Parcial DGIIM 2021 T.3y4 (Grupo 2)

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Sea $f: \mathbb{R}^4 \longrightarrow A_2(\mathbb{R})$ dada por

$$f(1,-1,0,0) = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix} \quad f(1,1,1,0) = \begin{pmatrix} 0 & 3 & 0 \\ -3 & 0 & 2 \\ 0 & -2 & 0 \end{pmatrix}$$

$$f(1,0,0,1) = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ -2 & -2 & 0 \end{pmatrix} \quad f(0,1,1,1) = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}$$

Base usual $\mathbb{R}^4$: $B_u = \{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$

Base usual $A_3(\mathbb{R}) = B_u' = \left\{ \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \right\}$

1) ¿$M(f; B_u' \leftarrow B_u)$? $= A$

Con las imágenes que nos dan, podemos crear una base ya que son L.I.

$\bar{B} = \{(1,-1,0,0),(1,1,1,0),(1,0,0,1),(0,1,1,1)\}$

y sabemos que:

$$\left.\begin{array}{l} f(1,-1,0,0) = (1,-1,-1)_{B_u'} \\ f(1,1,1,0) = (3,0,2)_{B_u'} \\ f(1,0,0,1) = (0,2,2)_{B_u'} \\ f(0,1,1,1) = (1,1,-1)_{B_u'} \end{array}\right\} \begin{pmatrix} 1 & 3 & 0 & 1 \\ -1 & 0 & 2 & 1 \\ -1 & 2 & 2 & -1 \end{pmatrix} = M(f; B_u' \leftarrow \bar{B}) = C$$

$(1,0,0,0) = a(1,-1,0,0) + b(1,1,1,0) + c(1,0,0,1) + d(0,1,1,1)$

$$\begin{array}{l} 1 = a + b + c \Longrightarrow -2d = 1 \Longrightarrow d = -\frac{1}{2} \\ 0 = -a + b + d \Longrightarrow a = 0 \\ 0 = b + d \Longrightarrow b = -d \Longrightarrow b = 1/2 \\ 0 = c + d \Longrightarrow c = -d \Longrightarrow c = 1/2 \end{array} \qquad (1,0,0,0) = (0, 1/2, 1/2, -1/2)_{\bar{B}}$$

$(0,1,0,0)$:

$$\begin{array}{l} 0 = a + b + c \Longrightarrow -1 - 2d = 0 \Longrightarrow \boxed{d = -1/2} \\ 1 = -a + b + d \Longrightarrow \boxed{a = -1} \\ 0 = b + d \Longrightarrow b = -d \Longrightarrow \boxed{b = 1/2} \\ 0 = c + d \Longrightarrow c = -d \Longrightarrow \boxed{c = 1/2} \end{array} \qquad (0,1,0,0) = (-1, 1/2, 1/2, -1/2)_{\bar{B}}$$

$(0,0,1,0)$:

$$\begin{array}{l} 0 = a + b + c \Longrightarrow 1 + 1 - d - d = 0 \Longrightarrow \boxed{d = 1} \\ 0 = -a + b + d \Longrightarrow -a + 1 = 0 \Longrightarrow \boxed{a = 1} \\ 1 = b + d \Longrightarrow b = 1 - d \Longrightarrow \boxed{b = 0} \\ 0 = c + d \Longrightarrow c = -d \Longrightarrow \boxed{c = -1} \end{array} \qquad (0,0,1,0) = (1, 0, -1, 1)_{\bar{B}}$$

$(0,0,0,1)$:

$$\begin{array}{l} 0 = a + b + c \Longrightarrow -d + 1 - d = 0 \Longrightarrow \boxed{d = 1/2} \\ 0 = -a + b + d \Longrightarrow \boxed{a = 0} \\ 0 = b + d \Longrightarrow b = -d \Longrightarrow \boxed{b = -1/2} \\ 1 = c + d \Longrightarrow c = 1 - d \Longrightarrow \boxed{c = 1/2} \end{array} \qquad (0,0,0,1) = (0, -1/2, 1/2, 1/2)$$

Luego tenemos $M_{\bar{B} \leftarrow B_u} = \begin{pmatrix} 0 & -1 & 1 & 0 \\ 1/2 & 1/2 & 0 & -1/2 \\ 1/2 & 1/2 & -1 & 1/2 \\ -1/2 & -1/2 & 1 & 1/2 \end{pmatrix} = D$

Con lo que $M(f; B_u' \leftarrow B_u) = M(f; B_u' \leftarrow \bar{B}) \cdot M_{\bar{B} \leftarrow B_u}$

$$A = C \cdot D$$

$$A = \begin{pmatrix} 1 & 0 & 2 & -1 \\ 0'5 & 1'5 & -2 & 1'5 \\ 2'5 & 3'5 & -4 & -0'5 \end{pmatrix} \Longrightarrow A^t = \begin{pmatrix} 1 & 0'5 & 2'5 \\ 0 & 1'5 & 3'5 \\ 2 & -2 & -4 \\ -1 & 1'5 & -0'5 \end{pmatrix} = M(f^t; B_u^* \leftarrow B_u'^*)$$

b) Bases de $an(\mathrm{Ker}(f))$ y $an(\mathrm{Im}(f))$

$an(\mathrm{Ker}(f)) = \mathrm{Im}(f^t) \qquad an(\mathrm{Im}(f)) = \mathrm{Ker}(f^t)$

$$\begin{vmatrix} 1 & 0'5 & 2'5 \\ 0 & 1'5 & 3'5 \\ 2 & -2 & -4 \end{vmatrix} = -3 \neq 0 \Longrightarrow \underline{rg}(A^t) = 3$$

$(B_u^*$ de $\mathbb{R}^4 = \{e_1, e_2, e_3, e_3\}$

Luego una base de $an(\mathrm{Ker}(f))$ será $\{e_1 + 2e_3 - e_4, 0'5e_1 + 1'5e_2 - 2e_3 + 1'5e_4, 2'5e_1 + 3'5e_2 - 4e_3 - 0'5e_4\}$

Por la fórmula de las dimensiones: $\dim_K(\cancel{S_2^*(\mathbb{R})})^{\to 3} = \dim_K(\cancel{\mathrm{Im}(f^t)})^{\to 3} + \dim(\mathrm{Ker}(f^t)) \Longrightarrow \dim_K(\mathrm{Ker}(f^t)) = 0$

Por lo que $an(\mathrm{Im}(f)) = \{e_0\}$ $\lightning$