, for constant time-delays, the energy of the transmission line $E(t)$ is given by

$$2\int_0^t(\dot{\mathbf{q}}_{ls}^\top\boldsymbol{\tau}_{ls} - \dot{\mathbf{q}}_{rs}^\top\boldsymbol{\tau}_{rs})\mathrm{d}\sigma = \int_{t-T_l}^t|\mathbf{s}_l^+|^2\mathrm{d}\sigma + \int_{t-T_r}^t|\mathbf{s}_r^-|^2\mathrm{d}\sigma.$$

In the sequel, the scattering-based schemes share the following common function $V_3(\mathbf{q}_i, t)$

$$V_3 = \int_0^t(\beta_l\dot{\mathbf{q}}_{ls}^\top\boldsymbol{\tau}_{ls} - \beta_r\dot{\mathbf{q}}_{rs}^\top\boldsymbol{\tau}_{rs})\mathrm{d}\sigma + \Psi(\mathbf{q}_i, t), \tag{14}$$

where Ψ is designed for each scattering-based scheme.

In the symmetric approach, both local and remote manipulators decode the scattered velocities from the incoming signals $\mathbf{s}_l^-$ and $\mathbf{s}_r^+$, respectively, and encode their corresponding torques. The scattered velocities are obtained from (13) and (9) as

$$\begin{aligned} \dot{\mathbf{q}}_{ls} &= \frac{1}{b}[\boldsymbol{\tau}_{ls} - \sqrt{2b}\mathbf{s}_r^-(t-T_r)] \\ \dot{\mathbf{q}}_{rs} &= \frac{1}{b}[\sqrt{2b}\mathbf{s}_l^+(t-T_l) - \boldsymbol{\tau}_{rs}]. \end{aligned} \tag{15}$$

To achieve impedance matching, it is proposed in Niemeyer (1996) and Niemeyer and Slotine (1991) to add a PI action also on the local manipulator, leading to the following[4]:

[3] Notice that even though $\dot{q}_{ls} = \dot{q}_l$, the subindex s is kept to keep a consistent notation.

[4] In this case the reflection coefficient for the local manipulator is $\frac{b - K_{dl}}{b + K_{dl}}$.

Proposition 5. *Consider the teleoperator (1) controlled by (12) with*

$$\boldsymbol{\tau}_{ls} = K_l\tilde{\mathbf{q}}_l + K_{dl}\dot{\tilde{\mathbf{q}}}_l; \qquad \boldsymbol{\tau}_{rs} = -K_r\tilde{\mathbf{q}}_r - K_{dr}\dot{\tilde{\mathbf{q}}}_r, \tag{16}$$

where $\tilde{\mathbf{q}}_i = \int_0^t(\dot{\mathbf{q}}_i - \dot{\mathbf{q}}_{is})\mathrm{d}\sigma$. Suppose that the scattering transformation (13) is used to obtain the scattered velocities (15), and, for constant time-delays, the local and remote sites are interconnected by (9). Then, under assumptions A1 *and* A2*, $b, K_{di}, K_i, B_i \in \mathbb{R}_{>0}$ and any constant time-delays $T_i \in \mathbb{R}_{\geq 0}$, the teleoperator velocities asymptotically converge to zero.*

Proof. The proof follows *verbatim* the proof of Proposition 4 with V_3 in (14) with $\Psi = \frac{1}{2}K_l|\tilde{\mathbf{q}}_l|^2 + \frac{1}{2}K_r|\tilde{\mathbf{q}}_r|^2$ and $\beta_i = 1$. In this case, $\dot{V} = -B_l|\dot{\mathbf{q}}_l|^2 - B_r|\dot{\mathbf{q}}_r|^2 - K_{dl}|\dot{\tilde{\mathbf{q}}}_l|^2 - K_{dr}|\dot{\tilde{\mathbf{q}}}_r|^2$. □

Remark 1. Let us define the error signals $\mathbf{e}_i$, for constant time-delays, as

$$\mathbf{e}_l = \mathbf{q}_l - \mathbf{q}_r(t - T_r); \qquad \mathbf{e}_r = \mathbf{q}_r - \mathbf{q}_l(t - T_l). \tag{17}$$

Matching the impedances of the *virtual* transmission line and the controllers, i.e. making $K_{dl} = K_{dr} = b$, and then substituting (13) and (16) in (15) for $\dot{\mathbf{q}}_{ls}$, yields $2\dot{\mathbf{q}}_{ls} = \dot{\mathbf{q}}_r(t - T_r) + \dot{\mathbf{q}}_l + \frac{K_l}{b}\tilde{\mathbf{q}}_l + \frac{K_r}{b}\tilde{\mathbf{q}}_r(t - T_r)$. Note that, in this case, the scattered velocity $\dot{\mathbf{q}}_{ls}$ does not contain any twice delayed term, like $\dot{\mathbf{q}}_{ls}(t - T_l - T_r)$, thus reducing the effects of wave reflections. However, despite this important improvement in system performance, these controllers cannot ensure position tracking. Setting the controller gains as $\frac{K_r}{b} = \frac{K_l}{b} = K$, and integrating $\dot{\mathbf{q}}_{ls}$, the tracking error $\mathbf{e}_l$ can be expressed as $\mathbf{e}_l = \mathbf{e}_l(0) + 2\tilde{\mathbf{q}}_l - 2\tilde{\mathbf{q}}_l(0) + K\int_0^t[\tilde{\mathbf{q}}_l + \tilde{\mathbf{q}}_r(\theta - T_r)]\mathrm{d}\theta$. Notice that the error depends on the initial conditions and it increases with the integral term.

The following two subsections present two schemes that do provide position tracking.

4.4. Position tracking controllers

The principle of these schemes is to send, together with the scattering variables, explicit position information of each manipulator. Then, each controller is equipped with a term proportional to the position error (Chopra, Spong, Ortega, & Barabanov, 2004; Chopra et al., 2006). These controllers are given by

$$\boldsymbol{\tau}_l = \boldsymbol{\tau}_{ls} + K\mathbf{e}_l + B_l\dot{\mathbf{q}}_l; \qquad \boldsymbol{\tau}_r = \boldsymbol{\tau}_{rs} - K\mathbf{e}_r - B_r\dot{\mathbf{q}}_r \tag{18}$$

where $\boldsymbol{\tau}_{rs} = -K_{dr}\dot{\tilde{\mathbf{q}}}_r$. The input to the scattering transformation is $\dot{\mathbf{q}}_l$ and $\boldsymbol{\tau}_{rs}$, and the output $\boldsymbol{\tau}_{ls}$ and $\dot{\mathbf{q}}_{rs}$, respectively. Thus, from (13) and (9) the required $\boldsymbol{\tau}_{ls} = \sqrt{2b}\mathbf{s}_r^-(t - T_r) + b\dot{\mathbf{q}}_l$ and $\dot{\mathbf{q}}_{rs} = \frac{1}{b}[\sqrt{2b}\mathbf{s}_l^+(t - T_l) - \boldsymbol{\tau}_{rs}]$ can be obtained.

