Deducción recta regresión

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Sea $Y = aX + b$ tq $y_i = ax_i + b$

+ residuos : $ax_i + b + \varepsilon_i$

↳ $\varepsilon_i = y_i - \hat{y}_i \Rightarrow$ valor estimado

$$\sum \varepsilon_i = \sum y_i - \hat{y}_i \Rightarrow \sum \varepsilon_i = \sum (y_i - \hat{y})^2 = \sum (y_i - ax - b)^2 = S$$

Derivadas parciales sobre a y b:

$$\varphi(a,b) = \sum_{i=1}^{k}\sum_{j=1}^{p} f_{ij}\,[y_j - \overbrace{(bx_i + a)}^{\hat{y}_i}]^2$$

$$\frac{\partial\,\varphi(a,b)}{\partial a} = \sum_{i=1}^{k}\sum_{j=1}^{p} f_{ij}\,(y_j - bx_i - a)\cdot 2\cdot(-1)$$

$$\sum_{i=1}^{k}\sum_{j=1}^{p} f_{ij}\,(y_j - bx_i - a)\,\cancel{2}\,\cancel{(-1)} = 0$$

$$\sum_{i=1}^{k}\sum_{j=1}^{p} f_{ij}\,(y_j - bx_i - a) = 0$$

$$\sum_i\sum_j f_{ij}y_j - \sum_i\sum_j f_{ij}bx_i - a\underbrace{\cancel{\sum_i\sum_j f_{ij}}}_{1} = m_{01} - bm_{10} - a = 0$$

$$a = m_{01} - bm_{10} \quad ①$$

$$\frac{\partial\varphi}{\partial b} = \sum_i\sum_j f_{ij}\,(y_j - bx_i - a)\,x_i\cdot 2\cdot(-1)$$

$$\sum_i\sum_j f_{ij}\,(y_j - bx_i - a)\,x_i\cdot\cancel{2}\cdot\cancel{(-1)} = 0$$

$$\sum_i\sum_j f_{ij}x_iy_j - b\sum_i\sum_j f_{ij}x_i^2 - a\sum_i\sum_j f_{ij}x_i = 0$$

$$m_{11} - bm_{20} - am_{10} = 0$$

$$a = \frac{m_{11} - bm_{20}}{m_{10}} \quad ②$$

Con la expresión ① y ②:

$$\frac{m_{11} - bm_{20}}{m_{10}} = m_{01} - bm_{10}$$

$$m_{11} - bm_{20} = m_{01}m_{10} - bm_{10}^2$$

$$(\underbrace{m_{20} - m_{10}^2}_{\sigma_x^2})\,b = (\underbrace{m_{11} - m_{01}m_{10}}_{\sigma_{xy}})$$

$$b = \frac{\sigma_{xy}}{\sigma_x^2} \Rightarrow a = m_{01} - bm_{10} = \bar{y} - \frac{\sigma_{xy}}{\sigma_x^2}\bar{x}$$

luego $Y = a + bx = \bar{y} + (x - \bar{x})\dfrac{\sigma_{xy}}{\sigma_x^2}$ //