CÁLCULO I

1/10/21

① $a \leq b \iff -a \leq -b$

$\Longrightarrow\!\!|$ $a \leq b$ $\boxed{\text{Falso}}$

$a \leq b \iff -a \leq -b$ si $\boxed{a = b}$

Sería cierta esta proposición.

$a \leq b \iff -a \geq -b$

$\Longrightarrow\!\!|$

$a \leq b \Longrightarrow |a| \leq |b| \iff a, b \geq 0$

$a \leq b \Longrightarrow |a| \geq |b| \iff a, b \leq 0$

Si $a, b \geq 0 \Longrightarrow -a, -b \leq 0$

$|a| \leq |b| \equiv a \leq b$

$-|a| \geq -|b| \equiv -a \geq -b$

$$a \leq b \begin{cases} |a| \leq |b| & (\cancel{\text{si } a, b \geq 0}) \text{ Si } a \geq 0 \\ |a| \geq |b| & \text{si } b \leq 0 \end{cases}$$

$\underline{a \leq b}$ $a + (-a) = 0 \Longrightarrow$ Si $a \leq b \Longrightarrow -b \leq -a$ para que se cumpla $b + (-b) = 0$

② $\left.\begin{matrix} a \leq b \\ c \leq d \end{matrix}\right\} \Rightarrow a + c \leq b + d$

Suponiendo:

$a + c \leq b + d \Longrightarrow (-a) + c + a \leq (-a) + b + d$

$c \leq \underbrace{(-a + b)}_{\substack{b \geq a \\ -a+b \geq 0}} + d$

$c \leq d + \underbrace{(-a + b)}_{\geq 0}$ $\boxed{\text{Cierto}}$

$(c \leq d)$

~~③~~ ②' $\left.\begin{matrix} a < b \\ c \leq d \end{matrix}\right\} \Longrightarrow a + c \leq b + d$

Suponiendo: $a + c < b + d$

$(-a) + a + c < (-a) + b + d \Longrightarrow c < \underbrace{(-a + b)}_{>0} + d$

$(-c) + c < (-c) + (-a + b) + d$

$0 < \underbrace{(-a + b)}_{>0} + \underbrace{(-c + d)}_{\substack{c \leq d \\ -c+d \geq 0}}$ $\boxed{\text{Cierto}}$

① $a \leq b \Longleftrightarrow -b \leq a$ $\boxed{\text{Falsa}}$ $a = -500$, $b = 5$

$\Longrightarrow\!\!|$ $a \leq b$

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$$\left.\begin{array}{l} a \leq b \\ c \geq 0 \end{array}\right\} \Longrightarrow ac \leq bc$$

$$\cancel{a \leq b \Longrightarrow}$$

<u>Suponiendo</u>: $ac \leq bc$

$$ac \cdot \left(\frac{1}{c}\right) \leq bc\left(\frac{1}{c}\right) \Longrightarrow a\left(\cancel{c \cdot \frac{1}{c}}\right) \leq b\left(\cancel{c \frac{1}{c}}\right)$$

$$a \cdot 1 \leq b \cdot 1$$

$$\boxed{a \leq b}$$

3'

$$\left.\begin{array}{l} a < b \\ c > 0 \end{array}\right\} \Longrightarrow ac < bc$$

$$\text{Si } c = 0 \Longrightarrow a \cdot 0 \leq b \cdot 0$$

$$\boxed{0 \leq 0} \checkmark$$

4 $a \in \mathbb{R}^+ \Longrightarrow a \cdot a > 0$

$$a \cdot a = a_1 + a_2 \ldots + a_a$$

$$\text{Si } a > 0 \Longrightarrow a + a > 0 \Longrightarrow a_1 + a_2 + \ldots + a_a > 0$$

$$\boxed{a \cdot a > 0}$$

~~(5) $0 < 1 < 1 + 1 < 1 + 1 + 1 < 1 + 1 + 1 + 1 \ldots$)~~

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4 $a \in \mathbb{R}^* \Longrightarrow a \cdot a > 0$

(S) $a \cdot a > 0 \Longrightarrow a \cdot a \cdot \left(\frac{1}{a}\right) > 0 \cdot \frac{1}{a}$

$$a > 0$$

$$\frac{1}{a} \cdot a > 0 \cdot \frac{1}{a}$$

$$\boxed{1 > 0}$$

$$\text{si } a < 0 \Longrightarrow (-a)(-a) > 0$$

$$-(-a)a > 0$$

$$a \cdot a > 0$$

⑤ $0 < 1 < 1+1 < 1+1+1 \dots$

$\forall n \in \mathbb{R} \ \exists (n+1) \Longrightarrow n < n+1$

$\forall n \in \mathbb{R} ; \ n < (n+1)$

⑥ $a \in \mathbb{R}^* \Longleftrightarrow \frac{1}{a} \in \mathbb{R}^*$

$a \cdot \frac{1}{a} = 1 \Longrightarrow \mathbb{R} \times \mathbb{R} \longrightarrow \mathbb{R}$

$a \neq 0 \Longrightarrow \nexists \, 1/a$

⑦ $0 < a < b \Longleftrightarrow 0 < \frac{1}{b} < \frac{1}{a}$

$a \cdot 1/a = \cancel{0}\,1 = b \cdot 1/b \Longrightarrow a \cdot 1/a = b \cdot 1/b$

si $a < b$

$1/a > 1/b$ para que se cumpla la igualdad

$a \cdot 1/a = b \cdot 1/b$

$a/b = \frac{1/b}{1/a}$

$a/b < 1 \Longrightarrow \frac{1/b}{1/a} < 1 \Longrightarrow \boxed{1/b < 1/a}$

⑧ $a, b, c, d \in \mathbb{R}^+$

$\left.\begin{matrix} a < b \\ c < d \end{matrix}\right\} ac < bd$

Suponiendo $ac < bd$

$ac \cdot 1/b < bd \cdot 1/b$

$\frac{a}{b} \cdot c < d \Longrightarrow \frac{a}{b} \cdot c \cdot \frac{1}{d} < d \cdot 1/d$

$\left.\begin{matrix} a < b \\ c < d \end{matrix}\right\} \Longleftarrow \underbrace{\frac{a}{b}}_{<1} \cdot \underbrace{\frac{c}{d}}_{<1} < 1$ $\boxed{\text{Cierto}}$