Proposition 6 (*Chopra et al., 2006*)**.** *Consider the teleoperator (1) controlled by (18) using the scattering transformation (13) with (9). Set the control gains such that*

$$2B_lB_r > (T_l^2 + T_r^2)K^2 \tag{19}$$

holds for any arbitrary positive B_i, K, T_i. Then, under assumptions A1–A3,

(a) *velocities and position error are bounded. Moreover, velocities are in $\mathcal{L}_2$,*
(b) *when the human does not exert any force and the remote manipulator does not become in contact with the environment, i.e., $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$, velocities asymptotically converge to zero and position tracking is achieved: $|\dot{\mathbf{q}}_i| \to 0$, $|\mathbf{e}_i| \to 0$ as $t \to \infty$.*

Proof. Define Ψ in (14) as

$$\Psi = \frac{1}{2}K|\mathbf{q}_l - \mathbf{q}_r|^2, \tag{20}$$

together with the constants $\beta_i = 1$ for (3), (5) and (14). $V \geq 0$ and is radially unbounded in $\dot{\mathbf{q}}_i$ and $\mathbf{q}_l - \mathbf{q}_r$. Its time derivative along (1), (9), (13) and (18), and using Property P2, is given by $\dot{V} = K\dot{\mathbf{q}}_l^\top[\mathbf{q}_r(t - T_r) - \mathbf{q}_r] + K\dot{\mathbf{q}}_r^\top[\mathbf{q}_l(t - T_l) - \mathbf{q}_l] - B_l|\dot{\mathbf{q}}_l|^2 - B_r|\dot{\mathbf{q}}_r|^2 - K_{dr}|\dot{\tilde{\mathbf{q}}}_r|^2$. Using the following expression

$$\int_{-T_i}^0 \dot{\mathbf{q}}_i(t + \theta)\mathrm{d}\theta = \mathbf{q}_i - \mathbf{q}_i(t - T_i), \tag{21}$$

on the first two terms of $\dot{V}$, yields $\dot{V} = -B_l|\dot{\mathbf{q}}_l|^2 - B_r|\dot{\mathbf{q}}_r|^2 - K\dot{\mathbf{q}}_l^\top\int_{-T_r}^0\dot{\mathbf{q}}_r(t+\theta)\mathrm{d}\theta - K\dot{\mathbf{q}}_r^\top\int_{-T_l}^0\dot{\mathbf{q}}_l(t+\theta)\mathrm{d}\theta - K_{dr}|\dot{\tilde{\mathbf{q}}}_r|^2$. Integrating from 0 to t, and applying Corollary 3 to the double integral terms, returns $V(t) - V(0) \leq -K_{dr}\|\dot{\tilde{\mathbf{q}}}_r\|_2^2 - \lambda_l\|\dot{\mathbf{q}}_l\|_2^2 - \lambda_r\|\dot{\mathbf{q}}_r\|_2^2$, where $\lambda_l = B_l - \frac{K}{2}\left(\alpha_l + \frac{T_l^2}{\alpha_r}\right)$ and $\lambda_r = B_r - \frac{K}{2}\left(\alpha_r + \frac{T_r^2}{\alpha_l}\right)$. The signals $\dot{\mathbf{q}}_i, \dot{\tilde{\mathbf{q}}}_r \in \mathcal{L}_2$ if $B_l > \frac{K}{2}\left(\alpha_l + \frac{T_l^2}{\alpha_r}\right)$ and $B_r > \frac{K}{2}\left(\alpha_r + \frac{T_r^2}{\alpha_l}\right)$. These simultaneous inequalities have a positive solution for α_l and α_r if $2B_lB_r > (T_l^2 + T_r^2)K^2$. Furthermore, since V is bounded, from (2) and P1, we also conclude that $\dot{\mathbf{q}}_i, \mathbf{q}_l - \mathbf{q}_r \in \mathcal{L}_\infty$, and the proof of part (a) is finished.

In order to prove the asymptotic convergence of $\dot{\mathbf{q}}_i$ to the origin, one first has to prove that $\dot{\mathbf{q}}_{rs}$ is bounded. From (13), substituting and arranging terms for $\dot{\mathbf{q}}_{rs}$, yields

$$\dot{\mathbf{q}}_{rs} = \frac{K_{dr}}{b + K_{dr}}\Delta\dot{\mathbf{q}}_r + \frac{K_{dr} - b}{b + K_{dr}}\dot{\mathbf{q}}_{rs}(t - T_l - T_r) + \frac{2b}{b + K_{dr}}\dot{\mathbf{q}}_l(t - T_l). \tag{22}$$

Due to $\frac{K_{dr} - b}{b + K_{dr}} \leq 1$, this difference equation is stable with bounded input, hence, $\dot{\mathbf{q}}_{rs} \in \mathcal{L}_\infty$, and this fact makes $\boldsymbol{\tau}_{rs} \in \mathcal{L}_\infty$. $\dot{\mathbf{q}}_i, \mathbf{q}_l - \mathbf{q}_r \in \mathcal{L}_\infty$ imply that $\mathbf{e}_i \in \mathcal{L}_\infty$. Thus, $\boldsymbol{\tau}_r \in \mathcal{L}_\infty$. Similarly, it can be proved that $\boldsymbol{\tau}_l \in \mathcal{L}_\infty$. Hence, Lemma 1 ensures the asymptotic convergence of $\dot{\mathbf{q}}_i$ to zero. Differentiating (22), it can be shown that $\ddot{\mathbf{q}}_{rs} \in \mathcal{L}_\infty$, which also ensures asymptotic convergence of $\dot{\mathbf{q}}_{rs}$ to the origin.

Position coordination is established if we prove that $\ddot{\mathbf{q}}_i \to 0$. Lemma 1 establishes the conditions under which $\ddot{\mathbf{q}}_i \to 0$, and, in this case, the only need is that $\dot{\boldsymbol{\tau}}_i \in \mathcal{L}_\infty$, which indeed is true because $\dot{\boldsymbol{\tau}}_i$ only depends on bounded signals, i.e., $\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\tau}_i = \mathbf{g}(\ddot{\mathbf{q}}_i, \ddot{\mathbf{q}}_{rs}, \dot{\mathbf{q}}_i, \dot{\mathbf{q}}_{rs}, \mathbf{q}_l - \mathbf{q}_r)$. □

Remark 2. The novelty on the proof of Proposition 6 is that V is not a LF, i.e. $\dot{V}$ is not sign definite. However, its integral can be used to show boundedness and convergence of velocities and position errors. This method was first introduced by Chopra et al. (2006) and it is used in the proofs of most position tracking controllers, including the PD + d and P + d, in this tutorial.

4.5. Symmetric position tracking

As mentioned before, the fact that (22) has double delayed terms, such as $\dot{\mathbf{q}}_{rs}(t - T_l - T_r)$, induces wave reflections. The scheme of Namerikawa and Kawada (2006) aims to eliminate these effects with the use of a symmetric controller and by matching the impedances (see Remark 3).

