ASRR

Chuleta MNII

Tema 3

- Consistencia: $\lim\limits_{\substack{h\to 0\\ t=a+nh}} \frac{R_{n+k}}{h} \to 0 \iff \begin{cases} p(1)=0 \\ \Phi(\;)=p'(1)f(t_n,x(t_n)) \end{cases}$

- Estable: raíces de $p(\lambda)\in B(0,1)$ y si $|\lambda_{sol}|=1 \Rightarrow$ simple

- Euler
  - Explícito: $x_{n+1}=x_n+hf(t_n,x_n)$ $\quad R_{n+1}=\frac{h^2}{2}x''(t)+O(h^3)$
  - Implícito: $x_{n+1}=x_n+hf(t_{n+1},x_{n+1})$ $\quad R_{n+1}=-\frac{h^2}{2}x''(t)+O(h^3)$
  - Mejorado: $x_{n+1}=x_n+hf(t_n+h/2,\ x_n+h/2 f(t_n,x_n))$ $\quad p=2$
  - Modificado/Heun: $x_{n+1}=x_n+\frac{h}{2}(f(t_n,x_n)+f(t_{n+1},x_n+hf(t_n,x_n)))$ $\quad p=2$

- Taylor: $x_{n+1}=x_n+hx_n'+\frac{h^2}{2}x_n''+\dots+\frac{h^p}{p!}x_n^{(p)}$ Orden p

- Runge-Kutta:
  - $K_i(t,x)=f(t+c_ih,\ x+h\sum_{j=1}^{m}a_{ij}K_j(t,x))$ $\quad i=1,\dots,m$
  - $x_{n+1}=x_n+h\sum_{j=1}^{m}b_jK_j(t_n,x_n)$
  - Clásico: $x_{n+1}=x_n+\frac{h}{6}(K_1+2K_2+2K_3+K_4)$ $\quad p=4$
    - $K_4=f(t_n+h,\ x_n+hK_3)$
    - $K_3=f(t_n+h/2,\ x_n+h/2K_2)$
    - $K_2=f(t_n+h/2,\ x_n+h/2K_1)$
    - $K_1=f(t_n,x_n)$

- A-Estabilidad: con $R(\lambda)<0$ y $\begin{cases} x(0)=\mu \\ x'=\lambda x \end{cases} \Rightarrow \lim\limits_{n\to\infty} x_n=0$
  - $x_{n+1}=Q(w)x_n \Rightarrow |Q(w)|<1\ \forall w\in\mathbb{C}$

- Método k pasos: $x_{n+k}=\sum_{j=0}^{k-1}\alpha_j x_{n+j}+h\sum_{j=0}^{k}\beta_j f_{n+j}$

- Polinomios característicos
  - Primero: $p(\lambda)=\lambda^k-\alpha_{k-1}\lambda^{k-1}-\dots-\alpha_0$
  - Segundo: $q(\lambda)=\beta_k\lambda_k+\beta_{k-1}\lambda^{k-1}+\dots+\beta_0$

- Consistencia $\iff p(1)=0 \wedge p'(1)=q(1) \iff C_0=C_1=0$

- Orden p: si $C_0=C_1=\dots=C_p=0$ y $C_{p+1}\neq 0$ $\quad R_{n+1}=C_{p+1}h^{p+1}x^{(p+1)}(t_n)$

  $C_0=1-\sum_{j=0}^{k-1}\alpha_j \qquad C_1=k-\sum_{j=1}^{k-1}j\alpha_j-\sum_{j=0}^{k}\beta_j \qquad C_m=\frac{k^m}{m!}-\sum_{j=1}^{k-1}\frac{j^m}{m!}\alpha_j-\sum_{j=1}^{k}\frac{j^{m-1}}{(m-1)!}\beta_j$

- MMd Basados en Cuad. $x_{n+k} = x_{n+k-q} + h\sum_{j=m}^{k-r} \beta_j f_{n+j}$
  - Tipo Adams $q = 1,\ m = 0$
    - A-Bashforth $r = 1$
    - A-Moulton $r = 0$
  - Tipo Milnes-Simpson $q = 2 \quad m = 0 \quad r = 0$
  - Tipo Newton-Cotes $q = k$
    $x_{n+k} = x_n + h(\beta_0 f_n + \beta_1 f_{n+1} + \dots + \beta_k f_{n+k})$
    - Abiertos $m = r = 1$
    - Cerrados $m = r = 0$
  - Predictor-Corrector $\Rightarrow PC^m$ orden $P = p^*$, orden $C = p$
    - Orden $= \min\{p^* + m,\ p\}$