Relación II

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18.- $V = \mathbb{R}^4$ $U = \mathcal{L}(\{(1,-2,1,0),(2,3,-2,1),(4,-1,0,1)\})$

$$\begin{pmatrix} 1 & 2 & 4 \\ -2 & 3 & -1 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{pmatrix} \quad \dim_{\mathbb{R}}(U) = 2 \quad \begin{pmatrix} 1 & 2 & 0 & 0 \\ -2 & 3 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix} \quad W = \mathcal{L}(\{(0,0,1,0),(0,0,0,1)\})$$

$U \oplus W = \mathbb{R}^4$

$$\begin{vmatrix} 1 & 2 & 4 \\ -2 & 3 & -1 \\ 1 & -2 & 0 \end{vmatrix} = 16 - 2 - 12 - 2 = 0$$

$$\begin{vmatrix} 1 & 2 & 4 \\ -2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 3 - 8 + 1 + 4 = 0$$

b) $U = \{(x,y,z,t) \in \mathbb{R}^4 / x - 2y + z = 0,\ x + y + z + t = 0\}$

$z = -x + 2y$ $\qquad \mathrm{Dim}_{\mathbb{R}}(U) = 2$

$z + t = -x - y$

$t = -\cancel{x} - y + \cancel{x} - 2y = -3y$

Complementario

$\{(1,0,-1,0),(0,1,2,-3)\}$ Base de U

$W = \mathcal{L}(\{(0,0,1,0),(0,0,0,1)\})$

$U \oplus W = \mathbb{R}^4$

c) $V = \mathbb{R}_3[x]$ $\quad U = \{p(x) \in \mathbb{R}_3[x] / p'(1) = 0\}$

$p(x) = a + bx + cx^2 + dx^3 \Longrightarrow p'(x) = b + 2cx + 3dx^2$

$p'(1) = b + 2c + 3d = 0$

Base de U: $\{1,\ 2x - x^2,\ 3x - x^3\}$

$\dim_{\mathbb{R}}(U) = 2$

$W = \mathcal{L}(\{x\}) \Longrightarrow U \oplus W = \mathbb{R}_3[x]$

d) $V = \mathbb{R}_2[x]$; $U = \{p(x) \in \mathbb{R}_2[x] / \int_0^1 p(x)dx = 0\}$

$p(x) = a + bx + cx^2$ $\quad \int_0^1 a + bx + cx^2 = ax + \frac{bx^2}{2} + \frac{cx^3}{3}\Big]_0^1 = a + \frac{b}{2} + \frac{c}{3} = 0$

$\mathrm{Dim}_{\mathbb{R}}(U) = 1$ $\quad$ Base de U: $\{1 - 2x,\ 1 - 3x^2\}$ $\quad W = \mathcal{L}(\{1\})$

$U \oplus W = \mathbb{R}_2[x]$

27.- $k \in \mathbb{R}$

$U_k = \mathcal{L}(\{(0,-1,k,3),(0,k,-2-k,3),(k-2,-1,-2,3)\})$

$$\begin{pmatrix} 0 & 0 & k-2 \\ -1 & k & -1 \\ k & -2-k & -2 \\ 3 & 3 & 3 \end{pmatrix}$$

$\begin{vmatrix} -2-k & -2 \\ 3 & 3 \end{vmatrix} = -\cancel{6} - 3k + \cancel{6} = -3k \Longrightarrow k = 0;\ \dim_k(U_0) = 3$

$3\begin{vmatrix} -1 & k & -1 \\ k & -2-k & -2 \\ 1 & 1 & 1 \end{vmatrix} = 3(\cancel{2} + \cancel{k} - 2k - \cancel{k} - \cancel{2} - k - k^2 - 2) = -9k - 3k^2 - 6 \Longrightarrow k^2 + 3k + 2 \quad k = \frac{-3 \pm \sqrt{9-8}}{2} = \frac{-3 \pm 1}{2}$ $\boxed{k = -1}$ $\boxed{k = -2}$

Si $k = -1 \Longrightarrow \dim_k(U_{-1}) = 2$ $\qquad$ Si $k \neq -1, -2 \Longrightarrow 3$

$k = -2 \Longrightarrow \dim_k(U_{-2}) = 3$

$W = \mathcal{L}(\{(0,0,1,0),(0,0,0,1)\})$

8. c) $V = \mathbb{R}_n[x]$ $\quad U_1 = \{p(x) \in \mathbb{R}_n[x] / p(1) + p'(1) = 0\}$

$U_2 = \{p(x) \in \mathbb{R}_n[x] / p(0) + p'(0) = 0\}$

$\forall p(x) \in \mathbb{R}_n[x] \quad p(x) = a_0 + a_1 x + a_2 x^2 \dots + a_n x^n \Longrightarrow p(1) = a_0 + a_1 + a_2 \dots + a_n \;\; y \;\; p(0) = a_0$

$p'(x) = a_1 + a_2 2x \dots + a_n n x^{n-1} \Longrightarrow p'(1) = a_1 + 2a_2 \dots + a_n n$

$p''(x) = a_2 2 + a_3 6x + \dots + a_n n(n-1) x^{n-2} \Longrightarrow p''(0) = 2a_2$

$\forall p(x) \in U_1 \quad \sum_{i=0}^{n} a_n = -\sum_{i=1}^{n} i \cdot a_i$

$a_0 + \sum_{i=1}^{n} a_i = -\sum_{i=1}^{n} i a_i \Longrightarrow a_0 = -\sum_{i=1}^{n} a_i (1+i)$

$\forall p(x) \in U_2 \quad a_0 = 2a_2$

$\forall p(x) \in U_1 \cap U_2 \quad \begin{cases} a_0 = -\sum_{i=1}^{n} a_i (1+i) \\ a_0 = 2a_2 \end{cases} \quad \begin{cases} a_0 = -2a_1 - 3a_2 - \sum_{i=3}^{n} a_i (1+i) \\ a_0 = 2a_2 \end{cases}$

$-2a_1 - 5a_2 - \sum_{i=3}^{n} a_i (1+i) = 0$