Proposition 7 (*Namerikawa & Kawada, 2006*)**.** *Consider the teleoperator (1) controlled by (18) with*

$$\boldsymbol{\tau}_{ls} = K_{dl}\dot{\tilde{\mathbf{q}}}_l; \qquad \boldsymbol{\tau}_{rs} = -K_{dr}\dot{\tilde{\mathbf{q}}}_r \tag{23}$$

where $K_{di} > 0$. Additionally consider that the scattered velocities are codified using (13) and (9). Set the control gains s.t. (19) holds. Then, under assumptions A1–A3*, conclusions* (a) *and* (b) *of Proposition* 6 *hold.*

Proof. Take the same functional V as in the proof of Proposition 6. In this case, its time derivative is $\dot{V} = -K\dot{\mathbf{q}}_l^\top \int_{-T_r}^{0} \dot{\mathbf{q}}_r(t+\theta)\mathrm{d}\theta - K\dot{\mathbf{q}}_r^\top \int_{-T_l}^{0} \dot{\mathbf{q}}_l(t+\theta)\mathrm{d}\theta - B_l|\dot{\mathbf{q}}_l|^2 - B_r|\dot{\mathbf{q}}_r|^2 - K_{dl}|\ddot{\mathbf{q}}_l|^2 - K_{dr}|\ddot{\mathbf{q}}_r|^2$. The rest of the proof follows *verbatim* the proof of Proposition 6. □

Remark 3. Matching the impedances as $K_{dl} = K_{dr} = b$, and substituting the terms for the scattered velocities in (15) with (13) and (23), yields $\dot{\mathbf{q}}_{ls} = \frac{1}{2}\dot{\mathbf{e}}_l$ and $\dot{\mathbf{q}}_{rs} = \frac{1}{2}\dot{\mathbf{e}}_r$, it is clear that these expressions do not contain any double delayed term. Moreover, rewriting (18) with these scattered velocities, yields $\boldsymbol{\tau}_l = \frac{b}{2}\dot{\mathbf{e}}_l + K\mathbf{e}_l + B_l\dot{\mathbf{q}}_l$ and $\boldsymbol{\tau}_r = -\frac{b}{2}\dot{\mathbf{e}}_r - K\mathbf{e}_r - B_r\dot{\mathbf{q}}_r$, which coincide with the PD + d controllers for constant time-delays that are analyzed in Section 5.

Until now, the scattering transformation has been employed to render a constant time-delay communication passive. The following subsection deals with the variable time-delay case.

4.6. Classical scattering for variable time-delays

In general, the scattering transformation (13) is not passive in the presence of variable time-delays. In order to recover passivity, Lozano, Chopra, and Spong (2002) suggest the use of a time varying gain γ_i in the interconnection between the local and remote manipulators, as:

$$\mathbf{s}_r^+ = \gamma_l \mathbf{s}_l^+(t - T_l(t)); \qquad \mathbf{s}_l^- = \gamma_r \mathbf{s}_r^-(t - T_r(t)) \tag{24}$$

where $\gamma_i^2 \le 1 - \dot{T}_i(t)$. In this scenario, the energy storage function $E = \int_0^t (\dot{\mathbf{q}}_{ls}^\top \boldsymbol{\tau}_{ls} - \dot{\mathbf{q}}_{rs}^\top \boldsymbol{\tau}_{rs})\mathrm{d}\sigma$ becomes positive semi-definite, that is

$$\begin{aligned} E = {} & \frac{1}{2}\int_{t-T_l(t)}^{t} |\mathbf{s}_l^+|^2 \mathrm{d}\sigma + \frac{1}{2}\int_{t-T_r(t)}^{t} |\mathbf{s}_r^-|^2 \mathrm{d}\sigma \\ & + \frac{1 - \dot{T}_l(t) - \gamma_l^2}{2 - 2\dot{T}_l(t)} \int_0^{t-T_l(t)} |\mathbf{s}_l^+|^2 \mathrm{d}\sigma \\ & + \frac{1 - \dot{T}_r(t) - \gamma_r^2}{2 - 2\dot{T}_r(t)} \int_0^{t-T_r(t)} |\mathbf{s}_r^-|^2 \mathrm{d}\sigma . \end{aligned} \tag{25}$$

Therefore, if $\gamma_i^2 \le 1 - \dot{T}_i(t)$, the communications will not generate energy. Note that if $\gamma_i^2 = 1 - \dot{T}_i(t)$, (25) transforms to its counterpart for constant time-delays. Using (24), the functional $V_3 \ge 0$ for variable time-delays if Ψ is a positive function. The fact that decreasing the gains allows one to recover passivity is, of course, not surprising, and follows from the fact that the "zero operator" is passive and a simple continuity argument. On the other hand, although passivity is recovered, it is clear that decreasing the gains may cause deleterious effects on the performance.

Proposition 8. *Propositions 4 and 5 hold for variable time-delays, if the scattering transformation is transmitted using (24) with $\gamma_i^2 = 1 - \dot{T}_i(t)$.*

Remark 4. In order to compute $\dot{T}_i(t)$ at both ends, the value of a new function $f_i(t)$ can be sent through the communications together with the controllers data. Thus, when $f_i(t)$ arrives to its destination it has the value $f_i(t - T_i(t))$. Hence, we can estimate $\dot{T}_i(t)$, indirectly, from $\dot{f}_i(t - T_i(t)) = \dot{f}_i(t)[1 - \dot{T}_i(t)]$. Designing $f_i(t)$ s.t. $\dot{f}_i(t) = 1$, yields $\dot{T}_i(t) = 1 - \dot{f}_i(t - T_i(t))$. Hence, $\dot{T}_i(t)$ can be obtained without knowledge of $T_i(t)$ (Nuño, Basañez & Prada, 2009).

Remark 5. It is well known that when using a packet switched communications channel, like the Internet, besides variable time-delays, packet loss arises. In order to compensate for these losses, Berestesky, Chopra, and Spong (2004) and Chopra, Berestesky, and Spong (2008) propose a queue management strategy with the ability to expand and compress data queues; Hirche and Buss (2004) introduce a bounded rate approach in order to estimate the input energy and to bound the output energy in the communications, resulting on a passive data recovery algorithm and prove that the hold last sample algorithm is not passive.

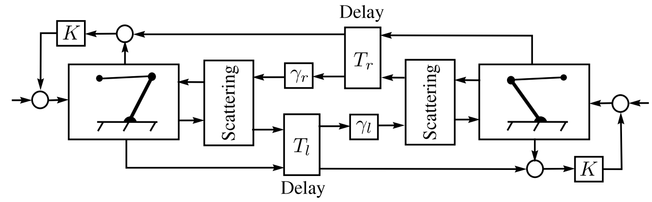

Fig. 4. Position tracking using the scattering transformation.

4.7. Position tracking for variable time-delays

In Nuño, Basañez, Ortega and Spong (2009) it is proved that it is possible to achieve position tracking for variable time-delays using the scattering transformation. The proposed controllers are given by

$$\boldsymbol{\tau}_l = \boldsymbol{\tau}_{ls} + K\boldsymbol{\varepsilon}_l + B_l\dot{\mathbf{q}}_l; \qquad \boldsymbol{\tau}_r = \boldsymbol{\tau}_{rs} - K\boldsymbol{\varepsilon}_r - B_r\dot{\mathbf{q}}_r \tag{26}$$

where

$$\boldsymbol{\varepsilon}_l = \mathbf{q}_l - \mathbf{q}_r(t - T_r(t)); \qquad \boldsymbol{\varepsilon}_r = \mathbf{q}_r - \mathbf{q}_l(t - T_l(t)), \tag{27}$$

and $\boldsymbol{\tau}_{is}$ defined as (23). The scattered velocities are codified using (13) and (24) with $\gamma_i^2 = 1 - \dot{T}_i(t)$ (see Fig. 4). It should be mentioned that a first approach to provide position tracking for variable time-delays has been reported in Chopra, Spong, Hirche, and Buss (2003). Boundedness of position error has been proved using a saturated position error term on the remote manipulator controller, that is $\boldsymbol{\tau}_r = \boldsymbol{\tau}_{rs} + K\,\mathbf{sat}(\boldsymbol{\varepsilon}_r) + B_r\dot{\mathbf{q}}_r$. However, in this case, asymptotic stability has not been guaranteed.

