Ejercicios cuádricas

1.- $4x^2 + y^2 + z^2 + 4xz - 2yz + 4x + 8z - 9 = 0$

$4(x^2 + xz + x)$

$x^2 + xz + x = (x + az + b)^2 - ab^2 - b^2 - 2abz$

$2a = 1$
$2b = 1$ $\Longrightarrow a = b = \frac{1}{2}$ $\boxed{x' = x + \frac{1}{2}z + \frac{1}{2}}$

$4(x')^2 - 4\left[\frac{1}{4}z^2 + \frac{1}{4} - \frac{1}{4}z\right] = 4(x')^2 - z^2 - 1 + 2z$

$\hookrightarrow \boxed{x'' = 2x'}$

$(x'')^2 - \cancel{z^2} - 1 - 2z + y^2 + \cancel{z^2} - 2yz + 8z - 9 = 0$

$(x'')^2 + 6z + y^2 - 2yz - 10 = 0$

$y^2 - 2yz = (y + az)^2 - a^2z^2 \Longrightarrow a = -1 \Longrightarrow \boxed{y' = y - z}$

$y^2 - 2yz = (y')^2 - z^2$

$(x'')^2 + 6z + (y')^2 - z^2 - 10 = 0$

$(z^2 - 6z) = (z + a)^2 - a^2 \Longrightarrow a = -3 \Longrightarrow \boxed{z' = z - 3}$

$(z^2 - 6z) = (z')^2 - 9 \Longrightarrow -(z^2 - 6z) = 9 - (z')^2$

$(x'')^2 + (y')^2 - (z')^2 - 1 = 0 \Longrightarrow$ Hiperboloide de una hoja

$\hookrightarrow (z')^2 - (x'')^2 - (y')^2 + 1 = 0$

Determina R donde representa forma reducida

$x'' = 2x' = 2(x + \frac{1}{2}z + \frac{1}{2}) = 2x + z + 1 \Longrightarrow x = \frac{x'' - z - 1}{2}$ $\quad z = z' + 3$

$z' = z - 3 \Longrightarrow \boxed{z = z' + 3}$ $\quad x = \frac{x'' - z' - 3 - 1}{2}$

$y' = y - z \Longrightarrow \boxed{y = y' + z = y' + z' + 3}$

$\boxed{x = \frac{x'' - z'}{2} - 2}$

Si $(x'', y', z') = (0,0,0) \Longrightarrow \quad a_0 = (-2,3,3)$

$= (1,0,0) \Longrightarrow \quad a_1 = (-\frac{3}{2},3,3)$

$= (0,1,0) \Longrightarrow \quad a_2 = (-2,4,3)$

$= (0,0,1) \Longrightarrow \quad a_3 = (-\frac{5}{2},4,4)$

Luego $R = \{(-2,3,3),(-\frac{3}{2},3,3),(-2,4,3),(-\frac{5}{2},4,4)\}$ será el sistema buscado.

(Cónica)

2.- $2x^2 - 4xy + 2y^2 + 12x - 18y + 11 = 0$. Clasifica y R

$2(x^2 - 2xy + 6x) \quad (x+ay+b)^2 = x^2 + axy + bx + ayx + a^2y^2 + aby + bx + aby + b^2$

$(x^2 - 2xy + 6x) = (x+ay+b)^2 - a^2y^2 - 2aby - b^2$

$2a = -2 \Rightarrow a = -1$

$2b = 6 \Rightarrow b = 3$

$\Big\} \; x' = (x - y + 3)$

$(x^2 - 2xy + 6x) = (x')^2 - y^2 + 6y - 9$

$2(x^2 - 2xy + 6x) = 2(x')^2 - 2y^2 + 12y - 18$

$2(x')^2 - \cancel{2y^2} + 12y - 18 + \cancel{2y^2} - 18y + 11 = 0$

$2(x')^2 - 6y - 7 = 0 \Longrightarrow \quad x'' = \sqrt{2}\,x'$

$(x'')^2 - 6y - 7 = 0$

$\frac{(x'')^2}{7} - \frac{6}{7}y - 1 = 0 \Longrightarrow \quad x''' = \frac{1}{\sqrt{7}}x'' \qquad 2(\frac{3}{7}y + \frac{1}{2}) = 2y' \Rightarrow y' = \frac{3}{7}y + \frac{1}{2}$

$(x''')^2 - 2y' = 0 \Longrightarrow$ Parábola

S.R:

$x''' = \frac{1}{\sqrt{7}}x'' = \frac{\sqrt{2}}{\sqrt{7}}x' = \frac{\sqrt{2}}{\sqrt{7}}(x - y + 3) \Longrightarrow x = \frac{\sqrt{7}}{\sqrt{2}}x''' + y - 3 = \frac{\sqrt{7}}{\sqrt{2}}x''' + \frac{7}{3}(y' - \frac{1}{2}) - 3$

$y' = \frac{3}{7}y + \frac{1}{2} \Longrightarrow y = \frac{7}{3}(y' - \frac{1}{2})$

Si $(x''', y') = (0,0) \Rightarrow (x,y) = \left(-\frac{25}{6}, -\frac{7}{6}\right) = a_0$

$(1,0) \Rightarrow (x,y) = \left(\frac{-25+3\sqrt{14}}{6}, -\frac{7}{6}\right) = a_1$

$(0,1) \Rightarrow (x,y) = \left(-\frac{11}{6}, \frac{7}{6}\right) = a_2$

Con lo que $R = \{a_0, a_1, a_2\}$ es el S.R buscado