Proposition 9. *Under assumptions* A1–A4, *conclusions* (a) *and* (b) *of Proposition 6 hold if the controllers (18) are replaced by (26) and (23) with $K, K_{di}, B_i \in \mathbb{R}_{>0}$ satisfying $4B_lB_r > ({}^*T_l^2 + {}^*T_r^2)K^2$, using the scattering transformation for variable time-delays given by (13) and (24) with $\gamma_i^2 = 1 - \dot{T}_i(t)$.*

Proof. The proof is established using the same Lyapunov-like function V as in Proposition 6. In this case, $\dot{V} = -B_l|\dot{\mathbf{q}}_l|^2 - K_{dl}|\ddot{\mathbf{q}}_l|^2 - B_r|\dot{\mathbf{q}}_r|^2 - K_{dr}|\ddot{\mathbf{q}}_r|^2 - K\dot{\mathbf{q}}_l^\top \int_{-T_r(t)}^{0} \dot{\mathbf{q}}_r(t+\theta)\mathrm{d}\theta - K\dot{\mathbf{q}}_r^\top \int_{-T_l(t)}^{0} \dot{\mathbf{q}}_l(t+\theta)\mathrm{d}\theta$. Integrating from 0 to t, and applying Lemma 2 to the double integral terms, yields:

$$V(t) - V(0) \le -\lambda_l\|\dot{\mathbf{q}}_l\|^2 - K_{dl}\|\ddot{\mathbf{q}}_l\|_2^2 - \lambda_r\|\dot{\mathbf{q}}_r\|_2^2 - K_{dr}\|\ddot{\mathbf{q}}_r\|_2^2.$$

Setting B_i and K fulfilling $4B_lB_r > ({}^*T_l^2 + {}^*T_r^2)K^2$ it can be proved that this inequality is satisfied for some positive λ_i. Thus, $\dot{\mathbf{q}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\mathbf{q}_l - \mathbf{q}_r \in \mathcal{L}_\infty$. This completes the proof of part (a). Part (b) of the proof follows the same directions as in Proposition 6. □

Remark 6. Performance of the scattering-based controller of Proposition 9 is improved by using the impedance matching of Niemeyer and Slotine (2004). Choosing $K_{dl} = K_{dr} = b$, in (23), the scattered velocities become $2\dot{\mathbf{q}}_{ls} = \dot{\mathbf{q}}_l + \gamma_r\dot{\mathbf{q}}_r(t - T_r(t))$ and $2\dot{\mathbf{q}}_{rs} = \dot{\mathbf{q}}_r + \gamma_l\dot{\mathbf{q}}_l(t - T_l(t))$. Substituting them in the control laws (26) yields the same expressions for the PD + d controllers for variable time-delays that are analyzed in the following section.

5. Damping injection schemes

Damping injection is an essential component in passivity based control of manipulators that allows to obtain asymptotic stability

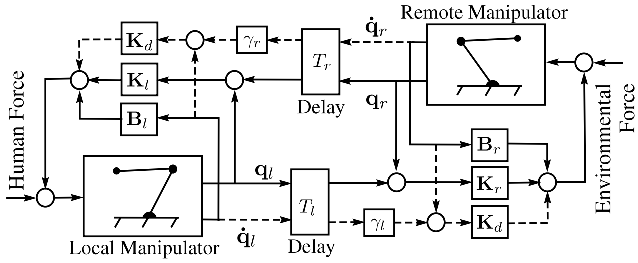

Fig. 5. General scheme for the P + d (dotted lines disabled) and PD + d (dotted lines enabled) controllers.

(Takegaki & Arimoto, 1981), playing a similar role in teleoperators. Along this line, some previous works have predicted that a P + d controller, often called position–position controller or symmetric position error controller, could stabilize the teleoperator with short time-delays if the controller gains are properly chosen Arcara and Melchiorri (2004), Hastrudi-Zaad and Salcudean (2001) and Lee and Lee (1993), and the use of a PD + d controller for teleoperators with constant time-delays has been proposed by Lee and Spong (2006). The first proof of stability of the simpler P + d controller has been reported in Nuño, Ortega, Barabanov et al. (2008), where, it is also shown that the same analysis applies to the PD + d controller. The physical interpretation of both, P + d and PD+d controllers is that the interconnection between the local and remote manipulators contains *virtual* springs and dampers. Such strategies provide position tracking for constant or variable time-delays.

Damping injection schemes share the following common Lyapunov–Krasovskiĭ functional

$$V_3 = \frac{K_l}{2}|\mathbf{q}_l - \mathbf{q}_r|^2 + \frac{K_d\beta_r}{2}\int_{t-T_l(t)}^{t}|\dot{\mathbf{q}}_l|^2 \mathrm{d}\theta + \frac{K_d}{2}\int_{t-T_r(t)}^{t}|\dot{\mathbf{q}}_r|^2 \mathrm{d}\theta. \tag{28}$$

For the PD + d controller, $K_l, K_d \in \mathbb{R}_{>0}$; for the P + d controller, $K_l \in \mathbb{R}_{>0}$ and $K_d = 0$; and for the passive output interconnection controller, $K_d \in \mathbb{R}_{>0}$ and $K_l = 0$.

5.1. PD + d *controller*

The PD + d controllers for variable time-delays are

$$\begin{aligned} \boldsymbol{\tau}_l &= K_d[\dot{\mathbf{q}}_l - \gamma_r\dot{\mathbf{q}}_r(t - T_r(t))] + K_l\boldsymbol{\epsilon}_l + B_l\dot{\mathbf{q}}_l \\ \boldsymbol{\tau}_r &= -K_d[\dot{\mathbf{q}}_r - \gamma_l\dot{\mathbf{q}}_l(t - T_l(t))] - K_r\boldsymbol{\epsilon}_r - B_r\dot{\mathbf{q}}_r, \end{aligned} \tag{29}$$

where $K_d, K_i, B_i \in \mathbb{R}_{>0}$ and γ_i are defined by $\gamma_i^2(t) = 1 - \dot{T}_i(t)$. The error signals $\boldsymbol{\epsilon}_i$ are defined in (27). Fig. 5 shows the schematics of the PD + d controller scheme. Notice that the gains γ_i only affect velocity signals.

Proposition 10 (*Nuño, Basañez et al., 2008; Nuño, Basañez, Ortega & Spong, 2009*). *Consider the teleoperator (1) controlled by (29). Then, under assumptions* A1–A4, *conclusions* (a) *and* (b) *of Proposition* 6 *hold, for variable time-delays $T_i(t)$ and any arbitrary positive K_d, if the gains B_i, K_i are set as $B_r \geq B_l$, $K_r \geq K_l$ and $4B_lB_r > ({}^*T_l^2 + {}^*T_r^2)K_lK_r$.*

Proof. Consider V defined by the sum of (3), (5) and (28), setting $\beta_l = 1$ and $\beta_r = \frac{K_l}{K_r}$. In this case, $V \geq 0$ and it is radially unbounded in $\dot{\mathbf{q}}_i$ and $\mathbf{q}_l - \mathbf{q}_r$. Its time derivative along (1) and (29), after applying Young's inequality in the crossed terms, yields $\dot{V} \leq -\left[B_l - \frac{K_dK_l}{2K_r} + \frac{K_d}{2}\right]|\dot{\mathbf{q}}_l|^2 - \left[B_r - \frac{K_dK_l}{2K_r} + \frac{K_d}{2}\right]|\dot{\mathbf{q}}_r|^2 - K_l\dot{\mathbf{q}}_l^\top\int_{-T_r(t)}^{0}\dot{\mathbf{q}}_r(t+\theta)\mathrm{d}\theta - K_l\dot{\mathbf{q}}_r^\top\int_{-T_r(t)}^{0}\dot{\mathbf{q}}_l(t+\theta)\mathrm{d}\theta$. Integrating from 0 to t and applying Lemma 2 to the double integral terms, with α_l and α_r, returns $V(t) - V(0) \leq -\lambda_l\|\dot{\mathbf{q}}_l\|_2^2 - \lambda_r\|\dot{\mathbf{q}}_r\|_2^2$ where $\lambda_l = B_l - \frac{K_d}{2}\left(\frac{K_l}{K_r} - 1\right) - \frac{K_l}{2}\left(\alpha_l + \frac{{}^*T_l^2}{\alpha_r}\right)$ and $\lambda_r = \frac{K_lB_r}{K_r} - \frac{K_d}{2}\left(\frac{K_l}{K_r} - 1\right) - \frac{K_l}{2}\left(\alpha_r + \frac{{}^*T_r^2}{\alpha_l}\right)$. It is clear that if $\lambda_i > 0$ then $\dot{\mathbf{q}}_i \in \mathcal{L}_2$. Solving for $\alpha_i > 0$, there exists a solution – if for any K_d, $K_r \geq K_l$ and $B_r \geq B_l$ – it holds that $4B_lB_r > ({}^*T_l^2 + {}^*T_r^2)K_lK_r$. Fulfilling this last inequality ensures that $\dot{\mathbf{q}}_i \in \mathcal{L}_2$ and that $V \in \mathcal{L}_\infty$, thus $\dot{\mathbf{q}}_i, \mathbf{q}_l - \mathbf{q}_r \in \mathcal{L}_\infty$. This completes the proof of part (*a*).

Up to this point, it can be also concluded that $\boldsymbol{\epsilon}_i \in \mathcal{L}_\infty$. Assumption A4 is sufficient to show that $0 < \gamma_i^2 = 1 - \dot{T}_i(t) < 2$, hence, $\gamma_i \in \mathcal{L}_\infty$. Thus $\boldsymbol{\tau}_i \in \mathcal{L}_\infty$, implying that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$. Now, $\dot{\boldsymbol{\tau}}_i \in \mathcal{L}_\infty$ due to the boundedness of $\dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i$, and A4, that allows one to show that $0 < \gamma_i < \sqrt{2}$, ensuring that, $\dot{\gamma}_i(t) = \frac{-\ddot{T}_i(t)}{[1-\dot{T}_i(t)]^{\frac{1}{2}}}$ is bounded. Under these conditions, Lemma 1 proves that $|\dot{\mathbf{q}}_i| \to 0$, $|\dot{\mathbf{q}}_i| \to 0$, and it is obvious that when $|\ddot{\mathbf{q}}_i| \to 0$, $|\boldsymbol{\epsilon}_i| \to 0$. This completes the proof. □

Corollary 11 (*Nuño, Ortega, Barabanov et al., 2008*). *For constant time-delays, ${}^*T_l = T_l$ and ${}^*T_r = T_r$, controllers (29) simplify to $\boldsymbol{\tau}_l = K_d\dot{\mathbf{e}}_l + K_l\mathbf{e}_l + B_l\dot{\mathbf{q}}_l$ and $\boldsymbol{\tau}_r = -K_d\dot{\mathbf{e}}_r - K_r\mathbf{e}_r - B_r\dot{\mathbf{q}}_r$. In this case, conclusions* (a) *and* (b) *of Proposition* 6 *hold for any arbitrary positive K_d and K_i, B_i satisfying $4B_lB_r > ({}^*T_l^2 + {}^*T_r^2)K_lK_r$.*

Remark 7. Even though PD + d controllers ensure asymptotic convergence of velocities and position errors, they are sensitive to abrupt changes in time-delays. This is due to the inclusion of the variable gains γ_i, that depend on the rate of change of the delays. To improve performance, the following P + d controller does not make use of this variable gain.

5.2. P + d *scheme*

In this scheme, the torques applied by the controllers on both the local and the remote manipulators, are proportional to their position error plus a damping injection term. The control laws are given by

$$\boldsymbol{\tau}_l = K_l\boldsymbol{\epsilon}_l + B_l\dot{\mathbf{q}}_l; \qquad \boldsymbol{\tau}_r = -K_r\boldsymbol{\epsilon}_r - B_r\dot{\mathbf{q}}_r, \tag{30}$$

where $K_i, B_i \in \mathbb{R}_{>0}$. $\boldsymbol{\epsilon}_i$ represent the position errors for variable time-delays defined in (27). Fig. 5, when the dotted lines are disabled, shows the interconnection of the teleoperator using this approach.

Proposition 12. *Consider the teleoperator (1) controlled by (30). Set the control gains such that $4B_lB_r > ({}^*T_l^2 + {}^*T_r^2)K_lK_r$ holds. Then, under assumptions* A1–A3, *conclusions* (a) *and* (b) *of Proposition* 6 *hold.*

Proof. Using the same procedure as in the proof of Proposition 10, we get $V(t) - V(0) \leq -\lambda_l\|\dot{\mathbf{q}}_l\|_2^2 - \lambda_r\|\dot{\mathbf{q}}_r\|_2^2$, for $\lambda_l = B_l - \frac{K_l}{2}\left(\alpha_l + \frac{{}^*T_l^2}{\alpha_r}\right)$, $\lambda_r = B_r - \frac{K_r}{2}\left(\alpha_r + \frac{{}^*T_r^2}{\alpha_l}\right)$. Solving simultaneously for $\alpha_i > 0$ and $\lambda_i > 0$ yields a solution if $4B_lB_r > ({}^*T_l^2 + {}^*T_r^2)K_lK_r$. The rest of the proof follows *verbatim* the proof of Proposition 10. For this control scheme, the boundedness of $\boldsymbol{\tau}_i$ and $\dot{\boldsymbol{\tau}}_i$ is ensured, only, by $\dot{\mathbf{q}}_i, \mathbf{q}_l - \mathbf{q}_r \in \mathcal{L}_\infty$. □

Corollary 13 (*Nuño, Ortega, Barabanov et al., 2008; Nuño et al., 2008*). *Proposition 12 holds for constant time-delays $T_i = {}^*T_i$.*

5.3. *Passive output interconnection*

A special case of the PD + d controllers, with only the D action, are the passive output interconnection schemes, which simply interconnect the delayed passive outputs $\dot{\mathbf{q}}_i$ of the local and remote manipulators (Fig. 6) (Chopra & Spong, 2007a,b,c; Nuño, Basañez, Ortega & Spong, 2009; Wang & Slotine, 2006). The main motivation

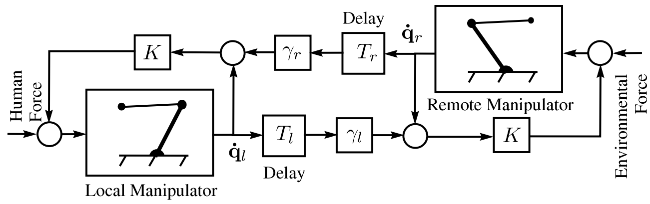

Fig. 6. Passive output synchronization scheme.

for using these schemes is their delay-independent stability property. The basic interconnection controllers for variable time-delays are

$$\boldsymbol{\tau}_l = K_d[\dot{\mathbf{q}}_l - \gamma_r\dot{\mathbf{q}}_r(t - T_r(t))]$$
$$\boldsymbol{\tau}_r = -K_d[\dot{\mathbf{q}}_r - \gamma_l\dot{\mathbf{q}}_l(t - T_l(t))], \tag{31}$$

where $\gamma_i^2 = 1 - \dot{T}_i(t)$ and $K_d \in \mathbb{R}_{>0}$, and for constant time-delays, simply by $\boldsymbol{\tau}_l = K_d\dot{\mathbf{e}}_l$ and $\boldsymbol{\tau}_r = -K_d\dot{\mathbf{e}}_r$.

Proposition 14. *Consider the teleoperator in (1) controlled by (31). Under assumptions A1–A4, the teleoperator's velocities asymptotically synchronize, in the sense of $|\dot{\mathbf{q}}_l - \gamma_r\dot{\mathbf{q}}_r(t - T_r(t))|^2 \to 0$ and $|\dot{\mathbf{q}}_r - \gamma_l\dot{\mathbf{q}}_l(t - T_l(t))|^2 \to 0$.*

Proof. In this case, the proof is established using the functional (28) with $K_l = 0$ and constants $\beta_i = 1$. $\dot{V}$ along the system trajectories is given by $\dot{V} = -K_d|\dot{\mathbf{q}}_l - \gamma_r\dot{\mathbf{q}}_r(t - T_r(t))|^2 - K_d|\dot{\mathbf{q}}_r - \gamma_l\dot{\mathbf{q}}_l(t - T_l(t))|^2$, that is negative semi-definite. The fact that $V \geq 0$ and $\dot{V} \leq 0$ ensures that $V(t) \in \mathcal{L}_\infty$, thus $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty$. This allows one to prove that, due to A4, $\boldsymbol{\tau}_i \in \mathcal{L}_\infty$. From the teleoperator dynamics (1) it can be concluded that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$, this fact can be used to prove that $\ddot{V} \in \mathcal{L}_\infty$, thus $\dot{V}$ is uniformly continuous. Using Barbălat's lemma it can be shown that $\dot{V} \to 0$ as $t \to \infty$. □

Corollary 15. *Proposition 14 holds for any constant time-delays T_i. Thus, $|\dot{\mathbf{e}}_i| \to 0$.*

Remark 8. Proposition 14 only proves asymptotic convergence to zero of velocity errors. However, if damping injection is added, resulting on a D + d controller, it yields asymptotic convergence of velocities to zero.

Remark 9. These schemes may be also used to provide position tracking by defining a new interconnection variable, namely $\mathbf{r}_i = \dot{\mathbf{q}}_i + \boldsymbol{\lambda}\mathbf{q}_i$, where $\boldsymbol{\lambda} = \boldsymbol{\lambda}^\top > 0$. Chopra and Spong (2007a) proposed the model-based controllers

$$\boldsymbol{\tau}_l = \mathbf{K}[\mathbf{r}_l - \mathbf{r}_r(t - T_r)] + \mathbf{M}_l(\mathbf{q}_l)\boldsymbol{\lambda}\dot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\boldsymbol{\lambda}\mathbf{q}_l \tag{32}$$
$$\boldsymbol{\tau}_r = \mathbf{K}[\mathbf{r}_l(t - T_l) - \mathbf{r}_r] - \mathbf{M}_r(\mathbf{q}_r)\boldsymbol{\lambda}\dot{\mathbf{q}}_r - \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\boldsymbol{\lambda}\mathbf{q}_r.$$

Under assumption A2 and passive human operator and environment maps, w.r.t. $\mathbf{r}_i$, these controllers are capable of achieving position tracking independently of the magnitude of T_i for any $\mathbf{K} = \mathbf{K}^\top > 0$.

6. Adaptive schemes

Similarly to standard adaptive manipulator controllers (Ortega & Spong, 1989), adaptive controllers for nonlinear teleoperators exploit two well-known facts: the linearity if the control law in the parameters $\boldsymbol{\theta}_i$, which stems from Property P4, and the passivity of the operator defined by the parameter update law $\dot{\hat{\boldsymbol{\theta}}}_i$ (where $\hat{\boldsymbol{\theta}}_i \in \mathbb{R}^p$ stands for the estimated parameters). Replacing the true parameters in the control law by their estimate and pulling out the parameter error term $\tilde{\boldsymbol{\theta}}_i := \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$, yields a passive map, which interconnected with the estimator, preserves the required passivity properties. Some relevant references along these lines are Chopra, Spong, and Lozano (2004), Hung, Narikiyo, and Tuan (2003), Love and Book (2004), Nuño et al. (2010a), Polushin, Tayebi, and Marquez (2006), Rodriguez-Angeles and Nijmeijer (2004) and Zhu and Salcudean (2000).

6.1. Lyapunov-like function for adaptive control

For the adaptive schemes, the general Lyapunov-like function (2) has to be redesigned, but the analysis follows similar lines, and, in this special scenario, assumptions A1–A4 are not needed. V_1 is inspired by the kinetic energy of the local and remote manipulators, and is given by $V_1 = \frac{1}{2}\boldsymbol{\upsilon}_l^\top\mathbf{M}_l(\mathbf{q}_l)\boldsymbol{\upsilon}_l + \frac{1}{2}\boldsymbol{\upsilon}_r^\top\mathbf{M}_r(\mathbf{q}_r)\boldsymbol{\upsilon}_r$, where $\boldsymbol{\upsilon}_i \in \mathbb{R}^n$ are defined later and can be seen as new reference velocities—similar to the Slotine–Li variable $\mathbf{s}$ (Slotine & Li, 1988). Hence, for any $\beta_i \in \mathbb{R}_{>0}$ and by Property P1, V_1 is positive definite and radially unbounded in $\boldsymbol{\upsilon}_i$. The estimation laws can take the standard form

$$\dot{\hat{\boldsymbol{\theta}}}_i = \boldsymbol{\Gamma}_i\mathbf{Y}_i^\top\boldsymbol{\upsilon}_i, \tag{33}$$

where $\boldsymbol{\Gamma}_i$ are constant symmetric positive definite matrices. It is well-known that (33) defines a passive operator $\boldsymbol{\upsilon}_i \to \mathbf{Y}_i\tilde{\boldsymbol{\theta}}_i$ with storage function $\tilde{\boldsymbol{\theta}}_i^\top\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\theta}}_i$. In this case, instead of using V_2 to model the energetic behavior of the human and the environment, V_2 is employed as an storage function of the parameter estimation. Thus we select $V_2(\tilde{\boldsymbol{\theta}}_i) = \tilde{\boldsymbol{\theta}}_l^\top\boldsymbol{\Gamma}_l^{-1}\tilde{\boldsymbol{\theta}}_l + \tilde{\boldsymbol{\theta}}_r^\top\boldsymbol{\Gamma}_r^{-1}\tilde{\boldsymbol{\theta}}_r$, which yields $\dot{V}_2 = -\tilde{\boldsymbol{\theta}}_l^\top\mathbf{Y}_l^\top\boldsymbol{\upsilon}_l - \tilde{\boldsymbol{\theta}}_r^\top\mathbf{Y}_r^\top\boldsymbol{\upsilon}_r$. As in the previous sections, the functional V_3 describes the coupling between the local and remote manipulators and it is designed for each adaptive controller.

6.2. Asymptotic regulation

In the adaptive control scheme of Nuño et al. (2010a), the coordination errors $\mathbf{e}_i$, $\dot{\mathbf{e}}_i$, asymptotically synchronize, independent of the constant time-delay T and without using the scattering transformation. For simplicity, let us assume that $T_l = T_r = T$, and define $\boldsymbol{\upsilon}_i$ as

$$\boldsymbol{\upsilon}_i = \dot{\mathbf{q}}_i + \boldsymbol{\lambda}\mathbf{e}_i, \tag{34}$$

where $\mathbf{e}_i$ has been previously defined in (17) and $\boldsymbol{\lambda} > 0$ is a diagonal matrix. The controllers, in Fig. 7, are given by

$$\boldsymbol{\tau}_l^* = \mathbf{Y}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l, \mathbf{e}_l, \dot{\mathbf{e}}_l)\hat{\boldsymbol{\theta}}_l + \bar{\boldsymbol{\tau}}_l \tag{35}$$
$$\boldsymbol{\tau}_r^* = -\mathbf{Y}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r, \mathbf{e}_r, \dot{\mathbf{e}}_r)\hat{\boldsymbol{\theta}}_r - \bar{\boldsymbol{\tau}}_r$$

where $\mathbf{Y}_i\hat{\boldsymbol{\theta}}_i = \hat{\mathbf{M}}_i(\mathbf{q}_i)\boldsymbol{\lambda}\dot{\mathbf{e}}_i + \hat{\mathbf{C}}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\boldsymbol{\lambda}\mathbf{e}_i - \hat{\mathbf{g}}_i(\mathbf{q}_i)$. Substituting the controllers (35) on the teleoperator dynamics (1) and using (34), yields

$$\mathbf{M}_l(\mathbf{q}_l)\dot{\boldsymbol{\upsilon}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\boldsymbol{\upsilon}_l = \mathbf{Y}_l\tilde{\boldsymbol{\theta}}_l - \bar{\boldsymbol{\tau}}_l + \boldsymbol{\tau}_h \tag{36}$$
$$\mathbf{M}_r(\mathbf{q}_r)\dot{\boldsymbol{\upsilon}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\boldsymbol{\upsilon}_r = \mathbf{Y}_r\tilde{\boldsymbol{\theta}}_r - \bar{\boldsymbol{\tau}}_r - \boldsymbol{\tau}_e.$$

Finally, $\bar{\boldsymbol{\tau}}_i = \mathbf{K}_i\boldsymbol{\upsilon}_i + \mathbf{B}\dot{\mathbf{e}}_i$, where $\mathbf{K}_i = \mathbf{K}_i^\top > 0$ and $\mathbf{B}$ is diagonal positive definite.

The original proof of the following proposition, reported in Nuño et al. (2010a), employs incorrect convergence arguments that have been suitable corrected in the erratum (Nuño, Ortega, & Basañez, 2011).

Proposition 16. *Consider the bilateral teleoperator (1) in free motion ($\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$) controlled by (35) and using the parameter update law (33) together with (34). Then, for any constant time-delay T, all signals in the system are bounded. Moreover, position errors and velocities asymptotically converge to zero.*

Proof. Use $V_3 = \frac{1}{2}\sum_{i\in\{l,r\}}\left[\mathbf{e}_i^\top\boldsymbol{\lambda}\mathbf{B}\mathbf{e}_i + \int_{t-T}^{t}\dot{\mathbf{q}}_i^\top\mathbf{B}\dot{\mathbf{q}}_i d\sigma\right]$. In this case $V \geq 0$ and it is radially unbounded in $\boldsymbol{\upsilon}_i, \tilde{\boldsymbol{\theta}}_i, \mathbf{e}_i$. $\dot{V}$ evaluated along (33), (34) and (36), yields $\dot{V} = -\sum_{i\in\{l,r\}}\left[\boldsymbol{\upsilon}_i^\top\mathbf{K}_i\boldsymbol{\upsilon}_i + \frac{1}{2}\dot{\mathbf{e}}_i^\top\mathbf{B}\dot{\mathbf{e}}_i\right]$. Due to $V \geq 0$ and $\dot{V} \leq 0$, $\boldsymbol{\upsilon}_i, \dot{\mathbf{e}}_i \in \mathcal{L}_2$ and $\boldsymbol{\upsilon}_i, \tilde{\boldsymbol{\theta}}_i, \mathbf{e}_i \in \mathcal{L}_\infty$. From (34)

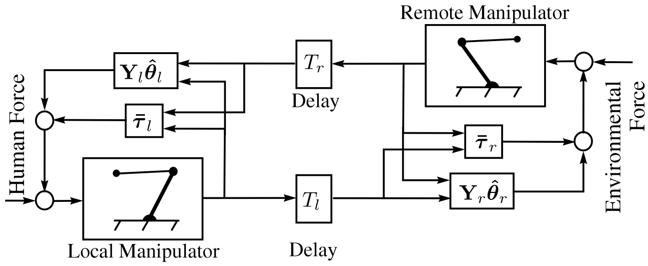

Fig. 7. A general adaptive controller.

it can be shown that $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty$, implying that $\dot{\mathbf{e}}_i \in \mathcal{L}_\infty$. All these guarantee that $\mathbf{Y}_i \in \mathcal{L}_\infty$. Now, from (36), using P1 and P2 together with the boundedness of $\bar{\boldsymbol{\tau}}_i$, $\mathbf{Y}_i$, $\tilde{\boldsymbol{\theta}}_i$, $\boldsymbol{\upsilon}_i$, $\dot{\mathbf{q}}_i$, it can be concluded that $\dot{\boldsymbol{\upsilon}}_i \in \mathcal{L}_\infty$. Hence, $\boldsymbol{\upsilon}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$, $\dot{\boldsymbol{\upsilon}}_i \in \mathcal{L}_\infty$ support that $|\boldsymbol{\upsilon}_i| \to 0$.

In order to prove that $|\dot{\mathbf{q}}_i| \to 0$, let us rewrite (34) in matrix form as

$$\dot{\mathbf{q}} = -(\mathbf{I}_2 \otimes \boldsymbol{\lambda})\mathbf{q} + (\mathbf{A}_0 \otimes \boldsymbol{\lambda})\mathbf{q}(t-T) + \dot{\boldsymbol{\upsilon}} \tag{37}$$

where $\boldsymbol{\upsilon} = [\boldsymbol{\upsilon}_i^\top, \boldsymbol{\upsilon}_r^\top]^\top$, $\mathbf{q} = [\mathbf{q}_l^\top, \mathbf{q}_r^\top]^\top$, $\mathbf{A}_0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\mathbf{I}_2$ is the identity matrix of size two and $\otimes$ is the standard Kronecker product. Using the change of variables, $\mathbf{z} = (\mathbf{D} \otimes \mathbf{I}_n)\mathbf{q}$, where $\mathbf{D} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ transforms (37) into two decoupled systems for which it can be proved that $|\dot{\mathbf{z}}| \to 0$, which in turn implies that $|\dot{\mathbf{q}}_i| \to 0$. Finally, convergence of $\boldsymbol{\upsilon}$ and $\dot{\mathbf{q}}$ to zero implies, from (34), convergence of $\mathbf{e}_i$ to zero. This completes the proof. □

Remark 10. It can be shown that, for any constant time-delay T, controllers (35) render the teleoperator (1) Input-to-State Stable (ISS) with states $\boldsymbol{\upsilon}_i$, $\mathbf{e}_i$ and inputs $\boldsymbol{\tau}_h$, $\boldsymbol{\tau}_e$. This conclusion can be easily proved using V_1 and V_2 together with $V_3 = \alpha|\mathbf{e}_l|^2 + \alpha|\mathbf{e}_r|^2$ for $\alpha \in \mathbb{R}_{>0}$.

Remark 11. When velocity is not measurable, Polushin et al. (2006) propose an adaptive approach that does not rely on velocity measurements. The controllers are similar to the adaptive version of (32), and their estimation law contains an extra term with the error of the estimate and a nominal value of the constant parameters. In their work they show that the teleoperator has finite ISS gains independent of constant time-delays (Polushin & Marquez, 2003). Their proofs are established using $V_3 = \frac{1}{2}\left(|\mathbf{q}_l|^2 + |\mathbf{e}_r|^2\right)$. Based on a modified version of the Input-to-Output Stability small gain theorem for interconnected nonlinear systems with delays, in Polushin, Marquez, Tayebi, and Liu (2009) and Polushin, Liu, and Lung (2006, 2007) propose a force-reflection algorithm that also renders the teleoperator ISS when the dynamical parameters of the teleoperator are unknown.

Remark 12. Instead of using the position errors, in $\boldsymbol{\upsilon}_i$, Chopra, Spong et al. (2008) designed $\boldsymbol{\upsilon}_i$ equal to $\mathbf{r}_i$, in Remark 9. The controllers are the adaptive version of (32) and employ the update law (33). Using assumption A2, this scheme ensures that $|\dot{\mathbf{q}}_i| \to |\mathbf{e}_i| \to 0$.

Remark 13. The use of the scattering transformation together with adaptive control has been exploited by Chopra, Spong et al. (2004). They propose to encode the variable $\boldsymbol{\upsilon}_i$ instead of velocities, as

$$\begin{bmatrix} \mathbf{s}_i^+ \\ \mathbf{s}_i^- \end{bmatrix} = \frac{1}{\sqrt{2b}}\begin{bmatrix} \mathbf{I} & b\mathbf{I} \\ \mathbf{I} & -b\mathbf{I} \end{bmatrix}\begin{bmatrix} \bar{\boldsymbol{\tau}}_i \\ \boldsymbol{\upsilon}_{is} \end{bmatrix}, \tag{38}$$

where $\bar{\boldsymbol{\tau}}_i = K(\boldsymbol{\upsilon}_{is} - \boldsymbol{\upsilon}_i)$, and for constant time-delays T_i, the local and remote manipulators are interconnected using (9). Under the assumption that the human and the environment are passive from input $\boldsymbol{\upsilon}$ to output $\boldsymbol{\tau}$, it is proved that the position errors (17) asymptotically converge to zero. In this case, $V_3 = \frac{K}{2}\mathbf{e}_l^\top\boldsymbol{\lambda}\mathbf{e}_l + \frac{K}{2}\mathbf{e}_r^\top\boldsymbol{\lambda}\mathbf{e}_r + \int_0^t \boldsymbol{\upsilon}_r^\top\boldsymbol{\tau}_e - \boldsymbol{\upsilon}_l^\top\boldsymbol{\tau}_h d\sigma + E(t)$ where $E(t)$ is the energy in the transmission line and it is given by $E = \int_0^t (\boldsymbol{\upsilon}_{ls}^\top\bar{\boldsymbol{\tau}}_l - \boldsymbol{\upsilon}_{rs}^\top\bar{\boldsymbol{\tau}}_r)d\sigma \geq 0$.

Table 1
Main capabilities and characteristics of the control schemes analyzed in this tutorial.

Scheme	Time-delays		Pos. Track.	Scatt. based
	Cons.	Var.		
PD + d	✓	✓	✓	
P + d	✓	✓	✓	
Passive Output Synch.	✓	✓		
Classical scattering	✓			✓
Symm. Imp. Matching	✓			✓
Pos. Track. Controller	✓		✓	✓
Symm. Pos. Tracking	✓		✓	✓
Classical Scatt. Var. T.-D.	✓	✓		✓
Pos. Track. Var. T.-D.	✓	✓	✓	✓
Asymp. regulation	✓		✓	
Scatt. State Synch.	✓	✓	✓	✓

Remark 14. The variable time-delays problem can be tackled using the transformation (38) with the interconnection (24), yielding the same conclusions as in the previous remark.

7. Comparative analysis

Table 1 depicts some of the features and characteristics of the control schemes analyzed in this tutorial, which can be summarized as follows.

- Stability of most of the scattering-based schemes is ensured because they are the result of interconnecting passive systems, which in its turn guarantees some nice robustness properties. Another important feature is that they are delay independent. Nevertheless, such schemes do not provide position tracking and, in order to achieve better tracking performance, the delay independence needs to be sacrificed.
- PD + d and P + d controllers are robust to variable time-delays and provide position tracking, however, their stability condition is not delay independent. Furthermore, since the stabilization mechanism relies on damping injection, sluggish responses should be expected.
- Passive output synchronization schemes ensure asymptotic stability of the interconnecting outputs, namely, velocities. They are robust to variable time-delays, without the need of knowing the value of such delays. However, position tracking is not guaranteed.
- Adaptive controllers can provide position tracking independent of the constant time-delay, however, only scattering-based schemes can guarantee stability in the presence of variable time-delays.

8. Conclusions and future directions

The stability problem that arises in teleoperators is to ensure velocity and position tracking as well as force reflection, despite constant or variable time-delays. Several approaches have been suggested that reach those goals, like the classical scattering approach, simple PD + d and P + d controllers, output synchronization techniques and adaptive control schemes. The objective of this tutorial is twofold, (i) to analyze and to prove, in a novel manner, the stability of different control schemes and, (ii) to show that these schemes share similar Lyapunov-like functions. This tutorial presents a *general* Lyapunov-like function V that can be used to analyze the stability of the aforementioned control schemes. The present work can be seen as a first step towards the unification of the stability analysis for teleoperators with time-delays.

The function V has a clear energy interpretation, it contains the energy terms associated to each component of the teleoperator, namely, the local and remote manipulators, the energy of the human and environment interactions, the interconnection of the local and remote sites and, for the adaptive cases, the classical quadratic in parametric errors terms. Thus, by replacing the interconnection term of V by a specific expression, the stability of such schemes can be analyzed. A key contribution of Chopra, Spong and Ortega et al. (2004) and Chopra et al. (2006), which is instrumental for the analysis of the scattering-based position tracking, the PD+d and the P+d controllers, is that, even if the sign of $\dot{V}$ is undefined, stability conclusions can still be drawn looking at its integral.

It should be underscored that in this tutorial it has been assumed that the teleoperator and its controllers are described by continuous time dynamics, while in computer-based control and in packet switched communications, as the Internet, such assumption does not always hold. Thus, a natural extension of this work is to consider sampling times in the analysis. Future research will also include the study of rate convergence and transient performance of the presented schemes.

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